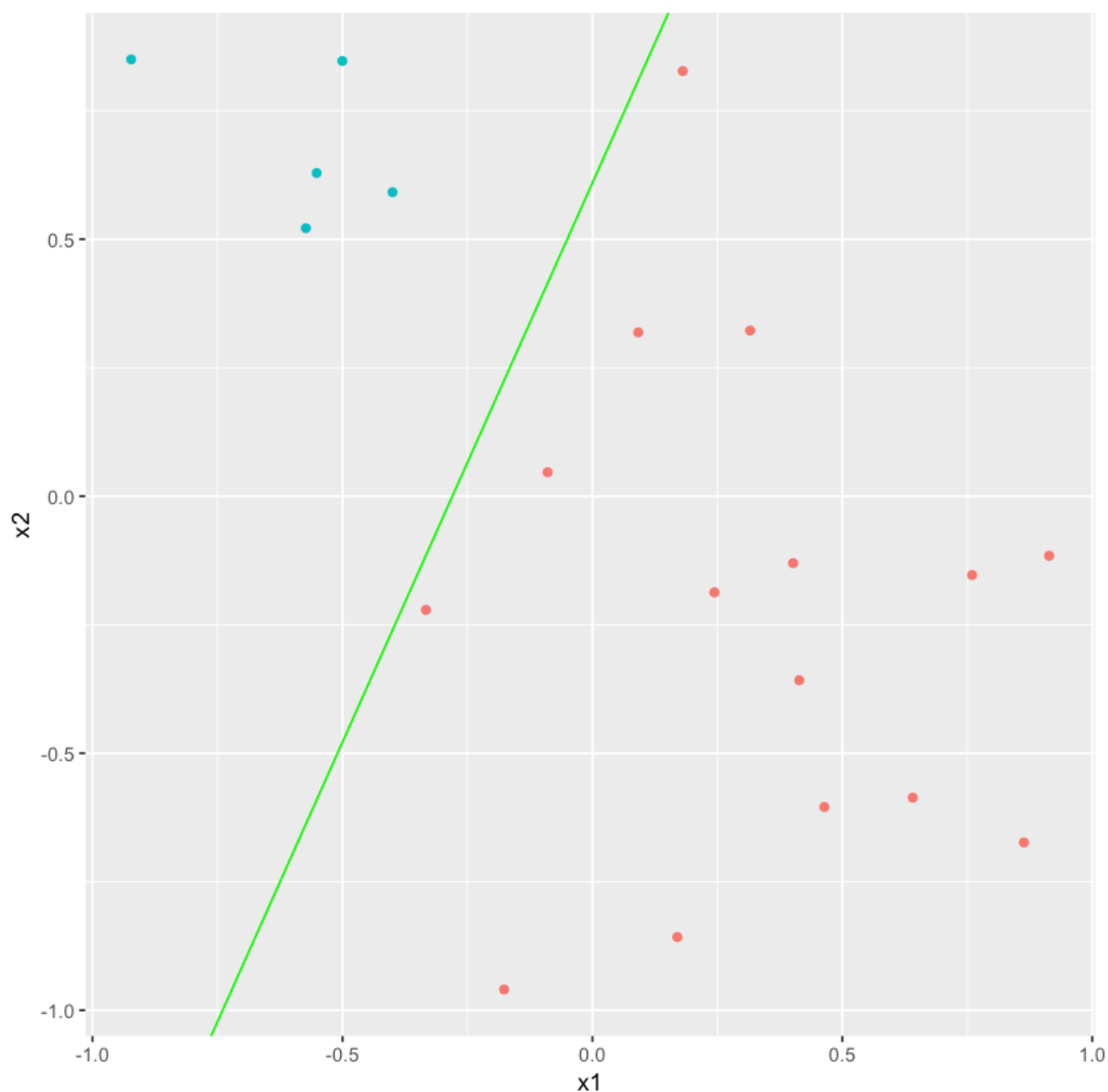


In [ ]: NOTE : The below code has been written **in** R kindly install R packages **in** jupyter notebook before running them.

