

# Unsupervised self-organized mapping: a versatile empirical tool for object selection, classification and redshift estimation in large surveys

James E. Geach<sup>★†</sup>

*Department of Physics, McGill University, Ernest Rutherford Building, 3600 rue University, Montréal, Québec H3A 2T8, Canada*

Accepted 2011 September 27. Received 2011 September 27; in original form 2011 August 15

## ABSTRACT

We present an application of unsupervised machine learning – the self-organized map (SOM) – as a tool for visualizing, exploring and mining the catalogues of large astronomical surveys. Self-organization culminates in a low-resolution representation of the ‘topology’ of a parameter volume, and this can be exploited in various ways pertinent to astronomy. Using data from the Cosmological Evolution Survey (COSMOS), we demonstrate two key astronomical applications of the SOM: (i) object classification and selection, using galaxies with active galactic nuclei as an example, and (ii) photometric redshift estimation, illustrating how SOMs can be used as totally empirical predictive tools. With a training set of  $\sim 3800$  galaxies with  $z_{\text{spec}} \leq 1$ , we achieve photometric redshift accuracies competitive with other (mainly template fitting) techniques that use a similar number of photometric bands [ $\sigma(\Delta z) = 0.03$  with a  $\sim 2$  per cent outlier rate when using  $u^*$  band to  $8 \mu\text{m}$  photometry]. We also test the SOM as a photo- $z$  tool using the PHoto- $z$  Accuracy Testing (PHAT) synthetic catalogue of Hildebrandt et al., which compares several different photo- $z$  codes using a common input/training set. We find that the SOM can deliver accuracies that are competitive with many of the established template fitting and empirical methods. This technique is not without clear limitations, which are discussed, but we suggest it could be a powerful tool in the era of extremely large – ‘petabyte’ – data bases where efficient data mining is a paramount concern.

**Key words:** methods: data analysis – methods: observational – methods: statistical.

## 1 INTRODUCTION

Extremely large surveys provide the means to take great steps forward in a wide range of astronomical fields, because they probe the large volumes required to detect those very rare objects that would otherwise be nearly impossible to find, and yield the immense sample sizes essential for robust statistical analyses. The crowning achievement of such an approach has undoubtedly been the Sloan Digital Sky Survey (SDSS; York et al. 2000), which at the time of writing is in its eighth data release, and now covers  $14\,000 \text{ deg}^2$  of imaging, and has obtained spectra for millions of objects (Aihara 2011). SDSS marked the beginning of an era of extremely large digital sky surveys and continues to demonstrate its power (in the form of ‘SDSS-III’) across a remarkably wide range of scientific areas, from Galactic studies to cosmology. The Panoramic Survey Telescope and Rapid Response System (Pan-STARRS), Large Synoptic Survey Telescope (LSST) and Dark Energy Survey (DES), amongst others, are poised to take up the mantle set by SDSS in the last decade. These surveys will provide deeper optical and near-infrared imaging over the majority of the solid angle of the sky.

The coming years and decades will see this panoramic approach spill over into other frequency domains; indeed, our view of the Universe is already being transformed by more sensitive large area surveys in the infrared and submillimetre [e.g. *Wide-Field Infrared Explorer* (Wright et al. 2010), *Herschel* (e.g. Eales et al. 2010), the Submillimetre Common-User Bolometer Array-2] and soon the radio regimes [e.g. LOW Frequency ARray (LOFAR) and Square Kilometre Array pathfinders (e.g. Norris et al. 2011)]. We are certainly entering exciting times in terms of our capability to survey the Universe across most of the electromagnetic spectrum, but these large surveys pose a common challenge: how does one efficiently mine the parameter volume when we move into the petabyte regime of information content? Innovative techniques that can efficiently sift and filter the myriad data will be vital, since often one is interested in selecting for a very specific subset of data, for example searching for rare objects (populations of high-redshift galaxies, quasars, low-mass stars or gravitational lenses for example) or events (supernovae, gamma-ray bursts and other transient phenomena).

By design, most of the next-generation panoramic surveys will involve simple continuum imaging. However, many projects will be complemented by ancillary sub-surveys that will accrue more (deeper) imaging and spectroscopic observations over smaller areas. The largest surveys that cover most of the sky will necessarily

<sup>★</sup>Banting Fellow.

<sup>†</sup>E-mail: jimgeach@physics.mcgill.ca

overlap with fields that have already undergone significant observational investments. Several of these [e.g. the Great Observatories Origins Deep Survey, *Chandra Deep Fields*, Cosmological Evolution Survey (COSMOS) field and the like] have been targeted with deep imaging from virtually all terrestrial and space-based facilities, and often have been subject to extensive spectroscopic campaigns, providing very large, deep redshift catalogues (e.g. Lilly et al. 2007). As time goes on, the interplay between overlapping surveys will become more important as we move towards a truly holistic picture of the sky.

Artificial neural networks, or more generally the technique of ‘machine learning’, have been used as a tool in astronomy (as well as other scientific disciplines and industrial applications) for some time (e.g. Storrie-Lombardi et al. 1992; Lahav et al. 1995); the estimation of galaxy redshifts from photometry is a classic example. Perhaps the most successful application of neural networks in an astronomical setting in the past few years has been the ‘crowd sourcing’ technique of Galaxy Zoo (Lintott et al. 2008; Raddick et al. 2008), which exploits thousands of humans to perform visual classification of galaxies from the SDSS, relying on the superior capabilities of the human brain (i.e. a *real* neural network) for pattern recognition. Many of the established machine learning techniques employ supervised learning, where the neural network is trained using a series of input pairs. A common example is the multi-layer perceptron, where each input pair consists of a vector (a set of photometry for example) and a known output (a spectroscopic redshift). The network then attempts to find the mapping that successfully converts the input vectors to the required output – in this sense, it is ‘supervised’ learning. After training, new input vectors (e.g. photometry) can be passed through the network to predict their output (redshifts; e.g. Collister & Lahav 2004).

An alternative approach is to use the input vectors themselves to find the mapping, since if such a mapping between parameters (or combinations of parameters) exists, this information should be latent in the input catalogue. In this paper, I will describe a specific type of unsupervised machine learning – the Kohonen self-organizing map (SOM; Kohonen 1982, 2001) – as a tool for data mining in astronomy. SOMs have found application in other scientific disciplines, notably geophysics and genetics, and other disparate areas, especially those where some form of pattern recognition is required. While there has been some use of self-organization and SOMs in astronomical applications (e.g. Núñez & Llacer 2003; Mahdi 2011), the technique is not in widespread use. In essence, an SOM is a neural network that takes as input a large training set (in this case large astronomical catalogues), and maps it by a process of competitive learning, where neurons compete to become more like members of the training set. The resulting map is a representation of the topology of the input parameter space, encoding correlations between parameters, and allowing one to visualize the high-dimensional properties of a parameter volume in a low-resolution, lower-dimensional way.

This self-organized mapping has often been described as a form of non-linear principle component analysis, and is a variant of  $k$ -means clustering algorithms. It allows one to identify special features (e.g. clustering) in the input catalogue. As the algorithm itself is effectively classifying new input data based on previously seen specimens, after training SOMs be used to ‘classify’ new inputs, even if they only contain a subset of the information used to train the original map. The method is unsupervised in the sense that the user is not required to specify the desired output, as the ‘mapping’ of components of the input vectors is a natural outcome of the learning. Indeed, perhaps the most fascinating aspect of

large SOMs is the potential for emergent behaviour (Ueltsch 2007) allowing one to discover new properties of the input data that would be imperceptible otherwise.

In summary, the SOM is an extremely versatile tool, and could have several possible uses, however in this paper we demonstrate two of its main applications: object selection and parameter estimation. In Section 2, we describe the algorithm, including a toy example, and in Section 3 we present two practical examples using data from the COSMOS field<sup>1</sup> (Scoville et al. 2007). At the end of the paper we provide a brief list of common SOM terminology for convenience. The SOM algorithm used here as a Python class is made available at <http://www.physics.mcgill.ca/~jimgeach/som>, or from the author on request.

## 2 SELF-ORGANIZATION

### 2.1 Learning philosophy

The SOM can be considered as a collection of ‘nodes’ arranged in a grid of arbitrary dimension, although for visualization purposes two dimensions are most common. Each node is attached to a vector of ‘weights’  $\mathbf{w}$  with the same dimension as an input ‘training’ vector,  $\mathbf{t}$ . In the case of a galaxy survey for example, a  $\mathbf{t}$  could comprise five measurements of *ugriz* photometry. In fact, the input data need not actually be vectorized; any input – provided it can be digitized – way could be considered. A further example to consider might be a digital astronomical image, where we might expect the SOM to help perform morphological classifications. Throughout this work, however, we will consider the case of inputs that are represented as vectors, with each vector component made up from standard ‘catalogue data’ such as photometry and redshift information. The map can be considered as a set of ‘component planes’, with a given node in the  $i$ th plane taking the value of  $w_i$ . In the two-dimensional representation, plotting two or more component planes next to each other provides a low-resolution visual representation of the higher-dimensional topology of the input data.

