Kalman Filtering for Orientation Estimation using IMUs

Filtrado Kalman para Estimación de la Orientación usando IMUs

Ph.D. thesis defense

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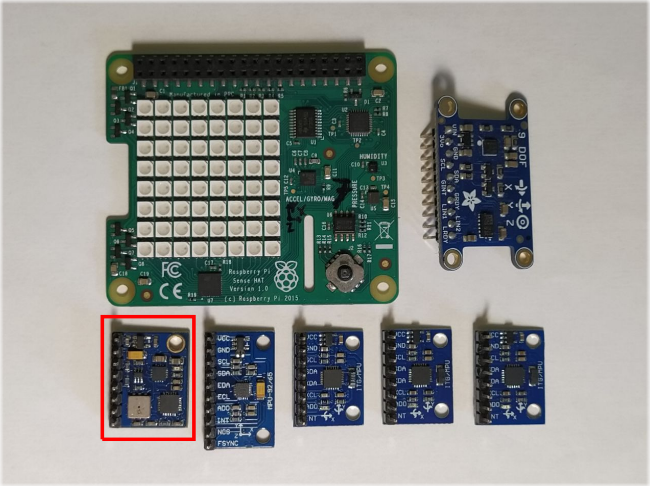
UNIVERSIDAD DE MURCIA

FACULTAD DE INFORMÁTICA

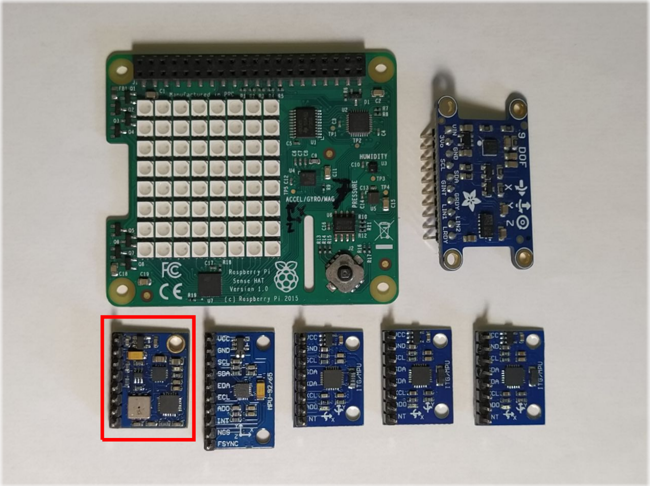
Outline of the defense

- Motivation
- Orientation estimation
  - Kalman filters
  - Quaternions as the orientation representation
- Triaxial sensor calibration
  - Temperature dependent calibration
  - Characterization of triaxial sensors
- Future work

Motivation

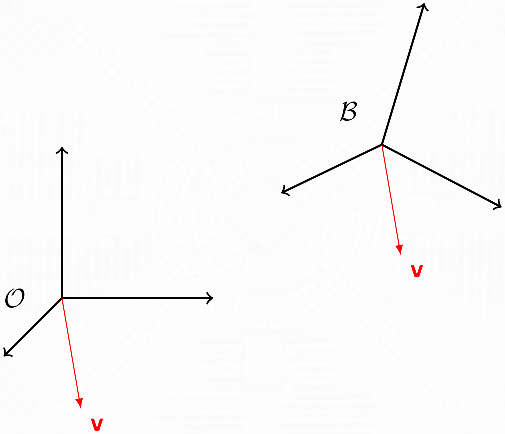

Manufactured in PRC
HUMIDITY
ACCEL/GYRO/MAG
PRESSURE
Raspberry Pi
Sense HAT
Version 1.0
(c) Raspberry Pi 2015
9-DOF
VCC
GND
SCL
SDA
EDA
ECL
ADO
INT
NCS
FSYNC
MPU-92/65
ITG/MPU

Motivation




```
./run_show_data.sh
```

Orientation of a system



Kalman filter

```
./run_test_EKF.sh
```

Orientation representation

Representation	Parameters	Continuous	Non-Singular	Linear Evolution
Euler angles	3	✗	✗	✗
Axis-angle	3–4	✗	✗	✗
Rotation matrix	9	✓	✓	✓
Unit quaternion	4	✓	✓	✓

Orientation representation

Representation	Parameters	Continuous	Non-Singular	Linear Evolution
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Axis-angle	3–4	✗	✗	✗
Rotation matrix	9	✓	✓	✓
Unit quaternion	4	✓	✓	✓

"...it is topologically impossible to have a global 3-dimensional parametrization without singular points for the rotation group."

