Chapter Four

Motion In a Plane (2D Motion)

* There are 2 types of ~~vector~~ Quantities :-

i) Scalar quantity
ii) vector quantity

i) **Scalar quantity** :- The quantity which have only magnitude is known as scalar quantity. for example distance, speed.

ii) **vector quantity** :- The quantity which has magnitude as well as the direction is known as vector quantity.

Eg:- Displacement, Velocity, acceleration and Force.

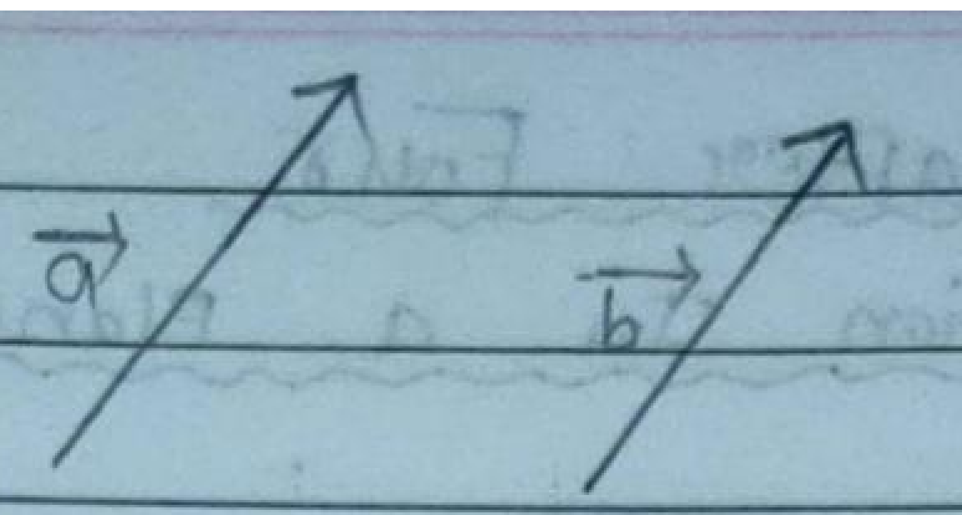
* **Representation of the vector quantity** :-

$\vec{a}$, $\vec{A}$, $\vec{b}$, $\vec{B}$ etc. but

in our ncert book vector quantity is shown as **A** , **B**

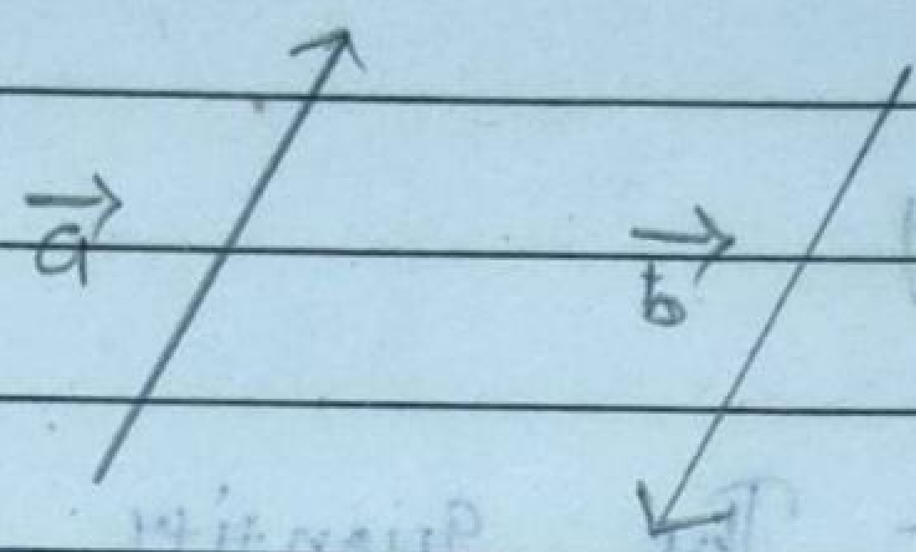
* **Equal vector** :- When 2 vectors having the Same magnitude as well as the same direction then the vectors are equal

Eg.:



, $\vec{a} = \vec{b}$

* Negative vector $\Rightarrow$
or Opposite vector



$\vec{a} = -\vec{b}$

here,

$\vec{a} \Rightarrow$ magnitude + direction

$\vec{b} \Rightarrow$ magnitude + direction

Magnitud of $\vec{a}$ and $\vec{b}$ is equal but direction are opposite. so,

$$\vec{a} = -\vec{b}$$

IMP.

Unit vector :- A vector whose magnitude is unity is called unit vector.

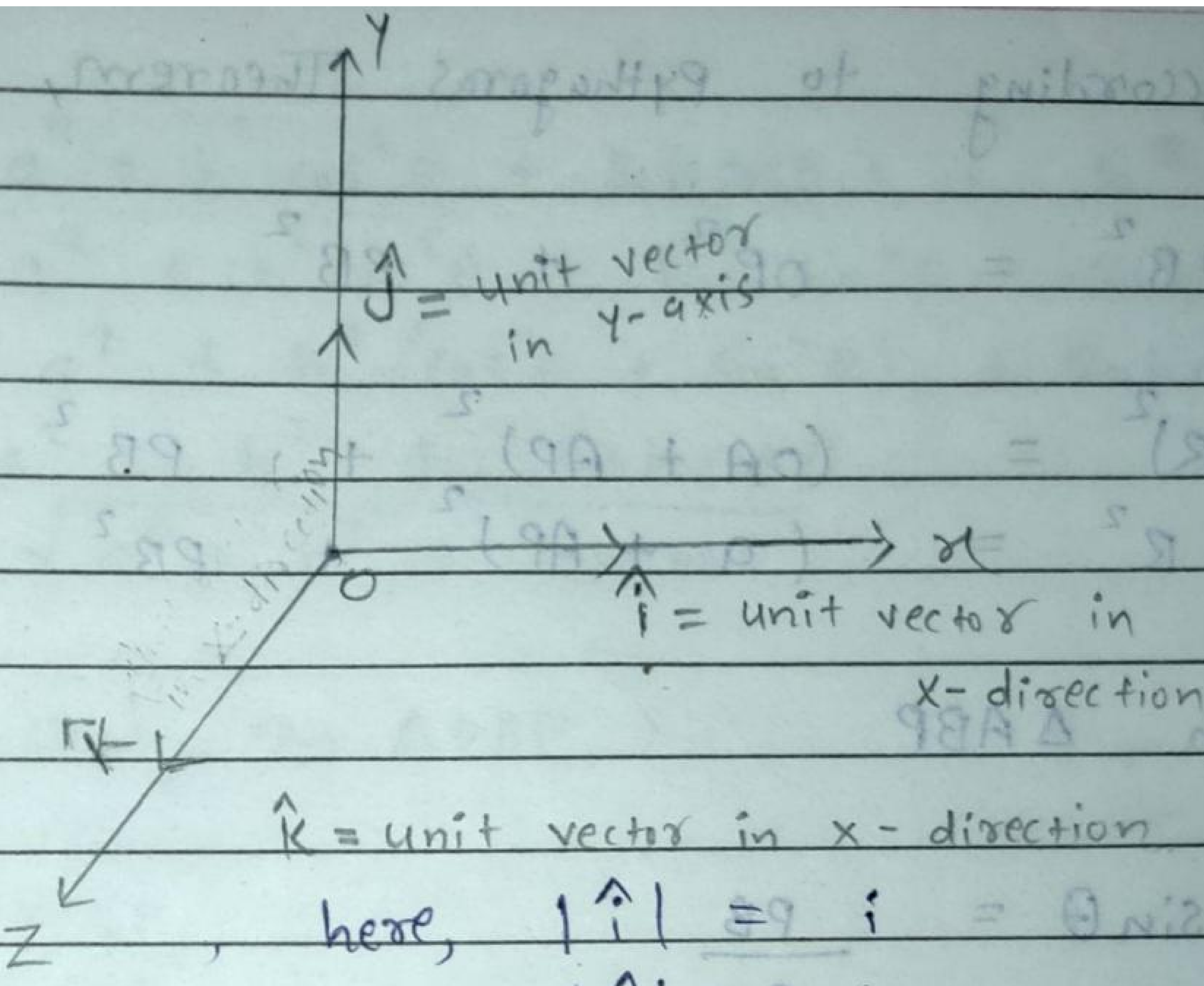
* $|\vec{a}|$, mod of vector a represents only Magnitude.

