Fabrication Oriented Quasi-developable Surface Design and Optimization(Draft)

Paul Yujian Zheng

Figure 1: *Four models designed by quasi-developable strips using my system.(This is an alternative picture. I'm now working on experiments of those four modles using our algorithms.)*

Abstract

Developable surfaces are important in plate design and forming. In the past two years, I mainly focused on the modeling of quasi-developable surfaces and its application in the manufacturing of sheet material. This article is a statement of my research during undergraduate days.

1 Research Overview

Developable surfaces can be fabricated by bending a flat sheet, which makes that they are important in manufacturing objects from sheet metal, cardboard, and plywood. An industry which uses developed surfaces extensively is shipbuilding. In some cases, a ship hull can be entirely designed and manufactured using developable surfaces[Pérez and Suárez 2007].

1.1 Single-strip quasi-developable surfaces modeling

Following the literature, I study how to construct a quasi-developable surface between two boundary curves as a start point. There are mainly three previous works about this problem[Pérez and Suárez 2007][Chen and Tang 2013b][Chen and Tang 2013a]. All of them are based on searching the corresponding relationships, i.e. rulings, between discrete sampling points of two boundaries. This will cause the problem that only finite rulings can be found each time, which makes that users need to run their algorithms again and again until finding the rulings with a suitable amount to construct a quasi-developable surface meeting their requirements.

To solve this problem, I implemented a method allowing continuous mapping between boundary curves, which enhances the ability of searching for better developable surfaces[Zheng and Bo 2018a]. This algorithm can be explained as a method for finding the parameterization of boundary curves driven by developability. It firstly finds a quasi-developable surface whose boundaries are close to the given two using a numerical optimization method with constraints about boundary distance, developability and surface smoothness. Then it samples sufficient points on given boundary curves and find interpolating parameters for those points utilizing the continuous corresponding relationships between the boundaries of that optimized surface. After curve interpolation, users can obtain a smoothing and quasi-developable surface meeting their requirements in one-algorithm-time. Please refer to the presentation https://paulyzheng.github.io/paper/2017-02-report.pdf.

Between two given space curves, there may be not a quasi-developable surface meeting the requirement of design or manufacture. Hence, designers need to modify the boundary curves according to the resulting surface and then reuse those method to find a more developable surface than the previous one. However, the modification of curves is time-consuming even for a experienced designer. So an optimization problem is solved to search for the most developable surface whose boundaries are close to the given two directly in my resent work[Zheng and Bo 2018b]. The optimization algorithm is the same as the one in my previous work[Zheng and Bo 2018a], but I replace the method for generating initial ruled surfaces. In my prior work, I paid too much attention to efficiency, which caused that the ability of our optimization algorithm did not be explored. I obtained some better result surfaces with smaller distance and ddevelopability errors via more iterations of optimization. Fig.2 illustrates the Hard Chine model. In Fig.2(c), the maximal and minimal developability errors of this model are 0.08° and 0° respectively, while the maximal value less than 6° can meet the requirement of the manufacturing of sheet metal[Oetter et al. 2002]. And the maximal distance error of this model is 4.6×10^{-4} in a unit bound box, which means the maximal value will be 2.3mm if the real length of this ship is 5m.

1.2 Multi-strip quasi-developable surfaces modeling

In some industrial fields, designers may need to design multi-strip developable surfaces whose adjacent strips share a same boundary like Fig.2(a). Although the single-strip optimization method can constrain that boundaries are close to given curves, the gap between seam-lines still leads to critical problems in some fields with large-size and high-precision. For example, in shipbuilding, this small gap between adjacent steel plates will make the welding more difficult and expensive. The chine of the result surface in Fig.2(c) is represented as two different B-spline curves actually. Those kind of curves will be mentioned as seam-lines in rest sections.