John Stuelpnagel. On the parametrization of the three-dimensional rotation group. SIAM review, 6(4):422–430, 1964

Unit quaternions

Complex numbers

$$z = y + x\, i \equiv$$
$$\equiv (y, x)$$

Quaternions

$$\boldsymbol{q} = q_0 + q_1\, \boldsymbol{i} + q_2\, \boldsymbol{j} + q_3\, \boldsymbol{k} \equiv$$
$$\equiv (q_0, q_1, q_2, q_3)$$

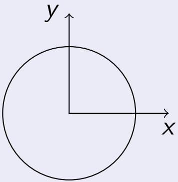
Unit quaternions

Complex numbers

$$z \;=\; y + x\,i \;\equiv$$

$$\equiv\; (\,y\,,\,x\,)$$

$$y^2 + x^2 \;=\; 1$$

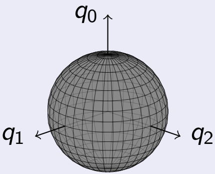


Quaternions

$$\boldsymbol{q} \;=\; q_0 + q_1\,\boldsymbol{i} + q_2\,\boldsymbol{j} + q_3\,\boldsymbol{k} \;\equiv$$

$$\equiv\; (\,q_0\,,\,q_1\,,\,q_2\,,\,q_3\,)$$

$$q_0^2 + q_1^2 + q_2^2 + q_3^2 \;=\; 1$$



Kalman filter equations

- State prediction:

$$
\begin{aligned}
\bar{\mathbf{x}}_{n|n-1} &= \mathbf{F}_n \, \bar{\mathbf{x}}_{n-1|n-1} + \bar{\mathbf{q}}_n \\
\mathbf{P}_{n|n-1} &= \mathbf{F}_n \, \mathbf{P}_{n-1|n-1} \, \mathbf{F}_n^T + \mathbf{Q}_n
\end{aligned}
$$

- Measurement prediction:

$$
\begin{aligned}
\bar{\mathbf{z}}_{n|n-1} &= \mathbf{H}_n \, \bar{\mathbf{x}}_{n|n-1} \\
\mathbf{S}_{n|n-1} &= \mathbf{H}_n \, \mathbf{P}_{n|n-1} \, \mathbf{H}_n^T + \mathbf{R}_n
\end{aligned}
$$

- Kalman update:

$$
\begin{aligned}
\mathbf{K}_n &= \mathbf{P}_{n|n-1} \, \mathbf{H}_n^T \, \mathbf{S}_{n|n-1}^{-1} \\
\bar{\mathbf{x}}_{n|n} &= \bar{\mathbf{x}}_{n|n-1} + \mathbf{K}_n \left(\mathbf{z}_n - \bar{\mathbf{z}}_{n|n-1} \right) \\
\mathbf{P}_{n|n} &= (\mathbf{I} - \mathbf{K}_n \mathbf{H}_n) \, \mathbf{P}_{n|n-1} \, (\mathbf{I} - \mathbf{K}_n \mathbf{H}_n)^T + \mathbf{K}_n \mathbf{R}_n \mathbf{K}_n^T
\end{aligned}
$$

Kalman filter adaptation for unit quaternions

Vectors

$$\mathbf{x} = \bar{\mathbf{x}} + \Delta\mathbf{x}$$
$$\Delta\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$$

$$\mathbf{P} = \mathrm{E}[(\Delta\mathbf{x})(\Delta\mathbf{x})^T]$$

$$\Delta\mathbf{x}_n = \mathbf{K}_n\left(\mathbf{z}_n - \bar{\mathbf{z}}_{n|n-1}\right)$$
$$\bar{\mathbf{x}}_{n|n} = \bar{\mathbf{x}}_{n|n-1} + \Delta\mathbf{x}_n$$

