



NAME: AMINA QADEER (359607)

DEPT-SYNDICATE: CE-42-A

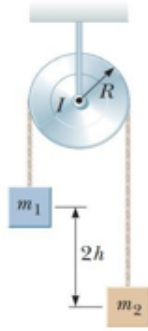
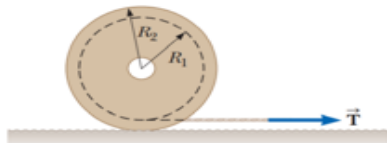
SUBJECT: AP ASSIGNMENT 2

GROUP MEMBER: 2

QUESTIONS ASSIGNED: Q1 (B,E,G), Q3 AND Q6

QUESTION 1:

b)	<p>A circular curve of highway is designed for traffic moving at 60 km/h. Assume the traffic consists of cars without negative lift.</p> <p>(a) If the radius of the curve is 150 m, what is the correct angle of banking of the road?</p> <p>(b) What would be the minimum coefficient of friction between tires and road that would keep traffic from skidding out of the turn when traveling at 70 km/h?</p>
g)	<p>Figure 10-48 shows a rigid assembly of a thin hoop (of mass m and radius $R = 0.150$ m) and a thin radial rod (of mass m and length $L = 2.00R$). The assembly is upright, but if we give it a slight nudge, it will rotate around a horizontal axis in the plane of the rod and hoop, through the lower end of the rod. Assuming that the energy given to the assembly in such a nudge is negligible, what would be the assembly's angular speed about the rotation axis when it passes through the upside-down (inverted) orientation?</p> <div data-bbox="890 1104 1225 1366"> <p>Figure 10-48 Problem 67.</p> </div>
e)	<p>Four particles, each of mass, 0.20 kg, are placed at the vertices of a square with sides of length 0.50 m. The particles are connected by rods of negligible mass. This rigid body can rotate in a vertical plane about a horizontal axis A that passes through one of the particles. The body is released from rest with rod AB horizontal (Fig. 10-64).</p> <p>(a) What is the rotational inertia of the body about axis A?</p> <p>(b) What is the angular speed of the body about axis A when rod AB swings through the vertical position?</p> <p>© Find rotational kinetic energy of the rigid object.</p> <div data-bbox="954 1518 1185 1787"> <p>Figure 10-64 Problem 104.</p> </div>

<p>Q3</p>	<p>Consider two objects with $m_1 > m_2$ connected by a light string that passes over a pulley having a moment of inertia of I about its axis of rotation as shown in Figure P10.50. The string does not slip on the pulley or stretch. The pulley turns without friction. The two objects are released from rest separated by a vertical distance $2h$.</p> <p>(a) <u>find</u> the translational speeds of the objects as they pass each other.</p> <p>(b) Find the angular speed of the pulley at this time.</p> <p>(c) Find Tensions in the string</p>	 <p>Figure P10.50</p>
<p>Q6</p>	<p>A spool of thread consists of a cylinder of radius R_1 with end caps of radius R_2 as depicted in the end view shown in Figure P10.91. The mass of the spool, including the thread, is m, and its moment of inertia about an axis through its <u>center</u> is I. The spool is placed on a rough, horizontal surface so that it rolls without slipping when a force T acting to the right is applied to the free end of the thread. (a) Show that the magnitude of the friction force exerted by the surface on the spool is given by</p> $f = \left(\frac{1 + mR_1R_2}{1 + mR_2^2} \right) T$ <p>(b) Determine the direction of the force of friction.</p>	 <p>Figure P10.91</p>

SOLUTIONS

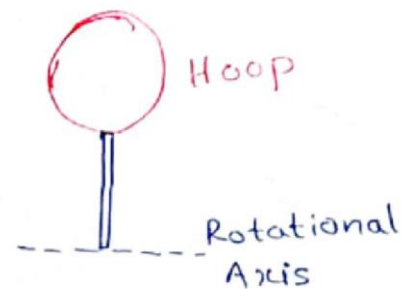
Amina Abdul-Qadeer (Member 2) (359607)

CE - 42 - A

AP - assignment 2

Q# 1(g)

Given that there is a assembly of a thin hoop and a rod.



- Length of rod = $L = 2R$
- Radius of thin hoop = $R = 0.150\text{m}$

Find, angular speed of assembly about the rotational axis?

$$I = \frac{1}{2} ML^2 \quad (\text{Moment of inertia of rod about its COM})$$

Mass of Hoop = mass of rod = m

Using Parallel axis theorem, that states that, "Moment of inertia of rigid body about any axis is equal to the sum of its moment of inertia about a parallel axis through its centre of mass and Product of mass of body and square of distance between the two axes."

