

# BASIC PROBABILITY

The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

## EXPECTED VALUES

Trial

Observing an event and record the outcome

Experiment

Collection of one or multiple trials

Experimental Probability

Probability of an event based on the experiment

Expected Value

Specific outcome we expect to occur when we run an experiment

Categorical

$$E(X) = n \cdot p$$

Numeric

$$E(X) = \sum_{i=1}^n n_i \cdot p_i$$

## FREQUENCY

Probability frequency distribution

Collection of the probabilities for each possible outcome of an event.

Why?

To try and predict future events when the expected value is unattainable.

Frequency

Number of times a given value or outcome appears in the sample space.

Frequency distribution table

Table that matches each distinct outcome in the sample space to its associated frequency.

How?

By dividing every frequency by the size of the sample space.

## COMPLEMENTS

Everything an event is **not**

$$A' = \text{Not } A$$

$$P(A') = 1 - P(A)$$

$$A + A' = \text{Sample space}$$

# COMBINATORICS

The likelihood of an event occurring

$$P(X) = \frac{\text{preferred outcomes}}{\text{sample space}}$$

$$P(A, B) = P(A) + P(B)$$

Without  
repetition

## PERMUTATIONS (arrange)

With  
repetition

$$P_n = P_n^n = n!$$

How many ways are  
to arrange 3 letters a,  
b, c?

How many ways are  
to arrange 2 letters a  
and 2 letters b?

$$P_{n_1, \dots, n_k} = \frac{(\sum n_i)!}{\prod n_i!}$$

## VARIATIONS (pick and arrange)

$$V_p^n = P_p^n = \frac{n!}{(n-p)!}$$

How many words of 2  
different letters can  
you make with 4  
letters a, b, c, d?

How many words of 2  
different letters can  
you make with 4  
letters a, b, c, d?

$$\bar{V}_p^n = n^p$$

## COMBINATIONS (pick)

$$C_p^n = \frac{n!}{p!(n-p)!}$$

How many ways are  
there to pick 2  
different letters out  
of 4 letters a, b, c, d?

How many ways are  
there to pick 2 letters  
out of 4 letters a, b, c,  
d?

$$\bar{C}_p^n = C_p^{n+p-1}$$

## FACTORIALS

$$0! = 1$$

If  $n < 0$ ,  $n!$  doesn't exist

If  $n > 0, n > k$

$$(n+k)! = n! \cdot (n+1) \cdot \dots \cdot (n+k)$$

$$(n-k)! = \frac{n!}{(n-k+1) \cdot \dots \cdot n}$$

$$\frac{n!}{k!} = (k+1) \cdot \dots \cdot n$$

# BAYESIAN INFERENCE

$x \in A, x \notin A$ : element  $x$  is a part of set  $A$  ( $x$  NOT in  $A$ )

$A \ni x$ : set  $A$  contains element  $x$

$\forall x$ : for all/any  $x$

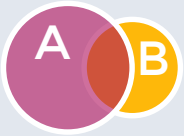
$A \subseteq B$ :  $A$  is a subset of  $B$



**Mutually exclusive**

$$A \cap B = \emptyset$$

All complements are mutually exclusive, but not all mutually exclusive sets are complements



**Intersect**

$$A \cap B$$

Satisfies all the events simultaneously

**Union**

$$A \cup B = A + B - A \cap B$$

Satisfies at least one of the events



**Completely overlap**

## Independent Events

$$P(A | B) = P(A)$$

Theoretically probability remains unaffected by other events

## Dependent Events

$$P(A | B) \neq P(A)$$

Probabilities of dependent events vary as conditions change

## Conditional Probability

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

- $B$  has occurred
- Only elements of the intersection can satisfy  $A$
- $P(A|B)$  not the same meaning as  $P(B|A)$

## Law of Total Probability

$$A = B_1 + \dots + B_n$$

$$P(A) = P(A|B_1) \cdot P(B_1) + \dots + P(A|B_n) \cdot P(B_n)$$

## Additive Law

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

## Multiplication Rule

$$P(A \cap B) = P(A|B) \cdot P(B)$$

## Bayes' Law

$$P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}$$

# DISTRIBUTIONS

Show the possible values a random variable can take and how frequently they occur.

