05/05

Chapter - 4
DETERMINANTS

1) $\Rightarrow \Delta = \begin{vmatrix} a & b \\ c & d \end{vmatrix}$

$\Rightarrow \Delta = ad - bc$

Note:- In 3x3 determinant
→ for even = Positive
→ for odd = negative

2) $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$\Rightarrow \Delta = \begin{vmatrix} a & b & c \\ d & e & f \\ x & y & z \end{vmatrix}$

Expanding with respect to "R_1"

$\Rightarrow \Delta = a \begin{vmatrix} e & f \\ y & z \end{vmatrix} - b \begin{vmatrix} d & f \\ x & z \end{vmatrix} + c \begin{vmatrix} d & e \\ x & y \end{vmatrix}$

$\Rightarrow \Delta = a(ez - fy) - b(dz - fx) + c(dy - ex)$

Ex:- = i) $A = \begin{vmatrix} 2 & 3 & 5 \\ -1 & 2 & 1 \\ 3 & 4 & 6 \end{vmatrix}$

Expanding w.r.t. "R_1"

$= 2 \begin{vmatrix} 2 & 1 \\ 4 & 6 \end{vmatrix} - 3 \begin{vmatrix} -1 & 1 \\ 3 & 6 \end{vmatrix} + 5 \begin{vmatrix} -1 & 2 \\ 3 & 4 \end{vmatrix}$

$= 2(2\times6 - 1\times4) - 3(-1\times6 - 1\times3) + 5(-1\times4 - 2\times3)$

$= -13$ Ans.

Exercise - 1

1) $\begin{vmatrix} 2 & 4 \\ -5 & -1 \end{vmatrix}$

Sol.

$$= 2 \times -1 - (-5) \times 4$$
$$= -2 + 20$$
$$= 18$$

4) If $A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$

Sol. Given,

$$A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow \quad 3A = 3 \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

$$\Rightarrow \quad 3A = \begin{bmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{bmatrix}$$

$$\Rightarrow \quad |3A| = \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$$

L.H.S $= |3A|$

$$= \begin{vmatrix} 3 & 0 & 3 \\ 0 & 3 & 6 \\ 0 & 0 & 12 \end{vmatrix}$$

Expanding w.r.t "R_1"

$$= 3(3\times12 - 6\times0) - 0(0\times12 - 6\times0) + 3(0\times0 - 3\times0)$$

$$= 3(36)$$

L.H.S $= 108$

R.H.S $= 27|A|$

$$= 27\begin{vmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 0 & 0 & 4 \end{vmatrix}$$

Expanding w.r.t "R_1"

$$= 27\left\{1(1\times4 - 2\times0) - 0(0\times0 - 2\times0) + 1(0\times0 - 1\times0)\right\}$$

$$= 27\{4\}$$

R.H.S $= 108$

$\Rightarrow$ L.H.S = R.H.S

Hence proved

H.W.

Q) i) $\begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

Sol. $A = \begin{vmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{vmatrix}$

$$= \cos\theta \cdot \cos\theta - (-\sin\theta \cdot \sin\theta)$$
$$= \cos^2\theta + \sin^2\theta$$
$$= 1$$

ii) $\begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$

Sol.

$$\Rightarrow A = \begin{vmatrix} x^2-x+1 & x-1 \\ x+1 & x+1 \end{vmatrix}$$

$$\Rightarrow A = \{(x^2-x+1)(x+1)\} - \{(x-1)(x+1)\}$$
$$\Rightarrow A = \{x^3 - \cancel{x^2} + \cancel{x} + \cancel{x^2} - \cancel{x} + 1\} - \{x^2-1\}$$
$$\Rightarrow A = x^3+1 - x^2+1$$
$$\Rightarrow A = x^3+2-x^2$$

$$\Rightarrow A = x^3 - x^2 + 2$$

3) If $A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$, then show that $|2A| = 4|A|$

Sol. Given

$$\Rightarrow A = \begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow 2A = 2\begin{bmatrix} 1 & 2 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow 2A = \begin{bmatrix} 2 & 4 \\ 8 & 4 \end{bmatrix}$$

$$\Rightarrow |2A| = \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$$

L.H.S $= |2A|$

$$= \begin{vmatrix} 2 & 4 \\ 8 & 4 \end{vmatrix}$$

$= (2\times 4 - 3\times 4)$

$= 8 - 32$

$= -24$

R.H.S $= 4|A|$

$= 4\begin{vmatrix} 1 & 2 \\ 4 & 8 \end{vmatrix}$

$= 4(8\times 1 - 4\times 2)$

$= 4(-6)$

$= -24$

$\Rightarrow$ L.H.S = R.H.S

Hence proved

5) Evaluate :-

i) $\begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$ ii) $\begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$

Sol. i)

Given, $A = \begin{vmatrix} 3 & -1 & -2 \\ 0 & 0 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding w.r.t "R_1"

$\Rightarrow A = 3\begin{vmatrix} 0 & -1 \\ -5 & 0 \end{vmatrix} - (-1)\begin{vmatrix} 0 & -1 \\ 3 & 0 \end{vmatrix} + (-2)\begin{vmatrix} 0 & 0 \\ 3 & -5 \end{vmatrix}$

$\Rightarrow A = 3(0\times 0 - (-1)(-5)) + 1(0\times 0 - (3)(-1)) - 2(0\times -5 - 3\times 0)$

$\Rightarrow A = 3(-5) + 3$

$\Rightarrow A = -12$ Ans

ii) Given

$$\Rightarrow A = \begin{vmatrix} 3 & -4 & 5 \\ 1 & 1 & -2 \\ 2 & 3 & 1 \end{vmatrix}$$

Expanding w.r.t "R_1"

$\Rightarrow A = 3\left(1\times1 - (-2)(3)\right) - (-4)\left(1\times1 - 2(-2)\right) + 5\left(1\times3 - 1\times2\right)$

$\Rightarrow A = 3(7) + 4(5) + 5(1)$

$\Rightarrow A = 21 + 20 + 5$

$\Rightarrow A = 46$ Ans

iii) $\begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$ iv) $\begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Sol. iii) $A = \begin{vmatrix} 0 & 1 & 2 \\ -1 & 0 & -3 \\ -2 & 3 & 0 \end{vmatrix}$

Expanding w.r.t "R_1"

$\Rightarrow A = 0\left(0\times0 - (-3)(3)\right) - 1\left(-1\times0 - (-2)(-3)\right) + 2\left(-1\times3 - 0(-2)\right)$

$\Rightarrow A = 6 - 6$

$\Rightarrow A = 0$ Ans.

iv) Given $A = \begin{vmatrix} 2 & -1 & -2 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$

Expanding w.r.t "R_1"

$\Rightarrow A = 2(2\times 0 - (-5)(-1)) - (-1)(0\times 0 - 5(-1)) + (-2)(0\times -5 - 3\times 2)$

$\Rightarrow A = -10 + 1(5) - 2(-6)$

$\Rightarrow A = -10 + 5 + 12$

$A = 5$

6) If $A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$ find $|A|$

Sol. Given

$$A = \begin{bmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & 1 & -2 \\ 2 & 1 & -3 \\ 5 & 4 & -9 \end{vmatrix}$$

Expanding w.r.t 'R_1'

$\Rightarrow |A| = 1(1\times -9 - 4\times 3) - 1(2\times -9 - 5(-3)) + (-2)(2\times 4 - 1\times 5)$

$\Rightarrow |A| = -21 + 3 - 6$

$\Rightarrow |A| = -24$

$$\Rightarrow |A| = 1\begin{vmatrix} 1 & -3 \\ 4 & -9 \end{vmatrix} - 1\begin{vmatrix} 2 & -3 \\ 5 & -9 \end{vmatrix} + (-2)\begin{vmatrix} 2 & 1 \\ 5 & 4 \end{vmatrix}$$

$\Rightarrow |A| = 1(1\cdot -9 - 4(-3)) - 1(2\cdot -9 - 5(-3)) - 2(2\cdot 4 - 5\cdot 1)$

$\Rightarrow |A| = 3 + 3 - 6$

$\Rightarrow |A| = 0$ Ans.

Date 06/05

7) find x.

i) $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

Sol. $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$

$\Rightarrow 2 \cdot 1 - 4 \cdot 5 = 2x \cdot x - 4 \cdot 6$

$\Rightarrow -18 = 2x^2 - 24$

$\Rightarrow 2x^2 = 6$

$\Rightarrow x^2 = 3$

$\Rightarrow x = \pm\sqrt{3}$

ii) $\begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

Sol. $\Rightarrow \begin{vmatrix} 2 & 3 \\ 4 & 5 \end{vmatrix} = \begin{vmatrix} x & 3 \\ 2x & 5 \end{vmatrix}$

$\Rightarrow 2 \cdot 5 - 3 \cdot 4 = x \cdot 5 - 2x \cdot 3$

$\Rightarrow -2 = 5x - 6x$

$\Rightarrow \boxed{x = 2}$ Ans.

8) If $\begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$, find x.

Sol.

$\Rightarrow \begin{vmatrix} x & 2 \\ 18 & x \end{vmatrix} = \begin{vmatrix} 6 & 2 \\ 18 & 6 \end{vmatrix}$

$\Rightarrow x \cdot x - 18 \cdot 2 = 6 \cdot 6 - 18 \cdot 2$

$\Rightarrow x^2 - \cancel{36} = 36 - \cancel{36}$

$\Rightarrow x = \pm 6$ Ans.

$\Rightarrow$ option B.

self

eg.1) $\begin{vmatrix} 2 & 4 \\ -1 & 2 \end{vmatrix}$

Sol. $= 2 \cdot 2 - (-1) \cdot 4$

$= 4 + 4$

$= 8$

eg.2) $\begin{vmatrix} x & x+1 \\ x-1 & x \end{vmatrix}$

Sol. $= x \cdot x - (x-1)(x+1)$

$= x^2 - x^2 + 1$

$= 1$

eg.3) $\Delta = \begin{vmatrix} 1 & 2 & 4 \\ -1 & 3 & 0 \\ 4 & 1 & 0 \end{vmatrix}$

Sol. Expanding w.r.t "R_1".

$\Rightarrow \Delta = 1 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} - 2 \begin{vmatrix} -1 & 0 \\ 4 & 0 \end{vmatrix} + 4 \begin{vmatrix} -1 & 3 \\ 4 & 1 \end{vmatrix}$

$\Rightarrow \Delta = 4(-1 \cdot 1 - 4 \cdot 3)$

$\Rightarrow \Delta = -52$

and.

Expanding w.r.t "C_1"

$\Rightarrow \Delta = 1 \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} + 1 \begin{vmatrix} 2 & 4 \\ 1 & 0 \end{vmatrix} + 4 \begin{vmatrix} 2 & 4 \\ 3 & 0 \end{vmatrix}$

$\Rightarrow \Delta = -4 - 4(12)$

$= -52$ Ans.

Eg. 4) $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

Sol.

$$\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$$

Expanding w.r.t "R_1"

$$\Rightarrow \Delta = 0\begin{vmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{vmatrix} - \sin\alpha \begin{vmatrix} -\sin\alpha & \sin\alpha \\ \cos\alpha & 0 \end{vmatrix} + (-\cos\alpha)\begin{vmatrix} -\sin\alpha & \\ \cos\alpha & \end{vmatrix}$$

$$\Rightarrow \Delta = -\sin\alpha(-\sin\alpha\cos\alpha) - \cos\alpha(\sin^2\alpha)$$

$$\Rightarrow \Delta = \sin^2\alpha\cos\alpha - \sin^2\alpha\cos\alpha$$

$$\Rightarrow \Delta = 0$$ Ans.

Eg. 5 find x, $\begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$

Sol.

$$\Rightarrow \begin{vmatrix} 3 & x \\ x & 1 \end{vmatrix} = \begin{vmatrix} 3 & 2 \\ 4 & 1 \end{vmatrix}$$

$$\Rightarrow 3 - x^2 = 3 - 8$$

$$\Rightarrow -x^2 = -8$$

$$\Rightarrow x = \pm 2\sqrt{2}$$ Ans.

Eg. 4. $\Delta = \begin{vmatrix} 0 & \sin\alpha & -\cos\alpha \\ -\sin\alpha & 0 & \sin\beta \\ \cos\alpha & -\sin\beta & 0 \end{vmatrix}$

Expanding w.r.t "C_1".

$$\Rightarrow \Delta = 0\begin{vmatrix} 0 & \sin\beta \\ -\sin\beta & 0 \end{vmatrix} + \sin\alpha \begin{vmatrix} \sin\alpha & -\cos\alpha \\ -\sin\beta & 0 \end{vmatrix} + \cos\alpha \begin{vmatrix} \sin\alpha & -\cos\alpha \\ 0 & \sin\beta \end{vmatrix}$$

$\Rightarrow \Delta = \sin\alpha(-\sin\beta\cos\alpha) + \cos\alpha(\sin\alpha\sin\beta)$

$\Rightarrow \Delta = \cancel{-\sin^2\alpha\cos\alpha} - \sin\alpha\sin\beta\cos\alpha + \sin\alpha\sin\beta\cos\alpha$

$\Rightarrow \Delta = 0$ Ans.

Properties of Determinants :-

1) $\Rightarrow P_1$:- The value of the determinant remains unchanged if its rows and columns are interchanged.

2) $\Rightarrow P_2$:- If any two rows (or column) of a determinant are interchanged, then sign of determinant changes.

3) $\Rightarrow P_3$:- If any two rows (or column) of a determinant are identical (all corresponding elements are same), then value of determinant is zero.

4) $\Rightarrow P_4$:- If each element of a row (or a column) of a determinant is multiplied by a constant K, then its value gets multiplied by K.

5) $\Rightarrow P_5$:- If some or all elements of a row or column of a determinant are expressed as sum of two (or more) terms, then the determinant can be expressed as sum of two (or more) determinants.

6) ⇒ P_6 : If, to each elements of any row or column of a determinant, the equimultiples of corresponding elements of other row (or column) are added, then value of determinant remains the same, i.e., the value of determinant remain same if we apply the operation

$R_i \to R_i + KR_j$ or $C_i + KC_j$

7) ⇒ P_7 : If all elements of any row or any column is zero then value of determinant is zero.

Date 07/06/21 C.W.

Exercise 4.2.

Using the property of determinant and without expanding in exercise 1 to 7, prove that

(3) $\begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix} = 0$

Sol.

let $\Delta = \begin{vmatrix} 2 & 7 & 65 \\ 3 & 8 & 75 \\ 5 & 9 & 86 \end{vmatrix}$

By proces $C_3 \to C_3 - 9C_2$

$= \begin{vmatrix} 2 & 7 & 2 \\ 3 & 8 & 3 \\ 5 & 9 & 5 \end{vmatrix}$

$= 0$ {∵ C_1 and C_3 are identical}

Proved

(5) $\begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix} = 0$

Sol: Let

$\Rightarrow \quad \Delta = \begin{vmatrix} a-b & b-c & c-a \\ b-c & c-a & a-b \\ c-a & a-b & b-c \end{vmatrix}$

By process $C_1 \to C_1 + C_2 + C_3$

$\Rightarrow \quad \Delta = \begin{vmatrix} a-b+b-c+c-a & b-c & c-a \\ b-c+c-a+a-b & c-a & a-b \\ c-a+a-b+b-c & a-b & b-c \end{vmatrix}$

$\Rightarrow \quad \Delta = \begin{vmatrix} 0 & b-c & c-a \\ 0 & c-a & a-b \\ 0 & a-b & b-c \end{vmatrix}$

$\Rightarrow \quad \Delta = 0$ proved {because all elements of C_1 is zero}

(6) $\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} = 0$

Sol: Let,

$\Rightarrow \quad \Delta = \begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$

$\Delta = \begin{vmatrix} 0 & -a & b \\ a & 0 & c \\ -b & -c & 0 \end{vmatrix}$ {By R_1}

Date 07/05

$$\Rightarrow \Delta = (-)(-)(-)\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix} \quad \{By\ P_4\}$$

$$\Rightarrow \Delta = -\begin{vmatrix} 0 & a & -b \\ -a & 0 & -c \\ b & c & 0 \end{vmatrix}$$

$$\Rightarrow \Delta = -\Delta$$

$$\Rightarrow \Delta + \Delta = 0$$

$$\Rightarrow 2\Delta = 0$$

$$\Rightarrow \Delta = 0$$

show that

(9) $$\begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix} = (x-y)(y-z)(z-x)(xy+yz+zx)$$

Sol. let

$$\Rightarrow \Delta = \begin{vmatrix} x & x^2 & yz \\ y & y^2 & zx \\ z & z^2 & xy \end{vmatrix}$$

multiplying x in R_1, y in R_2 and z in R_3

$$\Rightarrow \Delta = \frac{1}{xyz}\begin{vmatrix} x^2 & x^3 & xyz \\ y^2 & y^3 & xyz \\ z^2 & z^3 & xyz \end{vmatrix}$$

Taking xyz common from C_3,

$$\Rightarrow \Delta = \frac{xyz}{xyz}\begin{vmatrix} x^2 & x^3 & 1 \\ y^2 & y^3 & 1 \\ z^2 & z^3 & 1 \end{vmatrix}$$

By process $R_1 \to R_1 - R_2$, $R_2 \to R_2 - R_3$

$$\Rightarrow \Delta = \begin{vmatrix} x^2-y^2 & x^3-y^3 & 0 \\ y^2-z^2 & y^3-z^3 & 0 \\ z^2 & z^3 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} (x-y)(x+y) & (x-y)(x^2+xy+y^2) & 0 \\ (y-z)(y+z) & (y-z)(y^2+yz+z^2) & 0 \\ z^2 & z^3 & 1 \end{vmatrix}$$

Taking common $(x-y)$ from R_1, and $(y-z)$ from R_2

$$\Rightarrow \Delta = (x-y)(y-z)\begin{vmatrix} x+y & x^2+xy+y^2 & 0 \\ y+z & y^2+yz+z^2 & 0 \\ z^2 & z^3 & 1 \end{vmatrix}$$

Expanding w.r.t C_3,

$$\Rightarrow \Delta = (x-y)(y-z)\left\{1\begin{vmatrix} x+y & x^2+xy+y^2 \\ y+z & y^2+yz+z^2 \end{vmatrix}\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{(x+y)(y^2+yz+z^2)-(y+z)(x^2+xy+y^2)\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{xy^2+xyz+xz^2+y^3+y^2z+yz^2+(-y-z)(x^2+xy+y^2)\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{\cancel{xy^2}+\cancel{xyz}+xz^2+\cancel{y^3}+y^2z+yz^2-yx^2-\cancel{xy^2}-\cancel{y^3}-zx^2-\cancel{zxy}-zy^2\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{xz^2-zx^2+\cancel{y^2z}+yz^2-yx^2-\cancel{zy^2}\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{xz(z-x)+y(z^2-x^2)\right\}$$

$$\Rightarrow \Delta = (x-y)(y-z)\left\{y(z-x)(z+x)+xz(z-x)\right\}$$

$= (x-y)(y-z)(z-x)(yz+yx+xz)$

$= (x-y)(y-z)(z-x)(xy+yz+zx)$

Proved.

Hold.

(1) $\begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix} = 0$

Sol. let

$\Rightarrow \quad \Delta = \begin{vmatrix} x & a & x+a \\ y & b & y+b \\ z & c & z+c \end{vmatrix}$

$\Rightarrow \quad \Delta = \begin{vmatrix} x & a & x \\ y & b & y \\ z & c & z \end{vmatrix} + \begin{vmatrix} x & a & a \\ y & b & b \\ z & c & c \end{vmatrix}$ $\{\because$ By $P_5\}$

$\Rightarrow \quad \Delta = 0 + 0$ $\{\because$ By P_3 in determinant 1, C_1 & C_3 is identical, and in 2, C_2 and C_3 is identical$\}$

$\Rightarrow \quad \Delta = 0$

Proved.

