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CE-42-A

Assignment # 4

Question 12:

(i)

$t^3 + \sin 2t$

$$f(t) = t^3 + \sin 2t$$

$$\mathcal{L}[f(t)] = \mathcal{L}[t^3 + \sin 2t]$$

$$f(s) = \mathcal{L}(t)^3 + \mathcal{L}(\sin 2t)$$

$$f(s) = \int_0^\infty e^{-st} t^3 dt + \int_0^\infty e^{-st} \sin 2t$$

$$\int_0^\infty e^{-st} t^3 dt = \lim_{A\to\infty} \left| \frac{t^3 e^{-st}}{-s} \right|_0^A - \lim_{A\to\infty} \int_0^A \frac{e^{-st}}{-s} \cdot 3t^2 dt$$

$$= \lim_{A\to\infty} \left[\left| \frac{-t^3 e^{-st}}{s} \right|_0^A + \frac{3}{s} \int_0^A e^{-st} t^2 dt \right.$$

$$= \lim_{A\to\infty} \left[\left| \frac{-t^3 e^{-st}}{s} \right|_0^A + \frac{3}{s} \left[\left| t^2 \cdot \frac{e^{-st}}{-s} \right|_0^A - \int_0^A \frac{e^{-st}}{-s} 2t dt \right. \right.$$

$$= \lim_{A\to\infty} \left[\left| \frac{-t^3 e^{-st}}{s} \right|_0^A - \left| \frac{3t^2 e^{-st}}{s^2} \right|_0^A + \frac{6}{s^2} \int_0^A t e^{-st} dt \right]$$

$$= \lim_{A\to\infty} \left[\left| \frac{-t^3 e^{-st}}{s} \right|_0^A - \left| \frac{3t^2 e^{-st}}{s^2} \right|_0^A + \frac{6}{s^2} \left[\left| \frac{t e^{-st}}{-s} \right|_0^A - \int_0^A \frac{e^{-st}}{-s} dt \right] \right]$$

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$$= \lim_{A\to\infty}\left[\left|-t^3\frac{e^{-st}}{s}\right|_0^A - \left|\frac{3}{s^2}t^2e^{-st}\right|_0^A + \frac{6}{s^2}\left[\left|\frac{te^{-st}}{-s}\right|_0^A - \int_0^n \frac{e^{-st}}{-s}dt\right]\right]$$

$$= \lim_{A\to\infty}\left[\left|-\frac{t^3e^{-st}}{s} - \frac{3t^2e^{-st}}{s^2} - \frac{6}{s^3} + \frac{6}{s^4}e^{-st}\right|_0^A\right]$$

$$= \lim_{A\to\infty}\left[\left|\frac{-s^3t^3e^{-st} - 3s^2te^{-st} - 6ste^{-st} - 6e^{-st}}{s^4}\right|_0^A\right]$$

- Applying limit, we get

$$0 - \left(\frac{-6}{s^4}\right)$$

$$= \frac{6}{s^4}$$

$$\int_0^\infty e^{-st}\sin 2t = \lim_{A\to\infty}\int_0^n e^{-st}\sin 2t$$

$$= \lim_{A\to\infty}\int_0^A e^{-st}\sin 2t = \lim_{A\to\infty}\left[\left|\frac{\sin 2t\, e^{-st}}{-s}\right|_0^A - \int_0^A \frac{e^{-st}}{-s}\cdot 2\cos 2t\cdot dt\right]$$

$$= \lim_{A\to\infty}\left[\left|-\frac{\sin 2t\cdot e^{-st}}{s}\right|_0 - \left|\frac{2\cos 2te^{-st}}{s^2}\right|_0^A - \frac{4}{s^2}\int_0 e^{-st}\sin 2t\,dt\right]$$

$$\lim_{A\to\infty}\int_0^A e^{-st}\sin 2t\,dt = \lim_{A\to\infty}\left[\left|-\frac{\sin 2t\, e^{-st}}{s}\right|_0^A - \left|\frac{2}{s^2}\cos 2t\, e^{-st}\right|_0^A\right]$$

$$-\lim_{A\to\infty}\frac{4}{s^2}\int_0^A e^{-st}\sin 2t\,dt.$$

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From (2)