- Y actual outcome
- Y one of the possible outcomes
- $P(Y = y) = p(y)$
- Probability function: function that assigns a probability to each distinct outcome in the sample space

**Mean**

**Variance**

**Standard Deviation**

**Population**

$\mu$

$\sigma^2$

$\sigma$

**Sample**

$\bar{x}$

$s^2$

$s$

## DISCRETE

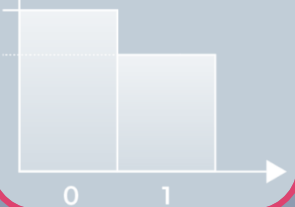
- Finite number of outcomes
- Can add up individual value to determine the probability of an interval
- Expressed with table, graph or piecewise function
- Expected values might be unattainable

### Uniform



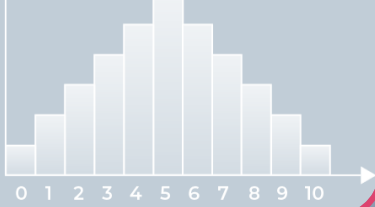
- $Y \sim U(a, b)$
- $Y \sim U(a)$  for categorical
- Outcomes are equally likely
- No predictive power

### Bernoulli



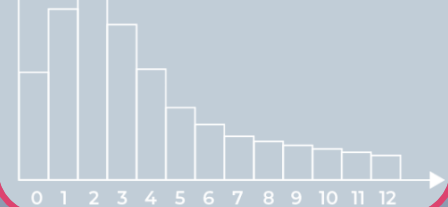
- $Y \sim \text{Bern}(p)$
- 1 trial, 2 possible outcomes
- $E(Y) = p$
- $\sigma^2(Y) = p \cdot (1-p)$

### Binomial



- $Y \sim B(n, p)$
- Measures the p 1 of the possible outcomes over n trials
- $P(Y=y) = p(y) = C(y, n) \cdot p^y \cdot (1-p)^{n-y}$
- $E(Y) = n \cdot p$
- $\sigma^2(Y) = n \cdot p \cdot (1-p)$

### Poisson



- $Y \sim \text{Po}(\lambda)$
- Measures the frequency over an interval of time or distance ( $\lambda \geq 0$ )
- $P(Y=y) = p(y) = \frac{\lambda^y}{y!} e^{-\lambda}$
- $E(Y) = \lambda$
- $\sigma^2(Y) = \lambda$

# DISTRIBUTIONS

- PDF: Probability Density Function
- CDF: Cumulative Density Function

## CONTINUOUS

- Infinitely many consecutive possible values
- Cannot add up individual value to determine the probability of an interval
- Expressed with graph or continuous function
- $P(Y=y) = p(y) = 0$  for any individual value  $y$  ( $P(Y < y) = P(Y \leq y)$ )

### Normal

- $Y \sim N(\mu, \sigma^2)$
- $E(Y) = \mu$
- $\sigma^2(Y) = \sigma^2$
- 68% of all values fall in the interval  $(\mu - \sigma, \mu + \sigma)$

- $Y \sim t(k)$
- Small sample size approximation of a Normal (accounts for extreme values better)
- If  $k > 1$ :  $E(Y) = \mu$  and  $\sigma^2(Y) = s^2 \cdot k / (k-2)$

### Students' T

### Chi-Squared

- $Y \sim \chi^2(\lambda)$
- Square of the t-distribution
- $E(Y) = k$
- $\sigma^2(Y) = 2k$

### Exponential

- $Y \sim \text{Exp}(\lambda)$
- $E(Y) = 1/\lambda$
- $\sigma^2(Y) = 1/\lambda^2$

### Logistic

- $Y \sim \text{Logistic}(\mu, s)$
- Continuous variable inputs and binary outcome
- CDF  $\uparrow$  when values near the mean
- $\downarrow s$ , the quicker it reaches values close to 1
- $E(Y) = \mu$
- $\sigma^2(Y) = s^2 \cdot \pi^2 / 3$