(4) $\begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix} = 0$

Sol. let

$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & ab \\ 1 & ca & bc \\ 1 & ab & ca \end{vmatrix} + \begin{vmatrix} 1 & bc & ac \\ 1 & ca & ba \\ 1 & ab & cb \end{vmatrix} \quad \{By\ P_5\}$$

By process $C_2 \to C_2 - C_3$ in determinant 1, in 2 ~~$C_2 \to C_2 + C_3$~~

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc-ab & ab \\ 1 & ca-bc & bc \\ 1 & ab-ca & ca \end{vmatrix} + \begin{vmatrix} 1 & bc+ac & ac \\ 1 & ca+ba & ba \\ 1 & ab+cb & cb \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & b(c-a) & ab \\ 1 & c(a-c) & bc \\ 1 & a(b-a) & ca \end{vmatrix}$$

$\Rightarrow$ multiply a in R_1, b in R_2, c in R_3 in both determinant

$$\Rightarrow \Delta = \frac{1}{abc}\begin{vmatrix} a & abc & a^2b \\ b & abc & b^2c \\ c & abc & c^2a \end{vmatrix} + \frac{1}{abc}\begin{vmatrix} a & abc & a^2c \\ b & abc & b^2a \\ c & abc & c^2b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a & 1 & a^2b \\ b & 1 & b^2c \\ c & 1 & c^2a \end{vmatrix} + \begin{vmatrix} a & 1 & a^2c \\ b & 1 & b^2a \\ c & 1 & c^2b \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2bc & abc & a^2b \\ b^2ac & abc & b^2c \\ c^2ab & abc & c^2a \end{vmatrix} + \begin{vmatrix} a^2bc & abc & a^2c \\ ab^2c & abc & b^2a \\ abc^2 & abc & c^2b \end{vmatrix}$$

$$\Rightarrow \Delta = abc\begin{vmatrix} abc & bc & ab \\ abc & ac & bc \\ abc & ab & ca \end{vmatrix} + abc\begin{vmatrix} abc & bc & ac \\ abc & ac & ba \\ abc & ab & cb \end{vmatrix}$$

08/05

$$\Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & bc & ab+ca \\ 1 & ca & ab+bc \\ 1 & ab & ac+ab \end{vmatrix}$$

$$\Delta = \begin{vmatrix} 1 & bc & ab \\ 1 & ca & ab \\ 1 & ab & bc \end{vmatrix} + \begin{vmatrix} 1 & bc & ca \\ 1 & ca & bc \\ 1 & ab & ac \end{vmatrix}$$

$$\Delta = b\begin{vmatrix} 1 & bc & a \\ 1 & ca & a \\ 1 & ab & c \end{vmatrix} + c\begin{vmatrix} 1 & bc & a \\ 1 & ca & b \\ 1 & ab & a \end{vmatrix}$$

$$\Delta = \begin{vmatrix} b & bc & a \\ b & ca & a \\ b & ab & c \end{vmatrix} + \begin{vmatrix} c & bc & a \\ c & ca & b \\ c & ab & a \end{vmatrix}$$

$$\Delta = \begin{vmatrix} ab & abc & a^2 \\ b^2 & abc & ab \\ ac & abc & c^2 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & a(b+c) \\ 1 & ca & b(c+a) \\ 1 & ab & c(a+b) \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & ab+ca \\ 1 & ca & bc+ab \\ 1 & ab & ac+bc \end{vmatrix}$$

By process $C_3 \to C_3 + C_2$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & ab+ac+bc \\ 1 & ca & bc+ab+ca \\ 1 & ab & ac+bc+ab \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & bc & ab+bc+ac \\ 1 & ca & ab+bc+ca \\ 1 & ab & ab+bc+ca \end{vmatrix}$$

$\Rightarrow \Delta = 0$ $\{ C_1$ and C_3 are identical $\}$

5) $$\begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix} = 2 \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$ (P_3)

Sol.

$$\Delta = \begin{vmatrix} b+c & q+r & y+z \\ c+a & r+p & z+x \\ a+b & p+q & x+y \end{vmatrix}$$

$$\Delta = \begin{vmatrix} b & q & y \\ c & r & z \\ a & p & x \end{vmatrix} + \begin{vmatrix} c & r & z \\ a & p & x \\ b & q & y \end{vmatrix}$$ $\{$By $P_4\}$

Date 08/06

for determinant 1 → process $C_3 \to C_3 \leftrightarrow C_1$
for determinant 2 → process $C_2 \to C_2 \leftrightarrow C_1$

$$\Rightarrow \Delta = -\begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix} - \begin{vmatrix} a & p & x \\ c & r & z \\ b & q & y \end{vmatrix} \quad \{BY\ P_2\}$$

In determinant 1 → By process $C_2 \leftrightarrow C_3$
—o—o— 2 → By process $C_2 \leftrightarrow C_3$

$$\Rightarrow \Delta = \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} + \begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix} \quad \{BY\ P_2\}$$

$$\Rightarrow \Delta = 2\begin{vmatrix} a & p & x \\ b & q & y \\ c & r & z \end{vmatrix}$$

Proved.

(7) $$\begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix} = 4a^2b^2c^2$$

Sol: let

$$\Delta = \begin{vmatrix} -a^2 & ab & ac \\ ba & -b^2 & bc \\ ca & cb & -c^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} -a^2 & ba & ca \\ ab & -b^2 & cb \\ ac & bc & -c^2 \end{vmatrix} \quad \{BY\ P_1\}$$

$$\Rightarrow \Delta = (-)(-)(-)\begin{vmatrix} a^2 & -ba & -ca \\ ab & b^2 & -cb \\ ac & bc & c^2 \end{vmatrix} \quad \{ by\ R_3 \}$$

$$\Rightarrow \Delta = -\begin{vmatrix} a^2 & -ba & -ca \\ -ab & b^2 & -cb \\ -ac & -bc & c^2 \end{vmatrix}$$

$$\Delta = \begin{vmatrix} -a^2 & -ba & -ca \\ ab & b^2 & -cb \\ ac & -bc & c^2 \end{vmatrix} \quad \{ \because - \text{ multiply in } C_1 \}$$

$$\Rightarrow \Delta = a^2 \begin{vmatrix} -1 & -b & -c \\ b & b^2 & -cb \\ c & -bc & c^2 \end{vmatrix} \quad \left\{ \begin{array}{l} \text{taking 'a' common from} \\ C_1 \text{ and } R_1 \end{array} \right\}$$

$$\Rightarrow \Delta = a^2 b^2 \begin{vmatrix} -1 & -1 & -c \\ 1 & 1 & -c \\ c & -c & c^2 \end{vmatrix} \quad \left\{ \begin{array}{l} \text{Taking 'b' common from} \\ R_2 \ , \ C_2 \end{array} \right\}$$

$$\Rightarrow \Delta = a^2 b^2 c^2 \begin{vmatrix} -1 & -1 & -1 \\ 1 & 1 & -1 \\ 1 & -1 & 1 \end{vmatrix} \quad \left\{ \begin{array}{l} \text{Taking 'c' common} \\ \text{from } R_3, C_3 \end{array} \right\}$$

$$\Rightarrow \Delta = (-)(-)\, a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix} \quad \left\{ \begin{array}{l} \text{Taking '}-\text{' sign common} \\ \text{from } R_2 \text{ and } R_3 \end{array} \right\}$$

$$\Rightarrow \Delta = a^2 b^2 c^2 \begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}$$

Rough

$-1(-1\cdot-1-1\cdot1) - 1(1\cdot-1-1\cdot1)$
$+1(1\cdot1--1\cdot1)$
$= +2 + 2$
$= 4$

(taking common ~~from~~ $(x+y+z)$ from R_2 & R_3

$$= \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} 2(x+y+z)^3$$

$$= 2(x+y+z)^3 \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding w.r.t C_1

$$= 2(x+y+z)^3 \left\{ 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\}$$

$$= 2(x+y+z)^3 \times 1$$

$$= 2(x+y+z)^3$$

Proved.

H.W. Date 04/05/21

(8) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol. det,

$$\Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

By process ~~$R_2 \to R_2 - R_1$~~ $R_1 \to R_1 - R_2$

$$\cancel{\Rightarrow \Delta = \begin{vmatrix} 0 & (a-b) & (a^2-b^2) \\ 1 & & \\ 1 & c & c^2 \end{vmatrix}} \Rightarrow \Delta = \begin{vmatrix} 0 & (a-b) & (a^2-b^2) \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

$$\Rightarrow \Delta = a^2b^2c^2 \overbrace{\begin{vmatrix} -1 & 1 & 1 \\ 1 & -1 & 1 \\ 1 & 1 & -1 \end{vmatrix}}^{(4)} \Rightarrow \Delta = a^2b^2c^2(4)$$

$$\Rightarrow \Delta = 4a^2b^2c^2$$

Proved

Contd.

Show that.

(11) ii) $\begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} = 2(x+y+z)^3$

Sol. Let,

$$\Rightarrow \Delta = \begin{vmatrix} x+y+2z & x & y \\ z & y+z+2x & y \\ z & x & z+x+2y \end{vmatrix} \cancel{= 2(x+y+z)^3}$$

By process $C_1 \to C_1 + C_2 + C_3$

$$= \begin{vmatrix} 2(x+y+z) & x & y \\ 2(x+y+z) & y+z+2x & y \\ 2(x+y+z) & x & z+x+2y \end{vmatrix}$$

Taking common $2(x+y+z)$ from C_1

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 1 & y+z+2x & y \\ 1 & x & z+x+2y \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$ & $R_3 \to R_3 - R_1$

$$= 2(x+y+z) \begin{vmatrix} 1 & x & y \\ 0 & y+z+x & 0 \\ 0 & 0 & z+x+y \end{vmatrix}$$

Taking common ~~from~~ $(x+y+z)$ from R_2 & R_3

$$= \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix} 2(x+y+z)^3$$

$$= 2(x+y+z)^3 \begin{vmatrix} 1 & x & y \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding w.r.t C_1

$$= 2(x+y+z)^3 \left\{ 1 \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \right\}$$

$$= 2(x+y+z)^3 \times 1$$

$$= 2(x+y+z)^3$$

proved.

H.W. Date 04/05/21

(5) $\begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix} = (a-b)(b-c)(c-a)$

Sol: Let,

$$\Rightarrow \Delta = \begin{vmatrix} 1 & a & a^2 \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

By process ~~$R_2 \to R_2 - R_1$~~ $R_1 \to R_1 - R_2$

$$\cancel{\Rightarrow \Delta = \begin{vmatrix} 0 & (a-b) & (a^2-b^2) \\ 1 & & \\ 1 & c & c^2 \end{vmatrix}} \Rightarrow \Delta = \begin{vmatrix} 0 & (a-b) & (a^2-b^2) \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

09/05

Taking common $(a-b)$ from R_1

$$\Rightarrow \Delta = (a-b)\begin{vmatrix} 0 & 1 & (a+b) \\ 1 & b & b^2 \\ 1 & c & c^2 \end{vmatrix}$$

By process $R_2 \to R_2 - R_3$

$$\Rightarrow \Delta = (a-b)\begin{vmatrix} 0 & 1 & a+b \\ 0 & b-c & b^2-c^2 \\ 1 & c & c^2 \end{vmatrix}$$

Taking common $(b-c)$ from R_2

$$\Rightarrow \Delta = (a-b)(b-c)\begin{vmatrix} 0 & 1 & a+b \\ 0 & 1 & b+c \\ 1 & c & c^2 \end{vmatrix}$$

Expanding w.r.t 'C_1'

$$\Rightarrow \Delta = (a-b)(b-c)\left| 1\left(1 \cdot a+b - (1 \cdot b+c)\right)\right|$$

$$\Rightarrow \Delta = (a-b)(b-c)(a+b-b-c)$$

$$\Rightarrow \Delta = (a-b)(b-c)(a-c)$$

Proved.

(10) i) $\begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix} = (5x+4)(4-x)^2$

Sol. Let,

$$\Delta = \begin{vmatrix} x+4 & 2x & 2x \\ 2x & x+4 & 2x \\ 2x & 2x & x+4 \end{vmatrix}$$

By process $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 5x+4 & 2x & 2x \\ 5x+4 & x+4 & 2x \\ 5x+4 & 2x & x+4 \end{vmatrix}$$

Taking common $(5x+4)$ from C_1

$$\Rightarrow \Delta = (5x+4) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 1 & 2x & x+4 \end{vmatrix}$$

By process $R_3 \to R_3 - R_2$

$$\Rightarrow \Delta = (5x+4) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 0 & x-4 & 4-x \end{vmatrix}$$

$$\Rightarrow \Delta = (5x+4) \begin{vmatrix} 1 & 2x & 2x \\ 1 & x+4 & 2x \\ 0 & -(4-x) & 4-x \end{vmatrix}$$

Taking common $(4-x)$ from R_3

$$\Rightarrow \Delta = (5x+4)(4-x) \begin{vmatrix} 1 & 2x & 2x \\ 1 & 4+x & 2x \\ 0 & -1 & 1 \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$

$$\Rightarrow \Delta = (5x+4)(4-x) \begin{vmatrix} 1 & 2x & 2x \\ 0 & 4-x & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

Taking common $(4-x)$ from R_2

$$\Rightarrow \Delta = (5x+4)(x-4)^2 \begin{vmatrix} 1 & 2x & 2x \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{vmatrix}$$

Expanding w.r.t 'C_1'

$\Rightarrow \Delta = (5x+4)(4-x)^2 \; (1)$

$\Rightarrow \Delta = (5x+4)(4-x)^2$

Proved.

ii) $\begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix} = k^2(3y+k)$

Sol.

$\Rightarrow$ let, $\Delta = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

By process $R_1 \to R_1 - R_2$

$\Rightarrow \Delta = \begin{vmatrix} k & k & 0 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

Taking common 'k' from 'R_1'

$\Rightarrow \Delta = k \begin{vmatrix} 1 & 1 & 0 \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$

By process $R_3 \to R_3 - R_2$

$\Rightarrow$

$\Rightarrow \Delta = k \begin{vmatrix} 1 & 1 & 0 \\ y & y+k & y \\ 0 & -k & k \end{vmatrix}$

Taking common k from R_3

$$\Rightarrow \Delta = k^2 \begin{vmatrix} 1 & 1 & 0 \\ y & y+k & y \\ 0 & -1 & 1 \end{vmatrix}$$

By process $C_3 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = k^2 \begin{vmatrix} 1 & 1 & 2 \\ y & y+k & 3y+k \\ 0 & -1 & 0 \end{vmatrix}$$

~~By process $C_1 \to 2C_1$~~ multiply 2 in C_1

$$\Rightarrow \Delta = \frac{k^2}{2} \begin{vmatrix} 2 & 1 & 2 \\ 2y & y+k & 3y+k \\ 0 & -1 & 0 \end{vmatrix}$$

Rough

$$\begin{vmatrix} 1 & 0 \\ 1 & 2 \\ 1 & -1 \end{vmatrix} \quad \begin{matrix} -1 \\ -1 \\ 2 \end{matrix} \quad \begin{matrix} 2 \\ -2 \\ 0 \end{matrix}$$

By process $C_2 \to C_2 + C_1$

$$\Rightarrow \Delta = \frac{k^2}{2} \begin{vmatrix} 2 & 3 & 2 \\ 2y & 3y+k & 3y+k \\ 0 & -1 & 0 \end{vmatrix}$$

multiplying $\frac{y}{2}$ in R_1

$$\Rightarrow \Delta = \frac{k^2}{2} \cdot \frac{y}{2} \begin{vmatrix} y & \frac{3}{2}y & y \end{vmatrix}$$

Rough

$4y - 6y + 2k$

$2k - 2y$

$2(k-y)$

$6y + 2k - 4y$

Expanding w.r.t (C_3)

$$\Rightarrow \Delta = \frac{k^2}{2} \left\{ +1 \left(2y \cdot 2 - (3y+k) 2 \right) \right\}$$

$$\begin{matrix} 1 & 2 \\ -1 & 0 \end{matrix} \Rightarrow 0 + 2$$

$k^2(y+k)$

$2 \cdot 3y + k - 2y \cdot 2$

$6y + 2k - 4y$

$2y + 2k$

$2(y+k)$

$\frac{k^2}{2} 2(3y+k)$

$k^2(3y+k)$

Expand w.r.t 'C_1'

$$\Rightarrow \Delta = k^2 \left\{ 1 \begin{vmatrix} y+k & 3y+k \\ -1 & 0 \end{vmatrix} - y \begin{vmatrix} 1 & 2 \\ -1 & 0 \end{vmatrix} \right\}$$

$$\Rightarrow \Delta = k^2 \left\{ 1(3y+k) - 2y \right\}$$

$$\Rightarrow \Delta = k^2 (y+k)$$

Sol. Let

$$\Rightarrow \Delta = \begin{vmatrix} y+k & y & y \\ y & y+k & y \\ y & y & y+k \end{vmatrix}$$

By process $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 3y+k & y & y \\ 3y+k & y+k & y \\ 3y+k & y & y+k \end{vmatrix}$$

Taking common '$3y+k$' from C_1

$$\Rightarrow \Delta = (3y+k) \begin{vmatrix} 1 & y & y \\ 1 & y+k & y \\ 1 & y & y+k \end{vmatrix}$$

By process $C_2 \to C_2 - C_1$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & y \\ 1 & k & y \\ 1 & -k & y+k \end{vmatrix} (3y+k)$$

~~By process~~

Taking common 'k' from C_2

$$\Rightarrow \Delta = \begin{vmatrix} 1 & x & y \\ 1 & 1 & y \\ 1 & -1 & y+k \end{vmatrix} (3y+k)\,k$$

Expanding w.r.t 1 'R'

$$\Rightarrow \Delta = k(3y+k)\left\{1(1\cdot y+k+1\cdot y)+y(1\cdot -1-1\cdot 1)\right\}$$

$$\Rightarrow \Delta = k(3y+k)\left\{\cancel{y}+k+\cancel{y}-\cancel{2y}\right\}$$

$$\Rightarrow \Delta = k(3y+k)\,k$$

$$\Rightarrow \Delta = k^2(3y+k)$$

<u>proved.</u>

(Y) i) $\begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix} = (a+b+c)^3$

<u>let</u>

$$\Delta = \begin{vmatrix} a-b-c & 2a & 2a \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

By process $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \Delta = \begin{vmatrix} a+b+c & a+b+c & a+b+c \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Taking common '$a+b+c$' from R_1

$$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 1 & 1 & 1 \\ 2b & b-c-a & 2b \\ 2c & 2c & c-a-b \end{vmatrix}$$

Date 09/05/21 7:12:00

By process $C_1 \to C_1 - C_2$

$$\Rightarrow \Delta = (a+b+c)\begin{vmatrix} 0 & 1 & 1 \\ b+c+a & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

Taking common $[b+c+a]$ from C_1,

$$\Rightarrow \Delta = (a+b+c)^2 \begin{vmatrix} 0 & 1 & 1 \\ 1 & b-c-a & 2b \\ 0 & 2c & c-a-b \end{vmatrix}$$

By process $C_3 \to C_3 - C_2$

$$\Rightarrow \Delta = (a+b+c)^2 \begin{vmatrix} 0 & 1 & 0 \\ 1 & b-c-a & b+c+a \\ 0 & 2c & -(c+a+b) \end{vmatrix}$$

Taking common $(a+b+c)$ from C_3

$$\Rightarrow \Delta = (a+b+c)^3 \begin{vmatrix} 0 & 1 & 0 \\ 1 & b-c-a & 1 \\ 0 & 2c & -1 \end{vmatrix}$$

Expanding w.r.t 'R_1'

$$\Rightarrow \Delta = (a+b+c)^3 \left\{ \cancel{-(b-c-a+1)} \quad -1(1-1-0-1) \right\}$$

$$\Rightarrow \Delta = (a+b+c)^3 (-1(-1))$$

$$\Rightarrow \Delta = (a+b+c)^3$$

Proved.

Date 09/05

(12) $\begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix} = (1-x^3)^2$

Sol: Let.