How does the SOM achieve this mapping? To start, each node is initialized with a random weight; this can be selected from a uniform distribution, or an arbitrary probability distribution (limited according to a sensible physical range of values), or even randomly sampled from the input training set. The learning process is then a set of iterations, and follows a simple philosophy: each node ‘competes’ to be the best match to a randomly selected vector from the training set. The winning node – called the best matching unit (BMU) – is rewarded by being allowed to become more like the input vector. In addition, nodes in the vicinity of the BMU,  $r_{\text{BMU}}$ , are also allowed to be altered in the same direction, but to a lesser extent than the BMU.<sup>2</sup>

After many samplings, the nodes can learn to become more like the training set, with the distribution of weights representing the probability distribution of the training set and the relationship between the components of individual weights encoding correlations between parameters. Most importantly, similar nodes get grouped together in the map. This allows one to examine the parameter space topology, and can be used to search for clusters within the parameter space of the training set, and thus provides a means of object classification. In addition, the BMU of any new test galaxy (for example)

<sup>1</sup> <http://cosmos.astro.caltech.edu/>

<sup>2</sup> For convenience, we provide a table of SOM nomenclature at the end of the paper.

contains the SOM's 'best guess' of what that galaxy's parameters should be, based on similar galaxies it has seen before. In the case of incomplete data for a new test galaxy (e.g. a missing redshift), the BMU can provide a prediction for what that missing parameter should be. Thus, the SOM can be a predictive tool.

The process of learning occurs over a series of  $N$  iterations. At each iteration  $t$ , nodes compete to be the best match to a randomly selected training vector, with the BMU being rewarded by changing its weight vector in the direction of the training vector. Crucially, nodes within some vicinity of the BMU ( $r < r_{\text{BMU}}$ ) are *also* allowed to adapt, but to a lesser extent than the BMU. The effect is that nodes with similar properties end up grouped close to each other on the map. The adaptation is set by a learning handicap, called the 'neighbourhood function'  $R$ , that falls off with  $r$ , and decays with learning time. The exact form of the neighbourhood function,  $R$ , is arbitrary, but a Gaussian function is often chosen as a suitable form:

$$R = e^{-r/\sigma} \quad (1)$$

where  $\sigma$  depends on time:

$$\sigma = r_{\text{BMU}} e^{-t/\tau}. \quad (2)$$

Here  $\tau$  is a decay constant, usually chosen to be equal to the number of iterations,  $N$ .

The region of influence around the BMU  $r_{\text{BMU}}$  shrinks over time  $t$ , such that ever smaller regions of the SOM are allowed to adapt as  $t \rightarrow N$ :

$$r_{\text{BMU}} = r_{\text{BMU}}^0 (1 - t/N), \quad (3)$$

where  $r_{\text{BMU}}^0$  is taken to be half of the size of the map. Finally, *all* nodes in the SOM have their learning handicapped over time, with an additional factor,

$$L = e^{-t/\tau}. \quad (4)$$

The effect of these decaying learning rates and neighbourhood function is the sequence of refinement, where the most dramatic and coarse organization of nodes occurs early in the learning process, with subsequent steps fine-tuning the SOM on smaller scales and resolving more subtle topology in the data.

## 2.2 The algorithm

From the learning sequence described in Section 2.1, the algorithm itself can be summarized as follows.

- (1) Initialize the SOM by randomly assigning vectors to each node. The vectors can be selected uniformly from within some suitably limited parameter volume, or take the values of vectors sampled randomly from the training set.
- (2) A training vector  $\mathbf{t}$  is picked randomly from the training set.
- (3) For the  $i$ th node in the map described by weight vector  $\mathbf{w}_i$ , the Euclidean distance of that weight from the  $j$ th training vector  $\mathbf{t}_j$  is assessed:  $d_i = |\mathbf{t}_j - \mathbf{w}_i|$ . The winning node has  $\min(d)$  – i.e. it was the closest to the input vector, and becomes the BMU.
- (4) Every node within the region of influence,  $r_{\text{BMU}}$ , is allowed to be pulled in the direction of the input vector, weighted by the learning factors (equations 1–4):  $\mathbf{w}'_i = \mathbf{w}_i + \delta_j \times R \times L \times (\mathbf{t}_j - \mathbf{w}_i)$ . The factor  $\delta_j$  is an optional additional weighting that can take into account the measurement uncertainty, or data quality of each element of the training vector. This penalizes unreliable data by not allowing it to contribute heavily to the development of the map.
- (5) Repeat Steps 2–4 for  $N$  iterations (where  $N$  is sufficiently large that it oversamples the training set several times), or until  $r_{\text{BMU}} = 1$  (or a user chosen minimum).

After many iterations, the SOM will evolve such that similar regions are geometrically close to each other on the map. Although the nodes of the SOM are distributed in a 2D grid, the boundaries of the grid are periodic, such that the 2D projection is effectively an unravelled toroid. Wrapping the boundaries ensures that trained nodes are not 'pushed off' the boundaries of the map. A plot of the 2D grid coloured by the value of the  $i$ th weight of each node is called a component plane, and a comparison of different component planes can be used to study relationships between parameters in the training set.

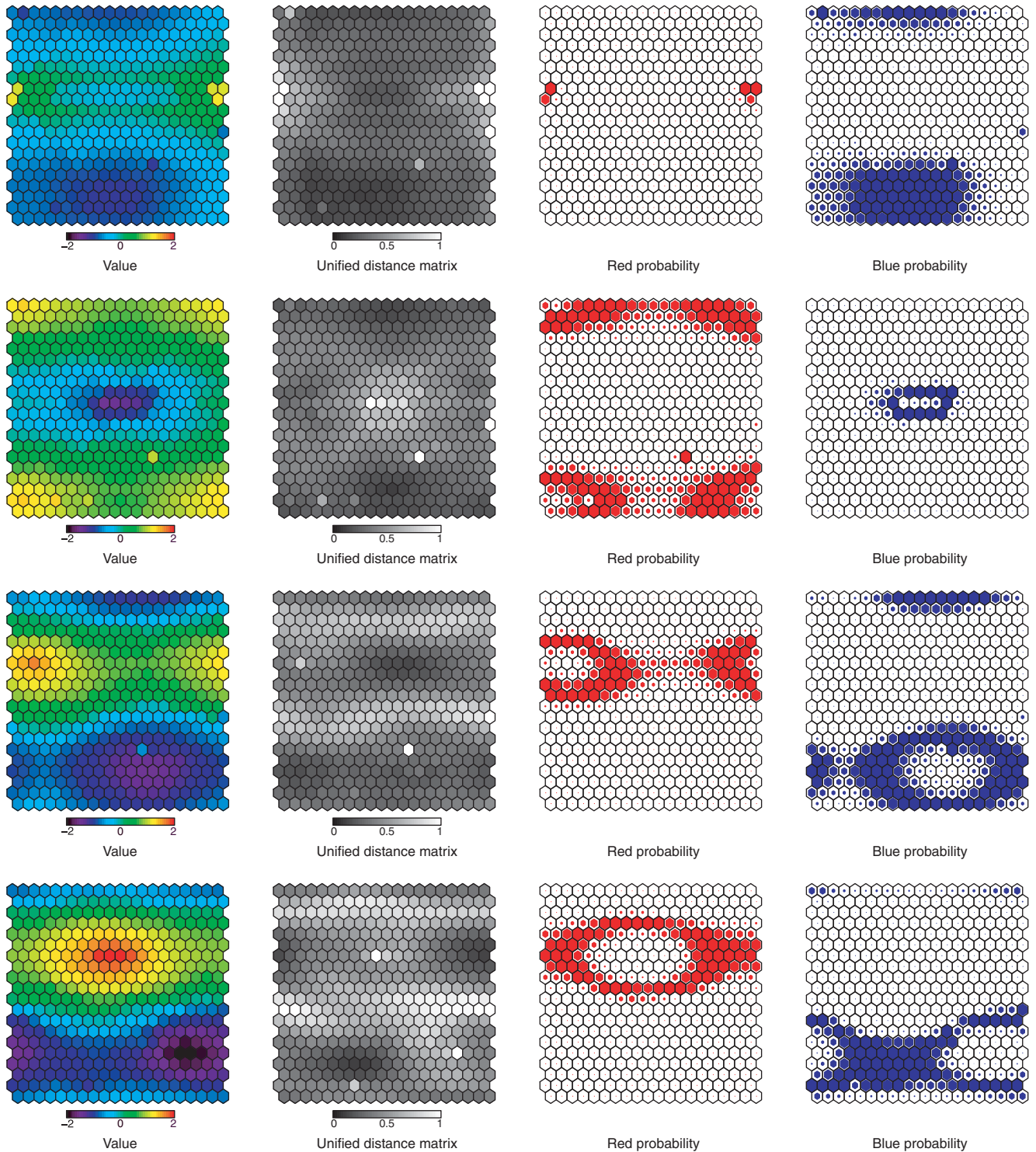
Restricting the learning rate of the SOM as a function of time, and only allowing it to change in ever finer regions, ensures that the introduction of new training vectors refines the SOM, rather than obliterating the learning of previous iterations. On this note, one requires the total learning time (i.e. how many training vectors are used in the learning) to sufficiently oversample the input training set so that all training vectors are given a chance to contribute to the learning at different stages of refinement. Note that since the SOM is initialized randomly, and training vectors are selected randomly, SOMs trained on the same input set will not 'look' identical; however the encoding of the map should be equivalent – all that matters is that similar nodes are close to each other (and distant from dissimilar nodes) on the toroidal surface. The key characteristic of the self-organization is that it retains the 'topology' of the input training set, revealing correlations between inputs that are not obvious. In fact, the SOM is often described as a form of non-linear principle component analysis.