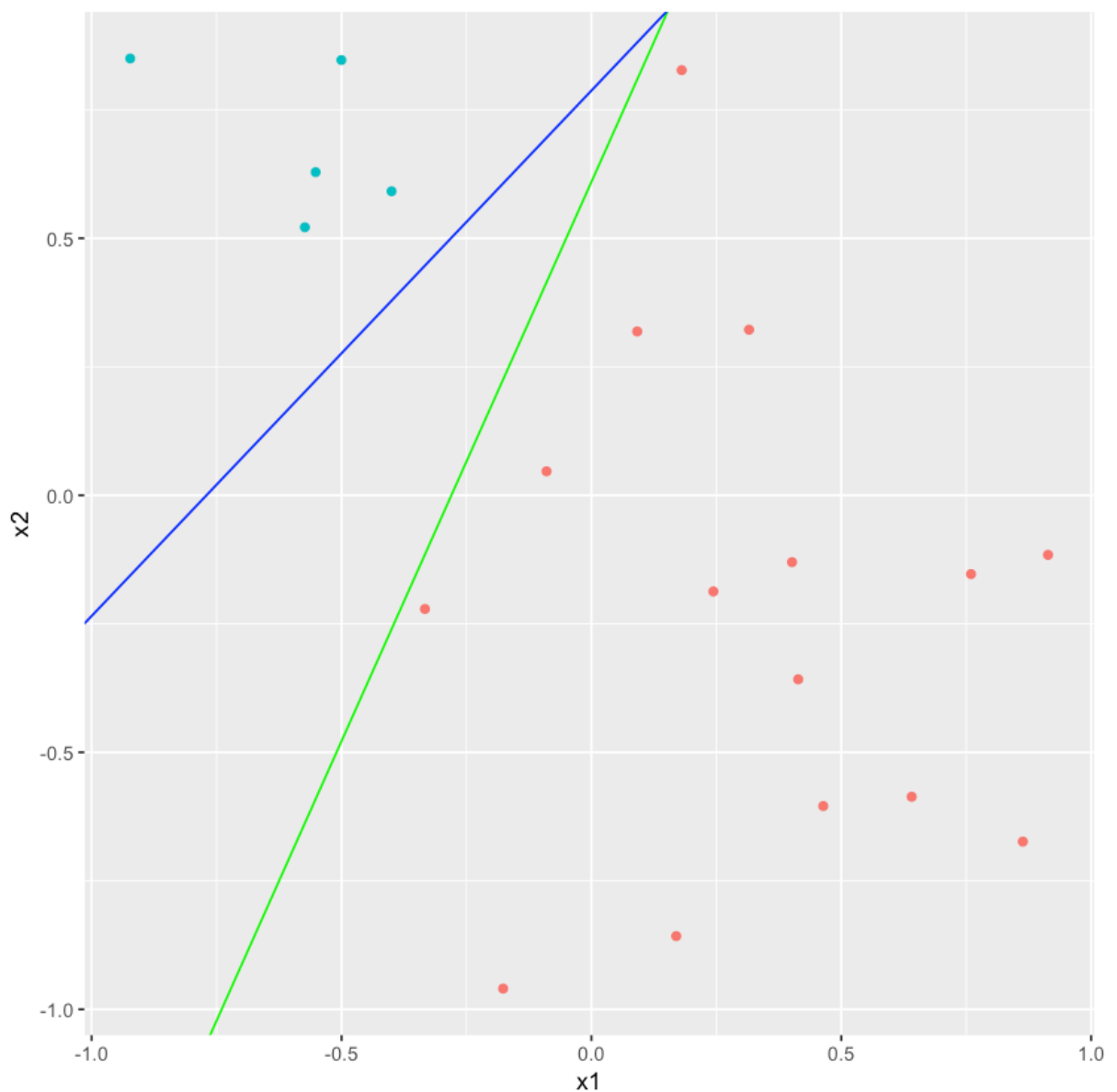
In [ ]: (a) Below, we generate a linearly separable data set of size 20 and the target **function**  $f$  (**in** red)

```
In [13]: library(ggplot2)
set.seed(101)
h <- function(x, w) {
  scalar_product <- cbind(1, x$x1, x$x2) %*% w
  return(as.vector(sign(scalar_product)))
}
w0 <- runif(1, min = -999, max = 999)
w1 <- runif(1, min = -999, max = 999)
w2 <- runif(1, min = -999, max = 999)
f <- function(x) {
  return(h(x, c(w0, w1, w2)))
}
D <- data.frame(x1 = runif(20, min = -1, max = 1), x2 = runif(20, min =
-1, max = 1))
D <- cbind(D, y = f(D))
dots <- ggplot(D, aes(x = x1, y = x2, col = as.factor(y + 3))) + geom_po
int() + theme(legend.position = "none")
p_f <- dots + geom_abline(slope = -w1 / w2, intercept = -w0 / w2, colour
= "green")
p_f
```