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

~~By~~ multiplying x in R_1

$$\Rightarrow \Delta = \frac{1}{x} \begin{vmatrix} x & x^2 & x^3 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

By process $C_1 \rightarrow C_1 - C_3$

$$\Rightarrow \Delta = \frac{1}{x} \begin{vmatrix} x-x^3 & x^2 & x^3 \\ x^2-x & 1 & x \\ x-1 & x^2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \frac{1}{x} \begin{vmatrix} x(1-x^2 \end{vmatrix}$$

~~By process~~
Multiplying x^2 in C_1,

$$\Rightarrow \Delta = \frac{1}{x^2} \begin{vmatrix} x^2 & x & x^2 \\ x^4 & 1 & x \\ x^3 & x^2 & 1 \end{vmatrix}$$

By process $C_3 \to C_3 - C_1$

$$\Rightarrow \Delta = \frac{1}{x^2}\begin{vmatrix} x^2 & x & 0 \\ x^4 & 1 & x(1-x^3) \\ x^3 & x^2 & 1-x^3 \end{vmatrix}$$

Taking common $(1-x^3)$ from C_3

$$\Rightarrow \Delta = \frac{1-x^3}{x^2}\begin{vmatrix} x^2 & x & 0 \\ x^4 & 1 & x \\ x^3 & x^2 & 1 \end{vmatrix}$$

multiply x in R_3

$$\Rightarrow \Delta = \frac{1-x^3}{x^3}\begin{vmatrix} x^2 & x & 0 \\ x^4 & 1 & x \\ x^4 & x^3 & x \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$

$$\Rightarrow \Delta = \frac{(1-x^3)}{x^3}\begin{vmatrix} x^2 & x & 0 \\ 0 & 1-x^3 & 0 \\ x^4 & x^3 & x \end{vmatrix}$$

Taking common $1-x^3$ from R_2

$$\Rightarrow \Delta = \frac{(1-x^3)^2}{x^3}\begin{vmatrix} x^2 & x & 0 \\ 0 & 1 & 0 \\ x^4 & x^3 & x \end{vmatrix}$$

Taking common x from R_3

$$\Rightarrow \Delta = \frac{(1-x^3)^2}{x^2}\begin{vmatrix} x^2 & x & 0 \\ 0 & 1 & 0 \\ x^3 & x^2 & 1 \end{vmatrix}$$

Let

$$\Delta = \begin{vmatrix} 1 & x & x^2 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

By process $R_1 \to R_1 + R_2 + R_3$.

$$\Delta = \begin{vmatrix} 1+x+x^2 & 1+x+x^2 & 1+x+x^2 \\ x^2 & 1 & x \\ x & x & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (1+x+x^2) \begin{vmatrix} 1 & 1 & 1 \\ x^2 & 1 & x \\ x & x^2 & 1 \end{vmatrix}$$

By process $C_2 - C_1$

$$\Rightarrow \Delta = (1+x+x^2) \begin{vmatrix} 1 & 0 & 1 \\ x^2 & 1-x^2 & x \\ x & x^2-x & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (1+x+x^2) \begin{vmatrix} 1 & 0 & 1 \\ x^2 & 1-x^2 & x \\ x & -x+x^2 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (1+x+x^2) \begin{vmatrix} 1 & 0 & 1 \\ x^2 & (1-x)(1+x) & x \\ x & -(x-x^2) & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = (1+x+x^2) \begin{vmatrix} 1 & 0 & 1 \\ x^2 & (1-x)(1+x) & x \\ x & -x(1-x) & 1 \end{vmatrix}$$

Taking common $(1-x)$ from C_2

$$\Rightarrow \Delta = (1+x+x^2)(1-x) \begin{vmatrix} 1 & 0 & 1 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

By process $C_3 \to C_3 - C_1$

$$\Rightarrow \Delta = 1+x+x^2-(x+x^2+x^3)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x-x^2 \\ x & -x & 1-x \end{vmatrix}$$

$$\Rightarrow \Delta = 1+\cancel{x}+\cancel{x^2}-\cancel{x}-\cancel{x^2}-x^3\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x(1-x) \\ x & -x & 1-x \end{vmatrix}$$

Taking common $(1-x)$ from C_3,

$$\Rightarrow \Delta = (1-x^3)(1-x)\begin{vmatrix} 1 & 0 & 0 \\ x^2 & 1+x & x \\ x & -x & 1 \end{vmatrix}$$

Expanding w.r.t 'R_1'

$$\Rightarrow \Delta = (1-x^3)(1-x)\left\{1(1+x)-(-x)(x)\right\}$$

$$\Rightarrow \Delta = (1-x^3)(1-x)(1+x+x^2)$$

$$\Rightarrow \Delta = (1-x^3)(1+\cancel{x}+\cancel{x^2}-\cancel{x}-\cancel{x^2}-x^3)$$

$$\Rightarrow \Delta = (1-x^3)(1-x^3)$$

$$\Rightarrow \Delta = (1-x^3)^2$$

Proved

(13) $$\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$$

Sol.

Let,

$$\Delta = \begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

By process $R_1 \to R_1 + bR_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2+b^2 & 0 & -b(1+a^2+b^2) \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Rough

$-2b + b(1-a^2-b^2)$

$-2b + b - ba^2 - b^3$

$-b - ba^2 - b^3$

$b(-1-a^2-b^2)$

$-b$

$2a - a - a^3 - b^2a$

$-a - a^3 - ab^2$

$-a(1+a^2+b^2)$

Taking common $(1+a^2+b^2)$ from R_1

$$\Delta = (1+a^2+b^2) \begin{vmatrix} 1 & 0 & -b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

By process $R_2 \to R_2 - aR_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & -b \\ 0 & 1+a^2+b^2 & a(1+a^2+b^2) \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} (1+a^2+b^2)$$

Taking common $(1+a^2+b^2)$ from R_2

$$\Rightarrow \Delta = \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} (1+a^2+b^2)^2$$

$$\Rightarrow \Delta = (1+a^2+b^2)^2 \begin{vmatrix} 1 & 0 & -b \\ 0 & 1 & a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix}$$

Expanding w.r.t C_1.

$$\Rightarrow \Delta = (1+a^2+b^2)^2 \left\{ 1\left(\overline{1 \cdot 1-a^2-b^2} - \overline{-2a \cdot a}\right) + 2b\left(0 \cdot a - 1 \cdot -b\right) \right\}$$

$$\Rightarrow \Delta = (1+a^2+b^2)^2 \left\{ (1+a^2-b^2) + 2b^2 \right\}$$

$$\Rightarrow \Delta = (1+a^2+b^2)^3$$ Proved.

09/05

Choose the correct Answer :-

15) Let A be a square matrix of order 3x3, then $|kA|$ is equal to

Ans → c) $k^3|A|$

Trick

$$\begin{vmatrix} ka & kb & kc \\ kd & ke & kf \\ kg & kh & ki \end{vmatrix} \rightarrow k^3 = k^3|A|$$

16) Which of the following is correct:-

Ans. c) Determinant is a no. associated to a square matrix.

(14) $$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1 + a^2 + b^2 + c^2$$

Sol.

Let,

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

By process $R_1 \rightarrow R_1 + R_2 + R_3$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+1+ab+ca & ab+b^2+1+cb & ac+bc+c^2+1 \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2+ab+ac & 1+ab+b^2+cb & 1+ac+bc+c^2 \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

$$\therefore \Delta = \begin{vmatrix} 1+a(a+b+c) & 1+b(a+b+c) & 1+c(a+b+c) \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking common $(a+b+c)$ from R_1

$$\Delta = a+b+c \begin{vmatrix} \frac{1}{a+b+c}+a & \frac{1}{a+b+c}+b & \frac{1}{a+b+c}+c \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking common a, b, c from R_1, R_2 & R_3 respectively

$$\Rightarrow \Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$

$$\Rightarrow \Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

By process $R_3 \to R_3 - R_1$

$$\Rightarrow \Delta = \begin{vmatrix} a+\frac{1}{a} & b & c \\ -\frac{1}{a} & \frac{1}{b} & 0 \\ -\frac{1}{a} & 0 & \frac{1}{c} \end{vmatrix}$$

multiplying a, b, c in C_1, C_2 and C_3 respectively

$$\Rightarrow \Delta = \frac{abc}{abc}\begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} a^2+1 & b^2 & c^2 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{vmatrix}$$

Expanding with respect to R_1

$$\Rightarrow \Delta = a^2+1\left(1\cdot1-0\cdot0\right) - b^2\left(-1\cdot1-\cdot-1\cdot0\right) + c^2\left(-1\cdot0-\cdot-1\cdot1\right)$$

$$\Rightarrow \Delta = a^2+1+b^2+c^2$$

$$\Rightarrow \Delta = 1+a^2+b^2+c^2$$

proved.

C.W.

(11)

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = 1+a^2+b^2+c^2$$

Sol. let,

$$\Delta = \begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix}$$

Taking common 'a' from R_1, b from R_2 and c from R_3

10/03

$$\Rightarrow \Delta = abc \begin{vmatrix} a+\frac{1}{a} & b & c \\ a & b+\frac{1}{b} & c \\ a & b & c+\frac{1}{c} \end{vmatrix}$$

multiply a, in C_1, b in C_2 and c in C_3,

$$\Rightarrow \Delta = \begin{vmatrix} a^2+1 & b^2 & c^2 \\ a^2 & b^2+1 & c^2 \\ a^2 & b^2 & c^2+1 \end{vmatrix}$$

By process $C_1 \to C_1 + C_2 + C_3$

$$\Rightarrow \Delta = \begin{vmatrix} 1+a^2+b^2+c^2 & b^2 & c^2 \\ 1+a^2+b^2+c^2 & b^2+1 & c^2 \\ 1+a^2+b^2+c^2 & b^2 & c^2+1 \end{vmatrix}$$

Taking common $(1+a^2+b^2+c^2)$ from C_1

$$\Rightarrow \Delta = 1+a^2+b^2+c^2 \begin{vmatrix} 1 & b^2 & c^2 \\ 1 & b^2+1 & c^2 \\ 1 & b^2 & c^2+1 \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$ and $R_3 \to R_3 - R_1$

$$\Rightarrow \Delta = 1+a^2+b^2+c^2 \begin{vmatrix} 1 & b^2 & c^2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$$

Expanding w.r.t C_1

$$\Rightarrow \Delta = 1+a^2+b^2+c^2 \left\{ 1(1.1 - 0.0) \right\}$$

$$\Rightarrow \Delta = 1+a^2+b^2+c^2 \ (1)$$

$$\Rightarrow \Delta = 1+a^2+b^2+c^2$$

Proved.

Date 10/05

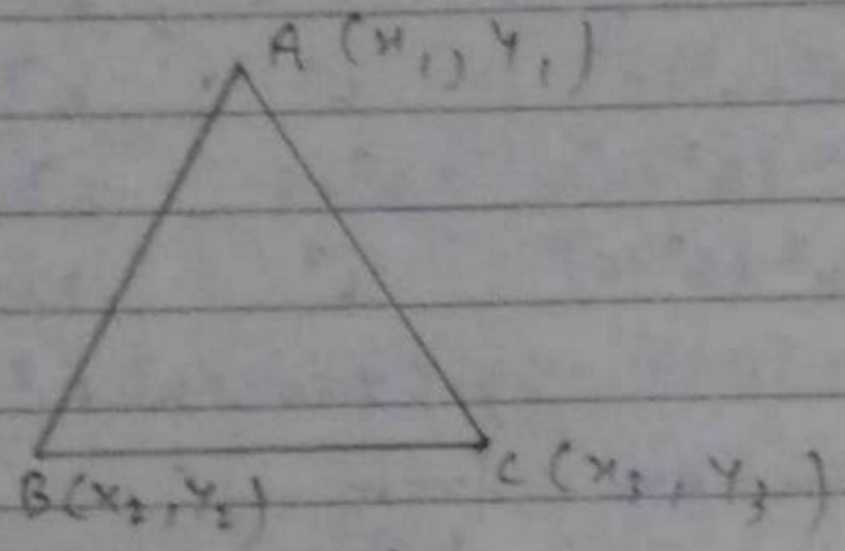
Note :-

If a determinant has 'n' order then

$$|KA| = K^{n} |A|$$

Area of Triangle :-

suppose the vertices of triangle be (x_1, y_1), (x_2, y_2) and (x_3, y_3)



$$\Rightarrow \text{Area of } \Delta = \frac{1}{2} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Note :- If The area of triangle be zero then the given points are collinear.

Date 10/05

Exercise 4.3

Find Area of Δ with vertices,

i) $(1,0)$, $(6,0)$, $(4,3)$

$(x_1, y_1) \equiv (1,0)$

$(x_2, y_2) \equiv (6,0)$

$(x_3, y_3) \equiv (4,3)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} 1 & 0 & 1 \\ 6 & 0 & 1 \\ 4 & 3 & 1 \end{vmatrix}$$

Expanding w.r.t R_1

$$= \frac{1}{2}\left\{1(0\cdot1 - 3\cdot1) + 1(6\cdot3 - 4\cdot0)\right\}$$

$$= -\frac{3}{2} + \frac{18}{2}$$

$$= \frac{15}{2} \text{ square unit}$$

10/05

Q) Show that points, $A(a, b+c)$, $B(b, c+a)$, $C(c, a+b)$ are collinear.

Sol.

$A(a, b+c) \equiv (x_1, y_1)$
$B(b, c+a) \equiv (x_2, y_2)$
$C(c, a+b) \equiv (x_3, y_3)$

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}.$$

Expanding w.r.t ~~R~~ C_1

$$= \frac{1}{2}\left\{a(c+a\cdot 1 - a+b\cdot 1) - b(b+c\cdot 1 - a+b\cdot 1) + c(b+c\cdot 1 - c+a\cdot 1)\right\}$$

$$= \frac{1}{2}\left\{a(c-b) - b(c-a) + c(b-a)\right\}$$

$$= \frac{1}{2}\left\{ac - ba - bc + ab + cb - ac\right\}$$

$$= \frac{1}{2}\left\{ac - ac\right\}$$

$$= \frac{1}{2}\left\{0\right\}$$

$$= 0$$

Therefore points are collinear.

Date 10/04

ii) $(2,7)$, $(1,1)$, $(10,8)$

let, $A(2,7) \equiv (x_1, y_1)$
$B(1,1) \equiv (x_2, y_2)$
$C(10,8) \equiv (x_3, y_3)$

Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$= \frac{1}{2}\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$

Expanding w.r.t R_1

$= \frac{1}{2}\left\{2(1\times1 - 8\times1) - 7(1\times1 - 10\times1) + 1(1\times8 - 10\times1)\right\}$

$= \frac{1}{2}\left\{-14 + 63 - 2\right\}$

$= \frac{47}{2}$ square unit.

iii) $(-2,-3)$, $(3,2)$, $(-1,-8)$

$(x_1, y_1) \equiv (-2,-3)$
$(x_2, y_2) \equiv (3,2)$
$(x_3, y_3) \equiv (-1,-8)$

Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$= \frac{1}{2}\begin{vmatrix} -2 & -3 & 1 \\ 3 & 2 & 1 \\ -1 & -5 & 1 \end{vmatrix}$$

Expanding w.r.t R_1

$$= \frac{1}{2}\left\{-2(2\times1+5\times1)+3(3\times1+1\times1)+1(3\times-5+2\times1)\right\}$$

$$= \frac{1}{2}\left\{-20+12-23\right\}$$

$$= \frac{-30}{2}$$

$= -15$ square unit.

$= 15$ square unit.

(8) Find values of K if area of triangle is 4 sq. units and vertices are

i) $(K,0), (4,0), (0,2)$

Sol

$$\text{Area of } \Delta = \frac{1}{2}\begin{vmatrix} K & 0 & 1 \\ 4 & 0 & 1 \\ 0 & 2 & 1 \end{vmatrix}$$

Expanding w.r.t R_1

$$\Rightarrow \pm 4 = \frac{1}{2}\left\{K(0\times1-2\times1)+1(4\times2-0\times0)\right\}$$

$$\Rightarrow \pm 8 = -2K+8$$

$\Rightarrow K = 0$ Ans. | $K = 8$ Ans.

[illegible]

[illegible]

$$\frac{1}{2}\begin{vmatrix} [illegible] & [illegible] & 1 \\ [illegible] & [illegible] & 1 \\ [illegible] & [illegible] & 1 \end{vmatrix}$$

$$\frac{1}{2}\begin{vmatrix} [illegible] & [illegible] & 1 \\ [illegible] & [illegible] & 1 \\ [illegible] & [illegible] & 1 \end{vmatrix}$$

[illegible]

[illegible]

[illegible]

$k = 8$ Ans | $k = 0$ Ans

5(a) Find eqn of line joining (1,8) and (3,6) using determinant

Let 3rd vertex $(x_3, y_3) = (x, y)$

$(x_2, y_2) = (3, 6)$

$(x_1, y_1) = (1, 8)$

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

$$\frac{1}{2}\begin{vmatrix} x & y & 1 \\ 1 & 8 & 1 \\ 3 & 6 & 1 \end{vmatrix} = 0$$

Expanding w.r.t R_1

$$\Rightarrow \frac{1}{2}\left\{1(2x-y\times x)-2(3\times x-x\times 1)+1(3y-x\times 1)\right\}=0$$

$$\Rightarrow \frac{1}{2}\left\{\cancel{2x}-y-\cancel{6x}+2x+3y-\cancel{6x}\right\}=0$$

$$\Rightarrow \frac{1}{2}\left(-4x+2y\right)=0$$

$$\Rightarrow -2x+y=0$$

$$\Rightarrow y=2x \quad \underline{\text{Ans.}}$$

(ii) find eq^n of line joining (3,1) and (9,3) using determinant

Sol. Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$$\Rightarrow 0 = \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ x & y & 1 \end{vmatrix}$$

Expanding w.r.t R_1

$$\Rightarrow 0 = \frac{1}{2}\left\{3(3\times 1-y\times 1)-1(9\times 1-x\times 1)+1(9\times y-x\times 3)\right\}$$

$$\Rightarrow 0 = \frac{1}{2}\left\{\cancel{9}-3y-\cancel{9}+x+9y-3x\right\}$$

$$\Rightarrow 0 = \frac{1}{2}\left\{6y-2x\right\}$$

$$\Rightarrow 3y=2x \quad \underline{\text{Ans.}}$$

$$\Rightarrow y=\frac{2}{3}x \Rightarrow x=\frac{3}{2}y$$

Date 10/5

(5) Area of Δ is 35 sq. units with vertices $(2,-6)$, $(5,4)$ and $(k,4)$. Then k is

Sol. Area of $\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$

$\Rightarrow 35 = \frac{1}{2}\begin{vmatrix} 2 & -6 & 1 \\ 5 & 4 & 1 \\ k & 4 & 1 \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow 70 = \{2(4\times1 - 4\times1) + 6(5\times1 - k\times1) + 1(5\times4 - k\times4)\}$

$\Rightarrow 70 = \{0 + 30 - 6k + 20 - 4k\}$

$\Rightarrow 70 = 50 - 10k$

$\Rightarrow -10k = 20$

$\Rightarrow k = -2$, <u>option B.</u>

Evl.

Date 11/05/21

<u>Minors</u> :- represented by 'M'

* $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & I \end{vmatrix}$

$\Rightarrow M_{11} = \begin{vmatrix} e & f \\ h & I \end{vmatrix}$

$\Rightarrow M_{22} = \begin{vmatrix} a & c \\ g & I \end{vmatrix}$

$\Rightarrow M_{32} = \begin{vmatrix} a & c \\ d & f \end{vmatrix}$

$\Rightarrow M_{33} = \begin{vmatrix} a & b \\ d & e \end{vmatrix}$

Cofactor :- represented by 'C'.

