Quasi-developable Surface Design and Optimization

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Figure 1: *Three models with quasi-developable strips designed in my system: ship, vase and lampshade form left to right.*

Abstract

Developable surfaces are important in design and plate forming. In the past two years, I mainly focused on the modeling of quasi-developable surfaces and their application. This article is a statement of my research work during my undergraduate days.

1 Research Overview

Developable surfaces can be flattened onto a plane without distortion. And it is easy to form a developable surface by rolling a flat sheet material, which makes them important in manufacturing objects from sheet metal, cardboard and plywood. One industry which uses developed surfaces extensively is shipbuilding[Friedman 1998]. In some cases, a ship hull can be designed and manufactured using developable surfaces entirely[Pérez and Suárez 2007]. An example produced using my system and algorithms is shown in Figure 1(left). It is designed by quasi-developable surfaces and can be fabricated using sheet materials.

1.1 Single-strip quasi-developable surface modeling

Following the literature, I study how to construct a quasi-developable surface between two boundary curves as a start point. There are mainly three previous works about this problem[Pérez and Suárez 2007][Chen and Tang 2013a][Chen and Tang 2013b]. All of them are based on searching the corresponding relationships i.e. rulings, between discrete sampling points of two boundaries. This causes the problem that only finite rulings can be found each time, which makes users need to run their algorithms many times until finding the rulings with a suitable amount to construct a quasi-developable surface meeting their requirements.

To solve this problem, I implement a method allowing continuous mapping between boundary curves, which enhances the ability to search for better developable surfaces[Zheng and Bo 2018b]. This algorithm can be explained as a method for finding the parameterization of boundary curves driven by developability. It firstly finds a quasi-developable surface whose boundaries are close to the given using a numerical optimization method with constraints of boundary distance, developability and surface smoothness. Then it samples sufficient points on given boundary curves and find interpolating parameters for those points utilizing the boundaries of the optimized surface. After curve interpolation, users can obtain

a smoothing and quasi-developable surface meeting their requirements in one-algorithm-time. The work flow of this method is illustrated in Figure 2. Please refer to my slides for more details https://paulyzheng.github.io/paper/2017-02-report.pdf.

Between two given space curves, there may not be a quasi-developable surface meeting the requirement of design or fabrication. Hence, designers need to modify the boundary curves according to the resulting surface and then reuse those method to find a more developable surface. However, the modification of curves is time-consuming even for a experienced designer. So we automate that tedious procedure by solving an optimization problem to search for the most developable surface whose boundaries are near to the given in my resent work[Zheng and Bo 2018a]. The optimization algorithm is the same as the one in [Zheng and Bo 2018b], but I replace the method of generating initial ruled surfaces. In my prior work, I pay too much attention to efficiency, which causes that the ability of our optimization algorithm is not fully explored. I obtain some better resulting surfaces with smaller distance and developability errors via more iterations in optimization. Figure 3 illustrates the Hard Chine model. In Figure 3(c), the maximal and minimal developability errors of this model are $0.08°$ and $0°$ respectively, while the maximal value less than $6°$ can meet the requirement of the manufacturing of sheet metal[Oetter et al. 2002]. And the maximal distance error of this model is 4.6×10^{-4} in a unit bound box, which means the maximal value will be 2.3mm if the real length of this ship is 5m.
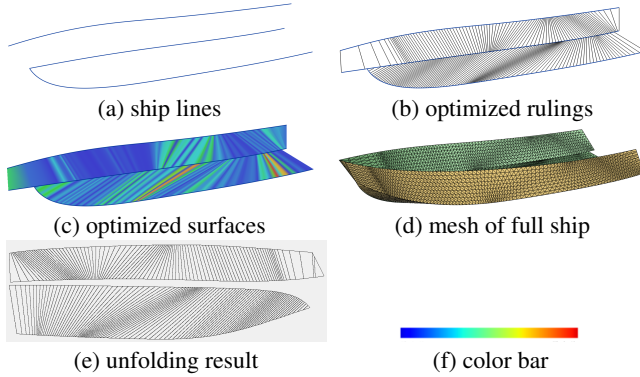
1.2 Multi-strip quasi-developable surfaces modeling

In some industrial fields, designers need to design multi-strip developable surfaces whose adjacent strips share same boundaries like Figure 3(a). Although the single-strip optimization method can constrain that boundaries are close to given curves, the gap between seam-lines still leads to critical problems in some fields with large-size and high-precision. For example, in shipbuilding, this small gap between adjacent steel plates will make the welding more difficult and expensive. Note that the chine of the result surface in Figure 3(c) is represented as two different B-spline curves actually.

