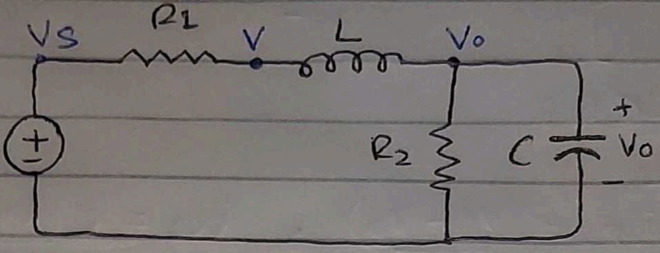
Muawiz Umar #335806

Electric Network Analysis
'Assignment 1'

Q1:



→ Applying KCL;

$$i_L = i_C + i_{R_2}$$

→ Applying $V_L = V - V_o$;

$$\boxed{\frac{1}{L}\int V_1 - V_o = C\frac{dv_o}{dt} + \frac{V_o}{R_2}}$$

$$\int (V_1 - V_o) = LC\frac{dv_o}{dt} + \frac{L}{R_2}V_o$$

$$\boxed{V_1 = LC\left(\frac{d^2v_o}{dt^2}\right) + \left(\frac{L}{R_2}\right)\left(\frac{dv_o}{dt}\right) + V_o}$$

→ Applying KVL; (At node V)

$$i_{R_1} = i_L$$

→ Applying $\frac{(V_S - V_R)}{R_1} = i_{R_1}$, $i_L = \frac{1}{L}\int V_1 - V_o$;

$$\boxed{\frac{V_S - V_1}{R_1} = \frac{1}{L}\int V_1 - V_o}$$

→ subsitute eq I and II in above eq;

$$\frac{V_S}{R_1} - \frac{1}{R_1}\left(LC\left(\frac{d^2v_o}{dt}\right) + \frac{1}{R_2}\left(\frac{dv_o}{dt}\right) + V_o\right) = C\frac{dv_o}{dt} + \frac{V_o}{R_2}$$

$$\frac{L}{R_2}\left(\frac{dv_o}{dt}\right) + LC\left(\frac{d^2v_o}{dt^2}\right) + V_o = V_S - CR_1\left(\frac{dv_o}{dt}\right) + R_1\left(\frac{V_o}{R_2}\right)$$

$$\boxed{\frac{d^2v_o}{dt^2} + \frac{dv_o}{dt}\left(\frac{1}{R_2C} + \frac{R_1}{L}\right) + V_o\left(\frac{R_1}{R_2LC} + \frac{1}{LC}\right) = \frac{V_S}{LC}}$$

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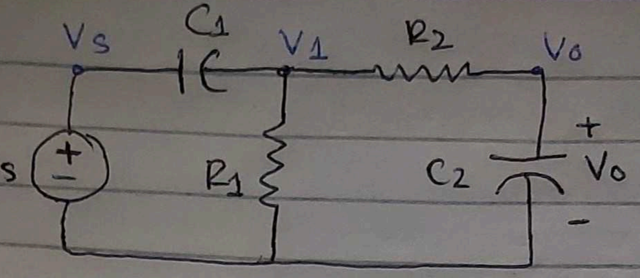
Q_2:



→ Applying KCL;

$$\boxed{i_{R2} = i_{C2}}$$

→ Applying $i_{R2} = \frac{(V_1 - V_o)}{R_2}$, $i_{C2} = C\frac{dv_o}{dt}$

$$\frac{V_1 - V_o}{R_2} = C_2\frac{dv_o}{dt}$$

$$\boxed{V_1 = R_2C_2\frac{dv_o}{dt} + V_o} \boxed{\text{I}}$$

→ Subsituting above equation in i_{R1};

$$i_{R1} = \frac{V_1}{R_1}$$

$$\boxed{i_{R2} = \frac{R_2C_2}{R_1} \times \left(\frac{dv_o}{dt}\right) + \frac{V_o}{R_1}} \boxed{\text{II}}$$

→ subsituting eq I now in i_{C2};

$i_{C1} = C_1\frac{d(V_c)}{dt}$ where $V_c = (V_s - V_1)$

$$i_{C1} = C_1\frac{d(V_s - V_1)}{dt}$$

$$\boxed{i_{C2} = C_1\left(\frac{dV_s}{dt} - R_2C_2\left(\frac{d^2v_o}{dt^2}\right) - \frac{dv_o}{dt}\right)} \boxed{\text{III}}$$

→ subsitute eq II and III in following equation;

