Chapter - 1

SETS

Date: 04 08 20

a set is a well - defined collection of objects.

Units of sets

1. N → The set of all natural numbers
2. Z → The set of all integers.
3. Q → The set of all rational numbers.
4. R → The set of real numbers.
5. Z^+ → The set of positive integers.
6. Q^+ → The set of positive rational numbers.
7. R^+ → The set of positive real numbers.

There are two methods of representing a set :-

(i) Roster or tabular form.
Example :- A = {~~A~~ a, e, i, o, u}

(ii) Set - builder form.
Example :- A = {x : x is vowels of eng. alphabet}

Exercise - 1.1

1. Which of the following are sets ? Justify your answer.

(i) The collection of all the months of a year beginning with the letter J.

Ans.→ yes, it is a set because it is well defined collection of months.

$$A = \{ January, June, July \}$$

(ii) The collection of ten most talented writers of India.

Ans.→ It is not a set because it is not well defined collections of writers.

(iii) A team of eleven best - cricket batsmen of the world.

Ans.→ It is not a set because it is not well defined team of best cricket batsmen.

(iv) The collection of all boys in your class.

Ans.→ yes, it is a set because it is well defined collection of boys in class.

(v) The collection of all natural no. less than 100.

Ans.→ yes, it is a set because the collection of all natural no. less than 100 is a Well-defined collection.

(vi) A collection of novels written by the writer Munshi prem Chand.

Ans. yes, it is a set because the collection of novels written by the writer Munshi Prem chand is well defined collection.

(vii) The collection of all even integers.

Ans. yes, it is a set because the collection of all even integers is a Well-defined collection.

(viii) The collection of questions in this chapter.

Ans.→ yes, it is a set because the collection of questions in this chapter is a Well-defined collection.

(ix) A collection of most dangerous animals of the world.

Ans.→ No, it is not a set because collection of most dangerous animals of the world is not-Well-defined.

Q. 2. Let $A = \{1, 2, 3, 4, 5, 6\}$. Insert the appropriate symbol $\in$ or $\notin$ in the blank spaces.

(i) $5 \in A$

(ii) $8 \notin A$

(iii) $0 \notin A$

(iv) $4 \in A$

(v) $2 \in A$

(vi) $10 \notin A$

Q. 3. Write the following sets in roster form:

(i) $A = \{x : x$ is an integer and $-3 \leq x < 7\}$

Sol. $A = \{-3, -2, -1, 0, 1, 2, 3, 4, 5, 6\}$

(ii) $B = \{x : x$ is a natural no. less than $6\}$

Ans. $B = \{1, 2, 3, 4, 5\}$

(iii) $C = \{x : x$ is a two digit natural no. such that the sum of its digit is $8\}$

Sol. $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

Ans. $C = \{17, 26, 35, 44, 53, 62, 71, 80\}$

(iv) $D = \{x : x$ is a prime no. which is divisor of $60\}$

Ans. $D = \{2, 3, 5\}$

(v) E = The set of all letters in the word TRIGONOMETRY.

Ans. $E = \{T, R, I, G, O, N, M, E, Y\}$

(vi) F = The set of all letters in the word BETTER.

Ans. $F = \{B, E, T, R\}$

Q.4. Write the following sets in the set-builder form:

~~(i) A = {x: x is an od~~

(i) $A = \{3, 6, 9, 12\}$

Ans. $A = \{x : x = 3n$, where $n \leq 4\}$

(v) $E = \{1, 4, 9 \ldots, 100\}$

Ans. $E = \{x : x = n^2$, where n is natural number and $x \leq 10\}$

(ii) $B = \{2, 4, 8, 16, 32\}$

Sol. $B = \{x : x = 2^n$, $\cancel{n \in}$ $n \in N$ and $1 \leq n \leq 4\}$

(iii) $C = \{5, 25, 125, 625\}$

Sol. $C = \{x : x = 5^n, n \in N \text{ and } 1 \leq n \leq 4\}$

(iv) $D = \{2, 4, 6 \ldots\}$

Sol. $D = \{x : x$ is an even natural no.$\}$

Q.5. List all the elements of the following sets:

(i) $A = \{x : x$ is an odd natural no.$\}$

Ans. → x is an odd natural no. and we know that natural odd no. are 1, 3, 5 ...

so,

$A = \{1, 3, 5, 7 \ldots\}$

(ii) $B = \{x : x$ is an integer, $-\frac{1}{2} < x < \frac{9}{2}\}$

Ans. Here, x is an integer where $\frac{-1}{2} < x < \frac{9}{2}$.

if we solve $\frac{-1}{2} = -0.5$ and $\frac{9}{2} = 4.5$.

so,

$B = \{0, 1, 2, 3, 4\}$

(iii) $C = \{x : x$ is an integer, $x^2 \leq 4\}$

Sol. Here, x is an integer and x^2 is less than or equal to 4.

so, $(-2)^2 = 4$,
$(-1)^2 = 1$,
$(1)^2 = 1$
$(2)^2 = 4$
$(0)^2 = 0$

Then,

$C = \{-2, -1, 0, 1, 2\}$

(iv) $D = \{x : x$ is a letter in the word "LOYAL" $\}$
sol. $D = \{L, O, Y, A\}$

(v) $E = \{x : x$ is a month of year not having 31 days$\}$
sol. $E = \{$February, April, June, september, November$\}$

(vi) $F = \{x : x$ is a consonant in the eng. alphabet which preceedes K. $\}$

sol. $F = \{b, c, d, f, g, h, j\}$

Q.6. Match each of the set on the left in the raster form with the same set on the right described in set builder form :

A — B — Ans.

(i) $\{1, 2, 3, 6\}$ — $\{x : x$ is a natural no. and divisor of 6 $\}$

(ii) $\{2, 3\}$ — $\{x : x$ is a prime no. and a divisor of 6 $\}$

(iii) $\{M, A, T, H, E, I, C, S\}$ — $\{x : x$ is letter of the word MATHEMATICS $\}$

(iv) $\{1, 3, 5, 7, 9\}$ — $\{x : x$ is an odd no. less than $10\}$

Notes

1) **Empty Set** → A set which does not contain any content is called empty set or null set. It is denoted by ϕ (phi).

2) **Finite set** → A set which is empty or consists of a definite number of elements is called finite set.

3) **Equal set** → Two sets A and B are said to be equal if they have exactly the same elements.

Exercise 1.2

Q. 1. Which of the following are examples of the null set.

(i) Set of odd natural no. divisible by 2.