## 2.3 A toy example

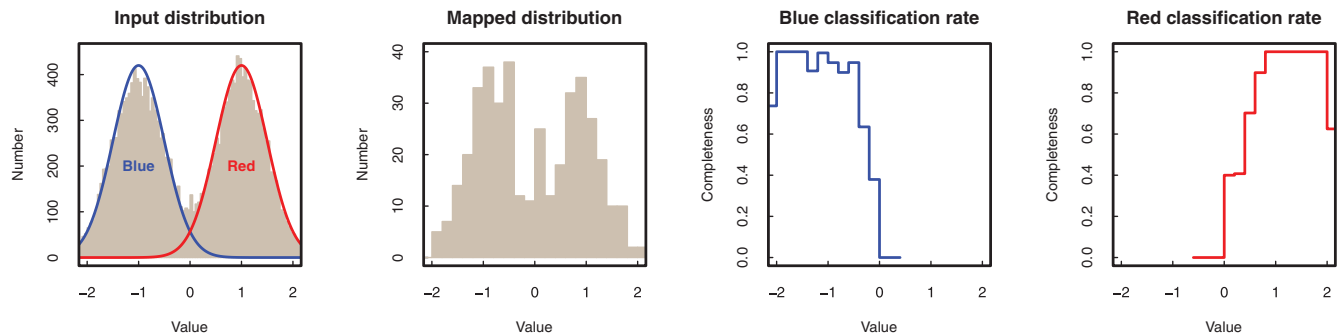
Before we move to real world data, to demonstrate the concept of self-organization, we consider a simple toy example. In this example, we have two 'populations' which are simply represented as two Gaussian distributions. We will label these as 'Red' and 'Blue'. Red and Blue have means of  $\mu_{\text{Red}} = 1$ ,  $\mu_{\text{Blue}} = -1$  and both have scales  $\sigma = 0.5$ . We now randomly draw 10 000 samples from Red and Blue and consider these as our training set – simply a list of 20 000 numbers. Can we use self-organization to separate these two populations and predict whether a new test value belongs to the Red or Blue population? Of course, this is a trivial example, because we could have achieved the same result by simply plotting a histogram of the parameter values, found the form of the toy distributions and therefore assigned a probability to any new value to determine the likelihood that it belongs to Red or Blue. Still, this is a good demonstrative example.

We create a  $20 \times 20$  node SOM, initialized with random weights selected uniformly from the 20 000 member training set. We set-up the initial SOM parameters as described in Section 2.1, and allow the total number of iterations to be 200 000, thus oversampling the training set by a factor of 10. The single component plane of the SOM seen at different stages of the learning process is shown in Fig. 1. The component plane is coloured by the 'value' of the (in this case single element) weight of each node, each of which has competed to represent the values of the training set. It is clear that the map has arranged itself into two distinct regions – this is because the training sample is itself distributed around two distinct values (the means of the distributions). Nodes that are close together (in terms of map distance) have similar values, and there is a clear interface region on the map where the distributions overlap. To reinforce this point, next to the component plane we also show the so-called unified distance matrix (UDM, or U-Matrix), which visualizes the average distance between the weights of neighbouring nodes. The





**Figure 1.** A toy example of self-organization. We consider two populations, ‘Red’ and ‘Blue’, defined by two Gaussian distributions with means of 1 and  $-1$ , respectively. We generate a catalogue containing 10 000 of each class, randomly drawing from the two Gaussians, and allow a  $20 \times 20$  node SOM to organize the values. The different rows show different stages of the learning (1000, 10 000, 100 000 and 200 000 iterations top to bottom). The bottom-left panel shows the clear segregation of map into two distinct regions representing the two populations; note how the maximum and minimum values are separated by a large grid distance (the hexagonal cell representation is traditional for SOM visualization). Boundaries defining the two clusters are made apparent by the UDM, which represents the average distance (in parameter space) between nodes (a grey-scale colour scheme is traditional for the UDM). Collections of nodes with low UDM values bordered by swathes with high UDM values can be considered distinct clusters. Finally, the two right-hand columns show a visualization where the size of the coloured nodes is scaled with the probability that the node value is drawn from the Red or Blue distributions, clearly showing how effective the SOM is at cleanly selecting the two classes in this low-resolution mapping.



**Figure 2.** From left: input toy training sample of ‘Red’ and ‘Blue’ objects drawn from two Gaussian distributions; distribution of node weight values in trained SOM (Fig. 1) showing how nodes are distributed in a way that represents training set. The two right-hand panels show the fraction of correctly identified Blue and Red objects from a new test sample of 2000 objects, where we have used the UDM division of the SOM into two main regions to classify new test data. The SOM can successfully identify new inputs, and the properties of the recovered distributions are well matched to the true underlying sample distributions.

U-Matrix is a means of identifying boundaries of ‘clusters’ within the map. Small UDM values indicate that neighbouring nodes have very similar weights and larger values indicate transition regions between clusters.

The segregation of the different parts of the map allows us to label certain nodes in the map as ‘Blue’ and some as ‘Red’ – in other words, it will allow us to classify new inputs (i.e. new samples that the SOM has never seen) based on their BMU (it will either be in the Red or Blue class). To further demonstrate this, in Fig. 1 we show two versions of the SOM but this time scale the size of nodes based on the probability that their weight value was drawn from the Red or Blue distributions. This clearly highlights how the different parts of the map defined by the UDM correspond to the two clusters in the input parameter space. In Fig. 2 we show the actual input distribution of Red and Blue objects, and the distribution of the values of the weights of nodes in the trained map. Note that the SOM has identified several nodes which define an ambiguous classification where the two abundance of the two populations is equal at values near zero.

We have labelled 101/400 nodes as ‘Red’ and 135/400 nodes as ‘Blue’ classifications based on the map division made apparent by the UDM. To test the SOM, we use these nodes to classify 1000 *new* inputs from each of the Red and Blue populations to find the identification rate, defined by the fraction of new Blue or Red objects that are correctly classified based on their BMU in the trained SOM. The results are shown in Fig. 2. Not only does this simple classification procedure successfully identify new test data, it correctly recovers the main properties of the underlying distribution: the mean and standard deviations of the values of objects classified as Red and Blue are  $\mu_{\text{Red}} = 1.03$ ,  $\mu_{\text{Blue}} = -1.01$ ,  $\sigma_{\text{Red}} = 0.49$  and  $\sigma_{\text{Blue}} = 0.51$ , compared to the input distribution of  $\mu_{\text{Red}} = 1$  and  $\mu_{\text{Blue}} = -1$  and  $\sigma = 0.5$ . Exactly the same principle can be applied to astronomical data sets, and so building on this trivial example we now demonstrate two real world applications of an SOM trained on galaxy data from the COSMOS, where we now include many more parameters in the training.

### 3 DEMONSTRATIONS USING REAL DATA

#### 3.1 Object classification and selection

Colour–colour plots are a traditional method of isolating objects of interest, since populations with similar spectral properties will have similar broad-band colours and therefore cluster together, or fol-

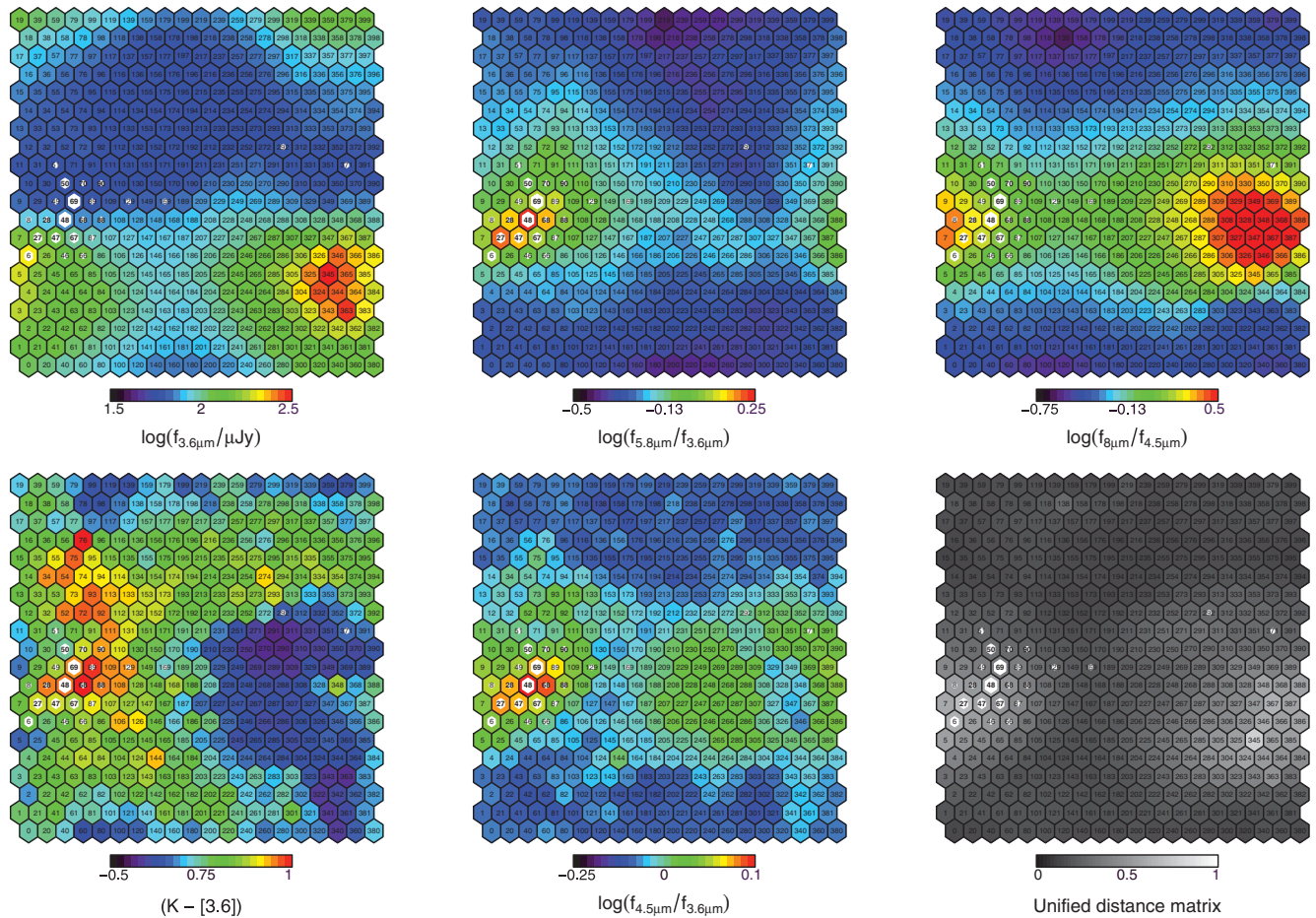
low loci in appropriate colour–magnitude or colour–colour planes. Perhaps the most successful example in extragalactic studies is the selection of distant galaxies by virtue of the Lyman break drop out, where UV–optical broad-band filters that straddle the redshifted Lyman break can efficiently sift  $z \sim 3$  galaxies from a field (Steidel & Hamilton 1993; Madau 1995; Steidel et al. 1996). There are many similar examples of highly effective selection of objects using simple colour criteria, and more recently this has been applied with great success for very high  $z$  ( $z \sim 6$ –9) galaxies that drop-out of optical bands altogether (e.g. Bouwens et al. 2010; McLure et al. 2010). More complicated selections can be constructed not only to pick-out galaxies at specific redshifts, but also isolate those with certain properties (e.g. the star-forming/passive  $z > 1.4$  galaxy selection of Daddi et al. 2004).

