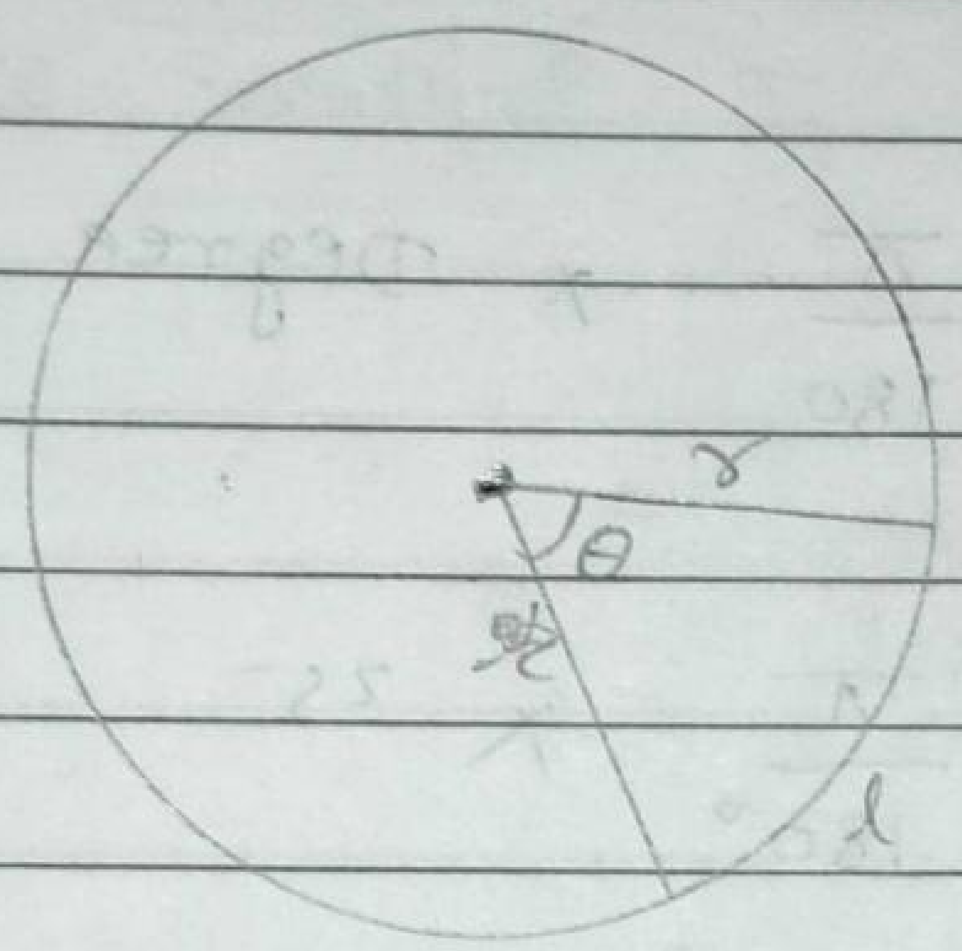
Chapter - 3
Trigonometric Functions

1) Radian ⇒



⇒ Angle at the centre = $\frac{\text{length of arc}}{\text{radius}}$

$$\boxed{\underset{\text{(radian)}}{\theta} = \frac{l}{r}}$$

⇒ 1 Radian = $57^\circ 16'$

⇒ π radian = 180° $\left(\because \pi = \frac{22}{7}\right)$

⇒ $1^\circ = 60'$

⇒ $1' = 60''$

Formula :-

a) Radian measure = $\frac{\pi}{180} \times$ degree measure

b) Degree = $\frac{180}{\pi} \times$ Radian measure

Exercise 3.1

Q.1) Find the radian measures corresponding to the following degree measures

i) 25°

Sol. ⇒ Radian Measure $= \frac{\pi}{180} \times$ Degree measure

⇒ Radian measure $= \frac{\pi}{180^\circ} \times 25$

⇒ Radian measures $= \frac{5\pi}{36}^{c}$

ii) $-47^\circ 30'$

Sol.

$D = -47^\circ 30'$

$D = -(47^\circ + 30')$

$D = -\left(47 + \frac{30}{60}\right)^\circ$

$D = -\left(47 + \frac{1}{2}\right)^\circ$

$D = \left(\frac{95}{2}\right)^\circ$

$D = \left(-\frac{95}{2}\right)^\circ$

⇒ Radian measure $= \frac{\pi}{180^\circ} \times$ degree measure

$= \frac{\pi}{180} \times \frac{-95}{2}$ ⇒ $\frac{\pi}{36} \times \frac{-19}{2}$

$\boxed{\text{Radian measure} = \frac{-19\pi}{72}}$ Ans.

Q.2. Find the degree measures corresponding to the following radian measures. (use $\pi = 22/7$).

i) $\theta = \frac{11}{16}$, $\pi = \frac{22}{7}$ (3.14)

Sol

Degree measure $= \frac{180}{\pi} \times$ radian measure

$= \frac{\overset{11.25}{\cancel{180}}}{(22/7)} \times \frac{11}{\cancel{16}}$

$= \frac{11.25 \times 11}{(22/7)}$

$= \frac{123.75}{22} \times 7$

$= 123.75 \times 0.31$

$= (39.37)^{\circ}$

Degree measure $= 39^{\circ} 22' 12''$ Ans.

Rough

$\Rightarrow$ 11.25×11

1125
1125x

123.75 $\Rightarrow \frac{7}{22} = 0.31$

$\Rightarrow$ 123.75×0.31

39.375

conversion $\rightarrow$

$\Rightarrow$ $.37 \times 60$

22.20

$\Rightarrow$ $.20 \times 60$

$= 12.00$

$= 12$

ii) $\theta^c = -4$

Sol. $\Rightarrow$ Degree measure $= \frac{180}{\pi} \times$ radian measure

$= \frac{180}{22/7} \times -4$

$= \frac{180 \times -4 \times 7}{22}$

$= 8.18 \times -28$

$\Rightarrow$ Degree measure $= (-229.09)^\circ$

$\Rightarrow$ Degree measure $= -229^\circ 5' 24''$

Ans.

Rough.

$\Rightarrow \dfrac{.09 \times 60}{5.40}$ $\Rightarrow \dfrac{.40 \times 60}{24.0\ 0}$

iii) $\theta^c = \dfrac{5\pi}{3}$.

Sol. $\Rightarrow$ Degree measure $= \dfrac{180}{\pi} \times$ radian measure

$= \dfrac{180}{22/7} \times \dfrac{5\pi}{3}$

$= \dfrac{60}{\pi} \times 5\pi$

Degree measure $= 300^\circ$ Ans.

iv) $\theta^c = \dfrac{7\pi}{6}$

Sol. $\Rightarrow$ Degree measure $= \dfrac{180}{\pi} \times$ radian measure

$= \dfrac{180}{\pi} \times \dfrac{7\pi}{6}$

$= 30 \times 7$

Degree measure $= 210^\circ$ Ans.

Q.1) Find the radian measures corresponding to degree measures-

iii) $D = 240^\circ$

Sol. $D = 240^\circ$

$\Rightarrow$ Radian measure $= \frac{\pi}{180} \times$ degree measure

$= \frac{\pi}{180} \times 240^\circ$

$= \frac{\pi}{3} \times 4$

$\Rightarrow$ Radian measure $= \left(\frac{4\pi}{3}\right)^c$ Ans.

iv) $D = 520^\circ$

Sol. $D = 520^\circ$

$\Rightarrow$ Radian measure $= \frac{\pi}{180^\circ} \times$ (degree measure)

$= \frac{\pi}{180} \times 520$

$= \frac{\pi}{90} \times 260 \Rightarrow \frac{\pi}{45} \times 130$

$= \frac{\pi}{9} \times 26$

$\Rightarrow$ Radian measure $= \left(\frac{26\pi}{9}\right)^c$ Ans.

Q.3. A wheel makes 360 revolutions in one minute. Through how many radians does in turn in 1 sec.?

Sol. Since, In 1 minute wheel makes 360 revolution.

$\therefore$ In 1 second, no. of revolution $= 360 \times \frac{1}{60}$

$= 6$ revolution

We know that, In one revolution wheel makes $(2\pi)^c$ angle at the centre.

$\therefore$ In 6 revolution $= 6 \times 2\pi = (12\pi)^c$

Hence In one second wheel makes $(12\pi)^c$.

Q.4. Find the degree measure of the angle subtended at the centre of radius 100 cm by an arc of length 22cm. (use $\pi = \frac{22}{7}$).

