

# Day / Date ENA Assignment 3

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359607

CE-42-A

(a) Power factor:

Q1 Find impedance Total ( $Z_T$ ):

$$Z_1 = (8 + j6) \parallel (10 - j5)$$

$$Z_1 = \frac{(8 + j6)(10 - j5)}{(8 + j6 + 10 - j5)} = \frac{40 + 20j}{18 - j} = \frac{110^2 + 20^2}{18^2 + (-1)^2} \angle \frac{10 - 30}{-3 - 17}$$

$$Z_1 = \frac{111.8 \angle 10.30}{18 \angle -3.17} = \boxed{6.21 + j0.77}$$

$$Z_T = Z_1 + 2\Omega = 6.21 + 2 + j0.77 = \boxed{8.15 + j0.77}$$

$$\bullet \text{ Rewrite in phasor form: } Z = 8.18 \angle 5.39^\circ$$

$\bullet$  Since Power factor angle is same as angle of ( $Z$ )

$$\therefore P_f = \cos(\theta_v - \theta_i)$$

$$= \cos(5.39^\circ)$$

$$\boxed{P_f = 0.995}$$

(b) Power delivered by the source / Complex Power:

$\bullet$  Using Ohm's law, Find  $I$  in circuit;

$$I = \frac{V}{Z_T} = \frac{16 \angle 45^\circ}{8.18 \angle 5.39^\circ} = \frac{16}{8.18} \angle 45 - 5.39 = \boxed{1.96 \angle 39.61^\circ \text{ A}}$$

$\bullet$  Taking Complex conjugate of  $I$ ,

$$I^* = 1.96 \angle -39.61^\circ$$

We know that,

$$\bullet S = \frac{1}{2} V I^*$$

$$S = 0.5 \times (16 \angle 45^\circ)(1.96 \angle -39.61^\circ)$$

$$S = 15.68 \angle 45 - 39.61$$

$$\boxed{S = 15.7 \angle 5.39^\circ \text{ (VA)}}$$



Day / Date (c) Average Power delivered by the Source:

We already have

$$V_m = 16V$$

$$I_m = 1.96A$$

$$\theta_v - \theta_i = 5.39^\circ$$

According to the formula,

$$P = \frac{1}{2} V_m I_m \cos(\theta_v - \theta_i)$$

Putting values,

$$P = 0.5 \times 16V \times 1.96A \times \cos(5.39^\circ)$$

$$P = 15.62 \text{ Watt}$$

(d) Reactive Power delivered by the Source (S):

We have already know complex power (S)

$$S = 15.7 \angle 5.39^\circ \text{ (VA)}$$

Write in vector form:

$$S = \left[ (15.6)^2 + (5.39)^2 \tan^{-1} \left( \frac{5.39}{15.6} \right) \right]^\circ$$

$$S = 15.7 \cos(5.39) + j \sin(5.39) \times 15.7$$

$$S = A \cos \theta + j A \sin \theta$$

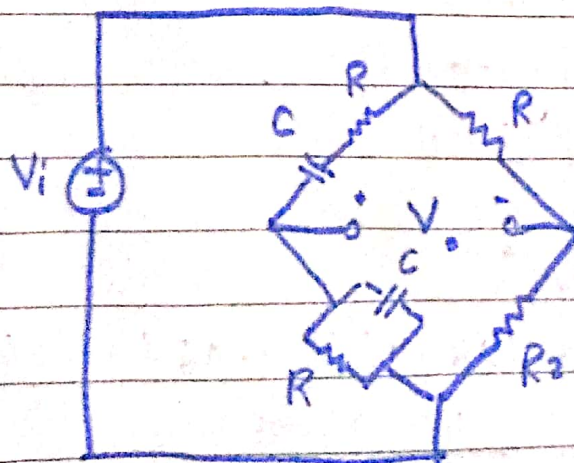
$$S = 15.7 \cos(5.39) + j (15.7) \sin(5.39)$$

$$S = 15.33 + 1.47j \text{ VA}$$

We know Imaginary Part of Complex Power S is Q the Reactive Power.  $\text{Im}(S) = Q$

Reactive Power =  $Q = 1.47 \text{ (VAR)}$   
by source

Q2)



