

Rappel de cours

Exercice 1**1.1**

Forme Trigonometrique: $re^{j\theta} = r \cos(\theta) + j \sin(\theta)$, avec $r = \sqrt{x^2 + y^2}$ et $\theta = \tan^{-1}(\frac{y}{x})$.

Pour $1 + j\sqrt{3}$, $r = 2$, $\theta = \tan^{-1}(\sqrt{3}) = 60$, $2e^{j60} = 2 \cos(60) + j2 \sin(60)$.
 Pour $1 + j$, $r = \sqrt{2}$, $\theta = \tan^{-1}(1) = 45$, $\sqrt{2}e^{j45} = \sqrt{2} \cos(45) + j\sqrt{2} \sin(45)$.

1.2

$$Z = \frac{1 + j\sqrt{3}}{1 + j} = \frac{1 + j\sqrt{3}}{(1 + j)(1 - j)} = \frac{1 + \sqrt{3} + j(\sqrt{3} - 1)}{2}$$

$$r_Z = \sqrt{\frac{(1 + \sqrt{3})^2}{4} + \frac{(\sqrt{3} - 1)^2}{4}} = \sqrt{2}$$

$$\theta_Z = \tan^{-1}\left(\frac{\sqrt{3} - 1}{1 + \sqrt{3}}\right) = \frac{\pi}{12}$$

1.3

On a, $\frac{1 + \sqrt{3} + j(\sqrt{3} - 1)}{2} = r_Z \cos(\theta_Z) + jr_Z \sin(\theta_Z)$. Donc

$r_Z \cos(\theta_Z) = \frac{1 + \sqrt{3}}{2}$ et $r_Z \sin(\theta_Z) = \frac{\sqrt{3} - 1}{2}$.

1.3

$$Z^{1000} = (r_Z e^{j\theta_Z})^{1000} = r_Z^{1000} e^{j1000\theta_Z}$$

$$Z = r_Z^{1000} \cos(1000\theta_Z) + jr_Z^{1000} \sin(1000\theta_Z) = r_Z^{1000} \cos\left(\frac{\pi}{3}\right) + jr_Z^{1000} \sin\left(\frac{\pi}{3}\right)$$