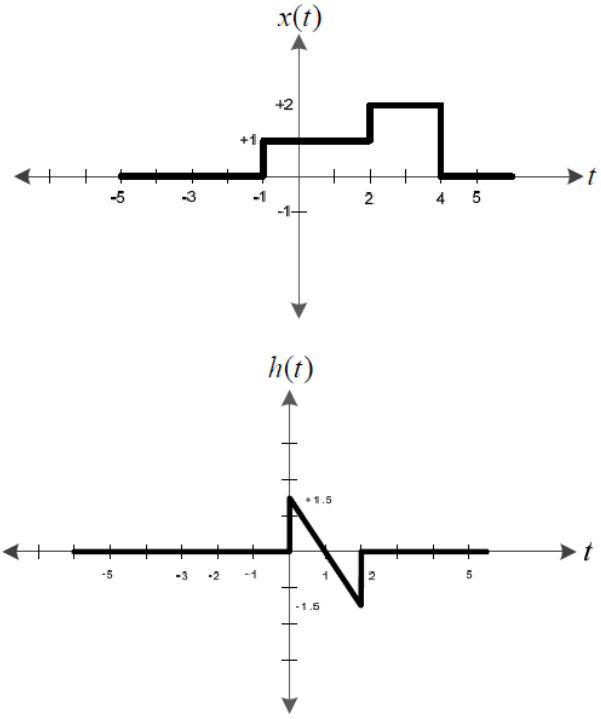
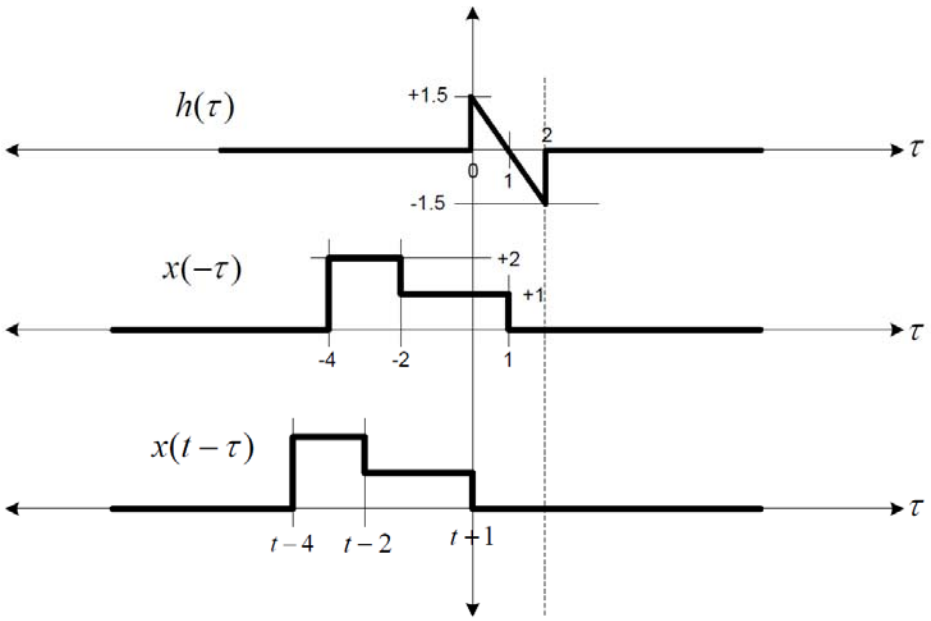
1. จงหาค่า Convolution Integral $y(t) = x(t) * h(t)$ โดยกำหนดให้ $x(t)$ และ $h(t)$ แสดงดังรูปข้างล่างนี้



