Chapter - 12
Thermodynamic

Thermodynamics is the relation b/w Heat energy and mechamical work.

* System & surrounding :-

system + surrounding = Universe.

* Thermodynamic variable :-

① Pressure (P) ③ Temperature (T)
② Volume (V) ④ Internal Energy (E)

* Thermodynamic Equilibrium :- if any system is in chemical, Mechamical and thermal Equillibrium then the system is in Thermodynamic Eqquilibrium.

i) The Net force of any body is Zero then this type of Equilibrium is known as Mechanical Equilibrium.

ii) The tem. of system is same as the tem. of surrounding then this equilibrium is known as Thermal equilibrium.

iii) If the chemical composition of a system does not change with time, then the system is in chemical equilibrium.

Zeroth law of Thermodynamics :-

If Two systems are in the thermal equilibrium with a third system individually then they will also be in Thermal equilibrium.

First law of Thermodynamics :-

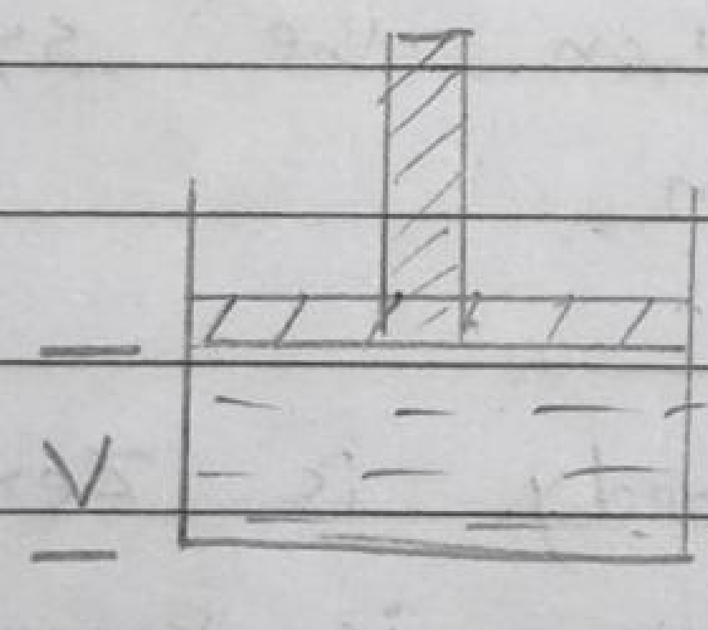
According to the first law of Thermodynamics,

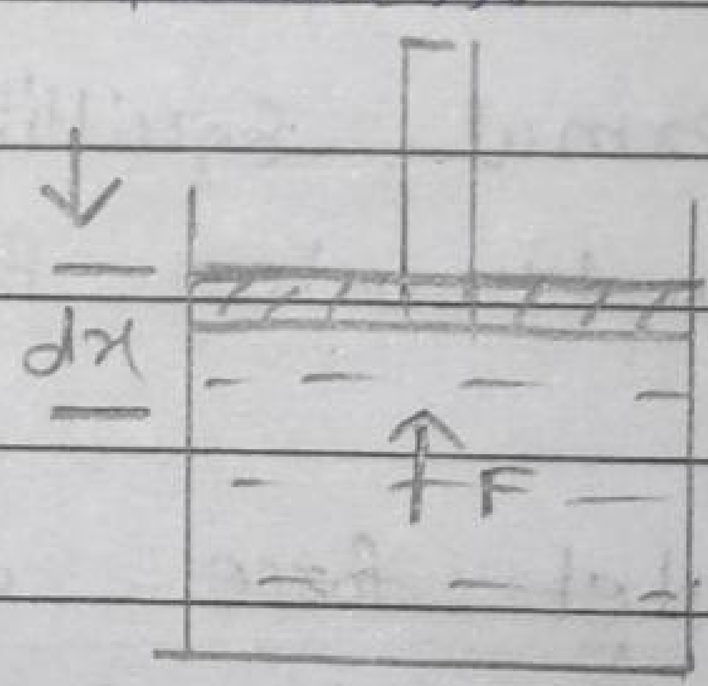
$$\boxed{\Delta Q = \Delta W + \Delta U}$$

where, ΔQ = Heat
ΔU = change in internal Energy

Q. Derive the Expression for work done in isothermal process.

Ans. The Type of process in which tem. is constant is called isothermal Process.





