Chapter - 2.

Relations And Functions

* order Pair — An order pair is a pair of objects. The order in which the objects appear in the pair is significant. The ordered pair is different from the ordered pair . ordered pairs are also called 2-tuples or sequences of length 2.

Notes — When 2 order pair are equal then

$(x, y) = (a, b)$

Example — $A = \{1, 3, 5\}$
$B = \{2, 4\}$

Sol. $A \times B = \{(1,2)\ (1,4), (3,2), (3,4), (5,2)\ (5,4)\}$

$B \times A = \{(2,1), (2,3), (2,5), (4,1), (4,3), (4,5)\}$

* Order Pair Remarks :-

i) Two ordered pairs are equal, if and only if the corresponding first elements are equal and the second elements are also equal.

ii) If there are p elements in A and q elements in B, then there will be pq elements in $A \times B$, i.e., if $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$.

iii) If A and B are non-empty sets and either A or B is an infinite set, then so is $A \times B$.

iv) $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$. Here (a, b, c) is called an ordered triplet.

Exercise 2.1.

Que.1. If $\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$, find the values of x and y.

Sol. Given,

$$\left(\frac{x}{3} + 1, y - \frac{2}{3}\right) = \left(\frac{5}{3}, \frac{1}{3}\right)$$

We know that,

The ordered pairs are equal, the corresponding elements will also be equal.

So,

$\Rightarrow \frac{x}{3} + 1 = \frac{5}{3}$, $\Rightarrow y - \frac{2}{3} = \frac{1}{3}$

$\Rightarrow \frac{x}{3} = \frac{5-3}{3}$, $\Rightarrow y = 1$

$\Rightarrow x = 2$, $\Rightarrow y = 1$

$\therefore x = 2$ and $y = 1$

Que. 2. If the set A has 3 elements and the set B = {3, 4, 5}, then find the no. of elements in (A × B)

Sol. Given,

$$n(A) = 3.$$
$$n(B) = 3$$

$$n(A \times B) = n(A) \times n(B)$$
$$= 3 \times 3$$
$$n(A \times B) = 9$$

Hence, the no. of elements in (A×B) is 9.

Que. 3 If G = {7, 8} and H = {5, 4, 2}; find G × H and H × G.

Sol. Given,

$$G = \{7, 8\}$$
$$H = \{5, 4, 2\}$$

According to question,

i) $G \times H = \{(7,5), (7,4), (7,2), (8,5), (8,4), (8,2)\}$

ii) $H \times G = \{(5,7), (5,8), (4,7), (4,8), (2,7), (2,8)\}$

Que. 4. State wheather each of the following statements ——— statement correctly.

i) If $P = \{m, n\}$ and $Q = \{n, m\}$, then $P \times Q = \{(m,n), (n,m)\}$

Sol. False

Given,

$$P = \{m, n\}, \quad Q = \{n, m\}$$

$$P \times Q = \{(m,n), (m,m), (n,n), (n,m)\}$$

ii) If A and B are non-empty sets, then $A \times B$ is a non-empty set of ordered pairs (x, y) such that $x \in A$ and $y \in B$.

Sol. True.

iii) If $A = \{1, 2\}$, $B = \{3, 4\}$, then $A \times (B \cap \phi) = \phi$.

Sol. True.

Que. 5. If $A = \{-1, 1\}$, ~~B =~~ find $A \times A \times A$.

Sol. We know that,

for any non-empty set A, $A \times A \times A$ is defined as $A \times A \times A = \{(a, b, c) : a, b, c \in A\}$

Given,

$$A = \{-1, 1\}$$

$\therefore A \times A \times A = \{(-1,-1,-1), (-1,-1,+1), (-1,1,-1), (-1,1,1), (1,-1,-1), (1,-1,1), (1,1,-1), (1,1,1)\}$

~~Que. 6. If $A \times B = \{(a,x)$~~

Que. 6. If $A \times B = \{(a,x), (a,y), (b,y)\}$. find A and B.

Sol. Given,

$\Rightarrow A \times B = \{(a,x), (a,y), (b,y)\}$

We know ~~that~~, The Cartesian product of two non - empty sets P and Q is defined as $P \times Q = \{(p,q) : p \in P, q \in Q\}$

$\therefore$ A is the set of all first elements and B is the set of all second elements.

Thus, $A = \{a, b\}$ and $B = \{x, y\}$

Que. 8. let $A = \{1, 2\}$ and $B = \{3, 4\}$. Write $A \times B$. How many subset will $A \times B$ have? List them.

Sol. Given,

$A = \{1,2\}$

$B = \{3,4\}$

According to question,

$\Rightarrow A \times B = \{(1,3), (1,4), (2,3), (2,4)\}$

$\Rightarrow n(A \times B) = 4$

We know that,

$\Rightarrow$ no. of subset of ~~elem~~ set $(A \times B) = 2^n$

$\Rightarrow$ no. of subset of set $(A \times B) = 2^4$

$= 16$ subset.

* subsets of $A \times B \Rightarrow \phi, \{(1,3)\}, \{(1,4)\}, \{(2,3)\}, \{(2,4)\}, \cancel{\{(1,3)\}} \{(1,3),(1,4)\}, \{(1,3),(2,3)\}, \{(1,3),(2,4)\}, \{(1,4),(2,3)\}, \{(1,4),(2,4)\}, \{(2,3),(2,4)\}, \{(1,3),(1,4),(2,3)\}, \{(1,3),(1,4),(2,4)\}, \{(1,3),(2,3),(2,4)\}, \{(1,4),(2,3),(2,4)\}, \{(1,3),(1,4),(2,3),(2,4)\}$

Que.9. Let A and B be ~~two~~ two sets such that ______ distinct elements.