Sol. Given,

radius $(r) = 100$ cm

length of arc $(l) = 22$ cm

We know that,

$\Rightarrow \theta \text{ (radian)} = \frac{l}{r}$

$\Rightarrow \theta = \left(\frac{22}{100}\right)^c$

$\Rightarrow$ Degree measure $= \frac{180}{\pi} \times$ radian measure

$$= \frac{180}{22/7} \times \frac{22}{100}$$

$$= \frac{90}{22} \times \frac{11}{10} \times 7$$

$$= 4.5 \times 7$$

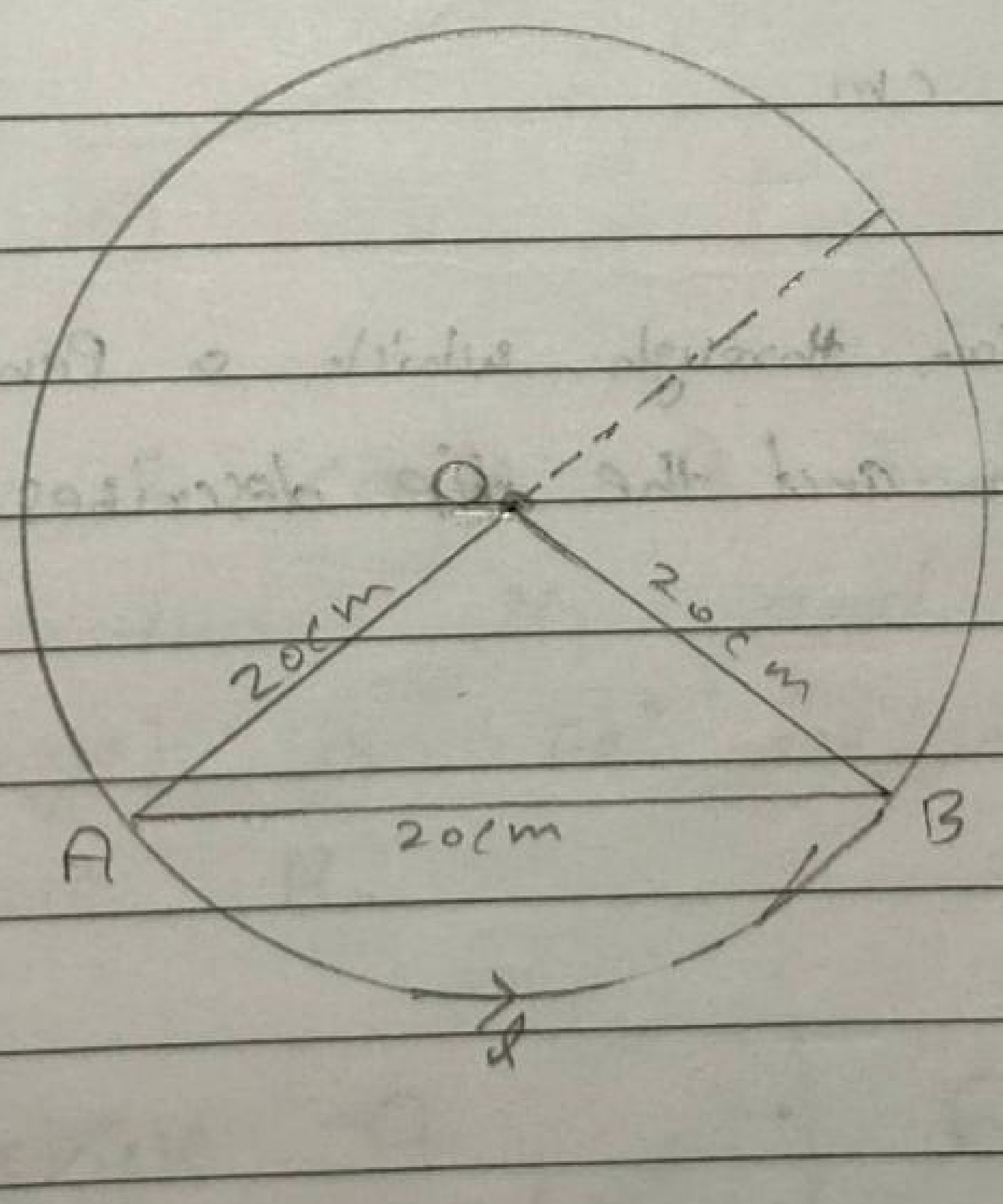
$\Rightarrow$ Degree measure $= 31.5$

$= 31^\circ 30'$

Rough –

$\Rightarrow .5 \times 60 = 30$

⑤ In a circle of diameter 40cm, the length of chord is 20cm. Find the length of minor arc of the chord.



Given,

diameter $= 40$ cm

$\therefore$ radius $= 20$ cm $= OA = OB$

$\Rightarrow$ and, lenghth of chord (AB) = 20 cm

Hence, ΔAOB is a equilateral triangle.

~~ΔAOB is~~ $\therefore$ $\angle AOB = 60^\circ = 60^\circ \times \frac{\pi}{180^\circ} = \left(\frac{\pi}{3}\right)^c$

Now,

$\Rightarrow \quad \theta = \frac{l}{r}$

$\Rightarrow \quad \frac{\pi}{3} = \frac{l}{20}$

$\Rightarrow \quad l = \frac{20\pi}{3}$

$\Rightarrow \quad l = \frac{20}{3} \times \frac{22}{7}$

$\Rightarrow \quad l = \frac{20}{3} \times 3.14$

$\Rightarrow \quad l = 20 \times 1.04$

$\Rightarrow \quad l = 22.80$ cm

Q.4. Find the angle in radian through which a Pendulam swings if its lenghth is 75 cm and the tip describes an arc of length

i) 10 cm

Sol. Given,

$l = 10$ cm

$r = 75$ cm

We know that,

$\theta = \frac{l}{r} = \frac{10}{75} = (0.13)^c$ or $= \left(\frac{2}{15}\right)^c$

ii) 15 cm

Sol. Given,

$\Rightarrow \quad l = 15 cm$

$\Rightarrow \quad r = 75 cm$

We know that,

$\Rightarrow \quad \theta = \frac{l}{r}$

$\Rightarrow \quad \theta = \frac{15}{75} = \left(\frac{1}{5}\right)^c$ Ans.

iii) 21 cm.