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$$Z_C = \frac{1}{C\omega j}, \text{ so,}$$

$$Z_1 = R + \frac{1}{C\omega j}$$

$\Rightarrow$

$$Z_1 = \frac{RC\omega j + 1}{RC\omega j}$$

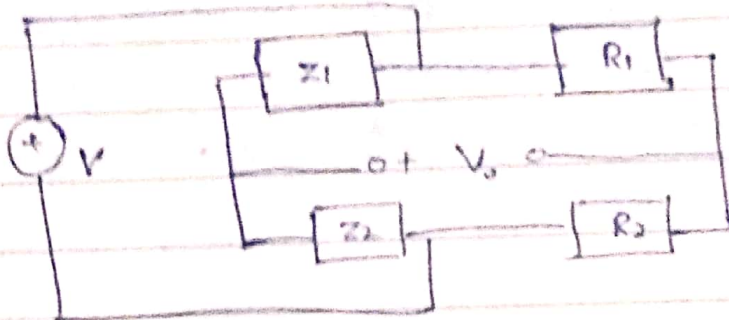
$$Z_2 = R \parallel \frac{1}{C\omega j}$$

$\Rightarrow$

$$Z_2 = \frac{R}{1 + RC\omega j}$$

The circuit becomes,

$\Rightarrow$  Applying VDR across  $Z_2$  and  $R_2$



$$V_1 = \frac{Z_2}{Z_1 + Z_2} V$$

$$= \frac{R / (1 + RC\omega j)}{R / (1 + RC\omega j) + \frac{1}{C\omega j}}$$

$$= \frac{R}{1 + RC\omega j} \div \frac{RC\omega j + (1 + RC\omega j)^2}{(1 + RC\omega j)(C\omega j)}$$

$$V_1 = \frac{RC\omega j}{RC\omega j + (1 + RC\omega j)^2} V$$

$$V_2 = \frac{R_2}{R_1 + R_2} V_1$$

Since  $V_0 = V_1$

$$V_0 = \frac{RC\omega j}{RC\omega j + (1 + RC\omega j)^2} V = \frac{R_1}{R_1 + R_2} V$$

$$V_0 = \left[ \frac{RC\omega j}{1 - \omega^2 R^2 C^2 + 3RC\omega j} \right] V = \frac{R_1}{R_1 + R_2} V$$

For  $V_0$  and  $V$  to be in phase,  $\frac{V_0}{V}$  must be purely real i.e., no complex part. To ensure the condition

$$1 - \omega^2 R^2 C^2 = 0$$

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$$\omega^2 = \frac{1}{R^2 C^2}$$

$$\omega = \frac{1}{RC}$$

• We know that,  $\omega = 2\pi f$ , so,

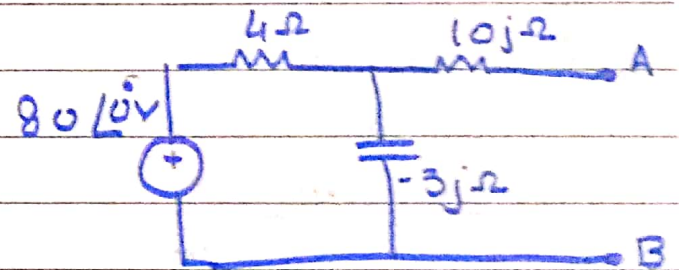
$$2\pi f = \frac{1}{RC}$$

$$f = \frac{1}{2\pi RC}$$

**Q4** Find the load Impedance that will receive the maximum power when connected across terminal A & B. Compute the value of max Power.

Ans)

For  $Z$  (Total impedance) of given circuit.



$$Z_T = \frac{(-3j)(4) + 10j}{4 - 3j} = \frac{-12 + 10j}{4 - 3j}$$

$$= \frac{-12j}{4 - 3j} \times \frac{4 + 3j}{4 + 3j} + 10j$$

$$= \frac{-48j}{25} + \frac{36}{25} + 10j$$

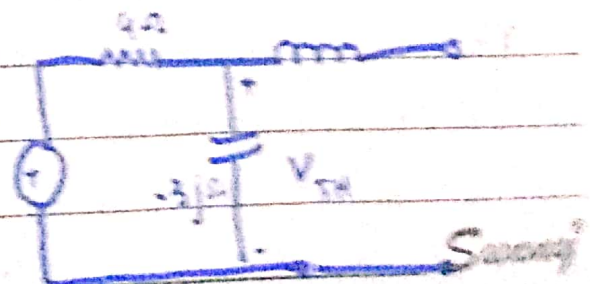
$$Z_T = 1.44 + 8.08j \text{ or } Z_T = 8.21 \angle 79.89^\circ$$