Eg. $\rightarrow$

$|\vec{a}| = 1$

$|\vec{b}| = 1$

$|\vec{c}| = 1$

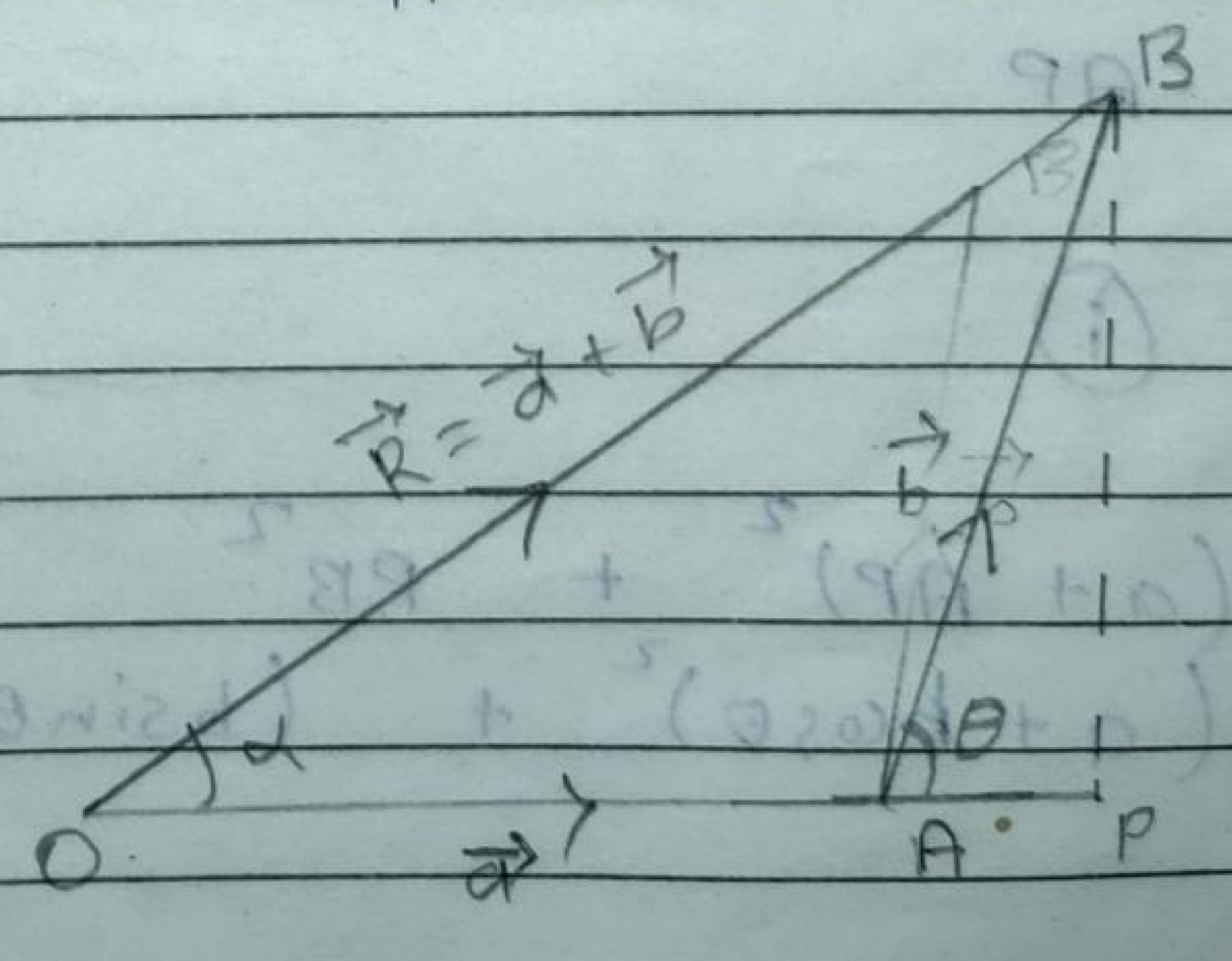


here, $|\hat{i}| = i$
$|\hat{j}| = j$
$|\hat{k}| = k$

Q. Write the triangle law of vector addition and derive the expression ~~for~~ it's resultant and direction.

Ans.

Triangle law of vector Addition :- If two vectors are represented both in magnitude and direction by the two sides of a triangle taken in one order, then there resultant is represented, both magnitude and direction, by the third side of the triangle taken in the opposite order



According to pythagoras Theorem,

$$\Rightarrow OB^2 = OP^2 + PB^2$$

$$\Rightarrow (R)^2 = (OA + AP)^2 + PB^2$$

$$\Rightarrow R^2 = (a + AP)^2 + PB^2 \quad \text{——} \quad (i)$$

In ΔABP

$$\therefore \sin\theta = \frac{PB}{AB}$$

$$\Rightarrow \sin\theta = \frac{PB}{b}$$

$$\Rightarrow b\sin\theta = PB$$

In ΔABP

$$\Rightarrow \cos\theta = \frac{AP}{AB}$$

$$\Rightarrow \cos\theta = \frac{AP}{b}$$

$$\Rightarrow b\cos\theta = AP$$

from eqn (i)

$$\Rightarrow R^2 = (a + AP)^2 + PB^2$$

$$\Rightarrow R^2 = (a + b\cos\theta)^2 + (b\sin\theta)^2$$

$$\Rightarrow R^2 = a^2 + b^2\cos^2\theta + 2ab\cos\theta + b^2\sin^2\theta$$

$$\Rightarrow R^2 = a^2 + b^2\cos^2\theta + b^2\sin^2\theta + 2ab\cos\theta$$

$$\Rightarrow R^2 = a^2 + b^2(\cos^2\theta + \sin^2\theta) + 2ab\cos\theta$$

$$\Rightarrow R^2 = a^2 + b^2 + 2ab\cos\theta$$

$$\Rightarrow R = \sqrt{a^2 + b^2 + 2ab\cos\theta}$$

For direction, In ΔOBP,

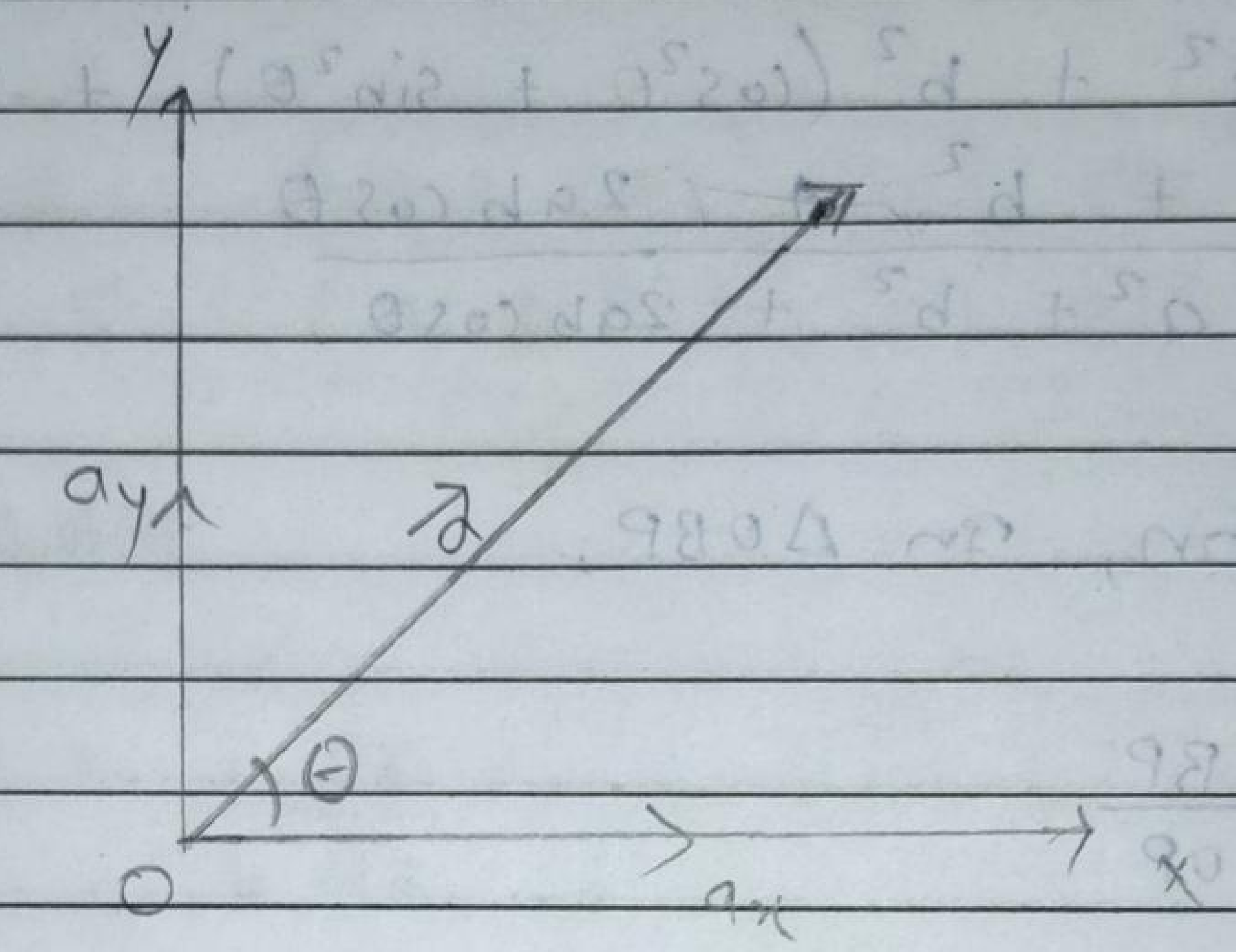
$$\Rightarrow \tan\alpha = \frac{BP}{OP}$$

$$\Rightarrow \tan\alpha = \frac{b\sin\theta}{OA + AP}$$

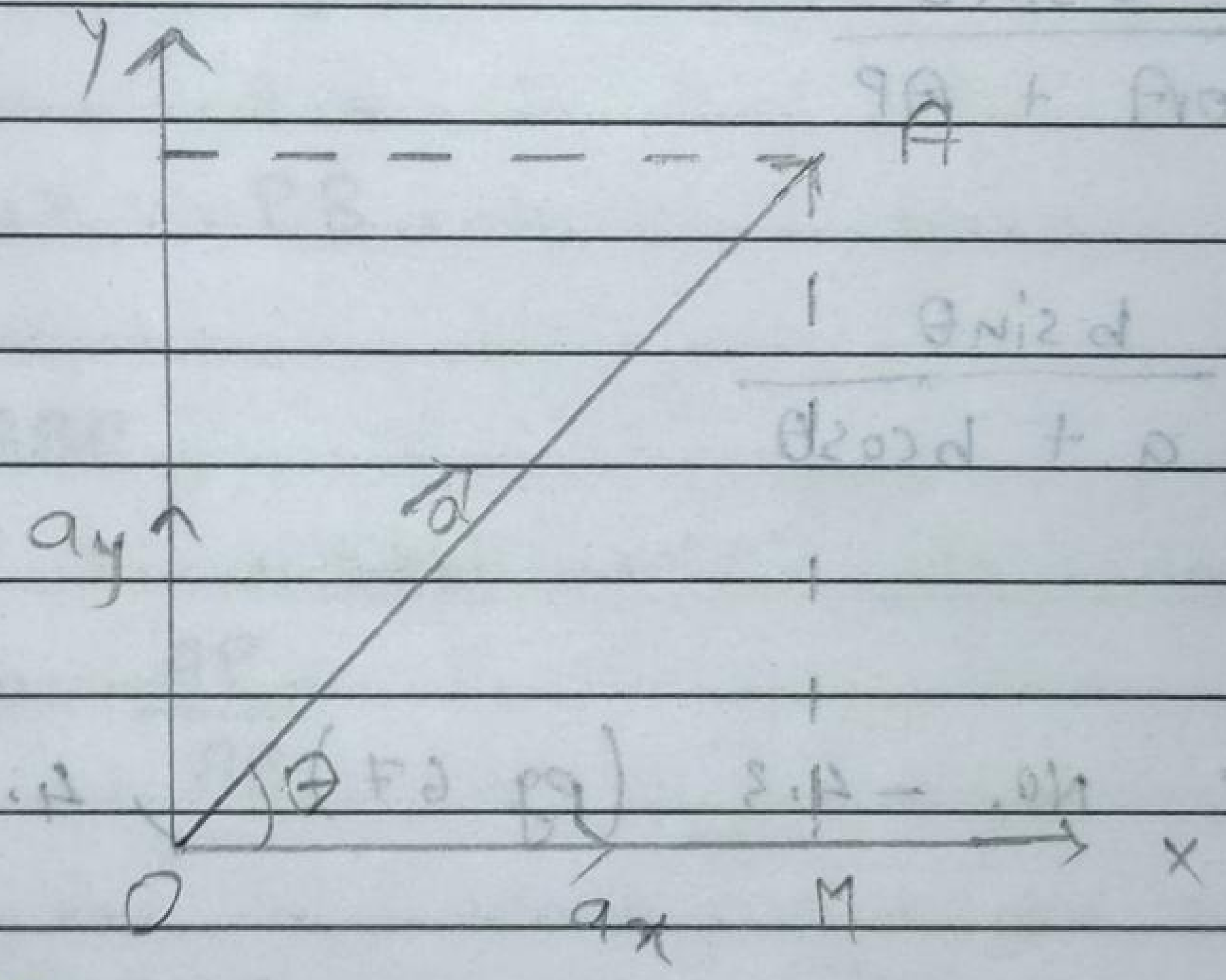
$$\Rightarrow \tan\alpha = \frac{b\sin\theta}{a + b\cos\theta}$$

