Bandits

Spyros Samothrakis
Research Fellow, IADS
University of Essex

February 7, 2017

About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

Bandits

- We will discuss bandits
- We are in effect revisiting some ideas from lecture two
  - Hypothesis testing
- I think this is a much easier to understand framework vs hypothesis testing

About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

Examples

- You send a user an e-mail
  - User clicks on the link you get $r = 1$
  - User fails to click on the link after 3 days $r = 0$
- Playing games
  - What is the next best action to take in Chess?
    - Chess has a sequential element - hence "Reinforcement Learning"
    - But close enough...
- Online adverts
  - User clicks on an advert ($r = 1$)
  - User clicks fails to click on an advert ($r = 0$)

The bandit problem

- Bandits are a tuple $< A, R >$
- Where $a \in A$ is a set of actions
  - Sometimes actions are called "arms"
- $r \in R$ is a set of rewards
- $R(a, r) = P(r|a)$
  - The probability of getting a reward r given that I have done action a
- "You do an action, you get some feedback"

The goal

- Find an optimal policy $\pi(a) = P(a)$ that maximises the long term sum of rewards
  - Long term sum is $\sum_{t=0}^{T} = r_t$
- The "action-value" function $Q(a)$ is the expected reward for taking action a
  - Q $= E[r|a]$
- The "value" function is $V = E_\pi[r]$
  - The average Q values, given a policy that I follow

Example problem

```
Dear Sir/Madam,

Best quality flasks and vials for your experiments

Click the link below to buy - discounted prices

(Link)
```

```
Dear <Name>,

This is Nick from www.MegaFlasksAndVials.com - super discounts below

(Link)
```

Let's simulate

- First e-mail is a_0
- Second e-mail is a_1
- Policy is $\pi(a_0) = 0.5, \pi(a_1) = 0.5$
- Let's manually calculate some Q's and V's on the e-mail sending problem

Goals (1)

- So our goal is to find the best action
- Optimal $V^* = \max_{a \in A} Q(a)$
- But these values can only be found through averages
  - $\hat{Q}(a)$, $\hat{V}$
- We could have done hypothesis testing...
  - But this would entail a random policy
  - Maybe we can do better

Goals (2)

- We would like to find the best action using the minimum amount of samples possible
- Keep focusing on the best action
  - While also checking making sure that other actions are sufficiently explored
- This is known as the "exploration/exploitation" dilemma

Regret (1)

- Regret is $I_t = E\left[\sum_{t=0}^{T} (V^* - Q(a_t))\right]$
  - Or, equivalently $E\left[\sum_{t=0}^{T} \left(\max_{a \in A} Q(a) - Q(a_t)\right)\right]$
- The count is $N_t(a)$, the number of times we took action a until time t
- The gap $\Delta_a = V^*(a) - Q(a)$, the difference between the optimal action and the action taken

Regret (2)

- It turns out that
  - $\sum_{a \in A} (E[N_t(a)\Delta_a])$
- We would like to minimise the times we have large gaps
- But we have no clue what the gaps are...

Another example

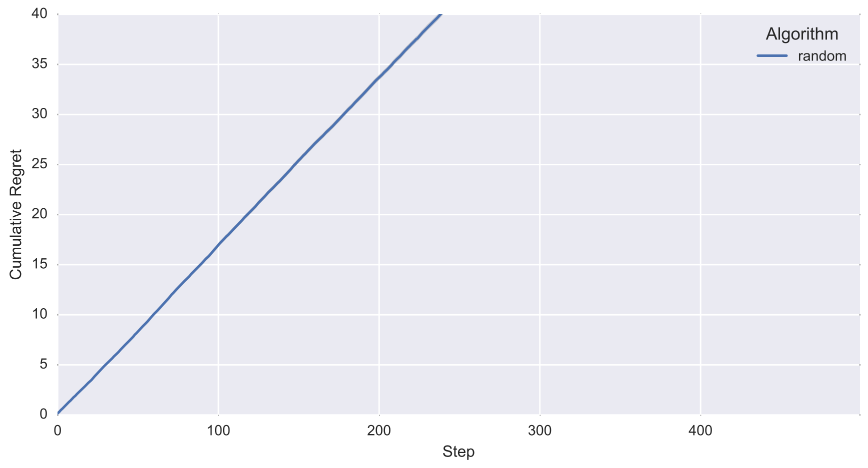
- Three actions to choose from
- Link in internal promo e-mail
  - Thus users more likely to click

```
n_actions = 3

def action_0():
    return np.random.choice([1,0], p=[0.5, 0.5])

def action_1():
    return np.random.choice([1,0], p=[0.6, 0.4])

def action_2():
    return np.random.choice([1,0], p=[0.2, 0.8])

rewards = [action_0, action_1, action_2]
```