Quaternions

$$\boldsymbol{q} = \bar{\boldsymbol{q}} * \boldsymbol{\delta}$$
$$\boldsymbol{\delta} = \bar{\boldsymbol{q}}^* * \boldsymbol{q}$$
$$\mathbf{e} = \varphi(\boldsymbol{\delta})$$
$$\mathbf{P} = \mathrm{E}[\mathbf{e}\,\mathbf{e}^T]$$

$$\mathbf{e}_n = \mathbf{K}_n\left(\mathbf{z}_n - \bar{\mathbf{z}}_{n|n-1}\right)$$
$$\bar{\boldsymbol{q}}_{n|n} = \bar{\boldsymbol{q}}_{n|n-1} * \varphi^{-1}(\mathbf{e}_n)$$

Kalman filter adaptation for unit quaternions

Vectors

$$\mathbf{x} = \bar{\mathbf{x}} + \Delta\mathbf{x}$$

$$\Delta\mathbf{x} = \mathbf{x} - \bar{\mathbf{x}}$$

$$\mathbf{P} = \mathrm{E}[(\Delta\mathbf{x})(\Delta\mathbf{x})^T]$$

$$\Delta\mathbf{x}_n = \mathbf{K}_n \left(\mathbf{z}_n - \bar{\mathbf{z}}_{n|n-1} \right)$$

$$\bar{\mathbf{x}}_{n|n} = \bar{\mathbf{x}}_{n|n-1} + \Delta\mathbf{x}_n$$

Quaternions

$$\boldsymbol{q} = \bar{\boldsymbol{q}} * \boldsymbol{\delta}$$

$$\boldsymbol{\delta} = \bar{\boldsymbol{q}}^* * \boldsymbol{q}$$

$$\mathbf{e} = \varphi(\boldsymbol{\delta})$$

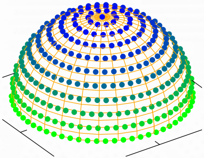
$$\mathbf{P} = \mathrm{E}[\mathbf{e}\,\mathbf{e}^T]$$

$$\mathbf{e}_n = \mathbf{K}_n \left(\mathbf{z}_n - \bar{\mathbf{z}}_{n|n-1} \right)$$

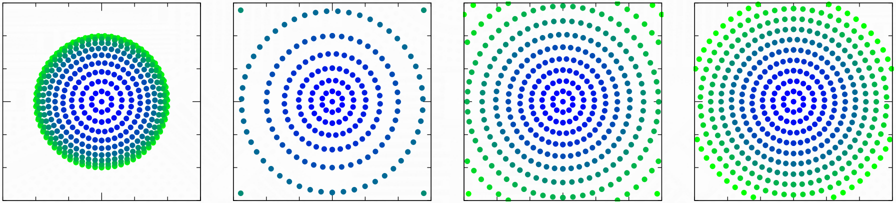
$$\bar{\boldsymbol{q}}_{n|n} = \bar{\boldsymbol{q}}_{n|n-1} * \varphi^{-1}(\mathbf{e}_n)$$

```
./run_show_charts.sh
```