Total moment of inertia can be calculated:

$$I = \left[\frac{mL^2}{12} + m\left(\frac{L}{2}\right)^2 \right] + \frac{1}{2}mR^2 + m(L+R)^2$$

$$I = \frac{mL^2}{3} + \frac{1}{2}mR^2 + 9mR^2 \quad \because L=2R$$

$$I = mR^2 \left(\frac{4}{3} + \frac{1}{2} + 9 \right)$$

$$I = 10.83 mR^2 \rightarrow \text{moment of inertia of assembly}$$

considering base of rod of 0.150m to be at origin $(0,0)$ then the centre of mass is at,

$$y_{\text{Position}} = \frac{\frac{mL}{2} + m(L+R)}{m+m} = 2R$$

Comparing the position, to its upside down position shows that the change in COM position is $|\Delta y| = 4R$

Amina Qadeer (359607)
CE-42-A

Aminaqadeer ce42.ceme

⇒ Corresponding loss in Gravitational P.E is converted into K.E.

$$K = \Delta U$$

$$K = (2m)g \times 4R$$

$$K = \frac{1}{2} I \omega^2 = 8mgR$$

$$10.83 \times \frac{1}{2} mR^2 \times \omega^2 = 8mgR$$

$$\omega = \sqrt{\frac{8 \times 2 \times 9.8}{10.83 \times 0.15}}$$

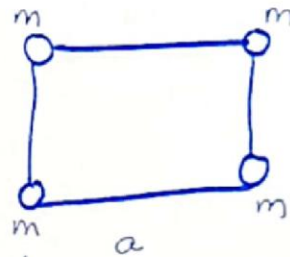
Result :-

$$\omega = 9.82 \text{ rad/s}$$

so angular speed of assembly is $\omega = 9.82 \text{ rad/s}$.

Q#1 Given,
 $m =$

⇒ Rotational inertia - About an axis passes through the midpoint of one of the sides and perpendicular to the plane of square.



According to parallel axes Theorem,

$$I = I_{\text{com}} + md^2$$

⇒ The moment of inertia (or rotational inertia) is.

$$I = 0.2 \times 0.562 + 0.2 \times 0.56 + 0.2 \times 0.7922$$

$$I = 0.251 \text{ kgm}^2$$

⇒ The square loses potential Energy as it drops and gain kinetic Energy.

$$\text{So, lost P.E is } 2mg\Delta h = 2 \times 0.2 \times 9.8 \times 1.12 = 4.39 \text{ joules}$$

⇒ So this must be K.E when AB is vertical. We know

$$K.E = \frac{1}{2} I \omega^2 \quad \text{or} \quad \omega^2 = \frac{2KI}{2}$$

(Aminaqadeer.ce.42-A-359607)

$$\omega^2 = \frac{2 \times 4.39}{0.251} = 35$$

$$\omega = 5.92 \text{ rad/sec}$$

Q1(b) Data :

Speed of Traffic = $v = 60 \text{ kmh}^{-1}$

a) Radius of curve = $R = 150 \text{ m}$

Angle of banking = $\theta = ?$

b) If speed increases to 70 kmh^{-1}

Coefficient of friction = $\mu = ?$
between tyre and road

Solution :

a) $v = 60 \text{ kmh}^{-1} = 16.67 \text{ ms}^{-1}$

$$\tan \theta = \frac{v^2}{rg}$$

$$\theta = \tan^{-1} \left(\frac{(16.67)^2}{150 \times 9.8} \right)$$

$$\theta = 10.70^\circ$$

b) Coefficient of friction $\mu = \frac{v^2}{rg}$

$$\mu = \frac{(16.67)^2}{150 \times 9.8}$$

$$\mu = 0.189$$

Solution. $\mu_s = \frac{v^2}{rg}$

$$\mu_s = \frac{(19.0)^2}{150 \times 9.8}$$

$$\mu_s = \frac{361}{1470}$$

$$\mu_s = 0.24$$

Amina Qadeer
CE-42-A (359607)

$$\therefore \frac{60 \times 1000}{3600}$$

$$\therefore \frac{70 \times 1000}{3600}$$

$$v = \sqrt{\mu rg}$$

$$\mu = \frac{v^2}{rg}$$

Amina Abdul-Q
adeer - CE42-A
359607

Q N03

DATA:

$$- m_1 > m_2$$

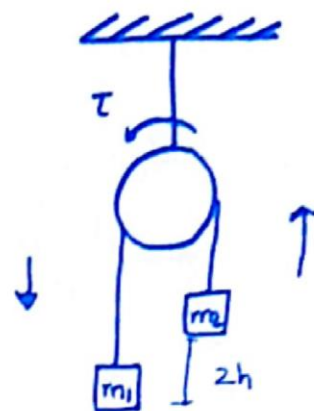
$$- v_1 = 0$$

$$- h = 2h$$

$$- v_f = ?$$

$$- w = ?$$

$$- T = ?$$



→ Conservation of energy states that the total energy is conserved. That is the sum of initial K.E energy and P.E energy is equal to the sum of final K.E and final P.E.