Sol. $\rightarrow$ Given,

$\Rightarrow \quad l = 21 cm$

$\Rightarrow \quad r = 75 cm$

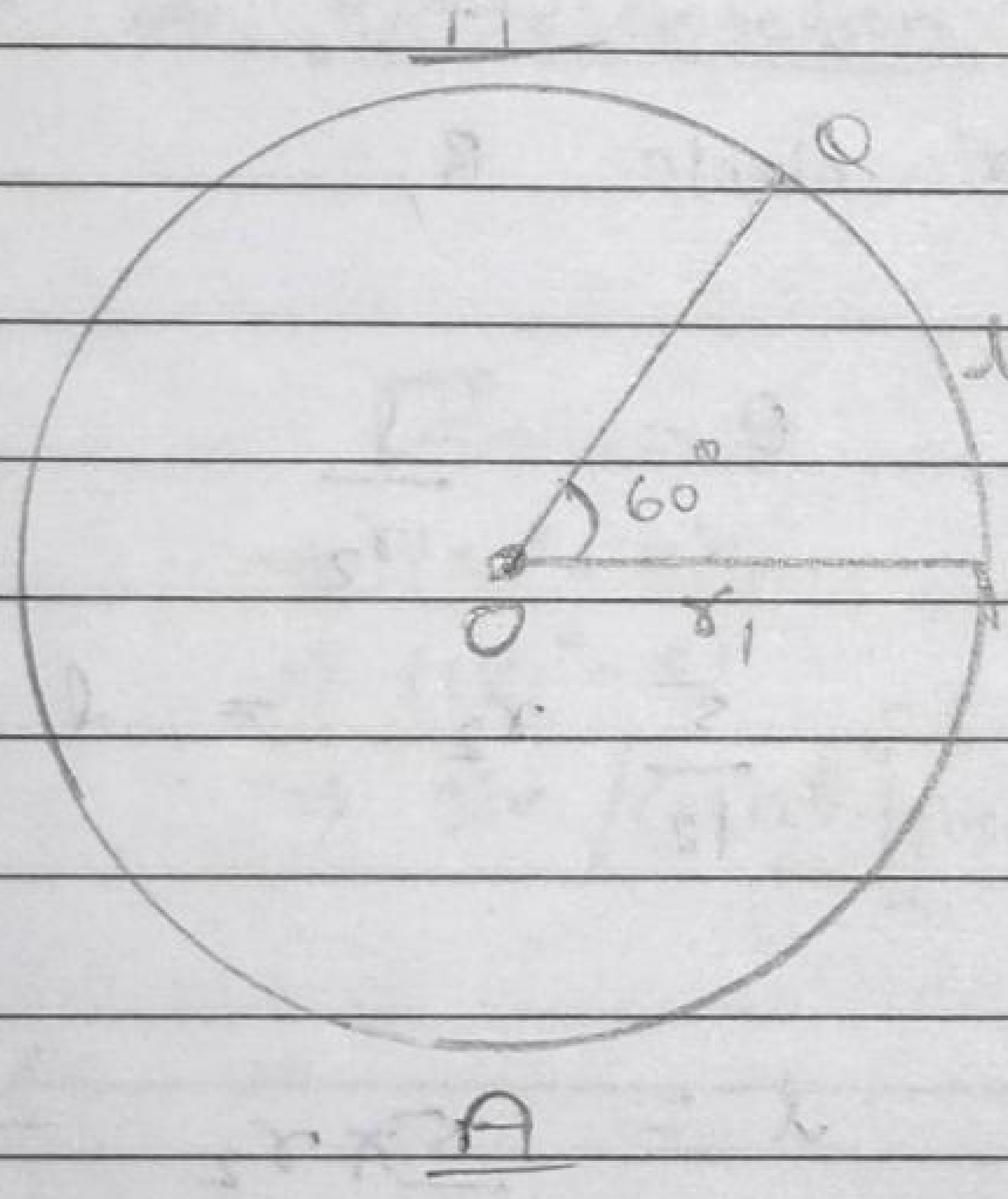
We know that,

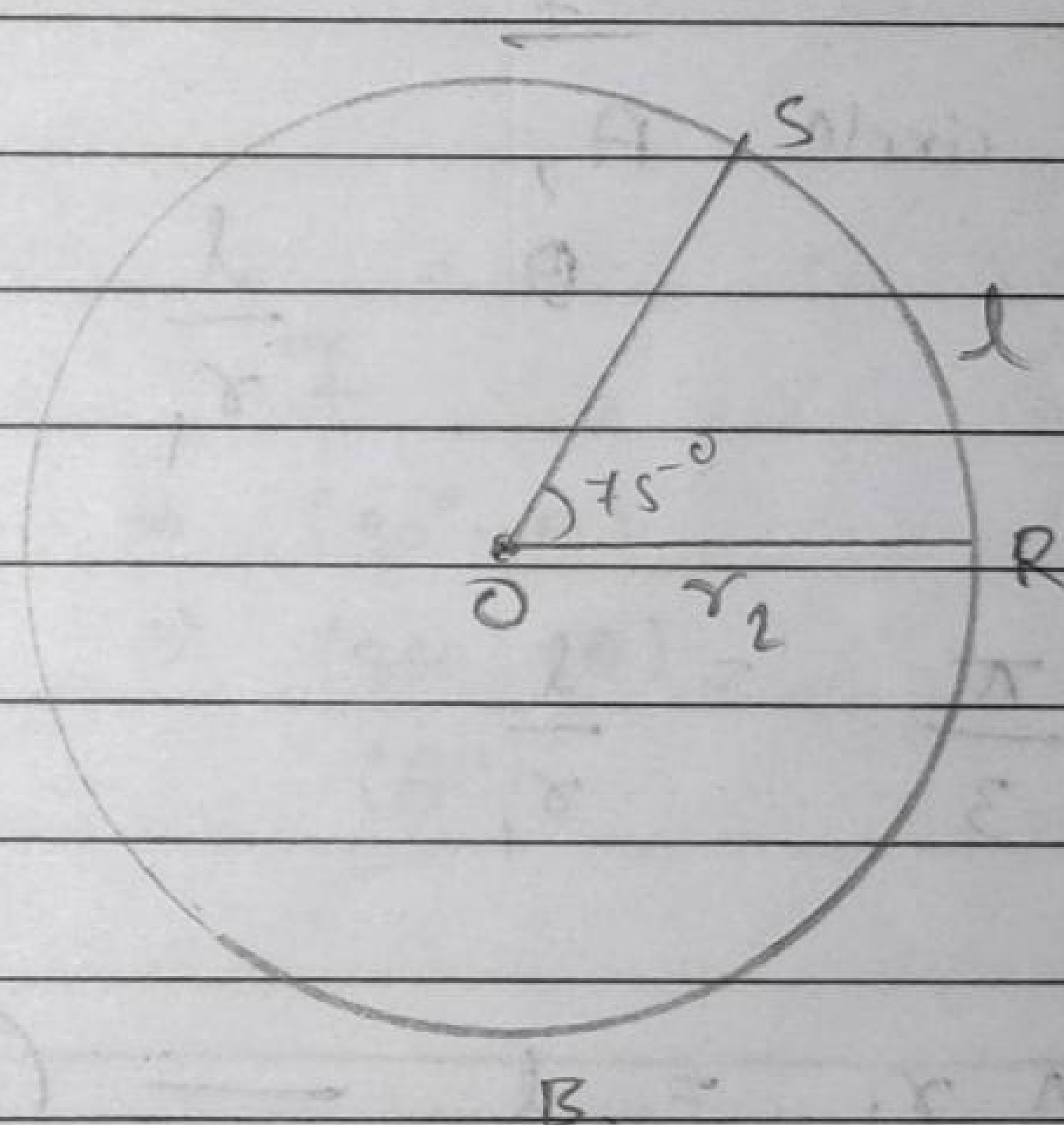
$\Rightarrow \quad \theta = \frac{l}{r}$

$\Rightarrow \quad \theta = \frac{21}{75} = \left(\frac{7}{25}\right)^c$ Ans.

Q.6. If in 2 circle, ~~of diameter~~ arcs of the same length subtended angles 60° and 75° at the centre, find the ratio of their radii.

Sol.





Given, 2 circle A and B of same length 'l' and radius r_1 and r_2 respectively. and θ (angle) is 60° for circle A and 75° for circle B.

For circle A,

$\Rightarrow$ degree $= \frac{180}{\pi} \times$ radian

For circle B,

$\Rightarrow$ degree $= \frac{180}{\pi} \times$ radian

$\Rightarrow 60^\circ = \frac{180}{\pi} \times \text{radian}$

$\Rightarrow \frac{60 \times \pi}{180} = \text{radian}$

$\Rightarrow \left(\frac{1}{3}\pi\right)^c = \text{radian}$

$\Rightarrow$ Radian measure(θ) $= \left(\frac{\pi}{3}\right)^c$

$\Rightarrow 75^\circ = \frac{180}{\pi} \times \text{radian}$

$\Rightarrow \frac{75 \times \pi}{180} = \text{radian}$

$\Rightarrow \frac{15\pi}{36} = \text{radian}$

$\Rightarrow$ Radian measure(θ) $= \left(\frac{5}{12}\pi\right)^c$

We know that,

$$\theta = \frac{l}{r}$$

for circle A,

$\Rightarrow \theta = \frac{l}{r_1}$

$\Rightarrow \frac{\pi}{3} = \frac{l}{r_1}$

$\Rightarrow \frac{\pi}{3} r_1 = l$ —— (i)

for circle B,

$\Rightarrow \theta = \frac{l}{r_2}$

$\Rightarrow \frac{5}{12} r_2 = l$

$\Rightarrow l = \frac{5\pi r_2}{12}$ —— (ii)

from eq^n (i) and (ii),

$\Rightarrow \frac{\pi}{3} r_1 = \frac{5}{12}\pi \cdot r_2$

$\Rightarrow r_1 = \frac{5}{4} r_2$

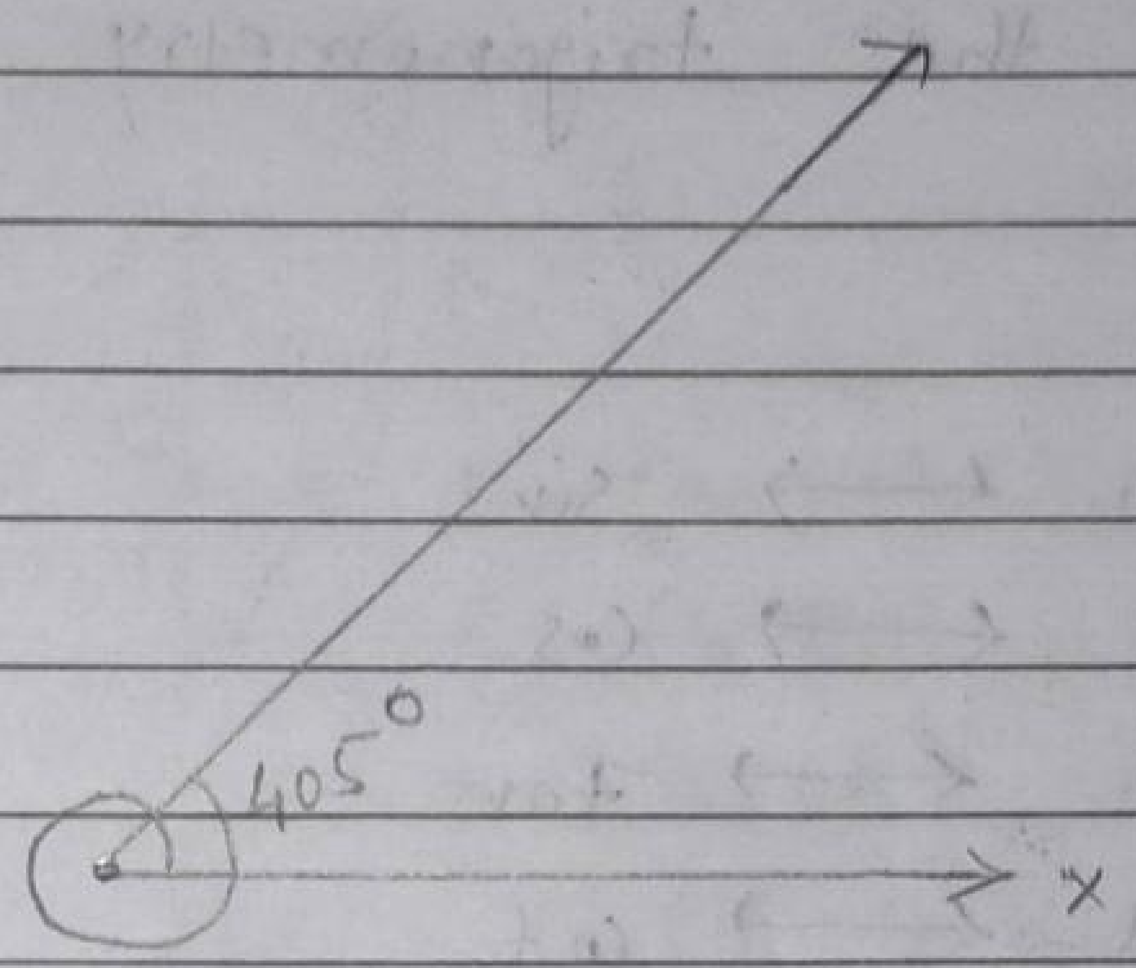
$\Rightarrow \frac{r_1}{r_2} = \frac{5}{4} = 5:4$,

Hence, ratio of the radii is $5:4$.

