Rappel de cours

Exercice 2.1

Exercice 2.1.1

$$\int_0^\pi x\sin(x)dx$$

Prenons $f = x$ et $g' = \sin(x)$ donc $f' = 1$ et $g = -\cos(x)$

$$\int fg' = fg - \int f'g = -x\cos(x) - \int -\cos(x) = \sin(x) - x\cos(x)$$

$$\int_0^\pi x\sin(x)dx = [\sin(x) - x\cos(x)]_0^\pi = \pi$$

Exercice 2.1.2

$$\int_0^{\ln(2)} \frac{e^x}{\sqrt{e^x+1}}dx$$

Prenons $u = e^x + 1$, on a $\frac{du}{dx} = e^x$ donc $dx = e^{-x}du$.

$$\int \frac{e^x}{\sqrt{e^x+1}}dx = \int \frac{e^x}{\sqrt{u}}e^{-x}du = \int \frac{1}{\sqrt{u}}du = 2\sqrt{u} = 2\sqrt{e^x+1}$$

$$\int_0^{\ln(2)} \frac{e^x}{\sqrt{e^x+1}}dx = \left[2\sqrt{e^x+1}\right]_0^{\ln(2)} = 2(\sqrt{3} - \sqrt{2})$$

Exercice 2.1.3

$$\int_0^1 \frac{4}{x^4-4}dx$$

Substitution $u = x^4 - 4$ -¿ Dead end.
On a $x^4 - 4 = x^{2^2} - 2^2 = (x^2-2)(x^2+2)$

$$\frac{1}{(x^2-2)(x^2+2)} = \frac{1}{4}(\frac{1}{x^2-2} - \frac{1}{x^2+2})$$

$$\int_0^1 \frac{4}{x^4-4}dx = \int_0^1 \frac{1}{x^2-2} - \frac{1}{x^2+2}dx = \int_0^1 \frac{1}{x^2-2}dx - \int_0^1 \frac{1}{x^2+2}dx$$

Donc

$$\int_0^1 \frac{1}{x^2-2}dx = \int_0^1 \frac{1}{x^2-\sqrt{2}^2}dx = \int_0^1 \frac{1}{(x+\sqrt{2})(x-\sqrt{2})}dx = \int_0^1 \frac{1}{2\sqrt{2}}\left(\frac{1}{x+\sqrt{2}} - \frac{1}{x-\sqrt{2}}\right)dx$$

Maintenant, prenons $u = x + \sqrt{2}$, $\frac{du}{dx} = 1$ donc $du = dx$ et

$$\int \frac{1}{x+\sqrt{2}}dx = \int \frac{1}{u}du = \ln(u) = \ln(x+\sqrt{2})$$

de même avec $u = x - \sqrt{2}$

$$\int \frac{1}{x-\sqrt{2}}dx = \int \frac{1}{u}du = \ln(u) = \ln(x-\sqrt{2})$$

Donc

$$\int_0^1 \frac{1}{x^2-2}dx = \frac{1}{2\sqrt{2}}\left(\left[\ln(x+\sqrt{2})\right]_0^1 - \left[\ln(x-\sqrt{2})\right]_0^1\right)$$

et pour la seconde partie

$$\int \frac{1}{x^2+2}dx$$

On connait l'intégrale de $\frac{1}{x^2+1}$, c'est presque pareil. Il faut juste éliminer le 2 avec un changement de variable judicieux (ou astucieux). Prenons $u = \frac{x}{\sqrt{2}}$, on a $\frac{du}{dx} = \frac{1}{\sqrt{2}}$ donc $dx = \sqrt{2}du$. Ce qui fait:

$$\int \frac{1}{x^2+2}dx = \int \frac{1}{(\sqrt{2}u)^2+2}\sqrt{2}du = \int \frac{\sqrt{2}}{2u^2+2} = \frac{\sqrt{2}}{2}\int \frac{1}{u^2+1} = \frac{\sqrt{2}}{2}\arctan(u) = \frac{\sqrt{2}}{2}\arctan(\frac{x}{\sqrt{2}})$$

Donc

$$\int_0^1 \frac{1}{x^2-2}dx = \left[\frac{\sqrt{2}}{2}\arctan\left(\frac{x}{\sqrt{2}}\right)\right]_0^1 = \frac{\sqrt{2}}{2}\arctan\left(\frac{1}{\sqrt{2}}\right)$$

Pour finir

$$\int_0^1 \frac{4}{x^4-4}dx = blabla - \frac{\sqrt{2}}{2}\arctan\left(\frac{1}{\sqrt{2}}\right)$$

Exercice 2.1.4

$$\int_{-1}^1 \ln(1+x^2)dx$$

Prenons $f = \ln(1+x^2)$ et $g' = 1$ donc $f' = \frac{2x}{x^2+1}$ et $g = x$

$$\int fg' = fg - \int f'g = x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}dx$$

Prenons $u = x^2+1$, on a $\frac{du}{dx} = 2x$ donc $dx = \frac{1}{2x}du$

$$\int \frac{2x^2}{x^2+1}dx = \int \frac{2x^2}{u}\frac{1}{2x}du = \int \frac{x}{u}du$$

Dead end.

On connait l'intégrale de $\frac{1}{x^2+1}$

$$\int \frac{2x^2}{x^2+1}dx = 2\int \frac{x^2+1-1}{x^2+1}dx = 2\int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}dx = 2\int 1 - \frac{1}{x^2+1}dx = 2\int 1dx - 2\int \frac{1}{x^2+1}dx = 2(x - \arctan(x))$$

donc

$$\int_{-1}^1 \ln(1+x^2)dx = \left[x\ln(x^2+1) + 2(\arctan(x) - x)\right]_{-1}^1 = \ln(2) + 2\pi/4 - 2 - (-ln(2) + 2 - 2\pi/4) = 2\ln(2) + \pi - 4$$

Exercice 2.1.5

$$\int_0^1 x^2e^x dx$$

Prenons $f = x^2$ et $g' = e^x$ donc $f' = 2x$ et $g = e^x$

$$\int fg' = fg - \int f'g = x^2e^x - \int 2xe^x dx$$

Prenons $f = 2x$ et $g' = e^x$ donc $f' = 2$ et $g = e^x$

$$\int fg' = fg - \int f'g = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x$$

Donc

$$\int_{-1}^1 \ln(1+x^2)dx = \left[x^2e^x - 2xe^x + 2e^x\right] = e - 5e^{-1}$$