Sol. Given,

$n(A) = 3$

$n(B) = 2.$

* $A \times B$ have $(x,1), (y,2), (z,1)$ elements

We know that,

A = set of first elements of the ordered pair elements of $A \times B$.

B = set of second elements of the ordered pair elements of $A \times B$.

$\therefore$ x, y and z are the elements of A; and 1 and 2 are the elements of B.

Since,

$n(A) = 3$

$n(B) = 2$

it is clear that $A = \{x, y, z\}$

and $B = \{1, 2\}$

Que. 7 Let $A = \{1, 2\}$, $B = \{1, 2, 3, 4\}$ ______ verify that :-

i) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

Sol. To verify :- $A \times (B \cap C) = (A \times B) \cap (A \times C)$

We have,

$\Rightarrow \quad B \cap C = \{1, 2, 3, 4\} \cap \{5, 6\}$

$= \phi$

$\therefore$ LHS $= A \times (B \cap C) = A \times \phi$

$= \phi$

$\Rightarrow A \times B = \{(1,1), (1,2), (1,3), (1,4), (2,1), (2,2), (2,3), (2,4)\}$

$\Rightarrow A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$\therefore$ R.H.S $= (A \times B) \cap (A \times C) = \phi$

$\therefore$ L.H.S = R.H.S

Hence, $A \times (B \cap C) = (A \times B) \cap (A \times C)$

ii) $A \times C$ is a subset of $B \times D$.

Sol. Given,

$$A = \{1, 2\}$$
$$B = \{1, 2, 3, 4\}$$
$$C = \{5, 6\}$$
$$D = \{5, 6, 7, 8\}$$

$\Rightarrow A \times C = \{(1,5), (1,6), (2,5), (2,6)\}$

$\Rightarrow B \times D = \{(1,5), (1,6), (1,7), (1,8), (2,5), (2,6), (2,7), (2,8), (3,5), (3,6), (3,7), (3,8), (4,5), (4,6), (4,7), (4,8)\}$

Here all elements of $A \times C$ is present in $B \times D$.
Hence, $A \times C$ is a subset of $B \times D$.

Que.10 The Cartesian Product ______ ______ elements of $A \times A$.

Sol. We know that,

$n(A) = p$ and $n(B) = q$
then $n(A \times B) = pq$

Given,

$n(A \times A) = 9$

$\Rightarrow n(A) = 3$

The ordered pairs $(-1, 0)$ and $(0, 1)$ are two of 9 elements of $A \times A$.

We know that,

$\Rightarrow \quad A \times A = \{(a,a) : a \in A\}$

Therefore,

-1, 0 and 1 are elements of A.

since $n(A) = 3$, it is clear that

$$A = \{-1, 0, 1\}$$

The remaining elements of set $A \times A$ are $(-1,-1)$, $(-1,1)$, $(0,-1)$, $(0,0)$, $(1,-1)$, $(1,0)$ and $(1,1)$.

25/08/20

Relations

* A relation ~~from~~ R from non-empty set A to a non-empty set B is a subset of the Cartesian product $A \times B$. The subset is derived by describing a relationship between the first element and the second element of the ordered pairs in $A \times B$. The second element is called the image of the first element.

Eg.

