Solution to IIT JEE 2019 (Advanced) : Paper - I

PART I – PHYSICS

PAPER –1 : INSTRUCTIONS TO CANDIDATES

- **Question Paper–1 has three (03) parts : Physics, Chemistry and Mathematics.**
- **Each part has a total eighteen (18) questions divided into three (03) sections (Section–1, Section–2 and Section–3)**
- **Total number of questions in Question Paper-1 are Fifty Four (54) and Maximum Marks are One Hundred Eighty Six (186)**

Type of Questions and Marking Scheme

SECTION 1 (Maximum Marks:12)

- This section contains FOUR **(04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	**+3** If ONLY the correct option is chosen.
Zero Marks	:	**0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** In all other cases.

SECTION 2 (Maximum Marks : 32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** in all other cases

SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	**+3** If ONLY the correct numerical value is entered.
Zero Marks	:	**0** In all other cases.

Answering Questions :

- To select the option(s), use the mouse to click on the corresponding button(s) of the option(s).
- To deselect the chosen option for the questions of SECTION−1, click on the button of the chosen option again or click on the Clear Response button to clear the chosen option.
- To deselect the chosen option(s) for the questions of SECTION−2, click on the button(s) of the chosen option(s) again or click on the Clear Response button to clear all the chosen options.
- To change the option(s) of a previously answered question of SECTION−1 and SECTION−2 first deselect as given above and then select the new option(s).
- To answer questions of SECTION−3, use the mouse to click on numbers (and/or symbols) on the on–screen virtual numeric keypad to enter the numerical value in the space provided for answer.
- To change the answer of a question of SECTION−3, first click on the Clear Response button to clear the entered answer and then enter the new numerical value.
- To mark a question ONLY for review (i.e. without answering it), click on the Mark for Review & Next button.
- To mark a question for review (after answering it), click on Mark for Review & Next button – the answered question which is also marked for review will be evaluated.
- To save the answer, click on the Save & Next button – the answered question will be evaluated.

PART I – PHYSICS

SECTION 1 (Maximum Marks:12)

- This section contains FOUR **(04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
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  Negative Marks : **–1** In all other cases.

1. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with small area $\alpha 4\pi R^2$ ($\alpha << 1$) is made on the shell without affecting the rest of the shell. Which one of the following statement is correct.

(A) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}R$ from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

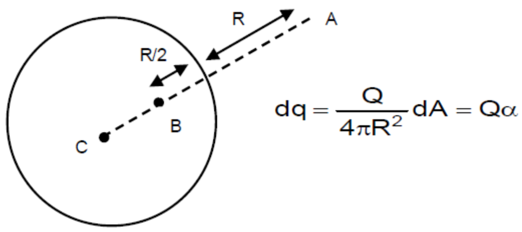
(B) The potential at the centre of shell is reduced by $2\alpha V_0$

(C) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(D) The magnitude of electric field at a point, located on a line passing thorugh the hole and shell's center, on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

1. **(A)**

Potential at surface, $V_0 = \frac{KQ}{R}$



Potential at C

$$V_C = \frac{KQ}{R} - \frac{K\alpha Q}{R} = V_0(1-\alpha)$$

Potential at B

$$V_B = \frac{KQ}{R} - \frac{K(\alpha Q)}{R/2} = V_0(1-2\alpha)$$

$$\therefore \quad \frac{V_C}{V_B} = \frac{1-\alpha}{1-2\alpha}$$

Electric field at A

$$E_A = \frac{KQ}{(2R)^2} - \frac{K\alpha Q}{R^2} = \frac{KQ}{4R^2} - \frac{\alpha V_0}{R}$$

So reduced by $\frac{\alpha V_0}{R}$

Electric field at C

$$E_C = \frac{K(\alpha Q)}{R^2} = \frac{\alpha V_0}{R}$$

So increased by $\frac{\alpha V_0}{R}$

2. In a radioactive sample $^{40}_{19}\text{Ar}$ nuclei either decay into stable $^{40}_{20}\text{Ca}$ nuclei with decay constant 4.5×10^{-10} per year or into stable $^{40}_{18}\text{Ar}$ nuclei with decay constant 0.5×10^{-10} per year. Given that in this sample all the stable $^{40}_{20}\text{Ca}$ and $^{40}_{18}\text{Ar}$ nuclei are produced by the $^{40}_{19}\text{K}$ nuclei only. In time $t \times 10^9$ years, if the ratio of the sum of stable $^{40}_{20}\text{Ca}$ and $^{40}_{18}\text{Ar}$ nuclei to the radioactive $^{40}_{19}\text{Ar}$ nuclei is 99, the value of t will be [Given ln 10 = 2.3]

(A) 9.2 (B) 1.15 (C) 4.6 (D) 2.3

2. (A)

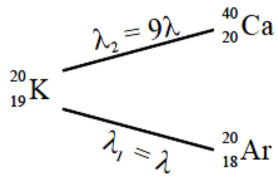
Parallel radioactive decay. So equivalent decay constant

$\lambda = \lambda_1 + \lambda_2 = 5 \times 10^{-10}$ per year

$N = N_0 e^{-\lambda t}$

$N_0 - N = N_{\text{stable}}$

$N = N_{\text{radioactive}}$



$$\frac{N_0}{N} - 1 = 99$$

$$\frac{N_0}{N} = 100$$

$$\frac{N}{N_0} = e^{-\lambda t} = \frac{1}{100} \quad \Rightarrow \lambda t = 2\,\ell n10 = 4.6$$

$t = 9.2 \times 10^9$ years

3. A current carrying wire heats a metal rod. The wire provides a constant power (P) to the rod. The metal rod is enclosed in an insulated container. It is observed that the temperature (T) in the metal rod changes with time (t) as

$T(t) = T_0(\,1 + \beta t^{1/4})$

where β is a constant with appropriate dimension while T_0 is a constant with dimension of temperature. The heat capacity of metal is :

(A) $\dfrac{4P\left(T(t)-T_0\right)^4}{\beta^4 T_0^5}$ (B) $\dfrac{4P\left(T(t)-T_0\right)^3}{\beta^4 T_0^4}$

(C) $\dfrac{4P\left(T(t)-T_0\right)}{\beta^4 T_0^2}$ (D) $\dfrac{4P\left(T(t)-T_0\right)^2}{\beta^4 T_0^3}$

3. (B)

Given, $P = \dfrac{dQ}{dt}$ and $T(t) = T_0(1+\beta t^{1/4})$

At equilibrium, $C\dfrac{dT}{dt} = P$

$$\frac{dT}{dt} = \frac{T_0\beta}{4} t^{-\frac{3}{4}}$$

So heat capacity $C = \dfrac{4P}{\beta T_0} t^{\frac{3}{4}}$

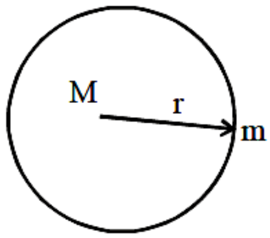
From the given equation $\dfrac{T(t)-T_0}{\beta T_0} = t^{\frac{1}{4}}$

So, $t^{\frac{3}{4}} = \dfrac{(T(t)-T_0)^3}{\beta^3 T_0^3}$. So, $C = \dfrac{4P}{\beta^4 T_0^4}(T(t)-T_0)^3$

4. Consider a spherical gaseous cloud of mass density ρ(r) in a free space where r is the radial distance from its centre. The gaseous cloud is made of particles of equal mass m moving in circular orbits about their common centre with the same kinetic energy K. The force acting on the particles is their mutual gravitational force. If ρ(r) is constant in time. The particle number density n(r) = ρ(r)/m : (G = universal gravitational constant)

(A) $\dfrac{K}{6\pi r^2 m^2 G}$ (B) $\dfrac{K}{\pi r^2 m^2 G}$ (C) $\dfrac{3K}{\pi r^2 m^2 G}$ (D) $\dfrac{K}{2\pi r^2 m^2 G}$

4. (D)
Let total mass included in a sphere of radius r be M.
For a particle of mass m,



$$\frac{GMm}{r^2} = \frac{mv^2}{r}$$

$$\Rightarrow \frac{GMm}{r} = 2K \quad \Rightarrow M = \frac{2Kr}{Gm}$$

$$\therefore \quad dM = \frac{2Kdr}{Gm}$$

$$\Rightarrow (4\pi r^2 dr)\rho = \frac{2Kdr}{Gm} \quad \Rightarrow \rho = \frac{K}{2\pi r^2 Gm}$$

$$\therefore \quad n = \frac{\rho}{m} = \frac{K}{2\pi r^2 m^2 G}$$

SECTION 2 (Maximum Marks:32)

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Negative Marks	:	**–1** in all other cases

1. Let us consider a system of units in which mass and angular momentum are dimensionless. If length has dimension of L, which of the following in statement(s) is/are correct?
(A) The dimension of force is L^{-3}
(B) The dimension of power is L^{-5}
(C) The dimension of energy is L^{-2}
(D) The dimension of linear momentum is L^{-1}

1. (A), (C), (D)
$[M^0L^0T^0] = [ML^2T^{-1}] \Rightarrow [L^2] = [T]$
$[\text{Energy}] = [MLT^{-2}L] = [L^{-2}]$
$[\text{Force}] = [MLT^{-2}] = [L^{-3}]$
$[\text{Power}] = [MLT^{-2}LT^{-1}] = [L^{-4}]$
$[\text{Linear momentum}] = [MLT^{-1}] = [L^{-1}]$

2. Two identical moving coil galvanometers have 10 Ω resistance and full scale deflection at 2μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into a Ammeter of 1mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R=1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ?

(A) The resistance of the Voltmeter will be 100 k Ω

(B) The resistance of the Ammeter will be 0.02 Ω (round off to 2nd decimal place)

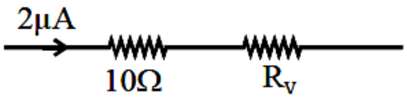
(C) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω

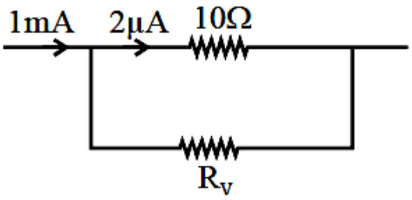
(D) The measured value of R will be 978 Ω < R < 982 Ω

2. (B), (D)

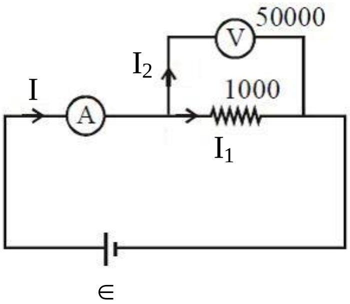
$0.1 = 2 \times 10^{-6} (10 + R_V)$

$\therefore \; R_V = 49990 \, \Omega$





$2 \times 10^{-6} \times 10 = 10^{-3} R_A \quad \therefore \; R_A = 0.02 \, \Omega$



So, $I_2 \, 50000 = (I - I_2) \, 1000$

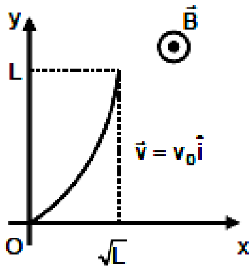
$\therefore \; 51 I_2 = I$

$\therefore \; \text{Reading} = \dfrac{I_2 50000}{I} = 980$

$I = \dfrac{51 \in}{5 \times 10^4} \; (\because R_A \to 0)$

$I_1 = \dfrac{\in}{1000}$

3. A conducting wire of parabolic shape, $y = x^2$, is moving with velocity $\vec{V} = V_0 \hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0 \left[1 + \left(\dfrac{y}{L} \right)^{\beta} \right] \hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and Δϕ is the potential difference developed between the ends of the wire, then the correct statements(s) is/are :



(A) $|\Delta\phi| = \dfrac{4}{3} B_0 V_0 L$ for $\beta = 2$

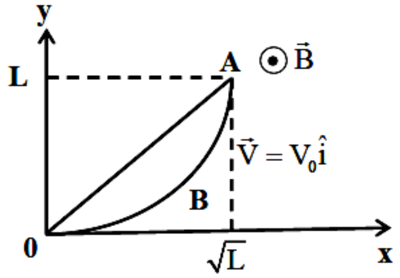
(B) $|\Delta\phi|$ remains same if the parabolic wire is replaced by a straight wire, y = x , initially, of length $\sqrt{2}\ell$

(C) $|\Delta\phi| = \dfrac{1}{2} B_0 V_0 L$ for $\beta = 0$

(D) $|\Delta\phi|$ is proportional to the length of wire projected on y–axis

3. (A), (B), (D)

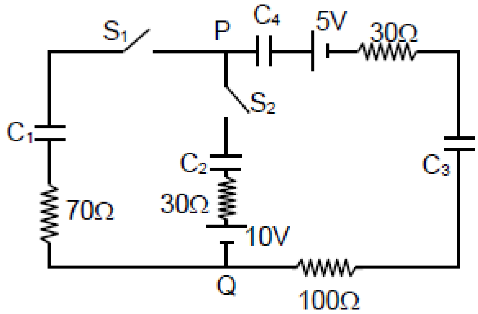
These is no change in flux through the loop OABO due to the movement of loop. So potential difference developed in curved wire and the straight wire OA is same.