Traditionally, researchers eliminate the gap between seam-lines by interpolating to given boundary curves. However, there may not be quasi-developable surfaces meeting the requirement of developability between the given curve shape. Hence, the sample points in poor-developability regions on given boundaries have been modified before interpolating in previous works[Pérez and Suárez 2007][Zheng and Bo 2018a]. I also found that those points to be modified often appear in regions with large distance errors[Zheng and Bo 2018a]. Another problem is that the modification of seam-

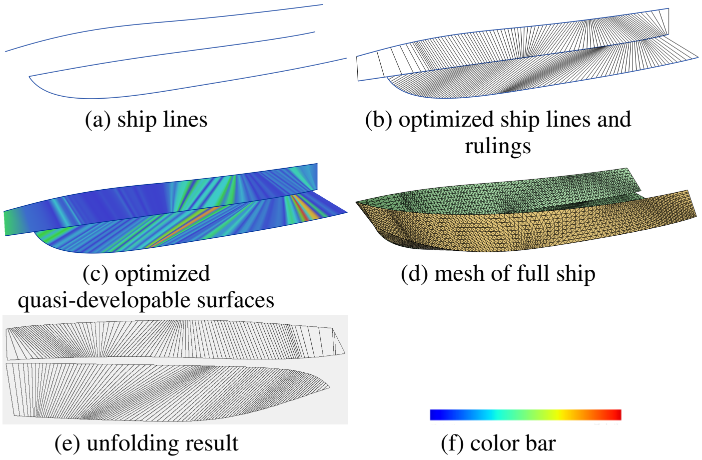

Figure 2: *An example of Hard Chine. (a) Input ship lines of half ship: sheer, chine and centreline respectively form top to bottom. (b) Ship lines and rulings after optimization. Note that only 100 of infinite rulings are drawn in this figure. (c) Optimized quasi-developable surfaces with developability colorization. (d) The triangular mesh of full ship after bow-cutting, centerline-seaming and meshing. (e) The result after unfolding to plane. (f) Color bar of developability. The maximal error is represented in red, while the minimal error is represented in blue.*

lines is influenced by two strips. So there may be still a gap after modification and interpolation. The reason of these problems is that the seam-line between two strips is represented by two curves. In order to eliminate the gap and ensure the developability of all strips, the most effective method is optimizing all strips together with one curve representing the seam-line between two adjacent strips. I implemented it in my resent work[Zheng and Bo 2018b], which has not been published. Please refer to the manuscript on my web site https://paulyzheng.github.io/paper/2018-02.pdf.

1.3 Toward ship hull design

To apply my above algorithms, I develop a small computer aided ship hull design system in my graduation project, which has been awarded one of the excellent graduation projects in my campus. Designers can use this system to design ship lines by dragging control points of B-spline curves handily and generate quasi-developable ship hulls using my optimization algorithm. After unfolding those hulls to plane, almost everyone can build their own ship according to the simple tutorial on https://www.wikihow.com/Build-a-Boat. I also implemented a cutting algorithm of symmetric ruled surfaces for bow-cutting [Zheng and Bo 2018b].

1.4 Cone spline surface fitting

In most industrial applications, the plane metal plate is rolled to be on-directional curved using three-roller-bending machine or four-roller-bending machine, which can roll sheet metal into conical surfaces with different radius according to the mathematical principle of three points determined a circle. Hence, for forming a developable or quasi-developable surface from plane plate, we need to divide it into many conical surfaces. The fewer the cone spline surfaces are, the higher the manufacturing efficiency is. To achieve this, we propose an approach which firstly generates a quasi-developable surface by using the method in [Zheng and Bo 2018a] and then computes a cone spline surface approximating the given target model[Zhang et al.]. Our method has the ability to divide the given surface to several cone regions automatically, while the method in [Leopoldseder and Pottmann 1998] needs manual division. Additionally, the boundary curves of given model can be arbitrary, yet they are constrained in two parallel planes in [Yang and Yang 2006].

2 Future Work

The modeling ability of developable surface is limited, as most of surfaces are doubly-curved. For example, although a ship hull can be entirely designed and manufactured using singly-curved surfaces, most ship hulls are designed and formed as doubly-curved surfaces for meeting some requirements in other fields like hydrodynamics and mechanics of materials. In terms of the fabrication of doubly-curved surfaces using sheet metal as materials, there are many forming methods, such as line heating[Zhang et al. 2005], stamping[Wang et al. 2015] and incremental forming[Dang et al. 2017]. But in the industrial fields using large-size metal plate like shipbuilding, line heating is the main method. There already is some methods for forming a doubly-curved surface only using line heating[Zhang et al. 2005]. However, line heating is expensive and time-consuming. So the plane plate is rolled to be singly-curved at first and then formed to be irregularly curved through line heating. Obviously, the process and workload of line heating will be affected by different rolling ways in the first step, which raises a interesting problem worth studying.

