# Dynamic Mode Decomposition: A Survey of SVD-Based Applications

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## Abstract

This paper discusses the dynamic mode decomposition method, its applications, and adaptations. Though there are two main approaches to computing modes, only the singular value decomposition-based derivation is considered below. Application areas include predictive modeling, video processing, and Bayesian inferencing. References to resources for further reading are provided throughout.

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## I. Introduction

Modal decomposition methods date back to the 1960's and have since been viewed as prime options for data-driven analysis[1]. A data set is characterized by its features of interest, called modes, that are extracted with basic linear algebra methods regardless of the data set's size or structure. The modes are then used in any number of calculations, dependent upon the application. Though *modal decomposition* itself may sound foreign, the means of achieving it likely are not. The most commonly used method of modal decomposition is principal component analysis (PCA), which has seen applications in nearly every scientific or mathematically-based field[2][3][4]. Though other modal decomposition methods may not share PCA's popularity, they all share one common denominator; modes are calculated by performing singular value decomposition (SVD) on the dataset[1].

Modal decomposition is used widely in experimental fluid dynamics, and it is within this field that dynamic mode decomposition (DMD) was developed. At the time of its inception in 2008[5], DMD was the only method of modal decomposition meant for nonlinearly evolving flows[1]. There are now many adaptations of the method, but all rely on baseline DMD theory.

This paper will explore the theory behind computations involved in DMD and applications of the original method, as well as a brief survey of adapted DMD methods and their applications.

# II. DMD Overview

The overall goals of DMD are to (i) reduce the dimension of a problem, and (ii) to solve the reduced problem efficiently.

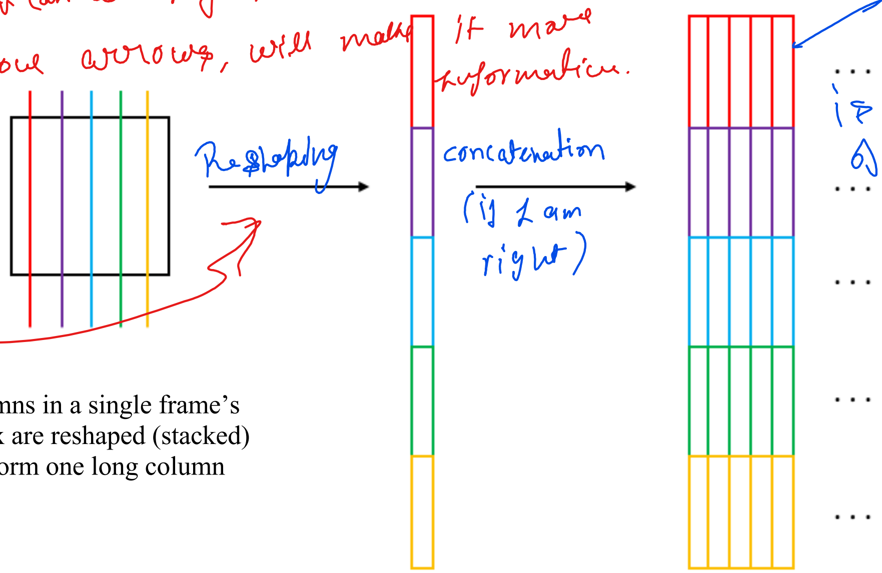
There are two methods for obtaining DMD modes. The SVD-based approach was developed second and results in fewer computational errors and better noise handling. It is explored below.

## i. Data formatting [5]

Operations are performed on a data matrix in which each column represents data from a single instance. For example, when using DMD to analyze flow in a video, each video frame is reshaped into a column. Each of the frame columns are then lined up next to each other to form the data matrix (visualization below).

You can add information about the transformation above arrows, will make it more information.

I believe this is due to concatenation of multiple time frame ??




columns in a single frame's matrix are reshaped (stacked) to form one long column

## ii. SVD-based algorithm [5]

This paper assumes prior knowledge of the singular value decomposition method. If needed, an explanation of this method can be found in the resources below ([6]). alternative is to replace it by just [6]

[6]

The full data matrix is defined as $V_1^N$, *1* and *N* representing indices of the first and last frames comprising the data matrix, respectively. The first step in computing DMD modes is to create two new matrices:

$V_1^{N-1}$ (all data matrix frames but the last)

$V_2^N$ (all data matrix frames but the first)

Compute the SVD of $V_1^{N-1}$, resulting in $V_1^{N-1} = U\Sigma V^T$. Note that $V^T$ contains eigenvectors of $V_1^{N-1}$, but otherwise bears no relation to the aforementioned $V_i^j$ matrices (i.e. it is not built from data matrix frames).

Lastly, construct $U^T V_2^N \mathrm{V}\Sigma^{-1}$ and calculate its eigenvalues $\lambda_i$ and eigenvectors $v_i$. DMD modes are given by $Uv_i$. The modes and eigenvalues are used to obtain information about the dataset.