(phasor form)

→ For  $V_{TH}$

Applying voltage divider Rule.

$$V_{TH} = \frac{-3j}{-3j + 4} \times 30 \angle 0^\circ$$



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$$= \begin{bmatrix} -3j \times 3j+4 \\ -3j+4 & 3j+4 \end{bmatrix} 30 \angle 0^\circ$$

$$= \begin{bmatrix} -12j + 9 \\ 25 & 25 \end{bmatrix} 30 \angle 0^\circ$$

$$= 0.6 \angle -53.13^\circ \times 30 \angle 0^\circ$$

$$V_{TH} = 18 \angle -53.13^\circ V$$

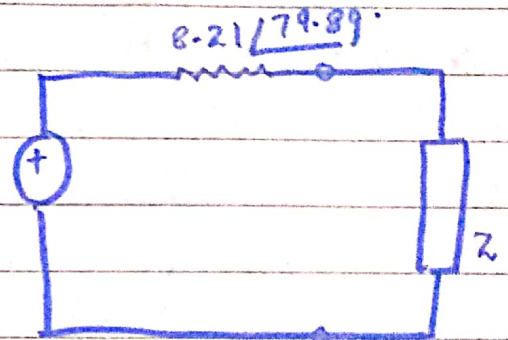
The return equivalent.

For maximum Power,

$$Z_L = Z_{TH}^*$$

$$Z_L = 8.21 \angle -79.89^\circ \Omega$$

$$18 \angle -53.13^\circ$$



⇒ For max Power,

$$\text{Average Power} = P = \frac{1}{2} \frac{|V_{TH}|^2 R_L}{(R_{TH} + R_L)^2 + (X_{TH} + X_L)^2}$$

From Previous Part of questions

$$Z_L = 1.44 - 8.08j \text{ so, } R_L = 1.44, X_L = +8.08j$$

$$Z_{TH} = 1.44 + 8.08j \text{ so, } R_{TH} = 1.44, X_{TH} = -8.08j$$

Substitution in eq of average Power,

$$P = \frac{1}{2} \frac{18^2 \times 1.44}{(1.44 + 1.44)^2 + (8.08j - 8.08j)^2}$$

$$P_{max} = \frac{466.56}{2(2.88)^2}$$

$$P_{max} = 28.125 \text{ Watt}$$

Q5) Load 2 draws 20kVA at pf of 0.85 lagging from 220V 50Hz rms, sinusoidal



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## (a) Average and relative Power delivered by the Source:

We know that,

$$\text{Average Power} = P = pf \times S$$

So,

$$P = 0.85 \times 20 \times 10^3$$

$$P = 17 \text{ kW}$$

⇒ For Reactive Power  $Q$ ,

$$Pf = \cos(\theta_v - \theta_i)$$

$$0.85 = \cos(\theta_v - \theta_i)$$

$$\theta_v - \theta_i = 31.79$$

$$Q = S \sin(\theta_v - \theta_i)$$

$$= (2 \times 10^3) (\sin(31.799))$$

$$Q = 10.54 \text{ kVAR}$$

## b) PEAK CURRENT:

We know that,

$$I_{rms} = \frac{S}{V_{rms}} = \frac{20 \times 10^3}{220}$$

$$I_{rms} = 90.91 \text{ A}$$

Since,

$$I_{rms} = I_{rms} / \sqrt{2}$$

$$= 90.91 \times \sqrt{2}$$

$$I_{rms} = 128.56 \text{ A}$$

## c) LOAD IMPEDENCE:

From Previous Parts of the question.