Here we demonstrate how the SOM can be used like an  $N$ -dimensional colour–magnitude diagram, and when trained using a large catalogue, exploited to identify those ‘clusters’ of interesting objects. The trained SOM can then be applied as a classification and filtering tool to extract objects of interest from a new input catalogue.

##### 3.1.1 Selecting active galactic nuclei using *Spitzer* IRAC colours

*Spitzer* Infrared Array Camera (IRAC; 3.6–8  $\mu\text{m}$ ) colours have been shown to be very effective at selecting active galactic nuclei (AGN), including those whose optical emission is obscured by dust, since these objects have a characteristic red power-law continuum in the near-/mid-infrared that starts to dominate over the stellar emission at a rest-frame wavelength of  $>2$   $\mu\text{m}$  (Lacy et al. 2004; Stern et al. 2005). This results in a red locus in IRAC colour space that stands out prominently from the general galaxy population. Can we identify this population by self-organizing a catalogue of galaxies with IRAC photometry?

We take the *Spitzer* component of COSMOS (S-COSMOS; Sanders et al. 2007) and set the training weights to be the 3.6  $\mu\text{m}$  flux and the 5.8  $\mu\text{m}$ /3.6  $\mu\text{m}$  and 8.0  $\mu\text{m}$ /4.5  $\mu\text{m}$  colours. A SOM is initialized with  $20 \times 20$  nodes. For the purposes of this demonstration we restrict the catalogue to detections in all four bands and a 3.6  $\mu\text{m}$  flux limit of 50  $\mu\text{Jy}$ . The training catalogue has 10 488 objects, and we allow the SOM to iterate 104 880 times in order to oversample the catalogue by a factor of 10 during the training. The three component planes of the trained SOM are shown in Fig. 3, clearly showing the structures representing the input catalogue (recall that the boundaries of the grid are periodic). To help interpret the



**Figure 3.** The component planes of an SOM trained on  $\sim 10^4$  IRAC-selected galaxies in the COSMOS field (Section 3.1). Note that the boundaries of each plane are periodic. Nodes have been labelled with a numeric index to ease the comparison between the component planes. The SOM was trained using the 5.8/3.6  $\mu\text{m}$  and 8/4.5  $\mu\text{m}$  colours and the 3.6  $\mu\text{m}$  flux density (top panels), whereas the  $K_s - [3.6]$  and 4.5/3.6  $\mu\text{m}$  colours were phantom vectors that did not contribute to the learning, but were allowed to follow the SOM adaption. Several structures are visible that represent real populations: for example the peak in the 3.6  $\mu\text{m}$  flux density map represents stars, and there is clear segregation of red and blue IRAC sources that can be matched to structures in the classic colour–colour plot. The red locus in the 5.8/3.6  $\mu\text{m}$  and 8/4.5  $\mu\text{m}$  colours can be identified, and we indicate the positions of the BMUs of 83 spectroscopically identified BLAGN from zCOSMOS as white hexagons (scaled in size to represent the number of BLAGN falling in each node). About half of the 83 BLAGN occupy just two nodes (48 and 69), and this helps us identify which nodes are best for our ‘AGN selection’. The identification of clusters of nodes for object selection is equivalent to colour-cuts in the traditional colour–colour plane, and we illustrate this in Fig. 4.

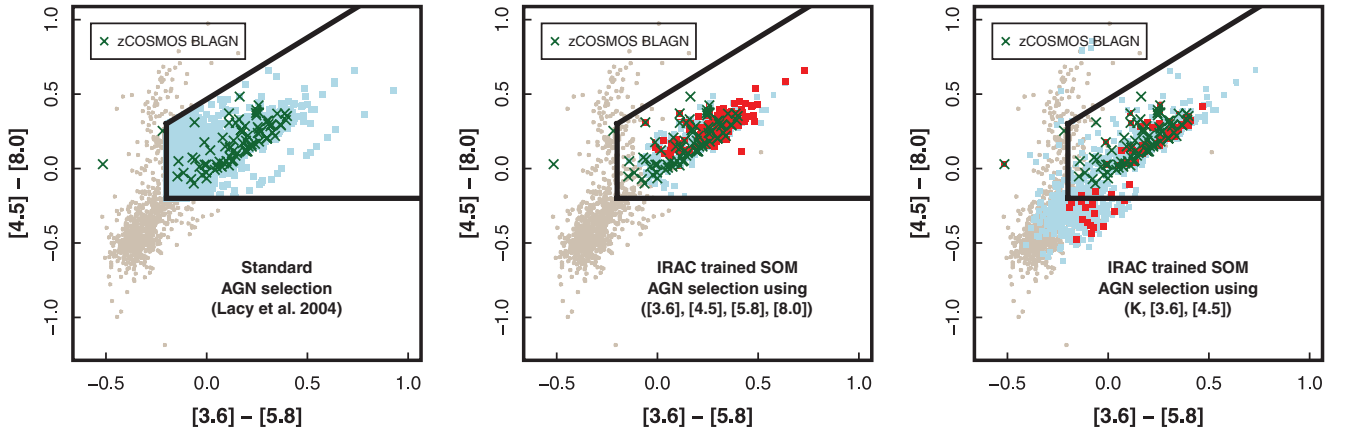
figure, the nodes have been labelled with a numeric index, and to understand the correlations between the component planes, one should compare the value of common nodes in each plane. Correlations between the two colours are apparent, and we can clearly identify the cluster of nodes that are red in both sets of colours; these nodes represent the locus of galaxies with power-law colours that would be apparent in the traditional colour–colour plot (Fig. 4). Therefore, rather than parametrizing the AGN selection as a series of cuts in colour space, we can simply use the map as a filter, classifying any input galaxy as an AGN if its BMU is one of these nodes.

To illustrate the accuracy of the selection, and to verify which are the correct nodes to use as robust ‘AGN selectors’, we have taken the 83 galaxies in the zCOSMOS catalogue (Lilly et al. 2007) identified as broad line AGN (BLAGN)<sup>3</sup> and found their BMU in the trained SOM. Nearly 50 per cent of the BLAGN fall in just two

nodes, and 80 per cent are described by 11 nodes, most of which are contiguous; we highlight these in Fig. 3. Although the SOM can ‘discover’ new classifications, the use of known objects (in this case spectroscopically identified BLAGN) can be of great use when trying to label nodes, and to assess the quality of subsequent selections. For example, the two nodes that successfully describe 50 per cent of the BLAGN could be taken as ‘high-confidence’ AGN nodes, with the remaining nodes being lower confidence selectors. Of course, as in normal techniques, there is a balance between completeness and contamination. As a guide to the contamination rate, we consider the two ‘high confidence’ nodes and find out how many of the zCOSMOS galaxies that are not classified as BLAGN (and have very secure redshifts, confidence class 3.5 or 4.5) fall into these nodes. Together, the two nodes 48 and 69 pick out 114 galaxies that are not BLAGN, in addition to the 39 (in the same redshift confidence class) that are. However, most of this contamination is from just one node (69); if we restrict our AGN selection node to 48 only (actually the reddest in IRAC colour; see Fig. 3), of the 19 zCOSMOS galaxies that match this node, *only one*

<sup>3</sup> zCOSMOS catalogue entries given the tag ‘13.x’ or ‘14.x’.





**Figure 4.** Traditional colour–colour plots for AGN selection. The first (left) panel shows the standard wedge colour selection defined by Lacy et al. (2004; see also Stern et al. 2005). We also show the position of spectroscopically identified BLAGN from zCOSMOS, highlighting the prominent colour–colour locus. The central panel shows the galaxies selected as AGN using the nodes identified in Fig. 3, where the BMU was calculated using both IRAC colours. Red points represent our ‘robust’ selection, utilizing just two nodes that best match  $\sim 50$  per cent of the 83 BLAGN (Fig. 3). Blue points represent less secure classifications from a further nine nodes. Together, these identify 80 per cent of the known BLAGN. The final (right) panel shows the same SOM selection, but this time the BMU was calculated using *only* the  $K_s$ – $3.6\ \mu\text{m}$ – $4.5\ \mu\text{m}$  photometry. This demonstrates that the SOM can be used as an effective selection tool, even when the key information required for the traditional selection is incomplete. In this case the 11 ‘AGN nodes’ correctly identified  $\sim 40$  per cent of the known BLAGN, but there is clearly some scatter away from the locus. In both cases, the completeness could be improved by including more nodes in the selection, but at the cost of contamination.

is not classified as a BLAGN. These contamination rates should only be taken as a guide, given the likely incompleteness in the spectroscopic selection and classification of galaxies in the input zCOSMOS catalogue. Nevertheless, it is clear that the SOM could provide a very clean method for selecting objects of interest.

