

1. $A \rightarrow 5 \times 3 \Rightarrow A^T = 3 \times 5$
 $B \rightarrow r \times s$
 $C \rightarrow 4 \times 3$

$$\Rightarrow A^T * C = 5 \times 5$$

$$A^T * B * C$$

$$(3 \times 5) * (r \times s) * (4 \times 3)$$

$$\quad \quad \quad \underline{5 \times 4}$$

Suppose A is 5×3 matrix, B is $r \times s$ matrix and C is 4×3 matrix. now if $A^T * B * C$ is defined then which is true?

- a. $r=5, s=4$
 b. $r=3, s=5$
 c. $r=3, s=4$
 d. $r=4, s=3$

2. Idempotent matrix $\rightarrow A^2 = A$

which of the following is true for an idempotent matrix?

- a. $A = A^{-1}$ c. $A^2 = A$
 b. $A = A^T$ d. none of them

3. What is the defn. of delta function $\{\delta(t)\}$ in time?

- a. $\delta(t) = \begin{cases} 1 & t=0 \\ -1 & t \neq 0 \end{cases}$
 b. $\delta(t) = \begin{cases} +\infty & t=0 \\ 1 & t \neq 0 \end{cases}$
 c. $\delta(t) = \begin{cases} +\infty & t=0 \\ 0 & t \neq 0 \end{cases}$
 d. none of them.

[delta function equals zero by definition for values of t other than zero. it must have infinite amt. when $t=0$.

4. Let u, v, w be three non-zero vectors which are linearly dependent, then

- a. u is linear combination of v, w
 b. v is " " " "
 c. w " " " "
 d. all of the above.

5. Consider the system $3x + ky = 3$
 $3x + 2y = 5$

for which value of k the system will not have any solution? $k=2$

6. When a matrix is symmetric. \rightarrow

- a) $A = A^T$
 b) $A = A^{-1}$
 c) $A^{-1} = A^T$
 d) none

7. $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$
 $B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$

Perform $A \times B$

- a. $\begin{bmatrix} 5 & 3 & 9 \\ 3 & 3 & 3 \end{bmatrix}$
 b. $\begin{bmatrix} 3, 7 & 9 \\ -3 & -3 & -3 \end{bmatrix}$
 c. undefined.
 d. None.

8. Eigen value calculation.

$$A = \begin{bmatrix} 3 & 0 & 0 \\ 0 & 4 & 5 \\ 0 & 4 & 3 \end{bmatrix}$$

What are the eigen values?

if A is a square matrix, then

$$Av = \lambda v$$

v is eigen vector

λ is eigen value.

$$Av = \lambda v$$

$$\Rightarrow Av - \lambda v = 0$$

$$\Rightarrow v[A - \lambda I] = 0$$

homogeneous system of linear equation.

So non-trivial solⁿ only when the det. of $A - \lambda I = 0$. so $|A - \lambda I| = 0$.

So eigen values of 3×3 matrix \rightarrow

$$\begin{bmatrix} 3-\lambda & 0 & 0 \\ 0 & 4-\lambda & 5 \\ 0 & 4 & 3-\lambda \end{bmatrix} \Rightarrow \begin{vmatrix} 3-\lambda & 0 & 0 \\ 0 & 4-\lambda & 5 \\ 0 & 4 & 3-\lambda \end{vmatrix} = 0$$

$$= (3-\lambda)[(4-\lambda)(3-\lambda) - 5 \times 4] + 0 + 0 = 0$$

$$\Rightarrow (3-\lambda)[12 - 4\lambda - 3\lambda + \lambda^2 - 20] = 0$$

$$\Rightarrow \lambda = 3 \left\{ \begin{array}{l} \lambda^2 - 7\lambda - 8 = 0 \\ \lambda^2 - 8\lambda + \lambda - 8 = 0 \end{array} \right.$$