$$2x + 3y = 6$$

$$y = \frac{6 - 2x}{3}$$

put $x = 0$

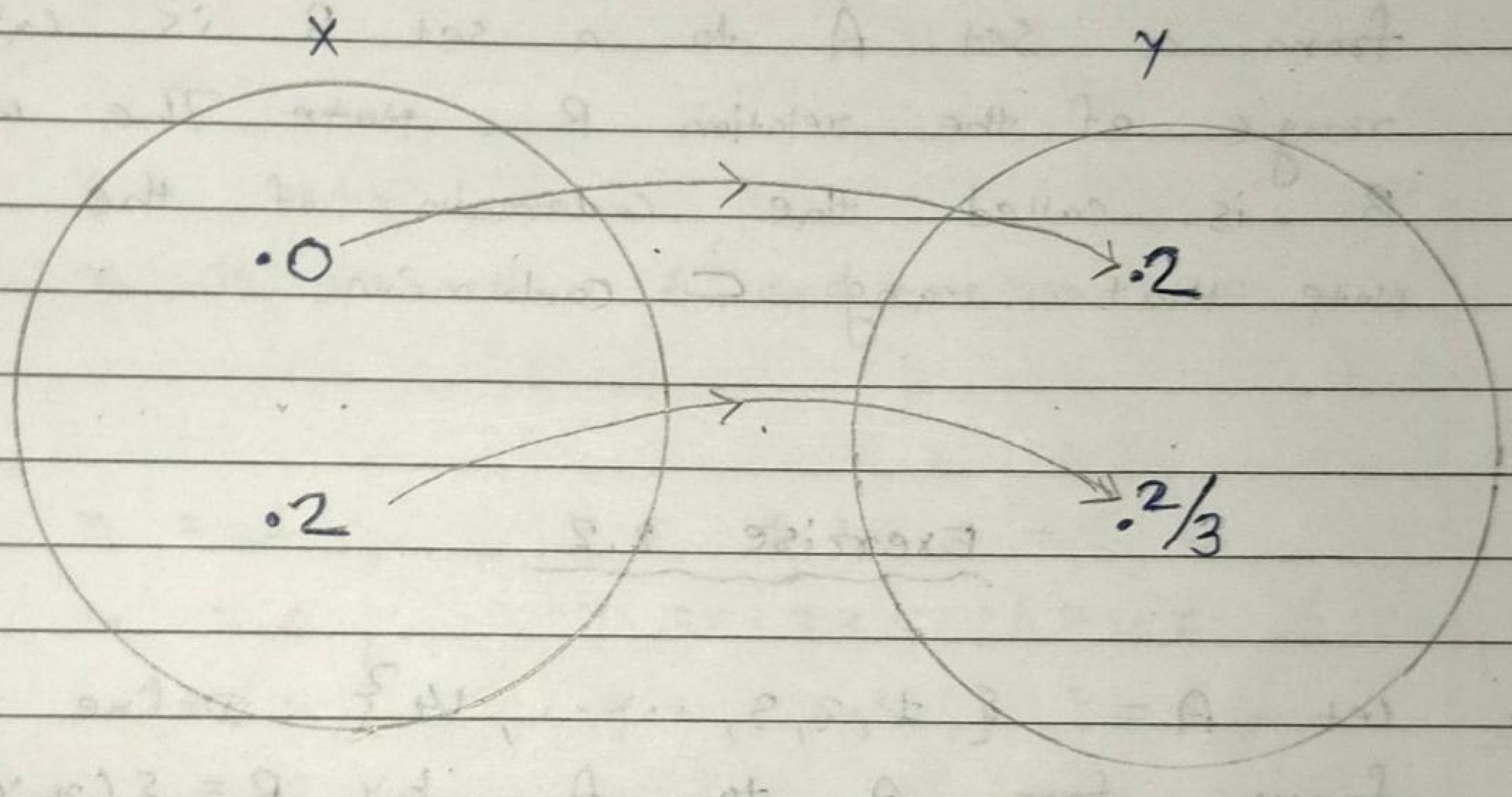
$$y = \frac{6 - 2(0)}{3}$$

$$\boxed{y = 2}$$

Put $x = 2$,

$$y = \frac{6 - 2(2)}{3}$$

$$\boxed{y = \frac{2}{3}}$$



<u>Note</u>:- The total no. of relations that can be defined from a set A to a set B is the no. of possible subsets of $A \times B$. If $n(A) = p$ and $n(B) = q$, then $n(A \times B) = pq$ and the total no. of relations is 2^{pq}.

* The set of all first elements of the ordered pairs in a relation R from a set A to a set B is called the domain of the relation R.

<u>Eg.</u> $y = x + 5$

<u>Put</u>
$x = 1$, $y = 6$
$x = 2$, $y = 7$
$x = 3$, $y = 8$

∞, $\frac{1}{0}$, $\frac{0}{0}$,
$\sqrt{-ve}$
<u>undefined.</u>

$x = 0$, $y = 5$
$x = -1$, $y = 4$

Value of x is called domain.

Domain $= \{1, 2, 3, 0, -1\}$

* The set of all second elements in a relation R from a set A to a set B is called the range of the relation R. ~~Note~~ The whole set B is called the codomain of the relation R.

Note that rang $\subset$ codomain.

Exercise 2.2

Q.1. Let $A = \{1, 2, 3, \ldots, 14\}$. Define a relation R from ~~for~~ A to A by $R = \{(x, y) : 3x - y = 0,$ where $x, y \in A\}$, Write down its domain, codomain and range.

Sol. $A = \{1, 2, 3, \ldots, 14\}$
$R = \{(x, y) : 3x - y = 0;\ \forall\ x, y \in A\}$

Here,
$\Rightarrow 3x - y = 0$
$\Rightarrow y = 3x$

when,
$x = 1$, $y = 3(1) = 3$
$x = 2$, $y = 3(2) = 6$
$x = 3$, $y = 3(3) = 9$
$x = 4$, $y = 3(4) = 12$

Domain $= \{1, 2, 3, 4\}$
Range $= \{3, 6, 9, 12\}$
Codomain $= \{1, 2, 3, \ldots, 14\}$

Q.2.

Sol. Given,

$$R = \{(x, y) : y = x + 5;\ x \text{ is a natural no. less than } 4;\ x, y \in N\}$$

Here,

$y = x + 5$

and x is less than 4 and natural no.,

when,

$x = 1, \quad y = 1 + 5 = 6$

$x = 2, \quad y = 2 + 5 = 7$

$x = 3, \quad y = 3 + 5 = 8$

$\therefore R = \{(1, 6), (2, 7), (3, 8)\}$

Domain $= \{1, 2, 3\}$

Range $= \{6, 7, 8\}$

Q.3.