In Fig. 4, we plot the traditional IRAC colour–colour plane, with the standard Lacy–Stern selection wedge indicated (although the Stern selection is actually defined slightly differently, the broad selection is effectively the same). As described above, we have chosen 11 nodes as our ‘AGN classification’, two of which we define as high-confidence. We then re-pass the input catalogue through the SOM, this time noting which sources have BMUs matching one of these classification nodes. The result is a clean selection of galaxies along the expected AGN locus, and as expected we identify 80 per cent of the BLAGN from the zCOSMOS sample. It should be possible to refine the efficacy of the selection by moving to a higher resolution SOM (more nodes), which would improve the ability of the SOM to resolve finer details in the topology of the data set (in fact it is not clear what the optimum SOM resolution is for a given training set, but ideally it should have many more nodes than there are parameters; see Section 3.3). Here we chose a fairly coarse  $20 \times 20$  SOM to better illustrate the component planes, and even at this low resolution, the SOM is a remarkably powerful and clean selection tool.

To improve the selection of different types of AGN, more information could be added to the training. For example, if one wanted to distinguish between obscured and unobscured AGN, an optical band could be introduced: the optical–NIR colours of obscured AGN are significantly redder than unobscured AGN (e.g. Hickox et al. 2007). The extra information carried by, say, the  $(R - [4.5])$  colour would allow the SOM to separate the two classes.

In this special case we already knew that we were classifying AGN, and could easily identify the part of the SOM that mapped this sub-population, a task that was made easier with the sample of robustly identified BLAGN. However, perhaps the most exciting possibility the SOM offers is the opportunity to detect *new* classifications based on clustering in the parameter volume that

become apparent in component planes that would otherwise be undetected using standard techniques. As described in Section 2.3, one technique of identifying significant clustering is to calculate the so-called UDM or U-Matrix, which visualizes the average ‘distance’ to neighbouring nodes. Clusters of nodes that are close to each other (i.e. similar UDM values), bordered by regions where the UDM values are large, could be considered as clusters, and therefore potentially new classifications. We show the UDM for the present example in Fig. 3, although this is not always an appropriate method of identifying clusters. Upcoming large surveys hold great promise for this type of data exploration; once new potential classifications are identified with the SOM, it should be possible to isolate those objects and properly assess their nature. The SOM provides a way of finding those key, potentially rare objects from the overwhelmingly large catalogues that are currently being produced.

### 3.1.2 Exploiting the map: the case of incomplete data

What if we did not have the full slew of IRAC photometry for a new test galaxy that we wish to classify, but instead have another photometric band? Here we consider how cleanly the SOM trained above can select those obscured AGN using just  $K_s$ ,  $3.6\ \mu\text{m}$  and  $4.5\ \mu\text{m}$  photometry (this is a practical example, now that *Spitzer* is operating in post-cryogenic ‘Warm mode’ it has lost the capability of its longer wavelength detectors, and so new fields will not have  $5.8$  and  $8\ \mu\text{m}$  photometry to perform the classic selection.).

To approach this challenge, during the training of the SOM we allowed two ‘phantom’ components to be added to each training vector:  $(K_s - [3.6])$  and  $([3.6] - [4.5])$ . The  $K_s$ -band data (from the Infrared Side Port Imager on the 4-m Cerro Tololo Inter-American Observatory and FLAMINGOS on the 4-m Kitt Peak National Observatory) are taken from the COSMOS photometric catalogue (2006 version; Capak et al. 2007). These additional components are not allowed to take part in the learning (i.e. they are not considered in part 3 of the algorithm in Section 2.2), but their values are still allowed to change, and thus they get mapped into the SOM. In ef-

fect, this tells us what set of  $(K_s - [3.6])$  and  $([3.6] - [4.5])$  values correspond to the full-band IRAC AGN selection described above. We can now introduce new test galaxies and find out what their BMU is on the basis of *just* their  $(K_s - [3.6])$  and  $([3.6] - [4.5])$  colours. Again, if they match any of the nodes we tagged as ‘AGN’ above, these galaxies can be sifted out, but we expect the selection to be less efficient, since it is now easier for galaxies to be scattered away from the selector nodes. We plot the result of this alternative selection in Fig. 4. As expected, the efficacy of the selection has been reduced, with only  $\sim 40$  per cent of the BLAGN identified, and more scatter away from the standard locus (especially in the case of the lower-confidence nodes). Nevertheless, the SOM is still effective at picking out AGN, even in this case where we are ‘missing’ some of the information used in the traditional selection.

As mentioned above, the performance could be improved by moving to an SOM with a larger number of nodes, and thus allowing finer mapping resolution. In the case of using  $(K_s - [3.6])$  and  $([3.6] - [4.5])$  colours, contamination could also be reduced by only using a subset of the nodes we have classified as AGN on the basis of their  $3.6\text{--}8\text{ }\mu\text{m}$  colours – i.e. just using the ‘highest quality’ nodes, as indicated in Fig. 4.

### 3.2 Photometric redshifts

#### 3.2.1 Setting up the problem

When estimating a photometric redshift, we assume that there is a mapping between a galaxy’s true redshift  $z$ , and photometry vector  $\mathbf{p}$  such that  $z = F(\mathbf{p})$ . If such a mapping exists, then the information to find  $F$  should be latent in a large galaxy catalogue where both photometry and spectroscopic redshifts are available. Self-organization of such a training set will naturally encode  $F$ , and therefore an SOM can be used to predict the redshifts (or indeed any other parameter that was involved in the training) of new galaxies where, for example, only a subset of photometry is known. This technique could be easily applied to a large imaging survey that contains a smaller spectroscopic component in order to robustly estimate redshifts for those galaxies lacking spectroscopic coverage. The advantage of using an SOM for photometric redshift estimation is that it is completely empirical, requires no assumptions about the spectral properties of the galaxies and involves no user intervention to guide the learning (i.e. the learning is unsupervised). However, there are two fundamental limitations to the method.

(1) The SOM cannot accurately extrapolate the properties of objects, should they fall outside of the parameter volume of the original training set. For example, if a catalogue limited to  $z \leq 1$  is used to train the SOM, it will catastrophically fail to predict the redshift of a  $z = 2$  galaxy, because a galaxy of this type has not been ‘seen’ by the SOM. Instances of such failures could be flagged, because their ‘distance’ from the BMU will be large. It is therefore essential that the training set is a representative sample, the larger the better, with a well-known redshift distribution that can aid in the interpretation of the reliability of predictions.

Actually, one *could* use information stored in the SOM to extrapolate photometric redshifts beyond the range of the training set, in the sense that the photometric weights of each node actually represent low-resolution versions of the spectral shapes of galaxies (the broad-band photometry could be simply interpolated to provide a continuous spectrum). Although more time-consuming, if these low-resolution spectra were trusted to be a representative sample of the full range of galaxy types, it should be possible to use them

in the usual  $\chi^2$  template fitting procedures of other photo- $z$  methods, allowing the spectra to redshift beyond the upper bound of the training set and convolving with the relevant filter transmissions.

(2) Related to (1), any biases in the training set will also bias the prediction of unknown parameters in new test data. In this example that bias might be the redshift distribution of the training set; the SOM will have seen more examples of galaxies at the peak of the distribution compared to the tails, potentially biasing the redshift estimates of galaxies in the tails towards the centre of the distribution. Similarly, if the training set contains exclusively red galaxies (classically selected luminous red galaxies for example), then the SOM will only be useful in predicting the properties of input galaxies with similar characteristics. In summary, the predictive power of the SOM is a strong function of the parameter distribution of the training set, and so a proper understanding of the statistical nuances of the training set is of critical importance when interpreting the SOM.