$\Rightarrow \quad dW = F \cdot dx$

$\because \quad P = \dfrac{F}{A}$

$\Rightarrow \quad F = PA$

$\Rightarrow \quad dW = PA \cdot dx$

$\Rightarrow \quad dW = P \cdot dV$

$\Rightarrow \quad \int dW = \int_{V_1}^{V_2} P \cdot dV$

$$\Rightarrow W = \int_{V_1}^{V_2} \frac{nRT}{V} \cdot dV \qquad \left[\because PV = nRT \\ P = \frac{nRT}{V} \right]$$

when, $n = 1$,

$$\Rightarrow W = \int_{V_1}^{V_2} \frac{RT}{V} \cdot dV$$

$$\Rightarrow W = RT \int_{V_1}^{V_2} \frac{dV}{V}$$

$$\Rightarrow \boxed{W = RT \log \frac{V_2}{V_1}}$$

Since, ~~$V \propto P$~~ $\boxed{V \propto \frac{1}{P}}$

then,

$$\Rightarrow \boxed{W = RT \log \frac{P_1}{P_2}}$$

Q. Derive the expression for work done in a Adiabetic Process

Sol.

$$\Rightarrow dw = F \cdot dx$$

$$\Rightarrow dw = PA \cdot dx$$

$$\Rightarrow dw = P \cdot dV$$

$$\Rightarrow \int dw = \int_{V_1}^{V_2} P \cdot dV.$$

$$\Rightarrow W = \int_{V_1}^{V_2} P \cdot dV,$$

$$\because PV^{\gamma} = K \quad \text{(adiabetic Process)}$$

$$P = \frac{K}{V^{\gamma}}$$

$$\Rightarrow W = \int_{V_1}^{V_2} \frac{K}{V^{\gamma}} dV$$

$$\Rightarrow \quad W = K \int_{V_1}^{V_2} V^{-\gamma}\, dV$$

$$\left[\because \quad \int x^n\, dx = \frac{x^{n+1}}{n+1} \right]$$

$$\Rightarrow \quad W = K \left[\frac{V^{-\gamma+1}}{-\gamma+1} \right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = K \left[\frac{V^{-(\gamma-1)}}{1-\gamma} \right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = \frac{K}{(1-\gamma)} \left[V^{-(\gamma-1)} \right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = \frac{K}{(1-\gamma)} \left[\frac{1}{V^{\gamma-1}} \right]_{V_1}^{V_2}$$

$$\Rightarrow \quad W = \frac{K}{(1-\gamma)} \left[\frac{1}{V_2^{\gamma-1}} - \frac{1}{V_1^{\gamma-1}} \right]$$

$$\Rightarrow \quad W = \frac{1}{1-\gamma} \left[\frac{K}{V_2^{\gamma-1}} - \frac{K}{V_1^{\gamma-1}} \right]$$

$$\because \quad P_1 V_1^{\gamma} = P_2 V_2^{\gamma} = K,$$

$$\Rightarrow \quad W = \frac{1}{1-\gamma} \left[\frac{P_2 V_2^{\gamma}}{V_2^{\gamma-1}} - \frac{P_1 V_1^{\gamma}}{V_1^{\gamma-1}} \right]$$

$$\Rightarrow \quad W = \frac{1}{1-\gamma} \left[\frac{P_2 \cancel{V_2^{\gamma}}}{\cancel{V_2^{\gamma}} \cdot V_2^{-1}} - \frac{P_1 \cancel{V_1^{\gamma}}}{\cancel{V_1^{\gamma}} \cdot V_2^{-1}} \right]$$