Sol. Given,

$A = \{1, 2, 3, 4, 5\}$

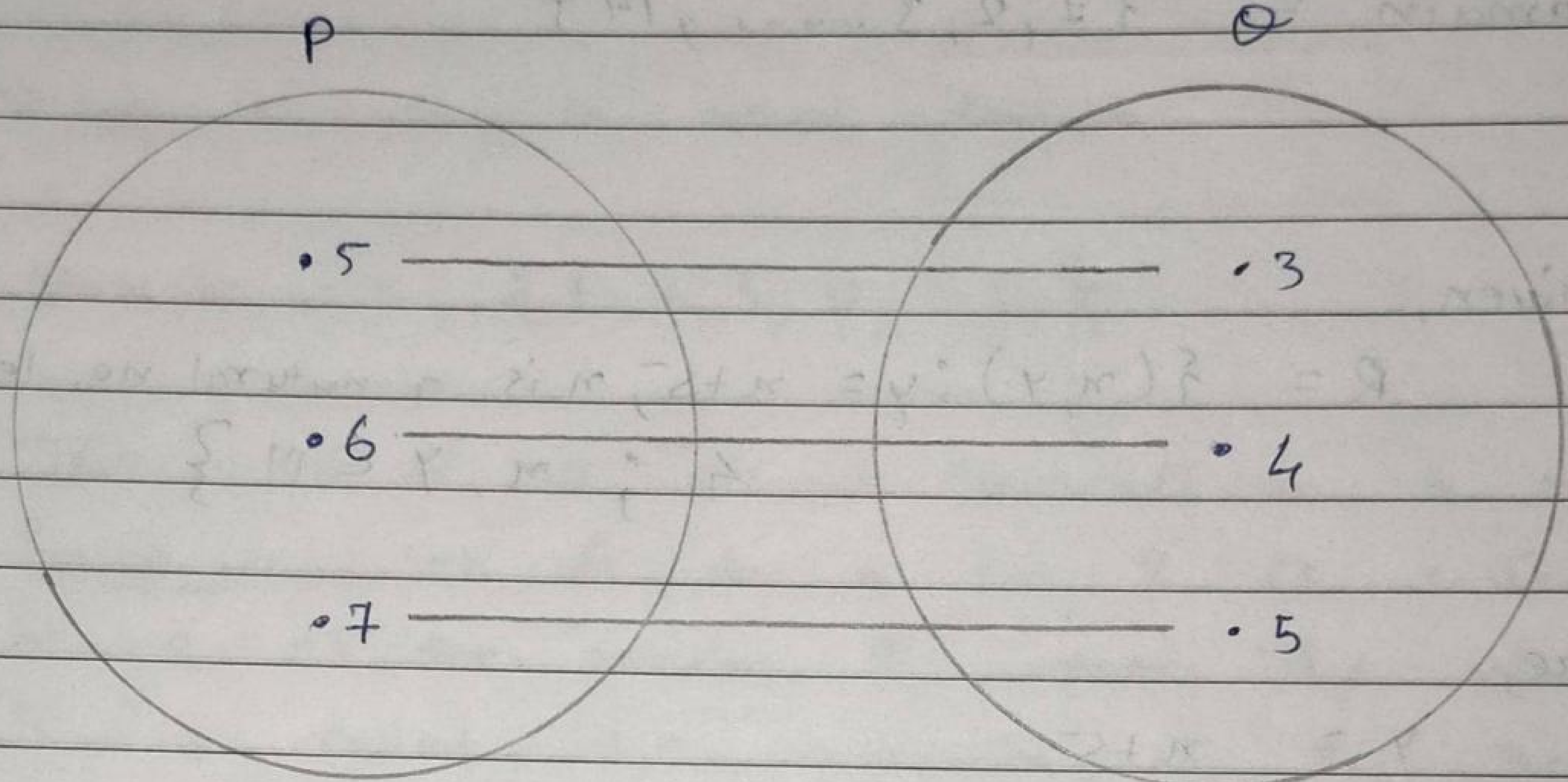
$B = \{4, 6, 9\}$

and $R = \{(x, y) :$ the difference between x and y is odd; $x \in A, y \in B\}$

$\therefore R = \{(1, 4), (1, 6), (2, 9), (3, 4), (3, 6), (5, 4), (5, 6)\}$

Q.4. ~~Giv~~

Sol. Given,



$P = \{5, 6, 7\}$

$Q = \{3, 4, 5\}$

Sol. i) $R = \{(x, y) : y = x - 2;\ x \in P\}$

or

$R = \{(x, y) : y = x - 2$ for $x = 5, 6, 7\}$

Sol. ii) $R = \{(5,3), (6,4), (7,5)\}$

Here,

Domain of $R = \{5, 6, 7\}$

~~Rage~~ Range of $R = \{3, 4, 5\}$

Q.5.

Sol. Given,

$A = \{1, 2, 3, 4, 6\}$

$R = \{(a, b) : a, b \in A$, b is exactly divisible by $a\}$

Sol. i) $R = \{(1,1), (1,2), (1,3), (1,4), (1,6), (2,2), (2,4), (2,6),$
$(3,3), (3,6), (4,4), (6,6)\}$

Sol. ii) Domain of $R = \{1, 2, 3, 4, 6\}$
Sol. iii) Range of $R = \{1, 2, 3, 4, 6\}$

Q.6.

Sol. Given,

$$R = \{(x, x+5) : x \in \{0, 1, 2, 3, 4, 5\}\}$$

Here,

$$y = x+5$$

when,

$x = 0$, $y = 0+5 = 5$
$x = 1$, $y = 1+5 = 6$
$x = 2$, $y = 2+5 = 7$
$x = 3$, $y = 3+5 = 8$
$x = 4$, $y = 4+5 = 9$
$x = 5$, $y = 5+5 = 10$

$$R = \{(0,5), (1,6), (2,7), (3,8), (4,9), (5,10)\}$$

Domain of $R = \{0, 1, 2, 3, 4, 5\}$
Range of $R = \{5, 6, 7, 8, 9, 10\}$

Q.7.

