

## Rappel de cours

k

## Exercice 1

### Exercice 1.1 pour A

On a

$$\det(A - \lambda I) = \det \begin{pmatrix} 0 - \lambda & 0 & 1 \\ 2 & i - \lambda & 2i \\ -1 & 0 & 0 - \lambda \end{pmatrix} = (i - \lambda)(\lambda^2 + 1)$$

Calculons  $\det(A - \lambda I) = 0$ ,  $(i - \lambda)(\lambda^2 + 1) = 0$ , donc  $sp(A) = \{i, -i\}$ .

### Exercice 1.2 pour A

Calculons

$$E_i(A) = \ker(A - iI) = \ker \begin{pmatrix} 0 - i & 0 & 1 \\ 2 & i - i & 2i \\ -1 & 0 & 0 - i \end{pmatrix}$$

Cherchons  $\lambda_1, \lambda_2, \lambda_3$  tel que

$$\begin{pmatrix} -i & 0 & 1 \\ 2 & 0 & 2i \\ -1 & 0 & -i \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = 0$$
$$\begin{cases} -i\lambda_1 + 0\lambda_2 + \lambda_3 & = 0 \\ 2\lambda_1 + 0\lambda_2 + 2i\lambda_3 & = 0 \\ -\lambda_1 + 0\lambda_2 - i\lambda_3 & = 0 \end{cases}$$

Echelonnage

$$\begin{cases} \lambda_1 + 0\lambda_2 + i\lambda_3 & = 0 (L_1 = L_1 * -i) \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 (L_2 = L_2 + 2iL_1) \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 (L_3 = L_3 + iL_1) \end{cases}$$

Donc, en fixant  $\lambda_3 = c_3$  et  $\lambda_2 = c_2$  on a  $\lambda_1 = -ic_3$

$$\ker(A - iI) = \{(-i, 0, 1), (0, 1, 0)\}$$

et  $\dim(E_i(A)) = \dim(\ker(A - iI)) = 2$ .

Calculons

$$E_{-i}(A) = \ker(A + iI) = \ker \begin{pmatrix} 0 + i & 0 & 1 \\ 2 & i + i & 2i \\ -1 & 0 & 0 + i \end{pmatrix}$$

Cherchons  $\lambda_1, \lambda_2, \lambda_3$  tel que

$$\begin{pmatrix} i & 0 & 1 \\ 2 & 2i & 2i \\ -1 & 0 & i \end{pmatrix} \cdot \begin{pmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{pmatrix} = 0$$
$$\begin{cases} i\lambda_1 + 0\lambda_2 + \lambda_3 & = 0 \\ 2\lambda_1 + 2i\lambda_2 + 2i\lambda_3 & = 0 \\ -\lambda_1 + 0\lambda_2 + i\lambda_3 & = 0 \end{cases}$$

Echelonnage

$$\begin{cases} -\lambda_1 + 0\lambda_2 + i\lambda_3 & = 0 (L_1 = L_1 \cdot i) \\ 0\lambda_1 + 2i\lambda_2 + 4i\lambda_3 & = 0 (L_2 = L_2 + 2iL_1) \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 (L_3 = L_1 + iL_3) \end{cases}$$

Donc, en fixant  $\lambda_3 = c_3$  on a  $\lambda_1 = ic_3$  et  $\lambda_2 = -2c_3$

$$\ker(A + iI) = \{(i, -2, 1)\}$$

et  $\dim(E_{-i}(A)) = \dim(\ker(A + iI)) = 1$ .

**Exercice 1.3 pour A**

Calculons  $\dim(A)$ .

$$\begin{vmatrix} 0 & 0 & 1 \\ 2 & i & 2i \\ -1 & 0 & 0 \end{vmatrix}$$

Echelonnage

$$\begin{vmatrix} 1 & 0 & 0 \\ 0 & i & 2i \\ 0 & 0 & 1 \end{vmatrix}$$

Donc  $\dim(A) = 3$  et  $\dim(E_i(A)) + \dim(E_{-i}(A)) = 3$  donc diagonalisable.

