ASSIGNMENT 1

Numerical Methods.

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Q1) The single positive zero of the function $f(x) = x \tan\left(\frac{x}{2}\right) - 1$ models the wavenumber of the water wave at a certain frequency in the water of depth 0.5. Find two points which bracket the zero of $f(x)$?

Ans1) Now we will be using few random values and put them in $f(x)$ to find the bracket

x	$f(x) = x\tan\left(\frac{x}{2}\right) - 1$
0	$f(0) = 0 \tan(0) - 1 = -1$
0.5	$f(0.5) = 0.5 \tan\left(\frac{0.5}{2}\right) - 1 = -0.877$
1	$f(1) = 1 \tan\left(\frac{1}{2}\right) - 1 = -0.537$
1.5	$f(1.5) = 1 \tan\left(\frac{1.5}{2}\right) - 1 = -0.047$
2	$f(2) = 2 \tan(1) - 1 = 0.583$

(2)

As we can see in the table above $f(x)$ changes sign at ~~f(x)~~ $x = 2$.

$\therefore$ possible interval and best choice is $x = 1.5$ and $x = 2$, as $f(1.5) \times f(2) = -ve$

Hence, $x = 1.5$ and $x = 2$ bracket 0 of $f(x)$.

Q2. Consider a simple electronic circuit with an input voltage, a resistor and a diode. It can be shown that the voltage across the diode can be found as the single positive zero of $x = 3^{-x}$. Use the Newton - Raphson method to approximate the zero by taking the initial value $x_0 = 0.5$. Perform three iterations using 8 decimal arthimetic rounding.

Ans 2) $f(x) = 3^{-x}$ $\qquad x_0 = 0.5$

$f'(x) = -x \ln(3)$ $\qquad f(x_0) = 0.577$

formula: $x_n = x_n - \frac{f_{n-1}}{f(n-1)}$

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n	x_{n-1}	$f(x_{n-1})$	$f'(x_{n-1})$	x_n
1	$x_{1-1} = x_0 = 0.5$	$f(0.5) = 0.57735027$	$f'(0.5) = -0.63428410$	$x_1 = x_0 - \frac{f_0}{f'_0} = 0.5 - \frac{0.57735027}{-0.63428410}$; $x_1 = 1.40417322$
2	$x_1 = 1.4041$	$f_1(x_1) = 0.21381546$	$f''(x_2) = -0.23490030$	$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$; $x_2 = 2.31441241$
3	$x_2 = 2.3144$	$f_2(x_2) = 0.07865832$	$f'(x_2) = -0.08641499$	$x_3 = x_2 - \frac{f(x_2)}{f'(x_2)}$; $x_3 = 3.22465171$

n	$f(x_n)$
1	$f(1.40417322) = 0.21381546$
2	$f(2.31441241) = 0.07865832$
3	$f(3.22465171) = 0.02893677$

Q3) Given $f(x) = 2x - 2\sin x - 0.5$

a) Give mathematical reason why Newton's method cannot be applied for the approximation of the root of $f(x)$ with $x_0 = 0$

Ans 3a) $f(x) = 2x - 2\sin x - 0.5$
$f'(x) = 2 - 2\cos x$
$x_0 = 0$

Now, ~~$f(0)$~~ x_0 in $f(x_0)$ & $f'(x_0)$

$f(0) = 2(0) - 2\sin(0) - 0.5$
$f(0) = -0.5$

$f'(0) = 2 - 2\cos(0)$
$f'(0) = 0$

Hence, Newton's formula:- $x_n = x_{n-1} - \frac{f(n-1)}{f'(n-1)}$

putting $n=1$

$$x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$$

$$x_1 = \frac{0 - (-0.5)}{0}$$

$$x_1 = \infty$$

Now we can see that root comes out to be infinity, undefined value hence, we can conclude from the result the Newton's method can't be applied for approx roots of $f(x)$.

b) Use Regula Falsi Method to approximate the root of $f(x)$ correct to 4-D places.

Ans 3b) Firstly we will find the bracket.