$$I_{rms} = 90.91 \text{ A}$$

$$V_{rms} = 220 \text{ V}$$

$$\theta_v - \theta_i = 31.79^\circ$$

## Impedance Matching - Q42-A

Given:

$$\text{Power} = \frac{V_{rms} I_{rms} \cos(\theta_v - \theta_i)}{2}$$

$$100 = \frac{220 \times 1.67 \times \cos(\theta_v - \theta_i)}{2}$$

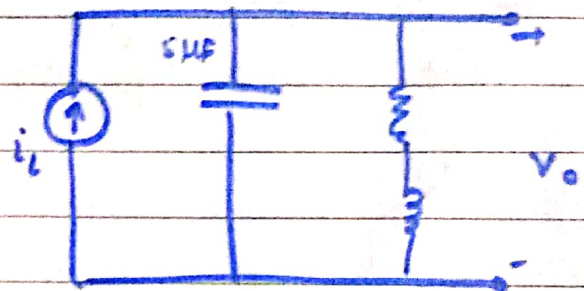
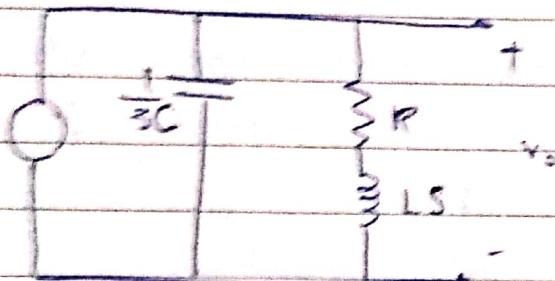
$$\cos(\theta_v - \theta_i) = 0.27$$

Since,

$$\cos(\theta_v - \theta_i) = \text{Pf}$$

$$\text{Pf} = 0.27$$

Q3 Re drawing circuit.



$$Z_T = \frac{(R + LS) \left( \frac{1}{sC} \right)}{(R + LS) + \left( \frac{1}{sC} \right)} = \frac{(R + LS) \left( \frac{1}{sC} \right)}{RSC + LSC + 1}$$

$$Z_T = \frac{R + LS}{(RSC + LSC + 1)}$$

Using Ohm's Law,

$$Z = \frac{V_o}{I_i}$$

$$\frac{V_o}{I_i} = \frac{R + LS}{RSC + LSC + 1}$$

Thus,

$$\text{Transfer function } H(s) = \frac{V_o}{I_i}$$

Sum



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$$H(s) = \frac{R + Ls}{RSC + Ls^2C + 1}$$

### \* DIMENSIONAL CHECK:

Each of numerator term has same dimensions as that of resistance.

Each of denominator term has same is dimensionless. Ratio of 2 sets is  $\Omega$ , the same dimensions ratio as  $\frac{V_o}{I_i}$ .

Our Function thus passes the dimension test

### \* ASYMPTOTIC CHECKING

Letting  $s \rightarrow 0$ , our transfer function,  $H(s)$

$$H(s) = R = 120\Omega$$

To verify it physically, At  $s=0$  our capacitance acts as an open circuit.

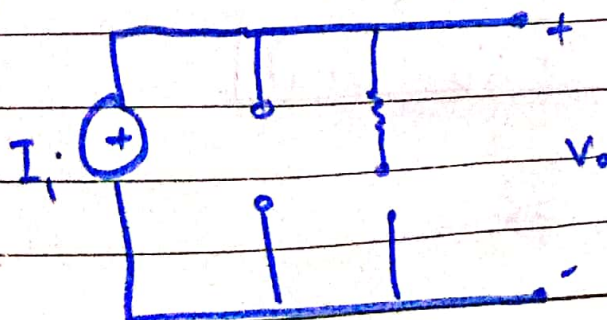
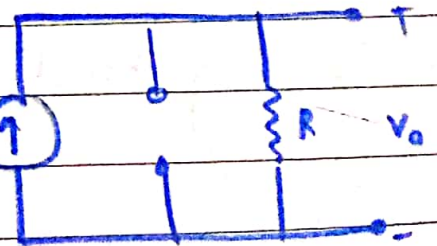
$$\boxed{\lim_{s \rightarrow \infty} Z_L = 0}$$

• Transfer function =  $R = 120\Omega$

$$\therefore H(s) = 0$$

$$\lim_{s \rightarrow \infty} Z_o = 0 \text{ — short circuit}$$

$$\lim_{s \rightarrow \infty} Z_L = \infty \text{ — Open circuit.}$$





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Any voltage through shortcircuit is Zero, Thus the transfer function is 0.