$$\Rightarrow W = \frac{1}{1-\gamma}\left[P_2V_2 - P_1V_1\right]$$

$$\Rightarrow W = \frac{1}{\gamma-1}\left[P_1V_1 - P_2V_2\right]$$

$\Rightarrow \because PV = nRT$

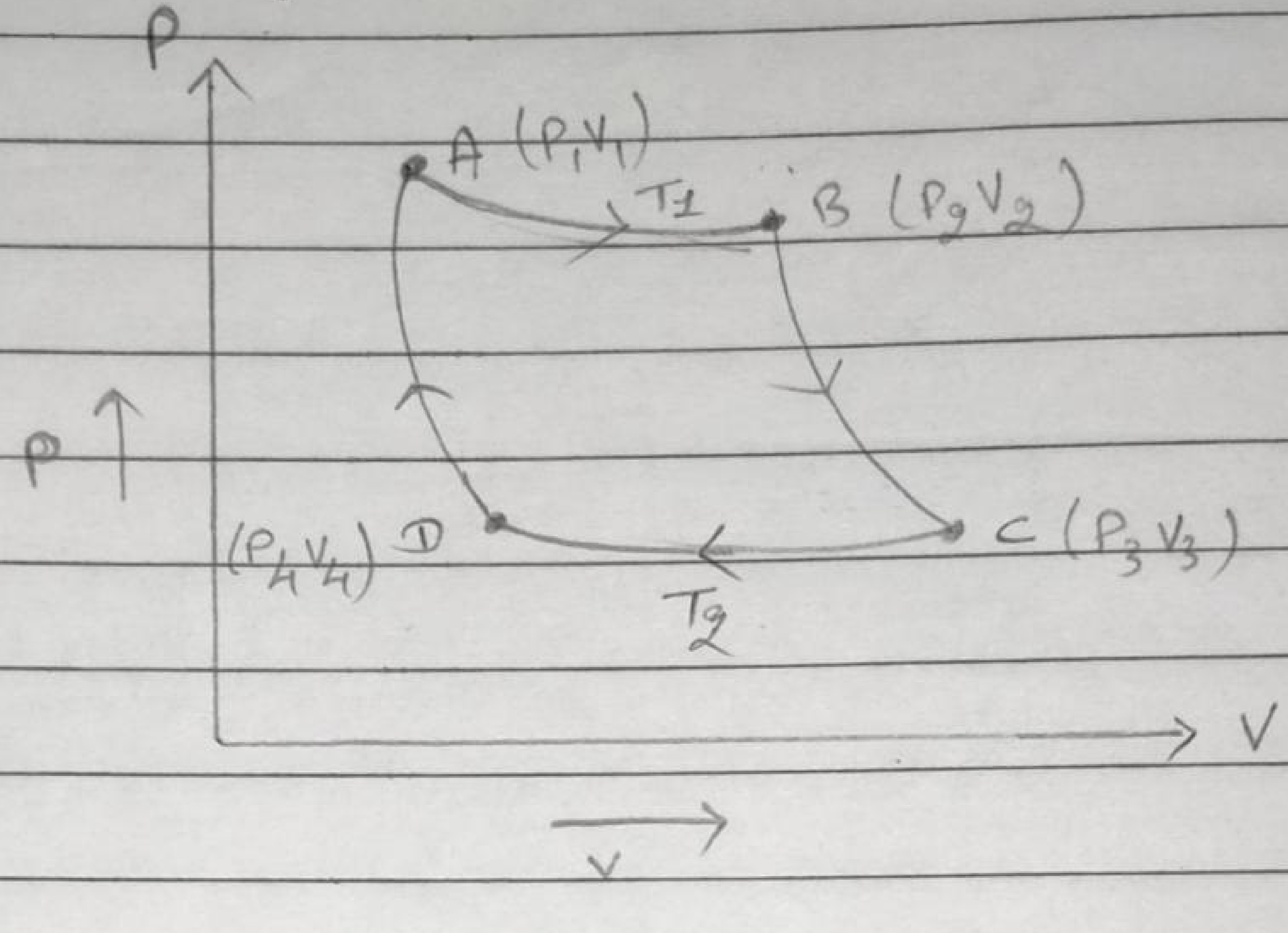
$P_1V_1 = RT_1$ (when $n=1$]