Sol. Set of odd natural no. divisible by 2 is null set because there is no any odd no. which is divisible by 2.

(ii) Set of even Prime numbers.

Sol. Set of even Prime numbers is not a Empty set because 2 is an even prime no.

(iii) $\{x : x$ is a natural numbers, $x < 5$ and $x > 7\}$

Sol. This is a empty set because there is no natural no. which is less than 5 and more than 7.

(iv) $\{y : y$ is a point common to any two parrallel lines $\}$

Sol. This is empty set because parrallel line do not intersect each other. so, there is no common point on that parrallel lines.

Q. 2 Which of the following sets are finite or infinite.

(i) The set of months of a year.

Ans. The set of months of a year is finite set because we can count the no. of months of a year.

(ii) $A = \{1, 2, 3 \ldots\ldots\}$

Ans. This set is infinite set because we can't count how much natural no. present more than 3.

(iii) $\{1, 2, 3 \ldots\ldots 99, 100\}$

Ans. This set is finite set because we are able to count natural no. till 100.

(iv) The set of positive integers greater than 100.

Ans. This is infinite set because we can't count positive integers more that 100.

(v) The set of prime no. less than 99.

Ans. This is finite set because we are able to count all prime no. less than 99.

3. State wheather each of the following set is finite or infinite :

(i) The set of lines which are parrallel to x-axis.

Ans. The set of lines which are parrallel to x-axis is infinite set because there are infinite line parrallel to x-axis.

(ii) The set of letters in the English alphabet.

Ans. The set of letters in the Eng. alphabet is Finite set because we are able to count no. of Eng. alphabet.

(iii) The set of no. which are multiple of 5.

Sol. The set of no. which are multiple of 5 is infinite set because we can't able to count the numbers which are divisible by 5.

(iv) The set of animals living on the earth.

Ans. The set of animals living on the earth is finite set because we are able to count all the animals present in the world.

(v) The set of circles passing through the origin (0,0)

Ans. The set of circles passing through the origin is infinite set. because infinite no. of circle can pass through the origin.

Q.4. In the following state whether $A = B$ or not:

(i) $A = \{a, b, c, d\}$, $B = \{d, c, b, a\}$

Ans. Here, $A = B$

Because all the element of set A is equally matched with the element of set B.

(ii) $A = \{4, 8, 12, 16\}$, $B = \{8, 4, 16, 18\}$

Ans. Here, $A \neq B$

Because element of set A is not matched with the element of set B.

(iii) $A = \{2, 4, 6, 8, 10\}$

$B = \{x : x \text{ is a positive even integer and } x \leq 10\}$

Ans. Here, $A = B$

Because the set A has positive even integer till 10 and In set B also said same thing. If we solve set B then :-

$B = \{2, 4, 6, 8, 10\}$

It is equal to set A.

(iv) $A = \{x : x \text{ is a multiple of } 10\}$
$B = \{10, 15, 20, 25, 30, \ldots\}$

Ans. Here, $A \neq B$

Because in set B 15, 25 is present which is not multiple of 10.

Q. 5. Are the following pair of sets equal? Give reason.

(i) $A = \{2, 3\}$
$B = \{x : x \text{ is solution of } x^2 + 5x + 6 = 0\}$

Sol. The above ~~following~~ pair of sets are not equal.

$A \neq B$

Because when we solve set then we get value of x is -2 and -3 but in set A value of x is 2 and 3 so both are unequal.

(ii) A = {x : x is a letter in the word FOLLOW}
B = {y : y is a letter in the word WOLF}

Ans. A = {F, O, L, W}
B = {W, O, L, F}

set A and set B contain same element so both sets are equal to each other.

A = B.

Q.6. From the set given below, select equal sets :

A = {2, 4, 8, 12}
B = {1, 2, 3, 4}
C = {4, 8, 12, 14}
D = {3, 1, 4, 2}
E = {-1, 1}
F = {0, a}
G = {1, -1}
H = {0, 1}

Ans. Here,
(i) set B = D
(ii) set E = G

Because element of set B is equal to element of set D and element of set E is equal to element of set G.

NOTES

1) Sub-Set :- A set A is Said to be a subset of a set B if every element of A is also an element of B

$A \subset B$ if $a \in A \Rightarrow a \in B$

Empty set ϕ is subset of every set.

2.

Interval

i) Closed interval :- The interval which contains the end points also is called closed interval and is denoted by $[a, b]$.

$$[a,b] = \{x : a \leq x \leq b\}$$

ii) Open interval :- The set of real numbers $\{y : a < y < b\}$ is called an open inteval and is denoted by (a,b).

iii) We can also have intervals closed at one end and open at the other.

Example :- $[a, b) = \{x : \cancel{a \leq b}\ a \leq x < b\}$.

is an open interval from a to b, including a but excluding b.

3. **Power Set** :- The Collection of all subsets of a set A is called Power set of that set A. It is denoted by P(A)

Example :- if $A = \{1, 2\}$
then $P(A) = \{\phi, \{1\}, \{2\}, \{1,2\}\}$

4. Formula for finding the total no. of subsets of a set :-

$$\boxed{\text{No. of subset} = 2^n}$$

Exercise 1.3.

Q.1. Make Correct statements by filling in the symbols $\subset$ or $\not\subset$ in the blank spaces :-

(i) $\{2, 3, 4\} \subset \{1, 2, 3, 4, 5\}$

(ii) $\{a, b, c\} \not\subset \{b, c, d\}$

(iii) {x: x is a student of class XI of your school} $\subset$ {x: x student of your school}

(iv) {x: x is a circle in the plane} $\not\subset$ {x: x is a circle in the same plane with radius 1 unit}

(v) $\{x$: is a triangle in a plane$\}$ $\not\subset$ $\{x : x$ is a rectangle in the plane$\}$

(vi) $\{x : x$ is an equilateral triangle in a plane$\}$ $\subset$ $\{x : x$ is a triangle in the same plane$\}$

(vii) $\{x : x$ is an even natural no.$\}$ $\subset$ $\{x : x$ is an integer$\}$

Q.2 Examine wheather the following statements are true or false :-

(i) $\{a, b\} \not\subset \{b, c, a\}$

Ans. False

(ii) $\{a, e\} \subset \{x : x$ is a vowel in the Eng. alphabet$\}$

Ans. True

(iii) $\{1, 2, 3\} \subset \{1, 3, 5\}$

Ans. False

(iv) $\{a\} \subset \{a, b, c\}$

Ans. True

(v) $\{a\} \in \{a, b, c\}$

Ans. False

(vi) $\{x : x$ is an even natural no. less than $6\}$ $\subset$ $\{x : x$

is an natural no. which divides 36 }

Ans. True.