In [ ]: (b) Below, we plot the training data, the target **function**  $f$  (in red) and the final hypothesis  $g$  (in blue) generated by PLA

```
In [9]: iter <- 0
w <- c(0, 0, 0)
repeat {
  y_prediction <- h(D, w)
  D_mis <- subset(D, y != y_prediction)
  if (nrow(D_mis) == 0)
    break
  x_t <- D_mis[1, ]
  w <- w + c(1, x_t$x1, x_t$x2) * x_t$y
  iter <- iter + 1
}
p_g <- p_f + geom_abline(slope = -w[2] / w[3], intercept = -w[1] / w[3],
  colour = "blue")
p_g
```



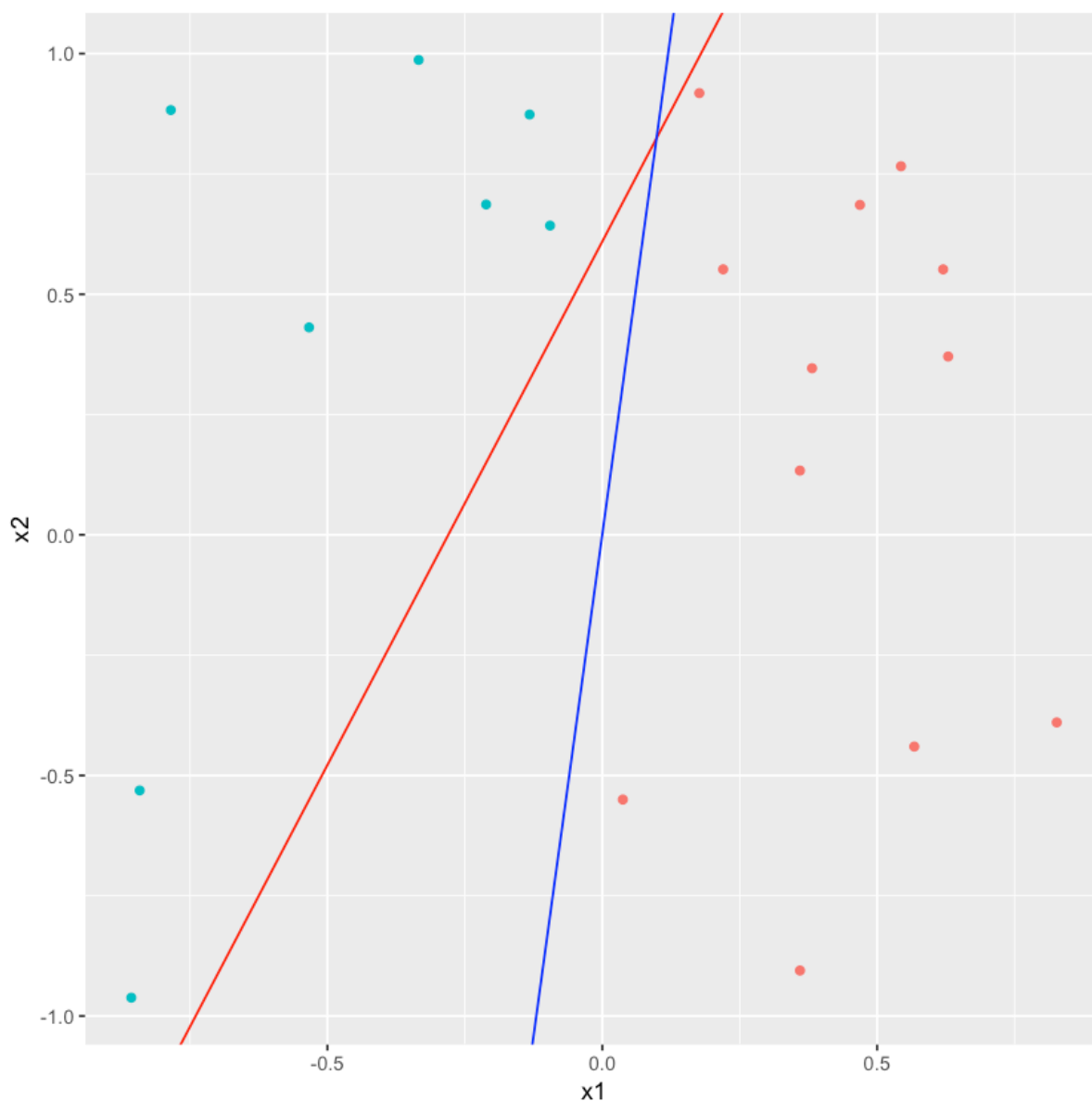
In [ ]: Here, the PLA took 5 iterations before converging.  
We may notice that although  $g$  is close to  $f$ , but are not converging

