

# Fabrication Oriented Quasi-developable Surface Design and Optimization

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**Figure 1:** *Three models with quasi-developable strips designed in my system.*

## Abstract

Developable surfaces are important in design and plate forming. In the past two years, I mainly focused on the modeling of quasi-developable surface and its application. This article is a statement of my research work during my undergraduate days.

## 1 Research Overview

Developable surfaces can be flattened onto a plane without distortion. And it is easy to form a developable surface by rolling a flat sheet material, which makes them important in manufacturing objects from sheet metal, cardboard and plywood. One industry which uses developed surfaces extensively is shipbuilding[Friedman 1998]. In some cases, a ship hull can be designed and manufactured using developable surfaces entirely[Pérez and Suárez 2007]. Three examples produced using my system and algorithms are shown in Figure 1. All of them are designed by quasi-developable surfaces and can be fabricated using sheet materials.

### 1.1 Single-strip quasi-developable surfaces modeling

Following the literature, I studied how to construct a quasi-developable surface between two boundary curves as a start point. There are mainly three previous works about this problem[Pérez and Suárez 2007][Chen and Tang 2013b][Chen and Tang 2013a]. All of them are based on searching the corresponding relationships i.e. rulings, between discrete sampling points of two boundaries. This will cause the problem that only finite rulings can be found each time, which makes users need to run their algorithms many times until finding the rulings with a suitable amount to construct a quasi-developable surface meeting their requirements.

To solve this problem, I implemented a method allowing continuous mapping between boundary curves, which enhances the ability to search for better developable surfaces[Zheng and Bo 2018b]. This algorithm can be explained as a method for finding the parameterization of boundary curves driven by developability. It firstly finds a quasi-developable surface whose boundaries are close to the given using a numerical optimization method with constraints of boundary distance, developability and surface smoothness. Then it samples sufficient points on given boundary curves and find interpolating parameters for those points utilizing the boundaries of the optimized surface. After curve interpolation,

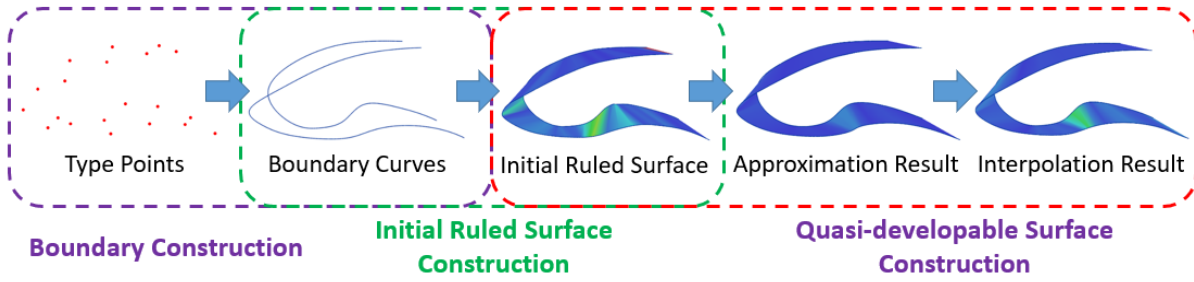
users can obtain a smoothing and quasi-developable surface meeting their requirements in one-algorithm-time. The work flow of this method is illustrated in Figure 2. Please refer to the presentation <https://paulyzheng.github.io/paper/2017-02-report.pdf> for more details.

Between two given space curves, there may not be a quasi-developable surface meeting the requirement of design or fabrication. Hence, designers need to modify the boundary curves according to the resulting surface and then reuse those method to find a more developable surface. However, the modification of curves is time-consuming even for a experienced designer. So we automated that tedious procedure by solving an optimization problem to search for the most developable surface whose boundaries are near to the given in my resent work[Zheng and Bo 2018a]. The optimization algorithm is the same as the one in [Zheng and Bo 2018b], but I replaced the method for generating initial ruled surfaces. In my prior work, I paid too much attention to efficiency, which caused that the ability of our optimization algorithm wasn't fully explored. I obtained some better resulting surfaces with smaller distance and developability errors via more iterations in optimization. Figure 3 illustrates the Hard Chine model. In Figure 3(c), the maximal and minimal developability errors of this model are  $0.08^\circ$  and  $0^\circ$  respectively, while the maximal value less than  $6^\circ$  can meet the requirement of the manufacturing of sheet metal[Oetter et al. 2002]. And the maximal distance error of this model is  $4.6 \times 10^{-4}$  in a unit bound box, which means the maximal value will be 2.3mm if the real length of this ship is 5m.