Q.3. Let $A = \{1, 2, \{3,4\}, 5\}$. Which of the following statements are incorrect and why?

(i) $\{3,4\} \subset A$

Ans. It is incorrect because $\{3,4\}$ is a element of set A not a subset of set A.

(ii) $\{3,4\} \in A$

Ans. It is correct, $\{3,4\}$ belongs to set A.

(iii) $\{\{3,4\}\} \subset A$

Ans. It is correct, $\{\{3,4\}\}$ is a subset of set A.

(iv) $1 \in A$

Ans. It is correct 1 is a element of set A.

(v) $1 \subset A$

Ans. It is incorrect because 1 is not a subset of set A. $\{1\} \subset A$ is correct statement.

(vi) $\{1,2,5\} \subset A$

Ans. It is correct because 1, 2, 5 are also elements of set A.

(vii) $\{1,2,5\} \in A$

Ans. It is incorrect because $\{1,2,5\}$ is not a element of set A.

(xiii) $\{1, 2, 3\} \subset A$

Ans. It is incorrect because 3 is not subset of A.

(ix) $\phi \in A$

Ans. It is incorrect because set A has not element like ϕ.

(x) $\phi \subset A$

Ans. It is correct because ϕ is a subset of set A.

(xi) $\{\phi\} \subset A$

Ans. It is incorrect ϕ is not belogs to set A.

Q.4. Write down all the subsets of the following sets:

(i) $\{a\}$

sol. The subsets of $\{a\}$ is ϕ and $\{a\}$.

(ii) $\{a, b\}$

sol. The subsets of $\{a, b\}$ is ϕ, $\{a\}$, $\{b\}$ and $\{a, b\}$.

(iii) $\{1, 2, 3\}$

sol. The subsets of $\{1, 2, 3\}$ is ϕ, $\{1\}$, $\{2\}$, $\{3\}$, $\{1, 2\}$, $\{1, 3\}$, $\{2, 3\}$, $\{1, 2, 3\}$.

(iv) ϕ

sol. It has only 1 subset that is ϕ

Q.5. How many elements has $p(A)$, if $A = \Phi$?

Sol. We know the formula for finding the total subsets of ~~A~~ set.

Here in set A ~~only~~ no element is present.

$\Rightarrow$ No. of element $= 2^n$

$\Rightarrow$ No. of element $= 2^0$

$\Rightarrow$ No. of element $= 1$

Hence, the no. of element in set A is 1.

Q.6. Write the following as intervals:

(i) $\{x : x \in R, -4 < x \leq 6\}$

Sol. $x = (-4, 6]$

(ii) $\{x : x \in R, -12 < x < -10\}$

Sol. $x = (-12, -10)$

(iii) $\{x : x \in R, 0 \leq x < 7\}$

Sol. $x = [0, 7)$

(iv) $\{x : x \in R, 3 \leq x \leq 4\}$

Sol. $x = [3, 4]$

Q.7. Write the following intervals in set-builder form:-

(i) $(-3, 0)$

Sol. $\{x : x \in R, -3 < x < 0\}$

(ii) $[6, 12]$

Sol. $\{x : x \in R, 6 \leq x \leq 12\}$

(iii) $(6, 12]$

Sol. $\{x : x \in R, 6 < x \leq 12\}$

(iv) $[-23, 5)$

Sol. $\{x : x \in R, x \leq -23 < 5\}$

Q.8. What Universal set(s) would you Propose for each of the following:

(i) The set of right triangles.

Ans. Universal set can be the set of polygons or triangles.

$U = \{x : x$ is a set of triangle$\}$

(ii) The set of isosceles triangles.

Ans. The Universal set can be the set of triangle / polygons or set of 2D figures.

$U = \{x : x$ is set of triangles / polygons or 2D figures$\}$

Q.2. Given the sets $A = \{1, 3, 5\}$, $B = \{2, 4, 6\}$ and $C = \{0, 2, 4, 6, 8\}$, which of the following may be considered as universal set (s) for all the three sets A, B and C.

(i) $D = \{0, 1, 2, 3, 4, 5, 6\}$

Ans. Here,

$A \subset D$

$B \subset D$

$C \not\subset D$

Hence, the set D is not universal set for set A, B & C.

(ii) $E = \phi$

Ans. $A \subset E$, $B \subset E$, $C \subset E$

But here all element of set A, B and C is not present so set E is also not universal set.

(iii) $Z = \{0, 1, 2, 3, 4, 5, 6, 7, 8\}$

Ans. Here,

$A \subset Z$

$B \subset Z$

$C \subset Z$

Hence, set Z is universal set for set A, B & C.

(iv) $y = \{1, 2, 3, 4, 5, 6, 7, 8\}$

Ans. Here,

$A \subset y$

$B \subset y$

$C \not\subset y$

Hence, set y is not a universal set for set A, B and C.

Union of Sets

Let A and B be any two sets. The union of A and B is the set which consists of all the elements of A and all the elements of B, the common elements being taken only once.

$$\boxed{A \cup B = \{x : x \in A \text{ or } x \in B}$$

Example :- $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6, 8, 10\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 8, 10\}$

Some properties of the operation of Union :-

(i) $A \cup B = B \cup A$ (Commulative law)

(ii) $(A \cup B) \cup C = A \cup (B \cup C)$
(Associative law)

(iii) $A \cup \phi = A$ (Law of identity element, ϕ is the identity of $\cup$)

(iv) $A \cup A = A$ (Idempotent law)

(v) $U \cup A = U$ (law of U)

Intersection of sets

The intersection of sets A and B is the set of all elements which are common to both A and B.

$$A \cap B = \{x : x \in A \text{ and } x \in B\}$$

Example :- $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6, 8, 10\}$

$$A \cap B = \{2, 4, 6\}$$

Some properties of operation of Intersection :-

(i) $A \cap B = B \cap A$ (commulative law)

(ii) $(A \cap B) \cap C = A \cap (B \cap C)$ (Associative law)

(iii) $\phi \cap A = \phi$, $U \cap A = A$ (Law of ϕ and U)

(iv) $A \cap A = A$ (Idempotent law)

(v) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$
(distributive law) i.e., $\cap$ distributes over $\cup$.