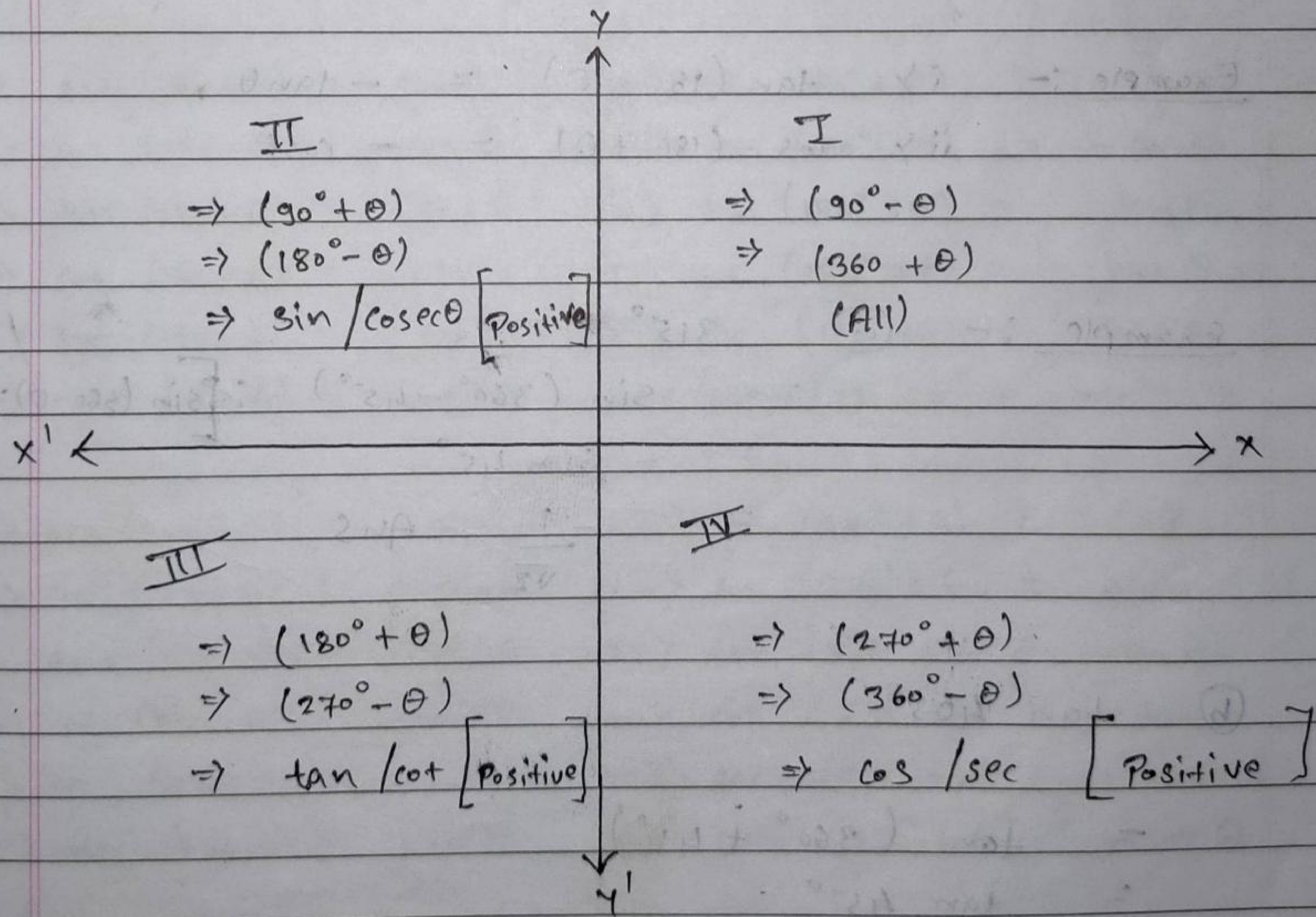
Notes

* Representation of angle more than 360° :-

i) 405°



⇒ Representation :-



NoTe :- i) when we convert with respect 90° or 270° then trigonometry ratio also changed.

sin $\longleftrightarrow$ $\cos\theta$

tan $\longleftrightarrow$ cot

sec $\longleftrightarrow$ cosec

Example :- $\sin(90^\circ+\theta) = \sin\theta$

ii) When we convert with respect of 180° and 360° the trigonometry ratio will not change, means :-

sin $\longleftrightarrow$ sin

cos $\longleftrightarrow$ cos

tan $\longleftrightarrow$ tan

cot $\longleftrightarrow$ cot

sec $\longleftrightarrow$ sec

cosec $\longleftrightarrow$ cosec

Example :- i) $\tan(180-\theta) = -\tan\theta$

ii) $\cos(180+\theta) = -\cos\theta$

(2) Example :- (a) 315°

$$= \sin(360^\circ - 45^\circ) \quad \because [\sin(360-\theta) = -\sin\theta]$$

$$= -\sin 45^\circ$$

$$= -\frac{1}{\sqrt{2}} \text{ Ans}$$

(b) $\tan 405^\circ$

$$= \tan(360^\circ + 45^\circ)$$

$$= \tan 45^\circ$$

$$= 1 \text{ Ans.}$$

Formulas :-

i) $\sin(-\theta) = -\sin\theta$
ii) $\cos(-\theta) = \cos\theta$
iii) $\tan(-\theta) = -\tan\theta$

iv) $\cot(-\theta) = -\cot\theta$
v) $\sec(-\theta) = \sec\theta$
vi) $\operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta$

i) $\sin(90-\theta) = \cos\theta$
ii) $\cos(90-\theta) = \sin\theta$
iii $\tan(90-\theta) = \cot\theta$
iv) $\cot(90-\theta) = \tan\theta$
v) $\sec(90-\theta) = \operatorname{cosec}\theta$
vi) $\operatorname{cosec}(90-\theta) = \sec\theta$

i) $\sin(90+\theta) = \cos\theta$
ii) $\cos(90+\theta) = -\sin\theta$
iii) $\tan(90+\theta) = -\cot\theta$
iv) $\cot(90+\theta) = -\tan\theta$
v) $\sec(90+\theta) = -\operatorname{cosec}\theta$
vi) $\operatorname{cosec}(90+\theta) = \sec\theta$

vii) $\sin(180-\theta) = \sin\theta$
viii) $\cos(180-\theta) = -\cos\theta$
ix) $\tan(180-\theta) = -\tan\theta$
x) $\cot(180-\theta) = -\cot\theta$
xi) $\sec(180-\theta) = -\sec\theta$
xii) $\operatorname{cosec}(180-\theta) = \operatorname{cosec}\theta$

vii) $\sin(180^\circ+\theta) = -\sin\theta$
viii) $\cos(180^\circ+\theta) = -\cos\theta$
ix) $\tan(180^\circ+\theta) = \tan\theta$
x) $\cot(180^\circ+\theta) = \cot\theta$
xi) $\sec(180^\circ+\theta) = -\sec\theta$
xii) $\operatorname{cosec}(180^\circ+\theta) = -\operatorname{cosec}\theta$

xiii) $\sin(270-\theta) = -\cos\theta$
xiv) $\cos(270-\theta) = -\sin\theta$
xv) $\tan(270-\theta) = \cot\theta$
xvi) $\cot(270-\theta) = \tan\theta$
xvii) $\sec(270-\theta) = -\operatorname{cosec}\theta$
xviii) $\operatorname{cosec}(270-\theta) = -\sec\theta$

xiii) $\sin(270^\circ+\theta) = -\cos\theta$
xiv) $\cos(270^\circ+\theta) = \sin\theta$
xv) $\tan(270^\circ+\theta) = -\cot\theta$
xvi) $\cot(270^\circ+\theta) = -\tan\theta$
xvii) $\sec(270^\circ+\theta) = \operatorname{cosec}\theta$
xviii) $\operatorname{cosec}(270^\circ+\theta) = -\sec\theta$

xix) $\sin(360^\circ-\theta) = -\sin\theta$
xx) $\cos(360^\circ-\theta) = \cos\theta$
xxi) $\tan(360^\circ-\theta) = -\tan\theta$

xix) $\sin(360^\circ+\theta) = \sin\theta$
xx) $\cos(360^\circ+\theta) = \cos\theta$
xxi) $\tan(360^\circ+\theta) = \tan\theta$

xxii) $\cot(360^\circ \mp \theta) = -\cot\theta$	xxii) $\cot(360^\circ + \theta) = \cot\theta$
xxiii) $\sec(360^\circ \mp \theta) = \sec\theta$	xxiii) $\sec(360^\circ + \theta) = \sec\theta$
xxiv) $\operatorname{cosec}(360^\circ \mp \theta) = -\operatorname{cosec}\theta$	xxiv) $\operatorname{cosec}(360^\circ + \theta) = \operatorname{cosec}\theta$

i) $\sin(n.360 - \theta) = -\sin\theta$

ii) $\cos(n.360 - \theta) = \cos\theta$

iii) $\tan(n.360 - \theta) = -\tan\theta$

iv) $\cot(n.360 - \theta) = -\cot\theta$

v) $\sec(n.360 - \theta) = \sec\theta$

vi) $\operatorname{cosec}(n.360 - \theta) = -\operatorname{cosec}\theta$, here $\forall\, n \in I$

vii) $\sin(n.360 + \theta) = \sin\theta$

viii) $\cos(n.360 + \theta) = \cos\theta$

ix) $\tan(n.360 + \theta) = \tan\theta$

x) $\cot(n.360 + \theta) = \cot\theta$

xi) $\sec(n.360 + \theta) = \sec\theta$

xii) $\operatorname{cosec}(n.360 + \theta) = \operatorname{cosec}\theta$

i) $30^\circ \rightarrow (\pi/6)^c$

ii) $45^\circ \rightarrow (\pi/4)^c$

iii) $60^\circ \rightarrow (\pi/3)^c$

iv) $90^\circ \rightarrow (\pi/2)^c$

v) $180^\circ \rightarrow (\pi)^c$

vi) $270^\circ \rightarrow (3\pi/2)^c$

vii) $360^\circ \rightarrow (2\pi)^c$

viii) $0^\circ \rightarrow 0$

Exercise 3.2

Q.1.

Sol. Given, x lies in 3^{rd} quadrant.

and $\cos x = \frac{-1}{2}$

here, $\therefore B = 1$, $H = 2$.

$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow (2)^2 = P^2 + (1)^2$

$\Rightarrow \boxed{P = \sqrt{3}}$

$\Rightarrow \sin x = \frac{P}{H} = -\frac{\sqrt{3}}{2}$, x - lies in III quadrant.

$\Rightarrow \tan x = \frac{P}{B} = \sqrt{3}$

$\Rightarrow \cot x = \frac{B}{P} = \frac{1}{\sqrt{3}}$

$\Rightarrow \sec x = \frac{H}{B} = -\frac{2}{\cancel{\sqrt{3}}}$, x lies in 3^{rd} quadrant.

$\Rightarrow \operatorname{cosec} x = \frac{H}{P} = -\frac{2}{\sqrt{3}}$, x lies in 3^{rd} quadrant.

Q.2.