For $\beta = 0$, $|\Delta\phi| = 2B_0V_0L$

For $\beta = 2$, $|\Delta\phi| = \int_0^L B_0\left(1+\frac{y^2}{L^2}\right)V_0 dy = \frac{4}{3}B_0V_0L$

4. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\mu F$, $C_2 = 30\mu F$, and $C_3 = C_4 = 80\ \mu F$. Which statements is/are correct :



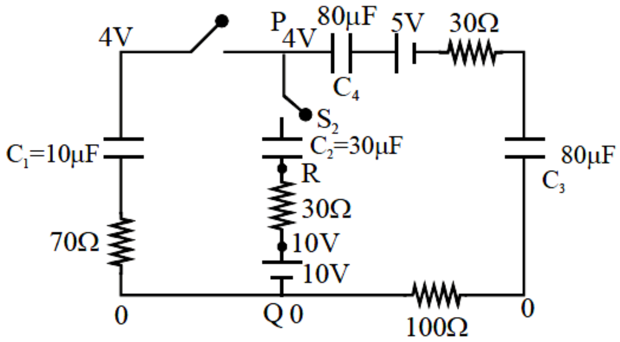
(A) The key S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time the instantaneous current across 30Ω resistor (between points P & Q) will be 0.2A round off to 1st decimal place).

(B) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across C_1 will be 4V.

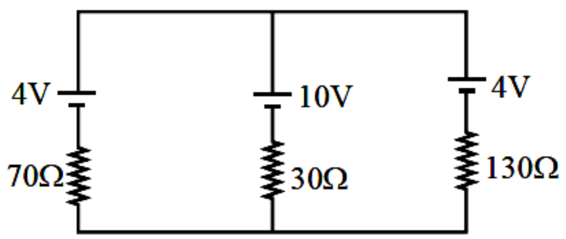
(C) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA

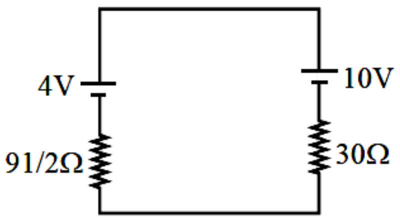
(D) If S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between P and Q will be 10V.

4. (B), (C)



When S_2 is closed at $t = 0^+$, capacitor C_1 acts as a battery of 4V, C_4 and C_3 of $\frac{1}{2}$V each, C_2 is shorted circuit will look like



$\Rightarrow$ 

$\Rightarrow i = \dfrac{6}{30 + \dfrac{91}{2}} = 0.079$ A

At steady state,
When capacitor is fully charged it behave as open circuit and current through it zero.
Hence, charge on each capacitor is same.
$Q = C_{eq} V = (8\ \mu F) \times 5$
$Q = 40\ \mu C$
Now, $V_p - \dfrac{40}{10} = V_Q$
$V_P - V_Q = 4V$
At t = 0, when S_1 is closed, capacitor act as short circuit.
$i = \dfrac{V}{R_{eq}} = \dfrac{5}{200} = 25$ mA

5. A charged shell of radius R carries a total charge Q. Given ϕ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here center of cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is are correct [ε_0 is the permittivity of free space]

(A) If $h > 2R$ and $r = \dfrac{4R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

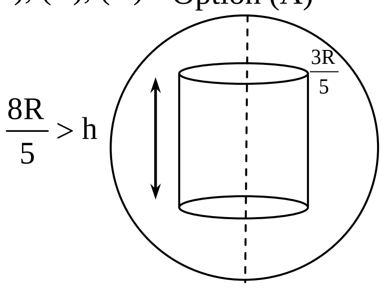
(B) If $h > 2R$ and $r = \dfrac{3R}{5}$ then $\phi = \dfrac{Q}{5\varepsilon_0}$

(C) If $h < \dfrac{8R}{5}$ and $r = \dfrac{3R}{5}$ then $\phi = 0$

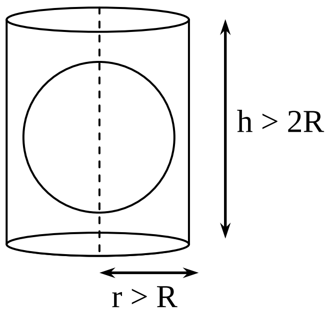
(D) If $h > 2R$ and $r > R$ then $\phi = \dfrac{Q}{\varepsilon_0}$

5. (B), (C), (D)

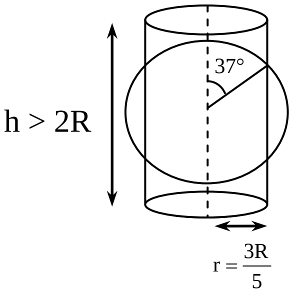
Option (A)



Option (B)

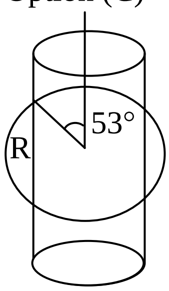


Option (D)



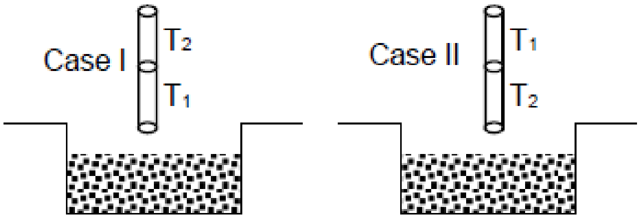
$\phi = \dfrac{Q}{4\pi\varepsilon_0}(2)2\pi(1-\cos 37°)$

Option (C)



$\phi = \dfrac{2Q}{2\varepsilon_0}(1-\cos 53°) = \dfrac{2Q}{5\varepsilon_0}$

6. A cylindrical capillary tube of 0.2 mm radius is made by joining two capillaries T_1 and T_2 of different materials having water contact angles of 0° and 60° respectively. The capillary tube is dipped vertically in water in two different configurations, case I and II as shown in figure. Which of the following option(s) is (are) correct?
[Surface tension of water = 0.075N/m, density of water = 1000 kg/m^3, take g = 10 m/s^2]



(A) For case I, if the joint is kept at 8 cm above the water surface, the height of water column in the tube will be 7.5 cm. (Neglect the weight of the water in the meniscus)
(B) For case I, capillary joint is 5cm above the water surface, the height of water column raised in the tube will be more than 8.75 cm. (Neglect the weight of the water in the meniscus)
(C) The correction in the height of water column raised in the tube, due weight of water contained in the meniscus, will be different for both cases.
(D) For case II, the capillary joint is 5 cm above the water surface, the height of water column raised in the tube will be 3.75 cm. (Neglect the weight of the water in the meniscus)

6. (A), (C), (D)

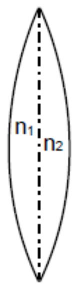
When T_1 is in contact with water, then h = $\frac{2T\cos\theta_1}{r\rho g}$ = 7.5 cm < 8 cm

But in option (B) height is insufficient.

When T_2 is in contact with water, then h = $\frac{2T\cos\theta_2}{r\rho g}$ = 3.75 cm < 5 cm

Since Angle of contact is different so correction in height will be different in both cases. Volume of water in the meniscus depends upon the angle of contact.'

7. A thin convex lens is made of two materials with refractive indices n_1 and n_2, as shown in figure. The radius of curvature of the left and right spherical surfaces are equal f is the focal length of the lens when $n_1 = n_2 = n$. The focal length is f +Δf when $n_1 = n$ and $n_2 = n + \Delta n$. Assuming $\Delta n << (n – 1)$ and $(1 < n < 2)$, the correct statement(s) is/are



(A) If $\frac{\Delta n}{n} < 0$ then $\frac{\Delta f}{f} > 0$

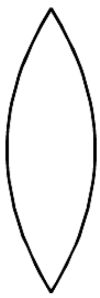
(B) For n = 1.5, Δn = 10^{-3} and f = 20 cm, the value of | Δf | will be 0.02 cm (round off to 2nd decimal place).

(C) $\left|\frac{\Delta f}{f}\right| < \left|\frac{\Delta n}{n}\right|$

(D) The relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ remains unchanged if both the convex surfaces are replaced by concave surfaces of the same radius of curvature.

7. (A), (B), (D)

When $n_1 = n_2 = n$



$$\frac{1}{f} = (n-1)\times\frac{2}{R}$$

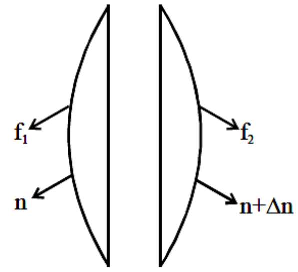
So, $f = \frac{R}{2(n-1)}$ … (1)

Second case :

$$\frac{1}{f_1} = \frac{n-1}{R}$$

$$\frac{1}{f_2} = \frac{(n+\Delta n)-1}{R}$$

$$\frac{1}{f_{eq}} = \frac{1}{f+\Delta f} = \left(\frac{n-1}{R}\right) + \frac{(n+\Delta n)-1}{R} = \frac{2(n-1)+\Delta n}{R}$$



$$\Delta f = \left(\frac{R}{2(n-1)+\Delta n}\right) - \left(\frac{R}{2(n-1)}\right)$$

$$= \frac{R}{2}\left[\frac{(n-1)-(n-1+\Delta n)}{(n-1+\Delta n)(n-1)}\right] = \frac{-\Delta n}{(n-1)^2} \times \frac{R}{2}$$

$$\frac{\Delta f}{f} = -\frac{\Delta n}{2(n-1)} \qquad \ldots\ (2)$$

Relation between $\frac{\Delta f}{f}$ and $\frac{\Delta n}{n}$ is independent of R

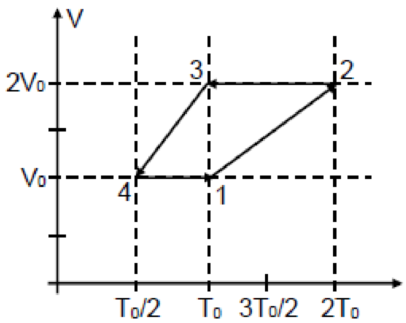
2n – 2 < n because n < 2

$$\Rightarrow \frac{\Delta f}{f} = \frac{1}{2}\left|\frac{\Delta n}{n-1}\right| > \frac{\Delta n}{n}. \text{ So, } \frac{\Delta f}{f} > \left|\frac{\Delta n}{n}\right|$$

Now, $|\Delta f| = \frac{f\Delta n}{(n-1)} = \frac{(20\times10^{-3})}{1.5-1} = 40 \times 10^{-3} = 0.04$

If $\frac{\Delta n}{n} < 0$, then $\frac{\Delta f}{f} > 0$ from equation (2).