Difference of Sets

The difference of the sets A and B in this order is the set of elements which belong to A but not to B. symbolically, we write A−B and read as "A Minus B".

$$\boxed{A - B = \{x : x \in A \text{ and } x \notin B\}}$$

Example :- $A = \{1, 2, 3, 4, 5, 6\}$
$B = \{2, 4, 6, 8, 10\}$

$A - B = \{1, 3, 5\}$
$B - A = \{8, 10\}$

Exercise 1.4.

1. Find the Union of each of the following pairs of sets :

(i) $X = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

sol. Given,

$$X = \{1, 3, 5\}$$
$$Y = \{1, 2, 3\}$$

$$X \cup Y = \{1, 3, 5\} \cup \{1, 2, 3\}$$
$$X \cup Y = \{1, 2, 3, 5\}$$

(ii) $A = [a, e, i, o, u\}$, $B = \{a, b, c\}$

sol. Given,

$\Rightarrow$ $A = \{a, e, i, o, u\}$

$\Rightarrow$ $B = \{a, b, c\}$

$\Rightarrow A \cup B = \{a, e, i, o, u\} \cup \{a, b, c\}$

$= A \cup B = \{a, b, c, e, i, o, u\}$

(iii) $A = \{x : x$ is a natural no. and multiple of 3 $\}$
$B = \{x : x$ is a natural no. less than 6 $\}$

sol. Given,

$A = \{x : x$ is a natural no. multiple of 3 $\}$

$B = \{x : x$ is a natural no. less than 6 $\}$

It means,

$A = \{3, 6, 9, 12, 15, 18 \ldots\}$

$B = \{1, 2, 3, 4, 5\}$

$A \cup B = \{1, 2, 3, 4, 5, 6, 9, 12, 15 \ldots\ldots\}$

$A \cup B = \{x : x$ is a natural no. less than 6 or than after multiple of 3$\}$

iv) $A = \{x : x$ is a natural no. and $1 < x \leq 6\}$
$B = \{x : x$ is a natural no. and $6 < x < 10\}$

sol. $\Rightarrow A = \{x : x$ is a natural no. and $1 < x \leq 6\}$
$\Rightarrow B = \{x : x$ is a natural no. and $6 < x < 10\}$

$\Rightarrow A = \{2, 3, 4, 5, 6\}$
$\Rightarrow B = \{7, 8, 9\}$

$\Rightarrow A \cup B = \{2, 3, 4, 5, 6\} \cup \{7, 8, 9\}$
$\Rightarrow A \cup B = \{2, 3, 4, 5, 6, 7, 8, 9\}$

v) $A = \{1, 2, 3\}$
$B = \phi$

sol. $\Rightarrow A = \{1, 2, 3\}$
$\Rightarrow B = \phi$

$\Rightarrow A \cup B = \{1, 2, 3\} \cup \phi$

$\Rightarrow A \cup B = \{1, 2, 3\}$

or.

$\Rightarrow A \cup B = A$

Q.2. Let $A = \{a, b\}$; $B = \{a, b, c\}$. Is $A \subseteq B$?
What is $A \cup B$?

Sol. Here, $A = \{a, b\}$

$\Rightarrow B = \{a, b, c\}$

Yes, $A \subseteq B$.

$\Rightarrow A \cup B = \{a, b\} \cup \{a, b, c\}$

$\Rightarrow A \cup B = \{a, b, c\}$

$\Rightarrow A \cup B = B$

Q.3. If A and B are two sets such that $A \subseteq B$, then what is $A \cup B$?

Sol. If A and B are two sets such that $A \subseteq B$. then the element of set A is present in set B.

It means ;-

$A \cup B = B$

Q.4. If $A = \{1,2,3,4\}$, $B = \{3,4,5,6\}$,
$C = \{5,6,7,8\}$ and $D = \{7,8,9,10\}$; find.

i) $A \cup B$

Sol. $A \cup B = \{1,2,3,4\} \cup \{3,4,5,6\}$
$A \cup B = \{1,2,3,4,5,6\}$

ii) $A \cup C$

Sol. $A \cup C = \{1,2,3,4\} \cup \{5,6,7,8\}$
$A \cup C = \{1,2,3,4,5,6,7,8\}$

iii) $B \cup C$

Sol. $B \cup C = \{3,4,5,6\} \cup \{5,6,7,8\}$
$B \cup C = \{3,4,5,6,7,8\}$

iv) $B \cup D$

Sol. $B \cup D = \{3,4,5,6\} \cup \{7,8,9,10\}$
$B \cup D = \{3,4,5,6,7,8,9,10\}$

v) $A \cup B \cup C$

Sol. $A \cup B \cup C = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{5,6,7,8\}$
$A \cup B \cup C = \{1,2,3,4,5,6\} \cup \{5,6,7,8\}$
$A \cup B \cup C = \{1,2,3,4,5,6,7,8\}$

vi) $A \cup B \cup D$

Sol. $A \cup B \cup D = \{1,2,3,4\} \cup \{3,4,5,6\} \cup \{7,8,9,10\}$

$A \cup B \cup D = \{1,2,3,4,5,6\} \cup \{7,8,9,10\}$

$A \cup B \cup D = \{1,2,3,4,5,6,7,8,9,10\}$

vii) $B \cup C \cup D$

Sol- $B \cup C \cup D = \{3,4,5,6\} \cup \{5,6,7,8\} \cup \{7,8,9,10\}$

$B \cup C \cup D = \{3,4,5,6,7,8\} \cup \{7,8,9,10\}$

$B \cup C \cup D = \{3,4,5,6,7,8,9,10\}$

Q.6. If $A = \{3,5,7,9,11\}$,

$B = \{7,9,11,13\}$,

$C = \{11,13,15\}$,

$D = \{15,17\}$; find.

i) $A \cap B$

Sol- $A = \{3,5,7,9,11\}$

$B = \{7,9,11,13\}$

$A \cap B = \{3,5,7,9,11\} \cap \{7,9,11,13\}$

$A \cap B = \cancel{\{3,5} \{7,9,11\}$

vi) $A \cap (B \cup C)$

Sol. $A = \{3,5,7,9,11\}$

$B = \{7,9,11,13\}$

$C = \{11,13,15\}$

ii) $B \cap C$

Sol. $\Rightarrow B = \{7, 9, 11, 13\}$

$\Rightarrow C = \{11, 13, 15\}$

$\Rightarrow B \cap C = \{7, 9, 11, 13\} \cap \{11, 13, 15\}$

$\Rightarrow B \cap C = \{11, 13\}$

iii) $A \cap C \cap D$

Sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow C = \{11, 13, 15\}$

$\Rightarrow D = \{15, 17\}$

$\Rightarrow A \cap C \cap D = \{3, 5, 7, 9, 11\} \cap \{11, 13, 15\} \cap \{15, 17\}$

$\Rightarrow A \cap C \cap D = \{11\} \cap \{15, 17\}$

$\Rightarrow A \cap C \cap D = \cancel{\{\phi\}} \quad \phi$

iv) $A \cap C$

Sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow C = \{11, 13, 15\}$

$\Rightarrow A \cap C = \{3, 5, 7, 9, 11\} \cap \{11, 13, 15\}$.