Sol. Given,

$R = \{(x, x^3) : x$ is a Prime no. less than $10\}$

Here, $x \in 2, 3, 5$ and 7.

$$\therefore R = \{(2,8), (3,27), (5,125), (7,343)\}$$

Q.8.

Sol. Given,

$$A = \{x, y, z\}$$
$$B = \{1, 2\}$$

$\therefore$ $A \times B = \{(x,1), (x,2), (y,1), (y,2), (z,1), (z,2)\}$

$n(A \times B) = 6$

$\because$ ~~Nor~~ We know that,

no. of subset of A set $= 2^n$

Here no. of subset of $A \times B = 2^6$
$= 64$

Hence, the no. of relations from A to B is 64 or 2^6.

Q.9.

Sol. Given,

$R = \{(a,b) : a, b \in Z, a-b$ is an integer$\}$

We know that,

Difference between two integer is always a integer.

Therefore, Domain of $R = Z$ (set of all integer)
Range of $R = Z$ (set of all integer)

Function

a function f is a relation from a non-empty set A to a non-empty set B such that the domain of f is A and no two distinct ordered pairs in f have the same first element.

If f is a function from A to B and $(a,b) \in f$, then $f(a) = b$, where b is called the image of a under f and a is called the preimage of b under f.

The function f from A to B is denoted by $f: A \rightarrow B$

Some function and their Graphs.

i) Identity function $\rightarrow$ $f: R \rightarrow R$ by $y = f(x) = x$ for each $x \in R$. such a function is called the identity function.

ii) Constant function $\longrightarrow$ Define the function $f: R \rightarrow R$ by $y = f(x) = C$, $x \in R$ where C is a constant and each $x \in R$. Here domain of f is R and its range is $\{C\}$.

iii) Polynomial function $\rightarrow$ a function $f: R \rightarrow R$ is said to be polynomial function if for each x in R, $y = f(x)$.

iv) Rational functions $\rightarrow$ Rational functions are functions of the type

$\frac{f(x)}{g(x)}$ where $g(x) \neq 0$

v) The Modulus function $\rightarrow$ The function $f: R \rightarrow R$ defined by $f(x) = |x|$ for each $x \in R$ is called modulus function.

$$f(x) = \begin{Bmatrix} x, & x \geq 0 \\ -x, & x < 0 \end{Bmatrix}$$

vi) Signum function $\rightarrow$ The function $f: R \rightarrow R$ defined by

$$f(x) = \begin{Bmatrix} 1, & \text{if } x > 0 \\ 0, & \text{if } x = 0 \\ -1, & \text{if } x < 0 \end{Bmatrix}$$

is called the signum function. The domain of the signum function is R and the range is the set $\{-1, 0, 1\}$.

vii) Greatest Integer Function $\rightarrow$ The function $f: R \rightarrow R$ defined by $f(x) = [x]$, $x \in R$.

From the defination of $[x]$, we can see that

$[x] = -1$ for $-1 \leq x < 0$
$[x] = 0$ for $0 \leq x < 1$

$[x] = 1$ for $1 \leq x < 2$
$[x] = 2$ for $2 \leq x < 3$ and so on.

Exercise 2.3

Q.1.

i) $\{(2,1), (5,1), (8,1), (11,1), (14,1), (17,1)\}$

Sol: Here, The set of all first element are unique.
Therefore the above set is a function.

Domain $= \{2, 5, 8, 11, 14, 17\}$
Range $= \{1\}$

ii) $\{(2,1), (4,2), (6,3), (8,4), (10,5), (12,6), (14,7)\}$

Sol: Here,
All first element are different from each other and can not be repeated so above set is a function.

Domain $= \{2, 4, 6, 8, 10, 12, 14\}$
Range $= \{1, 2, 3, 4, 5, 6, 7\}$

iii) $\{(1,3), (1,5), (2,5)\}$

Sol: Here,
The first element is repeated once and we know that, In a function first element of set is never repeated. Hence given set is not a function.

Q.2.

i) $f(x) = -|x|$

Sol. Given,

$\Rightarrow \quad f(x) = -|x|$

Let, i) $x = -1$, $f(-1) = -|-1|$

$\Rightarrow \quad f(-1) = -(1)$

$\Rightarrow \quad f(-1) = -1$

ii) $x = 5$, $f(5) = -|5|$

$\Rightarrow \quad f(5) = -(5)$

$\Rightarrow \quad f(5) = -5$

iii) $x = 1$, $f(1) = -|1|$

$\Rightarrow \quad f(1) = -(1)$

$\Rightarrow \quad f(1) = -1$

The Domain of f is R (all real no.

Domain $= \{R\}$

Range $= (-\infty, 0]$

ii) $f(x) = \sqrt{9-x^2}$

Sol. Given

$\Rightarrow \quad f(x) = \sqrt{9-x^2}$, ~~x~~ ..