Sol. Given, x - lies in 2^{nd} quadrant.

and $\sin x = \frac{3}{5}$

Here, $\sin x = \frac{P}{H}$

$\Rightarrow \quad P = 3, \quad H = 5$

According to pythagoras theorem,

$\Rightarrow \quad H^2 = P^2 + B^2$

$\Rightarrow \quad (5)^2 = (3)^2 + (B)^2$

$\Rightarrow \quad B = 4$

so, $\cos x = \frac{B}{H} = -\frac{4}{5}$

$\tan x = \frac{P}{B} = -\frac{3}{4}$

$\cot x = \frac{B}{P} = -\frac{4}{3}$

$\sec x = \frac{H}{B} = -\frac{5}{4}$

$\operatorname{cosec} x = \frac{1}{\sin x} = \frac{1}{3/5} = \frac{5}{3}$

Q.3.

Sol. Given $\cot x = \cancel{\frac{3}{5}}\ \frac{3}{4}$, x lies in 3^{nd} quadrant.

We know that,

$$\cot x = \frac{B}{P}$$

$\Rightarrow B = 3$ and $P = 4$

According to Pythagoras theorem :-

$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow H^2 = (4)^2 + (3)^2$

$\Rightarrow H = 5$

So, $\sin x = \frac{P}{H} = -\frac{4}{5}$

$\cos x = \frac{B}{H} = -\frac{3}{5}$

$\tan x = \frac{P}{B} = \frac{4}{3}$

$\sec x = \frac{H}{B} = -\frac{5}{3}$

$\operatorname{cosec} x = \frac{H}{P} = -\frac{5}{4}$

Q.4.

Sol. Given, x - lies in 4^{th} quadrant.

and $\sec x = \frac{13}{5}$

We know that,

$$\sec x = \frac{H}{B}$$

$\Rightarrow H = 13, \quad B = 5$

According to Pythagoras theorem,

$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow (13)^2 = P^2 + (5)^2$

$\Rightarrow P = 12$

So,

$\sin x = \frac{P}{H} = -\frac{12}{13}$, $\because x$ lies in 4^{th} quadrant

$\cos x = \frac{B}{H} = \frac{5}{13}$

$\tan x = \frac{P}{B} = -\frac{12}{5}$, —— o – o ——

$\cot x = \frac{B}{P} = -\frac{5}{12}$, — o – o —

$\operatorname{cosec} x = \frac{H}{P} = -\frac{13}{12}$, — o – o —

Q.5.

Sol. Given, x lies in second quadrant,

and $\tan x = -\frac{5}{12}$

$\Rightarrow$ We know that;

$$\tan x = \frac{P}{B}$$

$\Rightarrow P = 5, \quad B = 12$

According to Pythagoras theorem,

$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow H^2 = (5)^2 + (12)^2$

$\Rightarrow H = 13$

So,

$\sin x = \frac{P}{H} = \frac{5}{13}$,

$\cos x = \frac{B}{H} = -\frac{12}{13}$, $\because x$ lies in 2^{nd} quadrant

$\cot x = \frac{B}{P} = -\frac{12}{5}$, $\because x$ lies in 2^{nd} quadrant

$\sec x = \frac{H}{B} = -\frac{13}{12}$, $\because x$ lies in 2^{nd} quadrant

$\operatorname{cosec} x = \frac{H}{P} = \frac{13}{5}$

Q.6. $\sin 765^\circ$, value of trigonometric function?

Example) $\tan 225^\circ$

$\tan 225^\circ = \tan(180^\circ + 45^\circ)$

$[\because \tan(180^\circ + \theta) = \tan\theta]$

$\Rightarrow \tan 225^\circ = \tan 45^\circ$

$\Rightarrow \tan 225^\circ = 1$ $\quad [\because \tan 45^\circ = 1]$

Q. 7) $\operatorname{cosec}(-1410^\circ)$

Sol. $= -\operatorname{cosec} 1410^\circ$ $\quad [\because \operatorname{cosec}(-\theta) = -\operatorname{cosec}\theta]$

$= -\operatorname{cosec}(3 \times 2\pi + 330^\circ)$

$= -\operatorname{cosec} 330^\circ$ $\quad [\because \operatorname{cosec}(n \cdot 360 + \theta) = \operatorname{cosec}\theta]$

$= -\operatorname{cosec}(360^\circ - 30^\circ)$

$= (-)(-)\operatorname{cosec} 30^\circ$ $\quad [\because \operatorname{cosec}(360^\circ - \theta) = -\operatorname{cosec}\theta]$

$= \operatorname{cosec} 30^\circ$

$= 2$ Ans. $\quad [\because \operatorname{cosec} 30^\circ = 2]$

Q.9. $\sin\left(-\frac{11\pi}{3}\right)$

Sol. $= \sin(-660^\circ)$

$= -\sin 660^\circ$ $\quad [\because \sin(-\theta) = -\sin\theta]$

Rough.

$= \frac{11 \times 180}{3} = 660^\circ$

1st method

$= -\sin(360^\circ + 300^\circ)$

$= -\sin 300^\circ$ $\quad [\because \sin(360^\circ + \theta) = \sin\theta]$

$= -\sin(360^\circ - 60^\circ)$

$[\because \sin(360^\circ - \theta) = -\sin\theta]$

$= \sin 60^\circ$

$\sin\left(\frac{-11\pi}{3}\right) = \frac{\sqrt{3}}{2}$ Ans.

2nd method

$\sin\left(\frac{-11\pi}{3}\right) = -\sin(2.360-60)$ $[\because \sin(n.360-\theta) = -\sin\theta]$

$= (-)(-)\sin 60^\circ$

$\sin\left(\frac{-11\pi}{3}\right) = \frac{\sqrt{3}}{2}$ Ans.

Q.6. $\sin 765^\circ$

Sol. $\sin 765^\circ = \sin(2.360+45^\circ)$

$= \sin 45^\circ$ $[\because \sin(n.360^\circ+\theta) = \sin\theta]$

$\sin 765^\circ = \frac{1}{\sqrt{2}}$ Ans.

Q.8. $\tan\frac{19\pi}{3}$

Sol. $\tan\frac{19\pi}{3} = \tan\left(\frac{19\times 180^\circ}{3}\right)$ $[\because \pi^c = 180^\circ]$

$= \tan(1140^\circ)$

$= \tan(3.360+60^\circ)$

$= \tan 60^\circ$ $[\tan(n.360+\theta) = \tan\theta]$

$\tan\frac{19\pi}{3} = \sqrt{3}$ Ans.

Q.10 $\cot\left(-\frac{15\pi}{4}\right)$

Sol. $\cot\left(\frac{-15\pi}{4}\right) = -\cot\left(\frac{15\pi}{4}\right)$ $[\because \cot(-\theta) = -\cot\theta]$

$$= -\cot\left(\frac{15}{4} \times 180^\circ\right) \quad \left[\because \pi^c = 180^\circ\right]$$

$$= -\cot\left(675^\circ\right)$$

$$= -\cot\left(2 \cdot 360 - 45^\circ\right)$$

$$= (-)(-)\cot 45^\circ \quad \left[\because \cot(n \cdot 360 - \theta) = -\cot\theta\right]$$

$$= \cot 45^\circ$$

$$= 1 \quad \underline{\text{Ans.}}$$

Trigonometric Formulas

1. $\sin(A+B) = \sin A \cos B + \cos A \sin B$
2. $\sin(A-B) = \sin A \cos B - \cos A \sin B$
3. $\cos(A+B) = \cos A \cos B - \sin A \sin B$
4. $\cos(A-B) = \cos A \cos B + \sin A \sin B$
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A * \tan B}$
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}$
7. $\cot(A+B) = \dfrac{\cot A \cot B - 1}{\cot A + \cot B}$
8. $\cot(A-B) = \dfrac{\cot A \cot B + 1}{\cot B - \cot A}$
9. $2 \sin A \cos B = \sin(A+B) + \sin(A-B)$
10. $2 \cos A \sin B = \sin(A+B) - \sin(A-B)$
11. $2 \cos A \cos B = \cos(A+B) + \cos(A-B)$
12. $2 \sin A \sin B = \cos(A-B) - \cos(A+B)$
13. $\sin A + \sin B = 2 \sin\left(\dfrac{A+B}{2}\right) \cos\left(\dfrac{A-B}{2}\right)$
14. $\sin A - \sin B = 2 \cos\left(\dfrac{A+B}{2}\right) \sin\left(\dfrac{A-B}{2}\right)$
15. $\cos A + \cos B = 2 \cos\left(\dfrac{A+B}{2}\right) \cos\left(\dfrac{A-B}{2}\right)$
16. $\cos A - \cos B = -2 \sin\left(\dfrac{A+B}{2}\right) \sin\left(\dfrac{A-B}{2}\right)$

(17) $\sin 2A = 2\sin A \cos A$

(18) $\cos 2A = \cos^2 A - \sin^2 A$

$\cos 2A = 2\cos^2 A - 1$

$\cos 2A = 1 - 2\sin^2 A$

(19) $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$

(20) $\cot 2A = \dfrac{\cot^2 A - 1}{2\cot A}$

(21) $\sin 3A = 3\sin A - 4\sin^3 A$

(22) $\cos 3A = 4\cos^3 A - 3\cos A$

(23) $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

(24) $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A} \Rightarrow \dfrac{2\tan A}{\sec^2 A}$

(25) $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$

(26) a) $\sin^2\theta + \cos^2\theta = 1$

b) $\sec^2\theta - \tan^2\theta = 1$

c) $\operatorname{cosec}^2\theta - \cot^2\theta = 1$

(27) a) $1 + \cos 2\theta = 2\cos^2\theta$

doubt

b) $1 - \cos 2\theta = 2\sin^2\theta$

c) $1 + \cos\theta = 2\cos^2\dfrac{\theta}{2}$

doubt

d) $1 - \cos\theta = \cancel{2\sin^2\theta}\ 2\sin^2\dfrac{\theta}{2}$

(28) $\sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}$

(29) $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$

(30) $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$

Exercise 3.3

Q.1. $\sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4} = -\frac{1}{2}$

Sol. L.H.S $= \sin^2\frac{\pi}{6} + \cos^2\frac{\pi}{3} - \tan^2\frac{\pi}{4}$

$= \cancel{\sin}\left(\frac{1}{2}\right)^2 + \left(\frac{1}{2}\right)^2 - (1)^2$

$= \frac{1}{4} + \frac{1}{4} - 1$

$= \frac{1}{2} - 1$

L.H.S $= -\frac{1}{2}$

L.H.S $=$ R.H.S

Hence Proved.