* $A = \begin{vmatrix} 1 & 2 & 5 \\ 5 & 3 & 2 \\ 1 & 6 & 2 \end{vmatrix}$

~~Cofact~~ Cofactor :- i) $C_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 2 \\ 6 & 2 \end{vmatrix} = -6$

ii) $C_{12} = (-1)^{1+2} \begin{vmatrix} 5 & 2 \\ 1 & 2 \end{vmatrix} = -(10-2) = -8$

iii) $C_{13} = (-1)^{1+3} \begin{vmatrix} 5 & 3 \\ 1 & 6 \end{vmatrix} = 27$

iv) $C_{21} = (-1)^{2+1} \begin{vmatrix} 2 & 5 \\ 6 & 2 \end{vmatrix} = -(4-30) = 26$

v) $C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 5 \\ 1 & 2 \end{vmatrix} = (2-5) = -3$

vi) $C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 2 \\ 1 & 6 \end{vmatrix} = -(6-2) = -4$

vii) $C_{31} = (-1)^{3+1} \begin{vmatrix} 2 & 5 \\ 3 & 2 \end{vmatrix} = 4-15 = -11$

viii) $C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 5 \\ 5 & 2 \end{vmatrix} = -(2-25) = 23$

ix) $C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 2 \\ 5 & 3 \end{vmatrix} = (3-10) = -7$

then,

$$\text{cof } A = \begin{vmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{vmatrix} = \begin{vmatrix} -6 & -8 & 27 \\ 26 & -3 & -4 \\ -11 & 23 & -7 \end{vmatrix}$$

11/05

Value of Determinants :-

$$\Rightarrow \Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$$

Expanding w.r.t R_1

$$= a_{11}C_{11} + a_{12}C_{12} + a_{13}C_{13}$$

Expanding w.r.t R_1

$$= a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$

Date → 05/21

Exercise 4.4

(1) (i) $\begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$

Sol.

$$A = \begin{vmatrix} 2 & -4 \\ 0 & 3 \end{vmatrix}$$

Minors

$\Rightarrow M_{11} = 3$

$\Rightarrow M_{12} = 0$

$\Rightarrow M_{21} = -4$

$\Rightarrow M_{22} = 2$

Cofactors

$\Rightarrow C_{11} = (-1)^{(1+1)}(3) = 3$

$\Rightarrow C_{22} = (-1)^{(2+2)}(2) = 2$

$\Rightarrow C_{12} = (-1)^{(1+2)}(0) = 0$

$\Rightarrow C_{21} = (-1)^{(2+1)}(-4) = 4$

$\Rightarrow$ Cof $A = \begin{vmatrix} 3 & 0 \\ 4 & 2 \end{vmatrix}$

$\Rightarrow$ Min $A = \begin{vmatrix} 3 & 0 \\ -4 & 2 \end{vmatrix}$

(ii) $\begin{vmatrix} a & c \\ b & d \end{vmatrix} = A$

Sol.

Minors

$\Rightarrow M_{11} = d$

$\Rightarrow M_{12} = b$

$\Rightarrow M_{21} = c$

$\Rightarrow M_{22} = a$

cofactors

$\Rightarrow C_{11} = (-1)^{1+1}(d) = d$

$\Rightarrow C_{12} = (-1)^{1+2}(b) = -b$

$\Rightarrow C_{21} = (-1)^{2+1}(c) = -c$

$\Rightarrow C_{22} = (-1)^{2+2}(a) = a$

Min $A = \begin{vmatrix} d & b \\ c & a \end{vmatrix}$

cof $A = \begin{vmatrix} d & -b \\ -c & a \end{vmatrix}$

(2) (i) $A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Sol. Minors,

$\Rightarrow M_{11} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow (1\times1 - 0\times0) = 1$

$\Rightarrow M_{12} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow 0$

$\Rightarrow M_{13} = \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} \Rightarrow 0$

$\Rightarrow M_{21} = \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow 0$

$\Rightarrow M_{22} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} \Rightarrow 1$

$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{31} = \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{32} = \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow M_{33} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow \text{Min } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

Cofactors.

$\Rightarrow C_{11} = (-1)^{1+1} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow C_{12} = (-1)^{1+2} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\Rightarrow C_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 1 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow C_{21} = (-1)^{2+1} \begin{vmatrix} 0 & 0 \\ 0 & 1 \end{vmatrix} = 0$

$\Rightarrow C_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow C_{31} = (-1)^{3+1} \begin{vmatrix} 0 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\Rightarrow C_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 0 \\ 0 & 0 \end{vmatrix} = 0$

$\Rightarrow C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow \text{cof } A = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$

ii

$A = \begin{vmatrix} 1 & 0 & 4 \\ 3 & 5 & -1 \\ 0 & 1 & 2 \end{vmatrix}$

Date 15/05

Minors,

$\Rightarrow M_{11} = \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = (10+1) = 11$

$\Rightarrow M_{12} = \begin{vmatrix} 3 & -1 \\ 0 & 2 \end{vmatrix} = \cancel{(2-4\times 0)} = 6$

$\Rightarrow M_{13} = \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = (3-5\times 0) = 3$

$\Rightarrow M_{21} = \begin{vmatrix} 0 & 4 \\ 1 & 2 \end{vmatrix} = -4$

$\Rightarrow M_{22} = \begin{vmatrix} 1 & 4 \\ 0 & 2 \end{vmatrix} = 2$

$\Rightarrow M_{23} = \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow M_{31} = \begin{vmatrix} 0 & 4 \\ 5 & -1 \end{vmatrix} = -20$

$\Rightarrow M_{32} = \begin{vmatrix} 1 & 4 \\ 3 & -1 \end{vmatrix} = -13$

$\Rightarrow M_{33} = \begin{vmatrix} 1 & 0 \\ 3 & 5 \end{vmatrix} = 5$

$$\text{Min } A = \begin{vmatrix} 11 & 2 & 3 \\ -4 & 2 & 1 \\ -20 & -13 & 5 \end{vmatrix}$$

Cofactors

$\Rightarrow C_{11} = (-1)^{1+1} \begin{vmatrix} 5 & -1 \\ 1 & 2 \end{vmatrix} = 11$

$\Rightarrow C_{12} = (-1)^{1+2} M_{12} = -6$

$\Rightarrow C_{13} = (-1)^{1+3} M_{13} = 3$

$\Rightarrow C_{21} = (-1)^{2+1} M_{21} = 4$

$\Rightarrow C_{22} = (-1)^{2+2} M_{22} = 2$

$\Rightarrow C_{23} = (-1)^{2+3} M_{23} = -1$

$\Rightarrow C_{31} = (-1)^{3+1} M_{31} = -20$

$\Rightarrow C_{32} = (-1)^{3+2} M_{32} = 13$

$\Rightarrow C_{33} = (-1)^{3+3} M_{33} = 5$

Date 12/05

cof $A = \begin{vmatrix} 11 & -6 & 3 \\ 4 & 2 & -1 \\ -20 & 13 & 5 \end{vmatrix}$

(3) using cofactors of elements of second row,

evaluate $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

Sol.

$\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$\Rightarrow C_{11} = (-1)^{1+1} \begin{vmatrix} 0 & 1 \\ 2 & 3 \end{vmatrix} = -2$

$\Rightarrow C_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 1 \\ 1 & 3 \end{vmatrix} = -5$

$\Rightarrow C_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 0 \\ 1 & 2 \end{vmatrix} = 4$

$\Rightarrow C_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 8 \\ 2 & 3 \end{vmatrix} = 7$

$\Rightarrow C_{22} = (-1)^{2+2} \begin{vmatrix} 5 & 8 \\ 1 & 2 \end{vmatrix} = 2$

$\Rightarrow C_{23} = (-1)^{2+3} \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = -7$

$\Rightarrow C_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 8 \\ 0 & 1 \end{vmatrix} = 3$

$\Rightarrow C_{32} = (-1)^{3+2} \begin{vmatrix} 5 & 8 \\ 2 & 1 \end{vmatrix} = 11$

$\Rightarrow C_{33} = (-1)^{3+3} \begin{vmatrix} 5 & 3 \\ 2 & 0 \end{vmatrix} = -6$

$$\Rightarrow \Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$$

Expanding w.r.t r_2

$$= a_{21}C_{21} + a_{22}C_{22} + a_{23}C_{23}$$
$$= 2(7) + 0(2) + 1(-7)$$
$$= 14 - 7$$
$$= 7$$

(4) Using cofactors of elements of third column evaluate,

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Sol.

$$\Rightarrow C_{13} = \begin{vmatrix} 1 & y \\ 1 & z \end{vmatrix} (-1)^{1+3} = (z-y)$$

$$\Rightarrow C_{23} = (-1)^{2+3} \begin{vmatrix} 1 & x \\ 1 & z \end{vmatrix} = -(z-x)$$
$$= (x-z)$$

$$\Rightarrow C_{33} = (-1)^{3+3} \begin{vmatrix} 1 & x \\ 1 & y \end{vmatrix} = (y-x)$$

$$\Delta = \begin{vmatrix} 1 & x & yz \\ 1 & y & zx \\ 1 & z & xy \end{vmatrix}$$

Expanding w.r.t to C_3,

$$= a_{13}C_{13} + a_{23}C_{23} + a_{33}C_{33}$$
$$= (yz)(z-y) + (zx)(x-z) + (xy)(y-x)$$

$= yz^2 - y^2z + zx^2 - z^2x + xy^2 - x^2y$

$\cancel{= -xy^2 - y^2z + zx^2 - x^2y + yz^2 - z^2x}$

$\cancel{= y^2(x-z) + x^2(z-y) + z^2(y-x)}$

$= (x^2z - y^2z) + (yz^2 - xz^2) + (xy^2 - x^2y)$

$= z(x^2 - y^2) + z^2(y-x) + xy(y-x)$

$= z(x+y)(x-y) - z^2(x-y) - xy(x-y)$

$= (zx + zy)(x-y) - z^2(x-y) - xy(x-y)$

$= (x-y)[zx + zy - z^2 - xy]$

$= (x-y)[zx - z^2 + zy - xy]$

$= (x-y)[z(x-z) + y(z-x)]$

$= (x-y)[-z(z-x) + y(z-x)]$

$= (x-y)(z-x)[y-z]$

$= (x-y)(z-x)(y-z)$ Ans.

5) If $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$ And A_{ij} is cofactors of a_{ij}, then value of Δ is given by

E) $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$

Sol. $\Delta = \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

Here A_{ij} is cofactors (C) of a_{ij}

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} a_{12} & a_{13} \\ a_{22} & a_{23} \end{vmatrix} = a_{12}a_{23} - a_{13}a_{22}$ $\cancel{a_{22}a_{23} - a_{22}a_{31}}$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{vmatrix} = -(a_{11}a_{33} - a_{13}a_{31})$

$= a_{13}a_{31} - a_{11}a_{33}$

12/05

$$\Rightarrow a_{33} = (-1)^{3+3} \begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}a_{22} - a_{12}a_{21}$$

N=(4)

$$= a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$$

$$= a_{11}(a_{12}a_{23} - a_{13}a_{22}) + a_{12}(a_{13}a_{31} - a_{11}a_{33}) + a_{13}(a_{11}a_{22} - a_{12}a_{21})$$

$$= a_{11}a_{12}a_{23} - a_{11}a_{13}a_{22} + a_{12}a_{13}a_{31} - a_{11}a_{12}a_{33} + a_{13}a_{11}a_{22} - a_{13}a_{12}a_{21}$$

$$= a_{12}a_{13}a_{31} - a_{13}a_{12}a_{21}$$

Ans. $D > a_{11}A_{11} + a_{21}A_{21} + a_{31}A_{31}$

C.W.

Adjoint of a matrix :- represent

Adjoint is transpose of cofactor, represented by 'adj'

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{cof } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

$$\text{Adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

12/05

Theorem:-

① $A.(\text{Adj } A) = |A| I = (\text{adj } A).A$

② $|\text{adj. } A| = |A|^{n-1}$

③ a square matrix 'A' is said to be singular, if ~~deter~~ $|A| = 0$.

④ a square matrix 'A' is said to be invertable if $|A| \neq 0$

⑤ Inverse matrix 'A^{-1}' is

$$\boxed{A^{-1} = \frac{\text{adj. } A}{|A|}}$$

Exercise 4.5

② $\begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$

Sol. Let;

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 2 & 3 & 5 \\ -2 & 0 & 1 \end{bmatrix}$$

$\Rightarrow A_{11} = (-1)^{1+1} \begin{vmatrix} 3 & 5 \\ 0 & 1 \end{vmatrix} = 3$

$\Rightarrow A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 5 \\ -2 & 1 \end{vmatrix} = -(2\times1-(-2\times5)) = -12$

Date 12/05

$\Rightarrow A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 3 \\ -2 & 0 \end{vmatrix} = 6$

$\Rightarrow A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 1 \end{vmatrix} = 1$

$\Rightarrow A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ -2 & 1 \end{vmatrix} = 5$

$\Rightarrow A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ -2 & 0 \end{vmatrix} = 2$

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 3 & 5 \end{vmatrix} = -11$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 2 & 5 \end{vmatrix} = -1$

$\Rightarrow A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & 3 \end{vmatrix} = 5$

$$\Rightarrow \text{adj } A = \begin{bmatrix} A_{11} & A_{21} & A_{31} \\ A_{12} & A_{22} & A_{32} \\ A_{13} & A_{23} & A_{33} \end{bmatrix}$$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 3 & 1 & -11 \\ -12 & 5 & -1 \\ 6 & 2 & 5 \end{bmatrix}$$ Ans.

(7) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$

Sol. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{bmatrix}$$

Date: 12/05

$\Rightarrow |A| = \begin{vmatrix} 1 & 2 & 3 \\ 0 & 2 & 4 \\ 0 & 0 & 5 \end{vmatrix}$

Expanding w.r.t C_1

$\Rightarrow |A| = 1(2\times5 - 4\times0)$

$\Rightarrow |A| = 10$

$\Rightarrow |A| \neq 0$

Hence A^{-1} exists.

<u>For adjoint A,</u>

$\Rightarrow A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & 4 \\ 0 & 5 \end{vmatrix} = 10$

$\Rightarrow A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & 4 \\ 0 & 5 \end{vmatrix} = \cancel{-1}\ 0$

$\Rightarrow A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 0 & 0 \end{vmatrix} = \cancel{1}\ 0$

$\Rightarrow A_{21} = (-1)^{2+1}\begin{vmatrix} 2 & 3 \\ 0 & 5 \end{vmatrix} = -10$

$\Rightarrow A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 3 \\ 0 & 5 \end{vmatrix} = 5$

$\Rightarrow A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 2 \\ 0 & 0 \end{vmatrix} = \cancel{-1}\ 0$

$\Rightarrow A_{31} = (-1)^{3+1}\begin{vmatrix} 2 & 3 \\ 2 & 4 \end{vmatrix} = 2$

$\Rightarrow A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 3 \\ 0 & 4 \end{vmatrix} = -4$

$\Rightarrow A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 2 \\ 0 & 2 \end{vmatrix} = 2$

Date 13/05

$$\Rightarrow \therefore \quad \text{Adj } A = \begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

Now,

$$\Rightarrow \quad A^{-1} = \frac{\text{Adj } A}{|A|}$$

$$\Rightarrow \quad A^{-1} = \frac{\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}}{10}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{10}\begin{bmatrix} 10 & -10 & 2 \\ 0 & 5 & -4 \\ 0 & 0 & 2 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \begin{bmatrix} 1 & -1 & 1/5 \\ 0 & 1/2 & -2/5 \\ 0 & 0 & 1/5 \end{bmatrix}$$ Ans.

H.W.

Date 13/05-/21

(1) $\begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$

Sol. let

$$A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}$$

$\Rightarrow A_{11} = (-1)^{1+1}(4) = 4$

$\Rightarrow A_{12} = (-1)^{1+2}(3) = -3$

$\Rightarrow A_{21} = (-1)^{2+1}(2) = -2$

$\Rightarrow A_{22} = (-1)^{2+2}(1) = 1$

Date 13/05

$$\text{Adj } A = \begin{bmatrix} 4 & -2 \\ -3 & 1 \end{bmatrix}$$

② Verify $A(\text{adj. } A) = (\text{adj } A)A = |A| I$

③ $\begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$

Sol:

let,

$$A = \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \quad ; \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$A_{11} = (-1)^{1+1}(-6) = -6$

$A_{12} = (-1)^{1+2}(-4) = 4$

$A_{21} = (-1)^{2+1}(3) = -3$

$A_{22} = (-1)^{2+2}(2) = 2$

$$\Rightarrow (\text{adj } A) = \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & 3 \\ -4 & -6 \end{vmatrix}$$

$= (2\times-6 - -4\times3)$

$= -12 + 12$

$|A| = 0$

Now,

$= A(\text{adj. } A)$

$$= \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix} \cdot \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times-6 + 3\times4 & 2\times-3 + 3\times2 \\ -4\times-6 + -6\times4 & -4\times-3 + -6\times2 \end{bmatrix}$$

Date 13/05
Page

$$A(\text{adj. } A) = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow (1)$$

Now,

$$= (\text{adj. } A)(A)$$

$$= \begin{bmatrix} -6 & -3 \\ 4 & 2 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 \\ -4 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -6\times 2 + -3\times -4 & -6\times 3 + -3\times -6 \\ 4\times 2 + 2\times -4 & 4\times 3 + 2\times -6 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \longrightarrow (2)$$

Now,

$$= |A| I$$

$$= 0 \cdot \begin{vmatrix} 1 & 0 \\ 0 & 1 \end{vmatrix}$$

$$= 0 \longrightarrow (3)$$

from (1), (2) and (3),

$$\Rightarrow A(\text{adj. } A) = (\text{adj. } A)A = |A| I$$

Proved.

(4) $\begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$

Sol. Let,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{vmatrix}$$

Expanding w.r.t C_2

$$\Rightarrow |A| = (-1)(3\times 3 - -2\times 1)$$

$$= -(-11)$$

$$\Rightarrow |A| = 11$$

19/05

$\Rightarrow A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -2 \\ 0 & 3 \end{vmatrix} = 0$

$\Rightarrow A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -2 \\ 1 & 3 \end{vmatrix} = -11$

$\Rightarrow A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 0 \\ 1 & 0 \end{vmatrix} = 0$

$\Rightarrow A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ 0 & 3 \end{vmatrix} = 3$

$\Rightarrow A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 1 & 3 \end{vmatrix} = 1$

$\Rightarrow A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 0 \end{vmatrix} = -1$

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -2 \end{vmatrix} = 2$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -2 \end{vmatrix} = 8$

$\Rightarrow A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 0 \end{vmatrix} = 3$

$\Rightarrow \text{adj. } A = \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

<u>Now,</u>

$= A(\text{adj } A)$

$= \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix} \cdot \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix}$

Date 13/05
Page

$$= \begin{bmatrix} 1\times0 + -1\times-11 + 2\times0 & 1\times3 + -1\times1 + 2\times1 & 1\times2 + -1\times8 + 2\times3 \\ 3\times0 + 0\times-11 + -2\times0 & 3\times3 + 0\times1 + -2\times1 & 3\times2 + 0\times8 + -2\times3 \\ 1\times0 + 0\times-11 + 3\times0 & 1\times3 + 0\times1 + 3\times-1 & 1\times2 + 0\times8 + 3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (1)$$

Now,

$$= (\text{adj } A)\, A.$$

$$= \begin{bmatrix} 0 & 3 & 2 \\ -11 & 1 & 8 \\ 0 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & -1 & 2 \\ 3 & 0 & -2 \\ 1 & 0 & 3 \end{bmatrix}$$

$$= \begin{bmatrix} 0\times1 + 3\times3 + 2\times1 & 0\times-1 + 3\times0 + 2\times0 & 0\times2 + 3\times-2 + 2\times3 \\ -11\times1 + 1\times3 + 8\times1 & -11\times-1 + 1\times0 + 8\times0 & -11\times2 + 1\times-2 + 8\times3 \\ 0\times1 + -1\times3 + 3\times1 & 0\times-1 + -1\times0 + 3\times0 & 0\times2 + -1\times-2 + 3\times3 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= 11 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (2)$$

Date: 13/05

Now,

$$= |A| I$$

$$= 14 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \longrightarrow (3)$$

from (1), (2) & (3),

$$\Rightarrow A(\text{adj } A) = (\text{adj. } A)A = |A| I$$

proved

Find Inverse

(5) $\begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$

Sol: let,

$$A = \begin{bmatrix} 2 & -2 \\ 4 & 3 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} 2 & -2 \\ 4 & 3 \end{vmatrix}$$

$$\Rightarrow |A| = 14$$

$\Rightarrow A^{-1}$ Exists

$\Rightarrow A_{11} = 3(-1)^{1+1} = 3$ | $A_{21} = -2(-1)^{2+1} = 2$

$\Rightarrow A_{12} = 4(-1)^{1+2} = -4$ | $A_{22} = 2(-1)^{2+2} = 2$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 3 & +2 \\ -4 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

Date 13/05
Page

$$= \frac{\begin{bmatrix} 3 & +2 \\ -4 & 2 \end{bmatrix}}{14}$$

$$= \frac{1}{14}\begin{bmatrix} 3 & +2 \\ -4 & 2 \end{bmatrix} \quad \text{Ans}$$

$$= \begin{bmatrix} \frac{3}{14} & +\frac{1}{7} \\ -\frac{2}{7} & \frac{1}{7} \end{bmatrix} \quad \underline{\text{Ans}}$$

(6) $\begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$

Sol. let,

$$A = \begin{bmatrix} -1 & 5 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow |A| = \begin{vmatrix} -1 & 5 \\ -3 & 2 \end{vmatrix}$$

$$\Rightarrow |A| = 13$$

A^{-1} Exists.

