Rappel de cours

Exercice 2.1

Exercice 2.1.1

$$\int_0^\pi x\sin(x)dx$$

Prenons $f = x$ et $g' = \sin(x)$ donc $f' = 1$ et $g = -\cos(x)$

$$\int fg' = fg - \int f'g = -x\cos(x) - \int -\cos(x) = \sin(x) - x\cos(x)$$

$$\int_0^\pi x\sin(x)dx = [\sin(x) - x\cos(x)]_0^\pi = \pi$$

Exercice 2.1.2

$$\int_0^{\ln(2)} \frac{e^x}{\sqrt{e^x+1}}dx$$

Prenons $u = e^x + 1$, on a $\frac{du}{dx} = e^x$ donc $dx = e^{-x}du$.

$$\int \frac{e^x}{\sqrt{e^x+1}}dx = \int \frac{e^x}{\sqrt{u}}e^{-x}du = \int \frac{1}{\sqrt{u}}du = 2\sqrt{u} = 2\sqrt{e^x+1}$$

$$\int_0^{\ln(2)} \frac{e^x}{\sqrt{e^x+1}}dx = \left[2\sqrt{e^x+1}\right]_0^{\ln(2)} = 2(\sqrt{3}-\sqrt{2})$$

Exercice 2.1.3

$$\int_0^1 \frac{4}{x^4-4}dx$$

Substitution $u = x^4 - 4$ -¿ Dead end Ona $x^4 - 4 = x^{2^2} - 2^2 = (x^2-2)(x^2+2)$ mais après??

Exercice 2.1.4

$$\int_{-1}^1 \ln(1+x^2)dx$$

Prenons $f = \ln(1+x^2)$ et $g' = 1$ donc $f' = \frac{2x}{x^2+1}$ et $g = x$

$$\int fg' = fg - \int f'g = x\ln(x^2+1) - \int \frac{2x^2}{x^2+1}dx$$

Prenons $u = x^2 + 1$, on a $\frac{du}{dx} = 2x$ donc $dx = \frac{1}{2x}du$

$$\int \frac{2x^2}{x^2+1}dx = \int \frac{2x^2}{u}\frac{1}{2x}du = \int \frac{x}{u}du$$

Dead end.

On connait l'intégrale de $\frac{1}{x^2+1}$

$$\int \frac{2x^2}{x^2+1}dx = 2\int \frac{x^2+1-1}{x^2+1}dx = 2\int \frac{x^2+1}{x^2+1} - \frac{1}{x^2+1}dx = 2\int 1 - \frac{1}{x^2+1}dx = 2\int 1dx - 2\int \frac{1}{x^2+1}dx = 2(x - \arctan(x))$$

donc

$$\int_{-1}^1 \ln(1+x^2)dx = \left[x\ln(x^2+1) + 2(\arctan(x) - x)\right]_{-1}^1 = \ln(2) + 2\pi/4 - 2 - (-ln(2) + 2 - 2\pi/4) = 2\ln(2) + \pi - 4$$

Exercice 2.1.5

$$\int_0^1 x^2 e^x dx$$

Prenons $f = x^2$ et $g' = e^x$ donc $f' = 2x$ et $g = e^x$

$$\int fg' = fg - \int f'g = x^2 e^x - \int 2xe^x dx$$

Prenons $f = 2x$ et $g' = e^x$ donc $f' = 2$ et $g = e^x$

$$\int fg' = fg - \int f'g = 2xe^x - \int 2e^x dx = 2xe^x - 2e^x$$

Donc

$$\int_{-1}^1 \ln(1+x^2)dx = \left[x^2e^x - 2xe^x + 2e^x\right] = e - 5e^{-1}$$