Assignment #2 – Integer Programming Fundamentals

Case study: PM Computer Services

Mathematical Programming Formulation

Decision Variables:

Let:

- nt = Number of employees in month t
- ht = Number of new hires in month t
- ft = Number of layoffs in month t
- xt = Regular production in month t (units)
- yt = Overtime production in month t (units)
- it = Inventory at the end of month t (units)

Objective Function:

Minimize total cost:

$$\boldsymbol{MIN\ Z = 1280(\sum n) + 12(\sum y) + 320(\sum f) + 200(\sum h) + 15(\sum i)}$$

where:

- **1280** = Cost per employee
- **12** = Cost per overtime unit
- **320** = Layoff cost per employee
- **200** = Hiring cost per employee
- **15** = Inventory holding cost per unit

Constraints:

1. Workforce Balance:

$$nt = nt - 1 + ht - ft$$

Where t ={1,2,3,4,5,6}

2. Product Capacity

$$xt \leq 12.7nt$$

Where t ={1,2,3,4,5,6}

12.7 is the regular production rate per worker.

$$yt \leq 0.6nt$$

Where t ={1,2,3,4,5,6}

3. Demand satisfaction

$$xt + yt + it - 1 = Dt + it$$

Where Dt is the Demand of that month
Where t ={1,2,3,4,5,6}

4. Non negativity and Integer Constraints:

$$yt, nt, ft, xt, it, \geq 0$$

Discussion of Model Sensitivities

The study reveals that workforce management by hiring and layoffs generates the largest impact on total expenses. The 480.75 shadow price for layoffs indicates large financial savings from employee retention while the 320-520 hiring shadow price range reveals workforce adjustments significantly impact expenses. The marginal values indicate negative costs for overtime production thus showing it costs more than standard production operations. The workforce controls standard production levels, yet inventory adjustments do not influence total expenses since inventory adjustments show no sensitivity.

If hiring restrictions were removed the company would need less expensive overtime production thus establishing a more efficient cost structure. The company achieves greater output levels from each worker which reduces the need for continuous hiring and firing of employees. The wide sensitivity ranges show inventory constraints have minimal impact so workforce efficiency improvements would create better cost savings than stock adjustments. The implementation of stable workforce management through balanced employee recruitment and termination practices together with reduced overtime work will establish an economical and sustainable production system.

Executive Summary of Solution Approach and Recommendations

Presenting Problem

Workforce planning and production scheduling problems at PM Computer Services require resolution to meet customer requirements while managing expenses related to employee hiring and firing and standard and overtime manufacturing and inventory control. The key goal exists to discover top staffing and production methods that will stabilize operations while maintaining minimal expenses. Three major constraints affect the solution because workforce numbers are limited, and employee acquisition and dismissal expenses are variable and workers have set output boundaries.

Solution Approach

Implementation of an Integer Programming model to achieve minimum total costs by producing products to satisfy customer demand requirements. The model functions based on three key requirements that include unchanging worker output rates and limited overtime hours and inventory usage for stabilizing customer demand. Total expenses are heavily influenced by layoff and hiring costs while overtime production expenses exceed regular production costs by a significant margin. The model results demonstrate workforce optimization serves as the primary cost reduction factor because inventory management has minimal impact on overall cost reduction.

Recommendations

- The use of overtime production should be minimized to cut costs because overtime expenses exceed regular production costs.
- The organization should strike a proper balance between employee recruitment and retention to achieve workforce stability.
- The high shadow price indicates layoffs should be avoided because they would create unnecessary cost increases.
- Organizations should boost individual productivity levels because this will enable them to produce more with fewer workers and minimize workforce fluctuations.
- A followable staffing design combined with strategic production planning creates an operation system that works sustainably while keeping costs affordable.