1st Solution:

let,

i) $x = 4$, $f(4) = \sqrt{9-4^2}$

$f(4) = \sqrt{-7}$

$f(4) = \infty$

ii) $x = -4$, $f(-4) = \sqrt{9-(-4)^2}$

$\Rightarrow$ $f(-4) = \sqrt{9-16}$

$\Rightarrow$ $f(-4) = \sqrt{-7}$

$\Rightarrow$ $f(-4) = \infty$

iii) $x = 3$, $f(3) = \sqrt{9-3^2}$

$\Rightarrow$ $f(3) = \sqrt{0}$

$\Rightarrow$ $f(3) = 0$

iv) $x = -3$, $f(-3) = \sqrt{9-(-3)^2}$

$\Rightarrow$ $f(-3) = \sqrt{9-9}$

$\Rightarrow$ $f(-3) = \sqrt{0}$

$\Rightarrow$ $f(-3) = 0$

From the above observation,

Domain = $\{-3, -2, -1, 0, 1, 2, 3\}$

Range = $\{-3, -2, -1, 0, 1, 2, 3\}$

or.

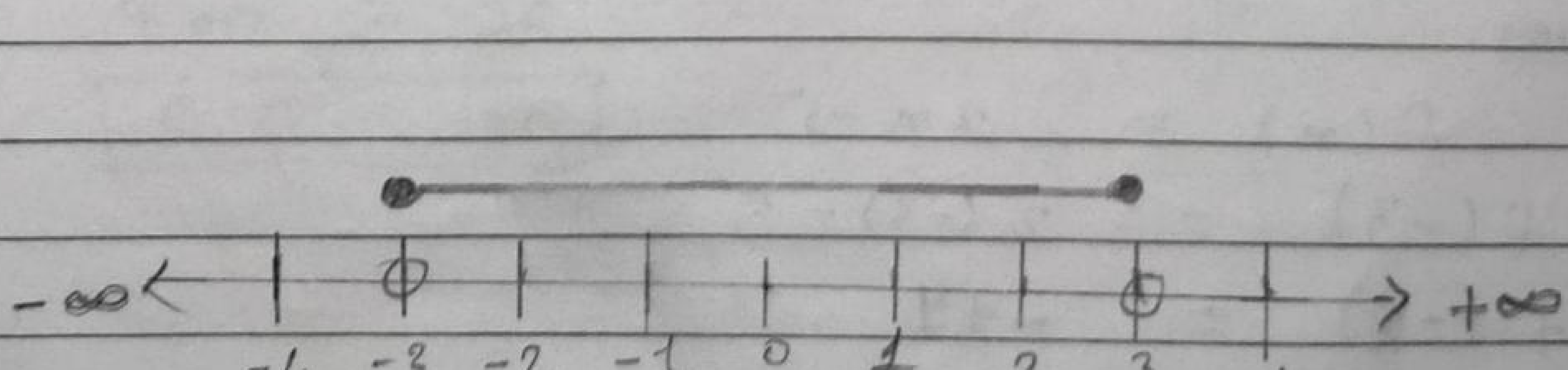
2nd solution.

$\Rightarrow$ $f(x) = \sqrt{9-x^2}$

$9 - x^2 \geq 0$

$x^2 - 9 \leq 0$

$(x+3)(x-3) \leq 0$



Domain $\Rightarrow$ $[-3, 3]$

~~let $f(x) = y$~~

then,

$\Rightarrow \quad f(x) = \sqrt{9-x^2}$

~~$y^2 = 9 - x^2$~~

If we put value of $x = -3, -2, -1, 0, 1, 2, 3$ then we get $0, \sqrt{5}, \sqrt{8}, 3, \sqrt{8}, \sqrt{5}, 0$ respectively.

Hence Range $= [0, 3]$

Q.3.

i) $f(0)$

Sol. Given,

$\Rightarrow \quad f(x) = 2x - 5$

$\Rightarrow \quad f(0) = 2(0) - 5$

$\Rightarrow \quad f(0) = -5$

ii) $f(7)$

Sol. Given,

$\Rightarrow \quad f(x) = 2x - 5$

$\Rightarrow \quad f(7) = 2(7) - 5$

$\Rightarrow \quad f(7) = 9$

iii) $f(-3)$

Sol. Given,

$\Rightarrow \quad f(x) = 2x - 5$

$\Rightarrow \quad f(-3) = 2(-3) - 5$

$\Rightarrow \quad f(-3) = -11$

Q.4.

Sol. Given,

$\Rightarrow \quad t(c) = \frac{9c}{5} + 32.$

i) $t(0)$

$\Rightarrow \quad t(0) = \frac{9(0)}{5} + 32$

$\Rightarrow \quad \boxed{t(0) = 32}$

ii) $t(28)$

$\Rightarrow \quad t(28) = \frac{9(28)}{5} + 32$

$\Rightarrow \quad t(28) = 50.4 + 32$

$\Rightarrow \quad \boxed{t(28) = 82.4}$

iii) $t(-10)$

$\Rightarrow \quad t(-10) = \frac{9(-10)}{5} + 32$

$\Rightarrow \quad \boxed{t(-10) = 14}$

iv) $t(c) = 212.$

$\Rightarrow \quad 212 = \frac{9c}{5} + 32$

$\Rightarrow 5(212) = 9c + 160$

$\Rightarrow 1060 - 160 = 9c$

$\Rightarrow 900 = 9c$

$\Rightarrow \boxed{c = 100}$

Q.5.

i) $f(x) = 2-3x \quad , \quad x \in R \quad , \quad x > 0.$

Sol. let,

$\Rightarrow \quad x = 0$

$\Rightarrow \quad f(0) = 2-3(0)$

$\Rightarrow \quad f(0) = 2.$

$x = 1$

$\Rightarrow \quad f(1) = 2-3(1)$

$\Rightarrow \quad f(1) = -1$

According to above observation,

~~Domo~~ Range $= [2, -\infty)$

ii) $f(x) = x^2 + 2 \quad , \quad x$ is a real no.