Resolution of vectors.

1st diagram.



2nd diagram.



In ΔOMA,

$$\cos\theta = \frac{OM}{OA} = \frac{a_x}{a}$$

$$\Rightarrow \boxed{a\cos\theta = a_x}$$

$$\sin\theta = \frac{AM}{OA}$$

$$\sin\theta = \frac{a_y}{a_x}$$

$$\Rightarrow \boxed{a \sin\theta = a_y}$$

We have,

$$a_x = a\cos\theta \quad \text{— (1)}$$
$$a_y = a\sin\theta \quad \text{— (2)}$$

Adding and squaring both eqⁿ,

$$\Rightarrow (a_x)^2 + (a_y)^2 = (a\cos\theta)^2 + (a\sin\theta)^2$$

$$\Rightarrow a_x^2 + a_y^2 = a^2\cos^2\theta + a^2\sin^2\theta$$

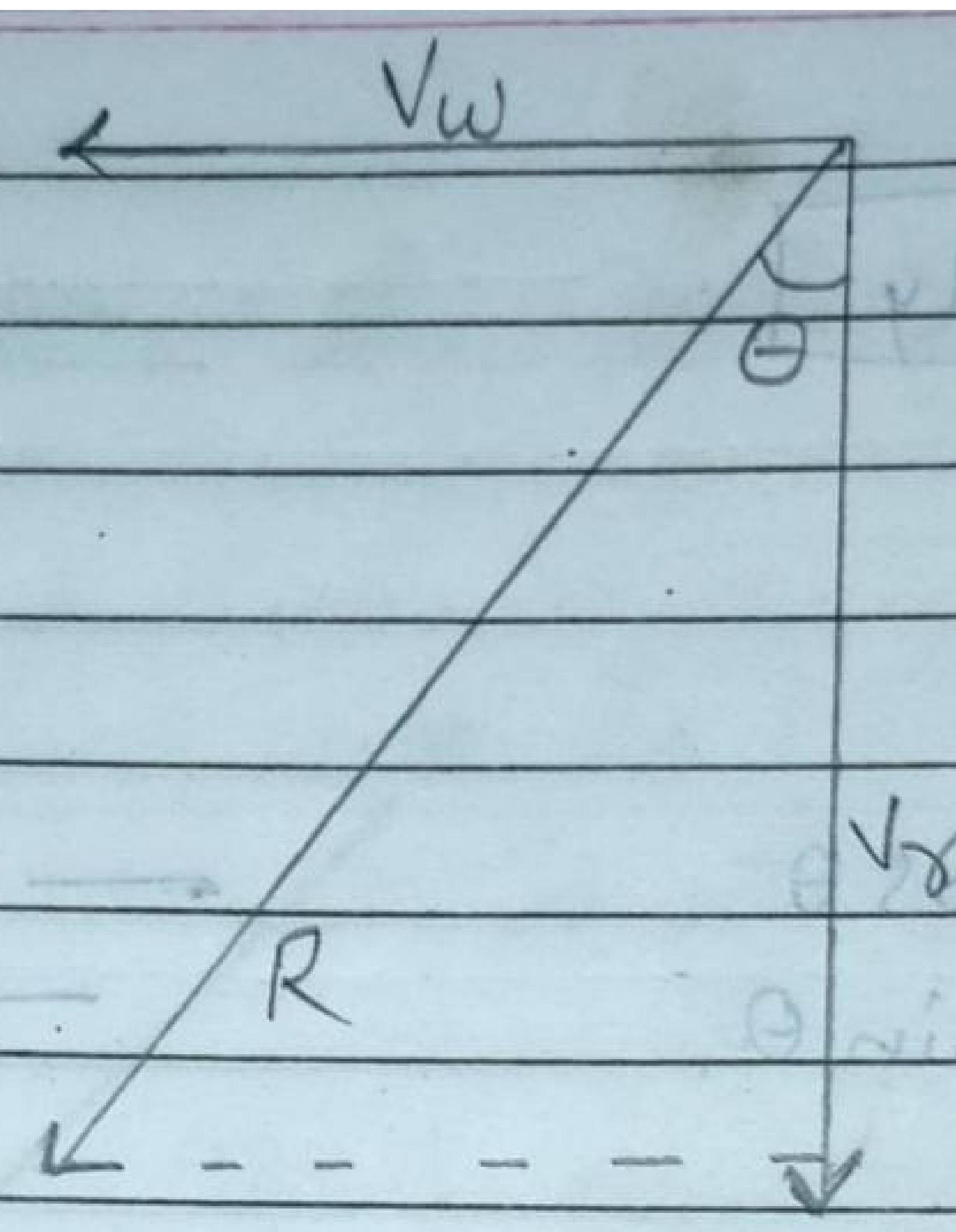
$$\Rightarrow a_x^2 + a_y^2 = a^2(\cos^2\theta + \sin^2\theta)$$

$$\Rightarrow a_x^2 + a_y^2 = a^2$$

$$\Rightarrow a = \sqrt{a_x^2 + a_y^2}$$

Eg. 4.1 Rain is falling vertically with a speed of 35 m/s. Winds starts blowing after some time with a speed of 12 m s^{-1} in east to west direction. In which direction should a boy waiting at a bus stop hold his umbrella?

Ans.



Given,

$V_r = 35\ m/s$

$V_w = 12\ m/s$

We know that,

$\Rightarrow R = \sqrt{V_r^2 + V_w^2}$

$\Rightarrow R = \sqrt{(35)^2 + (12)^2}$

$\Rightarrow R = \sqrt{1369}$

$\Rightarrow \boxed{R = 37}$

$\Rightarrow$ ~~In~~ $\tan\theta = \frac{P}{B} = \frac{V_w}{V_r}$

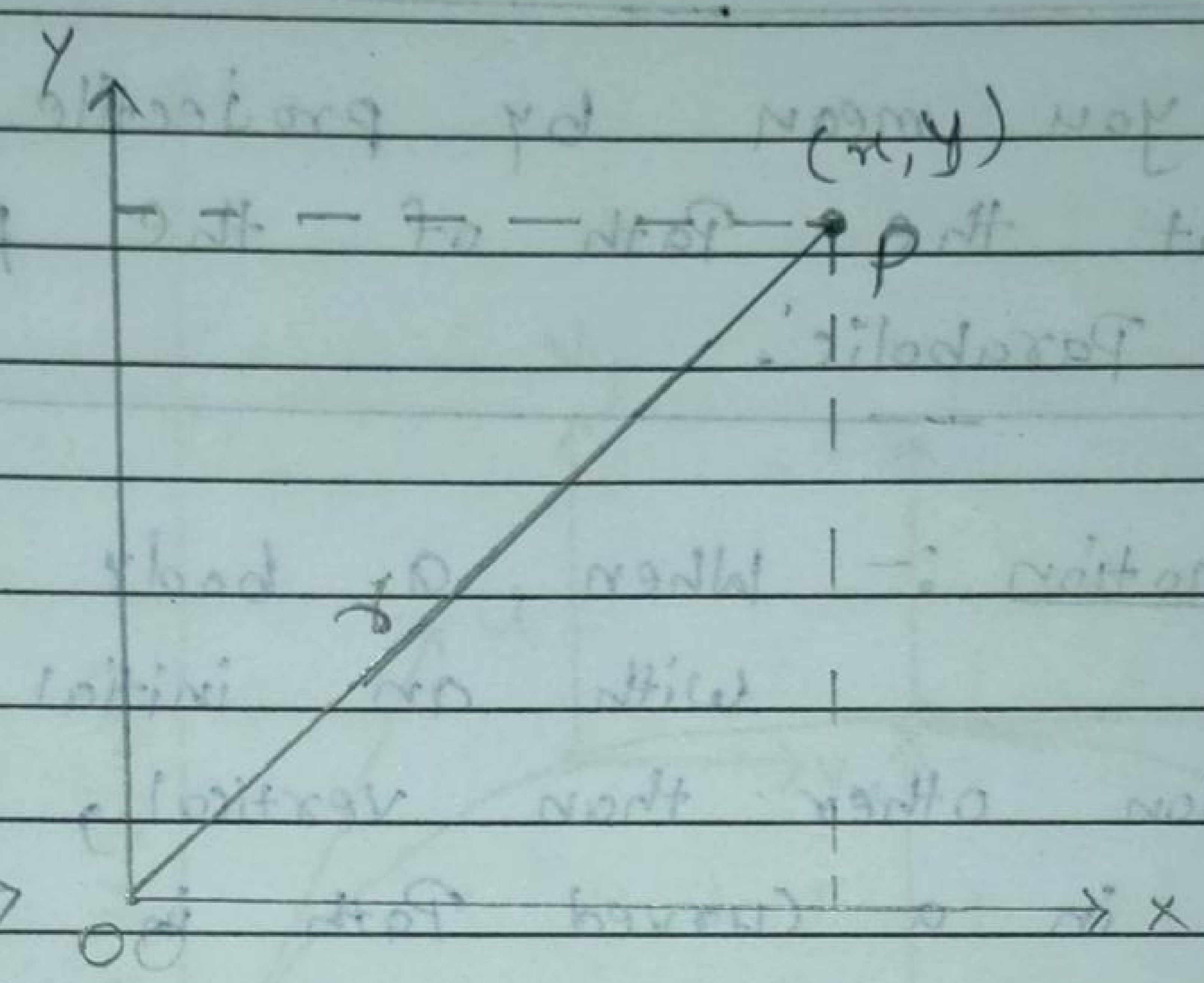
$\Rightarrow \tan\theta = \frac{12}{35}$

$\Rightarrow \tan\theta = 0.343$

$\Rightarrow \tan\theta = \tan 19°$

$\Rightarrow \boxed{\theta = 19°}$ Ans.