→ Apply KCL;

$$i_{C1} = i_{R1} + i_{C2}$$

$$C_1\left(\frac{dv_s}{dt} - R_2C_2\left(\frac{d^2v_o}{dt^2}\right) - \frac{dv_o}{dt}\right) =$$

$$\frac{R_2C_2}{R_1}\left(\frac{dv_o}{dt}\right) + \frac{V_o}{R_1} + C_2\frac{dv_o}{dt}$$

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$$\frac{dV_s}{dt} - R_2C_2\left(\frac{d^2V_o}{dt^2}\right) - \frac{dV_o}{dt} = \frac{R_2C_2}{R_1C_1}\left(\frac{dV_o}{dt}\right) + \frac{V_o}{R_1C_1} + \frac{C_2}{C_1}\frac{dV_o}{dt}$$

→ divide both sides R_2C_2;

$$\frac{dV_s}{R_2C_2\,dt} - \frac{d^2V_o}{dt^2} - \left(\frac{dV_o}{dt}\right)\frac{1}{R_2C_2} =$$

$$\left(\frac{dV_o}{dt}\right)\left(\frac{1}{R_1C_1}\right) + \frac{1}{R_1R_2C_1C_2}(V_o) + \frac{1}{C_1R_2}\left(\frac{dV_o}{dt}\right)$$

$$\boxed{\frac{d^2V_o}{dt^2} + \frac{dV_o}{dt}\left(\frac{R_1C_1+R_2C_2+R_1C_2}{R_1R_2C_1C_2}\right) + \left(\frac{1}{R_1R_2C_1C_2}\right)V_o = \frac{1}{R_2C_2}\left(\frac{dV_s}{dt}\right)}$$

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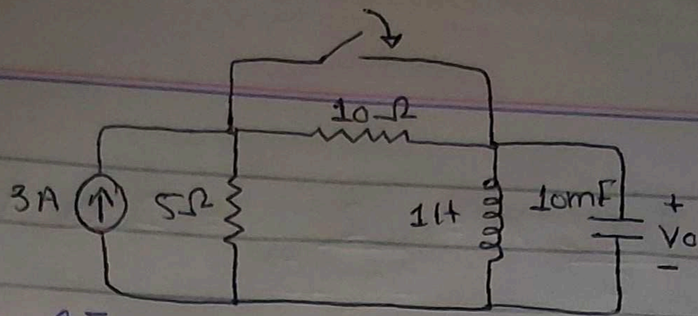
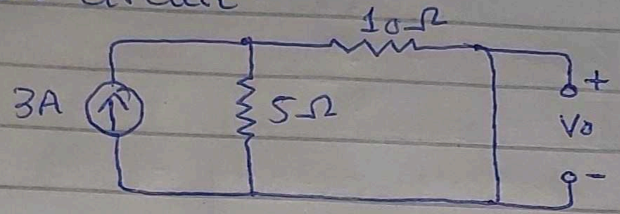
Q3 :



At $t = 0^-$;

→ During the steady state, before the switch Activation Capacitor acts as an open circuit and inductor as a short circuit.



→ Apply CDR;

$$i_L(0^-) = \frac{5}{10+5} \times 3$$

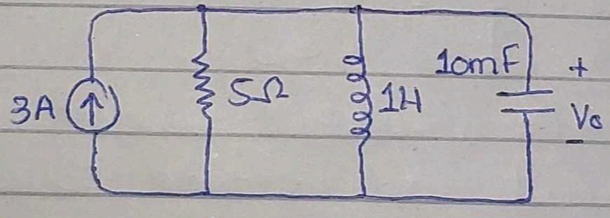
$$i_L(0^-) = 1\ A$$

→ $V_o(0^-) = 0$

→ By Continuity rules of capacitance and inductance;

$$i_L(0^-) = \boxed{I_L(0^+) = 1\ A}$$
$$V_o(0^-) = \boxed{V_o(0^+) = 0}$$

At $t = 0^+$;



→ By capacitance law;

$$i_c(0^+) = C\frac{dv(0^+)}{dt} \Rightarrow \frac{dv(0^+)}{dt} = \frac{i_c(0^+)}{C}$$