Pure exploration

- Somewhat similiar to the A/B case
  - But in A/B you should have set a cut-off point
- You send more or less the equal number of e-mails
- Very simple setup

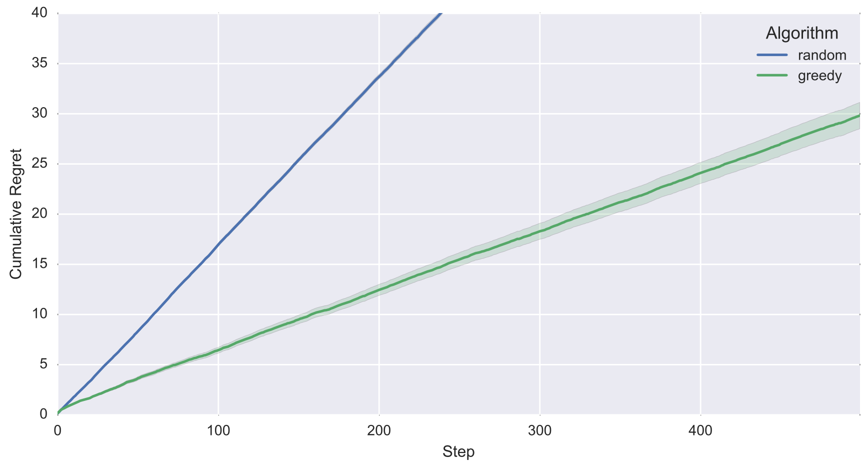
Regret of pure exploration



Greedy

- You choose the action with the highest $\hat{Q}(a)$
- Can you see a problem with this?
  - It might get stuck in suboptimal actions
- Let's try it out

Regret of greedy



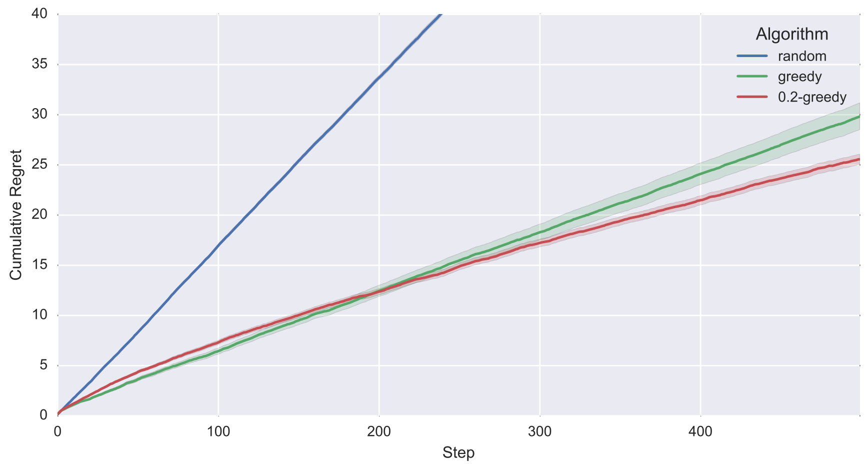
About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

ϵ-greedy

- You set a small probability ϵ with which you act randomly
- The rest of the time you add greedily
  - i.e. you choose the best action
- This is a very common (but inefficient setup)
- What is the optimal ϵ?

About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

Regret of ϵ-greedy



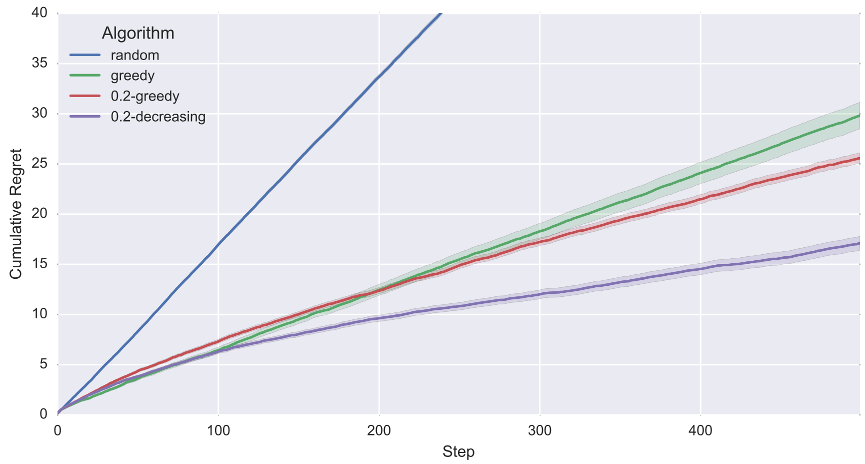
About Bandits Adapting to changing rewards regimes The adversarial case Contextual Bandits Conclusion