$\Rightarrow A \cap C = \{11\}$

v) $B \cap D$

Sol. $\Rightarrow B = \{7, 9, 11, 13\}$

$D = \{15, 17\}$

$\Rightarrow B \cap D = \{7, 9, 11, 13\} \cap \{15, 17\}$

$\Rightarrow B \cap D = \phi$

vii) $A \cap D$

sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow D = \{15, 17\}$

$\Rightarrow A \cap D = \{3, 5, 7, 9, 11\} \cap \{15, 17\}$

$\Rightarrow A \cap D = \phi$

viii) $A \cap (B \cup D)$

sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow B = \{7, 9, 11, 13\}$

$\Rightarrow D = \{15, 17\}$

$\Rightarrow A \cap (B \cup D) = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\} \cup \{15, 17\}$

$\Rightarrow A \cap (B \cup D) = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13, 15, 17\}$

$\Rightarrow A \cap (B \cup D) = \{7, 9, 11\}$

ix) $(A \cap B) \cap (B \cup C)$

sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow B = \{7, 9, 11, 13\}$

$\Rightarrow C = \{11, 13, 15\}$

$\Rightarrow (A \cap B) \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cap \{7, 9, 11, 13\} \cap \{7, 9, 11, 13\}$

$\cup \{11, 13, 15\}$

$\Rightarrow (A \cap B) \cap (B \cup C) = \{7, 9, 11\} \cap \cancel{\cup} \{7, 9, 11, 13, 15\}$

$\Rightarrow (A \cap B) \cap (B \cup C) = \{7, 9, 11\}$

x) $(A \cup D) \cap (B \cup C)$

sol. $\Rightarrow A = \{3, 5, 7, 9, 11\}$

$\Rightarrow B = \{7, 9, 11, 13\}$

$\Rightarrow C = \{11, 13, 15\}$

$\Rightarrow D = \{15, 17\}$

$\Rightarrow (A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11\} \cup \{15, 17\} \cap \{7, 9, 11, 13\}$

$\cup \{11, 13, 15\}$

$\Rightarrow (A \cup D) \cap (B \cup C) = \{3, 5, 7, 9, 11, 15, 17\} \cap \{7, 9, 11, 13, 15\}$

$\Rightarrow (A \cup D) \cap (B \cup C) = \{7, 9, 11, 15\}$

Q.5. Find the intersection of each pair of sets :-

i) $\cancel{X} = \{1, 3, 5\}$, $Y = \{1, 2, 3\}$

sol. $\Rightarrow X = \{1, 3, 5\}$

$\Rightarrow Y = \{1, 2, 3\}$

$\Rightarrow X \cap Y = \{1, 3, 5\} \cap \{1, 2, 3\}$

$\Rightarrow X \cap Y = \{1, 3\}$

ii) $A = \{a, e, i, o, u\}$, $B = \{a, b, c\}$

sol. $\Rightarrow A \quad \{a, e, i, o, u\}$

$\Rightarrow B = \{a, b, c\}$

$\Rightarrow A \cap B = \{a, e, i, o, u\} \cap \{a, b, c\}$

$\Rightarrow A \cap B = \{a,\}$

iii) $A = \{x : x$ is a natural no. and $1 < x < 6\}$

$B = \{x : x$ is a natural no. and $6 < x < 10\}$

sol. $\Rightarrow A = \{x : x$ is a natural no. and $1 < x \leq 6\}$

$\Rightarrow A = \{2, 3, 4, 5, 6\}$

$\Rightarrow B = \{x : x$ is a natural no. and $6 < x < 10\}$

$\Rightarrow B = \{7, 8, 9\}$

$\Rightarrow A \cap B = \{2, 3, 4, 5, 6\} \cap \{7, 8, 9\}$

$\Rightarrow A \cap B = \phi$

iv) $A = \{x : x$ is a natural no. and multiple of $3\}$

$B = \{x : x$ is a natural no. less than $6\}$

sol. $\Rightarrow A = \{x : x$ is a natural no. ~~less the~~ multiple of $3\}$

$\Rightarrow B = \{x : x$ is a natural no. less than $6\}$

$\Rightarrow A = \{3, 6, 9, 12, 15, 18 \ldots\ldots\}$

$\Rightarrow B = \{1, 2, 3, 4, 5\}$

$\Rightarrow A \cap B = \{3, 6, 9, 12, 15 \ldots\} \cap \{1, 2, 3, 4, 5\}$

$\Rightarrow A \cap B = \{3\}$

(v) $A = \{1, 2, 3\}$, $B = \phi$

sol. $\Rightarrow A = \{1, 2, 3\}$, $B = \phi$

$\Rightarrow A \cap B = \{1, 2, 3\} \cap \phi$

$\Rightarrow A \cap B = \phi$

Q.7. If $A = \{x : x \text{ is a natural no.}\}$,
$B = \{x : x \text{ is an even natural no.}\}$,
$C = \{x : x \text{ is an odd natural no.}\}$;
and $D = \{x : x \text{ is a prime no.}\}$, find;

(i) $A \cap B$

Sol: Here,

$\Rightarrow A = \{x : x \text{ is a natural no.}\}$
$\Rightarrow A = \{1, 2, 3, 4, 5 \ldots \infty\}$

$\Rightarrow B = \{x : x \text{ is an even natural no.}\}$
$\Rightarrow B = \{2, 4, 6, 8, 10 \ldots \infty\}$

$\Rightarrow A \cap B = \{1, 2, 3, 4, 5 \ldots \infty\} \cap \{2, 4, 6, 8, 10 \ldots \infty\}$

$\Rightarrow A \cap B = \{2, 4, 6, 8, 10 \ldots \infty\}$

$\Rightarrow A \cap B = \{x : x \text{ is an even natural no.}\}$

ii) $A \cap C$

Sol $\Rightarrow A = \{x : x \text{ is a natural no.}\}$
$\Rightarrow C = \{x : x \text{ is an odd natural no.}\}$

$\Rightarrow A = \{1, 2, 3, 4, 5 \ldots \infty\}$
$\Rightarrow C = \{1, 3, 5, 7, 9 \ldots \infty\}$