Sol. let, $x = 0,$

$\Rightarrow \quad f(0) = (0)^2 + 2$

$\Rightarrow \quad f(0) = 2$

$x = 1,$

$\Rightarrow \quad f(1) = (1)^2 + 2$

$\Rightarrow \quad f(1) = 3$

Range $= [2, \infty)$

iii) $f(x) = x \quad , \quad x$ is a real no.

Sol. Here we are able to put all the value in the place of x either that is positive, Negative or zero.

Hence, Range $= \{R\}$

Miscellaneous Exercise.

Q.5.

Sol. Given,

$\Rightarrow \quad f(x) = \dfrac{x^2 + 2x + 1}{x^2 - 8x + 12}$

Given function is defined only when $x^2 - 8x + 12 \neq 0$.

If,

$\Rightarrow \quad x^2 - 8x + 12 = 0$

$\Rightarrow \quad x^2 - 6x - 2x + 12 = 0$

$\Rightarrow \quad x(x-6) - 2(x-6) = 0$

$\Rightarrow \quad (x-2)(x-6) = 0$

$\Rightarrow \quad \boxed{\begin{matrix} x = 2 \\ x = 6 \end{matrix}}$

Domain $= R - \{2, 6\}$

Q.6.

Sol. Given,

$$f = \left\{ \left(x, \frac{x^2}{1+x} \right) : x \in R \right\}$$

Here,

$\Rightarrow \quad y = \dfrac{x^2}{1+x^2}$

$\Rightarrow \quad y + yx^2 = x^2$

$\Rightarrow \quad y = x^2 - yx^2$

$\Rightarrow \quad y = x^2(1-y)$

$\Rightarrow \quad x^2 = \dfrac{y}{1-y}$

$\Rightarrow \quad x = \sqrt{\dfrac{y}{1-y}}$

$\because \quad x \in R$

$\therefore \quad 1-y > 0$

$1 > y$

or $\quad y < 1$

Range $= [0, 1)$

Q.1.

Sol. Given,

$$f(x) = \begin{cases} x^2, & 0 \le x \le 3 \\ 3x, & 3 \le x \le 10, \end{cases}$$

$$g(x) = \begin{cases} x^2, & 0 \le x \le 2 \\ 3x, & 2 \le x \le 10 \end{cases}$$

$\because \quad f(x) = x^2$

at $x = 3, \quad f(3) = 3^2$

$= 9$

at $x = 3$

$\Rightarrow \quad f(x) = 3x$

$\Rightarrow \quad f(3) = 3(3)$

$\Rightarrow \quad f(3) = 9$

$(3, 9)$

$\because$ 3 has only one image 9 hence $f(x)$ is a function.

at $x = 2$, $g(x) = x^2$

$\Rightarrow$ $\therefore g(2) = 2^2$

$\Rightarrow$ $g(2) = 4$

at $x = 2$, $g(\cancel{x})\ 3x$

$g(\cancel{2}) = 3(2) = 6$

$g(2)$ has two imaged hence $g(x)$ is not a function it is a relation.

Q.2.

Sol. Given, $f(x) = x^2$

$$\frac{f(1.1) - f(1)}{(1.1 - 1)}$$

$$= \frac{(1.1)^2 - 1^2}{1.1 - 1}$$

$$= \frac{0.21}{0.1}$$

$$= 2.1$$

Q.4.

Sol. $f(x) = \sqrt{x-1}$

$f(x)$ is not defined if

$x - 1 < 0$

$x < 1$

So if (x) is defined

when $x > 1$

Domain $= [1, \infty)$

let $y = f(x)$

$\Rightarrow \quad y = \sqrt{x-1}$

$\Rightarrow \quad y^2 = x - 1$

$\Rightarrow \quad y^2 + 1 = x$

$y^2 + 1 \geq 1 \quad [\because x \geq 1]$

$\Rightarrow \quad \cancel{x = y^2 + 1} \quad y^2 \geq 0 \Rightarrow y \geq 0$

x is defined for all real value of y.

Range $= R^+$ or $[0, \infty)$ or N

Q.5.

Sol. Given real function,

$f(x) = |x-1|$

Clearly, the function $|x-1|$ is defined for all real no.

Hence,

Domain of $\cancel{R}$ $f = R$

Here, ~~Range~~ $= |x-1| \geq 0$, Range $= R^+$.

Also, for $x \in R$, $|x-1|$ assumes all real no.

Therefore the range of f is the set of all non - negative real no.

Q.7.

Sol. Given,

$f(x) = x + 1$

$g(x) = 2x - 3$

$\Rightarrow \quad f + g = f(x) + g(x)$

$\Rightarrow \quad = x + 1 + 2x - 3$

$= 3x - 2$

$\Rightarrow \quad f - g = f(x) - g(x)$

$= x + 1 - (2x - 3)$

$= x + 1 - 2x + 3$

$= -x + 4$

$\Rightarrow \quad f = f(x) = x + 1$

$\Rightarrow \quad g = g(x) = 2x - 3$

$\Rightarrow \quad \left(\frac{f}{g}\right)x = \frac{f(x)}{g(x)} = \frac{x+1}{2x-3}, \; x \neq \frac{3}{2}$

Q.8.