Developable surface is a special case of ruled surface without intersecting rulings between two boundaries and all points on a same ruling share a same tangent plane. So the curvature along a ruling direction of an arbitrary point on a developable surface must be one of two principle curvatures at that point and its value is zero. Therefore, the Gaussian curvature which is the production of two principle curvatures is also zero. As a result, researchers often chose Gaussian curvature as the standard of evaluating the developability of a surface[Pérez and Suárez 2007]. However, what we normally find between two given boundary curves are quasi-developable surfaces, which means there are some double-curved regions on obtained surfaces. The two principle curvatures may both be not equal to zero in those regions, which will lead to that the Gaussian curvature is related to the principle curvature not only along or nearly along the ruling direction, but also along the other bending direction. That makes the Gaussian curvature inappropriate to measure the developability of quasi-developable ruled surfaces. To ensure the developability unrelated to the degree of bending in the other direction, we use the warp angle θ of two normal vectors at the end points of each ruling to estimate the developable error on quasi-developable surfaces:

$$\theta((C_{1i} - C_{2i}) \times T_{1i}, (C_{1i} - C_{2i}) \times T_{2i})$$

where C_{1i} and C_{2i} are two end points of a ruling, T_{1i} and T_{2i} are tangent vector at C_{1i} and C_{2i} respectively. For a strict developable surface, the warp angle is zero at each ruling, as the two normal vectors at the end points are both perpendicular to a same tangent plane. In terms of quasi-developable surfaces, the smaller the angle is, the better the developability can be achieved. The warp angle shows more intuitive geometric feature of developable surfaces than Gaussian curvature. Mental sheet materials can stand a warp angle smaller than six degree according to [Oetter et al. 2002]. The thickness also has influence on the maximal warp angle the mental can stand, but a six degree angle is small enough for most aluminium alloys of standard thickness[Pérez and Suárez 2007]. While there is no such benchmark for Gaussian curvature. However, the warp angle is related to the thickness, material of plane sheet and the length of ruling. A detailed research should be done for analysing current standard measurement of developability of a surface and finding a better one.

Acknowledgements

Many thanks to Dr. Pengbo Bo.

References

CHEN, M., AND TANG, K. 2013. G^2 quasi-developable Bezier surface interpolation of two space curves. *Computer-Aided Design 45*, 11, 1365–1377.

CHEN, M., AND TANG, K. 2013. Quasi-developable surface modeling of contours with curved triangular patches. *Computers & Graphics 37*, 7, 851–861.

DANG, X., HE, K., LI, W., ZUO, Q., AND DU, R. 2017. Incremental bending of three-dimensional free form metal plates using minimum energy principle and model-less control. *Journal of Manufacturing Science and Engineering 139*, 7, 071009.

LEOPOLDSEDER, S., AND POTTMANN, H. 1998. Approximation of developable surfaces with cone spline surfaces. *Computer-Aided Design 30*, 7, 571–582.

OETTER, R., BARRY, C., DUFFY, B., AND WELTER, J. 2002. Block construction of small ship and boats through use of developable panels. *Journal of Ship Production 18*, 2, 6572.

PÉREZ, F., AND SUÁREZ, J. 2007. Quasi-developable B-spline surfaces in ship hull design. *Computer-Aided Design 39*, 10, 853–862.

WANG, C., JIAO, H., HU, Y., ZHANG, C., LI, J., FAN, Z., YUAN, P., MA, J., XIANG, Z., MAO, Y., AND LI, P. 2015. Curved surface forming device for adjustable segmented mold board with square rams. *U.S. Patent*, No. 8,939,754.

YANG, X., AND YANG, W. 2006. Cone spline approximation via fat conic spline fitting. *Computer-Aided Design 38*, 6, 703–712.

ZHANG, X., BO, P., ZHENG, Y., AND ZHANG, C. Cone spline surface fitting. *Accepted to Chinagraph 2018*.

ZHANG, X., JI, Z., AND LIU, Y. 2005. Curvature analysis method for ship-hull plate forming. *Journal of ship production 21*, 2, 65–72.

ZHENG, Y., AND BO, P. 2018. Quasi-developable surface construction based on boundary curve and its application in ship hull design. *Jounal of Computer-Aided Design & Computer Graphics 30*, 7, 1243–1250.

ZHENG, Y., AND BO, P. 2018. Quasi-developable surface modeling between boundaries. *Unpublished*.