Q.2) $2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3} = \frac{3}{2}$

Sol. L.H.S. $= 2\sin^2\frac{\pi}{6} + \operatorname{cosec}^2\frac{7\pi}{6}\cos^2\frac{\pi}{3}$

$= 2\left(\frac{1}{2}\right)^2 + \operatorname{cosec}^2\left(\pi + \frac{\pi}{6}\right)\left(\frac{1}{2}\right)^2$

$= \frac{1}{2} + \left(-\operatorname{cosec}\frac{\pi}{6}\right)^2\left(\frac{1}{4}\right)$

$= \frac{1}{2} + \left(\cancel{\operatorname{cosec}}\,2\right)^2\left(\frac{1}{4}\right)$

$$= \frac{1}{2} + 4\left(\frac{1}{4}\right)$$

$$= \frac{1}{2} + 1$$

$$\text{L.H.S} = \frac{3}{2}$$

L.H.S = R.H.S

Hence Proved.

Q.5) i) $\sin 75^\circ$

$$= \sin(45^\circ + 30^\circ)$$

$$= \sin 45^\circ \cos 30^\circ + \cos 45^\circ \sin 30^\circ$$

$$[\because \sin(A+B) = \sin A \cos B + \cos A \sin B]$$

$$= \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \cdot \cancel{\frac{\sqrt{3}}{2}}\ \frac{1}{2}$$

$$= \frac{\sqrt{3}}{2\sqrt{2}} + \frac{\cancel{\sqrt{2}}\ 1}{2\sqrt{2}}$$

$$= \frac{\sqrt{3}+1}{2\sqrt{2}}$$

Q.8 $$\frac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)} = \cot^2 x$$

Sol. $\underline{L.H.S} = \dfrac{\cos(\pi + x)\cos(-x)}{\sin(\pi - x)\cos\left(\frac{\pi}{2} + x\right)}$

$$= \frac{(-\cos x)(\cos x)}{(\sin x)(-\sin x)}$$

$$\left[\begin{aligned} \because \cos(\pi+\theta) &= -\cos\theta \\ \cos(-\theta) &= \cos\theta \\ \sin(180-\theta) &= \sin\theta \\ \cos(90+\theta) &= -\sin\theta \end{aligned}\right]$$

$$= \frac{+\cos^2 x}{+\sin^2 x}$$

$$= \frac{\cos^2 x}{\sin^2 x}$$

$L.H.S = \cancel{\cot^2 x}\ \cot^2 x$

$L.H.S = R.HS$

Hence Proved.

Q.10.

Sol. $\underline{L.H.S} = \sin(n+1)x \sin(n+2)x + \cos(n+1)x \cos(n+2)x = \cos x$

~~let~~ we consume,

$(n+1)x = A$ $\quad = \cancel{\cos(-x)}$

& $(n+2)x = B$ $\quad =$

$= \sin A \sin B + \cos A \cos B$

$= \cos A \cos B + \sin A \sin B$

$= \cos(A-B)$

$= \cos\{(n+1)x - (n+2)x\}$

$= \cos\{nx + x - (nx + 2x)\}$

$= \cos\{nx + x - nx - 2x\}$

$= \cos\{-x)$

$= \cos x \quad [\because \cos(-\theta) = -\cos\theta]$

L.H.S = R.H.S

Hence proved.

Q.3. $\cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6} = 6$

Sol. L.H.S. $= \cot^2\frac{\pi}{6} + \operatorname{cosec}\frac{5\pi}{6} + 3\tan^2\frac{\pi}{6}$

$= \cot^2(30^\circ) + \operatorname{cosec}(150^\circ) + 3\tan^2 30^\circ$

$= (\sqrt{3})^2 + \operatorname{cosec}(90^\circ + 60) + 3\left(\frac{1}{\sqrt{3}}\right)^2$

$= 3 + \sec 60^\circ + 3 \cdot \frac{1}{3}$

$= 3 + 2 + 1$

$= 6$

L.H.S = R.H.S

Hence proved.

Q.4.

Sol. L.H.S. $= 2\sin^2\frac{3\pi}{4} + 2\cos^2\frac{\pi}{4} + 2\sec^2\frac{\pi}{3} = 10$

$= 2\sin^2(135^\circ) + 2\cos^2 45^\circ + 2\sec^2 60^\circ$

$= 2\sin^2(90^\circ + 45^\circ) + 2 \cdot \frac{1}{2} + 8$

$= 2\cos^2 45^\circ + 1 + 8$

$= 2 \cdot \frac{1}{2} + 9$

$= 1 + 9$

$= 10$

L.H.S = R.H.S

Hence proved.

Q.5, ii) $\tan 15^\circ$

$= \tan(60^\circ - 45^\circ)$

$= \dfrac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \cdot \tan 45^\circ}$

$\left[\because \tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \cdot \tan B}\right]$

$= \dfrac{\sqrt{3} - 1}{1 + (\sqrt{3} \cdot 1)}$

$= \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}}$

rationalising the denominator,

$$= \left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)\left(\frac{\sqrt{3}-1}{\sqrt{3}-1}\right)$$

$$= \frac{(\sqrt{3}-1)^2}{(\sqrt{3})^2-(1)^2} \quad \left[\because (a+b)(a-b) = a^2-b^2\right]$$

$$= \frac{3+1-2\sqrt{3}}{3-1}$$

$$= \frac{4-2\sqrt{3}}{2}$$

$$= \frac{4}{2} - \frac{2\sqrt{3}}{2}$$

$\tan 15^\circ = 2-\sqrt{3}$ Ans.

Q.6.

Sol. L.H.S. $= \cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right) - \sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)$

$$= \frac{1}{2}\left[2\cos\left(\frac{\pi}{4}-x\right)\cos\left(\frac{\pi}{4}-y\right)\right] + \frac{1}{2}\left[-2\sin\left(\frac{\pi}{4}-x\right)\sin\left(\frac{\pi}{4}-y\right)\right]$$

$$= \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} + \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right] +$$

$$\frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4}-x\right)+\left(\frac{\pi}{4}-y\right)\right\} - \cos\left\{\left(\frac{\pi}{4}-x\right)-\left(\frac{\pi}{4}-y\right)\right\}\right]$$

$\because$ i) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

ii) $-2\sin A\sin B = \cos(A+B) - \cos(A-B)$

$$= 2 \times \frac{1}{2}\left[\cos\left\{\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right\}\right]$$

$$= \cos\left[\left(\frac{\pi}{4} - x\right) + \left(\frac{\pi}{4} - y\right)\right]$$

$$= \cos\left[\frac{\pi}{4} - x + \frac{\pi}{4} - y\right]$$

$$= \cos\left[\frac{2\pi}{4} - x - y\right]$$

$$= \cos\left[\frac{\pi}{2} - (x + y)\right]$$

$$= \sin(x + y)$$

L.H.S = R.H.S

Hence Proved.

Q.7.

Sol. L.H.S. $= \dfrac{\tan\left(\frac{\pi}{4} + x\right)}{\tan\left(\frac{\pi}{4} - x\right)}$

$$= \frac{\dfrac{\tan\frac{\pi}{4} + \tan x}{1 - \tan\frac{\pi}{4}\tan x}}{\dfrac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4} \cdot \tan x}}$$

$\because$ i) $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A \cdot \tan B}$

ii) $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A \tan B}$

$$= \frac{\dfrac{1 + \tan x}{1 - 1\tan x}}{\dfrac{1 - \tan x}{1 + 1\tan x}}$$

$$= \frac{\dfrac{1 + \tan x}{1 - \tan x}}{\dfrac{1 - \tan x}{1 + \tan x}}$$

$$= \frac{1 + \tan x}{1 - \tan x}\left(\frac{1 + \tan x}{1 - \tan x}\right)$$

$$L.H.S = \left(\frac{1 + \tan x}{1 - \tan x}\right)^2$$

L.H.S = R.H.S

Hence proved

Q9

Sol. L.H.S. $= \cos\left(\frac{3\pi}{2} + x\right)\cos(2\pi + x)\left[\cot\left(\frac{3\pi}{2} - x\right) + \cot(2\pi + x)\right]$

$$= \cos(270^\circ + x)\cos(360^\circ + x)\left[\cot(270^\circ - x) + \cot(360 + x)\right]$$

$$= \sin x \cos x\left[\tan x + \cot x\right]$$

$$= \sin x \cos x\left(\frac{\sin x}{\cos x} + \frac{\cos x}{\sin x}\right)$$

$$= \sin x \cos x\left(\frac{\sin^2 x + \cos^2 x}{\cos x \cdot \sin x}\right)$$

$L.H.S = 1$

$L.H.S = R.H.S$

Hence Proved.