8. One mole of a monatomic ideal gas goes through a thermodynamic cycle, as shown in the volume versus temperature (V-T) diagram. The correct statement(s) is/are : [R is the gas constant]



(A) Work done in this thermodynamic cycle

$(1 \to 2 \to 3 \to 4 \to 1)$ is $|W| = \frac{1}{2}\ RT_0$

(B) The ratio of heat transfer during processes

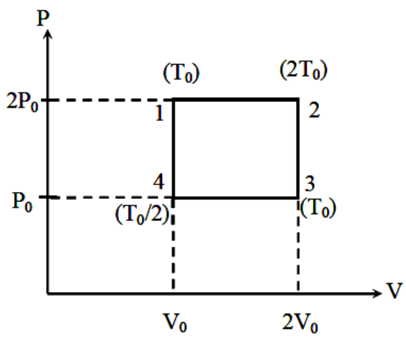
$1 \to 2$ and $2 \to 3$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{5}{3}$

(C) The above thermodynamic cycle exhibits only isochoric and adiabatic processes.

(D) The ratio of heat transfer during processes $1 \to 2$ and $3 \to 4$ is $\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \frac{1}{2}$

8. (A), (B)

From P–V diagram



$$W_{cycle} = P_0V_0 = \frac{RT_0}{2}$$

$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{{}_nC_P(T_2-T_1)}{{}_nC_V(T_3-T_2)}\right| = \left|-\frac{5}{3}\right| = \frac{5}{3}$$

$$\left|\frac{Q_{1\to2}}{Q_{2\to3}}\right| = \left|\frac{{}_nC_P(T_2-T_1)}{{}_nC_V(T_4-T_3)}\right| = 2$$

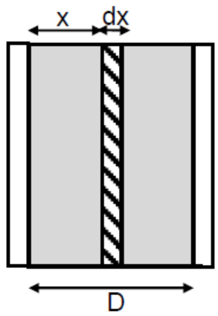
SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : **+3** If ONLY the correct numerical value is entered.
  Zero Marks : **0** In all other cases.

1. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the mth s layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N(> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d\,\ell n\, 2}\right)$. The value of α will be _____ . [$\in_0$ is the permittivity of free space]

1. [1]

$$K_m = K\left(1+\frac{m}{N}\right) = K\left(1+\frac{x}{D}\right)$$



$$\frac{x}{m} = \frac{D}{N}$$

$$d\left(\frac{1}{C}\right) = \frac{dx}{K_m\varepsilon_0 A} = \frac{dx}{K\varepsilon_0 A\left(1+\frac{x}{D}\right)}$$

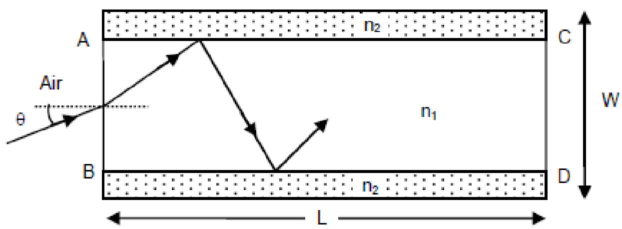
$$\frac{1}{C_{eq}} = \int d\left(\frac{1}{C}\right) = \int_0^D \frac{Ddx}{K\varepsilon_0 A(D+x)}$$

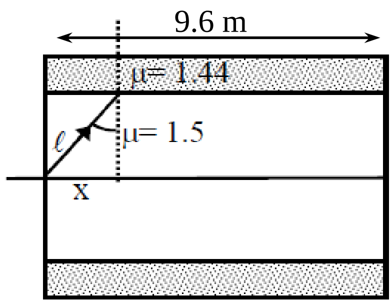
$$\frac{1}{C_{eq}} = \frac{D}{K\varepsilon_0 A}\ell n2$$

$C_{eq} = \frac{K\varepsilon_0 A}{D\ell n2}$. Therefore $\alpha = 1$.

2. A planar structure of length L and width W is made of two different optical media of refractive indices $n_1 = 1.5$ and $n_2 = 1.44$ as shown in figure. If L >> W, a ray entering from end AB will emerge from end CD only if the total internal reflection condition is met inside the structure. For L = 9.6 m, if the incident angle θ is varied, the maximum time taken by a ray to exit the plane CD is $t \times 10^{-9}$ s, where t is _____ .
[Speed of light $c = 3 \times 10^8$ m/s]

2. [50]





$1.5 \sin \theta_C = 1.44 \sin 90°$

$\sin \theta_C = \dfrac{24}{25}$

$\ell = \dfrac{x}{\sin \theta_C} = \dfrac{25}{4}x$

Total length for light to travel

$\ell' = \dfrac{25}{4} \times 9.6 = 10$ m

$\therefore$ time $= \dfrac{\ell'}{C/1.5} = 5 \times 10^{-8}$ s $\Rightarrow 50 \times 10^{-9}$ s

t = 50

3. A liquid at 30° C is poured very slowly into a Calorimeter that is at temperature of 110°C. The boiling temperature of the liquid is 80°C. It is found that the first 5 gm of the liquid completely evaporates. After pouring another 80 gm of the liquid the equilibrium temperature is found to be 50°C. The ratio of the Latent heat of the liquid to its specific heat will be _____ °C. (Neglect the heat exchange with surrounding]

3. [270]

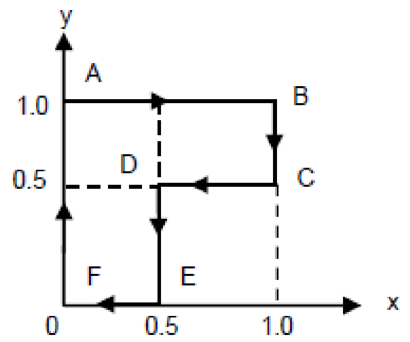
5(s)(50) + 5L = C(30) … (i)

80(s) (20) = C(30) … (ii)

$\therefore$ from (i) and (ii)

$\dfrac{L}{S} = 270°C$ $\therefore$ 270.00

4. A particle is moved along a path AB–BC–CD–DE–EF–FA, as shown in figure in presence of a force $\vec{F} = (\alpha y\hat{i} + 2\alpha x\hat{j})$ N, where x and y are in meter and α= = –1 Nm^{-1}. The work done on the particle by this force $\vec{F}$ will be _____ joule



4. [0.75]

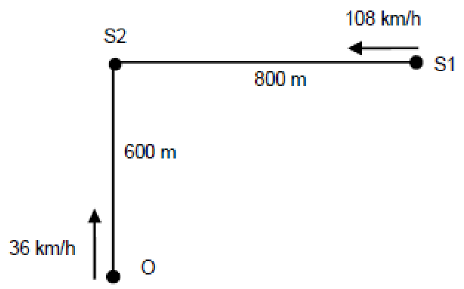
When α = –1

$W_{AB} = \int_0^1 \alpha y dx = -1;$ $W_{BC} = \int_1^{0.5} 2\alpha x dy = +1$

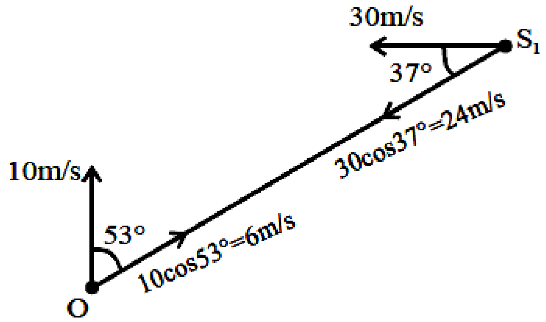
$W_{CD} = \int_1^{0.5} \alpha y dx = +0.25;$ $W_{DE} = \int_{0.5}^{0} 2\alpha x dy = +0.5$

$W_{EF} = W_{FA} = 0;$ $W_{net} = 0.75$

5. A train S1, moving with a uniform velocity of 108 km/h, approaches another train S2 standing on a platform. An observer O moves with a uniform velocity of 36 km/h towards S2, as shown in figure. Both the trains are blowing whistles of same frequency 120 Hz. When O is 600 m away from S2 and distance between S1 and S2 is 800 m, the number of beats heard by O is ______ .
(Speed of the sound = 330 m/s)



5. [8.128]



Frequency observed by O from S_2

$$f_2 = \frac{330+10}{330}\times 120 = \frac{340}{330}\times 120 = 123.63 \text{ Hz}$$

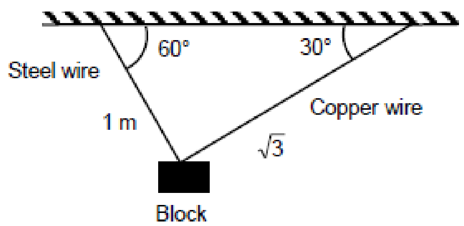
frequency observed by O from S_1

$$f_1 = \frac{330+6}{330-24}\times 120 = \frac{336}{306}\times 120 \approx 131.76 \text{ Hz}$$

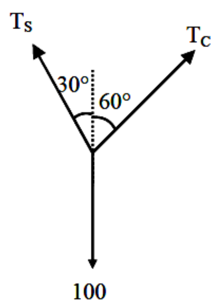
beat frequency = 131.76 – 123.63 = 8.128

6. A block of weight 100 N is suspended by copper and steel wires of same cross sectional area 0.5 cm^2 and, length $\sqrt{3}$ m and 1 m, respectively. Their other ends are fixed on a ceiling as shown in figure. The angles subtended by copper and steel wires with ceiling are 30° and 60°, respectively. If elongation in copper wire is $(\Delta\ell_c)$ and elongation in steel wire is $(\Delta\ell_s)$, then the ratio $\frac{\Delta\ell_c}{\Delta\ell_s}$ is ___ .

(Young's modulus for copper and steel are 1 × 10^{11} N/m^2 and 2 × 10^{11} N/m^2 , respectively)



6. [2]



$T_S \sin 30° = T_C \sin 60°$

$$\frac{\Delta\ell_C}{\Delta\ell_S} = \frac{T_C\ell_C}{A_CY_C}\left(\frac{A_SY_S}{T_S\ell_S}\right) = 2.00 \qquad [\because A_C = A_S]$$

❑ ❑

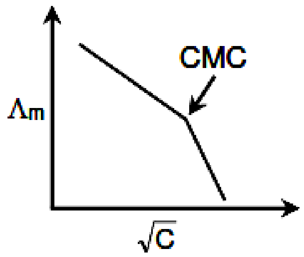
PART II : CHEMISTRY

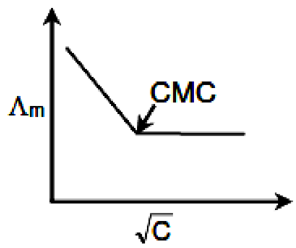
SECTION 1 (Maximum Marks : 12)

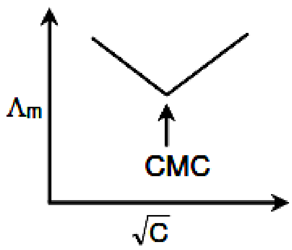
- This section contains FOUR **(04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :

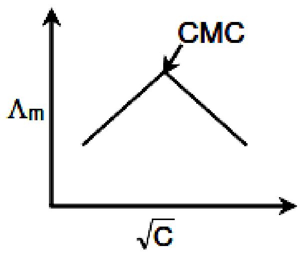
Full Marks	:	**+3** If ONLY the correct option is chosen.
Zero Marks	:	**0** If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** In all other cases.

1. Molar conductivity (Λ_m) of aqueous solution of sodium stearate, which behaves as a strong electrolyte is recorded at varying concentrations (c) of sodium stearate. Which one of the following plots provides the correct representation of micelle formation in the solution ? (critical micelle concentration (CMC) is marked with an arrow in the figures)

(A)


(B)


(C)


(D)


1. (A)

As the concentration of sodium stearate increases beyond CMC, stearate ions get clubbed together and form micelles. This abruptly causes the concentration of the current carrier anions to decreases. This is reflected by the sharp change in Λ_m at CMC, followed by greater rate of decrease of Λ_m with $\sqrt{C}$.