In [ ]: (c) Below, we **repeat** what we did **in** point (b) with another randomly generated data set of size 20

```

In [10]: D1 <- data.frame(x1 = runif(20, min = -1, max = 1), x2 = runif(20, min =
-1, max = 1))
D1 <- cbind(D1, y = f(D1))
iter <- 0
w <- c(0, 0, 0)
repeat {
  y_prediction <- h(D1, w)
  D_mis <- subset(D1, y != y_prediction)
  if (nrow(D_mis) == 0)
    break
  x_t <- D_mis[1, ]
  w <- w + c(1, x_t$x1, x_t$x2) * x_t$y
  iter <- iter + 1
}
ggplot(D1, aes(x = x1, y = x2, col = as.factor(y + 3))) + geom_point() +
  theme(legend.position = "none") +
  geom_abline(slope = -w1 / w2, intercept = -w0 / w2, colour = "red") + ge
om_abline(slope = -w[2] / w[3], intercept = -w[1] / w[3], colour = "blu
e")

```



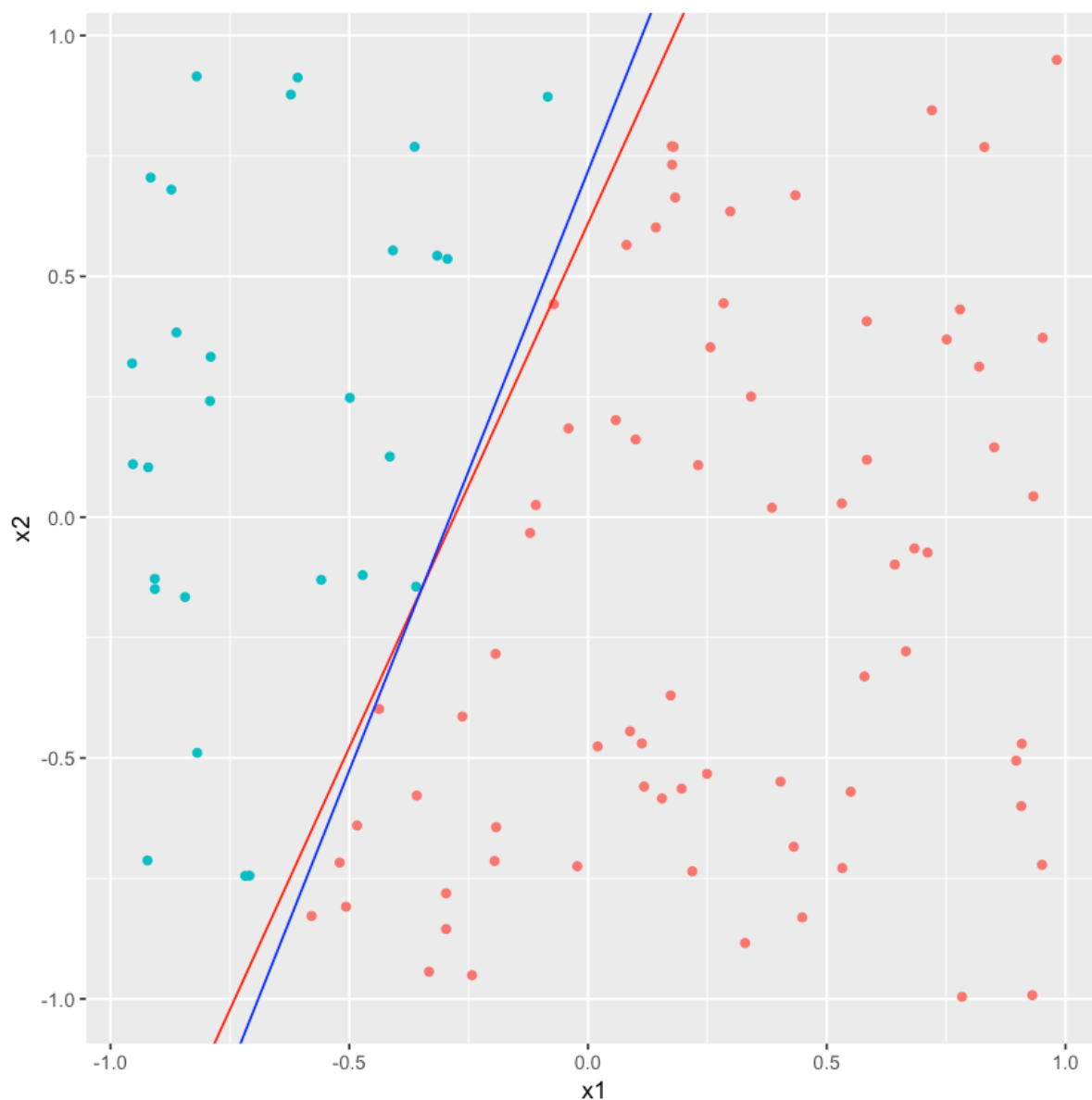
In [ ]: In this case, the PLA took 12 iterations (which is greater than **in** (b)) before converging.  
We may notice that, as **in** point (b), although  $g$  is pretty close to  $f$ , they are not quite identical