Q.11

Sol. $L.H.S. = \cos\left(\frac{3\pi}{4}+x\right) - \cos\left(\frac{3\pi}{4}-x\right)$

$= -2\sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)+\left(\frac{3\pi}{4}-x\right)}{2}\right\} \cdot \sin\left\{\frac{\left(\frac{3\pi}{4}+x\right)-\left(\frac{3\pi}{4}-x\right)}{2}\right\}$

$\left[\because \cancel{\cos(A+B)}\ \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$\cancel{= -2\sin\left\{3\times 45^\circ + x\right\}}$

$= -2\sin\left\{\frac{\frac{3\pi}{4}+x+\frac{3\pi}{4}-x}{2}\right\} \cdot \sin\left\{\frac{\frac{3\pi}{4}+x-\frac{3\pi}{4}+x}{2}\right\}$

$= -2\sin\left\{\frac{\frac{3\pi+3\pi}{4}}{2}\right\} \cdot \sin\left\{\frac{\cancel{\frac{3\pi+3\pi}{4}}\ x+x}{2}\right\}$

$= -2\sin\left(\frac{\frac{6\pi}{4}}{2}\right) \cdot \sin\left(\frac{2x}{2}\right)$

$= -2\sin\frac{3\pi}{2} \cdot \sin x$

$= -2\sin\left(\pi - \frac{\pi}{4}\right) \cdot \sin x$

$= -2\sin\frac{\pi}{4} \cdot \sin x$

$= -2 \cdot \frac{1}{\sqrt{2}} \cdot \sin x$

$= -\sqrt{2}\sin x$

L.H.S = R.H.S

Hence proved.

Q.12.

Sol. L.H.S = $\sin^2 6x - \sin^2 4x$

$= (\sin 6x + \sin 4x)(\sin 6x - \sin 4x)$

$= \left[2\sin\left(\frac{6x+4x}{2}\right)\cos\left(\frac{6x-4x}{2}\right)\right]\left[2\cos\left(\frac{6x+4x}{2}\right)\sin\left(\frac{6x-4x}{2}\right)\right]$

$\left[\because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)\right]$

$= [2\sin 5x \cos x][2\cos 5x \sin x]$

$= (2\sin 5x \cos 5x)(2\sin x \cos x)$

$= \sin 2(5x) \quad \sin 2(x)$

$[\because \sin 2A = 2\sin A \cos A]$

$= \sin 10x \quad \sin 2x$

L.H.S = R.H.S

Hence proved.

Q. 13.

Sol. $\Rightarrow \cos^2 2x - \cos^2 6x = \sin 4x \sin 8x$

$\Rightarrow$ L.H.S $= \cos^2 2x - \cos^2 6x$

$= (\cos 2x + \cos 6x)(\cos 2x - \cos 6x)$

~~$= (1-2\sin^2 2x) - 1 - 2\sin$~~

$= \left[2\cos\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right]\left[-2\sin\left(\frac{2x+6x}{2}\right)\sin\left(\frac{2x-6x}{2}\right)\right]$

$= [2\cos 4x \cos(-2x)][-2\sin 4x \sin(-2x)]$

$= [2\cos 4x \cos 2x][-2\sin 4x(-\sin 2x)]$

$= [2\sin 4x \cos 4x][2\sin 2x \cos 2x]$

$= \sin 8x \sin 4x.$

L.H.S = R.H.S

Hence Proved.

Q. 14.

Sol. $\sin 2x + 2\sin 4x + \sin 6x = 4\cos^2 x \sin 4x.$

L.H.S $= \sin 2x + 2\sin 4x + \sin 6x$

$= (\sin 2x + \sin 6x) + 2\sin 4x$

$= \left[2\sin\left(\frac{2x+6x}{2}\right)\cos\left(\frac{2x-6x}{2}\right)\right] + 2\sin 4x$

$= [2\sin 4x \cos(-2x)] + 2\sin 4x.$

$= 2\sin 4x \cos 2x + 2\sin 4x$

$= 2\sin 4x(\cos 2x + 1)$

$= 2\sin 4x \, 2\cos^2 x$ $[\because 1 + \cos 2\theta = 2\cos^2\theta]$

$= 4\cos^2 x \sin 4x$

Hence Proved

Q.15.

Sol. $\cot 4x(\sin 5x + \sin 3x) = \cot x(\sin 5x - \sin 3x)$

$\Rightarrow$ L.H.S. $= \cot 4x(\sin 5x + \sin 3x)$

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)\right]$$

$$= \frac{\cos 4x}{\sin 4x}\left[2\sin 4x \cos x\right]$$

$$= 2\cos 4x \cos x.$$

L.H.S = R.H.S

Hence Proved.

Q.18.

Sol. $\dfrac{\sin x - \sin y}{\cos x + \cos y} = \tan \dfrac{x-y}{2}$

$\Rightarrow$ L.H.S $= \dfrac{\sin x - \sin y}{\cos x + \cos y}$

$$= \frac{2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)}{2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)}$$

$$= \frac{\sin\left(\frac{x-y}{2}\right)}{\cos\left(\frac{x-y}{2}\right)}$$

$$= \tan\left(\frac{x-y}{2}\right)$$

L.H.S = R.H.S

Hence Proved.

Q.19.

Sol. $\frac{\sin x + \sin 3x}{\cos x + \cos 3x} = \tan 2x.$

$\Rightarrow$ L.H.S $= \frac{\sin x + \sin 3x}{\cos x + \cos 3x}$

$= \frac{\cancel{2}\sin\left(\frac{x+3x}{2}\right) \cancel{\cos\left(\frac{x-3x}{2}\right)}}{\cancel{2}\cos\left(\frac{x+3x}{2}\right) \cancel{\cos\left(\frac{x-3x}{2}\right)}}$

$= \frac{\sin 2x}{\cos 2x}$

$= \tan 2x$

$\Rightarrow$ L.H.S = R.H.S

Hence Proved.

Q.20.

Sol. $\frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x} = 2\sin x.$

L.H.S $= \frac{\sin x - \sin 3x}{\sin^2 x - \cos^2 x}$

$= \frac{2\cos\left(\frac{x+3x}{2}\right)\sin\left(\frac{x-3x}{2}\right)}{-\cos 2x}$

$= \frac{2\cos 2x \sin(-x)}{-\cos 2x}$

$= -2 \{\sin(-x)\}$

$= 2\sin x.$

L.H.S = R.H.S

Hence Proved.

Q.16. $\dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x} = -\dfrac{\sin 2x}{\cos 10x}$

Sol. L.H.S. $= \dfrac{\cos 9x - \cos 5x}{\sin 17x - \sin 3x}$

$$= \frac{-2\sin\left(\frac{9x+5x}{2}\right)\sin\left(\frac{9x-5x}{2}\right)}{2\cos\left(\frac{17x+3x}{2}\right)\sin\left(\frac{17x-3x}{2}\right)}$$

$$\left[\begin{array}{l} \because \cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \\ \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \end{array}\right]$$

$$= \frac{-2\sin(7x)\sin 2x}{2\cos(10x)\sin(7x)}$$

$$= -\frac{\sin 2x}{2\cos 10x}$$

L.H.S = R.H.S

Hence Proved.

Q.21.

Sol. L.H.S. $= \dfrac{\cos 4x + \cos 3x + \cos 2x}{\sin 4x + \sin 3x + \sin 2x}$

$$= \frac{\cos 4x + \cos 2x + \cos 3x}{\sin 4x + \sin 2x + \cos 3x}$$

$$= \frac{2\cos\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \cos 3x}{2\sin\left(\frac{4x+2x}{2}\right)\cos\left(\frac{4x-2x}{2}\right) + \sin 3x}$$

$$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right), \quad \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$$

$$= \frac{2\cos 3x \cos x + \cos 3x}{2\sin 3x \cos x + \sin 3x}$$

$$= \frac{\cos 3x\,(2\cos x + 1)}{\sin 3x\,(2\cos x + 1)}$$

$$= \frac{\cos 3x}{\sin 3x}$$

$$= \cot 3x \quad \left[\because \frac{\cos\alpha}{\sin\alpha} = \cot\alpha\right]$$

L.H.S = R.H.S

Hence Proved.

Q.25.

Sol. L.H.S $= \cos 6x$

$= \cos 3(2x)$

$= 4\cos^3 2x - 3\cos 2x$

$[\because \cos 3\phi = 4\cos^3\phi - 3\cos\phi]$

$= 4(2\cos^2 x - 1)^3 - 3(2\cos^2 x - 1)$

$[\cos 2\theta = 2\cos^2\theta - 1]$

$= 4[(2\cos^2 x)^3 - (1)^3 - 3(2\cos^2 x)^2 + 3(2\cos^2 x)] - 6\cos^2 x + 3$

$= 4[8\cos^6 x - 1 - 12\cos^4 x + 6\cos^2 x] - 6\cos^2 x + 3$

$= 32\cos^6 x - 4 - 48\cos^4 x + 24\cos^2 x - 6\cos^2 x + 3$

$= 32\cos^6 x - 48\cos^4 x + 18\cos^2 x - 1$

L.H.S = R.H.S

Hence proved.

22

Sol. $\cot x \cot 2x - \cot 2x \cot 3x - \cot 3x \cot x = 1$

$\Rightarrow \cot x \cot 2x - \cot 2x \cot 3x = 1 + \cot 3x \cot x$

$\Rightarrow \cot 2x(\cot x - \cot 3x) = 1 + \cot 3x \cot x$

$\Rightarrow \cot 2x = \dfrac{1 + \cot 3x \cot x}{\cot x - \cot 3x}$

$\Rightarrow \cot 2x = \cot(3x - x)$

$\left[\because \cot(A-B) = \dfrac{1 + \cot A \cot B}{\cot B - \cot A}\right]$

$\Rightarrow \quad \cot 2x = \cot 2x$

L.H.S = R.H.S

Hence Proved.

Q.23.

Sol. L.H.S $= \tan 4x$

$= \tan 2(2x)$

$= \dfrac{2\tan 2x}{1 - \tan^2 2x}$ $\quad \left[\because \tan 2\theta = \dfrac{2\tan\theta}{1-\tan^2\theta}\right]$

$= \dfrac{2\left(\dfrac{2\tan x}{1-\tan^2 x}\right)}{1 - \left(\dfrac{2\tan x}{1-\tan^2 x}\right)^2}$

$= \cancel{\dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{1 - \left(\dfrac{4\tan^2 x}{1+\tan^2 x - 2\tan x}\right)}}$

$= \dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{1 - \dfrac{4\tan^2 x}{(1-\tan^2 x)^2}}$

$= \dfrac{\dfrac{4\tan x}{1-\tan^2 x}}{\dfrac{(1-\tan^2 x)^2 - 4\tan^2 x}{(1-\tan^2 x)^2}}$

$$= \frac{4\tan x}{1-\tan^2 x} \times \frac{(1-\tan^2 x)^{\cancel{2}}}{1-2\tan^2 x+\tan^4 x-4\tan^2 x}$$

$$\text{L.H.S.} = \frac{4\tan x(1-\tan^2 x)}{1-6\tan^2 x+\tan^4 x}$$

Hence Proved.