Solution



$$y(t) = x(t) * h(t) = \int_0^2 x(\tau)h(t-\tau)d\tau$$

$$x(\tau) = m\tau + b = 1.5(1-\tau)$$

$$y(t) = x(t) * h(t) = 1.5\int_0^2 (1-\tau)h(t-\tau)d\tau$$

$t < -1$

$$y(t) = 1.5\int_0^2 (1-\tau)0d\tau = 0$$

$-1 \le t < 1$

$$y(t) = 1.5\int_0^{t+1} (1-\tau)(1)d\tau + 1.5\int_{t+1}^2 (1-\tau)(0)d\tau$$

$$= 1.5\left((t+1-\frac{(t+1)^2}{2}\right) = -0.75(t^2+1)$$

$1 \le t < 2$

$$y(t) = 1.5\int_0^2 (1-\tau)(1)d\tau = 1.5\tau - 1.5\frac{\tau^2}{2}\Big|_0^2$$

$$= 1.5\left(2-\frac{4}{2}\right) = 0$$

$2 \le t < 4$

$$y(t) = 1.5\int_0^t (1-\tau)(2)d\tau + 1.5\int_{t-2}^2 (1-\tau)(1)d\tau = 3\tau - 1.5\tau^2\Big|_0^{t-2} + 1.5\tau - 1.5\frac{\tau^2}{2}\Big|_{t-2}^2$$

$$y(t) = 3(t-2) - 1.5(t-2)^2 + 1.5(2) - 1.5\frac{(2^2)}{2} - 1.5(t-2) + \frac{1.5}{2}(t-2)^2$$

$$= 3t-6-1.5(t-2)^2 + 0 - 1.5(t-2) + \frac{1.5}{2}(t-2)^2$$

$$= 1.5t - 3 - 1.5(t-2)^2 + \frac{1.5}{2}(t-2)^2 = 1.5t - 3 - 0.75(t-2)^2$$

$$= 1.5t - 3 - 0.75t^2 + 1.5t - 3$$

$$= -0.75t^2 + 3t - 6$$

$= 1.5t - 3 + (t-2)^2(-1.5 + \frac{1.5}{2})$ $= 1.5t - 3 + (t-2)^2(-0.75)$

$= 1.5t - 3 + (-0.75)(t^2 - 4t + 4)$

$= 1.5t - 3 - 0.75t^2 - 3t - 3$

$= -0.75t^2 - 1.5t - 6$ ✓

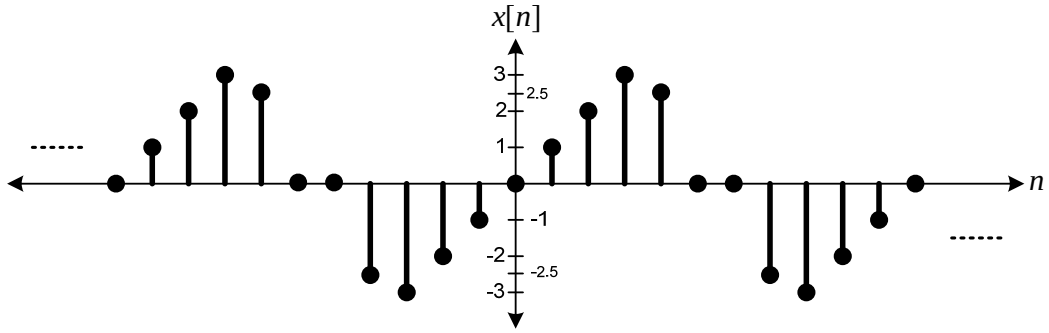
$(t-2)^2 = (t-2)(t-2) = t^2 - 4t + 4$

$4 \le t < 6$

$$y(t) = 1.5\int_{t-4}^{2}(1-\tau)(2)d\tau = 3\tau - \frac{3\tau^2}{2}\Big|_{t-4}^{2} = 3(2) - 1.5(4) - 1.5(t-4) + 1.5(t-4)^2$$
$$= -1.5t + 6 + 1.5t^2 - 12t + 24$$
$$= 1.5t^2 - 13.5t + 30$$