On a

$$P = \begin{vmatrix} i & -i & 0 \\ -2 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

$$P^{-1} = \begin{vmatrix} -\frac{i}{2} & 0 & \frac{1}{2} \\ \frac{i}{2} & 0 & \frac{1}{2} \\ -i & 1 & 1 \end{vmatrix}$$

$$P^{-1}AP = \begin{vmatrix} -i & 0 & 0 \\ 0 & i & 0 \\ 0 & 0 & i \end{vmatrix}$$

**Exercice 1.1 pour B**

On a

$$\det(A - \lambda I) = \det \left( \begin{vmatrix} i - \lambda & 0 & 2 \\ 0 & 0 - \lambda & 1 \\ 0 & -1 & 0 - \lambda \end{vmatrix} \right) = (i - \lambda)(\lambda^2 + 1)$$

Calculons  $\det(A - \lambda I) = 0$ ,  $(i - \lambda)(\lambda^2 + 1) = 0$ , donc  $sp(A) = \{i, -i\}$ .

**Exercice 1.2 pour B**

Calculons

$$E_i(B) = \ker(B - iI) = \ker \left( \begin{vmatrix} i - i & 0 & 2 \\ 0 & 0 - i & 1 \\ 0 & -1 & 0 - i \end{vmatrix} \right)$$

Cherchons  $\lambda_1, \lambda_2, \lambda_3$  tel que

$$\begin{vmatrix} 0 & 0 & 2 \\ 0 & -i & 1 \\ 0 & -1 & -i \end{vmatrix} \cdot \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{vmatrix} = 0$$

$$\begin{cases} 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 \\ 0\lambda_1 - i\lambda_2 - \lambda_3 & = 0 \\ 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0 \end{cases}$$

Echelonnage

$$\begin{cases} 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0(L_1 = L_3) \\ 0\lambda_1 - i\lambda_2 - \lambda_3 & = 0 \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0(L_3 = L_1) \end{cases}, \begin{cases} 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0 \\ \lambda_3 & = -i\lambda_2 \end{cases}, \begin{cases} 2\lambda_1 + \lambda_2 - i(-i\lambda_2) & = 0 \\ \lambda_3 & = -i\lambda_2 \end{cases}$$

Donc, en fixant  $\lambda_2 = c_2$  on a  $\lambda_1 = 0$ ,  $\lambda_3 = ic_2$

$$\ker(B - iI) = \{(0, 1, i)\}$$

et  $\dim(E_i(B)) = \dim(\ker(B - iI)) = 1$ .

Calculons

$$E_{-i}(B) = \ker(B + iI) = \ker \left( \begin{vmatrix} i+i & 0 & 2 \\ 0 & 0+i & 1 \\ 0 & -1 & 0+i \end{vmatrix} \right)$$

Cherchons  $\lambda_1, \lambda_2, \lambda_3$  tel que

$$\begin{vmatrix} 2i & 0 & 2 \\ 0 & i & 1 \\ 0 & -1 & i \end{vmatrix} \cdot \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{vmatrix} = 0$$

$$\begin{cases} 2i\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 \\ 0\lambda_1 + i\lambda_2 - \lambda_3 & = 0 \\ 2\lambda_1 + \lambda_2 + i\lambda_3 & = 0 \end{cases}$$

$$\begin{cases} \lambda_1 & = 0 \\ i\lambda_2 & = \lambda_3 \\ \lambda_2 & = -i\lambda_3 \end{cases}$$

Donc, en fixant  $\lambda_2 = c_2$  on a  $\lambda_1 = 0$ ,  $\lambda_3 = ic_2$

$$\ker(B - iI) = \{(0, 1, i)\}$$

et  $\dim(E_{-i}(B)) = \dim(\ker(B + iI)) = 1$ .

### Exercice 1.3 pour B

Calculons  $\dim(B)$ .

$$\begin{vmatrix} -i & 0 & 2 \\ 0 & 0 & 1 \\ 0 & -1 & 0 \end{vmatrix}$$

Echelonnage

$$\begin{vmatrix} -i & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Donc  $\dim(B) = 3$  et  $\dim(E_i(B)) + \dim(E_{-i}(B)) = 2$  donc pas diagonalisable.

QED