ϵ-decreasing

- Same as epsilon greedy, but now you decrease epsilon as you choose actions
- We do

```
e *= 0.99
```

Regret of ϵ-decreasing



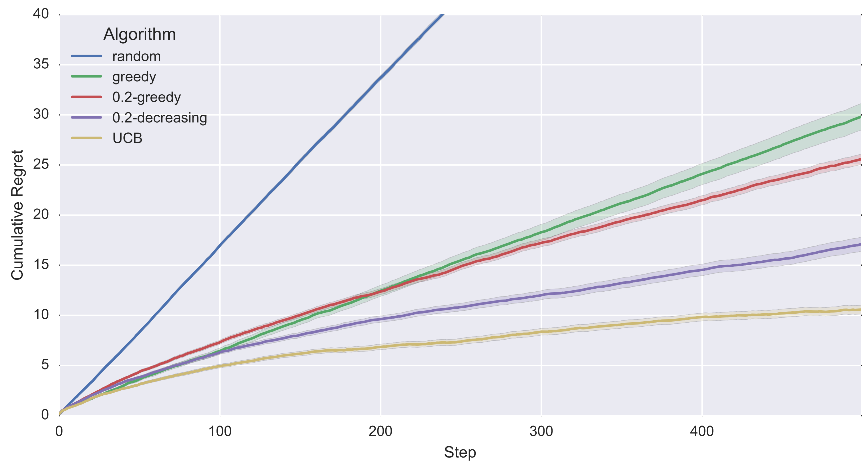
Optimism in the face of uncertainty

- There is a principle termed "optimism in face of uncertainty"
- In practical terms this means that you should try actions with highly uncertain outcomes
  - You believe the best action is the one you haven't explored enough
- Works well in practice

Upper Confidence Bounds

- A very popular algorithm
- Fairly robust
- $UCB(a) = \hat{Q}(a) + U(a)$
- $UCB1(a) = \hat{Q}(a) + C\sqrt{\frac{log(t)}{N_t(a)}}$
- $N_t(a)$ is the times action a was executed
- t is the current timepoint/time
- $C \in [0, \inf]$ is a constant - I set it to 0.5 for the plots below
  - Can you guess what the effect of C is?

Regret of upper confidence bounds



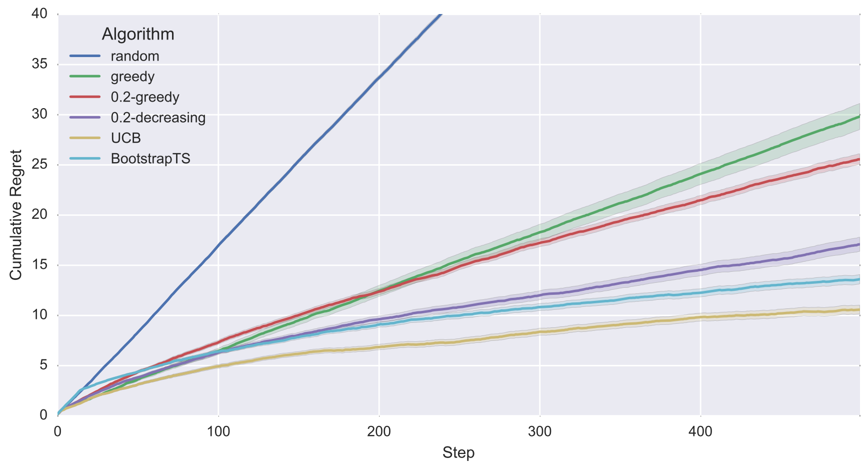
Bootstrap thompson sampling

- What if you could take bootstrap samples of action rewards that we have collected?
- You would have incorporated the uncertainty within your bootstrap samples
- If you have a large number of bootstrap samples you have a distribution over possible $\hat{Q}(s)$
- Sample from this distribution
- A version of probability matching
  - $\pi(a) = P[\hat{Q}(a) > \hat{Q}(a'), \quad \forall a' \in A]$

Priors

- You can get stuck here as well (like greedy)
- Add some pseudo-rewards
- Or act randomly a bit