$\Rightarrow A_{11} = 2(-1)^{1+1} = 2$ | $A_{21} = 5(-1)^{2+1} = -5$

$\Rightarrow A_{12} = -3(-1)^{1+2} = 3$ | $A_{22} = -1(-1)^{2+2} = -1$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 2 & -5 \\ +3 & -1 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{13}\begin{bmatrix} 2 & -5 \\ 3 & -1 \end{bmatrix} \quad \underline{\text{Ans}}$$

Date 13/05

3. $\begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$

Sol: let;

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{bmatrix}$$

$\Rightarrow |A| = \begin{vmatrix} 1 & 0 & 0 \\ 3 & 3 & 0 \\ 5 & 2 & -1 \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow |A| = 1(3\times-1 - 2\times0)$

$\Rightarrow |A| = -3$

A^{-1} Exists.

$\Rightarrow A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 0 \\ 2 & -1 \end{vmatrix} = -3$

$\Rightarrow A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & 0 \\ 5 & -1 \end{vmatrix} = 3$

$\Rightarrow A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 3 \\ 5 & 2 \end{vmatrix} = -9$

$\Rightarrow A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ 2 & -1 \end{vmatrix} = 0$

$\Rightarrow A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 5 & -1 \end{vmatrix} = -1$

$\Rightarrow A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 5 & 2 \end{vmatrix} = -2$

$\Rightarrow A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$\Rightarrow A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 3 & 0 \end{vmatrix} = 0$

$\Rightarrow A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 3 & 3 \end{vmatrix} = 3$

$\text{adj } A = \begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ 9 & -2 & 3 \end{bmatrix}$

$A^{-1} = \dfrac{\text{adj} A}{|A|}$

$A^{-1} = \dfrac{1}{-3}\begin{bmatrix} -3 & 0 & 0 \\ 3 & -1 & 0 \\ 9 & -2 & 3 \end{bmatrix}$

$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1/3 & 0 \\ -3 & \frac{2}{3} & -1 \end{bmatrix}$ Ans.

(9) $\begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$

Sol. Let,

$$A = \begin{bmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{bmatrix}$$

$\Rightarrow |A| = \begin{vmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow |A| = 2(-1\times1 - 2\times0) - 1(4\times1 - -7\times0) + 3(4\times2 - -7\times-1)$

$\Rightarrow |A| = -2 - 4 + 3$

$\Rightarrow |A| = -3$

A^{-1} Exists.

$\Rightarrow$

$\Rightarrow A_{11} = (-1)^{1+1} \begin{vmatrix} -1 & 0 \\ 2 & 1 \end{vmatrix} = -1$

$\Rightarrow A_{12} = (-1)^{1+2} \begin{vmatrix} 4 & 0 \\ -7 & 1 \end{vmatrix} = -4$

$\Rightarrow A_{13} = (-1)^{1+3} \begin{vmatrix} 4 & -1 \\ -7 & 2 \end{vmatrix} = 1$

$\Rightarrow A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 3 \\ 2 & 1 \end{vmatrix} = 5$

$\Rightarrow A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 3 \\ -7 & 1 \end{vmatrix} = 23$

$\Rightarrow A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ -7 & 2 \end{vmatrix} = -11$

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 3 \\ -1 & 0 \end{vmatrix} = 3$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 3 \\ 4 & 0 \end{vmatrix} = 12$

$\Rightarrow A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 4 & -1 \end{vmatrix} = -6$

$\Rightarrow \text{adj } A = \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{\text{adj } A}{|A|}$

$\Rightarrow A^{-1} = -\dfrac{1}{3} \begin{bmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{bmatrix}$

(10) $\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

Sol: Let,

$A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$

Expanding w.r.t C_1,

$\Rightarrow |A| = 1(2\times4 - 2\times3) + 3(-1\times-3 - 2\times2)$

$\Rightarrow |A| = 2 + (-3)$

$\Rightarrow |A| = -1$

$\Rightarrow A_{11} = (-1)^{1+1} \begin{vmatrix} 0 & -3 \\ -2 & 4 \end{vmatrix} = 2$

$\Rightarrow A_{12} = (-1)^{1+2} \begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = -9$

$\Rightarrow A_{13} = (-1)^{1+3} \begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = -6$

$\Rightarrow A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = 0$

$\Rightarrow A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = -2$

$\Rightarrow A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = -1$

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 0 & -3 \end{vmatrix} = -1$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = 3$

$\Rightarrow A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = 2$

$\Rightarrow \operatorname{adj} A = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{\operatorname{adj} A}{|A|}$

$\Rightarrow A^{-1} = -1 \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

Ans

(11) $\begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Sol. Let,

$\Rightarrow A = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

$\Rightarrow |A| = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow |A| = 1(\cos\alpha \times -\cos\alpha - \sin\alpha \cdot \sin\alpha)$

$\Rightarrow |A| = -\cos^2\alpha - \sin^2\alpha$

$\Rightarrow |A| = -(\cos^2\alpha + \sin^2\alpha)$

$\Rightarrow |A| = -1$

A^{-1} Exists.

$\Rightarrow A_{11} = (-1)^{1+1}\begin{vmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{vmatrix} = -\cos^2\alpha - \sin^2\alpha = -1$

$\Rightarrow A_{12} = (-1)^{1+2}\begin{vmatrix} 0 & \sin\alpha \\ 0 & -\cos\alpha \end{vmatrix} = 0$

$\Rightarrow A_{13} = (-1)^{1+3}\begin{vmatrix} 0 & \cos\alpha \\ 0 & \sin\alpha \end{vmatrix} = 0$

$\Rightarrow A_{21} = (-1)^{2+1}\begin{vmatrix} 0 & 0 \\ \sin\alpha & -\cos\alpha \end{vmatrix} = 0$

$\Rightarrow A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 0 \\ 0 & -\cos\alpha \end{vmatrix} = -\cos\alpha$

$\Rightarrow A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -\sin\alpha$

$\Rightarrow A_{31} = (-1)^{3+1}\begin{vmatrix} 0 & 0 \\ \cos\alpha & \sin\alpha \end{vmatrix} = 0$

$\Rightarrow A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 0 \\ 0 & \sin\alpha \end{vmatrix} = -\sin\alpha$

$\Rightarrow A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & 0 \\ 0 & \cos\alpha \end{vmatrix} = \cos\alpha$

$\text{adj } A = \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{\text{adj } A}{|A|}$

$\Rightarrow A^{-1} = -1\begin{bmatrix} -1 & 0 & 0 \\ 0 & -\cos\alpha & -\sin\alpha \\ 0 & -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\alpha & \sin\alpha \\ 0 & \sin\alpha & -\cos\alpha \end{bmatrix}$

Ans.

Date 13/05
Page

Contd.

(15) For the matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$

Show that $A^3 - 6A^2 + 5A + 11I = 0$, Hence find A^{-1}

Sol.

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\times1 + 1\times1 + 1\times2 & 1\times1 + 1\times2 + 1\times-1 & 1\times1 + 1\times-3 + 1\times3 \\ 1\times1 + 2\times1 + -3\times2 & 1\times1 + 2\times2 + -3\times-1 & 1\times1 + 2\times-3 + -3\times3 \\ 2\times1 + -1\times1 + 3\times2 & 2\times1 + -1\times2 + 3\times-1 & 2\times1 + -1\times-3 + 3\times3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 4\times1 + 2\times1 + 1\times2 & 4\times1 + 2\times2 + 1\times-1 & 4\times1 + 2\times-3 + 1\times3 \\ -3\times1 + 8\times1 + -14\times2 & -3\times1 + 8\times2 + -14\times-1 & -3\times1 + 8\times-3 + -14\times3 \\ 7\times1 + -3\times1 + 14\times2 & 7\times1 + -3\times2 + 14\times-1 & 7\times1 + -3\times-3 + 14\times3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix}$$

$$\begin{array}{r} 84 \\ 19 \\ \hline 26 \end{array} \qquad \begin{array}{r} 84 \\ 67 \\ \hline 15 \end{array} \qquad \begin{array}{r} 48 \\ 27 \\ \hline 21 \end{array} \qquad \boxed{= 13/-5}$$

Now,

$$= A^3 - 6A^2 + 5A + 11I$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - 6\begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} + 5\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 11\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 & 1 \\ -23 & 27 & -69 \\ 32 & -13 & 58 \end{bmatrix} - \begin{bmatrix} 24 & 12 & 6 \\ -18 & 48 & -84 \\ 42 & -18 & 84 \end{bmatrix} + \begin{bmatrix} 5 & 5 & 5 \\ 5 & 10 & -15 \\ 10 & -5 & 15 \end{bmatrix} + \begin{bmatrix} 11 & 0 & 0 \\ 0 & 11 & 0 \\ 0 & 0 & 11 \end{bmatrix}$$

$$= \begin{bmatrix} -16 & -5 & -5 \\ -5 & -21 & 15 \\ -10 & 5 & -36 \end{bmatrix} + \begin{bmatrix} 16 & 5 & 5 \\ 5 & 21 & -15 \\ 10 & -5 & 136 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= O$$

Again,

$$\Rightarrow A^3 - 6A^2 + 5A + 11I = O$$

pre-multiplying A^{-1} both side,

$$\Rightarrow A^{-1}(A^3 - 6A^2 + 5A + 11I) = A^{-1}(O)$$

$$\Rightarrow A^{-1}A^3 - 6A^{-1}A^2 + 5A^{-1}A + 11A^{-1}I = O$$

$$\Rightarrow A^{-1}A.A^2 - 6A^{-1}A.A + 5I + 11A^{-1} = O$$

$$\Rightarrow IA^2 - 6IA + 5I + 11A^{-1} = O$$

$$\Rightarrow A^2 - 6A + 5I + 11A^{-1} = O \qquad \{\because A^{-1}A = I\}$$

Date 13/05

$$\Rightarrow A^2 - 6A + 5I = -11A^{-1}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - 6\begin{bmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{bmatrix} + 5\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -11A^{-1}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 6 & 6 & 6 \\ 6 & 12 & -18 \\ 12 & -6 & 18 \end{bmatrix} + \begin{bmatrix} 5 & 0 & 0 \\ 0 & 5 & 0 \\ 0 & 0 & 5 \end{bmatrix} = -11A^{-1}$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 1 \\ -3 & 8 & -14 \\ 7 & -3 & 14 \end{bmatrix} - \begin{bmatrix} 11 & 6 & 6 \\ 6 & 17 & -18 \\ 12 & -6 & 23 \end{bmatrix} = -11A^{-1}$$

$$\Rightarrow \begin{bmatrix} -7 & -4 & -5 \\ -9 & -9 & 4 \\ -5 & 3 & \end{bmatrix} =$$

$$\Rightarrow \begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = -11A^{-1}$$

$$\Rightarrow \frac{1}{-11}\begin{bmatrix} 3 & -4 & -5 \\ -9 & 1 & 4 \\ -5 & 3 & 1 \end{bmatrix} = A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{11}\begin{bmatrix} -3 & 4 & 5 \\ 9 & -1 & -4 \\ 5 & -3 & -1 \end{bmatrix}$$ Ans.

H.W.

12) let $A = \begin{bmatrix} 2 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. Verify that $(AB)^{-1} = B^{-1}A^{-1}$

Sol

$A = \begin{bmatrix} 2 & 7 \\ 2 & 5 \end{bmatrix}$, $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$\Rightarrow |A| = 10 - 14$

$\Rightarrow |A| = -4$

$\Rightarrow |B| = 54 - 56$

$\Rightarrow |B| = -2$

$\Rightarrow A_{11} = (-1)^{1+1}(5) = 5$

$\Rightarrow A_{12} = (-1)^{1+2}(2) = -2$

$\Rightarrow A_{21} = (-1)^{2+1}(7) = -7$

$\Rightarrow A_{22} = (-1)^{2+2}(2) = 2$

$\Rightarrow A'_{11} = (-1)^{1+1}(9) = 9$

$\Rightarrow A'_{12} = (-1)^{1+2}(7) = -7$

$\Rightarrow A'_{21} = (-1)^{2+1}(8) = -8$

$\Rightarrow A'_{22} = (-1)^{2+2}(6) = 6$

$\Rightarrow \text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 2 \end{bmatrix}$

$\Rightarrow \text{adj. } B = \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{\text{adj. } A}{|A|}$

$\Rightarrow A^{-1} = -4 \begin{bmatrix} 5 & -7 \\ -2 & 2 \end{bmatrix}$

$\Rightarrow B^{-1} = \left\{ \dfrac{\text{adj } B}{|B|} \right\}$

$B^{-1} = -2 \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$\Rightarrow$ R.H.S $= B^{-1} A^{-1}$

$= -2 \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix} \cdot -4 \begin{bmatrix} 5 & -7 \\ -2 & 2 \end{bmatrix}$

$= \begin{bmatrix} -18 & 16 \\ 14 & -12 \end{bmatrix} \cdot \begin{bmatrix} -20 & 28 \\ 8 & -8 \end{bmatrix}$

$= \begin{bmatrix} -18 \times -20 + 16 \times 8 & -18 \times 28 + 16 \times -8 \\ 14 \times -20 + -12 \times 8 & 14 \times 28 + -12 \times 8 \end{bmatrix}$

$= \begin{bmatrix} 360 + 128 & -504 - 128 \\ -280 - 96 & 392 - 96 \end{bmatrix}$

Hotel.

$$= \begin{bmatrix} 488 & -612 \\ -376 & 296 \end{bmatrix}$$

(13) Let $A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$. verify that $(AB)^{-1} = B^{-1}A^{-1}$

Sol.

$$A = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix}$$

$\Rightarrow |A| = 15 - 14$

$\Rightarrow |A| = 1$

$\Rightarrow A_{11} = (-1)^{1+1}(5) = 5$

$\Rightarrow A_{12} = (-1)^{1+2}(2) = -2$

$\Rightarrow A_{21} = (-1)^{2+1}(7) = -7$

$\Rightarrow A_{22} = (-1)^{2+2}(3) = 3$

$$\Rightarrow \text{adj } A = \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj. } A}{|A|}$$

$$\Rightarrow A^{-1} = 1 \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$$

$$\Rightarrow B = \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$$

$\Rightarrow |B| = 54 - 56$

$\Rightarrow |B| = -2$

$\Rightarrow A_{11} = (-1)^{1+1}(9) = 9$

$\Rightarrow A_{12} = (-1)^{1+2}(7) = -7$

$\Rightarrow A_{21} = (-1)^{2+1}(8) = -8$

$\Rightarrow A_{22} = (-1)^{2+2}(6) = 6$

14/05

$\Rightarrow$ adj. B $= \begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$\Rightarrow B^{-1} = \dfrac{\text{adj. } B}{|B|}$

$\Rightarrow B^{-1} = \dfrac{1}{-2}\begin{bmatrix} 9 & -8 \\ -7 & 6 \end{bmatrix}$

$\Rightarrow B^{-1} = \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix}$

R.H.S $= B^{-1}A^{-1}$

$= \begin{bmatrix} -\frac{9}{2} & 4 \\ \frac{7}{2} & -3 \end{bmatrix} \begin{bmatrix} 5 & -7 \\ -2 & 3 \end{bmatrix}$

$= \begin{bmatrix} -\frac{9}{2}\times 5 + 4\times -2 & -\frac{9}{2}\times -7 + 4\times 3 \\ \frac{7}{2}\times 5 + -3\times -2 & \frac{7}{2}\times -7 + -3\times 3 \end{bmatrix}$

$= \begin{bmatrix} -\frac{45}{2} - 8 & \frac{63}{2} + 12 \\ \frac{35}{2} + 6 & -\frac{49}{2} - 9 \end{bmatrix}$

$B^{-1}A^{-1} = \begin{bmatrix} -\frac{61}{2} & \frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \longrightarrow (1)$

Now,

~~L.H.S $= (AB)^{-1}$~~

$AB = \begin{bmatrix} 3 & 7 \\ 2 & 5 \end{bmatrix} \begin{bmatrix} 6 & 8 \\ 7 & 9 \end{bmatrix}$

$AB = \begin{bmatrix} 3\times 6 + 7\times 7 & 3\times 8 + 7\times 9 \\ 2\times 6 + 5\times 7 & 2\times 8 + 5\times 9 \end{bmatrix}$

Date 14/05
Page

$\Rightarrow AB = \begin{bmatrix} 18+49 & 24+63 \\ 12+35 & 16+45 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 67 & 87 \\ 47 & 61 \end{bmatrix}$

$\Rightarrow |AB| = (61 \times 67 - 87 \times 47)$

$\Rightarrow |AB| = 4087 - 4089$

$\Rightarrow |AB| = -2$

$\Rightarrow A_{11} = (-1)^{1+1}(61) = 61$

$\Rightarrow A_{12} = (-1)^{1+2}(47) = -47$

$\Rightarrow A_{21} = (-1)^{2+1}(87) = -87$

$\Rightarrow A_{22} = (-1)^{2+2}(67) = 67$

$\Rightarrow \cancel{A^{-1}} = \text{adj } AB = \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \dfrac{\text{adj. } A}{|A|}$

$\Rightarrow (AB)^{-1} = -\dfrac{1}{2} \begin{bmatrix} 61 & -87 \\ -47 & 67 \end{bmatrix}$

$\Rightarrow (AB)^{-1} = \begin{bmatrix} -\frac{61}{2} & +\frac{87}{2} \\ \frac{47}{2} & -\frac{67}{2} \end{bmatrix} \longrightarrow (2)$

from (1) and (2)

$\Rightarrow \boxed{(AB)^{-1} = B^{-1}A^{-1}}$

Verified.

14/05

(13) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$. Hence find A^{-1}

Sol.