From manifold to charts



S^2

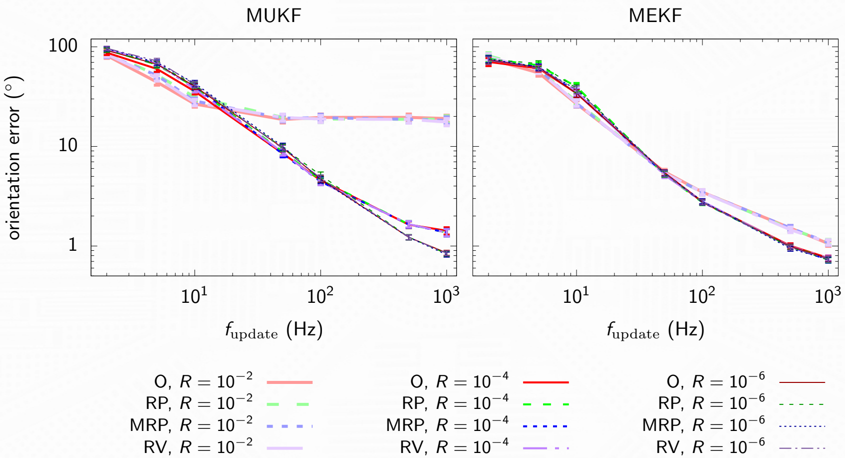


O RP MRP RV

Manifold Kalman Filters

```
./run_test_MKF.sh
```

Chart comparison



MKF comparison

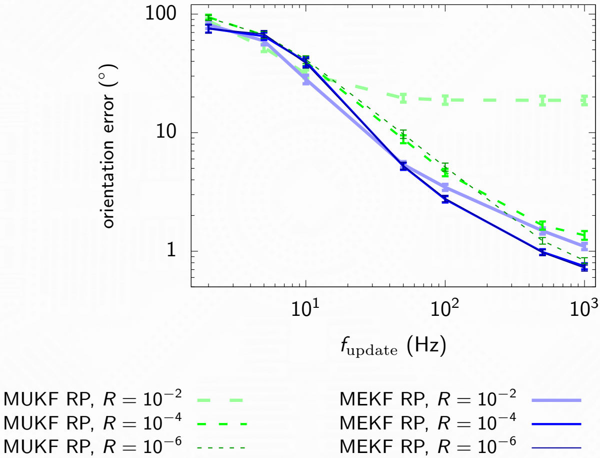
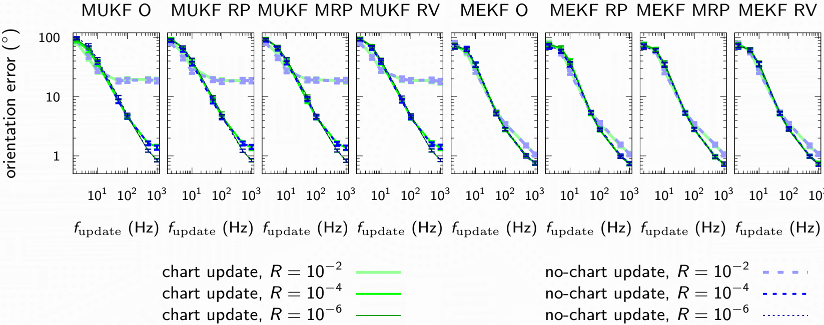


Chart update vs no-chart update



Position estimation

```
./run_test_MKF.sh
```

Usual steps to calibrate a scalar sensor

Data pairs

$$\{\,(\,x_i\,,\,y_i\,)\,\}_{i=1}^{M}$$

Model

$$y \;=\; f(\,x\,,\,\boldsymbol{\theta}\,)$$

Cost function

$$C(\,\boldsymbol{\theta}\,) \;=\; \sum_i \big(\,y_i \,-\, f(\,x_i\,,\,\boldsymbol{\theta}\,)\,\big)^2$$

Optimization

$$\boldsymbol{\theta}^* \;=\; \arg\min_{\boldsymbol{\theta}} \; C(\,\boldsymbol{\theta}\,)$$

Usual steps to calibrate a scalar sensor

Data ~~pairs~~ triplets

$$\{ (x_i , y_i) \}_{i=1}^{M} \quad \longrightarrow \quad \{ (\mathbf{x}_i , T_i , y_i) \}_{i=1}^{M}$$

Model

$$y = f(\mathbf{x}, \boldsymbol{\theta})$$

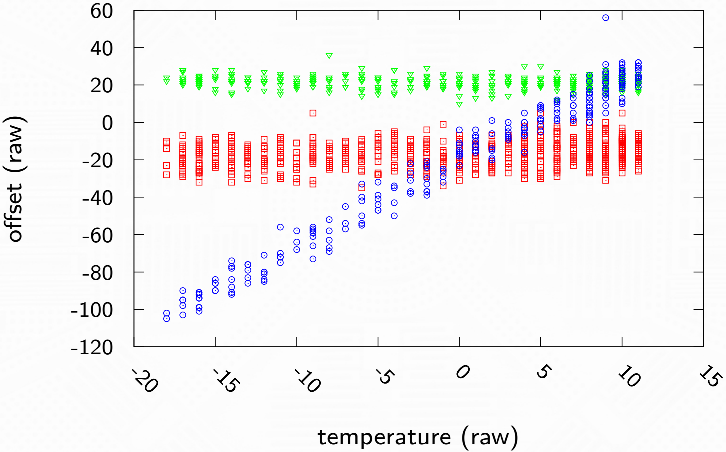
Cost function

$$C(\boldsymbol{\theta}) = \sum_i \big(y_i - f(\mathbf{x}_i , \boldsymbol{\theta}) \big)^2$$