### iii. Initial applications of DMD

Peter Schmid, creator of DMD, released papers in 2010[7] and 2011[8] demonstrating the method's wide range of applications. Schmid's testing included experimental fluids data, simulated data, and various data types with differing degrees of content. He highlights the capability of DMD to extract flow features from the data itself, without reliance upon an underlying physics model[7]. Though the classification of mode (global, normal, dynamic, etc.) changes with the source of the data, the method's ability to perform remains constant. Schmid's two publications on the applications of DMD are recommended reading, and section 2.4 of [1] further details the applications DMD was originally tested on while in development. Topics include flames, jets, and stochastic dynamical systems.

In 2012, Schmid verified[13] that DMD can also perform on 3-dimensional fluid flows.

## III. Adaptations

The aim of this section is to provide a brief overview of the many ways DMD has been adapted to improve efficiency, error distribution, and function.

| | |
|---|---|
| Algorithm name: | Exact DMD |
| Algorithm function: | Generalization of the original DMD algorithm, allows for the analysis of multiple datasets at once among other improvements[9] |
| | |
| Algorithm name: | Multi-Resolution DMD |
| Algorithm function: | Combination of multi-resolution analysis with Exact DMD to incorporate multiple dataset functionality with differing timescales[10] |
| Sample application: | Used with oceanic temperature data to identify a mode specific to El Niño, making it a predictive tool for affected years[10] |
| | |
| Algorithm name: | Optimized and Optimal Mode DMD |
| Algorithm function: | Reduces noise and truncation error sensitivity of the original DMD algorithm[11] and allows for user-defined SVD rank options within the framework of an optimization problem[11][12] |

# IV. Modern Applications

Like PCA, DMD has found its way into nearly every scientific area of study. There are countless examples of its use in neuroscience, finance, fluid dynamics, and other natural sciences. Below, however, are examples of DMD (or DMD adaptations) being applied to recent research in computational science.

## i. Computer vision, 2015 [14]

Applications of DMD to non-fluid flow videos have expanded in the last half-decade. The referenced paper calculates modal values according to the multi-resolution DMD algorithm, and uses those modes to classify different velocities in the frame. The algorithm separates foreground objects from the background, and can even track the targets it detects.

Here is an example of foreground separation calculated with DMD from [14]:

what's foreground matter

(a) Original frame (b) Ground truth (c) Modeled background (d) Foreground mask

Surprisingly, the DMD algorithm has shown to out-perform some classic computer vision algorithms in this task. Not only are computation speed and memory usage improved, but the average accuracy of results also shows a strong increase.

## ii. Bayesian inferencing, 2017 [15]

The referenced paper develops a method that combines Bayesian statistics with DMD, called Bayesian DMD. Posterior inferencing—probabilistic estimation based on evidence or prior experience—is done with a Gibbs sampler[17]. Bayesian DMD is shown to out-perform original DMD, particularly with noisy data[15].

Section 5 of the referenced paper also contains resources for further DMD applications and a brief history of its evolution.

## iii. Predictive modeling, 2019 [16]

This recent study looks at DMD as a potential predictive modeling tool and runs several tests to explore the validity of the idea. Due to its in-depth analysis, this paper can be considered a near-comprehensive review of DMD performance and applications.

# V. Conclusion

The survey conducted in this paper touches upon a small portion of the vast pool of DMD applications. Since its advent in 2008, DMD has proven to be an invaluable computational method that has earned its place in the applied sciences. It is clear from each resource referenced that there are open questions regarding DMD and countless avenues for expansion.

I think you did a great job writing this paper, I couldn't find any grammatical error (my english sucks!!)

Your article is an easy read, and gives a good detail about DMD. I really enjoyed it.

You can add well-informed caption, that will make the figure self-explanatory.

Suggestion

Also, if possible, you can add a brief algorithm to perform DMD for the most basic case.

## References

(hyperlinks active)


[1] DMD Theory and Applications, Tu, 2013

[2] Applications of PCA

[3] Application of PCA to Image Compression

[4] Principal Components of PCA

[5] DMD of Numerical and Experimental Data, Schmid, 2010

[6] SVD Tutorial

[7] Applications of DMD, Schmid, 2010

[8] Application of DMD to Experimental Data, Schmid, 2011

[9] On DMD: Theory and Applications, Tu, 2014

[10] Multi-Resolution DMD, Kutz, 2015

[11] Variants of DMD: Boundary Condition, Koopman, and Fourier Analysis, Chen, 2012

[12] Optimal Mode Decomposition for Unsteady Flows, Wynn, 2013

[13] Decomposition of Time-Resolved Tomographic PIV, Schmid, 2012

[14] Multi-Resolution DMD for Foreground/Background Separation and Object Tracking, Kutz, 2015

[15] Bayesian DMD, Takeishi, 2017

[16] Predictive Accuracy of DMD, Lu, 2019

[17] Bayesian Inference: Gibbs Sampling, Yildirim, 2012