→ Apply KCL;

$$I_s = I_R(0^+) + I_L(0^+) + i_c(0^+)$$
$$3 = \frac{V_o(0^+)}{R} + 1 + i_c(0^+)$$

$$i_c(0^+) = 3 - 1 + 0$$

$$\boxed{i_c(0^+) = 2\ A}$$

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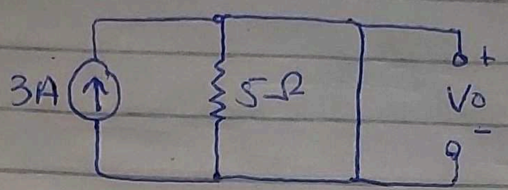
→ There fore ;

$$\frac{d(V(0^+))}{dt} = \frac{i_c(0^+)}{C} = \frac{2}{10\times10^{-3}} = 200 \ V/s$$

$$\boxed{\frac{d\,V(0^+)}{dt} = 200 \ V/s}$$

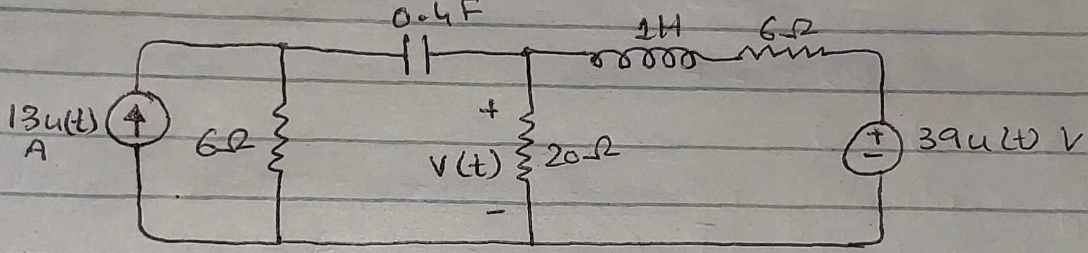
At $t = \infty$;



→ At $t=\infty$, circuit reaches steady state again.

$$\boxed{V_o(\infty) = 0}$$

Qu :

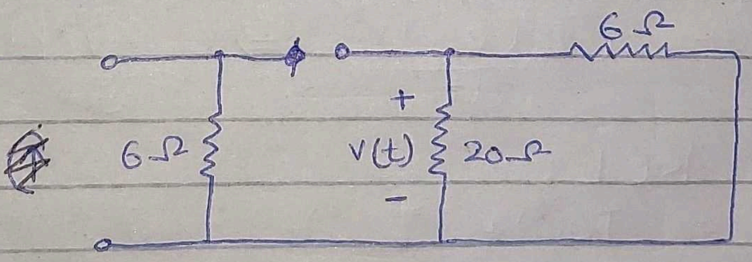


At $t = 0^-$;

→ u(t) is the unit step function defined as;

$$u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$$

So, at $t = 0^-$, current source becomes /acts as open circuit and voltage source acts as short circuit.



→ A circuit is not active ;

$V_c(0^-) = 0$

$i_L(0^-) = 0$

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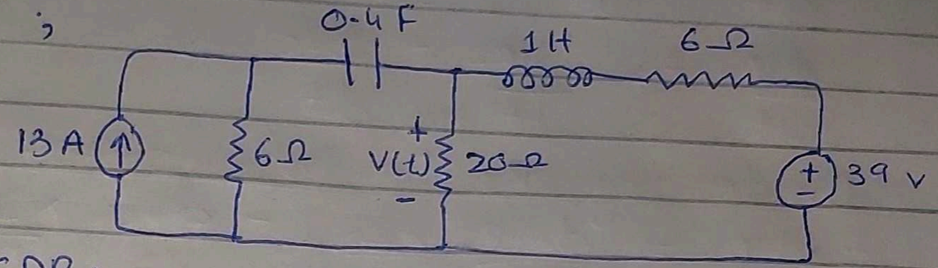
→ By continuity rules of capacitor and inductor;

$v_c(0^-) = v_c(0^+) = 0$

$i_L(0^-) = i_L(0^+) = 0$

At $t = 0^+$;



→ By CDR;

$$i(0^+) = \frac{6}{20+6} \times 13$$

$$i(0^+) = 3\ A$$

→ By Ohm's law;

$$v(0^+) = R \times i(0^+)$$

$$= 20 \times 3$$

$$\boxed{v(0^+) = 60\ V}$$

→ ~~$V(t) = 20\, i_c$~~

→ ~~Taking diffrentiation on both sides;~~

$$\cancel{\frac{d\,v(0^+)}{dt} = 20\,\frac{d\,i_c}{dt}}$$

→ ~~Letting $i_c = C\,\frac{d\,V_c(0^+)}{d}$~~

→ For $\frac{d\,v(0^+)}{dt}$;