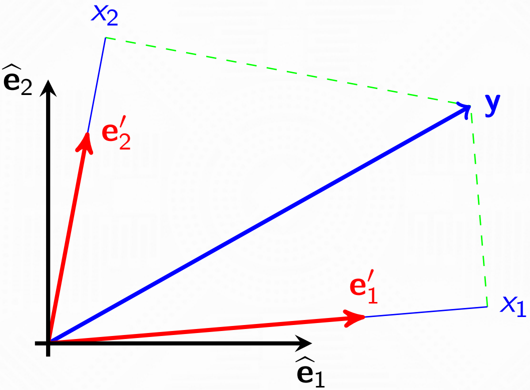
Optimization

$$\boldsymbol{\theta}^* = \arg\min_{\boldsymbol{\theta}} C(\boldsymbol{\theta})$$

Temperature dependence of triaxial sensors



Triaxial sensor model



Triaxial sensor calibration

Sensor model

$$\mathbf{x} \quad = \quad \mathbf{S}(T)\,\mathbf{y} \; + \; \mathbf{b}(T) \; + \; \mathbf{r}$$

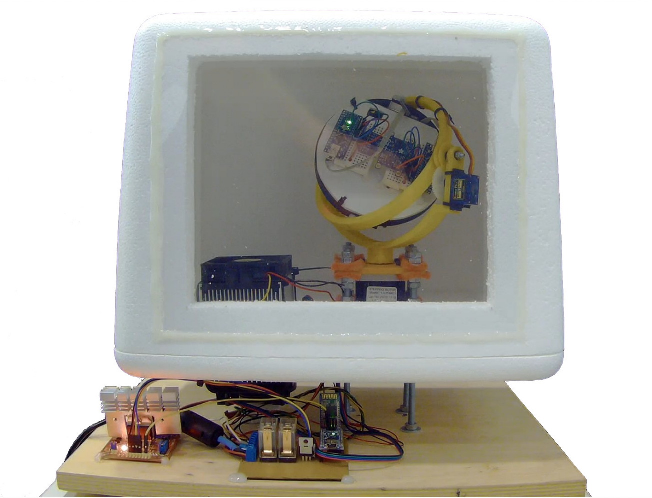
Module of the vector

$$\begin{aligned} \|\mathbf{y}\| \quad &= \quad \|\,\mathbf{S}^{-1}(T)\,[\,\mathbf{x} - \mathbf{b}(T) - \mathbf{r}\,]\,\| \quad = \\ &= \quad \|\,\mathbf{K}(T)\,\mathbf{x} + \mathbf{c}(T) + \mathbf{r}'\,\| \quad = \\ &= \quad \|\,\mathbf{A}(T)\,\widetilde{\mathbf{x}} + \mathbf{r}'\,\| \end{aligned}$$

Cost function

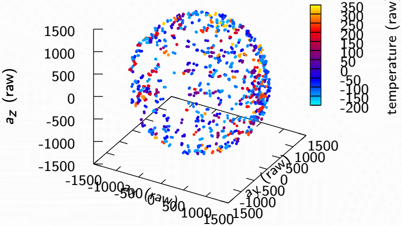
$$F(\,\mathbf{A}\,) \; = \sum_{m=1}^{M} \left[\; y_m^2 \; - \; \|\,\mathbf{A}(T_m)\,\widetilde{\mathbf{x}}_m\,\|^2 \;\right]^2$$