2. The green colour produced in the borax bead test of a chromium (III) salt is due to

(A) CrB (B) Cr_2O_3 (C) $Cr(BO_2)_3$ (D) $Cr_2(B_4O_7)_3$

2. (C)

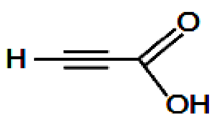
$$\text{Chromium (III) salt} \xrightarrow{\Delta} Cr_2O_3$$

$$\text{Borax} \xrightarrow{\Delta} B_2O_3 + NaBO_2$$

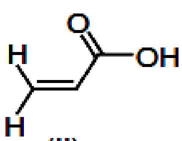
$$2Cr_2O_3 + 6B_2O_3 \longrightarrow 4Cr(BO_2)_3$$

So correct answer is option (C)

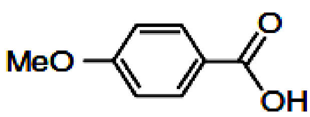
3. The correct order of acid strength of the following carboxylic acids is :



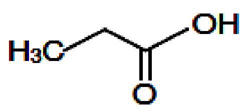
(I)



(II)



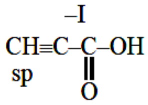
(III)



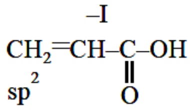
(IV)

(A) I > II > III > IV (B) II > I > IV > III
(C) I > III > II > IV (D) III > II > I > IV

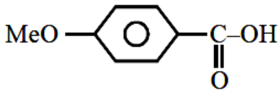
3. (A)
I > II > III > IV



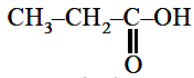
(I)
(pKa value 1.86)



(II)
(pKa value 4.3)



(III)
(pKa value 4.5)



(IV)
(pKa value 4.88)

4. Calamine, malachite, magnetite and cryolite, respectively, are
(A) $ZnSO_4$, $Cu(OH)_2$, Fe_3O_4, Na_3AlF_6
(B) $ZnCO_3$, $CuCO_3$, Fe_2O_3, Na_3AlF_6
(C) $ZnSO_4$, $CuCO_3$, Fe_2O_3, AlF_3
(D) $ZnCO_3$, $CuCO_3.Cu(OH)_2$, Fe_3O_4, Na_3AlF_6

4. (D)
Compound – Formula
Calamine – $ZnCO_3$
Malachite – $CuCO_3$. $Cu(OH)_2$
Magnetite – Fe_3O_4
Cryolite – Na_3AlF_6

SECTION 2 (Maximum Marks:32)

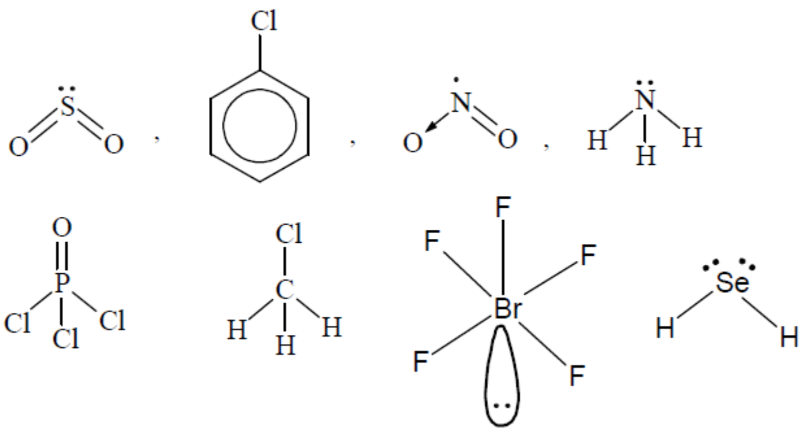
- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** In all other cases.

1. Each of the following options contains a set of four molecules, Identify the option(s) where all four molecules passes permanent dipole moment at room temperature.
(A) NO_2, NH_3, $POCl_3$, CH_3Cl (B) BF_3, O_3, SF_6, XeF_6
(C) $BeCl_2$, CO_2, BCl_3, $CHCl_3$ (D) SO_2, C_6H_5Cl, H_2Se, BrF_5

1. (A), (D)

Polar Molecule	Non–polar molecule
$CHCl_3$, SO_2, C_6H_5Cl,	$BeCl_2$, CO_2, BCl_3, SF_6
H_2Se, BrF_5, O_3, XeF_6,	
NO_2, NH_3, $POCl_3$, CH_3Cl	



2. Which of the following statement(s) is(are) correct regarding the root mean square speed (U_{rms}) and average translational kinetic energy (ε_{av}) of a molecule in a gas at equilibrium

(A) U_{rms} is inversely proportional to the square root of its molecular mass

(B) ε_{av} is doubled when its temperature is increased four times.

(C) U_{rms} is doubled when its temperature is increased four times.

(D) ε_{av} at a given temperature does not depend on its molecular mass.

2. (A), (C), (D)

$$\varepsilon_{av} = \frac{3}{2}RT \qquad U_{rms} = \sqrt{\frac{3RT}{M}} \qquad \text{and } U_{rms} \propto \frac{1}{\sqrt{M}}$$

$\therefore$ ε_{av} doesn't depend on its molecular mass

3. Which of the following statements(s) is(are) true ?

(A) The two six–membered cyclic hemiacetal forms of D–(+)–glucose are called anomers.

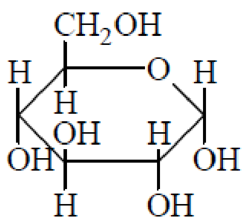
(B) Hydrolysis of sucrose gives dextrorotatory glucose and laevorotatory fructose.

(C) Monosaccharides cannot be hydrolysed to give polyhydroxy aldehydes and ketones.

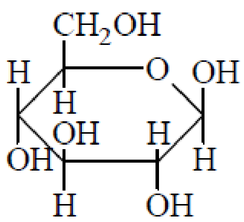
(D) Oxidation of glucose with bromine water gives glutamic acid.

3. (A), (B) , (C)

(A) True



α – D – glucopyranose



β – D – glucopyranose

α – D – glucopyranose and β – D – glucopyranose are anomers of each other.

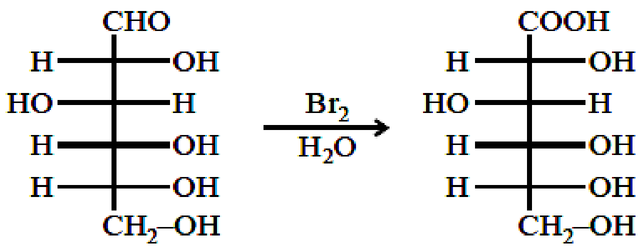
(B) True

$$C_{12}H_{22}O_{11} + H_2O \xrightarrow{\text{Invertase}} \underset{\substack{\text{Glucose} \\ (+)}}{C_6H_{12}O_6} + \underset{\substack{\text{Fructose} \\ (-)}}{C_6H_{12}O_6}$$

(C) True

Monosaccharide cannot be hydrolysed to give polyhydroxy aldehydes and ketones.

(D) False



D(+) glucose → Gluconic acid

4. In the decay sequence,

$${}^{238}_{92}U \xrightarrow{-x_1} {}^{234}_{90}Th \xrightarrow{-x_2} {}^{234}_{91}Pa \xrightarrow{-x_3} {}^{234}Z \xrightarrow{-x_4} {}^{230}_{90}Th$$

x_1, x_2, x_3 and x_4 are particles/radiation emitted by the respective isotopes. The correct options(s) is(are):

(A) x_1 will deflect towards negatively charged plate.

(B) x_2 is β^-

(C) x_3 is γ–ray

(D) z is an isotope of uranium

4. (A), (B), (D)

$${}_{92}U^{238} \longrightarrow {}_{90}Th^{234} + {}_{2}He^{4}(\text{or } \alpha) \text{ or } x_1$$

$$\downarrow$$

$${}_{91}Pa^{234} + \beta^{-}(\text{or } {}_{-1}e^{\circ}) \text{ or } x_2$$

$$\downarrow$$

$${}_{92}Z^{234} + \beta^{-}(\text{or } {}_{-1}e^{\circ}) \text{ or } x_3$$

$$\downarrow$$

$${}_{90}Th^{230} + {}_{2}He^{4}(\text{or } \alpha) \text{ or } x_4$$

5. A tin chloride Q undergoes the following reaction (not balanced) Q + Cl^- → X is monoanion having pyramidal geometry. Both Y and Z are neutral compounds. Choose the correct options(s)

(A) The central atom in X is sp^3 hybridized.

(B) There is a coordinate bond in Y

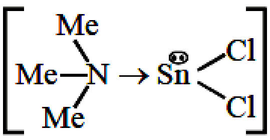
(C) The oxidation state of the central atom in Z is +2

(D) The central atom in Z has one lone pair of electrons.

5. (A), (B)

$$\underset{(Q)}{SnCl_2} + Cl^- \longrightarrow \underset{(X)}{SnCl_3^-}$$

$$\underset{(Q)}{SnCl_2} + Me_3N \longrightarrow \underset{(Y)}{SnCl_2.NMe_3}$$



$$\underset{(Q)}{SnCl_2} + 2CuCl_2 \longrightarrow \underset{(Z)}{SnCl_4} + 2CuCl$$

6. Choose the reaction(s) from the following options, for which the standard enthalpy of reaction is equal to the standard enthalpy of formation

(A) $\frac{1}{8}S_{8(s)} + O_2(g) \rightarrow SO_2(g)$

(B) $2H_2(g) + O_2(g) \rightarrow 2H_2O(I)$

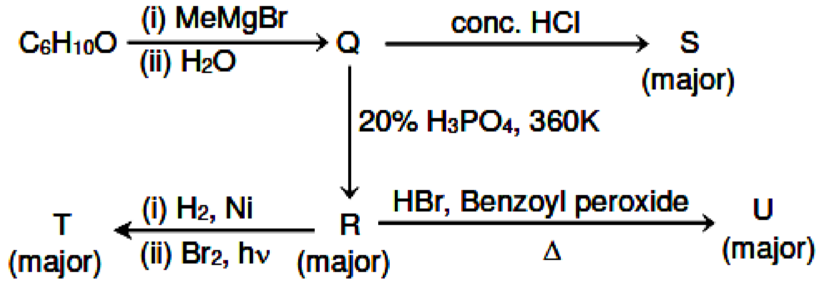
(C) $\frac{3}{2}O_2(g) \rightarrow O_3(g)$

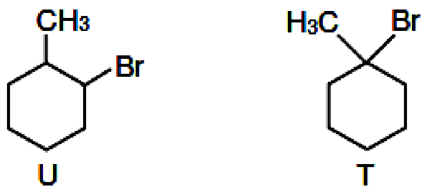
(D) $2C(g) + 3H_2(g) \rightarrow C_2H_6(g)$

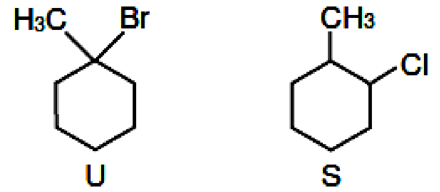
6. (A), (C)

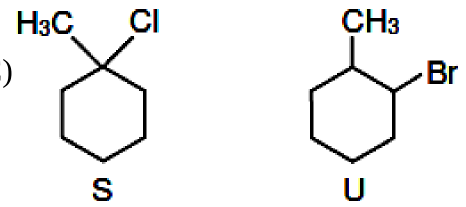
Standard enthalpy of formation of a compound is the standard enthalpy when one mole of a compound is formed from the elements in their stable state.