• Apply KCL;

$$i_s = i_{R1} + i$$

$$13 = \frac{V_{R1}}{R_1} + \frac{V}{R_2}$$

$$\boxed{\frac{V_{R1}}{R_1} = 13 - \frac{V}{R_2}} \quad (1)$$

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- Apply KVL;

$$V_{R1} = V_c + V$$

$$\frac{dV_{R1}}{dt} = \frac{dV_c}{dt} + \frac{dV}{dt}$$

$$\frac{dV}{dt} = \frac{dV_{R1}}{dt} - \frac{dV_c}{dt}$$

- Divide by R_1 on both sides

$$\boxed{\frac{1}{R_1} \times \frac{dV}{dt} = \frac{dV_{R1}}{dt} \times \frac{1}{R_1} - \frac{dV_c}{dt} \times \frac{1}{R_1}}$$

- Apply capacitance law, $\frac{dV_c}{dt} = \frac{i_c}{C}$;

and put eq 1 in above eq.

$$\frac{dV}{dt} \times \frac{1}{R_1} = \frac{d}{dt}\left(13 - \frac{dV}{dt} \times \frac{1}{R_2}\right) - \frac{i_c}{CR_1}$$

$$\frac{dV}{dt}\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = \frac{-i_c}{CR_1}$$

$$\frac{dV}{dt} = \frac{-i_c}{C} \times \frac{R_2}{(R_1 + R_2)}$$

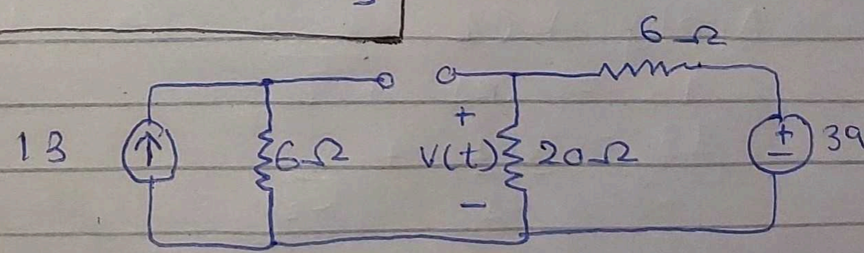
$$\frac{dV(0^+)}{dt} = \frac{-i_c R_2}{C(R_1 + R_2)}$$

→ subsitute Respective values;

$$\frac{dV(0^+)}{dt} = -\frac{3 \times 20}{(20+6)(0.4)}$$

$$\boxed{\frac{dV(0^+)}{dt} = 5.77 \ V/s}$$

At $t = \infty$;



→ By VDR;

$$V(\infty) = \frac{20}{26} \times 39$$

$$\boxed{V(\infty) = 30 \ V}$$

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Q5: $\frac{d^2i}{dt^2} + \frac{2di}{dt} + 5i = 10$

→ Source free or natural response of circuit is that response with $V_s = 0$ and $i_s = 0$

$$\frac{d^2i}{dt^2} + 2\frac{di}{dt} + 5i = 0 \quad -(1)$$

→ $$\frac{d^2y(t)}{dt^2} + 2\xi\omega_o\frac{dy(t)}{dt} + \omega_o^2 y(t) = 0 \quad -(2)$$

→ Comparing eq (1) with eq (2);

$2\xi\omega_o = 2$ and $\omega_o^2 = 5$

$\boxed{\xi = \frac{1}{\sqrt{5}}}$ $\boxed{\omega_o = \sqrt{5}}$

→ A suggested exponential solution for eq (1);

- $i(t) = Ae^{st}$
- $\frac{di(t)}{dt} = Ase^{st}$
- $\frac{d^2i(t)}{dt^2} = As^2e^{st}$

Thus,

$$As^2e^{st} + 2Ase^{st} + 5Ae^{st} = 0$$

$$\boxed{Ae^{st}(s^2 + 2s + 5) = 0}$$

→ As $\xi = 1/\sqrt{5}$, $0 < \xi < 1$, so an under damped response;

$$\boxed{S_{1,2} = -\xi\omega_o \pm (\sqrt{1-\xi^2})\,\omega_o}$$

OR

$$\boxed{S_{1,2} = -a \pm j\,\omega_d}$$

where,

$a = \xi\omega_o$, $\omega_d = \omega_o\sqrt{1-\xi^2}$

$= (1/\sqrt{5})(\sqrt{5})$ $= \sqrt{5}\sqrt{1-(1/\sqrt{5})^2}$

$\boxed{a = 1 \text{ Np/s}}$ $\boxed{\omega_d = 2 \text{ rad/s}}$

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→ $i(t)$ and it's derivative at $t=0^+$;