$$F = 1 + 2A - B - 2E$$

$$F = 1 + 2\left(-\frac{1}{8}\right) - \left(-\frac{3}{4}\right) + 2\left(\frac{1}{5}\right)$$

$$F = \frac{2}{5}$$

Putting values in (I)

$$f(s) = \frac{-7}{8s} - \frac{3}{4s^2} - \frac{1}{2s^2} + \frac{27}{40(s-2)} + \frac{\frac{1}{5}s + \frac{2}{5}}{s^2+1}$$

$$= \frac{-7}{8s} - \frac{3}{4s^2} - \frac{1}{2s^2} + \frac{27}{40(s-2)} + \frac{s}{5(s^2+1)} + \frac{2}{5(s^2+1)}$$

Applying Lapalace inverse:

$$f(t) = \frac{-7}{8} \mathcal{L}^{-1}\frac{1}{s} - \frac{3}{4}\mathcal{L}^{-1}\frac{1}{s^2} - \frac{1}{2}\mathcal{L}^{-1}\frac{1}{s^2} + \frac{27}{40}\mathcal{L}^{-1}\frac{1}{s-2} + \frac{1}{5}\mathcal{L}^{-1}\frac{s}{s^2+1} + \frac{2}{5}\mathcal{L}^{-1}\frac{1}{s^2+1}$$

$$= \frac{-7}{8} - \frac{3}{4}t - \frac{1}{2}\frac{t^2}{2} + \frac{27}{40}e^{2t} + \frac{1}{5}\cos t + \frac{2}{5}\sin t$$

$$\boxed{= \frac{-7}{8} + \left(\frac{-3t - t^2}{4}\right) + \frac{27}{40}e^{2t} + \frac{1}{5}\cos t + \frac{2}{5}\sin t}$$

--- The END ----

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Applying limit $\rightarrow \frac{1}{s}$

$$\int_0^\infty \cos 6t\, e^{-st}\, dt = \lim_{A\to\infty} \int_0^A e^{-st} \cos 6t\, dt$$

$$= \lim_{A\to\infty} \left[\left| \frac{e^{-st}}{-s} \cos 6t \right|_0^A - \int_0^A \frac{6e^{-st}}{-s} \cdot -\sin 6t\, dt \right]$$

$$= \lim_{A\to\infty} \left[\left| \frac{e^{-st}}{-s} \cos 6t \right|_0^A - \frac{6}{s} \left[\left| \frac{e^{-st}}{-s} \sin 6t \right|_0^A - \int_0^A \frac{e^{-st}}{-s} \cdot 6\cos 6t\, dt \right] \right]$$

$$= \lim_{A\to\infty} \left[\left| \frac{e^{-st}}{-s} \cos 6t \right|_0^A + \left| \frac{6}{s^2} e^{-st} \sin 6t \right|_0^A \right] - \frac{3}{s^6} \lim_{A\to\infty} \int_0^A e^{-st} \cos 6t\, dt$$

$$= \lim_{A\to\infty} \left[\left| \frac{e^{-st}}{-s} \cos 6t \right|_0^A + \left| \frac{6}{s^2} \sin 6t\, e^{-st} \right|_0^A \right] - \frac{3}{s6} \lim_{A\to\infty} \int_0^A e^{-st} \cos 6t\, dt$$

$$\int_0^\infty \cos 6t\, e^{-st}\, dt + \frac{36}{s^2} \int_0^\infty \cos 6t\, e^{-st}\, dt = \lim_{A\to\infty} \left[\left| \frac{-s e^{-st} \cos 6t + 6 e^{-st} \sin 6t}{s^2} \right|_0^A \right]$$

$$\left(\frac{s^2+36}{s^2} \right) \int_0^\infty \cos 6t\, e^{-st}\, dt = \lim_{A\to\infty} \left[\left| \frac{-s e^{-st} \cos 6t + 6 e^{-st} \sin 6t}{s^2+36} \right|_0^A \right]$$

Applying limit

$$\int_0^\infty \cos 6t\, e^{-st}\, dt = \left[0 - \left(-\frac{s}{s^2+36} \right) \right]$$

$$= \frac{s}{s^2+36}$$