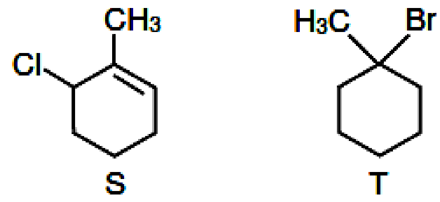
7. Choose the correct option(s) for the following set of reactions



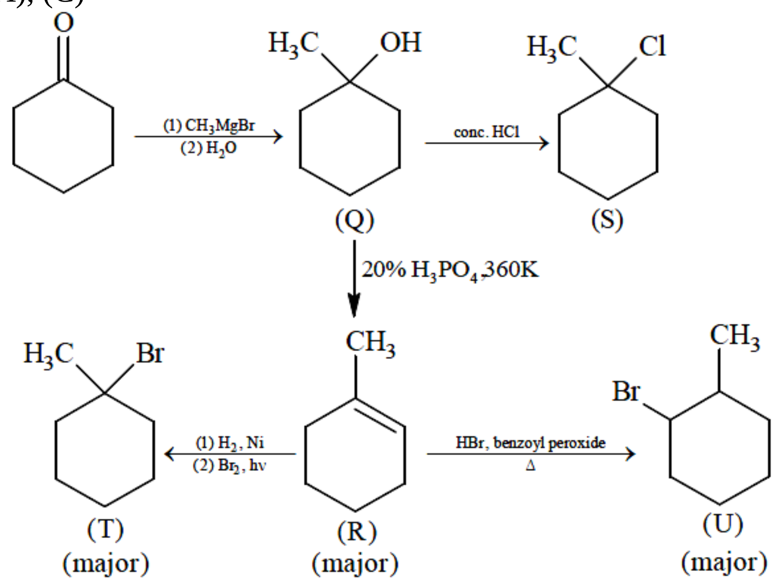
(A) 

(B) 

(C) 

(D) 

7. (A), (C)



8. Fusion of MnO_2, with KOH in presence of O_2 produces a salt W. Alkaline solution of W upon electrolytic oxidation yields another salt X. The manganese containing ions present in W and X, respectively are Y and Z. Correct statement(s) is (are)

(A) In both Y and Z, π–bonding occurs between p–orbitals of oxygen and d–orbitals of manganese

(B) Both Y and Z are coloured and have tetrahedral shape

(C) In aqueous acidic solution, Y undergoes disproportionation reaction to give Z and MnO_2

(D) Y is diamagnetic in nature while Z is paramagnetic

8. (A), (B) , (C)

$$MnO_2 + 2KOH + \frac{1}{2}O_2 \xrightarrow{\Delta} \underset{(W)}{K_2MnO_4} + H_2O$$

$$\left[(W) = K_2MnO_{4(aq)} \rightleftharpoons 2K^{\oplus}_{(aq)} + \underset{(Y)}{MnO^{2-}_{4(aq)}}\right]$$

$$K_2MnO_4 + H_2O \xrightarrow[\text{Redox}]{\text{Electrolytic}} H_2 + KOH + \underset{(X)}{KMnO_4}$$

$$\left[\text{anion of } X = \underset{(Z)}{MnO_4^-}\right]$$

$$\left[\because \underset{(Y)}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{(Z)}{MnO_4^-} + e^-\right]$$

$\because$ In acidic solution ; Y undergoes disproportionation reaction

$$\left[3MnO^{2-}_{4(aq)} + 4H^{\oplus} \longrightarrow \underset{(Z)}{2MnO_4^-} + MnO_2 + 2H_2O\right]$$

$$MnO_2 \xrightarrow{KOH/O_2} \underset{w}{MnO_4^{2-}} \xrightarrow[\text{Oxidation}]{\text{Electrolytic}} \underset{x}{MnO_4^-}$$

$$MnO_4^{2-} \xrightarrow{\text{acidic medium}} MnO_2 + MnO_4^-$$

SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : **+3** If ONLY the correct numerical value is entered.
  Zero Marks : **0** In all other cases.

1. For the following reaction, equilibrium constant K_c at 298 K is 1.6×10^{17}

$$Fe^{2+}(aq) + S^{2-}(aq) \rightleftharpoons FeS(s)$$

When equal volume of 0.06 M Fe^{+2}(aq) and 0.2 M S^{-2}(aq) solution are mixed, then equilibrium concentration of Fe^{+2}(aq) is found to be $Y \times 10^{-17}$ M. The value of Y is ____

1. [8.92 or 8.93]

$$Fe^{+2}_{(aq.)} + S^{-2}_{(aq.)} \rightleftharpoons FeS(s)$$

0.03 M	0.1 M	
(0.03 – x)	(0.1 – x)	
$\simeq$ y	$\simeq$ 0.07	

$K_c >> 10^3 \Rightarrow 0.03 - x \simeq 0 \simeq y \quad \Rightarrow x = 0.03$

$$K_c = 1.6 \times 10^{17} = \frac{1}{y \times 0.07}$$

$$y = \frac{10^{-17}}{1.6 \times 0.07} = 8.928 \times 10^{-17} = Y \times 10^{-17}$$

$y \approx 8.93$

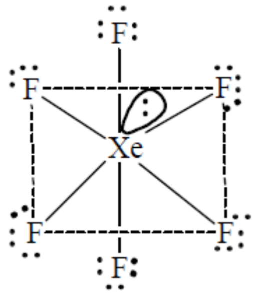
2. At 143 K, the reaction of XeF_4 with O_2F_2 produces a xenon compound Y. The total number of lone pair(s) electrons present on the whole molecule of Y is ________ .

2. [19]

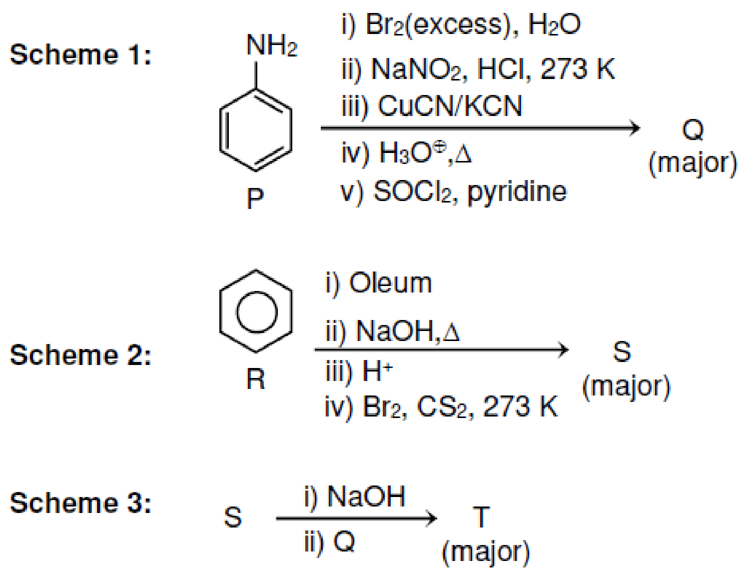
$$\underset{}{XeF_4 + O_2F_2} \longrightarrow \underset{Y}{XeF_6} + O_2$$

Y has 3 lone pair of electron in each fluorine and one lone pair of electron in xenon.

Hence total lone pair of electrons is 19.

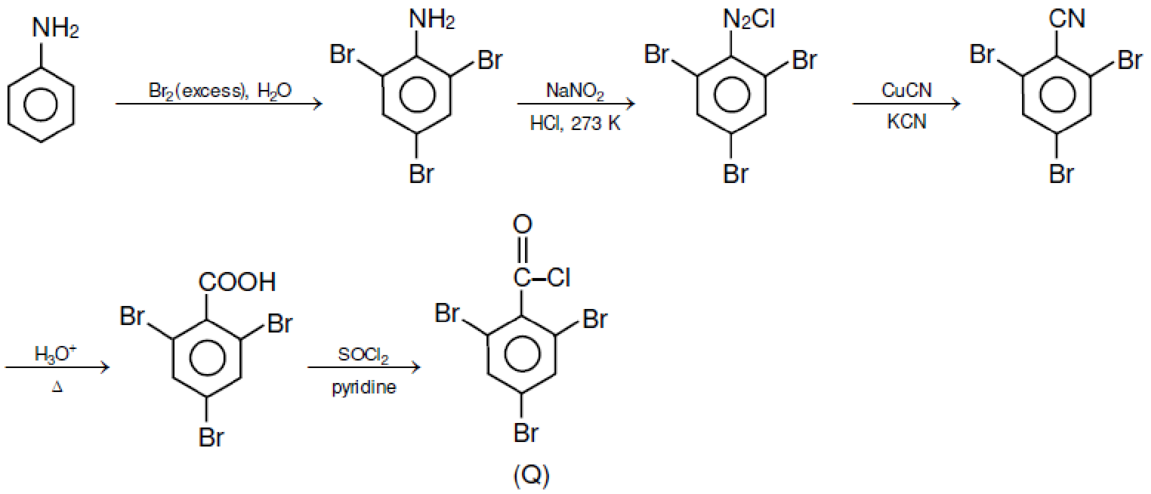


3. Schemes 1 and 2 describe the conversion of P to Q and R to S, respectively. Scheme 3 describes the synthesis of T from Q and S. The total number of Br atoms in a molecule of T is _________ .

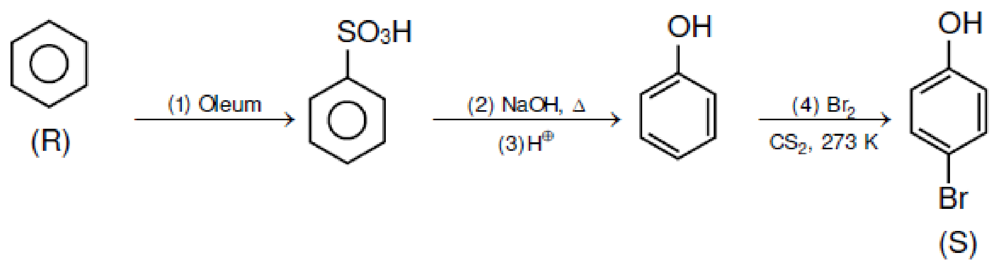


3. [4]

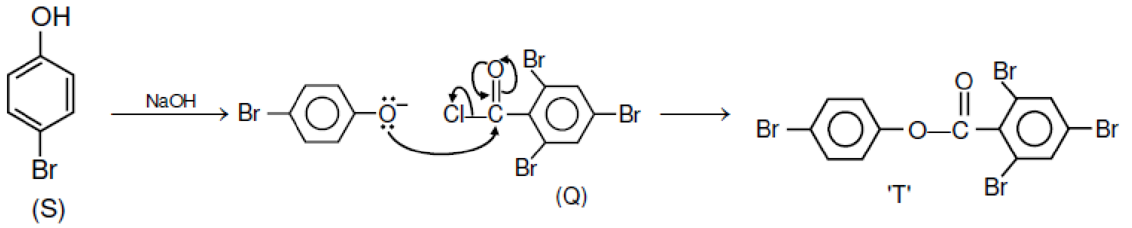
Scheme 1 :



Scheme 2 :



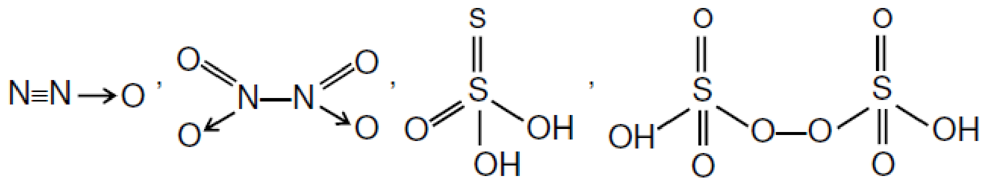
Scheme 3 :



4. Among B_2H_6, $B_3N_3H_6$, N_2O, N_2O_4, $H_2S_2O_3$ and $H_2S_2O_8$, the total number of molecules containing covalent bond between two atoms of the same kind is ________

4. [4]

N_2O_4 , $H_2S_2O_3$, N_2O, $H_2S_2O_8$



5. Consider the kinetic data given in the following table for the reaction :

$A + B + C \rightarrow$ product.