$$\lambda = 8$$

$$\lambda = -1$$

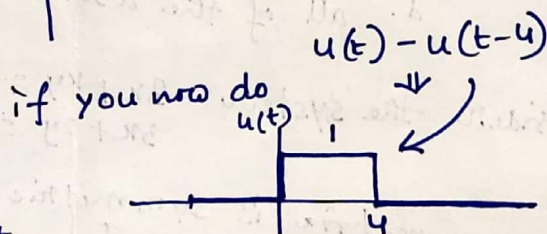
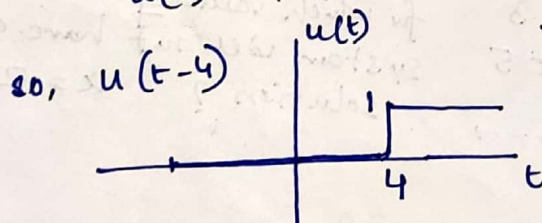
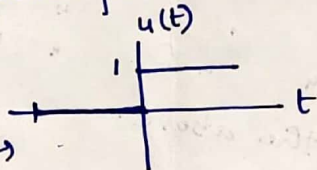
Ans

9. Continuous time impulse function in terms of step function...

- unit step

$$u(t) = 1, t > 0$$

$$u(t) = 0, t < 0$$



unit impulse is the derivative of unit step.

$$\text{so } \delta(t) = \frac{du(t)}{dt}$$

1. $\delta(t) = \int u(t) dt$
2. $\delta(t) = |u(t)|$
3. $\delta(t) = \frac{du(t)}{dt}$
4. none.

Supervised Learning. Categories of Pattern Recog. Problem.

24.07.2024
Pattern Recog.

- Example of Pen & USB.
- a priori knowledge of structure of Pen & USB.
- from early childhood → Pen is known.
- gradually → USB ...
- So for a new object like a pen structure will be identified as a pen as prior knowledge of pen structure is known to us.
- that is a priori knowledge. → based on this → Supervised learning.
 - Must have a priori knowledge
 - use this a priori knowledge to recognize or to classify unknown patterns.
- antonym is posteriori
 - apple is a fruit
 - apples are sweet.
 - a specific case, speaker knows from experience.
 - a fact based on logical reasoning.

→ Unsupervised learning. Mix those pens & USBs.
now try to pick one by one object, check the similarity of pattern and put them in one group. Pick another object of different pattern and try to keep in another group.

⇒ So in case of supervised learning based on a priori knowledge you make group of different object whereas in case of unsupervised learning based on a mixture of objects groups are created without any a priori knowledge, similar pattern objects are kept in similar groups.

→ "According to Plato - 'a priori knowledge is stored in our brain in a mystic way'.
So what we usually try - to train a system with similar level of intelligence so that the system can in the same way in which we work.

→ So now consider an USB device and a pen there should be some description associated with each device, based on which the system can identify that USB or Pen.
So that description or instance of description will be termed as 'feature'.

→ So from here we will go to ⇒ Feature Extraction.

Let us try to think about the feature of USB and Pen. consider two boundaries.



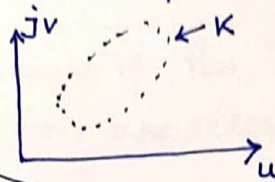
- boundary features. - can be extracted from boundary of object.

- Region Features

this boundary is nothing but a set of points.

whenever we cannot discriminate for shape of object different object or patterns with boundary features then it is required to utilize the region features. most of the information is actually available in boundary feature.

if you represent a 2D space for the boundary, the boundary can be represented by



→ in this form. where horizontal axis is real no axis & vertical is imaginary axis.

so every K^{th} point can be represented

by the complex no. $s(k) = u(k) + jv(k)$

$s(k) = 0, 1, \dots, n-1$

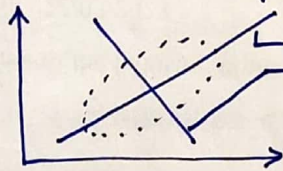
so $s(k)$ is nothing but sequence of complex no.