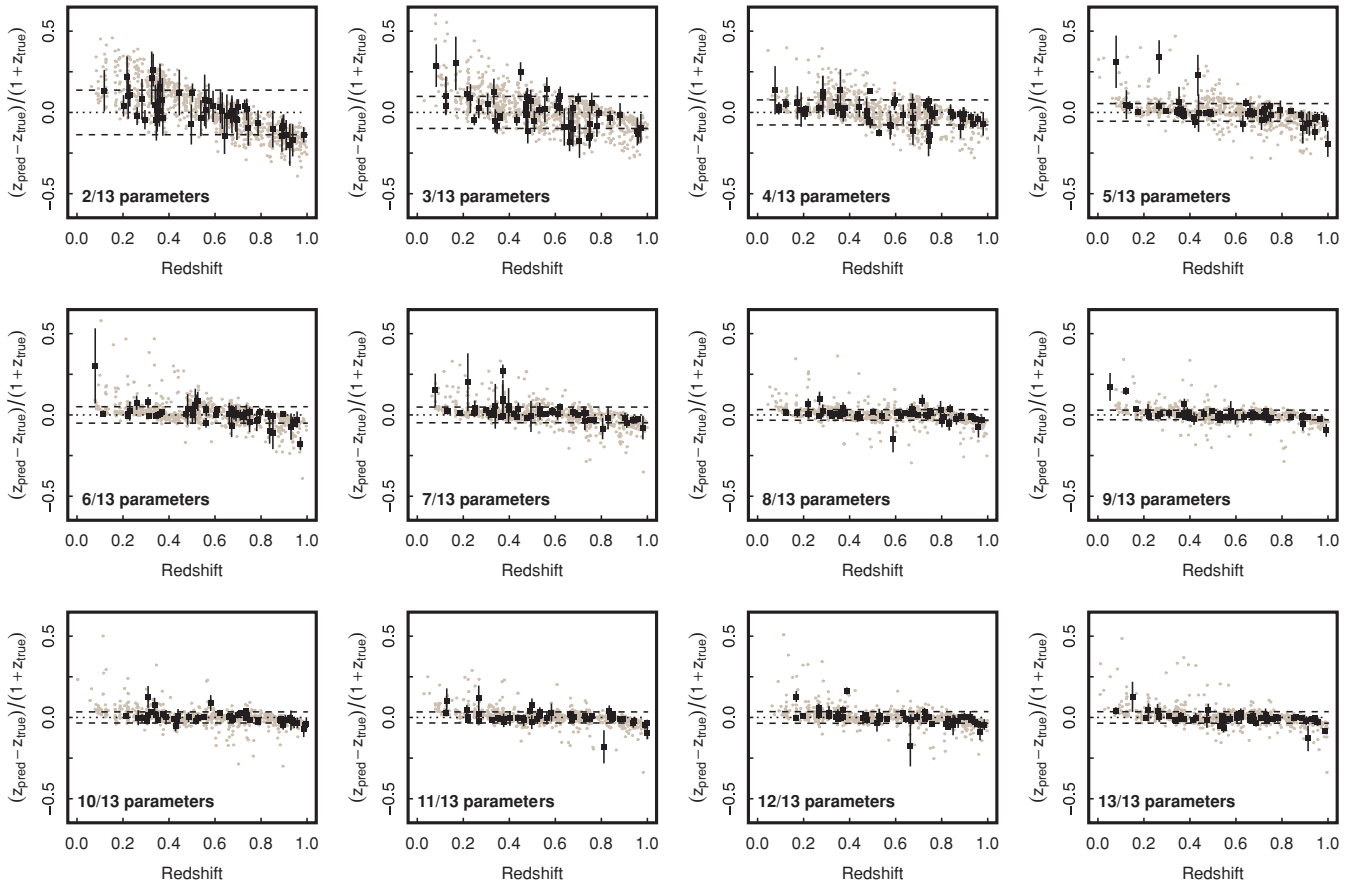
In this demonstration, we again use the photometric data from COSMOS (Capak et al. 2007) and S-COSMOS (Sanders et al. 2007), but merge it with 8910 spectroscopic redshifts from version 3.5 of the zCOSMOS (bright) sample, the  $i < 22.5$  mag limited spectroscopic branch of the survey (Lilly et al. 2007). The training subset is limited to galaxies with  $z \leq 1$  that are detected in each of the  $u^*BgVrizK_s$  and  $[3.6]$ ,  $[4.5]$ ,  $[5.8]$ ,  $[8.0]$  bands. In general, missing data (e.g. lack of coverage in a particular band for a galaxy, perhaps due to masking or contamination) could be dealt with, for example, by not allowing the missing weight to contribute to the learning and/or handicapping the learning coefficient for that particular test vector. Similarly, upper detection limits can be treated as equivalent to measurements at the relevant significance, but for the purposes of clarity in this demonstration, we require detection in all bands.

The number of objects in the catalogue after enforcing these constraints is 7651, with a median redshift of  $z = 0.55$ . In order to test the predictive power of the SOM, the sample is randomly split in two, such that one half of the catalogue can be used for training, and the other for testing (where the SOM has not had an opportunity to see those galaxies). The training set therefore consists of 3825 unique inputs.

Multiple neural networks, or ‘committees’, are often used to increase the robustness of predictions (e.g. Collister & Lahav 2004). Committees introduce an extra level of stochasticity that provide a measure of uncertainty through an examination of the fidelity of predictions made by committee members. Here we initialize 10 SOMs, each with  $100 \times 100$  nodes, and set the total number of iterations per SOM to 382 500, thus oversampling the input training set by a factor of 10 for an individual map, and a factor of 100 over the committee. To introduce an extra level of randomness, the initial choice of learning coefficient  $L$  (equation 4) is selected from a Gaussian distribution centred at unity with a scale of 0.1; this allows each SOM to learn at slightly different rates. The final predicted value is taken to be the mean of the individual predictions from the committee members, and we take the standard deviation of these predictions to be the uncertainty in the estimate. If we had used many more SOMs in the committee, it should be possible to collect the results together to form a probability density function for the parameter prediction, which might provide a better representation of the uncertainty (note that SOMs can be trained in parallel for this purpose).

In our example, for each training vector we have a set of broad-band photometry and a spectroscopic redshift. We set these as the weights of each training vector. To reduce the parameter space,





**Figure 5.** The photometric redshift accuracy recovered from a committee of 10 SOMs trained on 3825 galaxies from the zCOSMOS. Here we present the predictions for another 3825 galaxies from the catalogue that *did not* participate in the training (a randomly selected subset of 1000 points are plotted for clarity). In each panel we highlight 50 randomly selected galaxies and show their uncertainties, determined by the standard deviation of the predictions made by the committee. The progression of panels row-wise from the top-left to bottom-right shows the improvement of performance when increasingly complete subsets of photometry for each galaxy. The first panel just uses  $u^*$ - and  $r$ -band monochromatic fluxes. The second panel introduces the colour ( $u^* - B$ ), and subsequent panels include more colours up to  $[5.8] - [8.0]$ . The bias affecting predictions in the tails of the redshift distribution can clearly be seen, and interestingly the accuracy tends to asymptote after nine parameters – that is, all bands up to  $K_s$  – have been used. The projection of these plots as histograms is shown in Fig. 6.

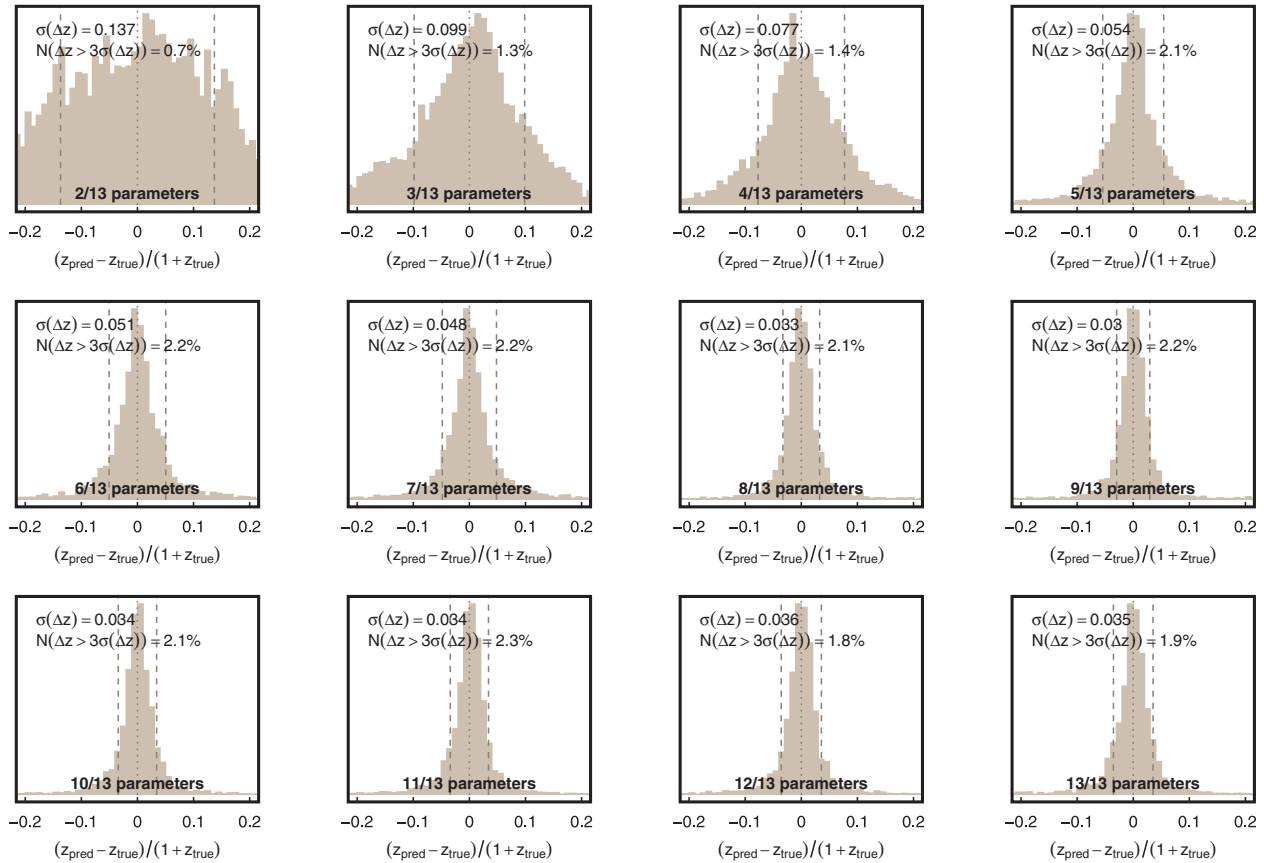
we assign the photometry as a set of colours in consecutive bands, ( $u^* - B$ ), ( $B - g$ ), ( $g - V$ ), and so on up to  $[5.8] - [8.0]$ . We also include the single  $u^*$  and  $r$  magnitudes as monochromatic flux measurements, and finally the spectroscopic redshift from zCOSMOS. In total, each training vector contains 14 elements. After training all SOMs in the committee, we test the predictive power of the SOM ensemble using the half of the catalogue that did not participate in the training, calculating the BMU for each object using subsets of the photometry [e.g. just ( $u^* - B$ ), then adding ( $B - g$ ), ( $g - V$ ), and so on until we include all photometry weights up to the IRAC bands]. In this case, the spectroscopic redshift component of the weight is not considered when calculating the BMU. In each trial, the redshift weight tagged to the BMUs provides the ‘photometric’ redshift, and these are averaged over the committee to give the final prediction. As we know what the true redshift of each test galaxy is, we can assess the accuracy of the method.