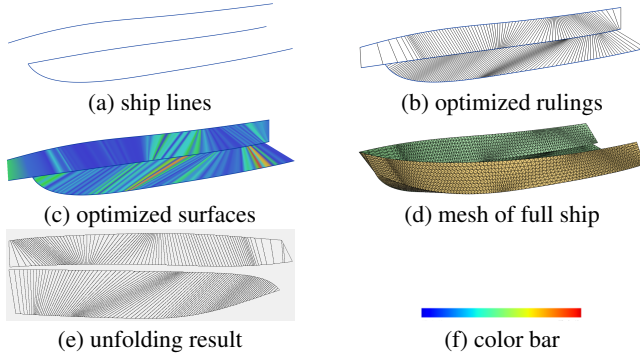
### 1.2 Multi-strip quasi-developable surfaces modeling

In some industrial fields, designers may need to design multi-strip developable surfaces whose adjacent strips share same boundary like Figure 3(a). Although the single-strip optimization method can constrain that boundaries are close to given curves, the gap between seam-lines still leads to critical problems in some fields with large-size and high-precision. For example, in shipbuilding, this small gap between adjacent steel plates will make the welding more difficult and expensive. Note that the chine of the result surface in Figure 3(c) is represented as two different B-spline curves actually.

Traditionally, researchers eliminate the gap between seam-lines by interpolating given boundary curves. However, there may not be quasi-developable surfaces meeting the requirement of developability between the given curve shape. Hence, the sample points



**Figure 2:** The work flow of our single-strip quasi-developable surface modeling method.



**Figure 3:** An example of Hard Chine. (a) Input ship lines of half ship: sheer, chine and centreline respectively form top to bottom. (b) Ship lines and rulings after optimization. Note that only 100 of infinite rulings are drawn in this figure. (c) Optimized quasi-developable surfaces with developability colorization. (d) The triangular mesh of full ship after bow-cutting, centerline-seaming and meshing. (e) The result after unfolding to plane. (f) Color bar of developability. The maximal error is represented in red, while the minimal error is represented in blue.

in poor-developability regions on given boundaries have been modified before interpolating in previous works[Pérez and Suárez 2007][Zheng and Bo 2018b]. I also found that those points to be modified often appear in regions with large distance errors[Zheng and Bo 2018b]. Another problem is that the modification of seam-lines is influenced by two strips. So there may be still a gap after modification and interpolation. The cause of these problems is that the seam-line between two strips is represented by two curves. In order to eliminate the gap and ensure the developability of all strips, the most effective method is optimizing all strips together with one curve representing the seam-line between two adjacent strips. I implemented it in my recent work[Zheng and Bo 2018a], which has not been published. Please refer to the manuscript on my web site <https://paulyzheng.github.io/paper/2018-02.pdf>.

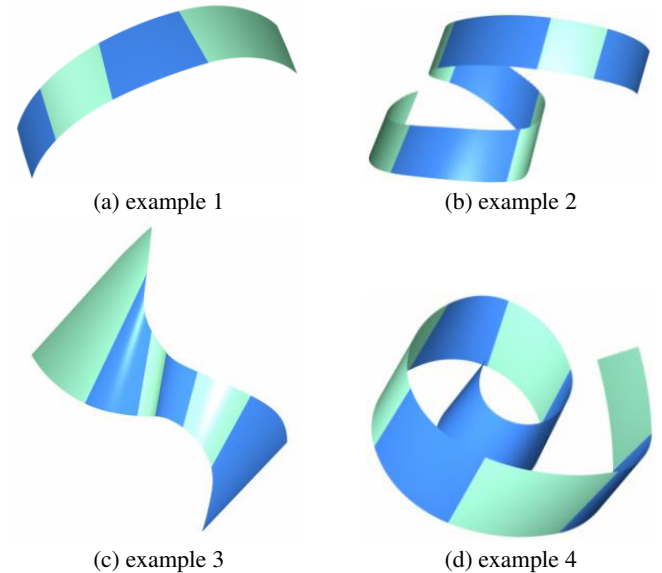
### 1.3 Toward ship hull design

To apply my algorithms mentioned above, I developed a simple computer aided ship hull design system in my graduation project, which has been awarded one of the excellent graduation projects in my campus. Designers can use this system to design ship lines by dragging control points of B-spline curves handily and generate quasi-developable ship hulls using my optimization algorithm. After unfolding those hulls to plane, almost everyone can build their own ship according to the simple tutorial on <https://www.wikihow.com/Build-a-Boat>. Other simple art works

also can be designed with this system, such as the vase and the lampshade in Figure 1.