Sol. Given,

$f = \{(1,1), (2,3), (0,-1), (-1,3)\}$

$f(x) = ax + b$

let $y = ax + b$

when $x = 1$ then $1 = a(1) + b$

$y = 1$ $\quad 1 = a + b$

or $a + b = 1 \longrightarrow$ (i)

when $x = 2$ then, $3 = a(2) + b$

$y = 3$ $\quad 3 = 2a + b$

or $2a + b = 3 \longrightarrow$ (ii)

from eq^n (i) and (ii), on substracting -

$$\begin{array}{rcl} a + b &=& 1 \\ 2a + b &=& 3 \\ (-) \quad (-) && (-) \\ \hline -a &=& -2 \end{array}$$

$\boxed{a = 2}$

from (i),

$\Rightarrow \quad a + b = 1$

$\Rightarrow \quad 2 + b = 1$

$\Rightarrow \quad b = -1$

Hence, $x = 2$ and $y = -1$ Ans.

$y = ax + b$

$y = 2(x) + (-1)$

$y = 2x - 1$

$1 = 2x - y$

or $2x - y = 1$

for $x = 0$; $2 \times 0 - y = 1$

$\Rightarrow \quad y = -1$

for $x = -1$, $2(-1) - y = 1$

$\Rightarrow \quad -2 - y = 1$

$\Rightarrow \quad y = -3$

Hence, $a = 2$

$b = -1$,

and $2x - y = 1$ will be true.

Q.9.

Sol. Given,

$R : N \longrightarrow N$

$R = \{ (a, b) : a, b \in N \text{ and } a = b^2 \}$

i) $(a, a) \in R$, for all $a \in N$.

Sol. ~~let~~, Given, $a = b^2$

let, $a = \{1, 2, 3, 4, 5 \ldots\}$

$(a,a) = \{(1,1), (2,2), (3,3) \ldots\ldots\}$

A.T.Q.

$(1,1) \in R$, $1 = 1^2$ (True)

$(2,2) \in R$, $2 \neq 2^2$ (false)

Hence the given statement is false.

ii) $(a,b) \in R$, implies $(b,a) \in R$

Sol. Given, $a = b^2$

let,

$(a,b) = (4, 2) \in R$

$(b,a) = (2,4) \in R$

$(a,b) \in R \leftarrow$ imply $(2,4) \in R$

$(4,2) \in R \leftarrow$ imply $(2,4) \in R$ (False)

Hence the given statement is false.

iii) $(a,b) \in R$, $(b,c) \in R$ implies $(a,c) \in R$.

Sol. Given, $a = b^2$

let, $a = 16$, $b = 4$, $c = 2$

$(a,b) \in R$, $(b,c) \in R \leftarrow$ imply $(a,c) \in R$

$(16,4) \in R$, $(4,2) \in R \leftarrow$ implyy $(16,2) \notin R$

Hence, above statement is false.

Q.10.

Sol. Given,

$A = \{1, 2, 3, 4\}$, $B = \{1, 5, 9, 11, 15, 16\}$

$f = \{(1,5), (2,9), (3,1), (4,5), (2,11)\}$

Are the following true :-

i) f is a relation from A to B.

Sol. $f : A \rightarrow B$

Here all the 1st element of f is belong to set A and second element's belong to set B.

Hence, f is a relation from A to B.

ii) f is a function from A to B. (false)

Sol. Here, $(2,9)$ $(2,11) \in f$ (false)

and for function no two distinct ordered pair can have same first element.

Hence above statement is false.

Q.11

Sol. Given, $f \subseteq Z \times Z$

$f : Z \rightarrow Z$

$f = \{(ab, a+b) : a, b \in Z\}$

Case I :- let, $a = 1$, $b = 8$.

$f = \{(ab, a+b)\}$

$f = \{(1 \times 8, 1+8)\}$

$f = \{(8, 9)\}$

Case II :- let, $a = 2$, $b = 4$

$f = \{(2 \times 4, 2 + 4)\}$
$f = \{(8, 6)\}$

According to both case, two ordered pairs have same first element. Therefore f is not function from Z to Z.

Q.12
Sol. Given, $A = \{9, 10, 11, 12, 13\}$
$f : A \rightarrow N$
$f(n)$ = highest prime factor of n.

According to question,

let, i) $n = 9$ $\Rightarrow$ factor of $9 = 3, 3$
$\boxed{f(9) = 3}$

ii) $n = 10$ $\Rightarrow$ factor of $10 = 2, 5$
$\boxed{f(10) = 5}$

iii) $n = 11$ $\Rightarrow$ factor of $11 = 11$
$\boxed{f(11) = 11}$

iv) $n = 12$ $\Rightarrow$ factor of $12 = 2, 2, 3$
$\boxed{f(12) = 3}$

v) $n = 13$ $\Rightarrow$ factor of $13 = 13$
$\boxed{f(13) = 13}$

So,

$f = \{(9,3), (10,5), (11,11), (12,3), (13,13)\}$

$\boxed{\text{Range} = \{3, 5, 11, 3, 13\}}$