### 3.2.2 Photometric redshift accuracy

We define the figure of merit for the photometric redshift accuracy in the usual way as the root mean square of the difference between the true and estimated redshift  $\sigma(\Delta z) = \sqrt{\langle \Delta z^2 \rangle}$ , where  $\Delta z = (z_{\text{spec}} - z_{\text{phot}})/(1 + z_{\text{spec}})$ .

Figs 5 and 6 show the results, where we have interrogated the committee of 10 SOMs for the photometric redshift of a test galaxy with increasingly complete subsets the full range of photometry. There is a clear decline in  $\sigma(\Delta z)$  as more photometric information is added, asymptoting at  $\sigma(\Delta z) \sim 0.03$ . Surprising accuracy can be achieved with a rather sparsely sampled input vector; however in these cases one can clearly see the bias described above which results in the overestimation of redshifts for galaxies at  $z < 0.5$  and vice versa. Where shown, the error bars are the standard deviation of redshifts recovered from the 10 SOMs. This is certainly an underestimation of the true error; one could also incorporate the formal photometric uncertainties by running the SOM interrogation several times and allowing each photometry value to randomly scatter about its mean according to its  $1\sigma$  measured uncertainty. In this example, large error bars simply reflect cases where galaxies with similar characteristics were poorly represented in the training set, and thus are scattered between dissimilar BMUs in each committee member.

Using the  $u^*$ - to  $K_s$ -band photometry, we can achieve  $\sigma(\Delta z) = 0.03$  after rejecting  $\sim 2$  per cent  $>3\sigma$  outliers. Including the IRAC bands does not significantly improve the accuracy, despite the fact that they were included in the training:  $\sigma(\Delta z)$  no longer improves



**Figure 6.** Distributions of redshift accuracy for the data shown in Fig. 5. The vertical lines indicate the rms value which we take to be the figure of merit  $\sigma(\Delta z)$ . Surprising accuracy can be achieved with a handful of parameters, and the improvement in accuracy reaches  $\sigma(\Delta z) \simeq 0.03$  after nine parameters are used, after rejecting a small outlier fraction of  $\sim 2$  per cent. This suggests that most of the information driving the predictions, and encoded in the SOM, are in the optical–near-IR bands; inclusion of the IRAC bands for this sample, which is limited to  $z < 1$ , does not significantly improve the accuracy.

after the ninth parameter ( $z - K_s$ ) is added. This reflects the fact that for  $z < 1$  it is the  $\lambda < 2 \mu\text{m}$  photometry that ‘carries’ most of the information required for the photometric redshift (as expected; the 4000 Å and Balmer breaks are still bluewards of the  $J$  band at  $z < 1$ , and the 1.6  $\mu\text{m}$  stellar bump, another good redshift discriminant, is just redwards of  $K_s$ ).

This accuracy is comparable to, or rivals, that which can be achieved with traditional spectral template fitting techniques. Pertinent to this data set, Mobasher et al. (2007) achieved  $\sigma(\Delta z) = 0.031$  with a template fitting technique to 16 photometric bands in the COSMOS field. This was found to be in good agreement with photo- $z$ s derived from the independent codes Le Phare (Arnouts et al. 1999), Bayesian photometric redshift estimation (BPZ; Benítez 2000) and Zurich Extragalactic Bayesian Redshift Analyzer (ZEBRA; Feldmann et al. 2006). Several of these methods use Bayesian inference to derive photometric redshifts. It should be noted that more recently Ilbert et al. (2009) achieved much higher photometric redshift accuracies [ $\sigma(\Delta z) < 0.01$ ] in the COSMOS field using the Le Phare code (Arnouts et al. 1999; Ilbert et al. 2006) with 30 broad, medium and narrow bands for template fitting. Given the improvement seen in the SOM photo- $z$  technique when more photometric bands are introduced, we would anticipate an improvement in our reported accuracy if we re-trained the SOM with a similar large number of bands.

One of the main benefits of the SOM technique, aside from the non-reliance on assumptions of spectral properties, is the speed at

which photometric redshifts can be calculated once training has completed. The time to calculate the photometric redshift and error is simply the computational time to query each SOM to find the BMU – less than a few hundredths of a second per galaxy on a typical modern desktop machine.<sup>4</sup>

### 3.2.3 Additional photometric redshift tests and comparisons: PHAT

Hildebrandt et al. (2010) present a system for the consistent testing of different photo- $z$  codes: ‘PHAT: PHoto- $z$  Accuracy Testing’.<sup>5</sup> PHAT provides a standard mock catalogue containing galaxies represented by the empirical spectral energy distribution templates of Coleman, Wu & Weedman (1980) and Kinney et al. (1996), together covering the full range of galaxy spectral types from passive ellipticals to starburst systems. Synthetic colour information for each galaxy is calculated for each template for photometric bands spanning the ultraviolet to mid-infrared, specifically: the Canada–France–Hawaii Telescope MEGACAM  $ugriz$ -bands, the United Kingdom Infrared Telescope  $YJHK$ -bands and the 3.6  $\mu\text{m}$  and 4.5  $\mu\text{m}$  *Spitzer* IRAC bands.

<sup>4</sup> The computations presented in this paper were performed on a 3.2 GHz Intel Core i3 iMac with 16 GB RAM, and the coding was certainly not optimal.

<sup>5</sup> [http://www.astro.caltech.edu/twiki\\_phat/bin/view/Main/WebHome](http://www.astro.caltech.edu/twiki_phat/bin/view/Main/WebHome)

As ours is an empirical method and requires a training set where the redshift is known, we use the ‘large’ PHAT catalogue of 170 000 objects with noise included (where a parametric model for the signal-to-noise ratio as a function of source flux is used, and photometry perturbed accordingly according to a Gaussian distribution). We create a training subset by randomly sampling 10 per cent of the full catalogue. In this example, we initialize a  $200 \times 200$  SOM and set the number of iterations to oversample the training set by a factor of 5.

Hildebrandt et al. (2010) define the photo- $z$  accuracy figure of merit as the mean and scatter (rms) in  $\Delta z = z_{\text{model}} - z_{\text{phot}}$ , and the outlier rate as the fraction of objects with  $|\Delta z| > 0.1$ . For comparison with the results presented in Hildebrandt et al. (2010) for PHAT testing of 16 recent photo- $z$  codes (several of which are in widespread use), we calculate the same statistics on the predicted redshifts retrieved for the galaxies that did not participate in the training of our SOM. The best codes tested by Hildebrandt et al. (2010) typically have  $\langle |\Delta z| \rangle \leq 0.005$ , scatters of  $\sigma(\Delta z) \sim 0.01$ – $0.02$  and small outlier rates of  $< 0.1$  per cent. Testing the trained SOM on a sub-sample of 100 000 galaxies from the large catalogue that did not participate in training we find an average  $\langle \Delta z \rangle = -7 \times 10^{-4}$ ,  $\sigma(\Delta z) = 0.016$  and outlier rate of 0.13 per cent. The relatively large outlier rate (compared to some of the codes tested in Hildebrandt et al. 2010) is driven by the poorer accuracy at the tails of the redshift distribution, which is a natural bias in this method. When considering only galaxies in the range  $0.1 < z < 0.5$ , although the rms accuracy is the same, the outlier rate drops to 0.06 per cent. Thus, the empirical SOM method for photo- $z$  prediction is competitive with established photo- $z$  codes. It is likely that the accuracy could be improved further by using an even larger training sample with a longer learning period, at the expense of computational time.

### 3.3 Limiting factors

Aside from the limitations discussed above regarding the choice training set, and the natural biases that are encoded into the SOM, there are several other important issues to consider, and we briefly review these here.

The rate of learning, or how quickly the SOM adapts during training, is set by (a) two learning coefficients (equation 1 and 4) which vary as a function of node distance and learning time; (b) the rate of decay of these coefficients; (c) the shape of and the rate of decay of the region of influence around the BMU where neighbouring nodes are allowed to change; (d) the size, or resolution, of the SOM and (e) the total learning time. It is not clear what the optimum combination of these factors is that would produce the best mapping is, and it would take a long time to do so. So, the exact choice of training parameters might be the main limiting factor in the SOM technique. However, during the course of this investigation, we have found some simple configurations that appear to produce robust results.

First, the number of nodes in the SOM should be initialized such that the total number scales roughly with the number of training parameters,  $\sim 2^N$ , and a good minimum is  $\sim 400$  nodes arranged in a  $20 \times 20$  grid.<sup>6</sup> This is to allow the mapping to ‘resolve’ possible correlations and clustering between several parameters. Clearly, when making predictions for new test data, the size of the SOM sets a fundamental limit on the accuracy, as the total input parameter

space is discretised into a finite number of bins. In the case of this photometric redshift example, we set the total number of nodes to be  $10^4$ , and this seems adequate to make accurate predictions whilst keeping down the training time. In the example of object selection, however, we were more interested in training the SOM to make selections of objects in rather broad swathes of the parameter space, and so in this case an SOM with fewer nodes was successful (and is also beneficial for visualization purposes). Note that there are variant SOM algorithms that allow the number of nodes in the map to be dynamic, growing according to the need of the training sample (Alahakoon & Halgamuge 1998).