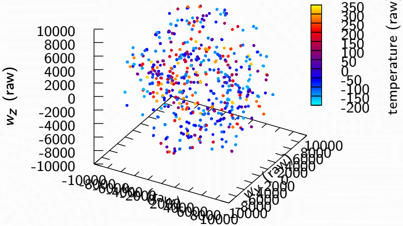
Triaxial calibration system

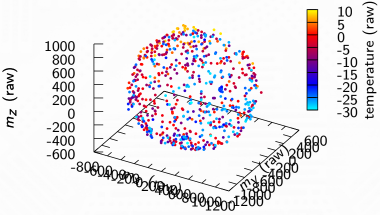


```
./show_calibrationVideo.sh
```

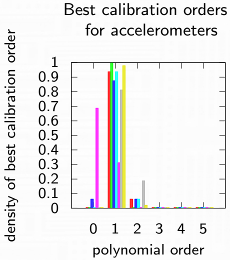
Examples of calibration data

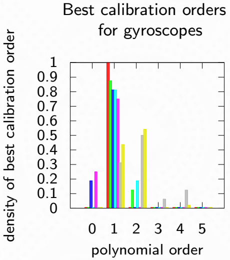


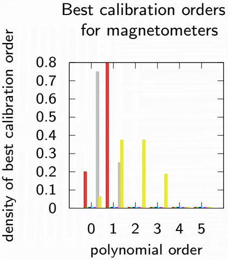




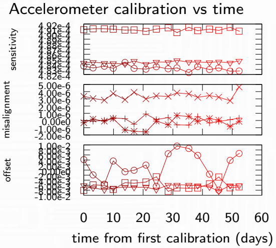
Density of calibration orders

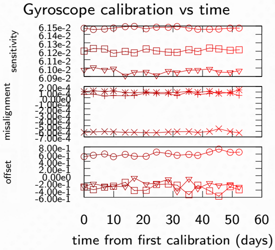


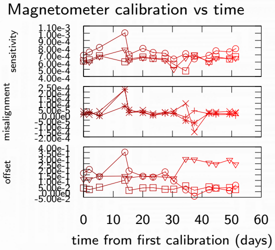




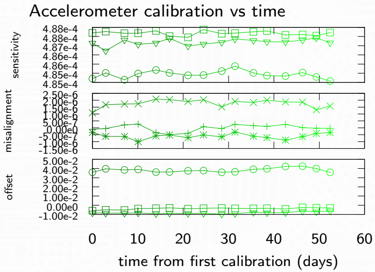
Examples of calibration data

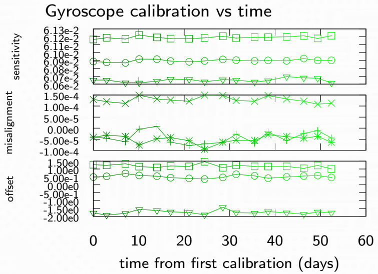




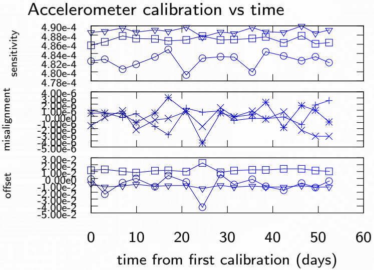


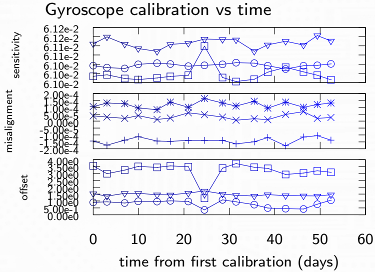
Examples of calibration data



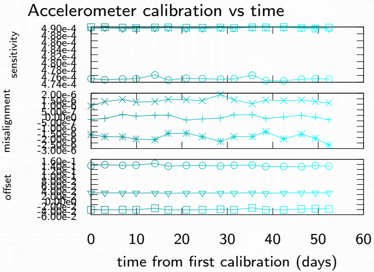


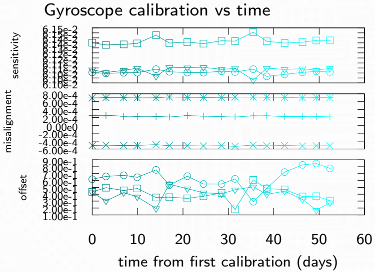
Examples of calibration data





Examples of calibration data





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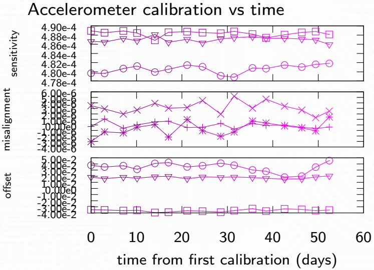
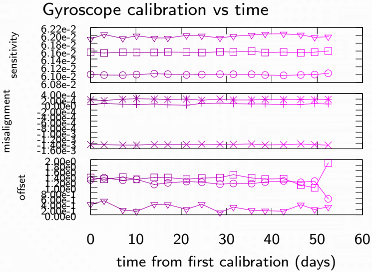
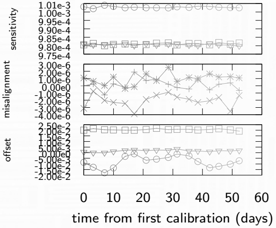