- $i(0^+) = 2A$
- $\frac{di(0^+)}{dt} = 4$ A/s

→ Let us now find, A and θ on the basis of initial condition,

$$i(t) = Ae^{-at}\cos(w_d t + \theta)$$

- $i(0^+) = Ae^{0}\cos(0+\theta)$

$$2 = A\cos\theta$$

$$\boxed{A\cos\theta = 2} \text{ (1)}$$

- $i(t) = Ae^{-at}(\cos(w_d t+\theta))$

$$\frac{di(t)}{dt} = A\left(e^{-at}(-a)(\cos(w_d t+\theta)) + \sin(w_d t+\theta)w_d e^{-at}\right)$$

→ put $a=1$ and $w_d = 2$

$$\frac{di(t)}{dt} = A[(e^{-t}(-)\cos(2t+\theta) + 2\sin(2t+\theta)e^{-t}]$$

$$\frac{di(0^+)}{dt} = A[e^{0}-\cos\theta + 2\sin\theta e^{0}]$$

$$4 = A(2\sin\theta - \cos\theta)$$

$$\boxed{2A\sin\theta - A\cos\theta = 4} \text{ (2)}$$

→ Subsitute eq (1) and eq (2);

[Equating A]

$$\frac{2}{\cos\theta} = \frac{4}{\cos\theta - 2\sin\theta}$$

$$\frac{-\cos\theta + 2\sin\theta}{\cos\theta} = 2$$

$$-1 + 2\tan\theta = 2$$

$$\tan\theta = +3/2$$

$$\boxed{\theta = 56.31}$$

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→ Put ② in ①;

$A \cos(-56.31) = 2$

$\boxed{A = 3.61}$

So,

$\boxed{i(t) = 3.61\, e^{-t} (\cos(2t - 56.31))}$

Q6: $\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i = 4$

→ Source free or natural response of circuit is that response in which $V_s = 0$ and $i_s = 0$.

$$\frac{d^2 i}{dt^2} + \frac{3di}{dt} + 2i = 0$$

$$\frac{d^2 y(t)}{dt^2} + 2\xi\omega_o \frac{dy(t)}{dt} + \omega_o^2 y(t) = 0$$

→ comparing eq ① with eq ②;

$2\xi\omega_o = 3$ and $\omega_o^2 = 2$

$\boxed{\xi = \frac{3}{2\sqrt{2}}}$ $\boxed{\omega_o = \sqrt{2}}$

→ A $\xi = \frac{3}{2\sqrt{2}} > 1$, so an over damped

Response. Thus,

- $\tau_1 = \frac{1}{(\zeta - \sqrt{\zeta^2 - 1})\omega_o} = \frac{1}{\left(\frac{3}{2\sqrt{2}} - \sqrt{\left(\frac{3}{2\sqrt{2}}\right)^2 - 1}\right)\sqrt{2}} = 1$

$\boxed{\tau_1 = 1}$

- $\tau_2 = \frac{1}{(\zeta + \sqrt{\zeta^2 - 1})\omega_o} = \frac{1}{\left(\frac{3}{2\sqrt{2}} + \sqrt{\left(\frac{3}{2\sqrt{2}}\right)^2 - 1}\right)\sqrt{2}} = \frac{1}{2}$

$\boxed{\tau_2 = 1/2}$

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→ $i(t)$ and its derivative at $t = 0^+$;

- $i(0^+) = 1$ A
- $\frac{di(0)}{dt} = -1$ A/s

→ Let us now find, on the basis of initial condition, A_1 and A_2 ;

- $i(0^+) = A_1 + A_2$

$$\boxed{1 = A_1 + A_2} \quad (1)$$

- $\frac{d(i(0^+))}{dt} = -\frac{1}{\tau_1} A_1 - \frac{1}{\tau_2} A_2$

$$-1 = -(1) A_1 - 2 A_2$$

$$\boxed{A_1 = 1 - 2 A_2} \quad (2)$$

→ subsitute eq (1) and (2) ;

- $1 = 1 - 2A_2 + A_2$

$$2A_2 - A_2 = 0$$

$$\boxed{A_2 = 0}$$

- $\boxed{A_1 = 1}$

→ $i(t) = A_1 e^{-t/\tau_1} + A_2 e^{-t/\tau_2} + i_{ss}$

As $i_{ss} = i(\infty)$

and From

$$\frac{d^2 i}{dt^2} + 3\frac{di}{dt} + 2i = 4$$

$$w_0 i(\infty) = 4$$

$$\sqrt{2}\, i(\infty) = 4$$

$$\boxed{i(\infty) = 2\sqrt{2}}$$

→ Therefore,

$$\boxed{i(t) = e^{-t} + 2\sqrt{2}}$$