Experiment No.	[A] (mol dm^{-3})	[B] (mol dm^{-3})	[C] (mol dm^{-3})	Rate of reaction (mol dm^{-3} s^{-1})
1	0.2	0.1	0.1	6.0×10^{-5}
2	0.2	0.2	0.1	6.0×10^{-5}
3	0.2	0.1	0.2	1.2×10^{-4}
4	0.3	0.1	0.1	9.0×10^{-5}

The rate of the reaction for [A] = 0.15 mol dm^{-3}, [B] = 0.25 mol dm^{-3} and [C] = 0.15 mol dm^{-3} is found to be $Y \times 10^{-5}$ mol dm^{-3}s^{-1}. The value of Y is __________

5. [6.75]

Rate = $k[A]^x [B]^y [C]^z$

By exp. No. 1 and 2 $\quad y = 0$

By exp. No. 1 and 3 $\quad z = 1$

By exp. No. 1 and 4 $\quad x = 1$

Rate = $k[A]^1 [B]^0 [C]^1$

From Exp. No. 1 : $\quad 6 \times 10^{-5} = k(0.2)(0.1)$

$\Rightarrow k = 3 \times 10^{-3}$

Now for [A] = 0.15 $\quad$ [B] = 0.25 $\quad$ [C] = 0.15

Rate $= k[A]^1 [B]^0 [C]^1 = 3 \times 10^{-3} \times 0.15 \times 1 \times 0.15$

6. On dissolving 0.5 g of a non–volatile non–ionic solute to 39 g of benzene, its vapor pressure decreases from 650 mm Hg to 640 mm Hg. The depression of freezing point of benzene (in K) upon addition of the solute is __________.

(Given data : Molar mass and the molal freezing point depression constant of benzene are 78 g mol^{-1} and 5.12 K kg mol^{-1}, respectively)

6. [1.02 or 1.03]

$$\frac{P^0 - P_s}{P^0} = \frac{n_{solute}}{n_{solute} + n_{solvent}} \quad ; \quad \frac{650-640}{650} = \frac{n_{solute}}{n_{solute} + 0.5}$$

$$n_{solute} = \left(\frac{5}{640}\right); \qquad \text{Molality} = \frac{5 \times 1000}{640 \times 39}$$

$$\Delta T_f = m \times K_b = \frac{5.12 \times 5 \times 1000}{640 \times 39} = 1.0256$$

$$\Delta T_f \approx 1.03$$

PART III – MATHEMATICS

SECTION 1 (Maximum Marks:12)

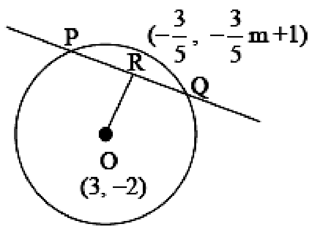
- This section contains FOUR **(04)** questions.
- Each question has **FOUR** options **ONLY ONE** of these four options is the correct answer.
- For each question, choose the correct option corresponding to the correct answer.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : **+3** If ONLY the correct option is chosen.
  Zero Marks : **0** If none of the options is chosen (i.e. the question is unanswered).
  Negative Marks : **–1** In all other cases.

1. A line y = mx + 1 intersect the circle $(x - 3)^2 + (y + 2)^2 = 25$ at points P and Q. If the midpoint of the line segment PQ has x–coordinate $-\frac{3}{5}$, then which one of the following options is correct.

(A) $6 \le m < 8$ (B) $2 \le m < 4$ (C) $4 \le m < 6$ (D) $-3 \le m < -1$

1. (B)

PQ ⊥ OR ⇒ Stope OR = $-\frac{1}{m} = \dfrac{-\frac{3}{5}m+1+2}{-\frac{3}{5}-3}$



$\Rightarrow\ m^2 - 5m + 6 = 0$

$\Rightarrow\ m = 2, 3$

2. Let $M = \begin{bmatrix} \sin^4\theta & -1-\sin^2\theta \\ 1+\cos^2\theta & \cos^4\theta \end{bmatrix} = \alpha I + \beta M^{-1}$

Where α = α (θ) and β = β (θ) are real numbers and I is the 2 × 2 identity matrix.

If α^* = is the minimum of the set {α (θ) : θ ∈ [0, 2π)} and

β^* = is the minimum of the set {β (θ) : θ ∈[0, 2π)}

Then the value of $\alpha^* + \beta^*$ is

(A) $\frac{-37}{16}$ (B) $\frac{-29}{16}$ (C) $\frac{-31}{16}$ (D) $\frac{-17}{16}$

2. (B)

$$M = \alpha\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \beta\begin{bmatrix} \cos^4\theta & 1+\sin^2\theta \\ -1-\cos^2\theta & \sin^4\theta \end{bmatrix}$$

On comparing we have

$$\sin^4\theta = \alpha + \frac{\beta\cos^4\theta}{|M|}$$

$$-1-\sin^2\theta = \alpha + \frac{\beta(1+\sin^2\theta)}{|M|}$$

$\alpha = \sin^4\theta + \cos^4\theta$

Now $\alpha = (\sin^2\theta + \cos^2)^2 - 2\sin^2\theta\cos^2\theta$

$= 1 - \dfrac{\sin^2 2\theta}{2}$

$\Rightarrow\ \alpha^* = \dfrac{1}{2}$

We have, $|M| = \sin^4\theta\cos^4\theta + (1+\sin^2\theta)(1+\cos^2\theta)$

$= 2 + \sin^2\theta\cos^2\theta + \sin^4\theta\cos^4\theta$

$= (\sin^2\theta\cos^2\theta + 1/2)^2 + 7/4$

$$\Rightarrow \beta = -|M| = -\frac{7}{4} - \left(\sin^2\theta\cos^2\theta + \frac{1}{2}\right)^2$$

$$\Rightarrow \beta^* = -\frac{7}{4} - \left(\frac{1}{4}+\frac{1}{2}\right)^2 = -\frac{7}{4} - \frac{9}{16} = -\frac{37}{16}$$

$$\alpha^* + \beta^* = -\frac{29}{16}.$$

3. Let S be the set of all complex numbers z satisfying $|z-2+i| \geq \sqrt{5}$. If the complex number z_0 is such that $\frac{1}{|z_0-1|}$ is the maximum of the set $\left\{\frac{1}{|z-1|} : z \in S\right\}$, then the principal argument of $\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+2i}$ is

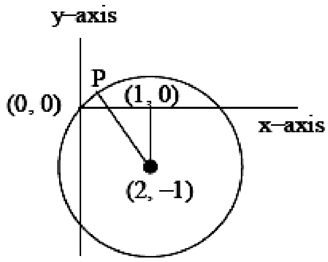
(A) $\frac{\pi}{4}$ (B) $\frac{3\pi}{4}$ (C) $-\frac{\pi}{2}$ (D) $\frac{\pi}{2}$

3. (C)

Clearly location of required point z_0 is at P with abscissa < 1 and ordinate > 0

Now $\arg\left[\frac{4-z_0-\bar{z}_0}{z_0-\bar{z}_0+zi}\right] = \text{Arg}\left(\frac{x-2}{y+1}\right)i = \text{Arg } ki$ and $k < 0$

$\Rightarrow$ Required argument $= -\pi/2$



4. The area of the region $\{(x, y) : xy \leq 8, 1 \leq y \leq x^2\}$ is

(A) $16\log_e 2 - 6$ (B) $8\log_e 2 - \frac{7}{3}$ (C) $16\log_e 2 - \frac{14}{3}$ (D) $8\log_e 2 - \frac{14}{3}$

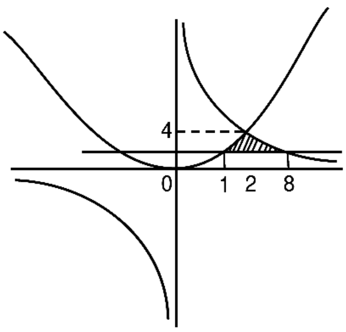
4. (C)

$xy \leq 8$

$1 \leq y \leq x^2$

$x^2 . x = 8$

$x = 2$



Required Area $= \int_1^4 \left(\frac{8}{y} - \sqrt{y}\right)dy = \left[8\ell n\, y - \frac{y^{3/2}}{3/2}\right]_1^4 = 8\ell n 4 - \frac{2}{3}.8 - 0 + \frac{2}{3} = 16\ell n 2 - \frac{14}{3}$

SECTION 2 (Maximum Marks:32)

- This section contains **EIGHT (08)** questions.
- Each question has **FOUR** options. **ONE OR MORE THAN ONE** of these four option(s) is (are) correct answer(s).
- For each question, choose the correct option(s) corresponding to (all) the correct answer(s).
- Answer to each question will be evaluated according to the following marking scheme :

Full Marks	:	**+4** If only (all) the correct option(s) is (are) chosen.
Partial Marks	:	**+3** If all the four options are correct but ONLY three options are chosen.
Partial Marks	:	**+2** If three or more options are correct but ONLY two options are chosen and both of which are correct.
Partial Marks	:	**+1** If two or more options are correct but ONLY one option is chosen and it is a correct option.
Zero Marks	:	0 If none of the options is chosen (i.e. the question is unanswered).
Negative Marks	:	**–1** In all other cases.

1. Define the collections $\{E_1, E_2, E_3, ...\}$ of ellipses and $\{R_1, R_2, R_3, ...\}$ of rectangles of follows :

E_1 : $\frac{x^2}{9}+\frac{y^2}{4}=1$;

R_1 : Rectangle of largest area with sides parallel to the axes, inscribed in E_1 :

E_n : ellipse $\frac{x^2}{a_n^2}+\frac{y^2}{b_n^2}=1$ of largest area inscribed in R_{n-1}, n > 1

R_n : rectangle of largest area, with sides parallel to the axes, inscribed in E_n, n >1

Then which of the following options is/are correct ?

(A) The eccentricities of E_{18} and E_{19} are NOT equal

(B) The length of latus rectum of E_9 is $\frac{1}{6}$

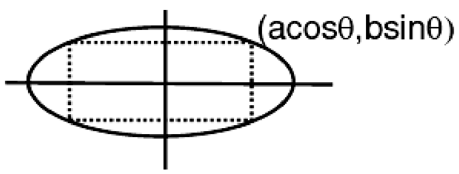
(C) $\sum_{n=1}^{N}\left(\text{area of } R_n\right) < 24,$ for each positive integer N

(D) The distance of a focus from the centre in E_9 is $\frac{\sqrt{5}}{32}$

1. (B), (C)

Area Maximum when θ = 45°

	a	b
E_1	3	2
E_2	$\frac{3}{\sqrt{2}}$	$\frac{2}{\sqrt{2}}$
E_3	$\frac{3}{(\sqrt{2})^2}$	$\frac{2}{\left(\sqrt{2}\right)^2}$
E_9	$\frac{3}{\left(\sqrt{2}\right)^8}$	$\frac{2}{\left(\sqrt{2}\right)^8}$



(A) $e_{18} = e_{19}$

(B) Length of LR is ellipse = $\dfrac{2b^2}{a} = 2.\dfrac{4.2^4}{2^8.3} = \dfrac{1}{6}$

(C) $R_1 + R_2 + \ldots.. + R_N < R_1 + R_2 + \ldots.. + R_N + \ldots\ldots = \dfrac{2ab}{1-\dfrac{1}{\sqrt{2}}\dfrac{1}{\sqrt{2}}} = 4ab = 4.3.2 = 24$

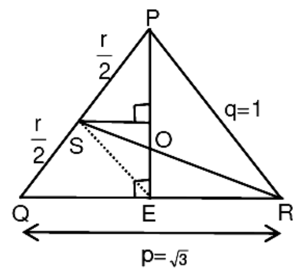
(D) Distance between focus and center of ellipse = $a_9\, e_9 = \dfrac{3}{2^4}\cdot\dfrac{\sqrt{5}}{3} = \dfrac{\sqrt{5}}{16}$

2. In a non-right-angled triangle ΔPQR, Let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median form R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, q = 1, and the radius of the circumcircle of the ΔPQR equals 1, then which of the following options is/are correct ?

