

Fibonacci Numbers Pattern

This pattern covers problems where the answer for position `n` depends on the answers to **previous positions**—typically, the last one or two (sometimes more).

The classic example is the **Fibonacci sequence**:

$$F(n) = F(n-1) + F(n-2)$$

Many dynamic programming problems fit this pattern:

- Counting ways to climb stairs (each step can be 1 or 2 at a time)
- Tiling problems
- House thief/robber (can't rob adjacent houses)
- Jumping stairs with variable jumps

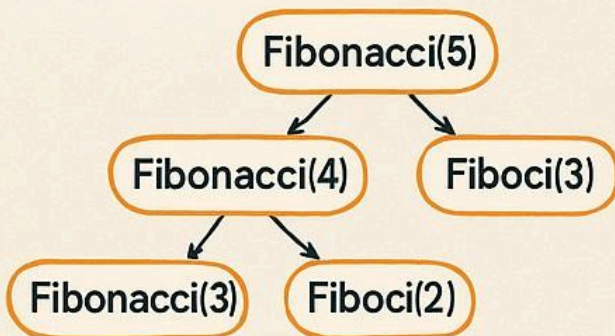
Fibonacci Numbers Pattern:

Fibonacci Sequence in Coding and DP

Given n, find nth Fibonacci number, where $F(0) = 0$, $F(1) = 1$, $F(n) = F(n-1) + F(n-2)$
Example: n = 7, Output; 13

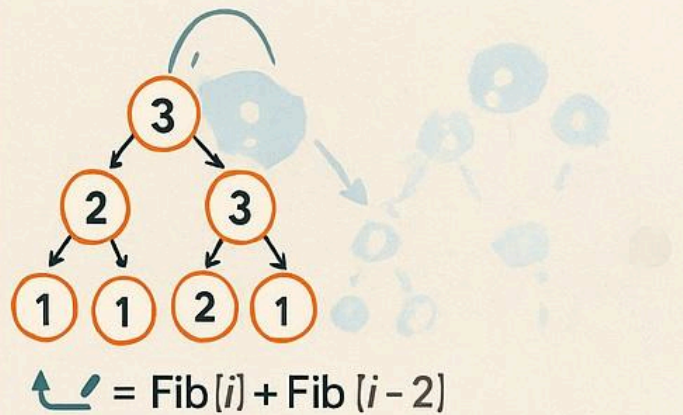
1. Naive Recursion

Break the problem into two smaller's subproblems



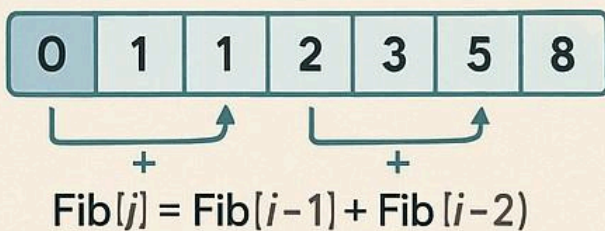
2. Efficient DP

Build up solutions and reuse them!



3. Efficient DP

Build up solutions and reuse them!

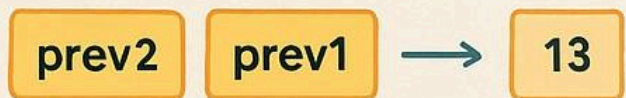


Bonus: This pattern appears in stair-climbing, tiling, and many DP problems where answers depend on previous results

The Fibonacci pattern teaches recursion, overlapping

4. Iterative DP

Just keep the last two numbers for max efficiency



BONUS: This pattern appears in stair-climbing, tiling, and many DP problems where answers depend on previous results!

Fib(7) = 13

Classic Problem: Fibonacci Sequence

Problem Statement:

Given n , compute the n th Fibonacci number ($F(0) = 0$, $F(1) = 1$, $F(n) = F(n-1) + F(n-2)$).

Explanation

1. Naive Recursive Solution:

Just call the function for $n-1$ and $n-2$, and add the results.

(But this is slow! $O(2^n)$)

2. Efficient Dynamic Programming (Bottom-Up):

- Use an array (or two variables) to keep track of already computed results.
- Build from 0 upwards to n .

C Code Example: Iterative Fibonacci (Efficient)

```
#include <stdio.h>
```

```
int fibonacci(int n) {  
    if (n == 0) return 0;  
    if (n == 1) return 1;  
  
    int prev2 = 0, prev1 = 1, curr;  
    for (int i = 2; i <= n; i++) {  
        curr = prev1 + prev2;  
        prev2 = prev1;  
        prev1 = curr;  
    }  
    return curr;  
}
```

```
int main() {  
    int n = 10;  
    printf("Fibonacci number %d is %d\n", n, fibonacci(n));  
    return 0;  
}
```

How does this work?

- Start with 0 and 1.
- For each position up to n , compute the next number as the sum of the previous two.
- This is $O(n)$ time and $O(1)$ space.

Where is this pattern useful?

• Staircase Problems:

Count ways to climb n stairs, taking 1 or 2 at a time.

• House Robber/Thief:

Maximum money if no two adjacent houses can be robbed.

- **Tiling:**
Number of ways to tile a floor with tiles of certain lengths.
- **Anything where the answer depends on the last few solutions.**

Practice Challenge

- Modify the code to **count the number of ways to climb n stairs** if you can take either 1 or 2 steps at a time.
- Try implementing the Fibonacci with **recursion + memoization** (top-down DP).