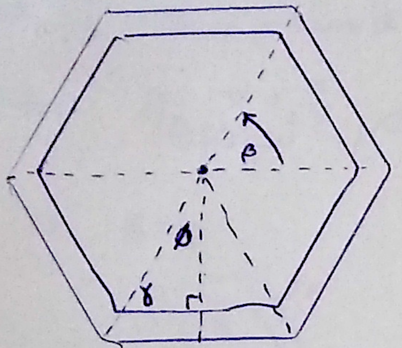
Take some Kresling module with N polygon edges:




The center of each face is located at (0,0)

The position of each point is

$R(\cos\beta, \sin\beta)$ where $\beta = [0, 2\pi]$

If determining the position of each point using the point number,

$$P_{jk} = R\left[\cos\left(\frac{2\pi jk}{N}\right), \sin\left(\frac{2\pi k}{N}\right)\right] \quad , \quad k = [0, N-1]$$

or using trig identity $\cos\left(\theta - \frac{\pi}{2}\right) = \sin(\theta)$,

$$P_k = R\left[\cos\left(\frac{2\pi k}{N}\right), \cos\left(\frac{2\pi k}{N} - \frac{\pi}{2}\right)\right]$$

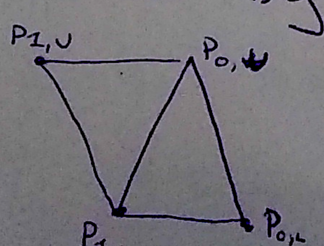
Within the Kresling, two important angles are defined:

$$\phi = \frac{\pi}{N} \quad \text{and} \quad \gamma = \frac{\pi}{2} - \phi$$

To define the points of the opposite face, shifted by some angle $\alpha$, we define the bottom face as "0" and then the top (upper) face is rotated by $-\alpha$ st.

$$P_{k,U} = R\left[\cos\left(\frac{2\pi k}{N} - \alpha\right), \cos\left(\frac{2\pi k}{N} - \alpha - \frac{\pi}{2}\right)\right]$$

The triangles of the Kresling edges can be defined by




$$T_0 = [P_{0L}, P_{0,U}, P_{1,L}]$$

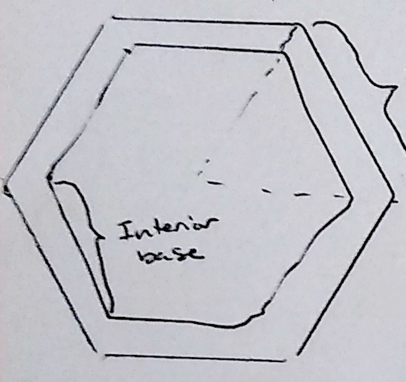
$$T_1 = [P_{0U}, P_{1L}, P_{1U}]$$

$$\ldots \quad T_{EVEN} = [P_{k,L}, P_{k,U}, P_{k+1,U}] \quad , \quad T_{ODD} = [P_{kU}, P_{k+1,L}, P_{k+1,U}]$$

The triangle faces can be converted to bodies by lofting each set of triangles:

$B_k$ = loft ( $T_{0,E}$ , $T_{0,I}$ )

where $T_{k,E}$ is the larger radius and $T_{k,I}$ is the smaller




Finally, a base may be added by drawing the Kresling base from the set of lower points and extruding it a distance $-t$ in $z$. (this describes Kresling.py)

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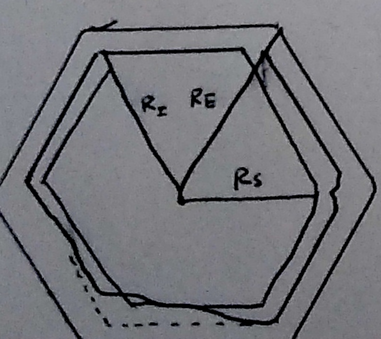
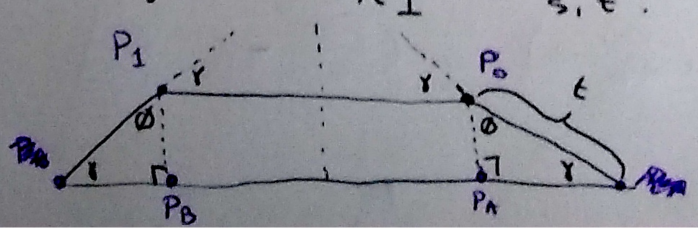
The above notes generate a Kresling module with uniform thickness $t = R_E - R_I$. However, if we wish to design a module with thicker walls / faces + thin hinges (ie, to replicate the behavior of paper) we need a third Kresling polygon with radius $R_s$ where $R_s < R_I < R_E$ (relaxed constraint) or $R_s + 2t = R_I + t = R_E$ (strict constraint)




To make thin hinges, we draw a line orthogonal to the $R_s$ Kresling polygon edge to $R_I$ s.t.: The

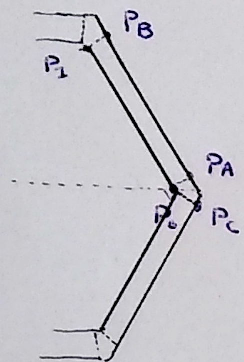


$$P_{A,x} = P_{0,x} + 0 \qquad P_{B,x} = P_{1,x} + 0$$

$$P_{A,y} = P_{0,y} + t\cos\phi \qquad P_{B,y} = P_{1,y} + t\cos\phi$$

for $\beta = \pi + \frac{\pi}{3}$

If $\beta = 0$...




$$P_{A,x} = P_{0,x} + t\cos\phi\cos\phi$$

$$P_{A,y} = P_{0,y} + t\cos\phi\sin\phi$$

$$P_{B,x} = P_{1,x} + t\cos\phi\cos\phi$$

$$P_{B,y} = P_{1,y} + t\cos\phi\sin\phi$$

$$P_{C,x} = P_{0,x} + t\cos\phi\cos\phi$$

$$P_{C,y} = P_{0,y} - t\cos\phi\sin\phi$$

The generalized expression for these points is

$$P_j = P_k + \underbrace{t\cos\phi}_{\text{offset}}\left[\cos\left(\frac{2\pi k}{N} \pm \phi\right), \sin\left(\frac{2\pi k}{N} \pm \phi\right)\right]$$

or written uniformly with cos:

$$P_j = \underbrace{(R_s)\left[\cos\left(\frac{2\pi k}{N} - \alpha\right), \cos\left(\frac{2\pi k}{N} - \alpha - \frac{\pi}{2}\right)\right]}_{\text{Points describing smallest Kresling}} + \underbrace{t\cos\phi\left[\cos\left(\frac{2\pi k}{N} \pm \phi\right), \cos\left(\frac{2\pi k}{N} \pm \phi - \frac{\pi}{2}\right)\right]}_{\text{2N points describing orthogonal projection offset}}$$

As above,

$$P_A = P_0 + t\cos\phi\left[\cos\left(\frac{2\pi(0)}{6} + \phi\right), \cos\left(\frac{2\pi(0)}{6} + \phi - \frac{\pi}{2}\right)\right]$$

$$P_C = P_0 + t\cos\phi\left[\cos\left(\frac{2\pi(0)}{6} - \phi\right), \cos\left(\frac{2\pi(0)}{6} + \phi - \frac{\pi}{2}\right)\right]$$