Traditionally, researchers eliminate the gap between seam-lines by interpolating given boundary curves. However, there may not be quasi-developable surfaces meeting the requirement of developability between the given curve shape. Hence, the sample points in poor-developability regions on given boundaries are modified be-
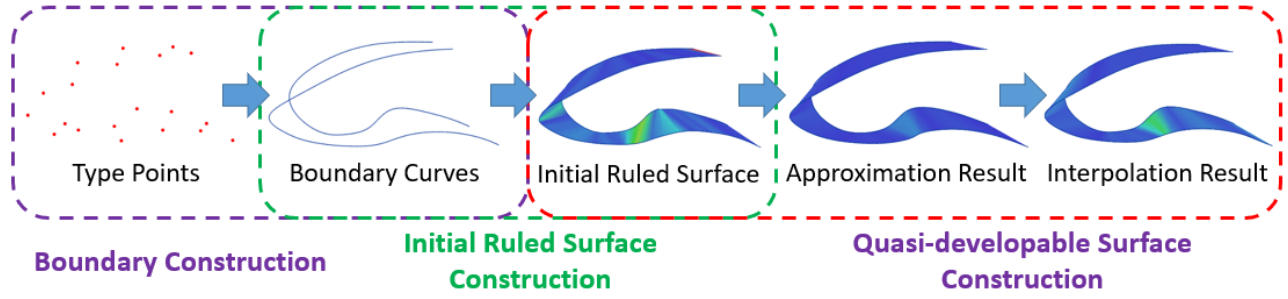
Figure 2: *Work flow of our single-strip quasi-developable surface modeling method.*

(a) ship lines

(b) optimized rulings

(c) optimized surfaces

(d) mesh of full ship

(e) unfolding result

(f) color bar

Figure 3: *An example of Hard Chine. (a) Input ship lines of the half ship: sheer, chine and centreline respectively form top to bottom. (b) Ship lines and rulings after optimization. Note that only 100 of infinite rulings are drawn in this figure. (c) Optimized quasi-developable surfaces with developability colorization. (d) The triangular mesh of full ship after bow-cutting, centerline-seaming and meshing. (e) The result after unfolding to plane. (f) Color bar of developability. The maximal error is represented in red, while the minimal error is represented in blue.*

fore interpolating in previous works[Pérez and Suárez 2007][Zheng and Bo 2018b]. I also find that those points to be modified often appear in regions with large distance errors[Zheng and Bo 2018b]. Another problem is that the modification of seam-lines is influenced by two strips. So there may be still a gap after modification and interpolation. The cause of these problems is that the seam-line between two strips is represented by two curves. In order to eliminate the gap and ensure the developability of all strips, the most effective method is optimizing all strips together with one curve representing the seam-line between two adjacent strips. I implement it in my resent work[Zheng and Bo 2018a], which has not been published. Please refer to the manuscript on my web site https://paulyzheng.github.io/paper/2018-02.pdf.

1.3 Toward ship hull design

To apply my algorithms mentioned above, I develop a simple computer aided ship hull design system in my graduation project, which has been awarded one of the excellent graduation projects in my campus. Designers can use this system to design ship lines by dragging control points of B-spline curves handily and generate quasi-developable ship hulls using my optimization algorithm. After unfolding those hulls to plane, almost everyone can build their own ship using plywood sheets referring to the simple tutori-

al[Anonymous 2018]. Other simple art works also can be designed with this system, such as the vase and the lampshade in Figure 1.

1.4 Developable surface reconstruction

Surface reconstruction is an important means in reverse engineering. However, the developability of surfaces is very sensitive to the noise which will be brought in when scanning real objects. Hence, we present an innovative method for developable surface reconstruction from noisy data[Bo et al. 2017]. In this work, I implement an optimization approach to smooth normal vector field of given model via $L0$-norm minimization[Sun et al. 2015]. This optimization method has the capability of preserving geometric information concerning developability. Based on that, a variational shape approximation procedure[Cohen-Steiner et al. 2004] is utilized to divide the target model into a sequence of planar segments. After each segment is assigned with an initial B-spline surface, a patch merging procedure is used to obtain a B-spline ruled surface which is nearly developable and covers the whole target model. Finally, a developable surface fitting process is used to improve the quality of fitting surface both in approximation error and developability. A simple example can be found in Figure 4.