Q.17.

Sol. $$\text{L.H.S} = \frac{\sin 5x + \sin 3x}{\cos 5x + \cos 3x}$$

$$= \frac{2\sin\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}{2\cos\left(\frac{5x+3x}{2}\right)\cos\left(\frac{5x-3x}{2}\right)}$$

$$= \frac{2\sin 4x \cdot \cos x}{2\cos 4x \cdot \cos x}$$

$$= \frac{\sin 4x}{\cos 4x}$$

$$= \tan 4x$$

Hence Proved.

$$\left[\begin{array}{l} \because \sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \\ \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right) \end{array}\right]$$

Q.24.

Sol. $\cos 4x = 1 - 8\sin^2 x \cos^2 x$.

$\Rightarrow$ L.H.S $= \cos 4x$

$= \cos 2(2x)$

$= 1 - 2\sin^2 2x$ $\quad [\because \cos 2A = 1 - 2\sin^2 A]$

$= 1 - 2(2\sin x \cos x)^2$ $\quad [\because \sin 2A = 2\sin A \cos A]$

$= 1 - 8\sin^2 x \cos^2 x$

L.H.S = R.H.S

Hence proved.

concept

Trignoometry Eq^n

i) If $\sin x = 0$ Then $x = n\pi$, where $n \in Z$.

ii) If $\cos x = 0$ Then $x = (2n+1)\frac{\pi}{2}$, where $n \in Z$.

iii) If $\tan x = 0$ then $x = n\pi$, where $n \in Z$.

iv) If $\sin x = \sin y$, then $x = n\pi + (-1)^n y$, $n \in Z$.

v) If $\cos x = \cos y$ then $x = 2n\pi \pm y$, $n \in Z$

vi) If $\tan x = \tan y$, then $x = n\pi + y$, $n \in Z$

Exercise 3.4

Q.1) $\tan x = \sqrt{3}$

Sol. $\tan x = \tan\frac{\pi}{3}$

Principal solution $x = \frac{\pi}{3}, \frac{4\pi}{3}$ (and also $\tan(\pi+\theta) = \tan\theta$ $\Rightarrow \tan(\pi + \frac{\pi}{3}) = \tan\frac{4\pi}{3}$

General solution $x = n\pi + y$

$= n\pi + \frac{\pi}{3}$

Q.2)

Sol. $\operatorname{cosec} x = -2$

$\sin x = -\frac{1}{2}$ $\left[\because \operatorname{cosec}\theta = \frac{1}{\sin}\right]$

$\Rightarrow \sin x = -\sin \frac{\pi}{6}$

$\Rightarrow \sin x = \sin\left(-\frac{\pi}{6}\right) \quad \left[\because \sin(-\theta) = -\sin\theta\right]$

Principal solⁿ, $x = -\frac{\pi}{6}$

General solⁿ, $x = n\pi + (-1)^n y$

General solⁿ, $x = n\pi + (-1)^n\left(-\frac{\pi}{6}\right)$

Q.6.

Sol. $\cos 3x + \cos x - \cos 2x = 0$

$\Rightarrow 2\cos\left(\frac{3x+x}{2}\right)\cos\left(\frac{3x-x}{2}\right) - \cos 2x = 0$

$\left[\because \cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)\right]$

$\Rightarrow 2\cos 2x \cos x - \cos 2x = 0$

$\Rightarrow \cos 2x\,(2\cos x - 1) = 0$

$\Rightarrow \cos 2x = 0$ or $2\cos x - 1 = 0$

$\Rightarrow 2x = (2n+1)\frac{\pi}{2}$

$\left[\cos x = 0, \text{ then } x = (2n+1)\frac{\pi}{2}\right]$

$\Rightarrow \cos x = \frac{1}{2}$

$\Rightarrow \cos x = \frac{\pi}{3}$

$\therefore x = 2n\pi \pm \frac{\pi}{3}$

$\Rightarrow \quad x = (2n+1)\frac{\pi}{4}$

So,

$$x = (2n+1)\frac{\pi}{4}$$

or

$$x = 2n\pi \pm \frac{\pi}{3}$$

Q.8. $\sec^2 2x = 1 - \tan^2 2x$

Sol. $\sec^2 2x = 1 - \tan 2x$

$\Rightarrow 1 + \tan^2 2x = 1 - \tan 2x$ $\quad [\because 1 + \tan^2\theta = \sec^2\theta]$

$\Rightarrow \tan^2 2x + \tan 2x = 0$

$\Rightarrow \tan 2x(\tan 2x + 1) = 0$

$\tan 2x = 0$

$\Rightarrow 2x = n\pi$

$x = \frac{n\pi}{2}$

or

$\tan 2x + 1 = 0$

$\Rightarrow \tan 2x = -1$

$\tan 2x = -\tan\frac{\pi}{4}$

$\tan 2x = \tan\left(\pi - \frac{\pi}{4}\right)$

$[\tan(\pi-\theta) = -\tan\theta]$

$\Rightarrow \tan 2x = \tan\frac{3\pi}{4}$

$\Rightarrow 2x = n\pi + \frac{3\pi}{4}$

$\Rightarrow x = \frac{n\pi}{2} + \frac{3\pi}{8}$

Miscellaneous Exercise.

Q. 1)

Sol. L.H.S. $= 2\cos\frac{\pi}{13}\cos\frac{9\pi}{13} + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= 2\cos\left(\frac{\pi}{13}+\frac{9\pi}{13}\right) + \cos\left(\frac{\pi}{13}-\frac{9\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$\left[\because 2\cos A\cos B = 2\cos\left(\frac{A+B}{2}\right) + \cos(A-B)\right]$

$= \cos\frac{10\pi}{13} + \cos\left(\frac{-8\pi}{13}\right) + \cos\frac{3\pi}{13} + \cos\frac{5\pi}{13}$

$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} + \cos\left(\pi - \frac{10\pi}{13}\right) + \cos\left(\pi - \frac{8\pi}{13}\right)$

$[\because \cos(-\theta) = \cos\theta]$

$= \cos\frac{10\pi}{13} + \cos\frac{8\pi}{13} - \cos\frac{10\pi}{13} - \cos\frac{8\pi}{13}$

$= 0$

L.H.S = R.H.S

Hence Proved.

Q.3.

Sol. L.H.S $= (\cos x + \cos y)^2 + (\sin x - \sin y)^2$

$$= \cos^2 x + \cos^2 y + 2\cos x \cos y + \sin^2 x + \sin^2 y - 2\sin x \sin y$$

$$\left[\because \begin{array}{l} (a+b)^2 = a^2 + b^2 + 2ab \\ (a-b)^2 = a^2 + b^2 - 2ab \end{array} \right]$$

$$= \cos^2 x + \sin^2 x + \cos^2 y + \sin^2 y + 2(\cos x \cos y - \sin x \sin y)$$

$$= 1 + 1 + 2\cos(x+y)$$

$$\left[\begin{array}{l} \because \sin^2\theta + \cos^2\theta = 1 \\ \because \cos(x+y) = \cos x \cos B - \sin x \sin B \end{array} \right]$$

$$= 2 + 2\cos(x+y)$$

$$= 2\{1 + \cos(x+y)\}$$

$$= 2 \times 2\cos^2\left(\frac{x+y}{2}\right)$$

$$\left[\because 1 + \cos\theta = 2\cos^2\left(\frac{\theta}{2}\right) \right]$$

$$= 4\cos^2\left(\frac{x+y}{2}\right)$$

L.H.S = R.H.S

Hence Proved

Q.7

Sol. L.H.S $= \sin 3x + \sin 2x - \sin x$

$$= \sin 3x - \sin x + \sin 2x$$

$$= 2\cos\left(\frac{3x+x}{2}\right)\sin\left(\frac{3x-x}{2}\right) + \sin 2x$$

$$\left[\because \sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right) \right]$$

$= 2\cos 2x \sin x + 2\sin x \cos x$

$\cancel{= 2\cos 2x}$

$= 2\sin x \, (\cos 2x + \cos x)$

$= 2\sin x \times 2\cos\left(\frac{2x+x}{2}\right)\cos\left(\frac{2x-x}{2}\right)$

$= 4\sin x \cos\frac{3x}{2} \cos\frac{x}{2}$

L.H.S = R.H.S

Hence Proved.

Q.8.

Sol. ~~L.H.S.~~ $\tan x = \frac{-4}{3}$, x lies in 2^{nd} quadrant.

We know that,

$\Rightarrow 1 + \tan^2 x = \sec^2 x$

$\Rightarrow 1 + \left(-\frac{4}{3}\right)^2 = \sec^2 x$

$\Rightarrow 1 + \left(\frac{16}{9}\right) = \sec^2 x$

$\Rightarrow \frac{25}{9} = \sec^2 x$

$\Rightarrow \sec x = \frac{5}{3}$

$$\cos x = \frac{1}{\sec x} = -\frac{3}{5}$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = 1 + \cos x$$

$$\Rightarrow 2\cos^2 \frac{x}{2} = 1 + \left(\frac{-3}{5}\right)$$

$$\Rightarrow \cos^2 \frac{x}{2} = 1 - \frac{3}{5}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{5-3}{5}$$

$$\Rightarrow \cos^2 \frac{x}{2} = \frac{2}{5}$$

$$\Rightarrow \cos \frac{x}{2} = \frac{1}{\sqrt{5}}$$