$\Rightarrow A \cap C = \{1, 2, 3, 4, 5 \ldots \infty\} \cap \{1, 3, 5, 7, 9 \ldots \infty\}$

$\Rightarrow A \cap C = \{1, 3, 5, 7, 9 \ldots \infty\}$
$\Rightarrow A \cap C = \{x : x \text{ is an odd natural no.}\}$
$\Rightarrow A \cap C = C$

iii) $A \cap D$

sol. ⇒ $A = \{x : x \text{ is a natural no.}\}$

⇒ $D = \{x : x \text{ is a Prime no.}\}$

⇒ $A = \{1, 2, 3, 4, 5 \dots \infty\}$

⇒ $D = \{2, 3, 5, 7 \dots \infty\}$

⇒ $A \cap D = \{1, 2, 3, 4, 5 \dots \infty\} \cap \{2, 3, 5, 7 \dots \infty\}$

⇒ $A \cap D = \{2, 3, 5, 7, \dots \infty\}$

⇒ $A \cap D = \{x : x \text{ is a prime no.}\}$

iv) $B \cap C$

sol. ⇒ $B = \{x : x \text{ is an even natural no.}\}$

⇒ $C = \{x : x \text{ is an odd natural no.}\}$

⇒ $B = \{2, 4, 6, 8, 10 \dots \infty\}$

⇒ $C = \{1, 3, 5, 7, 9 \dots \infty\}$

⇒ $B \cap C = \{2, 4, 6, 8, 10 \dots \infty\} \cap \{1, 3, 5, 7, 9 \dots \infty\}$

⇒ $B \cap C = \cancel{\{}\ \phi$

~~⇒ $B \cap C = \{x : x$~~

v) $B \cap D$

sol. ⇒ $B = \{x : x \text{ is an even natural no.}\}$

⇒ $B = \{2, 4, 6, 8, 10 \dots \infty\}$

⇒ $D = \{x : x \text{ is a Prime no.}\}$

⇒ $D = \{2, 3, 5, 7 \dots \infty\}$

⇒ $B \cap D = \{2, 4, 6, 8 \dots \infty\} \cap \{2, 3, 5, 7 \dots \infty\}$

⇒ $B \cap D = \{2\}$

vi) $C \cap D$.

sol. $\Rightarrow C = \{x : x \text{ is an odd natural no.}\}$

$\Rightarrow D = \{x : x \text{ is a Prime no.}\}$

$\Rightarrow C = \{1, 3, 5, 7, 9 \ldots \infty\}$

$\Rightarrow D = \{2, 3, 5, 7 \ldots \infty\}$

$\Rightarrow C \cap D = \{1, 3, 5, 7, 9 \ldots \infty\} \cap \{2, 3, 5, 7 \ldots \infty\}$

$\Rightarrow C \cap D = \{x : x \text{ is a odd prime no.}\}$

Q.8. Which of the following pairs of sets are disjoint.

i) $\{1, 2, 3, 4\}$ and $\{x : x \text{ is a natural no. and } 4 \leq x \leq 6\}$

sol. Let,

$\Rightarrow A = \{1, 2, 3, 4\}$

$\Rightarrow B = \{x : x \text{ is a natural no. and } 4 \leq x \leq 6\}$

$\Rightarrow B = \{4, 5, 6\}$

$\Rightarrow A \cap B = \{1, 2, 3, 4\} \cap \{4, 5, 6\}$

$\Rightarrow A \cap B = \{4\}$

Hence the pair of sets is not disjoint.

ii) $\{a, e, i, o, u\}$ and $\{c, d, e, f\}$

sol. Let,

$\Rightarrow A = \{a, e, i, o, u\}$

$\Rightarrow B = \{c, d, e, f\}$

$\Rightarrow A \cap B = \{a, e, i, o, u\} \cap \{c, d, e, f\}$

$\Rightarrow A \cap B = \{e\}$

Hence the pair of sets is not disjoint.

iii) $\{x : x$ is an even integer$\}$ and $\{x : x$ is an odd integer$\}$

sol. let.

$\Rightarrow A = \{x : x$ is an even integer$\}$

$\Rightarrow B = \{x : x$ is an odd integer$\}$

$\Rightarrow A \cap B = \{x : x$ is an even integer$\} \cap \{x : x$ is an odd integer$\}$

$\Rightarrow A \cap B = \phi$

Q.9. If $A = \{3, 6, 9, 12, 15, 18, 21\}$, $B = \{4, 8, 12, 16, 20\}$, $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$, $D = \{5, 10, 15, 20\}$, find -

i) $A - B$

sol. $\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow B = \{4, 8, 12, 16, 20\}$

$\Rightarrow A - B = \{3, 6, 9, 12, 15, 18, 21\} - \{4, 8, 12, 16, 20\}$

$\Rightarrow A - B = \{3, 6, 9, 15, 18, 21\}$

ii) $A - C$

sol. $\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$\Rightarrow A - C = \{3, 6, 9, 12, 15, 18, 21\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$

$\Rightarrow A - C = \{3, 9, 15, 18, 21\}$

iii) $A - D$

sol. $\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow D = \{5, 10, 15, 20\}$

$\Rightarrow A - D = \{3, 6, 9, 12, 15, 18, 21\} - \{5, 10, 15, 20\}$

$\Rightarrow A - D = \{3, 6, 9, 12, 18, 21\}$

iv) $B - A$

sol. $\Rightarrow B = \{4, 8, 12, 16, 20\}$

$\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow B - A = \{4, 8, 12, 16, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow B - A = \{4, 8, 16, 20\}$

v) $C - A$

sol. $\Rightarrow C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow C - A = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow C - A = \{2, 4, 8, 10, 14, 16\}$

vi) $D - A$

sol. $\Rightarrow D = \{5, 10, 15, 20\}$

$\Rightarrow A = \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow D - A = \{5, 10, 15, 20\} - \{3, 6, 9, 12, 15, 18, 21\}$

$\Rightarrow D - A = \{5, 10, 20\}$

vii) $B - C$

sol. ⇒ $B = \{4, 8, 12, 16, 20\}$

⇒ $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

⇒ $B - C = \{4, 8, 12, 16, 20\} - \{2, 4, 6, 8, 10, 12, 14, 16\}$

⇒ $B - C = \{20\}$

viii) $B - D$

sol. ⇒ $B = \{4, 8, 12, 16, 20\}$

⇒ $D = \{5, 10, 15, 20\}$

⇒ $B - D = \{4, 8, 12, 16, 20\} - \{5, 10, 15, 20\}$

⇒ $B - D = \{4, 8, 12, 16\}$

ix) $C - B$

sol. ⇒ $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

⇒ $B = \{4, 8, 12, 16, 20\}$

⇒ $C - B = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{4, 8, 12, 16, 20\}$