### 1.4 Cone spline surface fitting

In most industrial applications, plane metal plates are rolled to be on-directional curved using three-roller-bending machines or four-roller-bending machines, which can roll sheet metal into conical surfaces with different radius according to the mathematical principle that three points determine a circle. Hence, for forming a developable or quasi-developable surface from plane plate, we need to divide it into many conical surfaces. The fewer the cone spline surfaces are, the higher the manufacturing efficiency is. To achieve that, we proposed an approach which firstly generates a quasi-developable surface by using the method in [Zheng and Bo 2018b] and then computes a cone spline surface approximating the given target model[Zhang et al. ]. Our method has the ability to divide the given surface to several cone regions automatically, yet the method in [Leopoldseder and Pottmann 1998] needs manual division. Additionally, the boundary curves of given models are constrained in two parallel planes in [Yang and Yang 2006], while there isn't such strict constraint in our method.

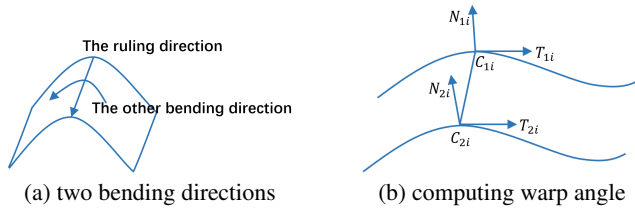


**Figure 4:** Four examples of cone spline surface produced by our algorithm.

## 2 Future Work

The modeling ability of developable surface is limited, as most of surfaces are doubly-curved. For example, although a ship hull can be entirely designed and manufactured using singly-curved surfaces, most ship hulls are still doubly-curved surfaces for meeting some requirements in other fields like hydrodynamics and mechanics of materials. In terms of the fabrication of doubly-curved surfaces using sheet metal as materials, there are many forming methods, such as line heating[Zhang et al. 2005], stamping[Wang et al. 2015] and incremental forming[Dang et al. 2017]. But in the industries using large-size metal plate like shipbuilding, line heating is the main method. There already are some methods for forming a doubly-curved surface only using line heating[Zhang et al. 2005]. However, line heating is expensive and time-consuming. So the plate is rolled to be singly-curved at first and then formed to be irregularly curved through line heating. Obviously, the process and workload of line heating will be affected by different rolling ways in the first step, which raises a interesting problem worth studying.

Developable surface is a special case of ruled surface without intersecting rulings between two boundaries and all points on a same ruling share a same tangent plane. So the curvature along a ruling direction of an arbitrary point on a developable surface must be one of two principle curvatures at that point and its value is zero. Therefore, the Gaussian curvature which is the production of two principle curvatures is also zero. As a result, researchers often chose Gaussian curvature as the standard of evaluating the developability of a surface[Pérez and Suárez 2007]. However, what we normally find between two given boundary curves are quasi-developable surfaces, which means there are some double-curved regions on obtained surfaces. The two principle curvatures may both be not equal to zero in those regions, which will lead to that the Gaussian curvature is related to the principle curvature not only along or nearly along the ruling direction, but also along the other bending direction, as showing in Figure 5(a). That makes the Gaussian curvature inappropriate to measure the developability of quasi-developable ruled surfaces.



**Figure 5:** Two bending directions at a ruling and the computing of warp angle  $\theta$  between two normal vectors.

To ensure the developability unrelated to the degree of bending in the other direction, we use the warp angle  $\theta$  of two normal vectors  $N_{1i}$  and  $N_{2i}$  at the end points of each ruling to estimate the developable error on quasi-developable surfaces(Figure 5(b)):

$$\theta((C_{1i} - C_{2i}) \times T_{1i}, (C_{1i} - C_{2i}) \times T_{2i})$$

where  $C_{1i}$  and  $C_{2i}$  are two end points of a ruling,  $T_{1i}$  and  $T_{2i}$  are tangent vector at  $C_{1i}$  and  $C_{2i}$  respectively. For a strict developable surface, the warp angle is zero at each ruling, as the two normal vectors at the end points are both perpendicular to a same tangent plane. In terms of quasi-developable surfaces, the smaller the angle is, the better the developability can be achieved. The warp angle shows more intuitive geometric feature of quasi-developable surfaces than Gaussian curvature. Metal sheet materials can stand a warp angle smaller than six degree according to [Oetter et al. 2002]. The thickness also has influence on the maximal warp angle, but a six

degree angle is small enough for most aluminium alloys of standard thickness[Pérez and Suárez 2007]. While there is no such benchmark for Gaussian curvature. However, the warp angle is related to the thickness, material of plane sheet and the length of ruling. It is only a relatively better representation of the developable error. A detailed research should be done for analysing current standard measurements of developability and finding a better one.

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