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A.A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3\times3 + 1\times-1 & 3\times1 + 1\times2 \\ -1\times3 + 2\times-1 & -1\times1 + 2\times2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix}$$

Now,

$$\Rightarrow A^2 - 5A + 7I = 0$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & 3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$$

$$\Rightarrow 0 = 0$$

Again

$$\Rightarrow A^2 - 5A + 7I = 0$$

pre-multiply in A^{-1}

$$\Rightarrow A^{-1}(A^2 - 5A + 7I) = A^{-1}(0)$$

$\Rightarrow A^{-1}A^2 - 5A^{-1}A + 7A^{-1}I = 0$

$\Rightarrow A^{-1}.A.A - 5I + 7A^{-1} = 0$

$\Rightarrow IA - 5I + 7A^{-1} = 0$

$\Rightarrow A - 5I + 7A^{-1} = 0$

$\Rightarrow A - 5I = -7A^{-1}$

$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - 5\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = -7A^{-1}$

$\Rightarrow \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix} = -7A^{-1}$

$\Rightarrow \begin{bmatrix} -2 & 1 \\ -1 & -3 \end{bmatrix} = -7A^{-1}$

$\Rightarrow \frac{1}{-7}\begin{bmatrix} -2 & 1 \\ -1 & 3 \end{bmatrix} = A^{-1}$

$\Rightarrow A^{-1} = \frac{1}{7}\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$

(14) for the matrix $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$, find the no.'s a and b such that

$A^2 + aA + bI = 0$

Sol. $A = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow AA = \begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 3 & 2 \\ 1 & 1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 3\times3 + 2\times1 & 3\times2 + 2\times1 \\ 1\times3 + 1\times1 & 1\times2 + 1\times1 \end{bmatrix}$

$\Rightarrow A^2 = \begin{bmatrix} 11 & 8 \\ 4 & 3 \end{bmatrix}$

Ans:

$A^2 + aA + bI = 0$

$\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + a\begin{bmatrix} 1 & 2 \\ 3 & 1 \end{bmatrix} + b\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\begin{bmatrix} 2a & 2a \\ 3a & 2a \end{bmatrix} + \begin{bmatrix} b & 0 \\ 0 & b \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -3 & -2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2a+b & 2a \\ 3a & 2a+b \end{bmatrix} = \begin{bmatrix} -2 & -2 \\ -3 & -2 \end{bmatrix}$

$\Rightarrow$ Comparing both sides,

$\Rightarrow \boxed{a = -4}$

$\Rightarrow 2a + b = -2$

$\Rightarrow -4 + b = -2$

$\Rightarrow b = 2$

(15) Q4 $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Verify that $A^3 - 6A^2 + 9A - 4I = 0$ and hence find A^{-1}

Sol $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$

$|A| =$ Expand

$\Rightarrow |A| = \begin{vmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow |A| = 2(2\times2 - -1\times-1) - 1(-1\times2 - 1\times-1) + 1(-1\times-1 - 1\times2)$

$\Rightarrow |A| = 6 + 1 - 1$

$\Rightarrow |A| = 6$

Date 14/05

$\Rightarrow A^2 - 6A + 9I - 4A^{-1} = 0$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$\Rightarrow A^2 - 6A + 9I = 4A^{-1}$

$$\Rightarrow \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - 6\begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix} + 9\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 4A^{-1}$$

$$\Rightarrow \begin{bmatrix} 6 & -5 & 5 \\ -5 & 6 & -5 \\ 5 & -5 & 6 \end{bmatrix} - \begin{bmatrix} 12 & -6 & 6 \\ -6 & 12 & -12 \\ 6 & -6 & 12 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 4A^{-1}$$

$$\Rightarrow \begin{bmatrix} -6 & 1 & -1 \\ 1 & -6 & 7 \\ -1 & 1 & -6 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 0 \\ 0 & 9 & 0 \\ 0 & 0 & 9 \end{bmatrix} = 4A^{-1}$$

$$\Rightarrow \begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 7 \\ -1 & 1 & 3 \end{bmatrix} = 4A^{-1}$$

$$\Rightarrow A^{-1} = \frac{1}{4}\begin{bmatrix} 3 & 1 & -1 \\ 1 & 3 & 7 \\ -1 & 1 & 3 \end{bmatrix}$$ Ans

$\Rightarrow A^{-1} =$

(17) Let A be a non singular square matrix of order 3x3. Then |adj. A| is equal to

Ans (B) $|A|^2$ Sol. $\Rightarrow |\text{adj. } A| = |A|^{n-1}$
$= |A|^{3-1} = |A|^2$ Ans.

(18) If A is an invertible matrix of order 2, then (A^{-1}) is equal to

Ans. (B) $\frac{1}{\det(A)}$

IMP Eg.

15/05

C.W.

Eg. → 15, 16 | Chapter 3 → Eg. 24

Eg. 15

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix} = 0$$, show that $1+xyz=0$

Sol.

$$\Delta = \begin{vmatrix} x & x^2 & 1+x^3 \\ y & y^2 & 1+y^3 \\ z & z^2 & 1+z^3 \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x & x^2 & 1 \\ y & y^2 & 1 \\ z & z^2 & 1 \end{vmatrix} + \begin{vmatrix} x & x^2 & x^3 \\ y & y^2 & y^3 \\ z & z^2 & z^3 \end{vmatrix}$$ {By P_5}

Taking ~~common~~ common x from R_1, y from R_2, z from R_3 in 2nd determinant and in 1st determinant $C_1 \longleftrightarrow C_2$

$$\Rightarrow \Delta = -\begin{vmatrix} x^2 & x & 1 \\ y^2 & y & 1 \\ z^2 & z & 1 \end{vmatrix} + \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} (xyz)$$ {By P_2}

⇒ By process $C_1 \longleftrightarrow C_2$

$$\Rightarrow \Delta = (-1)^2 \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix} + xyz \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

$$\Rightarrow \Delta = (1+xyz) \begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{vmatrix}$$

By process $R_3 \to R_3 - R_2$

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 0 & 1 & z+y \end{vmatrix}$

Taking common $(z-y)$ from R_3

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 0 & 1 & z+y \end{vmatrix}$

By $R_2 \to R_2 - R_1$

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 0 & y-x & y^2-x^2 \\ 0 & 1 & z+y \end{vmatrix}$

Taking common $y-x$ from R_2

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+y \end{vmatrix}(y-x)$

Given

x, y, z, all are different

$x-y \neq 0$

$y-x \neq 0$

$z-y \neq 0$

$y-z \neq 0$

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+y \end{vmatrix}(y-x)$

$\Rightarrow \Delta = (1+xyz)(z-y)\begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+y \end{vmatrix}(y-x)$

Date 15/05
Page

$$\Rightarrow (1+xyz) = \frac{0}{(z-y)(y-x)} \begin{vmatrix} 1 & x & x^2 \\ 0 & 1 & y+x \\ 0 & 1 & z+y \end{vmatrix}$$

$$\Rightarrow (1+xyz) = 0.$$

Eg. 16. Show that.

$$\begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix} = abc\left(1 + \frac{1}{a} + \frac{1}{b} + \frac{1}{c}\right)$$

$$= abc + bc + ca + ab.$$

Sol. let,

$$\Delta = \begin{vmatrix} 1+a & 1 & 1 \\ 1 & 1+b & 1 \\ 1 & 1 & 1+c \end{vmatrix}$$

Taking common a from R_1, b from R_2, c from R_3.

$$\Rightarrow \Delta = abc \begin{vmatrix} \frac{1}{a}+1 & \frac{1}{a} & \frac{1}{a} \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$\Rightarrow$ By process $R_1 \to R_1 + R_2 + R_3$.

$$\Rightarrow \Delta = abc \begin{vmatrix} \frac{1}{a}+1+\frac{1}{b}+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+1+\frac{1}{c} & \frac{1}{a}+\frac{1}{b}+\frac{1}{c}+1 \\ \frac{1}{b} & \frac{1}{b}+1 & \frac{1}{b} \\ \frac{1}{c} & \frac{1}{c} & \frac{1}{c}+1 \end{vmatrix}$$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 1 & 1 \\ 1/b & 1+\frac{1}{b} & 1/b \\ 1/c & 1/c & 1+\frac{1}{c} \end{vmatrix}$$

By processing $C_2 \to C_2 - C_1$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 1 \\ 1/b & 1 & 1/b \\ 1/c & 0 & 1+\frac{1}{c} \end{vmatrix}$$

By processing $C_3 \to C_3 - C_1$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\begin{vmatrix} 1 & 0 & 0 \\ 1/b & 1 & 0 \\ 1/c & 0 & 1 \end{vmatrix}$$

Expanding with R_1

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\left\{1\left(1\times 1 - 0\times 0\right)\right\}$$

$$\Delta = abc\left(1+\frac{1}{a}+\frac{1}{b}+\frac{1}{c}\right)\{1\}$$

$$\Delta = abc + bc + ac + ab$$

proved

Chapter - 4
DERMINANTS

4.7.1 # Solution of system of linear equation using inverse of a matrix :-

$$a_1x + b_1y + c_1z = d_1$$
$$a_2x + b_2y + c_2z = d_2$$
$$a_3x + b_3y + c_3z = d_3$$

which is in form of,

$$\boxed{AX = B}$$

where,

$$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$$

$$B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$

Solution of inverse of system :-

$$\boxed{X = A^{-1}B}$$

Consistent system :- A system of eq^n is said to be consistent if its solution (one or more) exist.

Inconsistent system :- A system of eq^n is said to be inconsistent if its solution does not exist.

Note:- ~~If~~ If $|A| = 0$, then

~~$= (adj. A) \cdot B$~~

Case i) if $(adj\ A) \cdot B \neq 0$, then solution does not exist and system of eq^n is called inconsistent.

case ii) if $(adj.\ A) \cdot B = 0$, then system may either consistent or inconsistent according as the system have either infinite many solutions or no solutions.

Exercise 4.6

Examine the consistency of the system {from Q.1 → Q.4}

(1) $x + 2y = 2$
$2x + 3y = 3$

Sol.
$$x + 2y = 2$$
$$2x + 3y = 3.$$

It can be written in form of

$$Ax = B$$

where,
$$A = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}, \quad x \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = 3 - 4 = -1 \neq 0$$

Hence it is inconsistence.

Page No.:
Date 02 06

(2) $2x - y = 5$
$x + y = 4$

Sol.

$$2x - y = 5$$
$$x + y = 4$$

It is in the form of,

$$AX = B.$$

where,

$$A = \begin{bmatrix} 2 & -1 \\ 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 4 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 2 & -1 \\ 1 & 1 \end{vmatrix} = 2+1$$
$$= 3 \neq 0$$

Hence it is inconsistence.

(8) $x + 3y = 5$
$2x + 6y = 8$

Sol.

$$x + 3y = 5$$
$$2x + 6y = 8$$

It is in the form of

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & 3 \\ 2 & 6 \end{bmatrix}, \quad X \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 8 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 3 \\ 2 & 6 \end{vmatrix} = 6 - 6$$
$$= 0 = 0$$

Hence it is consistence.

(4) $x + y + z = 1$
$2x + 3y + 2z = 2$
$ax + ay + 2az = 4$

Sol. $x + y + z = 1$
$2x + 3y + 2z = 2$
$ax + ay + 2az = 4$

It is in the form of

$$AX = B$$

where,
$$A = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 1 & 1 & 1 \\ 2 & 3 & 2 \\ a & a & 2a \end{vmatrix}$$

Expanding w.r.t R_1,

$$= 1(3 \times 2a - 2a) - 1(2 \times 2a - 2a) + 1(2a - 3a)$$
$$= 4a - 2a - a$$
$$= a$$
$$\neq 0$$

Hence it is inconsistence.

(5) $3x - y - 2z = 2$
$2y - z = -1$
$3x - 5y = 3$

Sol. $3x - y - 2z = 2$
$0x + 2y - z = -1$
$3x - 5y + 0z = 3$

It is in the form of

$$AX = B$$

where,

$$A = \begin{bmatrix} 3 & -1 & -1 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$$

Now,

$$|A| = \begin{vmatrix} 3 & -1 & -1 \\ 0 & 2 & -1 \\ 3 & -5 & 0 \end{vmatrix}$$

Expanding w.r.t C_1,

$$= 3(2\times 0 - (-1\times -5)) + 3(-1\times -1 - 2\times -1)$$
$$= -15 + 9$$
$$= -6$$
$$\neq 0$$

Hence it is inconsistence.

(6) $5x - y + 4z = 5$
$2x + 3y + 5z = 2$
$5x - 2y + 6z = -1$

Sol.

$$5x - y + 4z = 5$$
$$2x + 3y + 5z = 2$$
$$5x - 2y + 6z = -1$$

It is in the form of

$$AX = B$$

where,

$$A = \begin{bmatrix} 5 & -1 & 4 \\ 2 & 5 & 5 \\ 5 & -2 & 6 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ 2 \\ -1 \end{bmatrix}$$

Page No.:
Date 02 06

Now,

$$|A| = \begin{vmatrix} 5 & -1 & 4 \\ 2 & 3 & 5 \\ 5 & -2 & 6 \end{vmatrix}$$

Expanding w.r.t C_1,

$$= 5(3\times6 - (-2\times5)) - 2(-1\times6 - (-2\times4)) + 5(-1\times5 - 3\times4)$$
$$= 5(28) - 2(2) + 5(-17)$$
$$= 140 - 4 - 55$$
$$= 140 - 59$$
$$= 51$$
$$\neq 0$$

Hence it is inconsistence.

Date 02/06

Solve system of linear eq^n, using matrix method (Q.7)

(7) $5x + 2y = 4$
$7x + 3y = 5$

Sol. $5x + 2y = 4$
$7x + 3y = 5$

which can be written as

$$AX = B$$

where

$$A = \begin{bmatrix} 5 & 2 \\ 7 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 7 & 3 \end{vmatrix} = 5\times3 - 7\times2 = 1 \neq 0$$

$\therefore A^{-1}$ exist.

$A_{11} = (-1)^{1+1}(3) = 3$

$A_{12} = (-1)^{1+2}(7) = -7$

$A_{21} = (-1)^{2+1}(2) = -2$

$A_{22} = (-1)^{2+2}(3) = 5$

$$\text{adj } A = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{\begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}}{1}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix}$$

Now sol^n of eq^n,

$$\Rightarrow X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 3 & -2 \\ -7 & 5 \end{bmatrix} \begin{bmatrix} 4 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \end{bmatrix}$$

on comparing both matrix, then

$x = 2$, $y = -3$

(14) $x - y + 2z = 7$

$3x + 4y - 5z = -5$

$2x - y + 3z = 12$

Sol. Given system of eq^n

$$x - y + 2z = 7$$
$$3x + 4y - 5z = -5$$
$$2x - y + 3z = 12$$

system can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{vmatrix}$$
$$= 1(12-5) + 1(9+10) + 2(-3-8)$$
$$= 7 + 19 - 24$$
$$= 4$$
$$\neq 0$$

A^{-1} exist

For adj. A,

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 4 & -5 \\ -1 & 3 \end{vmatrix} = 7$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = -19$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 4 \\ 2 & -1 \end{vmatrix} = -11$$

$\Rightarrow A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 2 \\ -1 & 3 \end{vmatrix} = 1$

$\Rightarrow A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 2 \\ 2 & 3 \end{vmatrix} = -1$

$\Rightarrow A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 2 & -1 \end{vmatrix} = -1$

$\Rightarrow A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 2 \\ 4 & -5 \end{vmatrix} = -3$

$\Rightarrow A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 2 \\ 3 & -5 \end{vmatrix} = 11$

$\Rightarrow A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 3 & 4 \end{vmatrix} = 7$

$\Rightarrow \text{adj } A = \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

$\Rightarrow A^{-1} = \dfrac{\text{adj} \cdot A}{|A|} = \dfrac{\begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}}{4}$

$\Rightarrow A^{-1} = \dfrac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix}$

Also,

$\Rightarrow X = A^{-1} B$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{4} \begin{bmatrix} 7 & 1 & -3 \\ -19 & -1 & 11 \\ -11 & -1 & 7 \end{bmatrix} \begin{bmatrix} 7 \\ -5 \\ 12 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \dfrac{1}{4} \begin{bmatrix} 8 \\ 4 \\ 12 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$$

on comparing both matrix, then we get

$\Rightarrow x = 2$
$\Rightarrow y = 1$
$\Rightarrow z = 3$

Q. 16

Sol. Let cost of onion Rs. x /kg
cost of wheat Rs. y /-kg.
and cost of Rice Rs. z /-kg.

According to question,

$4x + 3y + 2z = 60$
$2x + 4y + 6z = 90$
$6x + 2y + 3z = 70$

system can be written as

$\Rightarrow \quad Ax = B$

where,
$$A = \begin{bmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & 3 & 2 \\ 2 & 4 & 6 \\ 6 & 2 & 3 \end{vmatrix} = 4(12-12) - 3(6-36) + 2(4-24)$$
$$= 0 + 90 - 40$$

$= 50$
$\neq 0$

A^{-1} exist

Now,

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 4 & 6 \\ 2 & 3 \end{vmatrix} = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & 6 \\ 6 & 3 \end{vmatrix} = 30$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 4 \\ 6 & 2 \end{vmatrix} = -20$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 3 & 2 \\ 2 & 3 \end{vmatrix} = -5$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 4 & 2 \\ 6 & 2 \end{vmatrix} = 0$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 4 & 3 \\ 6 & 2 \end{vmatrix} = 10$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 3 & 2 \\ 4 & 6 \end{vmatrix} = 10$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 4 & 2 \\ 2 & 6 \end{vmatrix} = -20$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 4 & 3 \\ 2 & 4 \end{vmatrix} = 10$$

$$\therefore \text{adj } A = \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{50} \begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0 & -5 & 10 \\ 30 & 0 & -20 \\ -20 & 10 & 10 \end{bmatrix}\begin{bmatrix} 60 \\ 90 \\ 70 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 0-450+700 \\ 1800+0-1400 \\ -1200+900+700 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{50}\begin{bmatrix} 250 \\ 400 \\ 400 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 8 \\ 8 \end{bmatrix}$$

on comparing both matrix, then we get

$x = 5$
$y = 8$
$z = 8$

Hence the cost of onions is Rs. 5 per kg, cost of wheat is Rs. 8 per kg and cost of Rice is Rs. 8 per kg.

Page No.
Date 05 06

(8) $2x - y = -2$

$3x + 4y = 3$.

Sol. $2x - y = -2$

$3x + 4y = 3$

system can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 2 & -1 \\ 3 & 4 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -1 \\ 3 & 4 \end{vmatrix}$$

$$= 8 - 3 \times -1$$

$$= 11$$

A^{-1} exist

$A_{11} = (-1)^{1+1}(4) = 4$

$A_{12} = (-1)^{1+2}(3) = -3$

$A_{21} = (-1)^{2+1}(-1) = 1$

$A_{22} = (-1)^{2+2}(2) = 2$

$$\text{adj. } A = \begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}$$

Now,

$\Rightarrow X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} 4 & 1 \\ -3 & 2 \end{bmatrix}\begin{bmatrix} -2 \\ 3 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11}\begin{bmatrix} -8 + (3) \\ 6 + 6 \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{11} \begin{bmatrix} -5 \\ 12 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \frac{-5}{11} \\ \frac{12}{11} \end{bmatrix}$$

or comparing both matrix,

$$\Rightarrow x = \frac{-5}{11}, \quad y = \frac{12}{11}$$

(9) $4x - 3y = 3$
$3x - 5y = 7$

Sol. $4x - 3y = 3$
$3x - 5y = 7$

system can be written as

$$\Rightarrow Ax = B.$$

where

$$A = \begin{bmatrix} 4 & -3 \\ 3 & -5 \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 7 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \end{bmatrix}$$

$$|A| = \begin{vmatrix} 4 & -3 \\ 3 & -5 \end{vmatrix} = -20 + 9$$
$$= -11$$

$\therefore A^{-1}$ exist

$A_{11} = (-1)^{1+1}(-5) = -5$
$A_{12} = (-1)^{1+2}(3) = -3$
$A_{21} = (-1)^{2+1}(-3) = 3$
$A_{22} = (-1)^{2+2}(4) = 4$

$$\text{adj. } A = \begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$$

$$A^{-1} = \frac{adj.A}{|A|} = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -5 & 3 \\ -3 & 4 \end{bmatrix}\begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} -15+21 \\ -9+28 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = -\frac{1}{11}\begin{bmatrix} 6 \\ 19 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -6/11 \\ -19/11 \end{bmatrix}$$

On comparing both matrix, then we get.

$$\Rightarrow x = \frac{-6}{11}, \quad y = \frac{-19}{11}$$

(10) $5x + 2y = 3$
$3x + 2y = 5$

Sol. $5x + 2y = 3$
$3x + 2y = 5$

System can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 5 & 2 \\ 3 & 2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \end{bmatrix}, \quad B = \begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 5 & 2 \\ 3 & 2 \end{vmatrix} = 4 \neq 0$$

$\therefore A^{-1}$ exist

$A_{11} = (-1)^{1+1}(2) = 2$
$A_{12} = (-1)^{1+2}(3) = -3$
$A_{21} = (-1)^{2+1}(2) = -2$
$A_{22} = (-1)^{2+2}(5) = 5$

$$\text{adj. } A = \begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}$$

<u>Now,</u>

$$X = A^{-1}B.$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 2 & -2 \\ -3 & 5 \end{bmatrix}\begin{bmatrix} 3 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} 6+(-2\times 5) \\ -6+25 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{4}\begin{bmatrix} -4 \\ 19 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} -1 \\ -19/4 \end{bmatrix}$$

on comparing both matrix,

Page No.:
Date 03 06

$x = -1 \quad , \quad y = \frac{-19}{4}$

(11)

$2x + y + z = 1$
$x - 2y - z = \frac{3}{2}$
$3y - 5z = 9$

Sol.

