**Topics: Normal distribution, Functions of Random Variables**

1. The time required for servicing transmissions is normally distributed with *μ* = 45 minutes and *σ* = 8 minutes. The service manager plans to have work begin on the transmission of a customer’s car 10 minutes after the car is dropped off and the customer is told that the car will be ready within 1 hour from drop-off. What is the probability that the service manager cannot meet his commitment?
2. 0.3875
3. 0.2676
4. 0.5
5. 0.6987

Ans: 0.2676

Z = , 40, σ = 8 , X = 60 minutes.

Z = = = 1.875

The table value for z= 1.875 is approximately 0.6987

P = 1-0.6987 = 0.3013

Closest answer is 0.2676

1. The current age (in years) of 400 clerical employees at an insurance claims processing center is normally distributed with mean *μ* = 38 and Standard deviation *σ* =6. For each statement below, please specify True/False. If false, briefly explain why.
2. More employees at the processing center are older than 44 than between 38 and 44.

Ans: Z = , 38, σ = 6, X = 44

Z44 = = 1

Z38 = = 0

A Z-score of 0 corresponds to the 50th percentile. So, about 50% of employees are between 38 and 44 years old.

Since 84% is greater than 50%, it is true that more employees at the processing center are older than 44 than between 38 and 44. Therefore, the statement is **True.**

1. A training program for employees under the age of 30 at the center would be expected to attract about 36 employees.

Z = , 38, σ = 6, X = 30

Z38 = = -4/3 = 0.0912

Number of employees under 30 = 0.0912\* 400

Number of employees under 30= 36.48

Ans: True

1. If *X1* ~ *N*(μ, σ2) and *X*2 ~ *N*(μ, σ2) are *iid* normal random variables, then what is the difference between 2 *X*1 and *X*1 + *X*2? Discuss both their distributions and parameters.

Ans: X1 ~ N(μ, σ2)

X2 ~ N(μ, σ2) As X1 and X2 are independent,identically distributed(iid) normal random variables, by Central Limit Theorem,

X1 + X2∼ N(2μ,2σ^2)

And, 2X1 ~ N(2μ,4σ^2)

So, 2X1-(X1+X2) ~ N(2μ-2μ, 2σ^2+4σ^2 ) ~ (0,6σ^2)

2X1 distribution has a mean of 2μ and variance=4σ^2 .

X1 + X2 distribution has mean= 2μ and variance= 2σ^2 .

And their difference has mean =0 and variance= 6σ^2.

1. Let X ~ N(100, 202). Find two values, *a* and *b*, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99.
2. 90.5, 105.9
3. 80.2, 119.8
4. 22, 78
5. 48.5, 151.5
6. 90.1, 109.9

Ans: D: 48.5, 151.5

X ~ N(100, 202)

P(a<X<b) = 0.99

Z = = 2.576

X = + Z.

* a = μ - z \* σ = 100 - 2.576 \* 14.20 ≈ 48.48
* b = μ + z \* σ = 100 + 2.576 \* 14.20 ≈ 151.52
* Therefore, the two values a and b, symmetric about the mean, such that the probability of the random variable taking a value between them is 0.99 are:a ≈ 48.48 and b ≈ 151.52

1. Consider a company that has two different divisions. The annual profits from the two divisions are independent and have distributions Profit1 ~ N(5, 32) and Profit2 ~ N(7, 42) respectively. Both the profits are in $ Million. Answer the following questions about the total profit of the company in Rupees. Assume that $1 = Rs. 45
2. Specify a Rupee range (centered on the mean) such that it contains 95% probability for the annual profit of the company.

Ans: 99 million to 982 million

1. Specify the 5th percentile of profit (in Rupees) for the company

Ans: 5th percentile of profit (in Rupees) for the company = 168.75

1. Which of the two divisions has a larger probability of making a loss in a given year?

Ans: Probability of loss making is greater for division 1.