Put

$$R = 120 \Omega$$

$$L = 20 \text{ mH}$$

$$C = 5 \mu\text{F}$$

$$H(s) = \frac{120 + 20ms}{(600\mu s) + (100n)s^2 + 1}$$

→ Zero:

$$N(s) = 120 + 20ms = 0$$

$$s = -\frac{120}{20m}$$

$$z = -6k \text{ NP/s}$$

→ Poles:

$$P_1 = -3k + 1kj$$

$$P_2 = -3k - 1kj$$

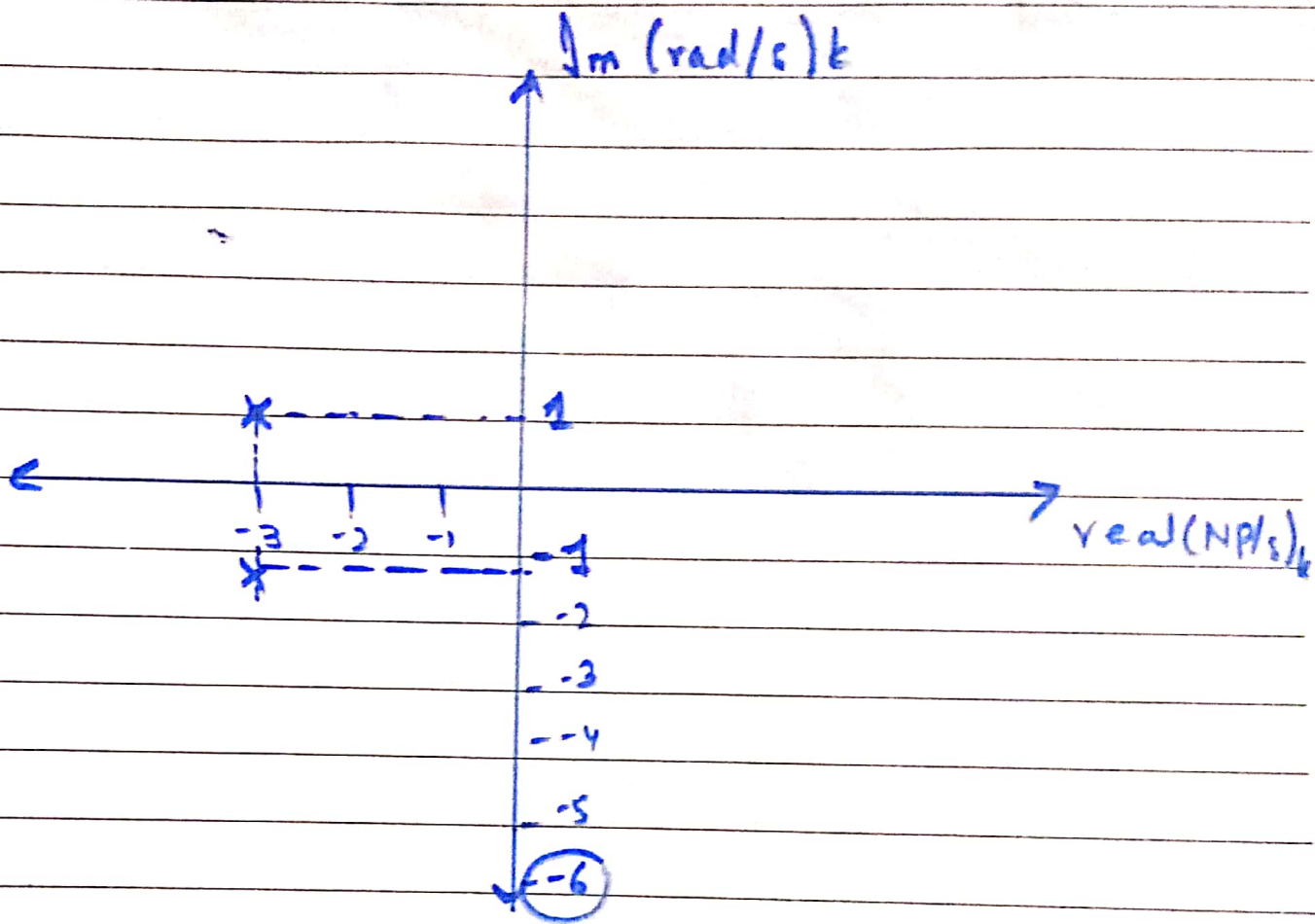
→ Scaling factor:

$$k = \frac{1s \cdot 9m}{bn} = \frac{20 \cdot m}{100n}$$

$$k = 200k$$

$$H(s) = \frac{200k (s + 6k)}{(100ns^2 + 600\mu s + 1)}$$

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THE END