$$(Total\ Energy)_{initial} = (T.E)_{final}$$

→ K.E of an object moving with velocity v is

$$K.E = \frac{1}{2}mv^2$$

Here m , is mass and v is velocity

→ Rotational K.E of an object is

$$K.E = \frac{1}{2}I\omega^2$$

Here I is moment of inertia and ω is angular speed.

→ In case of magnitudes of velocities of both masses are same but are in opposite direction.

K.E(initial) if both the masses are zero as both are released from the rest state.

$$K.E_{initial} = 0$$

→ The final K.E of object of mass m_1 travelling with velocity v is

$$K.E_1 = \frac{1}{2}mv_1^2$$

AMINA QADEER
359607

→ The final kinetic energy of the object of mass m_2 travelling with velocity v is

$$K.E = \frac{1}{2} m_1 v^2$$

→ The rotational K.E of Pulley is

$$K.E_{rot} = \frac{1}{2} I \omega^2$$

→ The linear and rotational speed is related as,

$$\omega = \frac{v}{R}$$

→ Here v is the velocity and R is the radius of the Pulley. Now substituting equation $\omega = \frac{v}{R}$ in equation

$$(K.E)_{rot} = \frac{1}{2} I \omega^2$$

$$(K.E)_{rot} = \frac{1}{2} I \left(\frac{v}{R} \right)^2$$

→ Here I is moment of inertia, v is velocity and R is radius of the pulley.

The final K.E of system is

$$(K.E)_{final} = \frac{1}{2} m_1 v^2 + \frac{1}{2} m_2 v^2 + \frac{1}{2} \frac{I v^2}{R^2}$$

→ The initial P-E is

$$(P.E)_{initial} = m_1 g (2h)$$

$$\therefore h = 2h$$

$$= 2m_1 g h$$

→ The final P-E energy of the system is,

$$(P.E)_{final} = m_1 g h + m_2 g h$$

→ The linear and rotational speed is related as:

$$\omega = \frac{v}{R}$$

→ Here v is the velocity and R is radius of the Pulley. Now

substituting $\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2) + \frac{I}{R^2}}}$ for v in eq

$$\omega = \frac{v}{R}$$

amina qadeer - CE42-A-359607

$$\omega = \frac{I}{R} \left[\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 + m_2 + \frac{I}{R^2})}} \right]$$

$$\omega = \sqrt{\frac{2(m_1 - m_2)gh}{(m_1 R^2 + m_2 R^2 + I)}}$$

Result:

⇒ Therefore, the angular speed of the pulley

is

$$\sqrt{\frac{2(m_1 - m_2)gh}{(m_1 R^2 + m_2 R^2 + I)}}$$

x — x — x — x — x

(amina qadeer - CE-42-A-359607)

Q#6 ① Visualizing motion of spool from figure on rough horizontal surface, and the friction exerted by the surface on the spool can be calculated.

② The problem is categorized under Newton's 2nd law of motion.

③ ANALYZE:

Given

Radius of spool R_1, R_2 .

a) from Newton's 2nd law of motion.

$$\sum F_x = ma_x$$

$$-f + T = ma \quad \text{--- ①}$$

• If we take torques around the centre of mass. we can use:

$$\sum \tau = I\alpha$$

$$+fR_2 - TR_1 = I\alpha$$

• For rolling without slipping we get,

$$\alpha = \frac{a}{R_2}$$

• On substitution we get,

$$fR_2 - TR_1 = \frac{I\alpha}{R_2}$$

$$= \frac{I}{R_2 m} (T - f) \rightarrow \text{from equation ①}$$

$$fR_2^2 m - TR_1 R_2 m = IT - If$$

$$f(1 + mR_2^2) = T(1 + mR_1 R_2)$$

So the frictional force becomes

$$f = \left[\frac{I + mR_1 R_2}{I + mR_2^2} \right] T$$

Amina Qadeer (359607)

⇒ From the answer, the frictional force is left so it is directed towards left.

⇒ FINALIZE:

Notice that we could use the impulse-momentum theorem for translational motion of the spool while ignoring that spool is rotating.

* ————— *

Amina Abdul-Qadeer

CE-42-A

359607

THE END