Rappel de cours

Definition 1. Bla bla

Exercice 2

Exercice 2.1

On a par définition

$$F_X(x) = \int_{-\infty}^{x} \lambda e^{-\lambda x} 1_{[0,\infty]} = \int_0^x \lambda e^{-\lambda x} = \left[-e^{-\lambda x}\right]_0^x = 1 - e^{-\lambda x}$$

Exercice 2.2

Calculons

$$F_X(F_X^{-1}(x)) = 1 - e^{-\lambda F_X^{-1}(x)} = x$$

$$e^{-\lambda F_X^{-1}(x)} = 1 - x$$

$$-\lambda F_X^{-1}(x) = \ln(1-x)$$

$$F_X^{-1}(x) = -\frac{1}{\lambda}\ln(1-x)$$

Exercice 2.4

$$\mathbb{P}(X > m_\lambda) = 1 - \mathbb{P}(X \leq m_\lambda) = 1 - F_X(m_\lambda) = 0.05$$

Donc

$$F_X(m_\lambda) = 1 - 0.05 = .95$$

et

$$m_\lambda = F_X^{-1}(.95) = -\frac{1}{\lambda}\ln(1-0.95) = -\frac{\ln(0.05)}{\lambda}$$

Exercice 3

Exercice 3.1

On a par définition:

$$F_X(x) = \int_{-\infty}^{x} \pi^{-1}\frac{1}{1+y^2}dy = \frac{1}{\pi}[arctan(x)]_{-\infty}^{x} = \frac{1}{\pi}(\arctan(x) - \arctan(-\infty)) = \frac{1}{\pi}\left(\arctan(x) + \frac{\pi}{2}\right) = \frac{\arctan(x)}{\pi} + \frac{1}{2}$$

Exercice 3.2

Calculons

$$F_X(F_X^{-1}(x)) = \frac{\arctan(F_X^{-1}(x))}{\pi} + \frac{1}{2} = x$$

$$\arctan(F_X^{-1}(x)) = \pi(x - \frac{1}{2})$$

$$F_X^{-1}(x) = \tan\left(\pi\left(x - \frac{1}{2}\right)\right)$$

Exercice 3.4

$$\mathbb{P}(|X| > m) = 1 - \mathbb{P}(|X| \leq m) = 1 - F_X(m) = 0.05$$

Donc

$$F_X(m) = 1 - 0.05 = .95$$

et

$$m_= F_X^{-1}(.95) = \tan\left(\pi\left(.95 - \frac{1}{2}\right)\right) = 6.31$$