Position vector and Displacement :-

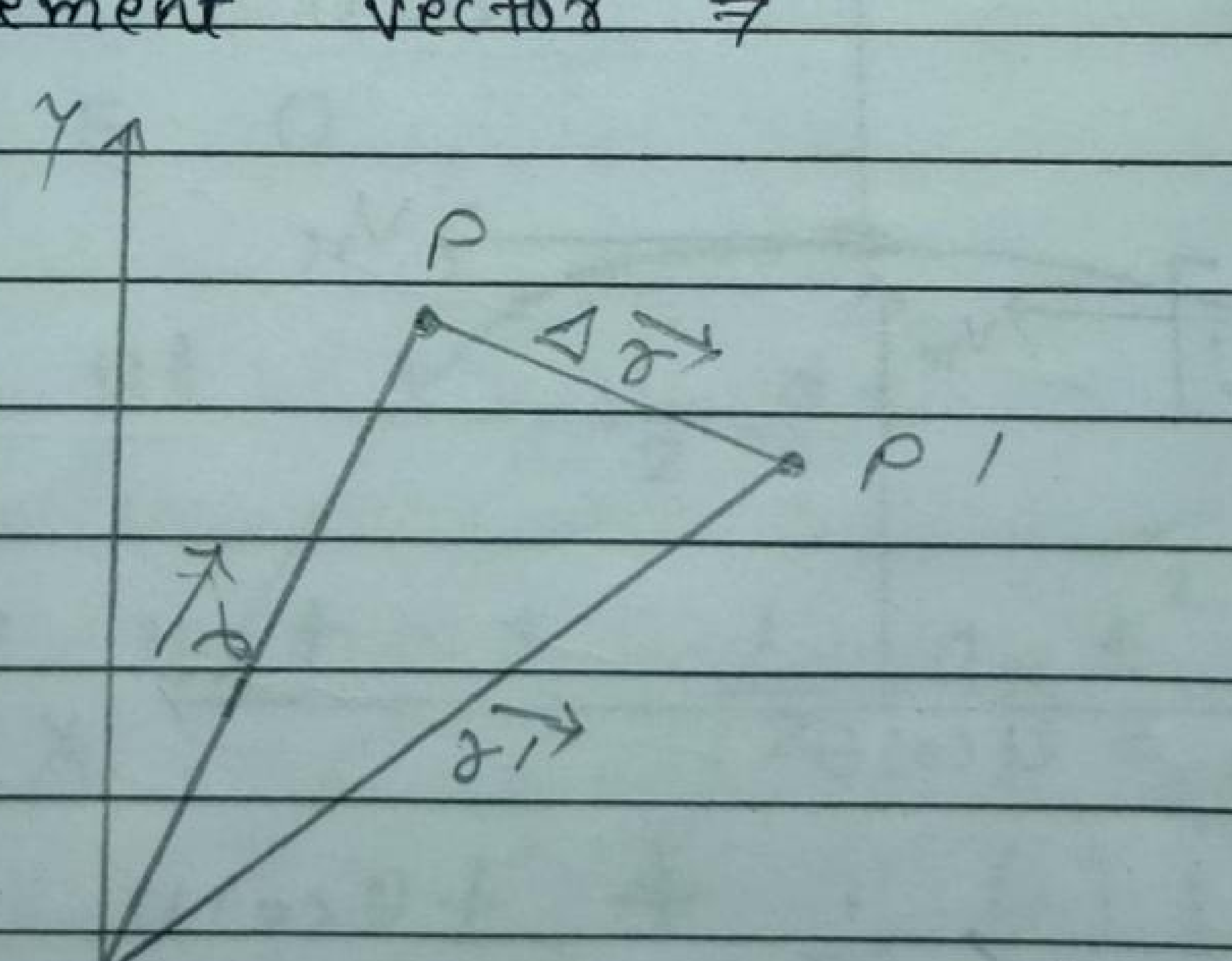


Position vector ⇒

↳ The Position vector "$\vec{r}$" of a Particle P located in a Plane With reference to the origin of an x-y reference frame is given by -

$$\vec{r} = x\hat{i} + y\hat{j}$$

Displacement vector ⇒

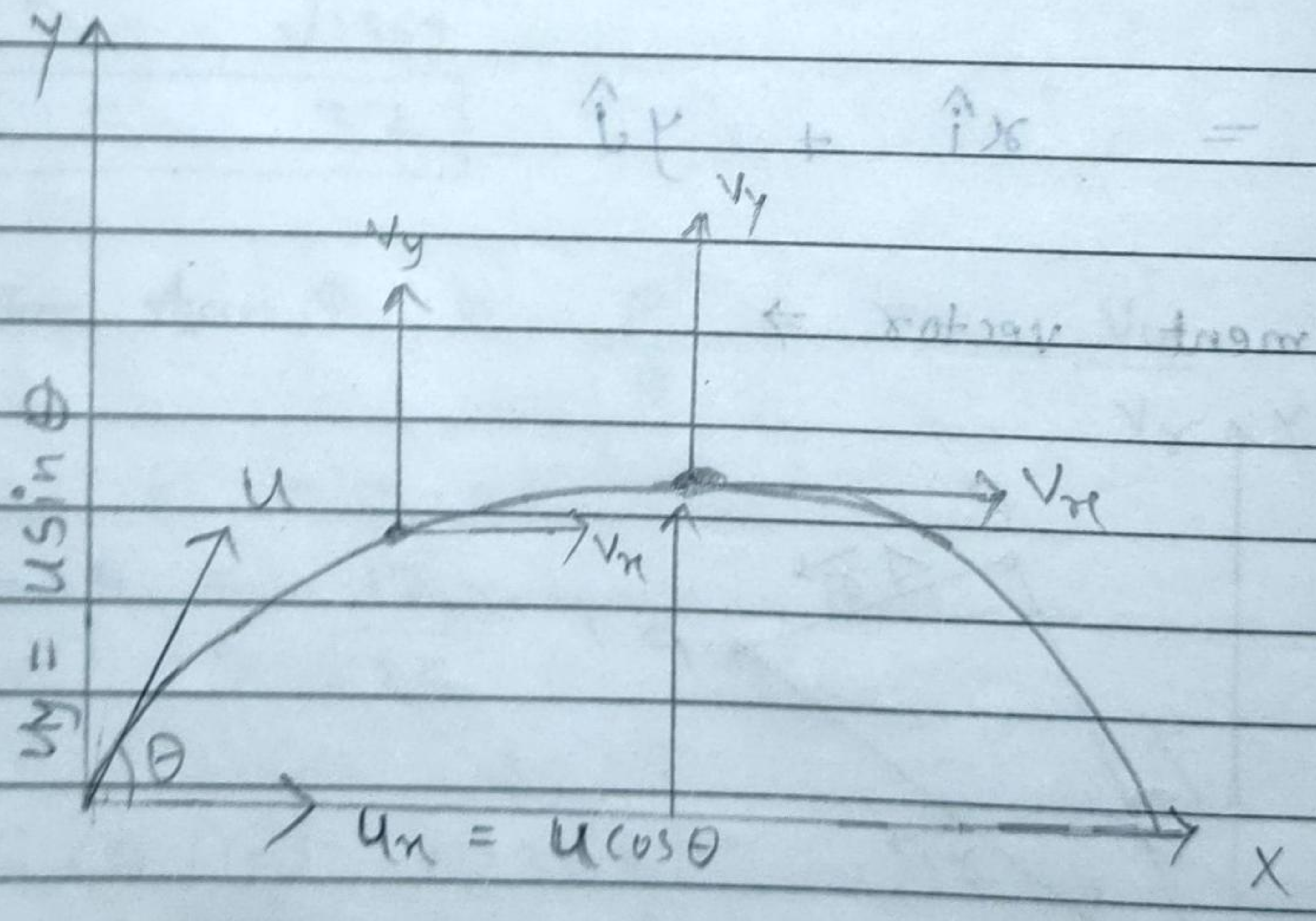


$$\Delta\vec{r} = \vec{r'} - \vec{r}$$

Q. What do you mean by projectile motion?
~~Ans~~ Prove that the Path of the projectile motion is Parabolic.

most IMP.

Ans. # Projectile motion :- When a body is thrown with an initial velocity in the direction other than vertical, then the body travels in a curved Path in a vertical Plane under acceleration due to gravity and falls on the ground ~~vert~~ at some other Point. This Type of motion is called projectile motion.