[complex nos. are used when there are phase & magnitude]

so one of the process to extract features that you have a sequence of complex no. and you can take FFT or DFT to extract features.

Imp: So after DFT if you have N no. of complex no. you will have N no. of Fourier coefficients. and the magnitude of the co-efficients will be able to describe the boundary drawn in the above-mentioned figure. Hence became a set of features.

→ If you consider another feature:

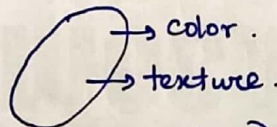


→ moment of the shape around this axis.

→ moment of the shape around another axis which is orthogonal to the principal axis and passing through the centre of gravity of obj. hence we got two components.

so this set of components will give you the shape features.

→ If you consider region feature →



so now we can see the object

is having - a set of property through boundary.

- a set of " " shape

- a set of " " color

- a set of " " texture, and so on...

- All these different feature actually give different numerical values which are actually representing some information about the object.

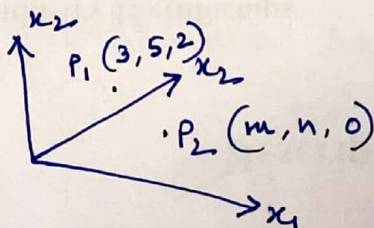
- now if you put these values in some specific order you will get a vector.

- that is called a feature vector. → this will simplify the classification.

↓ why

you can transfer the pattern from the spatial domain or time domain to a space which is feature space. As the vector is 'M' dimensional so the feature space is M dimensional.

$M=3$.



so when $P_1 = P_2 = ?$

→ chain code is nothing but a shape descriptor.
where the shape is obtained using the boundary information.
no information within the shape has been used.

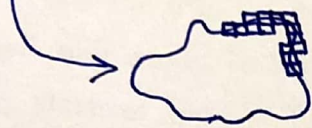
→ Another descriptor that can be used for Recognition purpose is
Polygonal Approximation.

↑
chain code is also
a type of polygonal approximation.

⇒ Polygonal Approximation:


→ Minimum Perimeter length polygonization.

→ Splitting → criterion technique.



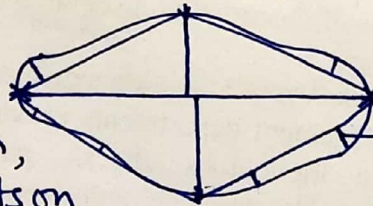
⇒ embed this boundary into a set of concatenated cells.
so inner and outer both wall can be there.
there is tension towards outer direction, inner direction
and there is some linking point.

- splitting technique:

consider any shape → 
on this shape if we can able
to point some vertices based on which the shape can be splitted
then those vertices will be vertices of polygon.

criterion function helps to decide the points on the boundary
which helps to create the boundary of polygon.

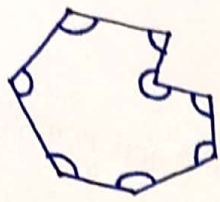
Say one shape:



according to a criterion function,
initially you decide two points on
boundary along which the boundary can be split.
straight line split the boundary into two halves.

these distances also
need to be checked.
if $<$ threshold no
separation of vertex.

criterion function says that the distance from boundary to split line
need to be checked. if the distance is greater than a threshold
in that case the maximum distance will give one vertex of polygon.



⇒ Suppose a polygon like this. - so how to generate feature vectors from this pattern.

- you can always find the angle, the inner angle of the polygon.
- if the n^{th} angle of the n^{th} vertex of the polygon is θ_n . then this n^{th} angle can be represented by a linear combination of k nos. of previous angles.