In [ ]: (d) Below, we **repeat** what we did **in** point (b) with another randomly generated data set of size 100.

```

In [12]: D1 <- data.frame(x1 = runif(100, min = -1, max = 1), x2 = runif(100, min
= -1, max = 1))
D1 <- cbind(D1, y = f(D1))
iter <- 0
w <- c(0, 0, 0)
repeat {
  y_prediction <- h(D1, w)
  D_mis <- subset(D1, y != y_prediction)
  if (nrow(D_mis) == 0)
    break
  x_t <- D_mis[1, ]
  w <- w + c(1, x_t$x1, x_t$x2) * x_t$y
  iter <- iter + 1
}
ggplot(D1, aes(x = x1, y = x2, col = as.factor(y + 3))) + geom_point() +
  theme(legend.position = "none") +
  geom_abline(slope = -w1 / w2, intercept = -w0 / w2, colour = "red") +
  geom_abline(slope = -w[2] / w[3], intercept = -w[1] / w[3], colour = "blue")

```



In [ ]: In this case, the PLA took 33 iterations (which is greater than **in** (b) and (c)) before converging.  
We may notice that, here  $f$  and  $g$  are very close to each other

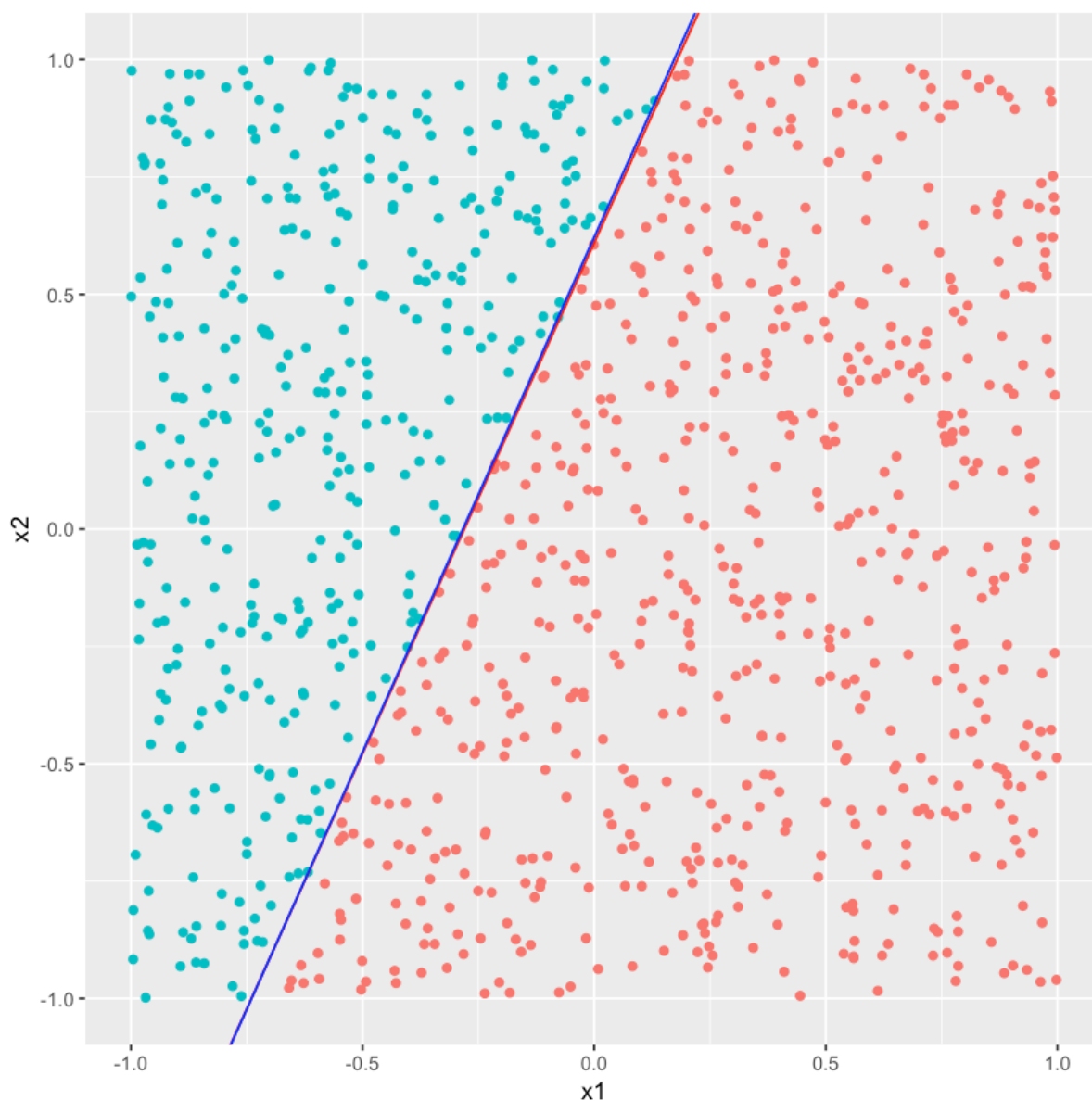
In [ ]: e) Below, we **repeat** what we did **in** point (b) with another randomly generated data set of size 1000



```

In [11]: D1 <- data.frame(x1 = runif(1000, min = -1, max = 1), x2 = runif(1000, m
in = -1, max = 1))
D1 <- cbind(D1, y = f(D1))
iter <- 0
w <- c(0, 0, 0)
repeat {
  y_prediction <- h(D1, w)