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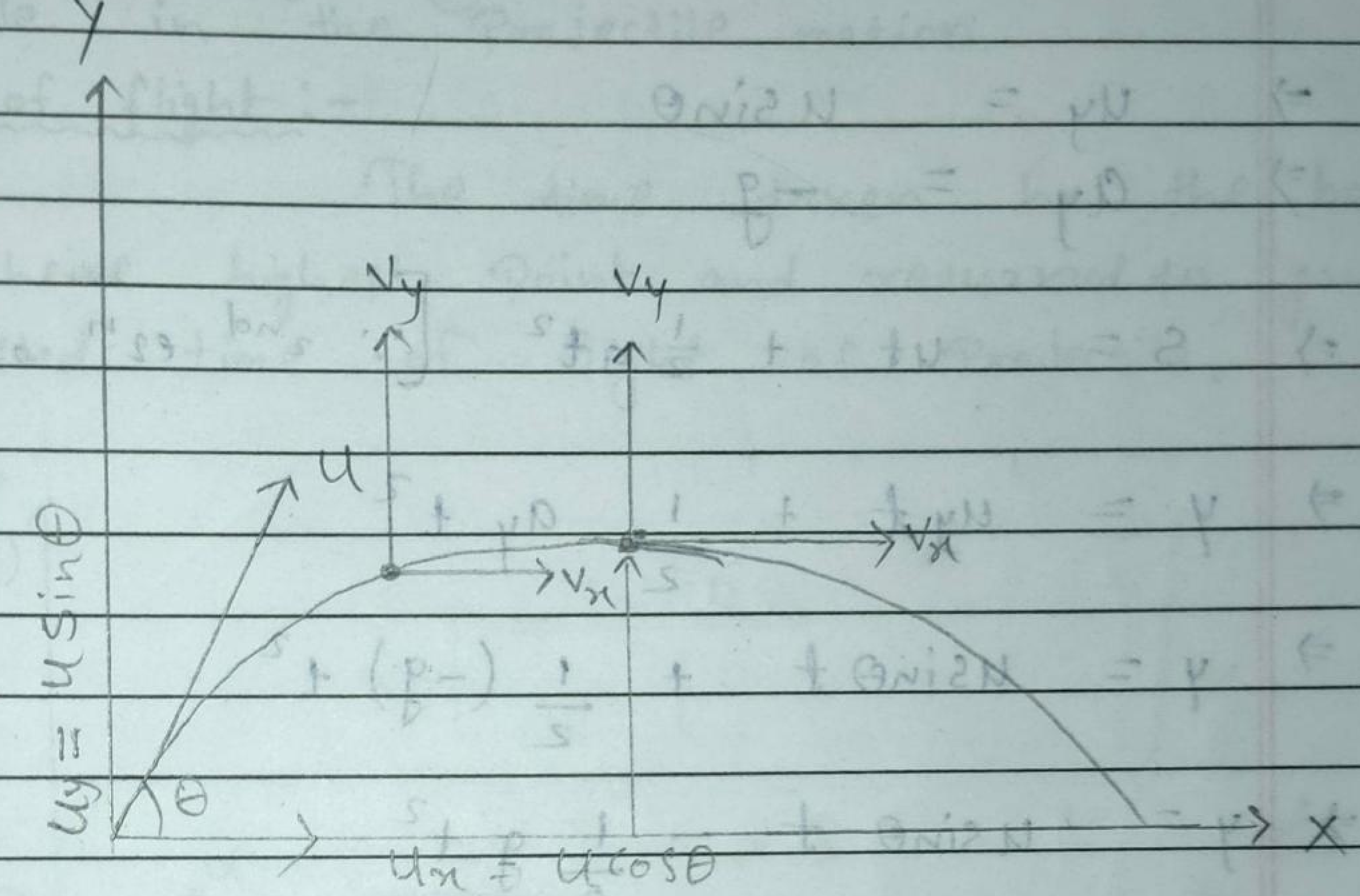

We know that,

(i) In x-axis, (only for x-axis).

$\Rightarrow \quad u_x = u\cos\theta$

$\Rightarrow \quad a_x = 0$

$\Rightarrow \quad S = ut + \frac{1}{2}at^2 \quad$ [∵ 2nd eqn of motion]

$\Rightarrow \quad x = u_x t + \frac{1}{2}a_x t^2$

$\Rightarrow \quad x = u\cos\theta\, t + \frac{1}{2}(0)t^2$

$\Rightarrow \quad x = u\cos\theta\, t + 0$

$\Rightarrow \quad x = u\cos\theta\, t$

$\Rightarrow \quad \boxed{t = \frac{x}{u\cos\theta}} \quad$ —— (i)

Now, In Y-axis, (only for Y)

$\Rightarrow u_y = u\sin\theta$

$\Rightarrow a_y = -g$

$\Rightarrow S = ut + \frac{1}{2}at^2$ [$\because$ 2nd eqn of motion]

$\Rightarrow y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow y = u\sin\theta\, t + \frac{1}{2}(-g)t^2$

$\Rightarrow y = u\sin\theta\, t - \frac{1}{2} g t^2$

$\Rightarrow y = u\sin\theta\left(\frac{x}{u\cos\theta}\right) - \frac{1}{2} g \left(\frac{x}{u\cos\theta}\right)^2$

[$\because$ from x eqn (i)]

$\Rightarrow y = \tan\theta\,(x) - \frac{1}{2}\frac{g}{u^2\cos^2\theta}\cdot x^2$

$\Rightarrow y = x\tan\theta - \frac{1}{2}\cancel{\frac{g}{u^2 x\cos}}\ \frac{g}{u^2\cos^2\theta}\cdot x^2$

$\Rightarrow y = ax - bx^2$ [$\because$ Where, $a = \tan\theta$, $b = \frac{1}{2}\frac{g}{u^2\cos^2\theta}$]

$y = ax - bx^2$ is a eqn of parabola so the path of the Projectile is Parabolic.

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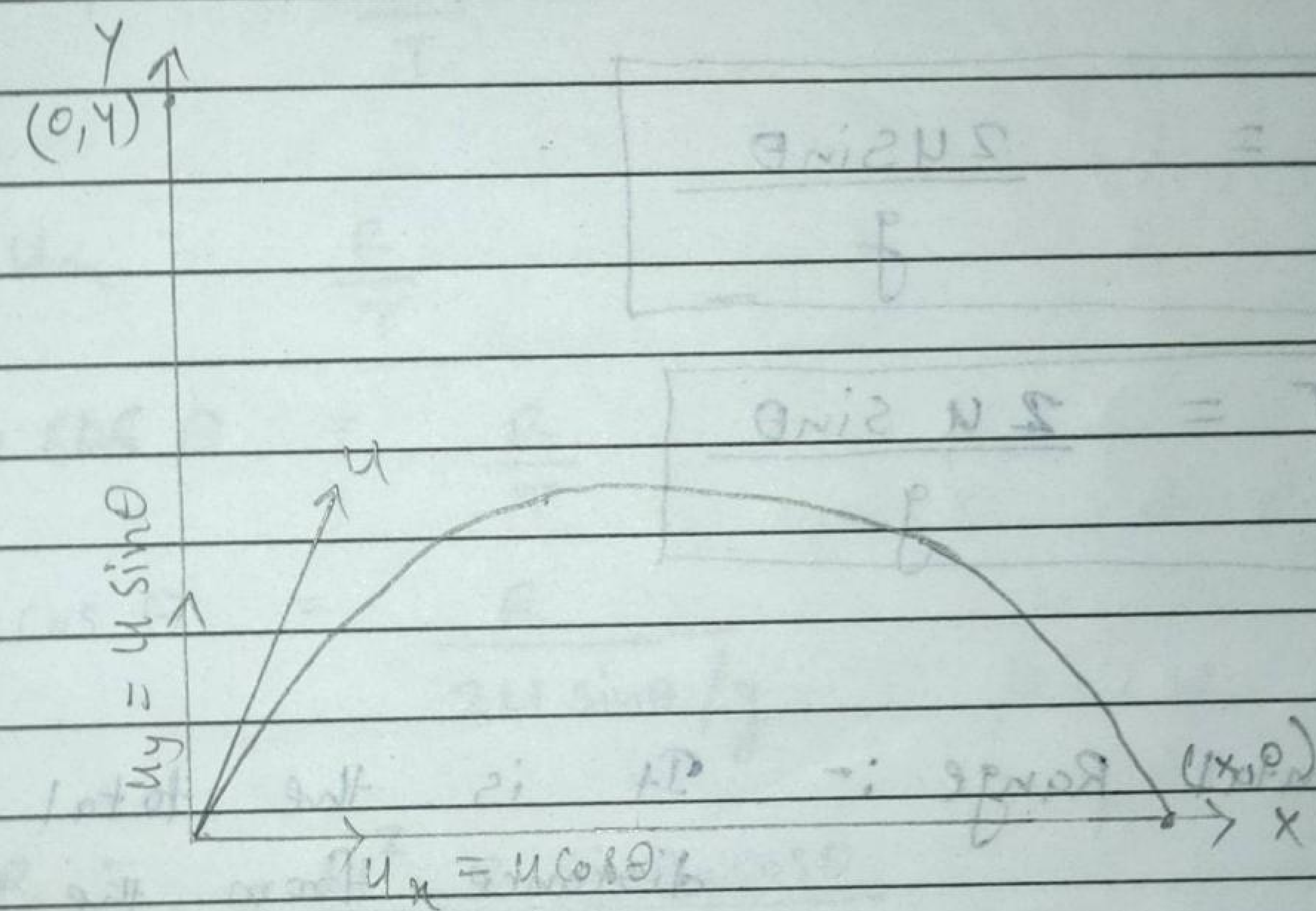
Q. Derive the expression for "time of flight" (a), (b) "horizontal range", (c) "maximum height" attained by projectile. in the projectile motion.

Ans (1) Time of flight :-

The time taken by the body to acheive highest point and returned to ground is called time of flight of projectile.



