Linearized 'Mindlin' tangential friction model

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The following algorithmic description provides a simple recipe to appropriately scale the tangential friction force with changes in Hertzian normal force for the 'linearized' case where, at constant normal force, the tangential friction force is assumed to vary linearly with tangential displacement from zero to the Coulomb friction limit μN, where μ is the coefficient of friction and N is the normal force. The assumed slope of the tangential friction force, $K(N)$ is the initial slope of a Mindlin micro-slip friction model [Mindlin, 1949]. The normal force N is assumed to vary with the 3/2 power of the normal displacement (*i.e.*, this is a simplified 'linearized' friction model for an elastic Hertzian contact. Similar to scheme provided to Elata & Berryman [1996]).

Model input quantities:

- $\hat{\mathbf{n}}$ = current contact (unit) normal (vector), determines the plane for 2D contact model.
- $\mathbf{T}$ = old friction force (vector)
- N_{old} = magnitude of old Hertzian normal force (from previous time step)
- N_{new} = magnitude of new Hertzian normal force (current time step)
- $\Delta\mathbf{s}$ = new increment of tangential surface displacement in the (current) contact plane
- $K(N)$ = (function) the initial slope of the Mindlin tangential force for normal force, N
- μ = coefficient of friction

<u>**Model Schematic** (pseudo-code)</u>

$\mathbf{T}' = \mathbf{T} - \mathbf{T}\cdot\hat{\mathbf{n}}$ — project old tangential force onto current tangent plane

(optional) $\mathbf{T}' = \dfrac{|\mathbf{T}|}{|\mathbf{T}'|}$ — This will scale the projected tangential force back to its original magnitude

(effectively making the 'projection' a 'rotation' onto the current tangent plane)

if ($N_{new} < N_{old}$)

$$\mathbf{T}' = \frac{K(N_{new})}{K(N_{old})}\mathbf{T}'$$

scale tangential force down if normal force has decreased

endif

$$\mathbf{T}_{new} = \mathbf{T}' + K(N_{new})\Delta\mathbf{s}$$

if ($|\mathbf{T}_{new}| > \mu N_{new}$)

$$\mathbf{T}_{new} = \frac{\mu N_{new}}{|\mathbf{T}_{new}|}\ \mathbf{T}_{new}$$

endif

All **bold** variables are assumed to be vectors, and *italic* variables are scalar quantities. Equations are vector.

References:

Mindlin, R.D. (1949) "Compliance of elastic bodies in contact," *J. Appl. Mech (trans ASME)* **16**, A-259.

Elata, D. and J.G. Berryman (1996) "Contact force-displacement laws and the mechanical behavior of random packs of identical spheres," *Mechanics of, Materials*, **24**, 229-240.