Rappel de cours

k

Exercice 1

Exercice 1.1 pour A

On a

$$\det(A-\lambda I) = \det\left(\begin{vmatrix} 0-\lambda & 0 & 1 \\ 2 & i-\lambda & 2i \\ -1 & 0 & 0-\lambda \end{vmatrix}\right) = (i-\lambda)(\lambda^2+1)$$

Calculons $\det(A-\lambda I) = 0$, $(i-\lambda)(\lambda^2+1) = 0$, donc $sp(A) = \{i\}$.

Exercice 1.2 pour A

Calculons

$$E_i(A) = ker(A-iI) = ker\left(\begin{vmatrix} 0-i & 0 & 1 \\ 2 & i-i & 2i \\ -1 & 0 & 0-i \end{vmatrix}\right)$$

Cherchons $\lambda_1, \lambda_2, \lambda_3$ tel que

$$\begin{vmatrix} -i & 0 & 1 \\ 2 & 0 & 2i \\ -1 & 0 & -i \end{vmatrix} . \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{vmatrix} = 0$$

$$\begin{cases} -i\lambda_1 + 2\lambda_2 - \lambda_3 & = 0 \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 \\ -\lambda_1 + 2i\lambda_2 - i\lambda_3 & = 0 \end{cases}$$

Echelonnage

$$\begin{cases} -i\lambda_1 + 2\lambda_2 - \lambda_3 & = 0 \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 (L_3 = L_1 - iL_3) \end{cases}$$

Donc, en fixant $\lambda_1 = c_1$ et $\lambda_2 = c_2$ on a $\lambda_3 = -ic_1 + 2c_2$

$$ker(A-iI) = \{(1,0,-i),(0,1,2)\}$$

et $\dim(ker(A-iI)) = 2$.

Exercice 1.1 pour B

On a

$$\det(A-\lambda I) = \det\left(\begin{vmatrix} i-\lambda & 0 & 2 \\ 0 & 0-\lambda & 1 \\ 0 & -1 & 0-\lambda \end{vmatrix}\right) = (i-\lambda)(\lambda^2+1)$$

Calculons $\det(A-\lambda I) = 0$, $(i-\lambda)(\lambda^2+1) = 0$, donc $sp(A) = \{i\}$.

Exercice 1.2

Calculons

$$E_i(B) = ker(B-iI) = ker\left(\begin{vmatrix} i-i & 0 & 2 \\ 0 & 0-i & 1 \\ 0 & -1 & 0-i \end{vmatrix}\right)$$

Cherchons $\lambda_1, \lambda_2, \lambda_3$ tel que

$$\begin{vmatrix} 0 & 0 & 2 \\ 0 & -i & 1 \\ 0 & -1 & -i \end{vmatrix} . \begin{vmatrix} \lambda_1 & \lambda_2 & \lambda_3 \end{vmatrix} = 0$$

$$\begin{cases} 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 \\ 0\lambda_1 - i\lambda_2 + \lambda_3 & = 0 \\ 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0 \end{cases}$$

Echelonnage

$$\left\{\begin{array}{ll} 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0 (L_1 = L_3) \\ 0\lambda_1 - i\lambda_2 + \lambda_3 & = 0 \\ 0\lambda_1 + 0\lambda_2 + 0\lambda_3 & = 0 (L_3 = L_1) \end{array}\right. , \left\{\begin{array}{ll} 2\lambda_1 + \lambda_2 - i\lambda_3 & = 0 \\ \lambda_3 & = i\lambda_2 \end{array}\right. , \left\{\begin{array}{ll} 2\lambda_1 + \lambda_2 - i(i\lambda_2) & = 0 \\ \lambda_3 & = i\lambda_2 \end{array}\right.$$

Donc, en fixant $\lambda_2 = c_2$ on a $\lambda_1 = -c_2$, $\lambda_3 = ic_2$

$$ker(B - iI) = \{(-1, 1, i)\}$$

et $\dim(ker(B - iI)) = 1$.

QED