In only Y-axis,

$$\Rightarrow \quad u_y = u\sin\theta \quad , \quad a_y = -g$$

$$\Rightarrow \quad S = ut + \frac{1}{2}at^2 \quad [2^{nd} \text{ eq}^n \text{ of motion}]$$

$$\Rightarrow \quad y = u_y t + \frac{1}{2}a_y t^2$$

$$\Rightarrow \quad y = u\sin\theta . t + \frac{1}{2}(-g)t^2$$

$$\Rightarrow \quad y = u\sin\theta . t - \frac{1}{2}gt^2$$

$$\Rightarrow 0 = u\sin\theta\, t - \frac{1}{2} g t^2$$

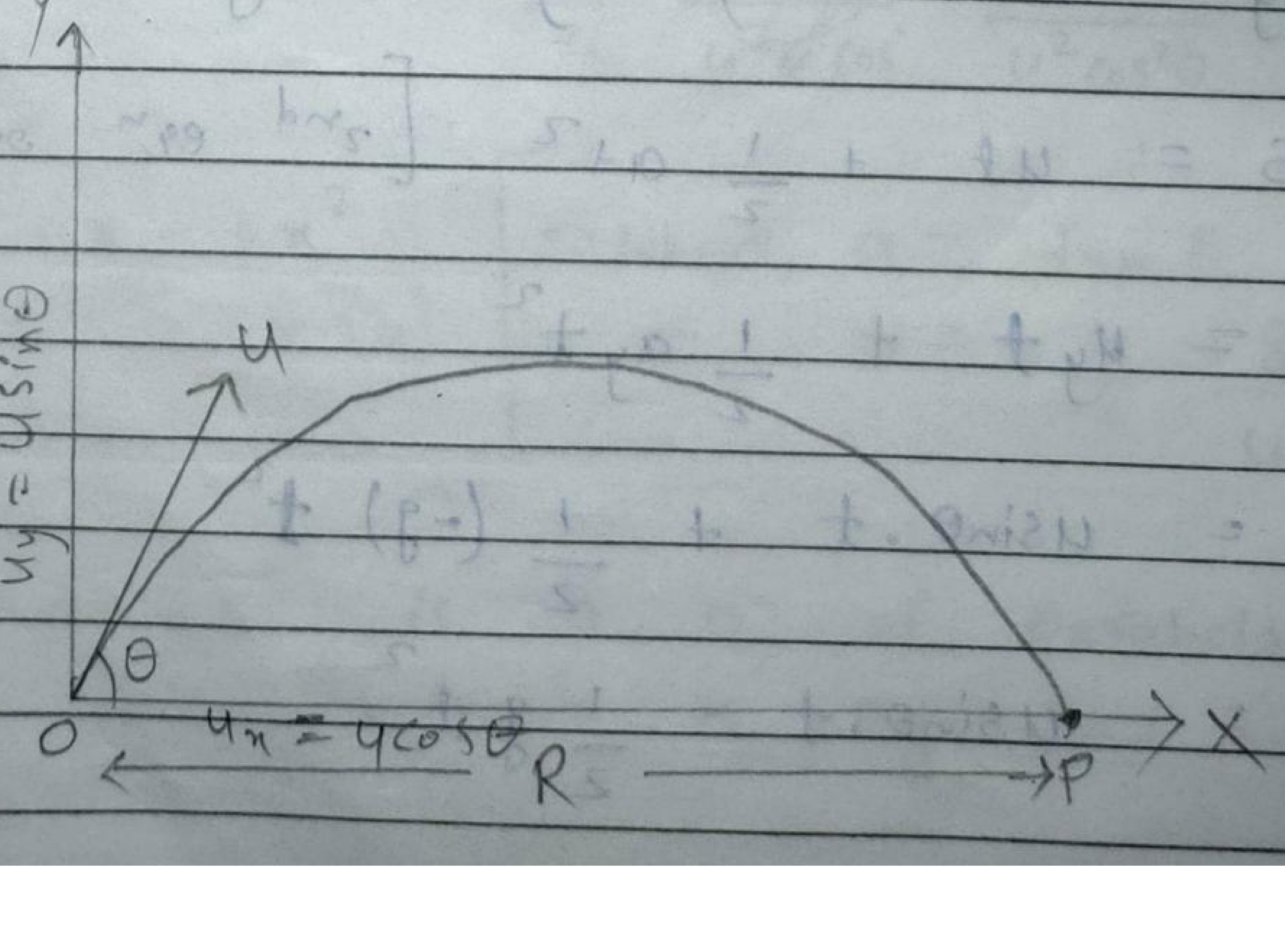
$$\Rightarrow \frac{1}{2} g t^2 = u\sin\theta . t$$

$$\Rightarrow \frac{1}{2} g t = u\sin\theta$$

$$\Rightarrow \boxed{t = \frac{2u\sin\theta}{g}}$$

$$\Rightarrow \boxed{T = \frac{2u\sin\theta}{g}}$$

(B) Horizontal Range :- It is the total horizontal distance from the Point of Projection to the Point where the Projectile come back to the Plane of Projection. It is denoted by 'R'.



We know that,

$\Rightarrow$ Speed $= \dfrac{\text{distance}}{\text{Time}}$

and, velocity $= \dfrac{\text{displacement}}{\text{time}}$

$$\Rightarrow \text{velocity} = \frac{R}{T}$$

$$\Rightarrow u_x = \frac{R}{T}$$

$$\Rightarrow u\cos\theta = \frac{R}{T}$$

$$\Rightarrow u\cos\theta = \frac{R}{2u\sin\theta/g}$$

$$\Rightarrow R = \frac{u^2 \cdot 2\sin\theta\cdot\cos\theta}{g}$$

$$\Rightarrow \boxed{R = \frac{u^2\sin 2\theta}{g}} \qquad \left[\because 2\sin\theta\cdot\cos\theta = \sin 2\theta\right]$$

for Maximum Range ~~(height)~~;

$$R = \frac{u^2\sin 2\theta}{g}$$

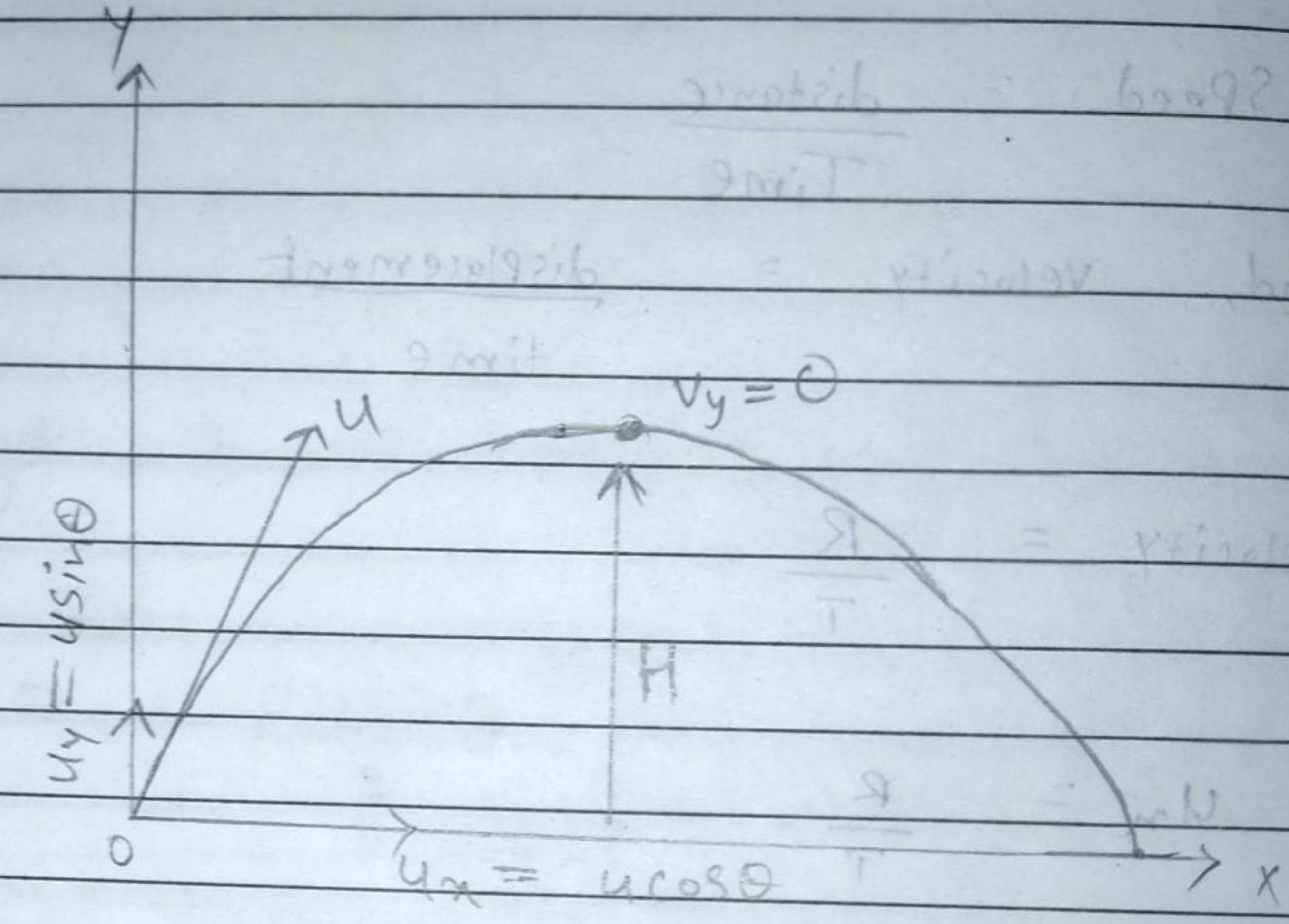
$\Rightarrow R_{max} = ?$

$\sin 2\theta = 1$

$\sin 2\theta = \sin 90^\circ$

$2\theta = 90^\circ \Rightarrow \theta = 45^\circ$

(c) Maximum Height ⇒



In only for y - axis,

$\Rightarrow u_y = u\sin\theta$

$\Rightarrow a_y = -g$

$\Rightarrow v = u + at$ [∵ first eqⁿ of motion]

$\Rightarrow v_y = u_y + a_y t$

$\Rightarrow 0 = u\sin\theta + (-g)t$

$\Rightarrow 0 = u\sin\theta - gt$

$\Rightarrow gt = u\sin\theta$

$$\Rightarrow \boxed{t = \frac{u\sin\theta}{g}}$$

again y - axis,

$\Rightarrow S = ut + \frac{1}{2} at^2$ [$\because$ 2nd eqn of motion]

$\Rightarrow y = u_y t + \frac{1}{2} a_y t^2$

$\Rightarrow H = u\sin\theta \left(\frac{u\sin\theta}{g}\right) + \frac{1}{2}(-g)\left(\frac{u\sin\theta}{g}\right)^2$

$\Rightarrow H = \frac{(u\sin\theta)^2}{g} - \frac{1}{2}\frac{(u\sin\theta)^2}{g}$

$\Rightarrow H = \frac{u^2\sin^2\theta}{g} - \frac{1}{2}\cdot\frac{u^2\sin^2\theta}{g}$

$\Rightarrow H = \frac{u^2\sin^2\theta}{g} - \frac{u^2\sin^2\theta}{2g}$

$\Rightarrow H = \frac{u^2\sin^2\theta}{g}\left(1 - \frac{1}{2}\right)$

$\Rightarrow H = \frac{u^2\sin^2\theta}{g} \times \frac{1}{2}$

$\Rightarrow \boxed{H = \frac{u^2\sin^2\theta}{2g}}$

Eg. 4.9 A cricket ball is thrown at a speed of 28 m/s in a direction 30° above the horizontal. Calculate

(a) → The maximum height (H)

(b) → The time taken by ball to return to the same level.