(a)   (b)   (c)   (d)

(e)   (f)
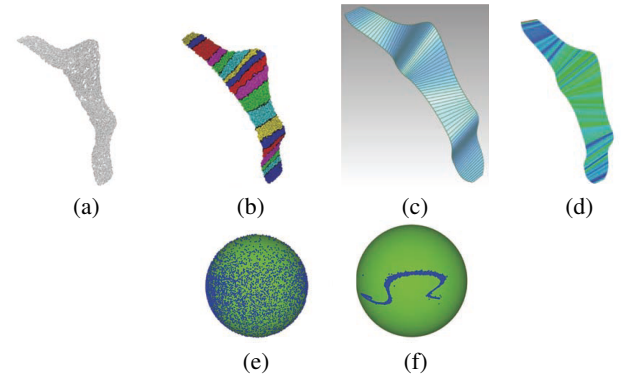
Figure 4: *A simple example for our reconstruction method. (a) Target noisy model. (b) Segmentation result based on optimized normal vector field. (c) Reconstructed developable surface. (d) Color coding of approximation error between reconstructed surface and target noisy model, with approximation error in [0.003, 0.007]. (e) Original normal vectors on Gaussian sphere. (f) Optimized normal vectors on Gaussian sphere.*

2 Future Work

The modeling ability of developable surface is limited, as most of surfaces are doubly-curved. For example, although a ship hul-

l can be entirely designed and manufactured using singly-curved surfaces, most ship hulls are still doubly-curved surfaces for meeting some requirements in other fields like hydrodynamics and mechanics of materials. In terms of the fabrication of doubly-curved surfaces using sheet metal as materials, there are many forming methods, such as line heating[Zhang et al. 2005], stamping with adjustable segmented mold[Wang et al. 2015] and incremental forming[Dang et al. 2017][Dang et al. 2018]. But in the industries using large-size metal plate like shipbuilding, line heating is the main method. There already are some methods for forming a doubly-curved surface only using line heating[Zhang et al. 2005]. However, line heating is expensive and time-consuming. So the plate is rolled to be singly-curved at first and then formed to be irregularly curved through line heating. Obviously, the process and workload of line heating will be affected by different rolling ways in the first step, which raises an interesting problem worth studying.

Developable surface is a special case of ruled surface without intersecting rulings between two boundaries and all points on a same ruling share a same tangent plane. So the curvature along a ruling direction of an arbitrary point on a developable surface must be one of two principle curvatures at that point and its value is zero. Therefore, the Gaussian curvature which equals to the production of two principle curvatures is also zero. As a result, researchers often chose Gaussian curvature as the standard of evaluating the developability of a surface[Pérez and Suárez 2007]. However, what we normally find between two given boundary curves are quasi-developable surfaces, which means there are some double-curved regions on obtained surfaces. The two principle curvatures may both be not equal to zero in those regions, which will lead to that the Gaussian curvature is related to the principle curvature not only along or nearly along the ruling direction, but also along the other bending direction, as showing in Figure 5(a). That makes the Gaussian curvature inappropriate to measure the developability of quasi-developable ruled surfaces.

(a) two bending directions   (b) computing of warp angle
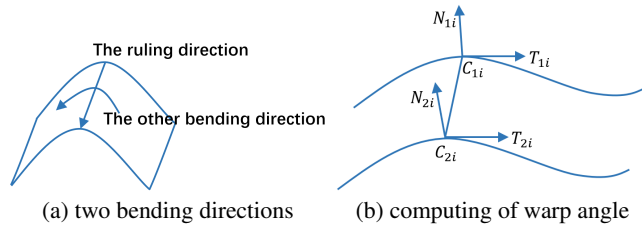
Figure 5: *Two bending directions at a ruling and the computing of warp angle θ between two normal vectors.*

To ensure the developability unrelated to the degree of bending in the other direction, we use the warp angle θ of two normal vectors N_{1i} and N_{2i} at the end points of each ruling to estimate the developable error on quasi-developable surfaces(Figure 5(b)):

$$\theta((C_{1i} - C_{2i}) \times T_{1i}, (C_{1i} - C_{2i}) \times T_{2i})$$

where C_{1i} and C_{2i} are two end points of a ruling, T_{1i} and T_{2i} are tangent vectors at C_{1i} and C_{2i} respectively. For a strict developable surface, the warp angle is zero at each ruling, as the two normal vectors are both perpendicular to a same tangent plane. In terms of quasi-developable surfaces, the smaller the angle is, the better the developability can be achieved. The warp angle shows more intuitive geometric feature of quasi-developable surfaces than Gaussian curvature. Metal can stand a warp angle less than six degree[**?**]. The thickness also has influence on the maximal warp angle, but a six degree angle is small enough for most aluminium alloys of standard thickness[Pérez and Suárez 2007]. While there is no such benchmark for Gaussian curvature. However, the warp

angle is related to the thickness, material of plane sheet and the length of ruling. It is only a relatively better representation of the developable error. A detailed research should be done for analysing current measurements of developability and finding a better one.

Acknowledgements

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