

## Rappel de cours

### Exercice 1

#### 1.1

Forme Trigonometrique:  $re^{j\theta} = r \cos(\theta) + j \sin(\theta)$ , avec  $r = \sqrt{x^2 + y^2}$  et  $\theta = \tan^{-1}(\frac{y}{x})$ .

Pour  $1 + j\sqrt{3}$ ,  $r = 2$ ,  $\theta = \tan^{-1}(\sqrt{3}) = 60^\circ$ ,  $2e^{j60} = 2 \cos(\frac{\pi}{3}) + j2 \sin(\frac{\pi}{3})$ .

Pour  $1 + j$ ,  $r = \sqrt{2}$ ,  $\theta = \tan^{-1}(1) = 45^\circ$ ,  $\sqrt{2}e^{j45} = \sqrt{2} \cos(\frac{\pi}{4}) + j\sqrt{2} \sin(\frac{\pi}{4})$ .

#### 1.2

$$Z = \frac{1 + j\sqrt{3}}{1 + j} = \frac{1 + j\sqrt{3}}{(1 + j)(1 - j)} = \frac{1 + \sqrt{3} + j(\sqrt{3} - 1)}{2}$$

$$r_Z = \sqrt{\frac{(1 + \sqrt{3})^2}{4} + \frac{(\sqrt{3} - 1)^2}{4}} = \sqrt{2}$$

$$\theta_Z = \tan^{-1}\left(\frac{\sqrt{3} - 1}{1 + \sqrt{3}}\right) = \frac{\pi}{12}$$

#### 1.3

On a,  $\frac{1 + \sqrt{3} + j(\sqrt{3} - 1)}{2} = \sqrt{2} \cos(\theta_Z) + j\sqrt{2} \sin(\theta_Z)$ . Donc

$\sqrt{2} \cos(\theta_Z) = \frac{1 + \sqrt{3}}{2}$  et  $\sqrt{2} \sin(\theta_Z) = \frac{\sqrt{3} - 1}{2}$ .

#### 1.4

$$Z^{1000} = (r_Z e^{j\theta_Z})^{1000} = r_Z^{1000} e^{j1000\theta_Z}$$

$$Z = r_Z^{1000} \cos(1000\theta_Z) + jr_Z^{1000} \sin(1000\theta_Z) = \sqrt{2}^{1000} \cos\left(\frac{4\pi}{3}\right) + j\sqrt{2}^{1000} \sin\left(\frac{4\pi}{3}\right) = 2^{500} \cos\left(\frac{4\pi}{3}\right) + j2^{500} \sin\left(\frac{4\pi}{3}\right)$$