(c) → The distance from the thrower to the point where the ball returns to the same level.

Sol. Given,

$u = 28\ m/s$, $g = 9.8\ m/s^2$ ~~(10)~~

$\theta = 30^\circ$

i) $H = ?$

ii) $T = ?$

iii) $R = ?$

For Maximum height (H),

$\Rightarrow H = \dfrac{u^2 \sin^2\theta}{2g}$

$\Rightarrow H = \dfrac{(28)^2 \times \sin^2 30}{2(10)}$ $\quad [\because g = 10\ m/s^2]$

$\Rightarrow H = \dfrac{\cancel{804}\ 784 \times \frac{1}{4}}{20}$

$\Rightarrow H = \dfrac{784 \times 0.25}{20}$

$\Rightarrow H = \dfrac{196}{20}$

$\Rightarrow H = 9.8$.

For T,

$$\Rightarrow T = \frac{2u\sin\theta}{g}$$

$$\Rightarrow T = \frac{2(28) \times \sin 30^\circ}{10} \quad [\because g = 10\ m/s^2]$$

$$= \frac{56 \times \frac{1}{2}}{10}$$

$$= \frac{28}{10}$$

$$\Rightarrow T = 2.8 \text{ sec.}$$

For R,

$$\Rightarrow R = \frac{u^2 \sin 2\theta}{g}$$

$$\Rightarrow R = \frac{(28)^2 \times \sin 2(30)^\circ}{10} \quad [\because g = 10]$$

$$= \frac{784 \times \cancel{\frac{1}{2}}\ \frac{\sqrt{3}}{2}}{10}$$

$$\cancel{= \frac{196}{10}} \qquad = \frac{392 \times \sqrt{3}}{10}$$

$$\cancel{R = 19.6} \qquad = 39.2 \times \sqrt{3}$$

$$= 39.2 \times 1.73$$

$$\boxed{R = 67.8\, m}$$

IMP.

Questions

Q.1) What do you mean by uniform circular motion?

Q.2) ~~txt~~ Define the following term :-

i) Angular displacement
ii) Angular Velocity
iii) Angular Acceleration.

Q.3. Derive the relation between linear velocity and angular velocity.

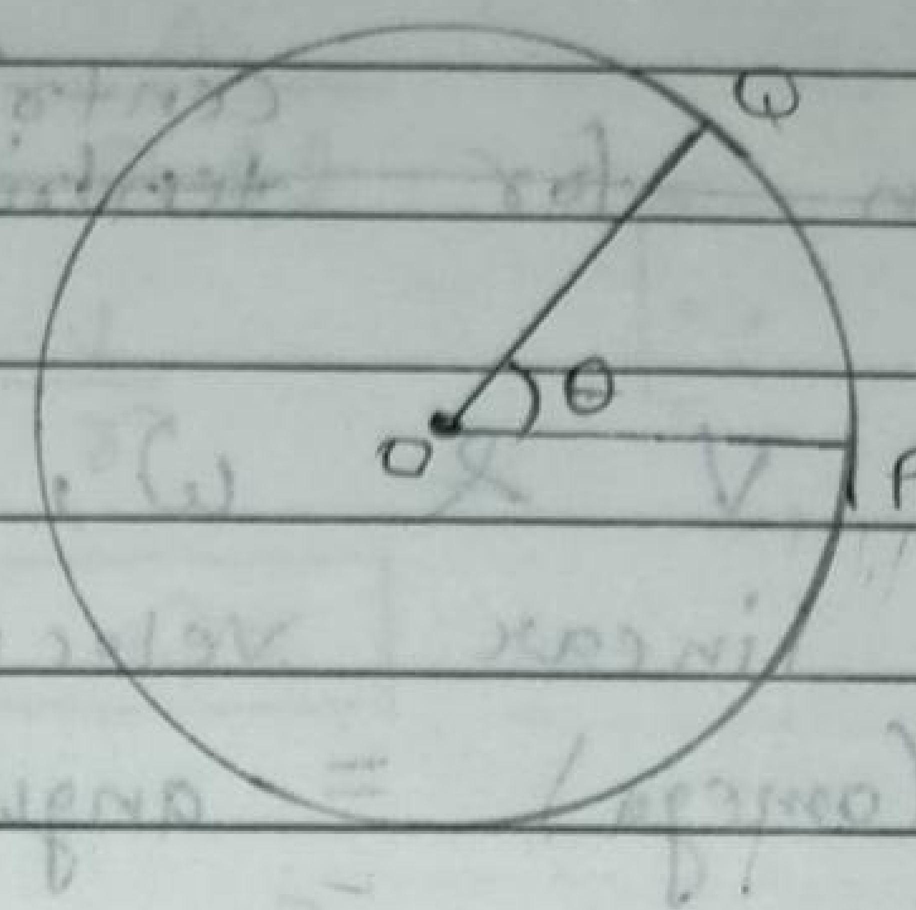
~~Ans.~~ **Q4)** Derive the relation between linear acceleration & Angular acceleration.

Q.5) Derive the expression for ~~angular~~ centripetal acceleration.

Ans. 1) Uniform circular motion :- If the body cover equal distance in equal time interval in circular Path then the motion of the body is known as uniform circular motion.

Ans. 2) i) Angular displacement (θ) :- The angular displacement of particle executing circular motion in a circular Path is defined as the angle subtended by the final position and initial position of the particle at the centre of that circular path.

It's SI unit is 'radian' and it is dimensionless quantity.



here,
OP = linear displacement,
θ = Angular displacement

⇒ Angular displacement $(\theta) = \dfrac{\text{arc}}{\text{radius}}$

→ Angular displacement $(\theta) = \dfrac{PQ}{OP}$

ii) Angular velocity :- The angle subtended at the centre in unit time by a Particle in circular motion is called angular velocity. It is denoted by 'ω' (omega) and SI unit is "radian/sec.".

⇒ Angular velocity $(\omega) = \dfrac{\theta}{t}$

here, ω = Angular velocity
θ = Angular displacement
t = time.

iii) Angular Acceleration :- The rate of change of angular velocity with time is called Angular Acceleration. It is denoted by 'α'. and SI unit is "radian/sec²"

⇒ $\alpha = \dfrac{d\omega}{dt}$, here, α = Angular Acceleration,
dω = change in angular velocity,
dt = ~~change in~~ time interval

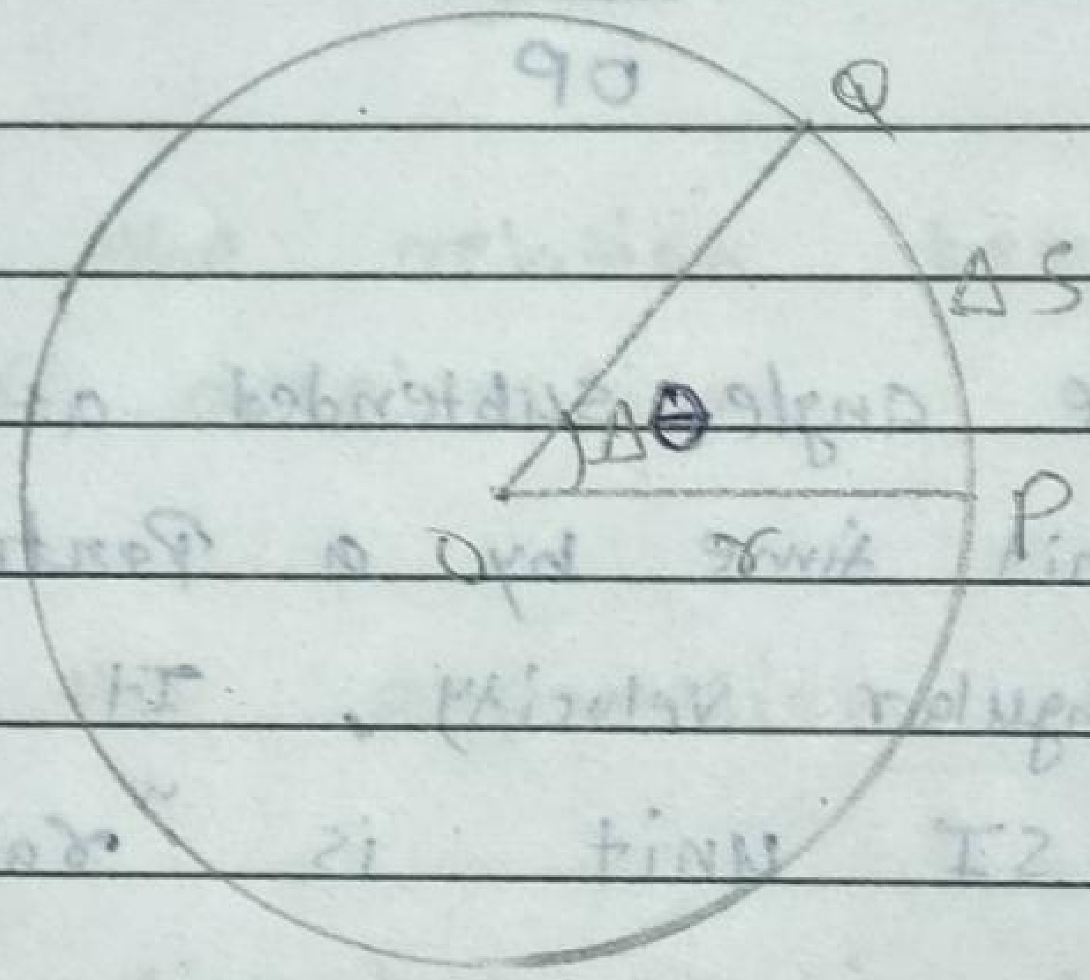
Ans-3) relation between V & ω,

here, V = linear velocity

and ω (omega) = angular velocity.