$x = 0$

$f(0) = -0.5$

$x = 1$

$f(1) = 2 - 2\sin(1) - 0.5$

$f(1) = -0.1829$

$x = -1$

$f(-1) = -2 - 2\sin(-1) - 0.5$

$= -0.8171$

$x = 2$

$f(2) = 2(2) - 2\sin(2) - 0.5$

$f(2) = 1.6814$

Formula used : $c = a - \dfrac{b-a}{f(b)-f(a)} \times f(a)$

n	a	f(a)	b	f(b)	c
1	1	f(1) = 0.1829	2	f(2) = 1.6815	1.098101266
2	1.0981	-0.0844	2	1.6815	1.141492837
3	1.1414	-0.0372	2	1.6815	1.159983767
4	1.1599	-0.0255	2	1.6815	1.162697715
5	1.1626	-0.0136	2	1.6815	1.166739425
6	1.1667	-0.0063	2	1.6815	1.169810434
7	1.6981	-0.00271	2	1.6815	1.171135852
8	1.7113	0.000356	2	1.6815	1.17127644

n	f(c)
1	= -0.084487
2	-0.0372669
3	-0.025527
4	-0.013606
5	-0.006365
6	-0.00271997
7	-0.000358414
8	-0.000049304 5

For $x = 1.7113$
f(c) has root upto
4 decimal.

Q4 Consider a simple electronic circuit with an input voltage of 2.0 V, a resistor of resistance 1000Ω and a diode. It can be shown that the voltage across the diode can be found as the single positive zero of:

$$f(x) = 1\times10^{-14}\, e^{\left(\frac{x}{0.026}\right)} - \frac{2-x}{1000}$$

Apply Secant's method to compute the root of $f(x)$, taking initial value of $x_0 = 0.75$. Do three iterations using 8 decimals arithmetic.

Ans 4) $x_0 = 0.75$

$$f(0.75) = 1\times10^{-14}\, e^{\left(\frac{0.75}{0.026}\right)} - \frac{2-(0.75)}{1000}$$

$$f(x_0) = 0.03245741$$

Now assuming $x_1 = 1$

$$f(1) = 1\times10^{-14}\, e^{\left(\frac{1}{0.026}\right)} - \frac{2-x}{1000}$$

$$f(x_1) = 505.3974$$

Secant's formula.

$$x_{n+1} = x_{n+1} - \frac{x_n - x_{n+1}}{f_n - f_{n-1}} \times f_{n-1}$$

n	x_{n-1}	$f(x_{n-1})$	x_n	$f'(x_n)$
1	$x_0 = 0.75$	$f(x_0) =$ 0.03245714	$x_1 = 1$	$f(x_1) =$ 505.3974659
2	$x_1 = 1$	$f(x_1) =$ 505.3974659	$x_2 = 0.749$ 98394	$f(x_2) = 0.03243658$
3	$x_2 = 0.7499$- 8394	$f(x_2) =$ ~~0.0~~ 0.03243658	$x_3 =$ 0.74996789	$f(x_3) =$ 0.3241578

	x_{n+1}	$f(x_{n+1})$
1	$0.74998394 = x_2$	$f(x_2) = 0.03243658$
2	$0.74996781 = x_3$	$f(x_3) = 0.03241578$
3	$0.73911032 = x_4$	$f(x_4) = 0.02091230$

Now it can be seen that after three iterations is $\boxed{0.73911032}$

Q5 Consider simple circuit with input voltage, resistor and a diode. It can be shown that the voltage across the diode can be found as the single positive zero of $f(x) = 2^x - 3$. Use the Newton - Raphson method to approximate the zero of $f(x)$, taking the initial value of $x_0 = 1.0$. Do three iterations using 8 decimal arithmetic

Ans 5) $f(x) = 2^x - 3$ $\quad x_0 = 1$

$f'(x) = 2^x \ln(2)$ $\quad f(x_0) = 2^1 - 3$

$= -1$

Newton - Raphson method formula

$$x_n = x_{n-1} - \frac{f_{n-1}}{f'_{n-1}}$$

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n	x_{n-1}	$f(x_{n-1})$	f'_{n-1}	x_n
1	$x_0 = 1$	-1	1.38629436	1.72134752
2	$x_1 = 1.721-34752$	0.29744254	2.28560932	1.59121043
3	$x_2 = 1.5912-121043$	0.01302038	2.08846658	1.58497601

n	$f(x_n)$
1	0.29744254
2	0.01302038
3	0.00002809

It can be seen on the left side that after three iterations the root is:

$x_3 = 1.58497601$

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