The total number of iterations was set to 10 times the number of elements in the training set. This was to allow the SOM to see each training vector about 10 times, and participate in the refining of the self-organization at different stages in the learning process. We could envision even better results if we allowed more oversampling of the training set, but this comes at the cost of longer training times.

Finally, we initialized the learning coefficients to unity, and set the size of the neighbourhood function to be approximately half the linear size of the SOM. This initial size allows test vectors selected at the start of the learning to influence large, unrefined sectors of the map. As iterations cumulate, new training vectors simply refine the map, contributing less drastic changes due to the decreasing size of the neighbourhood function and declining learning coefficients. We found that the shape of the neighbourhood function (a Gaussian), the rate of its decay (the size decreases linearly with time) and the decay of the learning coefficients produced excellent results in multiple SOM realizations involving different types of data. Again, a future study could investigate what the optimal learning parameters are, with the best results perhaps coming from a more extended committee of hundreds or thousands of SOMs (that could be trained in parallel), each with different self-organization styles and learning capabilities.

## 4 SUMMARY

SOMs are a class of neural network that employ unsupervised learning to map the topology of a multidimensional data set. This is a powerful method for exploring large catalogues of astronomical data; the method can discover correlations between parameters, can detect clustering within the parameter volume and can be exploited to predict the parameters of new test data in a completely empirical way. Here we have presented the SOM as a potential tool for current and future large astronomical surveys, highlighting two practical examples.

(i) Selection of galaxies with active nuclei trained on *Spitzer* IRAC colours (3.6–8  $\mu\text{m}$ ). The SOM trained on IRAC photometry naturally ‘finds’ the characteristic colours of obscured AGN, and the corresponding nodes can be used as a filter to select similar objects from a new data set. This filter can even be used where the information used in the training is incomplete, or unavailable. We demonstrate that the same SOM can be used to select known AGN using just  $K_s$ , [3.6] and [4.5] photometry. While we chose AGN as a demonstrative example, SOMs could be used to select a wide range of astronomical objects, with the exciting possibility that self-organization could discover ‘new’ classifications in upcoming large data surveys.

(ii) Estimation of redshifts from broad-band photometry, trained using a deep spectroscopic survey: zCOSMOS. The accuracy of the redshift estimation defined by the rms in  $\Delta z = (z_{\text{pred}} - z_{\text{true}})/(1 + z_{\text{true}})$  is  $\sigma(\Delta z) = 0.03$ , with a small outlier rate of  $\sim 2$  per cent,

<sup>6</sup> Note that there is nothing to preclude arranging the nodes in a three-dimensional (or higher) grid, but this would defeat the purpose of visualization.



competitive with other established photo- $z$  codes using alternative techniques for deriving the redshift from photometry. We also test the SOM as a photo- $z$  tool using the PHAT catalogue (Hildebrandt et al. 2010), which provides a much larger training set with model galaxies covering a range of spectral types, and 10 bands of broad-band photometry. We find that the photo- $z$  accuracy of the SOM is competitive with many established photo- $z$  codes, delivering an rms in  $(z_{\text{true}} - z_{\text{pred}}) = 0.016$  with a small outlier rate of 0.13 per cent.

Accuracies could be significantly improved by training on a larger training sample, but other factors also affect performance, including the ‘resolution’ of the SOM, the choice of learning coefficients and so on. Although not without its limitations, which are discussed, the advantages of using an SOM for predicting photometric redshifts (or any other parameter) are (a) it is a completely empirical method; (b) once training has completed, predictions can be achieved very quickly, since the only cost function that has to be evaluated is the location of the best matching node (BMU) for a new test galaxy.

We have demonstrated two simple examples here, using one of the most basic SOM algorithms, but there are many practical applications beyond what has been presented. One could envision more extravagant training scenarios, applications for and adaptations to the algorithm that might prove fruitful. In conclusion, however, we suggest that SOMs are versatile tools that could be used in data mining and visualization applications for existing and upcoming large surveys, where efficient techniques will be required to fully harness the power of the exceptionally large and intertwined data bases set to flood the community.

## ACKNOWLEDGMENTS

JEG is supported by a Banting Postdoctoral Fellowship, administered by the Natural Science and Engineering Research Council of Canada. We thank the anonymous referee for comments and suggestions that improved the paper, and Kristen Coppin, Duncan Farrah, Ryan Hickox and Phil Marshall for helpful comments and discussions.

This research has made use of the NASA/IPAC Infrared Science Archive, which is operated by the Jet Propulsion Laboratory, California Institute of Technology, under contract with the National Aeronautics and Space Administration.

## NOMENCLATURE AND NOTATION

SOM	Self-organized/organizing map
Node	Single ‘neuron’ in SOM; nodes are arranged on the surface of a 3D toroid, but visualized unravelled, in 2D
Training vector, $\mathbf{t}$	Example of single data element from training set (e.g. galaxy photometry)
Weight vector, $\mathbf{w}$	Vector of identical size to $\mathbf{t}$ attached to each node that competes to become more like the training vector
BMU	Best Matching Unit is the ‘winning node’ that is most similar to a randomly sampled training vector

U-Matrix	Unified Distance Matrix (UDM: a method of visualizing and detecting clustering in the map using the average distance to neighbouring nodes)
$r_{\text{BMU}}$	Radius of learning influence of BMU
Neighbourhood function	Form of spatial learning function within $r_{\text{BMU}}$
Component plane	2D representation of the values of the $i$ th element of the weight vector of nodes in the map
Learning rates	Coefficients determining the amount that weights can adapt to become more like training vectors; these vary spatially (relative to the BMU) and temporally, tending towards zero over the duration of training
Oversampling	Number of times given training vectors are ‘seen’ by the SOM during learning
Committee	Several SOMs trained independently on the same training set

## REFERENCES

- Aihara H., 2011, *ApJS*, 193, 29  
 Alahakoon D., Halgamuge S. K., 1998, *Proc. 5th Int. Conf. Soft Computing and Information/Intelligent Systems*. World Scientific Press, Singapore, p. 907  
 Arnouts S., Cristiani S., Moscardini L., Matarrese S., Lucchin F., Fontana A., Giallongo E., 1999, *MNRAS*, 310, 54  
 Benitez N., 2000, *ApJ*, 536, 571  
 Bouwens R. J. et al., 2010, *ApJ*, 709, L133  
 Capak P. et al., 2007, *ApJS*, 172, 99  
 Coleman G. D., Wu C.-C., Weedman D. W., 1980, *ApJS*, 43, 393  
 Collister A. A., Lahav O., 2004, *PASP*, 116, 345  
 Daddi E., Cimatti A., Renzini A., Fontana A., Mignoli M., Pozzetti L., Tozzi P., Zamorani G., 2004, *ApJ*, 617, 746  
 Eales S. et al., 2010, *PASP*, 122, 499  
 Feldmann R. et al., 2006, *MNRAS*, 372, 56  
 Hickox R. C. et al., 2007, *ApJ*, 671, 1365  
 Hildebrandt H. et al., 2010, *A&A*, 523, 31  
 Ilbert O. et al., 2006, *A&A*, 457, 841  
 Ilbert O. et al., 2009, *ApJ*, 690, 1236  
 Kinney A. L., Calzetti D., Bohlin R. C., McQuade K., Storchi-Bergmann T., Schmitt H. R., 1996, *ApJ*, 467, 38  
 Kohonen T., 1982, *Biological Cybernetics*, 43, 59  
 Kohonen T., 2001, in Huang T. S., Kohonen T., Schroeder M. R., eds, *Self-Organizing Maps*. Third, extended edition. Springer, Germany  
 Lacy M. et al., 2004, *ApJS*, 154, 166  
 Lahav O. et al., 1995, *Sci*, 267, 859  
 Lilly S. J. et al., 2007, *ApJS*, 172, 70  
 Lintott C. et al., 2008, *MNRAS*, 389, 1179  
 McLure R. J., Dunlop J. S., Cirasuolo M., Koekemoer A. M., Sabbi E., Stark D. P., Targett T. A., Ellis R. S., 2010, *MNRAS*, 403, 960  
 Madau P., 1995, *ApJ*, 441, 18  
 Mahdi B., 2011, preprint (arXiv:1108.0514)  
 Mobasher B. et al., 2007, *ApJS*, 172, 117  
 Norris R. et al., 2011, *Publ. Astron. Soc. Australia*, 28, 215  
 Núñez J., Llacer J., 2003, *Neural Networks*, 16, 411  
 Raddick J. et al., 2008, *BAAS*, 40, 240

- Sanders D. B. et al., 2007, ApJS, 172, 86  
Scoville N. et al., 2007, ApJS, 172, 1  
Steidel C. C., Hamilton D., 1993, AJ, 105, 2017  
Steidel C. C., Giavalisco M., Pettini M., Dickinson M., Adelberger K. L., 1996, ApJ, 462, L17  
Stern D. et al., 2005, ApJ, 631, 163  
Storrie-Lombardi M. C., Lahav O., Sodre L., Jr, Storrie-Lombardi L. J., 1992, MNRAS, 259, 8  
Ultsch A., 2007, Emergence in Self-Organizing Feature Maps, Proceedings Workshop on Self-Organizing Maps. Neuroinformatics Group, Bielefeld University, Germany  
Wright E. L. et al., 2010, AJ, 140, 1868  
York D. et al., 2000, AJ, 120, 1579

This paper has been typeset from a  $\text{\TeX/L\AA\TeX}$  file prepared by the author.