Regret of Bootstrap Thomson Sampling



Code

```
class Bandit(object):
    def __init__(self,n_actions):
        self.counts = np.zeros(n_actions)
        self.action_rewards = [[] for i in range(n_actions)]
        self.rewards = []
        self.n_actions = n_actions

    def select_action(self):
        """Selection which arm/action to pull"""
        pass

    def update(self,action,reward):
        """Update the actions"""
        self.counts[action] = self.counts[action] + 1
        self.action_rewards[action].append(reward)
        self.rewards.append(reward)


    def get_Q_values(self):
        Q_values = []
        for q_v in self.action_rewards:
             Q_values.append(np.array(q_v).mean())

        return np.array(Q_values)

    def get_V_value(self):
        return np.array(self.v_value.mean())
```

Change of rewards

- What if rewards just change
- Because people are bored of your e-mails
  - They talk to each other
  - Out of fashion
- You might want to have continuous adaptation
- Keeping all values and finding $\hat{Q}(a)$ is expensive
  - What happens in e-mail 1000? e-mail 100K?
  - How many additions?

The sequential case

- What if you are to take a series of actions?
- Surely your current action depends on your future actions
- Hence there is going to be a change in the distribution of rewards
  - Induced by the experimenter
- "Reinforcement Learning"

Example e-mail campaign

- You send your first e-mail
  - "Please buy this product"
- Send second e-mail
  - "Will you buy the add-on?"
- Send third e-mail
  - "Let us service your product"
- You want to maximise your rewards
- Creates a tree of possible actions

Tree

- Let's draw the tree of the above example
  - Three different actions for each "state"
- What do you observe?

Introducing state

- $s \in S$ can be used to differentiate between different "states", conditioning π, V and Q values on states
- $\pi(s, a)$, $V(s)$, $Q(s, a)$
- e.g. in the example above, you have $Q(\text{"}firstemail'', \text{"}emailtypeA'')$
- Let's write the rest of the states, the policies, V and Q-Values

Incremental calculation of a mean

$\mathrm{v_t}$ can be the reward or the sum of rewards you got at different steps

$$\hat{Q}_t(s,a) = \hat{Q}_{t-1}(s,a) + \overbrace{\frac{\mathrm{v_t} - \hat{Q}_{t-1}(s,a)}{t}}^{\textbf{Error}}$$

$$\hat{Q}_t(s,a) = \hat{Q}_{t-1}(s,a) + \frac{1}{t}\overbrace{\mathrm{v_t} - \hat{Q}_{t-1}(s,a)}^{\textbf{Error}}$$

$$\hat{Q}_t(s,a) = \hat{Q}_{t-1}(s,a) + \alpha\left[\mathrm{v_t} - \hat{Q}_{t-1}(s,a)\right]$$

Incremental bootstrap

Oza, Nikunj C., and Stuart Russell "Online bagging and boosting." Systems, man and cybernetics, 2005 IEEE international conference on. Vol. 3. IEEE, 2005.

- We will implement this in the labs

Equilibria

- We will discuss (very) briefly the notion of equilibria
  - Imagine you are putting up large advert banners on your website
  - They hide content
  - User can click on the top right corner and quit the banner
- Where should you put the banner?
- How often should the banner pop-up?

Adversarial bandits

- Most bandits we discussed until now assume the environment is indifferent
- i.e. the user will click in the link if she thinks it is interesting for her to click
- But quite often, people are annoyed by your efforts - so they will try to "adapt" around you
  - Close the advert-super fast without thinking
- Solution - put the advert in random places
  - Mixed policies
- Exp3 - but not now

Rethinking states

- States as we have defined them until now are black solid boxes
  - They can only be enumerated
- i.e. state s_0, state s_1
- What if a state could be decomposed into a set of features?
  - *sex*, *age*, *married*, *job*...
- Highly reminiscent of supervised learning
  - We are given features, we would like to predict the reward - i.e. the outcome!
- We could now do something that looks like regression!

Combining states and actions

- So you now have features that you can encode
- Various encoding strategies
  - One regressor per action
  - A single regressor with dummy encoded actions
- Let's do an example
- What could be a problem if you don't have separate regressors for each action?

ϵ-greedy and ϵ-decreasing

- Set ϵ to some small value
- Keep decreasing...
- Very popular because of its simplicity
- You need to be smart about your decreasing schedule
  - Possibly set some lower bound

Bootstrap Thomson Sampling

- Get a bootstrap sample of all your data
- Learn a regressor
- Act greedily using the regressor you learned
- Repeat

Conclusion

- First hit on bandits
- Super-exciting research area
- Used quite a bit on website optimisation and recommender systems
- We will delve deeper in the adversarial case and recommender systems in the future
- Again, the bootstrap saves the day