Rappel de cours:

- $\int_a^b f(u(x))u'(x)\,dx = \int_{x(a)}^{u(b)} u\,du$

Exercice 1

Exercice 1.a

$$\int tan(x)\,dx = \int \frac{sin(x)}{cos(x)}\,dx$$

Soit $u(x) = cos(x)$, $u'(x) = -sin(x)$ et $f(u) = -\frac{1}{u}$ donc

$$\int \frac{sin(x)}{cos(x)}\,dx = \int f(cos(x))sin(x)\,dx = \int -\frac{1}{u}\,du = -ln(u) = -ln(cos(x))$$

Exercice 1.b

$$\int \frac{ln(x+1)}{x+1}\,dx$$

Soit $u(x) = ln(x+1)$, $u'(x) = \frac{1}{x+1}$ et $f(u) = u$ donc

$$\int \frac{ln(x+1)}{x+1}\,dx = \int f(ln(x+1)).\frac{1}{x+1}\,dx = \int u\,du = \frac{u^2}{2} = \frac{ln^2(x+1)}{2}$$

Exercice 1.c

$$\int \frac{x}{\sqrt{2x^2+3}}$$

Soit $u(x) = 2x^2+3$, $u'(x) = 4x$ et $f(u) = \frac{1}{4\sqrt{u}}$ donc

$$\int \frac{x}{\sqrt{2x^2+3}}\,dx = \int f(2x^2+3).4x\,dx = \int \frac{1}{4\sqrt{u}}\,du = \frac{1}{4}\int \frac{1}{\sqrt{u}\,du} = \frac{\sqrt{u}}{2} = \frac{\sqrt{2x^2+3}}{2}$$

Exercice 1.d

$$\int \frac{cos(x)}{sin^2(x)}\,dx$$

Soit $u(x) = sin(x)$, $u'(x) = cos(x)$ et $f(u) = \frac{1}{u^2}$ donc

$$\int \frac{cos(x)}{sin^2(x)}\,dx = \int f(sin(x)).cos(x)\,dx = \int \frac{1}{u^2}\,du = -\frac{1}{u} = -\frac{1}{sin(x)}$$

Exercice 1.e

$$\int x\sqrt{x^2+1}\,dx$$

Soit $u(x) = x^2+1$, $u'(x) = 2x$ et $f(u) = \frac{\sqrt{(u)}}{2}$ donc

$$\int x\sqrt{x^2+1}\,dx = \int f(x^2+1).2x\,dx = \int \frac{\sqrt{(u)}}{2}\,du = \frac{1}{2}\int \sqrt{(u)}\,du = \frac{1}{2}\frac{2}{3}u^{\frac{3}{2}} = \frac{1}{3}u^{\frac{3}{2}} = \frac{(x^2+1)^{\frac{3}{2}}}{3}$$

Exercice 1.f

$$\int \frac{1}{xln(x)}\,dx$$

Soit $u(x) = ln(x)$, $u'(x) = \frac{1}{x}$ et $f(u) = \frac{1}{u}$ donc

$$\int \frac{1}{xln(x)}\,dx = \int f(ln(x)).\frac{1}{x}\,dx = \int \frac{1}{u}\,du = ln(|u|) = ln(|ln(x)|)$$

Exercice 1.g

$$\int sin^2(x)cos(x)\,dx$$

Soit $u(x) = sin(x)$, $u'(x) = cos(x)$ et $f(u) = u^2$ donc

$$\int sin^2(x)cos(x)\,dx = \int f(sin(x)).cos(x)\,dx = \int u^2\,du = \frac{u^3}{3} = \frac{sin^3(x)}{3}$$

Exercice 1.h

$$\int e^{sin(x)}cos(x)\,dx$$

Soit $u(x) = sin(x)$, $u'(x) = cos(x)$ et $f(u) = e^u$ donc

$$\int e^{sin(x)}cos(x)\,dx = \int f(sin(x)).cos(x)\,dx = \int e^u\,du = e^u = e^{sin(x)}$$

Exercice 1.i

$$\int \frac{ln^2(x)}{x}\,dx$$

Soit $u(x) = ln(x)$, $u'(x) = \frac{1}{x}$ et $f(u) = u^2$ donc

$$\int \frac{ln^2(x)}{x}\,dx = \int f(ln(x)).\frac{1}{x}\,dx = \int u^2\,du = \frac{u^3}{3} = \frac{ln^3(x)}{3}$$

Exercice 2

$t = u(x) = tan(\frac{x}{2})$ donc $x = u^{-1}(t) = 2arctan(t)$ et $(u^{-1})'(t) = \frac{dx}{dt} = \frac{2}{1+t^2}$. On a $sin(x) = \frac{2tan(\frac{x}{2})}{1+tan^2(\frac{x}{2})}$ donc $1 - sin(x) = 1 - \frac{2tan(\frac{x}{2})}{1+tan^2(\frac{x}{2})} = \frac{tan^2(\frac{x}{2})-2tan^2(\frac{x}{2})+1}{1+tan^2(\frac{x}{2})} = \frac{(tan(\frac{x}{2})-1)^2}{1+tan^2(\frac{x}{2})}$.

Donc $\frac{1}{1-sin(x)} = \frac{1+tan^2(\frac{x}{2})}{(tan(\frac{x}{2})-1)^2}$.

$$\int_0^{\pi/3} \frac{1}{1-sin(x)}\,dx = \int_0^{\pi/3} \frac{1+tan^2(\frac{x}{2})}{(tan(\frac{x}{2})-1)^2}\,dx$$

$$\int_{tan(0)}^{tan(\pi/6)} \frac{1+t^2}{(t-1)^2}.\frac{2}{1+t^2}\,dt = \int_{tan(0)}^{tan(\pi/6)} \frac{2}{(t-1)^2}\,dt$$

Soit $u(x) = x - 1$, $u'(x) = 1$ et $f(u) = \frac{1}{u^2}$ donc

$$\int_{tan(0)}^{tan(\pi/6)} \frac{2}{(t-1)^2}\,dt = 2\int_{tan(0)}^{tan(\pi/6)} f(t-1).1\,dt = 2\int_{tan(0)-1}^{tan(\pi/6)-1} \frac{1}{u^2}\,du = [\frac{2}{u}]_{tan(0)-1}^{tan(\pi/6)-1} = -2 - \frac{2}{tan(\pi/6)-1}$$

Exercice 3

QED