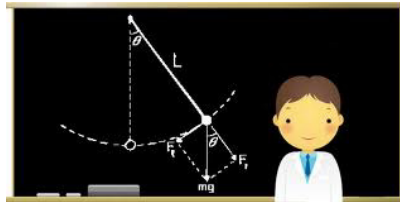


<In the Name of God>

Numerical Analysis Spring 2015

Instructor: Dr. Hamed Masnadi-Shirazi

HW #3

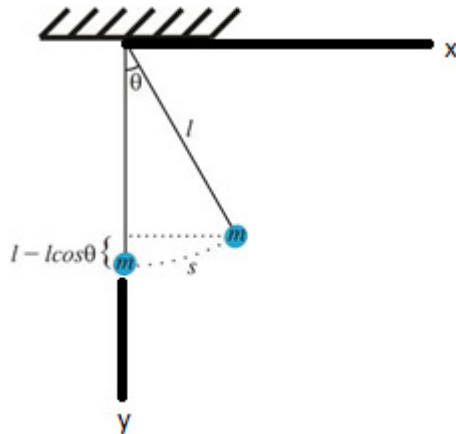


Due Date: (Late HWs will get a zero grade.)

(Note: getting help or helping others on this HW is NOT allowed!)

1) Pendulum:

Assume that we have a pendulum of length=L=10



We call “s” the arc-length as measured in the picture. $s=0$ corresponds to the pendulum at rest. We note the path of the pendulum over time as $s(t)$. We can also show the “x” and “y” coordinates of the pendulum as

$$x(t) = L \sin(s(t)/L)$$

$$y(t) = -L \cos(s(t)/L)$$

It can be shown that the pendulum is governed by the differential equation

$$\frac{d^2 s}{dt^2} = -g \sin\left(\frac{s(t)}{L}\right)$$

where $g=9.8(\text{m/s}^2)$. Use the 4th order Runge-Kutta method to find $s(t)$, $x(t)$ and $y(t)$ for time increments of $h=0.01$ and assume that the initial position of the pendulum is

$s(0)=7$. Plot $s(t)$, $x(t)$ and $y(t)$ from $t=0$ to $t=10$ seconds. What is the position of the pendulum at $t=10(s)$ ($s(10)=?$, $x(10)=?$ And $y(10)=?$).

2) **Rocket:**

It can be shown that the velocity of a rocket can be found from the following simplified rocket equation:

$$\frac{dv}{dt} = \frac{T - \left(\frac{1}{2} C_d \rho A v^2\right) - (mg \sin \theta)}{m}$$

Where :

T = rocket thrust (N)

v = rocket instantaneous velocity (m/s)

t = time (s)

$C_d = 0.75$ = rocket air drag coefficient

$\rho = 1$ = air pressure in Shiraz (kg/m^3)

A = cross sectional area of rocket (m^2)

m = mass of rocket (kg)

$g = 9.8$ (m/s^2)

θ = angle of rocket with horizon = 90 degrees if the rocket goes straight up.

What is the final velocity of the rocket after 10 seconds if it accelerates from stand still with a constant thrust $T=100\text{N}$ and has a constant mass of $m=1$ kg and $A = 0.000483$ (m^2)? When will the rocket break the sound barrier? Use time steps of $h=0.01$ (s) and plot velocity versus time for the rocket.

Submission Guide: submit your Matlab program. Submit the plot and the output of your program. Failing to follow the above guidelines will result in a ZERO grade!