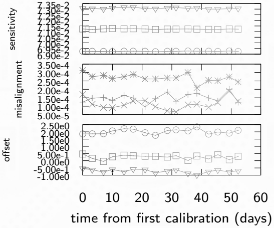


Examples of calibration data

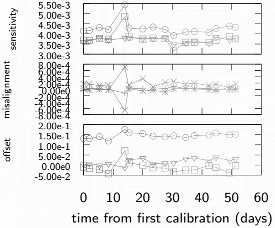
Accelerometer calibration vs time



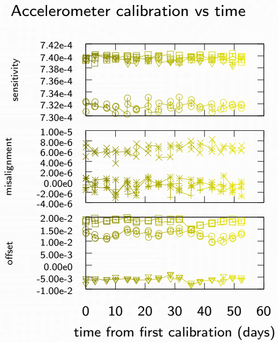
Gyroscope calibration vs time

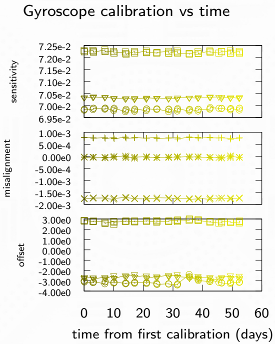


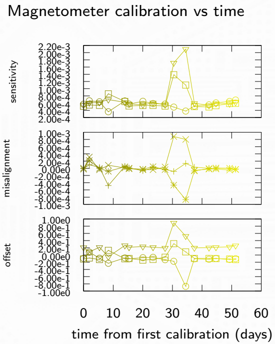
Magnetometer calibration vs time



Examples of calibration data







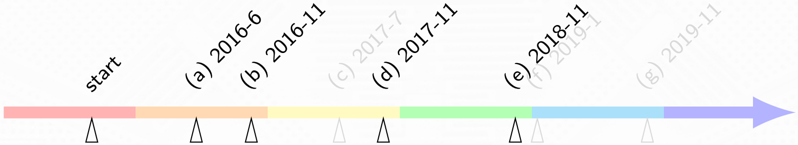
Results with calibrated sensors

```
./run_test_calibration.sh
```

Conclusion: contributions

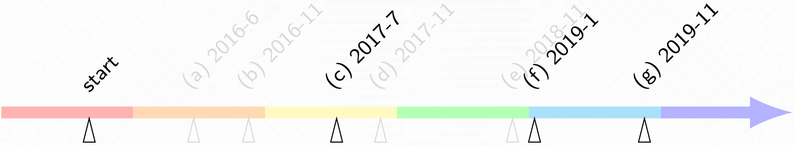
- Orientation estimation using the Kalman filter and quaternions to describe the orientation:
  - New perspective on the problem of orientation estimation.
  - Design of orientation estimation algorithms.
  - Solution to the problem of the "covariance correction step".
- Temperature-dependent calibration of triaxial sensors:
  - Temperature-dependent calibration algorithm for triaxial sensors.
  - Prototype to automatically collect calibration data.
  - Characterization of real triaxial sensors.

Conclusion: publications



(a) WAF'2016
(b) DESEi+d'2016
(c) Journal of Physical Agents; SJR: 0.176 (2016), Q4 in "Control and Systems Engineering"
(d) ROBOT'2017
(e) WAF'2018
(f) MDPI Sensors; JCR: 3.031 (2018), Q1 in "Instruments & Instrumentation"
(g) IEEE Sensors Journal; JCR: 3.076 (2018), Q1 in "Instruments & Instrumentation"

Conclusion: publications



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Future work

- Better IMUs?

```
./run_test_IMxs.sh
```

- Fusion of several IMUs

```
./run_test_IM.sh
```

- Fusion of several sensors

```
./run_test_IM.sh
```

Thank you