⇒ $C - B = \{2, 6, 10, 14\}$

x) $D - B$.

sol. ⇒ $D = \{5, 10, 15, 20\}$

⇒ $B = \{4, 8, 12, 16, 20\}$

⇒ $D - B = \{5, 10, 15, 20\} - \{4, 8, 12, 16, 20\}$

⇒ $D - B = \{5, 10, 15\}$

xi) $C - D$

sol. ⇒ $C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

⇒ $D = \{5, 10, 15, 20\}$

⇒ $C - D = \{2, 4, 6, 8, 10, 12, 14, 16\} - \{5, 10, 15, 20\}$

$\Rightarrow \quad C - D = \{2, 4, 6, 8, 12, 14, 16\}$

~~$\Rightarrow \quad C - D = 5$~~

xii) $D - C$

Sol. $\Rightarrow \quad D = \{5, 10, 15, 20\}$

$\Rightarrow \quad C = \{2, 4, 6, 8, 10, 12, 14, 16\}$

$\Rightarrow \quad D - C = \{5, 10, 15, 20\} - \{2, 4, 6, 8, 10, 12, 14\}$

$\Rightarrow \quad D - C = \{5, 15, 20\}$

Q.10. If $X = \{a, b, c, d\}$ and $Y = \{f, b, d, g\}$, find-

i) $X - Y$

Sol. $\Rightarrow \quad X = \{a, b, c, d\}$

$\Rightarrow \quad Y = \{f, b, d, g\}$

$\Rightarrow \quad X - Y = \{a, b, c, d\} - \{f, b, d, g\}$

$\Rightarrow \quad X - Y = \cancel{a, d} \quad \{a, c, \cancel{d}\}$

ii) $Y - X$

Sol. $\Rightarrow \quad Y = \{f, b, d, g\}$

$\Rightarrow \quad X = \{a, b, c, d\}$

$\Rightarrow \quad Y - X = \{f, b, d, g\} - \{a, b, c, d\}$

$\Rightarrow \quad Y - X = \{f, g\}$

iii) $X \cap Y$

Sol. $\Rightarrow \quad X = \{a, b, c, d\}$

$\Rightarrow \quad Y = \{f, b, d, g\}$

$\Rightarrow \quad X \cap Y = \{a, b, c, d\} \quad \{f, b, d, g\}$

$\Rightarrow \quad X \cap Y = \{b, d\}$

Q.11 If R is the set of real no. and θ is the set of rational no., then what is $R - \theta$?

sol. Given.

$\Rightarrow$ $R = \{x : x \text{ is real no.}\}$

$\Rightarrow$ $\theta = \{x : x \text{ is rational no. and } x = \frac{p}{q}\}$

$\Rightarrow$ $R - \theta = \{x : x \text{ is real no.}\} - \{x : x \text{ is rational no. and } x = \frac{p}{q}\}$

$\Rightarrow$ $R - \theta = \{x : x \text{ is set of irrational no.}\}$

Q.12. State wheather each of the following statement is true or false. Justify your answer.

(i) $\{2, 3, 4, 5\}$ and $\{3, 6\}$ are disjoint sets.

Ans. $\Rightarrow$ $A = \{2, 3, 4, 5\}$

$\Rightarrow$ $B = \{3, 6\}$

$\Rightarrow$ $A \cap B = \{2, 3, 4, 5\} \cap \{3, 6\}$

$\Rightarrow$ $A \cap B = \{3\}$

It is joint sets. so, above statement is false.

ii) $\{a, e, i, o, u\}$ and $\{a, b, c, d\}$ are disjoint sets.

sol. $\Rightarrow$ $A = \{a, e, i, o, u\}$

$\Rightarrow$ $B = \{a, b, c, d\}$

$\Rightarrow$ $A \cap B = \{a, e, i, o, u\} \cap \{a, b, c, d\}$

$\Rightarrow$ $A \cap B = \{a\}$

It is joint sets. so above statement is false

iii) $\{2, 6, 10, 14\}$ and $\{3, 7, 11, 15\}$ are disjoint sets.

sol. $\Rightarrow A = \{2, 6, 10, 14\}$

$\Rightarrow B = \{3, 7, 11, 15\}$

$\Rightarrow A \cap B = \{2, 6, 10, 14\} \cap \{3, 7, 11, 15\}$

$\Rightarrow A \cap B = \phi$

It is disjoint sets so above statement is True.

iv) $\{2, 6, 10\}$ and $\{3, 7, 11\}$ are disjoint sets.

