Lecture #2

Row and column operations in matrix:

Interchange of any 2 row/coulumns in a matrix

Multiplication of a row/column with a constant

Addition of scalar multiple of 1 row/col with another

Notation

R_{ij}
R_{12}
C_{13}

$5R_1$ (kR_i)

$(R_i + kR_j)$
$R_1 + 3R_2$

Pivot/leading entry of a matrix:

~~The~~ most left non-zero entry in a non-zero of a matrix is called pivot.

$$A = \begin{bmatrix} 2 & 3 & 7 \\ 0 & 4 & 3 \\ 0 & 0 & 3 \end{bmatrix} \begin{array}{l} \rightarrow \text{row \#1} \quad \text{most left is } 2 \\ \rightarrow \text{row \#2} \quad \text{most left is } 4 \\ \rightarrow \text{row \#3} \quad \text{most left is } 3 \end{array}$$

Echelon form of a Matrix

① The first non-zero element in each row is 1.

② The no. of zeros to the right hand of pivot increase row by row from up to down.

Reduced Echelon Form.

① A matric is in the echelon form

② The column in which 1 (pivot lies), all

In other words, the pivot of any non-zero row always comes to the right of the pivot of the row above it.

other entries of that column are zeros.

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

③ All the zero rows are at the bottom.

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$A = \begin{bmatrix} 1 & 3 & 4 \\ 0 & 1 & 7 \\ 0 & 0 & 1 \end{bmatrix}$$

Upper triangular Matrix is formed always.

$$B = \begin{bmatrix} 0 & 1 & 2 & 3 & 5 \\ 0 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

Row Equivalent Matrix

Two matrices A and B are said to be row equivalent if one can be obtained from the other by finite no. of row operations.

$$A \overset{R}{\sim} B$$

Rank of a matrix:

first convert a matrix into echelon form and then count the no. of non-zero rows.

Rank (A) = no. of non-zero rows in the echelon form of A.

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$$A = \begin{bmatrix} 6 & 3 & -4 \\ -4 & 1 & -6 \\ 1 & 2 & -5 \end{bmatrix}$$

need to get 1 inplace of 6 to make it a pivot divide row/colum by 6 or interchange with 3rd row.

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ -4 & 1 & -6 \\ 6 & 3 & -4 \end{bmatrix}$$ By R_{13}.

-4 and 6 must be zero.

$-4 + 4(1) = 0$

$1 + 4(2) = 9$

$-6 + 4(-5) = -26$

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 9 & -26 \\ 6 & 3 & -4 \end{bmatrix}$$ $R_2 + 4R_1$, R_3.

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 9 & -26 \\ 0 & -9 & 26 \end{bmatrix}$$ $R_3 - 6R_1$

$6 - 6(1) = 0$

$3 - 6(2) = -9$

$-4 - 6(-5) = 26$

Now making $9 \Rightarrow 1$. So, dividing row by 9.

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & -26/9 \\ 0 & -9 & 26 \end{bmatrix}$$ $\frac{1}{9}R_2$

Now making $-9 \Rightarrow 0$ so, ~~dividing R_3 by -9~~

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & -26/9 \\ 0 & 0 & 0 \end{bmatrix}$$ $R_3 + 9R_2$

~~Rank = 2~~ (echelon form)

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Now for reduced echelon form.

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 2 & -5 \\ 0 & 1 & -26/9 \\ 0 & 0 & 0 \end{bmatrix}$$

making $2 \Rightarrow 0$.

$R_1 - 2R_2$

$1 - 2(0) = 1 - 0 = 1$

$2 - 2(1) = 0$

$-5 - 2(-26/9) = 7/9$

making $-5 \Rightarrow 0$

$R_1 + 5R_2$

$1 + 5(0) = 1$

$2 + 5(1) = 2+5=7$

$-5 + 5(-2/9) =$

$$A \overset{R}{\sim} \begin{bmatrix} 1 & 0 & -7/9 \\ 0 & 1 & -26/9 \\ 0 & 0 & 0 \end{bmatrix} \rightarrow \text{reduced echelon form}$$

rank is 2

Inverse of a matrx:

A square matrix B is salled to be inverse of other matrix if.

$$\boxed{BA = I}$$

B is called right Inverse of A if $AB = I$

B is called left Inverse of A if $BA = I$

Invertible / Non-singular matrix.