$2x + y + z = 1$
$x - 2y - z = \frac{3}{2}$
$0x + 3y - 5z = 9$

system can be written as

$$Ax = B$$

where,

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{bmatrix}, \quad x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 1 & 1 \\ 1 & -2 & -1 \\ 0 & 3 & -5 \end{vmatrix}$$

Expanding w.r.t C_1

$$= 2(-2\times-5 - 3\times-1) - 1(1\times-5 - 3\times1)$$
$$= 2(13) - 1(-8)$$
$$= 26 + 8$$
$$= 34$$
$$\neq 0$$

$\therefore A^{-1}$ exist.

$\because A_{11} = (-1)^{1+1} \begin{vmatrix} -2 & -1 \\ 3 & -5 \end{vmatrix} = 5 - (-3) = 8$

$A_{12} = (-1)^{1+2} \begin{vmatrix} 1 & -1 \\ 0 & -5 \end{vmatrix} = 5$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 1 & -1 \\ 0 & 3 \end{vmatrix} = 3$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} 1 & 1 \\ 3 & -5 \end{vmatrix} = -(-5-3) = 8$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 2 & 1 \\ 0 & -5 \end{vmatrix} = -10$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 2 & 1 \\ 0 & 3 \end{vmatrix} = -6.$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix} = -1+2 = 1$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 2 & 1 \\ 1 & -1 \end{vmatrix} = -(-2-1) = 3$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 2 & 1 \\ 1 & -2 \end{vmatrix} = -5$$

$$\text{adj. } A = \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix}$$

Now,

$$x = A^{-1} B$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13 & 8 & 1 \\ 5 & -10 & 3 \\ 3 & -6 & -5 \end{bmatrix} \begin{bmatrix} 1 \\ 3/2 \\ 9 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 13+12+0 \\ 5-15+27 \\ 3-9-45 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{34} \begin{bmatrix} 34 \\ 17 \\ -51 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \cancel{\begin{bmatrix} \frac{5}{6} \\ \frac{17}{24} \\ \frac{33}{24} \end{bmatrix}} \begin{bmatrix} 1 \\ \frac{1}{2} \\ -\frac{3}{2} \end{bmatrix}$$

on comparing both matrix, then we get

$$\Rightarrow x = \cancel{\frac{5}{6}}\ 1, \quad y = \cancel{\frac{17}{24}}\ \frac{1}{2}, \quad z = \cancel{\frac{33}{24}}\ -\frac{3}{2}$$

(12) $x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 9$

Sol. $x - y + z = 4$
$2x + y - 3z = 0$
$x + y + z = 9$

System can be written as

$$AX = B$$

where

$$A = \begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{vmatrix} =$$

Expand ind w.r.t R_1,

$$= (1\times1 - 1\times(-3)) + 1(2\times1 - 1\times(-3)) + 1(2\times1 - 1\times1)$$
$$= 4 + 5 + 1$$
$$= 10$$
$$\neq 0$$

Page No.:
Date 04 06

Rough

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{bmatrix}$$

$\therefore A^{-1}$ exist.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 1 & -3 \\ 1 & 1 \end{vmatrix} = 4$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -5$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 2 & 1 \\ 1 & 1 \end{vmatrix} = 1$$

$$A_{21} = (-1)^{2+1} \begin{vmatrix} -1 & 1 \\ 1 & 1 \end{vmatrix} = 2$$

$$A_{22} = (-1)^{2+2} \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 0$$

$$A_{23} = (-1)^{2+3} \begin{vmatrix} 1 & -1 \\ 1 & 1 \end{vmatrix} = -2$$

$$A_{31} = (-1)^{3+1} \begin{vmatrix} -1 & 1 \\ 1 & -3 \end{vmatrix} = 2$$

$$A_{32} = (-1)^{3+2} \begin{vmatrix} 1 & 1 \\ 2 & -3 \end{vmatrix} = 5$$

$$A_{33} = (-1)^{3+3} \begin{vmatrix} 1 & -1 \\ 2 & 1 \end{vmatrix} = 3$$

$$\text{adj } A = \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\Rightarrow A^{-1} = \frac{1}{10} \begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}$$

Page No.:
Date 04 06

Now,

$$X = A^{-1}B,$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{bmatrix}\begin{bmatrix} 4 \\ 0 \\ 2 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 16+0+4 \\ -20+0+10 \\ 4+0+6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{10}\begin{bmatrix} 20 \\ -10 \\ 10 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 1 \end{bmatrix}$$

on comparing both matrix, then we get

$x = 2$, $y = -1$, $z = 1$ Ans.

(13) $2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y - 2z = 3$

Sol. $2x + 3y + 3z = 5$
$x - 2y + z = -4$
$3x - y + 2z = 3$

system can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 3 \\ 1 & -2 & 1 \\ 3 & -1 & -2 \end{vmatrix}$$

Expanding w.r.t C_1

$$= 2(-2\times-2-(-1)\times1) - 1(3\times-2-(-1)\times3) + 3(3\times1-1\times-2)$$
$$= 2(4+1) - 1(-6+3) + 3(3+6)$$
$$= 10 + 3 + 27$$
$$= 40$$
$$\neq 0$$

$\therefore A^{-1}$ exist.

Now,

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -2 & 1 \\ -1 & -2 \end{vmatrix} = 4+1 = 5$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 1 & 1 \\ 3 & -2 \end{vmatrix} = -(-2-3) = 5$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 1 & -2 \\ 3 & -1 \end{vmatrix} = +(-1+6) = 5$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 3 \\ -1 & -2 \end{vmatrix} = -(-6+3) = 3$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 3 \\ 3 & -2 \end{vmatrix} = \cancel{-4-6}\ -4-9 = -13$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 3 \\ 3 & -1 \end{vmatrix} = -(-2-9) = 11$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 3 \\ -2 & 1 \end{vmatrix} = 3+6 = 9$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 3 \\ 1 & 1 \end{vmatrix} = -(2-3) = 1$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 3 \\ 1 & -2 \end{vmatrix} = -4-3 = -7$$

Page No.
Date 04 05

$$\text{adj. } A = \begin{bmatrix} 5 & 3 & 19 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 19 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}$$

Now $X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 5 & 3 & 19 \\ 5 & -13 & 1 \\ 5 & 11 & -7 \end{bmatrix}\begin{bmatrix} 5 \\ -4 \\ 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 25 - 12 + 57 \\ 25 + 52 + 3 \\ 25 - 44 - 21 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{40}\begin{bmatrix} 40 \\ 80 \\ -40 \end{bmatrix}$$

$$= \begin{bmatrix} \frac{40}{40} \\ \frac{80}{40} \\ \frac{-40}{40} \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

on comparing both matrix then we get

$x = 1$, $y = 2$, $z = -1$ Ans.

15 If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, find A^{-1}. Using A^{-1} solve the system of eq^n

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

Sol. Given,

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$$

Expanding w.r.t R_1,

$$= 2(2\times-2 - (-4)\times 1) + 3(3\times -2 - (-4)(1)) + 5(3\times 1 - 1\times 2)$$
$$= 2(-4+4) + 3(-6+4) + 5(3-2)$$
$$= 0 - 6 + 5$$
$$= -1$$
$$\neq 0$$

$\therefore A^{-1}$ exist.

$$A_{11} = (-1)^{1+1} \begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = -4+4 = 0$$

$$A_{12} = (-1)^{1+2} \begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} = -(-6+4) = 2$$

$$A_{13} = (-1)^{1+3} \begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = +(3-2) = 1$$

Page No.:
Date 04 06

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = -(6-5) = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = -4-5 = -9$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = -(2+3) = -5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = +(12-10) = 2$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = -(-8-15) = +23$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = 4+9 = 13$$

$$\text{adj. } A = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj. } A}{|A|} = -1\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

<u>Now,</u>

$$2x - 3y + 5z = 11$$
$$3x + 2y - 4z = -5$$
$$x + y - 2z = -3$$

system can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}, \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad B = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

Now,

$$X = A^{-1} B.$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = -1 \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 + (-5) + (6) \\ -22 - 45 + 69 \\ -11 - 25 + 39 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

on comparing both matrix, then we get

$x = 1$, $y = 2$ and $z = 3$.

Page No.

Date | a h | a G |

Miscellaneous

(4) without expanding determinants prove that $\begin{vmatrix} a & a^2 & bc \\ b & b^2 & ca \\ c & c^2 & ab \end{vmatrix} = \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$

$\begin{vmatrix} x & \sin\theta & \cos\theta \\ \sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

Sol. multiplying R_1 by a, R_2 by b and R_3 by c

$$\text{L.H.S} = \frac{1}{abc}\begin{vmatrix} a^2 & a^3 & abc \\ b^2 & b^3 & bca \\ c^2 & c^3 & cab \end{vmatrix}$$

Taking common abc from C_3,

$$= \frac{abc}{abc}\begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

$$= \begin{vmatrix} a^2 & a^3 & 1 \\ b^2 & b^3 & 1 \\ c^2 & c^3 & 1 \end{vmatrix}$$

By process $C_1 \longleftrightarrow C_3$ and $C_2 \longleftrightarrow C_3$

$$= \begin{vmatrix} 1 & a^2 & a^3 \\ 1 & b^2 & b^3 \\ 1 & c^2 & c^3 \end{vmatrix}$$

L.H.S = R.H.S

Hence proved.

(5) Solve the eqn. $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

Sol. $\begin{vmatrix} x+a & x & x \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$

By process $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 3x+a & 3x+a & 3x+a \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

Taking common $(3x+a)$ from R_1

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 1 & 1 \\ x & x+a & x \\ x & x & x+a \end{vmatrix} = 0$$

$\Rightarrow$ By process $C_2 \to C_2 - C_1$
and $C_3 \to C_3 - C_1$

$$\Rightarrow (3x+a) \begin{vmatrix} 1 & 0 & 0 \\ x & a & 0 \\ x & 0 & a \end{vmatrix} = 0$$

Expanding w.r.t R_1

$\Rightarrow (3x+a)(1 \times a^2) = 0$

$\Rightarrow a^2(3x+a) = 0$

$\Rightarrow 3x+a = 0$

$\Rightarrow x = -\frac{a}{3}$

(8) let $A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$, verify that

i) $[\text{adj } A]^{-1} = \text{adj}(A^{-1})$

ii) $(A^{-1})^{-1} = A$

Page No.:
Date 04 06

Sol. Given,

$$A = \begin{bmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

i) $|A| = \begin{vmatrix} 1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{vmatrix}$

$|A| = 1(3\times5 - 1\times1) + 2(-2\times5 - 1\times1) + 1(-2\times1 - 3\times1)$

$|A| = 14 - 22 - 5$

$|A| = -13$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 3 & 1 \\ 1 & 5 \end{vmatrix} = 14$

$A_{12} = (-1)^{1+2}\begin{vmatrix} -2 & 1 \\ 1 & 5 \end{vmatrix} = -(-10-1) = 11$

$A_{13} = (-1)^{1+3}\begin{vmatrix} -2 & 3 \\ 1 & 1 \end{vmatrix} = -2-3 = -5$

$A_{21} = (-1)^{2+1}\begin{vmatrix} -2 & 1 \\ 1 & 5 \end{vmatrix} = 11$

$A_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 1 \\ 1 & 5 \end{vmatrix} = 4$

$A_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -2 \\ 1 & 1 \end{vmatrix} = -(3) = -3$

$A_{31} = (-1)^{3+1}\begin{vmatrix} -2 & 1 \\ 3 & 1 \end{vmatrix} = -2-3 = -5$

$A_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$

$A_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -2 \\ -2 & 3 \end{vmatrix} = 3-4 = -1$

$$\text{adj. } A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj. } A}{|A|}$$

$$\Rightarrow A^{-1} = -\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

Now,

$$\text{adj. } A = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

Let adj. $A = K$

$$\Rightarrow K = \begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow |K| = \begin{vmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{vmatrix}$$

Expanding w.r.t R_1

$$= 14(4\times-1-(-3\times-3)) - 11(11\times-1-5\times3) - 5(11\times-3-$$

$$= 14(-4-9) - 11(-11-15) - 5(-33+20)$$

$$= -182 + 286 + 65$$

$$= \cancel{461-182}$$

$$= \cancel{279}$$

$$= 169$$

$$K_{11} = (-1)^{1+1}\cancel{4} \quad (-1)^{1+1}\begin{vmatrix} 4 & -3 \\ -3 & -1 \end{vmatrix} = -13$$

121
56
65
56
121

Page No.:
Date 04 06

$\Rightarrow K_{12} = (-1)^{1+2}\begin{vmatrix} 11 & -3 \\ -5 & -1 \end{vmatrix} = -(-26) = 26.$

$\Rightarrow K_{13} = (-1)^{1+3}\begin{vmatrix} 11 & 4 \\ -5 & -3 \end{vmatrix} = (-33+20) = -13$

$\Rightarrow K_{21} = (-1)^{2+1}\begin{vmatrix} 11 & -5 \\ -3 & -1 \end{vmatrix} = -(-11 \overset{-15}{\cancel{+20}}) = 26.$

$\Rightarrow K_{22} = (-1)^{2+2}\begin{vmatrix} 14 & -5 \\ -5 & -1 \end{vmatrix} = +(-14-25) = -39$

$\Rightarrow K_{23} = (-1)^{2+3}\begin{vmatrix} 14 & 11 \\ -5 & -3 \end{vmatrix} = -(-42+55) = -13$

$\Rightarrow K_{31} = (-1)^{3+1}\begin{vmatrix} 11 & -5 \\ 4 & -3 \end{vmatrix} = -13.$

$\Rightarrow K_{32} = (-1)^{3+2}\begin{vmatrix} 14 & -5 \\ 11 & -3 \end{vmatrix} = -(-42+55) = -13.$

$\Rightarrow K_{33} = (-1)^{3+3}\begin{vmatrix} 14 & 11 \\ 11 & 4 \end{vmatrix} = +(56-121) = -65$

$$\text{adj. } K = \begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$K^{-1} = \frac{\text{adj. } K}{|K|}$$

$$= \frac{1}{169}\begin{bmatrix} -13 & 26 & -13 \\ 26 & -39 & -13 \\ -13 & -13 & -65 \end{bmatrix}$$

$$K^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$[\text{adj.}A]^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix} \longrightarrow (1)$$

Now,

$$A^{-1} = -+\frac{1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$A^{-1} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$A^{-1}_{11} = (-1)^{1+1}\begin{vmatrix} -\frac{4}{13} & \frac{3}{13} \\ \frac{3}{13} & \frac{1}{13} \end{vmatrix} = -\frac{4}{169} - \frac{9}{169} = \frac{-13}{169}$$

$$A^{-1}_{12} = (-1)^{1+2}\begin{vmatrix} -\frac{11}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{1}{13} \end{vmatrix} = \frac{-11}{169} - \frac{15}{169} = \frac{+26}{169}$$

$$A^{-1}_{13} = (-1)^{1+3}\begin{vmatrix} -\frac{11}{13} & -\frac{4}{13} \\ \frac{5}{13} & \frac{3}{13} \end{vmatrix} = \frac{-13}{169} \quad \frac{-13}{169}$$

$$A^{-1}_{21} = (-1)^{2+1}\begin{vmatrix} -\frac{11}{13} & \frac{5}{13} \\ \frac{3}{13} & \frac{1}{13} \end{vmatrix} = \frac{-11}{169} - \frac{15}{169} = +\frac{26}{169}$$

$$A^{-1}_{22} = (-1)^{2+2}\begin{vmatrix} -\frac{14}{13} & \frac{5}{13} \\ \frac{5}{13} & \frac{1}{13} \end{vmatrix} = \frac{-39}{169}$$

$$A^{-1}_{23} = (-1)^{2+3} \begin{vmatrix} \frac{-14}{13} & \frac{-11}{13} \\ \frac{5}{13} & \frac{3}{13} \end{vmatrix} = \frac{-13}{169}$$

$$A^{-1}_{31} = (-1)^{3+1} \begin{vmatrix} \frac{-11}{13} & \frac{5}{13} \\ \frac{-4}{13} & \frac{3}{13} \end{vmatrix} = \frac{-13}{169}$$

$$A^{-1}_{32} = (-1)^{3+2} \begin{vmatrix} \frac{-14}{13} & \frac{5}{13} \\ \frac{-11}{13} & \frac{3}{13} \end{vmatrix} = \frac{-13}{169}$$

$$A^{-1}_{33} = (-1)^{3+3} \begin{vmatrix} \frac{-14}{13} & \frac{-11}{13} \\ \frac{-11}{13} & \frac{-4}{13} \end{vmatrix} = \frac{-5}{169}.$$

$$\text{adj.}(A^{-1}) = \begin{bmatrix} \frac{-13}{169} & \frac{26}{169} & \frac{-13}{169} \\ \frac{26}{169} & \frac{-39}{169} & \frac{-13}{169} \\ \frac{-13}{169} & \frac{-13}{169} & \frac{-65}{169} \end{bmatrix}$$

$$\text{adj}(A^{-1}) = \begin{bmatrix} \frac{-1}{13} & \frac{2}{13} & \frac{-1}{13} \\ \frac{2}{13} & \frac{-3}{13} & \frac{-1}{13} \\ \frac{-1}{13} & \frac{-1}{13} & \frac{-5}{13} \end{bmatrix}$$

$$\text{adj}(A^{-1}) = \frac{1}{13} \begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix} \longrightarrow (2)$$

from ① & ②,

$$[\text{adj. } A]^{-1} = \text{adj}(A^{-1}).$$

ii) $(A^{-1})^{-1} = A.$

Sol. $$A^{-1} = \frac{-1}{13}\begin{bmatrix} 14 & 11 & -5 \\ 11 & 4 & -3 \\ -5 & -3 & -1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -\frac{14}{13} & -\frac{11}{13} & \frac{5}{13} \\ -\frac{11}{13} & -\frac{4}{13} & \frac{3}{13} \\ \frac{5}{13} & \frac{3}{13} & \frac{1}{13} \end{bmatrix}$$

$$\Rightarrow |A^{-1}| = \frac{-14}{13}\left(\frac{-4}{13}\times\frac{1}{13} - \frac{3}{13}\times\frac{3}{13}\right) + \frac{11}{13}\left(\frac{-11}{13}\times\frac{1}{13} - \frac{5}{13}\times\frac{3}{13}\right)$$
$$+ \frac{5}{13}\left(\frac{-11}{13}\times\frac{3}{13} - \frac{5}{13}\times\left(\frac{-4}{13}\right)\right)$$

$$\Rightarrow |A^{-1}| = \frac{-14}{13}\left(\frac{-4}{169} - \frac{9}{169}\right) + \frac{11}{13}\left(\frac{-11}{169} - \frac{15}{169}\right)$$
$$+ \frac{5}{13}\left(\frac{-33}{169} + \frac{20}{169}\right)$$

$$\Rightarrow |A^{-1}| = \frac{-14}{13}\left(\frac{-13}{169}\right) + \frac{11}{13}\left(\frac{-26}{169}\right) + \frac{5}{13}\left(\frac{-13}{169}\right)$$

$$\Rightarrow |A^{-1}| = \frac{182}{2197} + \frac{(-286)}{2197} + \frac{(-65)}{2197}$$

$$\Rightarrow |A^{-1}| = \frac{182 - 286 - 65}{2197}$$

$$\Rightarrow |A^{-1}| = \frac{-169}{2197}$$

$$\Rightarrow |A^{-1}| = \frac{-(13)^2}{(13)^3}$$

$$\Rightarrow |A^{-1}| = \frac{-1}{13}$$

$$\Rightarrow \text{adj. } A^{-1} = \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$\Rightarrow (A^{-1})^{-1} = \frac{\text{adj. } A^{-1}}{|A^{-1}|}$$

$$= \frac{1}{\left(\frac{-1}{13}\right)} \times \frac{1}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$= \frac{-13}{13}\begin{bmatrix} -1 & 2 & -1 \\ 2 & -3 & -1 \\ -1 & -1 & -5 \end{bmatrix}$$

$$= \begin{bmatrix} +1 & -2 & 1 \\ -2 & 3 & 1 \\ 1 & 1 & 5 \end{bmatrix}$$

$$(A^{-1})^{-1} = A$$

Verified.