$\Rightarrow P_2V_2 = RT_2$

$$\Rightarrow W = \frac{1}{\cancel{1-}\gamma-1}\left(RT_2 - RT_1\right)$$

$$\Rightarrow \boxed{W = \frac{R(T_2 - T_1)}{\cancel{1-}\gamma-1}}$$

Cornot Engine / cornot cycle ⇒



where,

AB → Isothermal Expansion
BC → ~~Isott~~ Adiabetic Expansion
CD → Isothermal compression
DA → Adiabetic compression

Work done by AB,

$$\Rightarrow W_1 \cancel{AB} = RT_1 \log \frac{V_2}{V_1}$$

$$\Rightarrow \boxed{W_1 = Q_1}$$

work done by BC,

$$W_2 = \frac{R(T_1 - T_2)}{\gamma - 1}$$

work done by CD,

$$W_3 = -RT_2 \log \frac{V_4}{V_3}$$

$$w_3 = RT_2 \log \frac{V_3}{V_4}$$

$$\boxed{w_3 = Q_2}$$

Work done by DA,

$$w_4 = \frac{-R}{\gamma - 1} (T_2 - T_1)$$

$$w_4 = \frac{R}{\gamma - 1} (T_1 - T_2)$$

Total work done,

$$W = w_1 + w_2 - w_3 - w_4.$$

$$\because \; w_2 = w_4$$

$$\Rightarrow W = w_1 + w_4 - w_3 - w_4$$

$$\Rightarrow W = w_1 - w_3$$

$$\Rightarrow W = Q_1 - Q_2$$

$$\therefore \; \text{Efficiency } (\eta) = \cancel{1 - \frac{Q_2}{Q_1}} \quad \frac{W}{Q_1}$$

$$= \frac{Q_1 - Q_2}{Q_1}$$

$$= 1 - \frac{Q_2}{Q_1}$$

$$\eta = 1 - \left(\frac{RT_2 \log \frac{V_3}{V_4}}{RT_1 \log \frac{V_2}{V_1}} \right) \longrightarrow (A)$$

BC, $\rightarrow$ Adiabetic expansion

$$TV^{\gamma-1} = \text{constant.}$$

At BC,

$$\Rightarrow T_1 V_2^{\gamma-1} = T_2 V_3^{\gamma-1}$$

$$\Rightarrow \frac{T_1}{T_2} = \left(\frac{V_3}{V_2}\right)^{\gamma-1} \longrightarrow (1)$$

for DA,

$$\Rightarrow T_2 V_4^{\gamma-1} = T_1 V_1^{\gamma-1}$$

$$\Rightarrow \left(\frac{V_4}{V_1}\right)^{\gamma-1} = \frac{T_1}{T_2} \longrightarrow (2)$$

from (1) & (2),

$$\Rightarrow \left(\frac{V_4}{V_1}\right)^{\gamma-1} = \left(\frac{V_3}{V_2}\right)^{\gamma-1}$$

$$\Rightarrow \frac{V_4}{V_1} = \frac{V_3}{V_2}$$

$$\Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$$

$$\Rightarrow \log\frac{V_2}{V_1} = \log\frac{V_3}{V_4}$$

$$\Rightarrow \eta = 1 - \frac{T_2 \log\frac{V_3}{V_4}}{T_1 \log\frac{V_2}{V_1}}$$

$$\Rightarrow \boxed{\eta = 1 - \frac{T_2}{T_1}}$$