(A) Length of RS = $\dfrac{\sqrt{7}}{2}$ (B) Area of $\Delta SOE = \dfrac{\sqrt{3}}{12}$

(C) Radius of incircle of $\Delta PQR = \dfrac{\sqrt{3}}{2}\left(2-\sqrt{3}\right)$

(D) Length of OE = $\dfrac{1}{6}$



2. (A), (C), (D)

$\dfrac{p}{\sin P} = \dfrac{q}{\sin Q} = 2(1) \quad \Rightarrow \quad \sin P = \dfrac{\sqrt{3}}{2}, \sin Q = \dfrac{1}{2}$

$\Rightarrow$ $\angle P = 60°$ or $120°$ and $\angle Q = 30°$ or $150°$

because $\angle P + \angle Q$ must be less than 180° but not equal to 90°

$\angle P = 120°$ and $\angle Q = 30°$ and $\angle R = 30°$ $\quad \dfrac{r}{\sin R} = 2 \Rightarrow r = 1$

Now length of median $RS = \dfrac{1}{2}\sqrt{2p^2+2q^2-r^2} = \dfrac{1}{2}\sqrt{6+2-1} = \dfrac{\sqrt{7}}{2} \Rightarrow$ option (A) is correct

In radius = $\dfrac{2\Delta}{p+q+r} = \dfrac{\dfrac{2pqr}{4\times(1)}}{p+q+r} = \dfrac{1}{2}\left(\dfrac{1\times1\times\sqrt{3}}{1+1+\sqrt{3}}\right) = \dfrac{\sqrt{3}}{2}\left(\dfrac{2-\sqrt{3}}{1}\right) \Rightarrow$ option (C) is correct

$\Rightarrow \quad \dfrac{1}{2}\times\sqrt{3}\times PE = \dfrac{pqr}{4(1)}$ (equal area of Δ) $\Rightarrow$ $PE = \dfrac{1\times1\times\sqrt{3}}{4}\times\dfrac{2}{\sqrt{3}} = \dfrac{1}{2}$

$\Rightarrow \quad OE = \dfrac{2(\text{Area of } \Delta OQR)}{QR} = \dfrac{2\times\dfrac{1}{3}\left(\dfrac{1}{2}.1.\sqrt{3}\sin 30°\right)}{\sqrt{3}} = \dfrac{1}{6}$

3. Let $f : R \to R$ be given by

$$f(x) = \begin{cases} x^5+5x^4+10x^3+10x^2+3x+1, & x<0 \\ x^2-x+1 & 0\le x<1 \\ (2/3)x^3-4x^2+7x-(8/3) & 1\le x<3 \\ (x-2)\ell n(x-2)-x+(10/3) & x\ge 3 \end{cases}$$

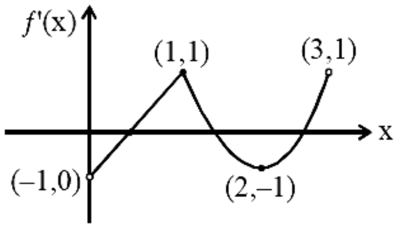
Then which of the following options is/are correct?

(A) f' is NOT differentiable at x = 1 (B) f is increasing on $(-\infty, 0)$

(C) f is onto (D) f' has a local maximum at x = 1

3. (A), (C), (D)

$$f(x)=\begin{cases}(x+1)^5-2x, & x<0;\\ x^2-x+1, & 0\le x<1;\\ \dfrac{2}{3}x^3-4x^2+7x-\dfrac{8}{3}, & 1\le x<3;\\ (x-2)\log_e(x-2)-x+\dfrac{10}{3}, & x\ge 3\end{cases}$$



$$f'(x)=\begin{cases}5(x+1)^4-2 & x<0\\ 2x-1 & 0\le x<1\\ \dfrac{2}{3}x^3-4x^2+7x-\dfrac{8}{3} & 1\le x<3\\ (x-2)\ln(x-2)-x+\dfrac{10}{3} & x\ge 3\end{cases}$$

$x^5 + 5x^4 + 10x^3 + 10x^2 + 3x + 1$ takes value between $-\infty$ to 1

Also $(x-2)\,\text{Ln}(x-2) - x + \dfrac{10}{3}$ takes value between $\dfrac{1}{3}$ to ∞

So, range of f(x) is R.

$f''(1^-) = 2$ and $f''(1^+) = -4$. So $f'(x)$ is non-diff at x = 1

$f'(x)$ has local maxima at x = 1.≤

4. Let α and β be the roots of $x^2 - x - 1 = 0$ with $\alpha > \beta$. For all positive integers n. define

$a_n = \dfrac{\alpha^n - \beta^n}{\alpha-\beta}, n \ge 1$ and $b_1 = 1$ and $b_n = a_n = a_{n-1} + a_{n+1}, n \ge 2$

the which of the following options is/are correct?

(A) $\displaystyle\sum_{n=1}^{\infty}\frac{a_n}{10^n} = \frac{10}{89}$ (B) $b_n = \alpha^n + \beta^n$ for all $n \ge 1$

(C) $a_1 + a_2 + \ldots + a_n = a_{n+2} - 1$ for all $n \ge 1$ (D) $\displaystyle\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \frac{8}{89}$

4. (A), (B), (C)

(A) $$b_n = a_{n+1} + a_{n-1} = \frac{\alpha^{n+1}-\beta^{n+1}}{\alpha-\beta} + \frac{\alpha^{n-1}-\beta^{n-1}}{\alpha-\beta} = \frac{\alpha^{n-1}\left(\alpha^2+1\right)-\beta^{n-1}(\beta^2+1)}{\alpha-\beta}$$

$$= \frac{\alpha^{n-1}(\alpha+2)-\beta^{n-1}(\beta+2)}{\alpha-\beta} = \frac{\alpha^{n-1}\left(\dfrac{5+\sqrt{5}}{2}\right)-\beta^{n-1}\left(\dfrac{5-\sqrt{5}}{2}\right)}{\alpha-\beta}$$

$$= \frac{\sqrt{5}\alpha^{n-1}\left(\dfrac{\sqrt{5}+1}{2}\right)-\sqrt{5}\,\beta^{n-1}\left(\dfrac{\sqrt{5}-1}{2}\right)}{\alpha-\beta} = \frac{\sqrt{5}(\alpha^n+\beta^n)}{\alpha-\beta} = \alpha^n+\beta^n \qquad \because\ \alpha-\beta=\sqrt{5}$$

(B) $$\sum_{n=1}^{\infty}\frac{b_n}{10^n} = \sum\left(\frac{\alpha}{10}\right)^n + \sum\left(\frac{\beta}{10}\right)^n = \frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}} + \frac{\frac{\beta}{10}}{1-\frac{\beta}{10}} = \frac{\alpha}{10-\alpha}+\frac{\beta}{10-\beta}$$

$$= \frac{10(\alpha+\beta)-2\alpha\beta}{100-10(\alpha+\beta)+\alpha\beta} = \frac{10+2}{89} = \frac{12}{89}$$

(C) $\sum_{n=1}^{\infty}\frac{a_n}{10^n}=\sum\frac{\alpha^n-\beta^n}{(\alpha-\beta)10^n}=\frac{1}{\alpha-\beta}\left(\frac{\frac{\alpha}{10}}{1-\frac{\alpha}{10}}-\frac{\frac{\beta}{10}}{1-\frac{\beta}{10}}\right)\frac{1}{\alpha-\beta}\left(\frac{\alpha}{10-\alpha}-\frac{\beta}{10-\beta}\right)$

$=\frac{1}{\alpha-\beta}\cdot\frac{(10(\alpha-\beta)-\alpha\beta+\alpha\beta)}{100-10(\alpha+\beta)+\alpha\beta}=\frac{10}{89}$ Option (C) is correct.

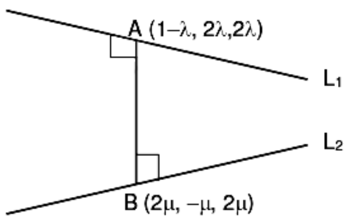
(D) $a_1 + a_2 + \ldots + a_n = \sum a_i=\frac{\sum\alpha^i-\sum\beta^i}{\alpha-\beta}=\frac{\frac{\alpha(1-\alpha^n)}{(1-\alpha)}-\frac{\beta(1-\beta^n)}{(1-\beta)}}{\alpha-\beta}$

$=\frac{(\alpha+1)(1-\alpha^n)-(\beta+1)(1-\beta^n)}{(1-\alpha)(1-\beta)(\alpha-\beta)}=\frac{\alpha^2-\alpha^{n+2}-\beta^2+\beta^{n+2}}{(1-\alpha)(1-\beta)(\alpha-\beta)}=\frac{\sqrt{5}+\beta^{n+2}-\alpha^{n+2}}{\beta-\alpha}=-1+a_{n+2}$

5. Let L_1 and L_2 denote the lines $\vec{r}=\hat{i}+\lambda\left(-\hat{i}+2\hat{j}+2\hat{k}\right)$, $\lambda\in R$ and $\vec{r}=\mu\left(2\hat{i}-\hat{j}+2\hat{k}\right)$, $\mu\in R$ respectively. If L_3 is a line which is perpendicular to both L_1 and L_2 and cuts both of them, then which of the following options describe (s) L_3?

(A) $\vec{r}=\frac{1}{3}\left(2\hat{i}+\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in R$
(B) $\vec{r}=\frac{2}{9}\left(4\hat{i}+\hat{j}+\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in R$
(C) $\vec{r}=\frac{2}{9}\left(2\hat{i}-\hat{j}+2\hat{k}\right)+t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in R$
(D) $\vec{r}=t\left(2\hat{i}+2\hat{j}-\hat{k}\right), t\in R$

5. (A), (B), (C)
Both given lines are skew lines.
So direction ratios of any line perpendicular to these lines are $6\hat{i}+6\hat{j}-3\hat{k}$
i.e 2, 2, −1
Points at shortest distance between given lines are



$\overrightarrow{AB}\perp \text{line } L_1$

$\overrightarrow{AB}\perp \text{line } L_2$

So $A\left(\frac{8}{9},\frac{2}{9},\frac{2}{9}\right)$

Now equation of required line $\vec{r}-=\left(\frac{8}{9}\hat{i}+\frac{2}{9}\hat{j}+\frac{2}{9}\hat{k}\right)+\alpha(2\hat{i}+2\hat{j}-\hat{k})$

6. There are three bags B_1, B_2 and B_3. The bag B_1 contains 5 red and 5 green balls. B_2 contains 3 red and 5 green balls and B_3 contains 5 red and 3 green balls. Bags B_1, B_2 and B_3 have probabilities 3/10, 3/10 and 4/10 respectively of being chosen**.** A bag is selected at random and a ball is chosen at random from the bag. Then which of the following options is/are correct?

(A) Probability that the chosen ball is green, given that the selected bag is B_3, equals $\frac{3}{8}$

(B) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{5}{13}$

(C) Probability that the chosen ball is green equals $\frac{39}{80}$

(D) Probability that the selected bag is B_3, given that the chosen ball is green, equals $\frac{3}{10}$

6. (A), (C)

	Bag – 1	Bag – 2	Bag – 3
Red Balls	5	3	5
Green Balls	5	5	3
Total	10	8	8

(A) P(Ball is Green) = $P(B_1)\,P(G/B_1) + P(B_2)P(G/B_2) + P(B_3)P(G/B_3)$

$$= \frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}=\frac{39}{80}$$

(B) P(Ball chosen is Green/Ball is from 3rd Bag) = $\frac{3}{8}$

(C), (D)

P(Ball is from 3rd Bag/Ball chosen is Green)

$$= \frac{P(B_3)P(G/B_3)}{P(B_1)P(G/B_1)+P(B_2)P(G/B_2)+P(B_3)P(G/B_3)}$$

$P(B_1) = \frac{3}{10}$

$P(B_2) = \frac{3}{10}$

$$P(B_3)=\frac{4}{10}=\frac{\frac{4}{10}\times\frac{3}{8}}{\frac{3}{10}\times\frac{5}{10}+\frac{3}{10}\times\frac{5}{8}+\frac{4}{10}\times\frac{3}{8}}=\frac{4}{13}$$

7. Let Γ denote a curve y = f (x) which is in the first quadrant and let the point (1, 0) lie on it. Let the tangent to Γ at a point P intersect the y-axis at Y_P. If PY_P has length 1 for each point P on Γ. Then which of the following options is/are correct?