$t \ge 6, y(t) = 0$

2. จงหาค่า DTFS Coefficients ของสัญญาณที่กำหนดให้ดังรูปต่อไปนี้



Solution

จากรูปภาพ เราจะได้ค่าคาบเท่ากับ $N = 11$

$x[n] = x[-5] = 0, x[-4] = -2.5, x[-3] = -3, x[-2] = -2, x[-1] = -1, x[0] = 0$
$, x[5] = 0, x[4] = 2.5, x[3] = 3, x[2] = 2, x[1] = 1.$

$$\Omega_0 = \frac{2\pi}{11}$$

$$X[k] = \frac{1}{N}\sum_{n=-5}^{5} x[n]e^{-jk\Omega_0 n}$$
$$= \frac{1}{11}\{-2.5e^{j8\pi k} + 2.5e^{-j8\pi k} - 3e^{j6\pi k} + 3e^{-j6\pi k} - 2e^{j4\pi k} + 2e^{-j2\pi k} - e^{j2\pi k} + e^{-j2\pi k}\}$$
$$X[k] = \frac{1}{j11}\{5\sin(8\pi k) + 6\sin(6\pi k) + 4\sin(4\pi k) + 2\sin(2\pi k)\}$$

3. จงหาค่า DTFS Coefficients ของสัญญาณ $x[n] = 10\{1 + \cos[2\pi n/120 + \pi/4]\}\cos[2\pi n/15]$ โดยใช้วิธี Method of Inspection

$$x[n] = 10\{1 + \cos[2\pi n/120 + \pi/4]\}\cos[2\pi n/15] = 10\cos[2\pi n/15] + 10\cos[2\pi n/120 + \pi/4]\cos[2\pi n/15]$$
$$= 10\cos[2\pi n/15] + 5\cos[18\pi n/120 + \pi/4] + 5\cos[14\pi n/120 - \pi/4]$$
$$= 5e^{j2\pi n/15} + 5e^{-j2\pi n/15} + \left(2.5e^{j\pi/4}\right)e^{j18\pi n/120} + \left(2.5e^{-j\pi/4}\right)e^{-j18\pi n/120} + \left(2.5e^{-j\pi/4}\right)e^{j14\pi n/120} + \left(2.5e^{j\pi/4}\right)e^{-j14\pi n/120}$$

เราจะได้ Fundamental Frequency และ Period เท่ากับ $\Omega_0 = \frac{2\pi}{120} \rightarrow N = 120$\

$$x[n] = \sum_{n=-60}^{59} X[k]e^{jk\Omega_0 n} = \sum_{n=-60}^{59} X[k]e^{j2\pi kn/120}$$

$$X[-8] = 5, X[8] = 5$$

$$X[-9] = 2.5e^{-j\pi/4}, X[9] = 2.5e^{j\pi/4}$$

$$X[-7] = 2.5e^{j\pi/4}, X[7] = 2.5e^{-j\pi/4}$$

4 จงหาค่า Inverse DTFS ของสมการ $X[k] = \sin\left[\frac{4\pi}{21}k\right] - \cos\left[\frac{8\pi}{21}k\right]$ โดยถ้าใครหาค่าที่ช่วง $k = -(N+1), ..., 0$ จะได้คะแนนมากกว่า

Solution

เราจะได้ค่า $\Omega_0 = \frac{4\pi}{21} \rightarrow N = 21, (m = 2)$ ดังนั้นเราจะหาค่าที่ช่วง $k = -20, ..., 0$

$$X[k] = \frac{1}{N}\sum_{n=-20}^{0} x[n]e^{-j4\pi kn/21} = \sin\left[\frac{4\pi}{21}k\right] - \cos\left[\frac{8\pi}{21}k\right]$$

$$= \left(\frac{1}{j2}\right)e^{j\frac{4\pi k}{21}} + \left(-\frac{1}{j2}\right)e^{-j\frac{4\pi k}{21}} + \left(-\frac{1}{2}\right)e^{j\frac{8\pi k}{21}} + \left(-\frac{1}{2}\right)e^{-j\frac{8\pi k}{21}}$$

$$= \left(\frac{1}{j2}\right)e^{j\frac{4\pi k}{21}} + \left(-\frac{1}{j2}\right)e^{j\frac{80\pi k}{21}} + \left(-\frac{1}{2}\right)e^{j\frac{8\pi k}{21}} + \left(-\frac{1}{2}\right)e^{j\frac{76\pi k}{21}}$$

$$x[-1] = \left(\frac{21}{j2}\right), \quad x[-2] = \left(-\frac{21}{2}\right), \quad x[-19] = \left(-\frac{21}{2}\right), \quad x[-20] = \left(-\frac{21}{j2}\right)$$