3×3 ✓.

$$A=\begin{bmatrix}1&0&3\\2&4&1\\1&3&0\end{bmatrix} \qquad B=\begin{bmatrix}-1&3&-4\\1/3&-1&-5/3\\2/3&1&4/3\end{bmatrix}$$

$$AB=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3. \qquad BA=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}=I_3$$

$$\boxed{AB=I=BA}$$

$$A=\begin{bmatrix}1&0&3\\2&4&1\\1&3&0\end{bmatrix} \qquad I=\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}$$

R_2-2R_1 and R_3-1R_1

$2-2(1)=0$ $\quad 1-1=0$

$4-2(0)=4$ $\quad 3-0=3$

$1-2(3)=-5$ $\quad 0-3=-3.$

R_2

$0-2(1)=-2$

$1-2(0)=1$

$0-2(0)=0.$

R_3

$0-1=-1$

$0-0=0$

$1-0=1$

$$\begin{bmatrix}1&0&3\\0&4&-5\\0&3&-3\end{bmatrix} \qquad \begin{bmatrix}1&0&0\\-2&1&0\\-1&0&1\end{bmatrix}$$

dividing R_2 by $4 \Rightarrow 1.$

$$\begin{bmatrix}1&0&3\\0&1&-5/4\\0&3&-3\end{bmatrix} \qquad \begin{bmatrix}1&0&0\\-\tfrac{1}{2}&1/4&0\\-1&0&1\end{bmatrix}$$

R_3-3R_2

$0-3(0)=0$

$3-3(1)=0$

$-3-3\left(\frac{-5}{4}\right)=\frac{3}{4}$

$-1-3\left(-\frac{1}{2}\right)=-1+3/2=1/2$

$0-3(1/4)=0-3/4=-3/4$

$1-3(0)=1-0=1$

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$$\left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & -5/4 \\ 0 & 0 & 3/4 \end{array}\right] = \left[\begin{array}{ccc} 1 & 0 & 0 \\ -1/2 & 1/4 & 0 \\ 1/2 & -3/4 & 1 \end{array}\right]$$

dividing R_3 by $3/4$

$$\left[\begin{array}{ccc} 1 & 0 & 3 \\ 0 & 1 & -5/4 \\ 0 & 0 & 1 \end{array}\right] \quad \left[\begin{array}{ccc} 1 & 0 & 0 \\ -1/2 & 1/4 & 0 \\ 2/3 & -1 & 4/3 \end{array}\right].$$

$\frac{1}{2} \div \frac{3}{4}$

$\frac{1}{2} \times \frac{4}{3}$

$1 \times \frac{4}{3}$

$R_1 - 3R_3$

$1 - 3(0) = 1 - 0 = 1$ $\quad 1 - 3(2/3) = 1 - 6/3 = \frac{-3}{3} = -1$

$0 - 3(0) = 0 - 0 = 0$ $\quad 0 - 3(-1) = +3$

$3 - 3(1) = 3 - 3 = 0$ $\quad 0 - 3(4/3) = -12/3 = -4.$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & -5/4 \\ 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc} -1 & 3 & -4 \\ -1/2 & 1/4 & 0 \\ 2/3 & -1 & 4/3 \end{array}\right]$$

$R_2 + 5/4 R_3$

$0 + 5/4(0) = 0$ $\quad -1/2 + 5/4(2/3) = -\frac{1}{2} + \frac{10}{12}$

$1 + 5/4(0) = 1$ $\quad 1/4 + 5/4(-1) = \frac{1}{4} - \frac{5}{4} =$

$-5/4 + 5/4(1) = 0$ $\quad 0 + 5/4(4/3) = \frac{20}{12}$

$$\left[\begin{array}{ccc} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{array}\right] = \left[\begin{array}{ccc} -1 & 3 & -4 \\ 1/3 & -1 & 5/3 \\ 2/3 & 1 & 4/3 \end{array}\right]$$

So $A^{-1} = \left[\begin{array}{ccc} -1 & 3 & -4 \\ 1/3 & -1 & 5/3 \\ 2/3 & 1 & 4/3 \end{array}\right]$