

# Fibonacci Numbers Pattern

This pattern covers problems where the answer for position  $n$  depends on the answers to **previous positions**—typically, the last one or two (sometimes more).

The classic example is the **Fibonacci sequence**:

$$F(n) = F(n-1) + F(n-2)$$

**Many dynamic programming problems fit this pattern:**

- Counting ways to climb stairs (each step can be 1 or 2 at a time)
- Tiling problems
- House thief/robber (can't rob adjacent houses)
- Jumping stairs with variable jumps

# Fibonacci Numbers Pattern: Fibonacci Sequence in Coding and DP

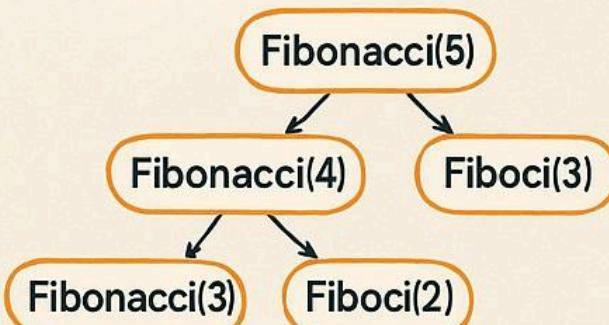
Given n, find n<sup>th</sup> Fibonacci number, where

$$F(0) = 0, F(1) = 1, F(n) = F(n-1) + F(n-2)$$

Example: n = 7, Output: 13

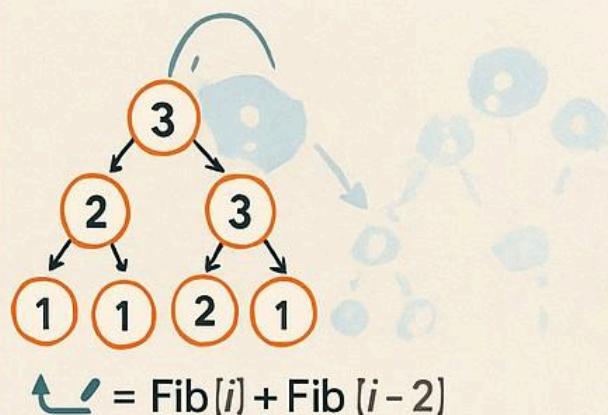
## 1. Naive Recursion

Break the problem into two smaller's subproblems



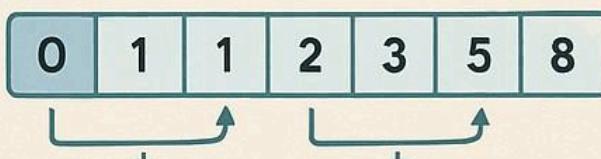
## 2. Efficient DP

Build up solutions and reuse them!



## 3. Efficient DP

Build up solutions and reuse them!



Bonus: This pattern appears in stair-climbing, tiling, and many DP problems where answers depend on previous results

The Fibonacci pattern teaches recursion, overlapping

## 4. Iterative DP

Just keep the last two numbers for max efficiency



BONUS: This pattern appears in stair-climbing, tiling, and many DP problems where answers depend on previous results!

$$\text{Fib}(7) = 13$$

## Classic Problem: Fibonacci Sequence

### Problem Statement:

Given  $n$ , compute the  $n$ th Fibonacci number ( $F(0) = 0$ ,  $F(1) = 1$ ,  $F(n) = F(n-1) + F(n-2)$ ).

## Explanation

### 1. Naive Recursive Solution:

Just call the function for  $n-1$  and  $n-2$ , and add the results.  
(But this is slow!  $O(2^n)$ )

### 2. Efficient Dynamic Programming (Bottom-Up):

- Use an array (or two variables) to keep track of already computed results.
- Build from 0 upwards to  $n$ .

## C Code Example: Iterative Fibonacci (Efficient)

```
#include <stdio.h>

int fibonacci(int n) {
    if (n == 0) return 0;
    if (n == 1) return 1;

    int prev2 = 0, prev1 = 1, curr;
    for (int i = 2; i <= n; i++) {
        curr = prev1 + prev2;
        prev2 = prev1;
        prev1 = curr;
    }
    return curr;
}

int main() {
    int n = 10;
    printf("Fibonacci number %d is %d\n", n, fibonacci(n));
    return 0;
}
```

## How does this work?

- Start with  $0$  and  $1$ .
- For each position up to  $n$ , compute the next number as the sum of the previous two.
- This is  $O(n)$  time and  $O(1)$  space.

## Where is this pattern useful?

- **Staircase Problems:**

Count ways to climb  $n$  stairs, taking 1 or 2 at a time.

- **House Robber/Theft:**

Maximum money if no two adjacent houses can be robbed.

- **Tiling:**  
Number of ways to tile a floor with tiles of certain lengths.
- **Anything where the answer depends on the last few solutions.**

## Practice Challenge

- Modify the code to **count the number of ways to climb n stairs** if you can take either 1 or 2 steps at a time.
- Try implementing the Fibonacci with **recursion + memoization** (top-down DP).