(A) $y=-\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)+\sqrt{1-x^2}$

(B) $xy'+\sqrt{1-x^2}=0$

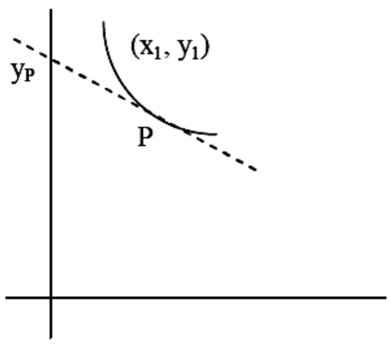
(C) $xy'-\sqrt{1-x^2}=0$

(D) $y=\ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)-\sqrt{1-x^2}$

7. (B), (D)

Equation tangent

$$y-y_1=\frac{dy_1}{dx_1}(x-x_1)$$



$$y_p\left(0, y_1-x_1\frac{dy_1}{dx_1}\right)$$

$$Py_p=\sqrt{x_1^2+\left(-x_1\frac{dy_1}{dx_1}\right)^2}=1$$

$$\Rightarrow \frac{dy}{dx}=\pm\frac{\sqrt{1-x^2}}{x}\Rightarrow dy=\pm\int\frac{\sqrt{1-x^2}}{x}dx,\ 1-x^2t^2, -x\,dx=t\,dt$$

$$dy=\pm\int\frac{-t^2}{1-t^2}dt$$

$$y=\pm\left(\int 1dt+\int\frac{1}{1-t^2}dt\right)$$

$$y = \pm\left(t - \frac{1}{2}\ell n\left|\frac{1+t}{1-t}\right|\right) + c$$

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\left|\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right|\right) + c$$

y is passing through (1, 0) so c = 0

$$y = \pm\left(\sqrt{1-x^2} - \frac{1}{2}\ell n\left(\frac{1+\sqrt{1-x^2}}{1-\sqrt{1-x^2}}\right)\right)$$

$$y = \pm\left(\sqrt{1-x^2} - \ell n\left(\frac{1+\sqrt{1-x^2}}{x}\right)\right)$$

8. Let $M = \begin{bmatrix} 0 & 1 & a \\ 1 & 2 & 3 \\ 3 & b & 1 \end{bmatrix}$ and $\operatorname{adj} M = \begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}$ where a and b are real numbers. Which of the following options is/are correct?

(A) det (adj M^2) = 81

(B) a + b = 3

(C) If $M\begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$, then $\alpha - \beta + \gamma = 3$

(D) $(\text{adj } M)^{-1} + \text{adj } M^{-1} = -M$

8. (B), (C), (D)

$(\text{adj}M)_{11} = 2 - 3b = -1 \Rightarrow b = 1$

Also, $(\text{adj}M)_{22} = -3a = -6 \Rightarrow a = 2$

Now, det M = $\begin{vmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{vmatrix} = -2$

$\Rightarrow \det(\text{adj}M^2) = (\det M^2)^2$

$= (\det M)^4 = 16$

Also $M^{-1} = \dfrac{\text{adj}M}{\det M}$

$\Rightarrow \text{adj}M = -2M^{-1}$

$\Rightarrow (\text{adj}M)^{-1} = \dfrac{-1}{2}M$

And, $\text{adj}(M^{-1}) = (M^{-1})^{-1}\det(M^{-1}) = \dfrac{1}{\det M}M = \dfrac{-M}{2}$

Hence, $(\text{adj}M)^{-1} + \text{adj}(M^{-1}) = -M$

Further, $MX = b$

$\Rightarrow X = M^{-1}b = \dfrac{-\text{adj}M}{2}b$

$$= \frac{-1}{2}\begin{bmatrix} -1 & 1 & -1 \\ 8 & -6 & 2 \\ -5 & 3 & -1 \end{bmatrix}\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} = \frac{-1}{2}\begin{bmatrix} -2 \\ 2 \\ -2 \end{bmatrix} = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

$\Rightarrow (\alpha, \beta, \gamma) = (1, -1, 1)$

SECTION 3 (Maximum Marks:18)

- This section contains **SIX (06)** questions. The answer to each question is a **NUMERICAL VALUE**.
- For each question, enter the correct numerical value of the answer using the mouse and the on-screen virtual numeric keypad in the place designated to enter the answer. If the numerical value has more than two decimal places **truncate/round-off** the value to **TWO** decimal placed.
- Answer to each question will be evaluated according to the following marking scheme :
  Full Marks : **+3** If ONLY the correct numerical value is entered.
  Zero Marks : **0** In all other cases.

1. Three lines are given by

$\vec{r} = \lambda\hat{i}, \lambda \in R$

$\vec{r} = \mu\left(\hat{i}+\hat{j}\right), n \in R$ and $\vec{r} = \nu\left(\hat{i}+\hat{j}+\hat{k}\right), \nu \in R$

Let the lines cut the plane x + y + z = 1 at the points A, B and C respectively. If the area of the triangle ABC is Δ then the value of $(6\Delta)^2$ equals ________.

1. [0.75]

Put $(\lambda, 0, 0)$ in $x + y + z = 1 \Rightarrow \lambda = 1 \Rightarrow P(1, 0, 0)$

Put $(\mu, \mu, 0) \Rightarrow 2\mu = 1 \Rightarrow Q\left(\frac{1}{2},\frac{1}{2},0\right)$

Put $(\gamma,\gamma,\gamma) \Rightarrow \gamma = \frac{1}{3} \Rightarrow R\left(\frac{1}{3},\frac{1}{3},\frac{1}{3}\right)$

Area of triangle PQR = $\frac{1}{2}\left|\overrightarrow{PQ}\times\overrightarrow{PR}\right| = \frac{1}{2}\left(\frac{\hat{i}-\hat{j}}{2}\right)\times\left(\frac{2\hat{i}-\hat{j}-\hat{k}}{3}\right) = \frac{1}{12}\left|\hat{i}+\hat{j}+\hat{k}\right| = \frac{\sqrt{3}}{12}$

$\Rightarrow (6\Delta)^2 = 0.75$

2. Let S be the sample space of all 3 × 3 matrices with entries from the set {0, 1}. Let the events

$E_1 = \{A \in S : \det A = 0\}$ and

$E_2 = \{A \in S : \text{Sum of entries of A is } 7\}$

If a matrix is chosen at random from S, then the conditional probability $P(E_1 \mid E_2)$ equals ________.

2. [0.50]

$n(E_2)$ = arrangement of 7, 1 and 2 or $= \frac{9!}{7!2!} = 36$

$n(E_1 \cap E_2)$ = both zero should be in a row or a column

$= \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 0 & 0 \end{bmatrix}$ (number of ways of arranging of (1, 0, 0) = 3 and arrangement of row = 3

Total = 9

In same way for (1, 0, 0) for columns number of ways will be = 9

Total ways = 18

$P\left(\frac{E_1}{E_2}\right) = \frac{P(E_1 \cap E_2)}{P(E_2)} = \frac{18}{36} = \frac{1}{2} = 0.50$

3. That $\omega \neq 1$ be a cube root of unity. Then the minimum of the set $\{|a + b\omega + c\omega^2|^2;$ a, b, c are distinct non zero integers$\}$ equals ________.

3. [3]

$$\left|a+b\omega+c\omega^2\right|^2 = (a+b\omega+c\omega)^2\left(\overline{a+b\omega+c\omega^2}\right)$$

$= (a + b\omega + c\omega^2)(a + b\omega^2 + c\omega)$

$= a^2 + b^2 + c^2 - ab - bc - ca$

$$= \frac{1}{2}\left[(a-b)^2+(b-c)^2+(c-a)^2\right]$$

$$\geq \frac{1+1+4}{2} = 3 \quad \text{(when a = 1, b = 2, c = 3)}$$

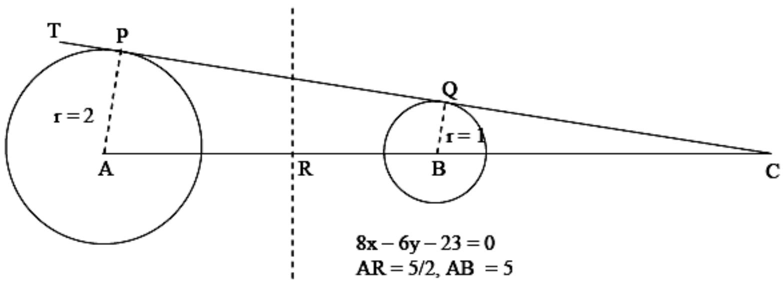
4. Let the point B be the reflection of the point A (2, 3) with respect to the line $8x - 6y - 23 = 0$. Let Γ_A and Γ_B be circles of radii 2 and 1 with centres A and B respectively. Let T be a common tangent to the circles Γ_A and Γ_B such that both the circles are on the same side of T. If C is the point of intersection of T and the line passing through A and B, then the length of the line segment AC is ________

4. [10]

Now ΔAPC and BQC are similarly

$$\frac{BC}{AC} = \frac{1}{2} \Rightarrow 2(AC - AB) = AC$$

AC = 2AB = 10



5. Let AP (a, d) denote the set of all the term of an infinite arithmetic progression with first term a and common difference $d > 0$. If AP (1, 3) $\cap$ AP (2, 5) $\cap$ AP (3, 7) = AP (a, d) then a + d equals ________.

5. [157]

AP (1, 3) $\equiv \{1, 4, 7, 10, \ldots\} = \{n/n = 3k + 1, k \in W\}$

AP (2, 5) $\equiv \{2, 7, 12, \ldots\} = \{n/n = 3k + 1, k \in W\}$

AP (3, 7) $\equiv \{3, 10, 17, \ldots\} = \{n/n = 7k + 3, k \in W\}$

Let common term is M

$M \equiv 1 \pmod 3$, $M \equiv 2 \pmod 5$, $M \equiv 3 \pmod 7$

$\Rightarrow M \equiv 52 \pmod{105}$

So, a = 52, d = 105 and a + d = 157

6. $I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}$ then find $27I^2$ equals ________.

6. [4]

$$I = \frac{2}{\pi}\int_{-\pi/4}^{\pi/4} \frac{dx}{(1+e^{\sin x})\left(2-\cos 2x\right)} \qquad \ldots(1)$$

By a + b – x property

$$I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{dx}{(1+e^{\sin x})(2-\cos 2x)} = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{e^{\sin x}dx}{(1+e^{\sin x})(2-\cos 2x)}dx \qquad \ldots(2)$$

Adding (1) and (2)

$$2I = \frac{2}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\left(1+e^{\sin x}\right)}{\left(1+e^{\sin x}\right)\left(2-\cos 2x\right)}dx \Rightarrow \quad I = \frac{1}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{1}{2-(2\cos^2 x-1)}dx = \frac{1}{\pi}\int_{-\frac{\pi}{4}}^{\pi/4} \frac{\sec^2 x}{3\sec^2 x-2}dx$$

Put, tanx = t, $\sec^2 x dx = dt$

$$= \frac{2}{\pi}\int_0^1 \frac{dt}{3t^2+1} = \frac{2}{3\pi}\frac{1}{\left(\frac{1}{\sqrt{3}}\right)}\left(\tan^{-1}\left(\frac{t}{1/\sqrt{3}}\right)\right)_0^1 = \frac{2}{\sqrt{3}\pi}\left(\tan^{-1}\left(\sqrt{3}\right)-\tan^{-1}(0)\right) = \frac{2}{\sqrt{3}\pi}\left(\frac{\pi}{3}\right) = \frac{2}{3\sqrt{3}}$$

Now, $27I^2 = 27 \times \frac{4}{27} = 4$

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