A linear regression equation can be formed which is given by

$$\theta_n = \sum_{i=1}^k \alpha_i \cdot \theta_{n-i}$$

where this i will vary from 1 to k .

so any angle is represented by some previous angles to form an autoregressive model.

and any angle is represented by $\alpha_1 \theta_{n-1} + \alpha_2 \theta_{n-2} + \dots + \alpha_k \theta_{n-k}$ and as the summation has been continued from 1 to k so this equation is a k^{th} order autoregressive model.

set of α is called as coefficients of auto regression.

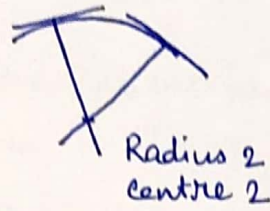
so if any polygon has a total of p nos. of vertices then we will have p nos. of linear equation like this.

now if we want to find out p nos. of solutions then we will have exact solution if p is exactly same as n , exactly same as k .

if $p > k$ we will have less no. of equations ^{than variables.} so system will be overspecified.

if $p < k$ we will have more no. of equations than the variables. so system will be underspecified.

⇒ Feature Extraction:



if $R_1 = R_2$ or
 $C_1 = C_2$ [centre 1 = centre 2]

then we can say centre 1. and
centre 2 along with radius 1 and
radius 2 are equal.

However in ideal cases it is almost impossible to find an exact match. there might be 1 or 2 pixel changes can be found, due to error of measurement, error of quantization.

In such cases the difference between radius 1 & radius 2 should be measured. Differences are as much as low, both radius will be that much of similar.

The feature vector for every pattern, depends on problem domain.

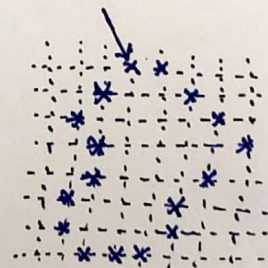
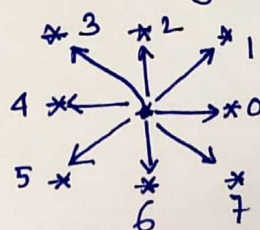
For any pattern, a unique feature vector can be generated. However given the same feature, from it the same pattern may not be generated.

So mapping from pattern to feature vector is unique, whereas the mapping from feature vector to pattern may not be unique.

⇒ chain code: In case of shape feature only boundary information can give the shape information. one of this kind of feature which is used as a descriptor for a particular shape is called a chain code.

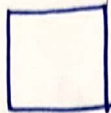
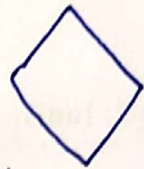
To represent any boundary, an arbitrary boundary by line segments, that is approximation or piece wise linear approximation of arbitrary boundaries, then it can be represented by some piecewise linear boundaries of specified length and directions.

Direction —→ 4-connectivity —→ 8-connectivity



identify the codes, check rotation - variable.

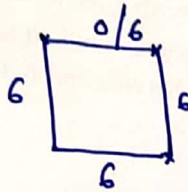
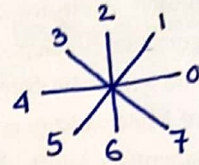
Previous sequence of movements represents the boundary information.

check the example of  and 

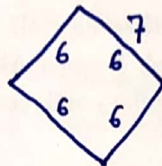
chain code will be different for that.

Hence it is not rotation invariant.

→ Differential chain code:

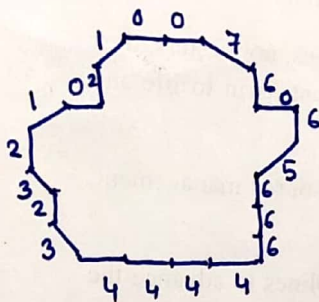


all 6.



all 6.

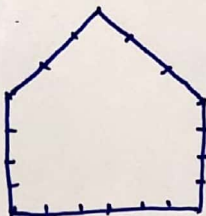
for the symmetric figure the code is very simple one.



Chain code

⇒ 0076065666444432321021
7077267100600077177727

last.
differential chain code



Chain code

⇒ 7776666444442222111
6007000600006000700

differential chain code