  D_mis <- subset(D1, y != y_prediction)
  if (nrow(D_mis) == 0)
    break
  x_t <- D_mis[1, ]
  w <- w + c(1, x_t$x1, x_t$x2) * x_t$y
  iter <- iter + 1
}
ggplot(D1, aes(x = x1, y = x2, col = as.factor(y + 3))) + geom_point() +
  theme(legend.position = "none") +
  geom_abline(slope = -w1 / w2, intercept = -w0 / w2, colour = "red") + ge
om_abline(slope = -w[2] / w[3], intercept = -w[1] / w[3], colour = "blu
e")

```



In [ ]: In this case, the PLA took 511 iterations (which is greater than **in** (b), (c) and (d)) before converging.  
We may notice that, here  $f$  and  $g$  are nearly undistinguishable.

In [ ]: (f) Here, we randomly generate a linearly separable data set of size 1000 with  $x_n \in \mathbb{R}^{10}$ .

```
In [20]: N <- 10
h <- function(x, w) {
  scalar_product <- cbind(1, x) %*% w
  return(as.vector(sign(scalar_product))) }
w <- runif(N + 1)
f <- function(x)
{
  return(h(x, w))
}
D2 <- matrix(runif(10000, min = -1, max = 1), ncol = N)
D2 <- cbind(D2, y = f(D2))
D2 <- data.frame(D2)
iter <- 0
w0 <- rep(0, N + 1)
repeat {
  y_pred <- h(as.matrix(D2[, 1:N]), as.numeric(w0))
  D_mis <- subset(D2, y != y_pred)
  if (nrow(D_mis) == 0)
    break
  x_t <- D_mis[1, ]
  w0 <- w0 + cbind(1, x_t[, 1:N]) * x_t$y
  iter <- iter + 1
}
```