Exercice 5

Question 5.A.1

$$\int_{-\infty}^{+\infty} \frac{m\theta^m}{x^{m+1}} 1_{\lceil\theta,\infty\lceil}(x)dx = \int_{-\infty}^{\theta} \frac{m\theta^m}{x^{m+1}} 1_{\lceil\theta,\infty\lceil}(x)dx + \int_{\theta}^{+\infty} \frac{m\theta^m}{x^{m+1}} 1_{\lceil\theta,\infty\lceil}(x)dx$$

$$0 + \int_{\theta}^{+\infty} \frac{m\theta^m}{x^{m+1}} dx = \left[\frac{\theta^m}{x^m}\right]_{\theta}^{+\infty} = \frac{\theta^m}{\theta^m} - \frac{\theta^m}{\infty^m} = 1 - 0 = 1$$

Question 5.A.2

$$\forall t \geq \theta, P(X \geq t) = \int_{\theta}^{t} \frac{m\theta^m}{x^{m+1}} dx = \left[\frac{\theta^m}{x^m}\right]_{\theta}^{t} = 1 - \left(\frac{\theta}{t}\right)^m$$

Question 5.A.2

$$E(X) = \int_{-\infty}^{+\infty} x\frac{m\theta^m}{x^{m+1}} 1_{\lceil\theta,\infty\lceil}(x)dx = 0 + \int_{\theta}^{+\infty} x\frac{m\theta^m}{x^{m+1}} dx = m\theta^m \int_{\theta}^{+\infty} \frac{1}{x^m} dx =$$

$$m\theta^m \left[-\frac{1}{(m-1)x^{m-1}}\right]_{\theta}^{+\infty} = -\frac{m\theta^m}{m-1}\left(0 - \frac{1}{\theta^{m-1}}\right) = \frac{m\theta}{m-1}$$

$$E(X^2) = \int_{-\infty}^{+\infty} x^2\frac{m\theta^m}{x^{m+1}} 1_{\lceil\theta,\infty\lceil}(x)dx = 0 + \int_{\theta}^{+\infty} x^2\frac{m\theta^m}{x^{m+1}} dx = m\theta^m \int_{\theta}^{+\infty} \frac{1}{x^{m-1}} dx =$$

$$m\theta^m \left[-\frac{1}{(m-2)x^{m-2}}\right]_{\theta}^{+\infty} = -\frac{m\theta^m}{m-2}\left(0 - \frac{1}{\theta^{m-2}}\right) = \frac{m\theta^2}{m-2}$$

Question 5.A.3

$$V(X) = E(X^2) - E(X)^2 = \frac{m\theta^2}{m-2} - \left(\frac{m\theta}{m-1}\right)^2 =$$

$$\frac{m(m-1)^2 - m^2(m-2)}{(m-2)(m-1)^2}\theta^2 = \frac{m}{(m-2)(m-1)^2}\theta^2$$

Question 5.B.1

Méthode des moments de niveau 1, $M_1 = \frac{1}{n}\sum_{i=1}^{n} X_i$, et $E(X) = M_1$ donc

$$\frac{1}{n}\sum_{i=1}^{n} X_i = \frac{m\theta}{m-1}$$

Donc l'estimateur estimateur

$$\hat{\theta}_1 = \frac{m-1}{mn}\sum_{i=1}^{n} X_i$$