Page No.:
Date 04 06

① Prove that determinant $\begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$ is independent of θ

Sol.

$$= \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$$

Expanding w.r.t R_1

$$= x(-x\times x - 1\times 1) - \sin\theta(-\sin\theta\times x - \cos\theta\times 1) + \cos\theta(-\sin\theta\cdot 1 - \cos\theta\cdot x)$$

$$= x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$$

$$= -x^3 - x + x\sin^2\theta + \cancel{\cos\theta\sin\theta} - \cancel{\sin\theta\cos\theta} + x\cos^2\theta$$

$$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$$

$$= -x^3 \cancel{-x} + \cancel{x}$$

$$= -x^3$$

It is independent of θ.

② Evaluate $\begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$

Sol: Let,

$$\Delta = \begin{vmatrix} \cos\alpha\cos\beta & \cos\alpha\sin\beta & -\sin\alpha \\ -\sin\beta & \cos\beta & 0 \\ \sin\alpha\cos\beta & \sin\alpha\sin\beta & \cos\alpha \end{vmatrix}$$

Expanding w.r.t C_1,

$$\Delta = \cos\alpha\cos\beta(\cos\beta\cos\alpha - 0) + \sin\beta(\cos\alpha\sin\beta\times\cos\alpha - \sin\alpha\sin\beta\times -\sin\alpha) + \sin\alpha\cos\beta(\cos\alpha\sin\beta\times 0 - \cos\beta(-\sin\alpha$$

Page No.:
Date 05 06

$\Rightarrow \Delta = \cos^2\alpha \cos^2\beta + \sin\beta(\sin\beta\cos^2\alpha + \sin^2\alpha\sin\beta) + \sin\alpha\cos\beta(\cos\beta\sin\alpha)$

$\Rightarrow \Delta = \cos^2\alpha\cos^2\beta + \sin^2\beta\cos^2\alpha + \sin^2\alpha\sin^2\beta + \sin^2\alpha\cos^2\beta$

$\Rightarrow \Delta = \cos^2\alpha(\cos^2\beta + \sin^2\alpha) + \sin^2\alpha(\sin^2\beta + \cos^2\beta)$

$\Rightarrow \Delta = \cos^2\alpha(1) + \sin^2\alpha(1)$

$\Rightarrow \Delta = \cos^2\alpha + \sin^2\alpha$

$\Rightarrow \Delta = 1$ Ans.

Date 05/06

(15) Prove that

$$\begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix} = 0$$

Sol. let

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos(\alpha+\delta) \\ \sin\beta & \cos\beta & \cos(\beta+\delta) \\ \sin\gamma & \cos\gamma & \cos(\gamma+\delta) \end{vmatrix}.$$

$$\Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta \\ \sin\beta & \cos\beta & \cos\beta\cos\delta - \sin\beta\sin\delta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta - \sin\alpha\sin\delta \end{vmatrix}$$

By process $C_3 \to C_3 + \sin\delta\, C_1 - \cos\delta\, C_2$

$$\Rightarrow \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & \cos\alpha\cos\delta - \sin\alpha\sin\delta + \sin\delta\sin\alpha - \cos\delta\cos\alpha \\ \sin\beta & \cos\beta & \cos\beta\cos\delta - \sin\beta\sin\delta + \sin\delta\sin\beta - \cos\delta\cos\beta \\ \sin\gamma & \cos\gamma & \cos\gamma\cos\delta - \sin\alpha\sin\delta + \sin\gamma\sin\delta - \cos\delta\cos\gamma \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} \sin\alpha & \cos\alpha & 0 \\ \sin\beta & \cos\beta & 0 \\ \sin\gamma & \cos\gamma & 0 \end{vmatrix}$$

$\Rightarrow \Delta = 0$

Proved.

(16) Solve the system of eq^n

$$\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$$

$$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$$

$$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$$

Sol. $\frac{2}{x} + \frac{3}{y} + \frac{10}{z} = 4$

$\frac{4}{x} - \frac{6}{y} + \frac{5}{z} = 1$

$\frac{6}{x} + \frac{9}{y} - \frac{20}{z} = 2$

let, $\frac{1}{x} = a$, $\frac{1}{y} = b$, $\frac{1}{z} = c$

$\Rightarrow$ $2a + 3b + 10c = 4 \rightarrow (1)$

$4a - 6b + 5c = 1 \rightarrow (2)$

$6a + 9b - 20c = 2 \rightarrow (3)$

system can be written as

$$AX = B$$

where,

$$A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, \quad X = \begin{bmatrix} a \\ b \\ c \end{bmatrix}, \quad B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$|A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

Expanding w.r.t to R_1,

$$= 2(-6\times-20-45) - 3(-80-30) + 10(36+36)$$
$$= 2(120-45) - 3(-110) + 720$$
$$= 150 + 330 + 720$$
$$= 1050 + 150$$
$$= 1200$$

$A_{11} = (-1)^{1+1}(-6\times-20 - 9\times5) = 75$

$A_{12} = (-1)^{1+2}(-80-30) = 110$

$A_{13} = (-1)^{1+3}(4\times9+36) = 72$

$A_{21} = (-1)^{2+1}(3\times-20-90) = 150$

$A_{22} = (-1)^{2+2}(2\times-20-60) = -100$

$A_{23} = (-1)^{2+3}(2\times9-6\times3) = 0$

$A_{31} = (-1)^{3+1}(3\times-20-9\times10) = 75$

$A_{32} = (-1)^{3+2}(2\times5-4\times10) = 30$

$A_{33} = (-1)^{3+3}(2\times-6-6\times3) = -24$

$$\text{adj. } A = \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

Now,

$$X = A^{-1}B$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -110 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

on comparing both matrix,

$\Rightarrow a = \frac{1}{2}$	$b = \frac{1}{3}$	$c = \frac{1}{5}$
$\Rightarrow \frac{1}{x} = \frac{1}{2}$	$\frac{1}{y} = \frac{1}{3}$	$\frac{1}{z} = \frac{1}{5}$
$\Rightarrow \boxed{x = 2}$	$y = 3$	$z = 5$

(17) If a, b, c are in A.P, then the determinant

$$\begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

Page No.:
Date 05 06 21

Sol: a, b, c are in A.P.

$\therefore \; b - a = c - b$

$\Rightarrow \; 2b = c + a$

$\Rightarrow \; b = \frac{a+c}{2} \longrightarrow (1)$

$$\Rightarrow \Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+2b \\ x+4 & x+5 & x+2c \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x+2 & x+3 & x+2a \\ x+3 & x+4 & x+(a+c) \\ x+4 & x+5 & x+2c \end{vmatrix} \quad \{\because \text{from } (1)\}$$

By process $R_1 \to R_1 - R_2$

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+a+c \\ x+4 & x+5 & x+2c \end{vmatrix}$$

By process $R_3 \to R_3 - R_2$

$$\Delta = \begin{vmatrix} -1 & -1 & a-c \\ x+3 & x+4 & x+a+c \\ 1 & 1 & c-a \end{vmatrix}$$

By process $R_1 \to R_1 + R_3$

$$\Delta = \begin{vmatrix} 0 & 0 & 0 \\ x+3 & x+4 & x+a+c \\ 1 & 1 & c-a \end{vmatrix}$$

$\Delta = 0$ Ans.

option A > 0.

Page No.:
Date 06 06

(19) $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$, where $0 \leq \theta \leq 2\pi$. Then

Sol

$\Rightarrow |A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$

Expanding w.r.t R_1

$\Rightarrow |A| = (1+\sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$\Rightarrow |A| = 1 + \sin^2\theta + \sin^2\theta + 1$

$\Rightarrow |A| = 2(1 + \sin^2\theta)$

put $\theta = 0$

$|A| = 2(1 + \sin^2 0^\circ)$
$= 2$

put $\theta = 2\pi$

$|A| = 2(1 + \sin^2 2\pi)$
$|A| = 2(1 + \sin^2(\pi + \pi))$
$= 2(1 + \sin^2\pi) = 2(1 + \sin^2(\frac{\pi}{2} + \frac{\pi}{2}))$
$= 2(1 + \cos^2\frac{\pi}{2})$
$= 2(1 + 1)$
$|A| = 4$

therefore
$0 \leq \theta \leq 4$

Ans. → option (D) $|A| \in [2, 4]$

(18) $A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$ then Inverse of A,

Sol.

$$A = \begin{bmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{bmatrix}$$

$$|A| = \begin{vmatrix} x & 0 & 0 \\ 0 & y & 0 \\ 0 & 0 & z \end{vmatrix}$$

Expanding w.r.t R_1

$|A| = xyz$

$\therefore A^{-1}$ exist.

$A_{11} = (-1)^{1+1} (yz - 0) = yz$

$A_{12} = 0$

$A_{13} = 0$

$A_{21} = 0$

$A_{22} = (-1)^{2+2} (xz) = xz = xz$

$A_{23} = 0$

$A_{31} = 0$

$A_{32} = 0$

$A_{33} = (-1)^{3+3} (xy) = xy$

$$\text{adj. } A = \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj. } A}{|A|} = \frac{1}{xyz} \begin{bmatrix} yz & 0 & 0 \\ 0 & xz & 0 \\ 0 & 0 & xy \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} \frac{1}{x} & 0 & 0 \\ 0 & \frac{1}{y} & 0 \\ 0 & 0 & \frac{1}{z} \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} x^{-1} & 0 & 0 \\ 0 & y^{-1} & 0 \\ 0 & 0 & z^{-1} \end{bmatrix}$$ Ans.

Option → (A)

(4) If a, b, c are real no.,

$$\Delta = \begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} =$$

Show that either $a+b+c=0$ or $a=b=c$.

Sol. Given,

$$\begin{vmatrix} b+c & c+a & a+b \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

By process $R_1 \to R_1 + R_2 + R_3$

$$\Rightarrow \begin{vmatrix} 2(a+b+c) & 2(a+b+c) & 2(a+b+c) \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = 0$$

Taking common $2(a+b+c)$ from R_1

$$\Rightarrow 2(a+b+c) \begin{vmatrix} 1 & 1 & 1 \\ c+a & a+b & b+c \\ a+b & b+c & c+a \end{vmatrix} = \begin{vmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{vmatrix}$$

By process $C_2 \to C_2 - C_1$
and $C_3 \to C_3 - C_1$

$$\Rightarrow 2(a+b+c)\begin{vmatrix} 1 & 0 & 0 \\ c+a & b-c & a-b \\ a+b & c-a & c-b \end{vmatrix} = 0$$

Expanding w.r.t R_1

$$\Rightarrow 2(a+b+c)\ 1\{(b-c)(c-b) - (c-a)(a-b)\} = 0$$

$$\Rightarrow 2(a+b+c) = 0$$

$$\Rightarrow a+b+c = 0$$

<u>Verified</u>

(6) Prove that $\begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix} = 4a^2b^2c^2$

<u>Sol.</u>

<u>L.H.S</u> $= \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Taking common a from C_1, b from C_2 and c from C_3

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

By process $R_2 \to R_2 - R_1$

$$= abc\begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ b & b+c & c \end{vmatrix}$$

By process $R_3 \to R_3 + R_2$

$$= abc\begin{vmatrix} a & c & a+c \\ b & b-c & -c \\ 0 & -2c & 2c \end{vmatrix}$$

Rough

$b-c \to b-c$

$-2c$

By process $C_3 \to C_3 + C_2$

$$= abc \begin{vmatrix} a-c & a & c \\ c & b & b-c \\ 0 & 0 & \end{vmatrix}$$

By process $C_2 \to C_2 + C_3$

$$= abc \begin{vmatrix} a & 2c+a & a+c \\ b & b-2c & -c \\ 0 & 0 & 2c \end{vmatrix}$$

Expanding w.r.t R_3,

$$= abc \cdot \{ 2c((a)(b-2c) - b(2c+a)) \}$$

$$= abc \{ 2c(ab - 2ac - 2bc - 2ab) \}$$

$$= abc \{ 2c(-ab - 2ac - 2bc) \}$$

$$= abc \{ -2abc - 4ac^2 - 4bc^2 \}$$

$$= -2a^2b^2c^2 - 4a^2c^3b - 4ab^2c^3$$

Expanding w.r.t C_1,

$$= abc \{ a((b-2c)2c - 0) - b((2c+a)2c - 0) \}$$

$$= abc \{ a(2bc - 4c^2 - \cancel{2bc}\, b(4c + 2a) \}$$

$$= abc \{ 2abc - 4ac^2 - 4bc - 2ab \}$$

$$= 2a^2b^2c^2 - 4a^2bc^3 - 4ab^2c^2 - 2a^2b^2c$$

Expanding w.r.t C_1

$$= abc\left[\{a((b-c)2c) - b(c(2c))\}\right]$$

$$= abc\left[a(2bc-2c^2) - b(2c^2)\right]$$

$$= abc\left[2abc - 2ac^2 - bc^2\right]$$

$$= 2a^2b^2c^2 - 2a^2bc^3 - 2b^2c^3a$$

Sol. 6

L.H.S $= \begin{vmatrix} a^2 & bc & ac+c^2 \\ a^2+ab & b^2 & ac \\ ab & b^2+bc & c^2 \end{vmatrix}$

Taking common a from C_1, b from C_2 and c from C_3.

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b & b+c & c \end{vmatrix}$$

By process $R_3 \to R_3 - R_1$

$$= abc\begin{vmatrix} a & c & a+c \\ a+b & b & a \\ b-a & b & -a \end{vmatrix}$$

By process $C_1 \to C_1 - C_3$

$$= abc\begin{vmatrix} -c & c & a+c \\ b & b & a \\ b & b & -a \end{vmatrix}$$

By process $C_1 \to C_1 - C_2$

$$= abc\begin{vmatrix} -2c & c & a+c \\ 0 & b & a \\ 0 & b & -a \end{vmatrix}$$

Expanding with respect to C_1,

$$= abc\{-2c(b.-a - ba)\}$$

$= abc\{-2c(-2ab)\}$

$= abc\{4abc\}$

$= 4a^2b^2c^2$

Hence Proved!

(Q) If, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$, find (AB)

Sol.

$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

$\Rightarrow |A^{-1}| = \begin{vmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{vmatrix}$

By process $C_2 \to C_2 + C_3$

$\Rightarrow |A| = \begin{vmatrix} 3 & 0 & 1 \\ -15 & 1 & -5 \\ 5 & 0 & 2 \end{vmatrix}$

Expanding w.r.t C_2,

$|A| = -1(3\times2 - 5\times1)$

$|A| = -1(6-5)$

$|A| = -1$

$\therefore (A^{-1})^{-1}$ exist.

$A^{-1}_{11} = (-1)^{1+1}(12-10) = 2$ | $A^{-1}_{13} = (-1)^{1+3}(30-30) = 0$

$A^{-1}_{12} = (-1)^{1+2}(-30+25) = 5$ | $A^{-1}_{21} = (-1)^{2+1}(-2+2) = 0$

Page No.:
Date

$\Rightarrow A^{-1}_{22} = (-1)^{2+2}(6-5) = -1$

$\Rightarrow A^{-1}_{23} = (-1)^{2+3}(-6+5) = 1$

$\Rightarrow A^{-1}_{31} = (-1)^{3+1}(5-6) = -1$

$\Rightarrow A^{-1}_{32} = (-1)^{3+2}(-15+15) = 0$

$\Rightarrow A^{-1}_{33} = (-1)^{3+3}(18-15) = 3$

$$\Rightarrow \text{adj. } A^{-1} = \begin{bmatrix} 2 & 0 & -1 \\ 5 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow (A^{-1})^{-1} = \frac{\text{adj. } A}{|A|}$$

$$\Rightarrow (A^{-1})^{-1} = -1 \begin{bmatrix} 2 & 0 & -1 \\ 5 & -1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 1 & 0 \\ 0 & -1 & -3 \end{bmatrix}$$

and $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$.

Now,

$$(AB) = \begin{bmatrix} -2 & 0 & 1 \\ -5 & 1 & 0 \\ 0 & -1 & -3 \end{bmatrix} \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} -2+0+0 & -4+0+(-2) & 4+0+1 \\ -5-1+0 & -10+3+0 & 10+0+0 \\ 0+1+0 & 0-3+6 & 0+0-3 \end{bmatrix}$$

(9) Evaluate $\begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$

Sol. Let,

$$\Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ x+y & x & y \end{vmatrix}$$

By process $R_3 \to R_3 + R_2 + R_1$

$$\Rightarrow \Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ 2x+2y & 2x+2y & 2x+2y \end{vmatrix}$$

$$\Rightarrow \Delta = \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ 2(x+y) & 2(x+y) & 2(x+y) \end{vmatrix}$$

Taking common $2(x+y)$ from R_3,

$$\Rightarrow \Delta = 2(x+y) \begin{vmatrix} x & y & x+y \\ y & x+y & x \\ 1 & 1 & 1 \end{vmatrix}$$

~~$\Rightarrow \Delta =$~~ Expanding w.r.t C_1,

$$\Rightarrow \Delta = 2(x+y) \left\{ x((x+y) - x) - y(y - (x+y)) + 1(yx - (x+y)^2) \right\}$$

$$\Rightarrow \Delta = 2(x+y) \left\{ x^2 + yx - x - y^2 - xy + y^2 + yx - (x^2 + y^2 + 2xy) \right\}$$

$$\Rightarrow \Delta = 2(x+y) \left\{ x^2 + xy - x - xy + xy - x^2 - y^2 - 2xy \right\}$$

$$\Rightarrow \Delta = 2(x+y) \left\{ -x + xy - y^2 - 2xy \right\}$$

$$\Rightarrow \Delta = (2x+2y)(xy - y^2 - x - 2xy)$$

$$\Rightarrow \Delta = 2x^2y - 2xy^2 - 2x^2 - 4x^2y + 2xy^2 - 2y^3 - 2xy - 4xy^2$$

$$\Rightarrow \Delta = -2x^2y - 2x^2 - 2y^3 - 4x^2y - 4xy^2 - 2xy$$

$$\Rightarrow \Delta = -2y^3 - 2x^2 - 2x^2y - 4xy^2 - 2xy$$

$$\Rightarrow \Delta = (2x+2y) \left\{ x^2 + xy - x - y(y - x - y) + (yx - (x^2 + y^2 + 2xy)) \right\}$$

$$\Rightarrow \Delta = (2x+2y) \left\{ x^2 + xy + xy - x + xy - x^2 - y^2 - 2xy \right\}$$

$\Rightarrow \Delta = (2x+2y)\{xy - x - y^2\}$

$\Rightarrow \Delta = 2x^2y - 2x^2 - \cancel{2xy^2} + \cancel{2xy^2} - 2xy - 2y^3$

$\Rightarrow \Delta = 2\{x^2y - x^2 \cancel{- 2xy^2 + xy^2} - xy - y^3\}$

$\Rightarrow \Delta = 2\{x^2y - x^2 - xy - y^3\}$

$\Delta = -2\{x^2 - x^2y + xy + y^3\}$

By process $C_2 \to C_2 - C_1$

and $C_3 \to C_3 - C_1$

$$\Rightarrow \Delta = \begin{vmatrix} x & y-x & y \\ y & x & x-y \\ 1 & 0 & 0 \end{vmatrix} 2(x+y)$$

Expanding w.r.t R_3

$$\Rightarrow \Delta = \left[1\{(y-x)(x-y) - x.y\}\right] 2(x+y)$$

$$\Rightarrow \Delta = 2(x+y)\left[\{-(x-y)^2 - x.y\}\right]$$

$\Rightarrow \Delta = 2(x+y)\{-(x^2+y^2-2xy) - xy\}$

$\Rightarrow \Delta = -2(x+y)\{\cancel{x^3y}\ x^2+y^2-2xy-xy\}$

$\Rightarrow \Delta = -2(x+y)\{x^2+y^2-3xy\}$

$\Rightarrow \Delta = -2\left[(x+y)(x^2+y^2-3xy)\right]$

$\Rightarrow \Delta = -2\left[x^3+xy^2-3x^2y+x^2y+y^3-3xy^2\right]$

$\Rightarrow \Delta = -2\left[x^3 - 3x^2y - 3xy^2 + y^3 + xy^2 + x^2y\right]$