We know that,

$\Rightarrow \quad V = \dfrac{\Delta S}{\Delta t}$ and, $\omega = \dfrac{\Delta \theta}{\Delta t}$



$\Rightarrow \quad \Delta\theta = \dfrac{arc}{radius} = \dfrac{QP}{OP}$

$\Rightarrow \quad \Delta\theta = \dfrac{\Delta S}{r}$

and ω (omeaga) $= \dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow \quad \omega = \dfrac{\frac{\Delta S}{r}}{\Delta t}$ $\left[\because \Delta\theta = \dfrac{\Delta S}{r} \text{ (we find)} \right]$

$$\Rightarrow \omega = \frac{\Delta s}{r.\Delta t}$$

$$\Rightarrow \omega = \frac{v}{r} \quad \left[\because \; v = \frac{\Delta s}{\Delta t} \text{ (given)}\right]$$

$$\Rightarrow \boxed{v = \omega r}$$

Hence $v = \omega r$ is relation between v and ω.

Ans. 4) Relation between 'a' and 'α'

here, a = acceleration (linear)

α = angular acceleration.

We know that,

$$v = r\omega \quad [\text{relation between } v \text{ and } \omega]$$

differenciating both side with respect of time,

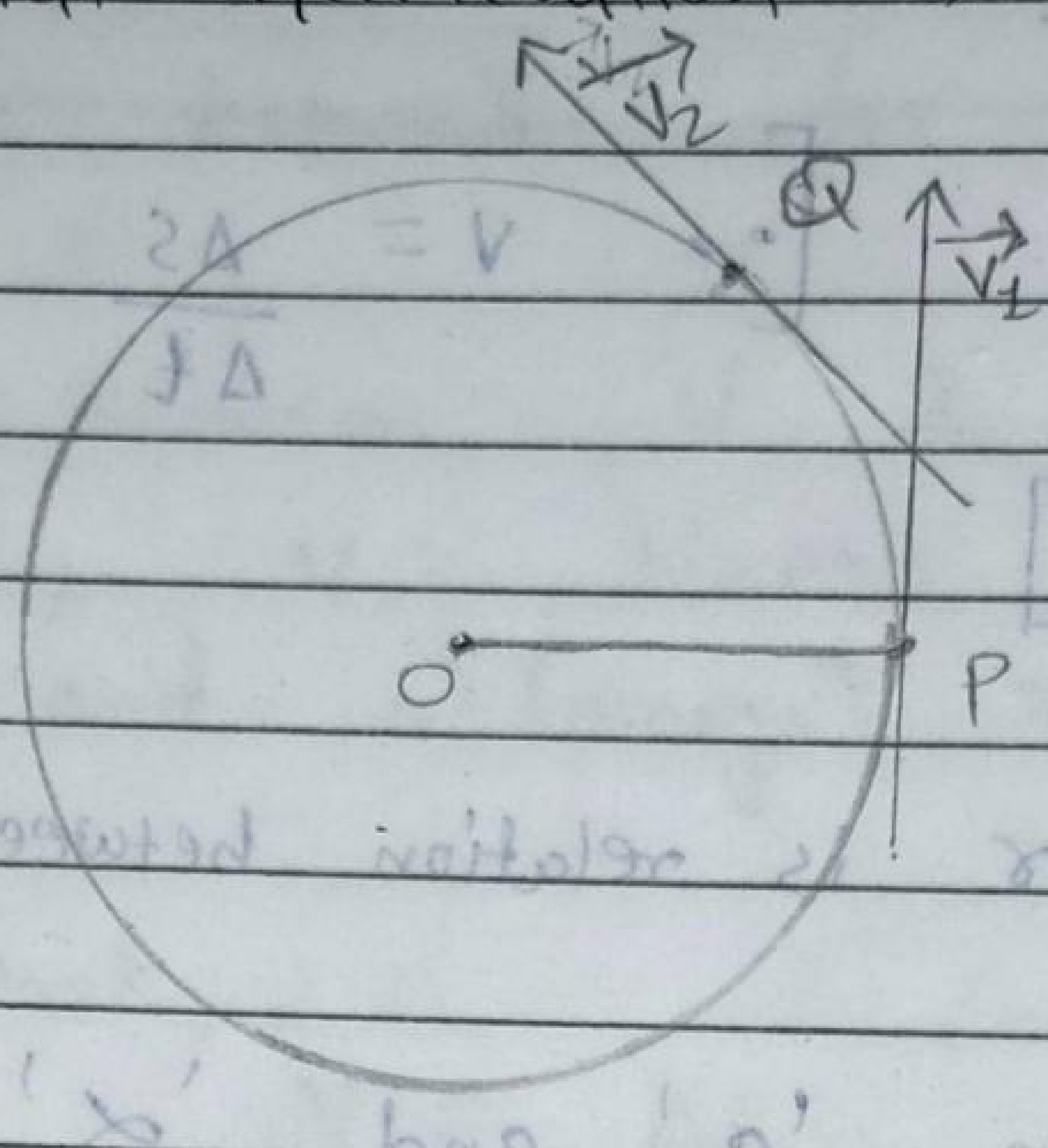
$$\Rightarrow \frac{d}{dt}(v) = \frac{d}{dt}(r\omega)$$

$$\Rightarrow \frac{dv}{dt} = \frac{d r\omega}{dt}$$

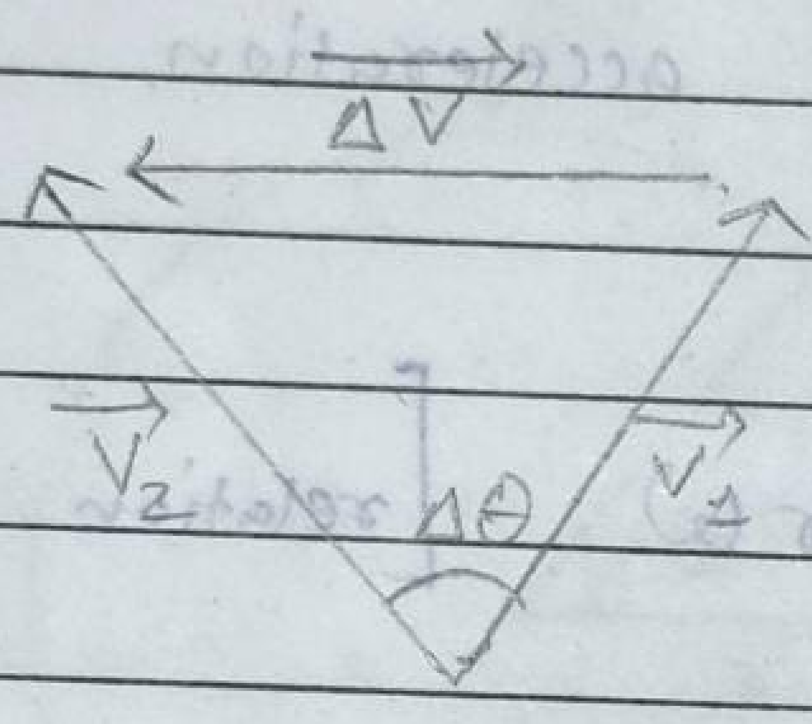
$$\Rightarrow \frac{dv}{dt} = \frac{r\,d\omega}{dt}$$

$$\Rightarrow \boxed{a = r\alpha} \quad \left[\because \text{ we know that, } \alpha = \frac{d\omega}{dt} \text{ and } a = \frac{dv}{dt}\right]$$

Ans.5) Centripetal Acceleration $\Rightarrow$



Drawing $\vec{V_2}$ and $\vec{V_1}$



$\Rightarrow \vec{V_1} + \Delta\vec{V} = \vec{V_2}$ [$\because$ triangle law of ~~motion~~ addition]

$\Rightarrow \vec{\Delta V} = \vec{V_2} - \vec{V_1}$

and $\Delta\theta = \dfrac{\text{arc}}{\text{radius}} = \dfrac{\Delta V}{V}$ —— (i)

[$\because |\vec{V_2}| = |\vec{V_1}| = V$]

We know that,

$\Rightarrow \omega = \dfrac{\Delta\theta}{\Delta t}$

$\Rightarrow \Delta\theta = \omega\,\Delta t$ —— (ii) from eq^n (i) and (ii)

$$\Rightarrow \frac{\Delta v}{v} = \omega \Delta t$$

$$\Rightarrow \frac{\Delta v}{\Delta t} = \omega v$$

$$\Rightarrow a = v\omega \qquad \left[\because a = \frac{\Delta v}{\Delta t}\right]$$

$$\Rightarrow a = v \cdot \frac{v}{r}$$

$$\Rightarrow a = \frac{v^2}{r} \qquad \left[\therefore \text{ we know that, } \omega = \frac{v}{r} \Rightarrow v = \omega r\right]$$

→ एक समान वित्तीय गति करने वाले कर्ण वेग सदीश लगातार परिवर्तित होते रहता है किन्तु उसका चाल स्थिर रहता है।

→ ~~जब कोई कर्ण एक समान चाल के वित्तीय गति करता है।~~

→ when the body is in uniform circular motion then the velocity is different in diff. points but speed is same.

→ when the body is in circular motion then is a acceleration towards the centre of the circular path. This acceleration is known as centripetal acceleration.