sol. $\Rightarrow A = \{2, 6, 10\}$

$\Rightarrow B = \{3, 7, 11\}$

$\Rightarrow A \cap B = \{2, 6, 10\} \cap \{3, 7, 11\}$

$\Rightarrow A \cap B = \phi$

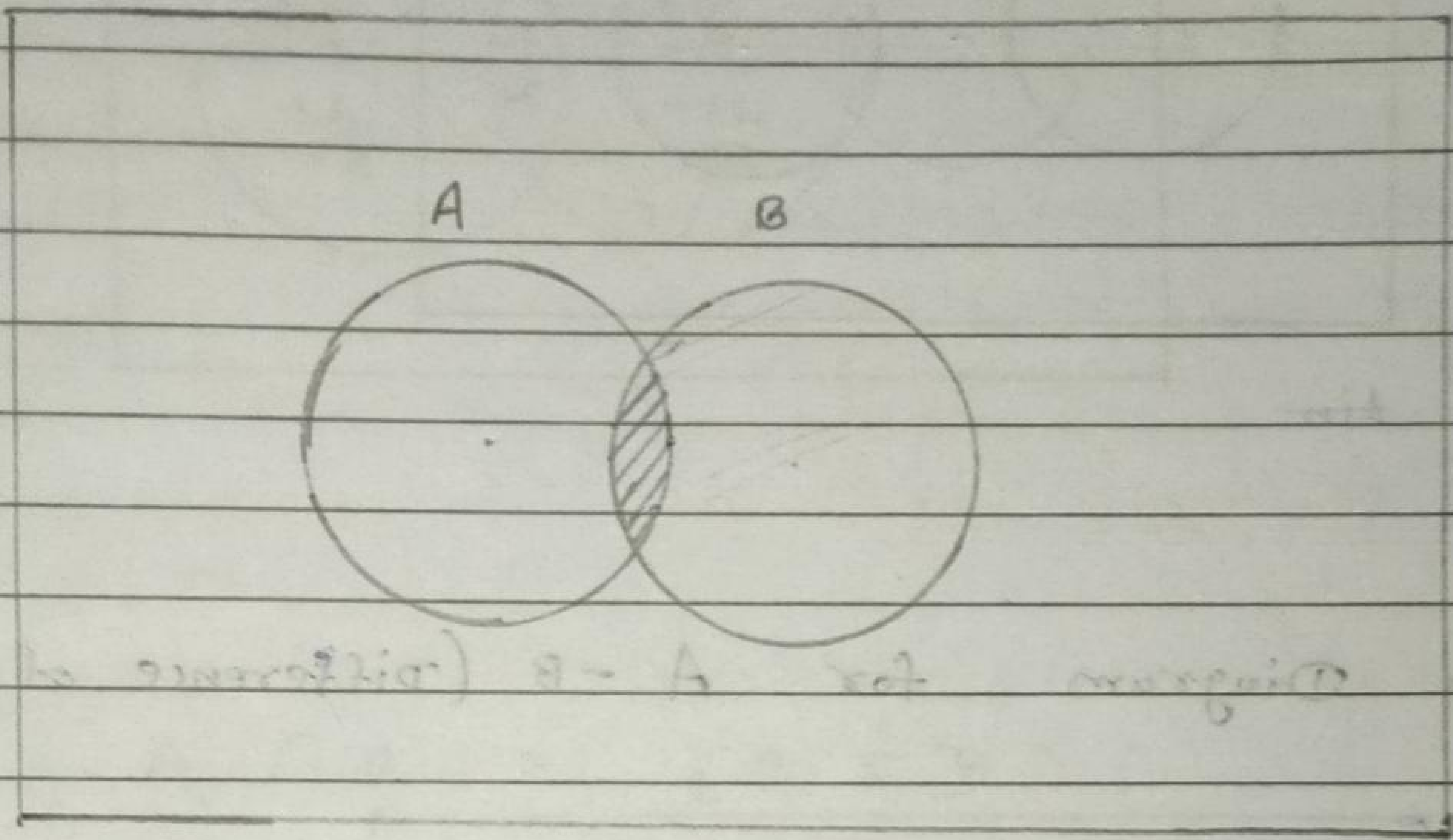
It is disjoint set so above statement is true.

Disjoint Set

The intersection of 2 sets A and B is a null set means no. of elements is zero then A and B are called disjoint set.

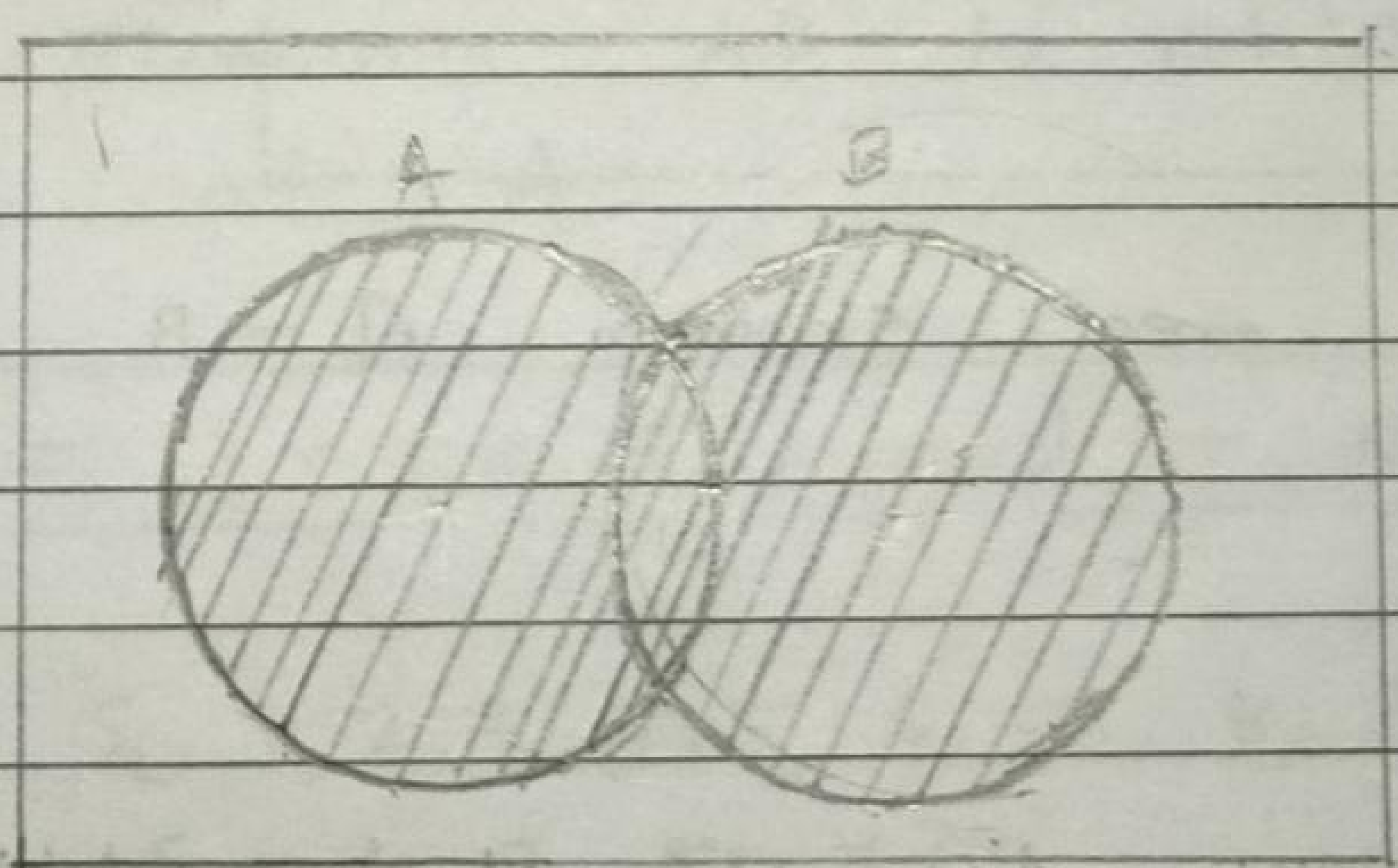
Venn Diagram.

1) venn Diagram for $A \cap B$ ⇒



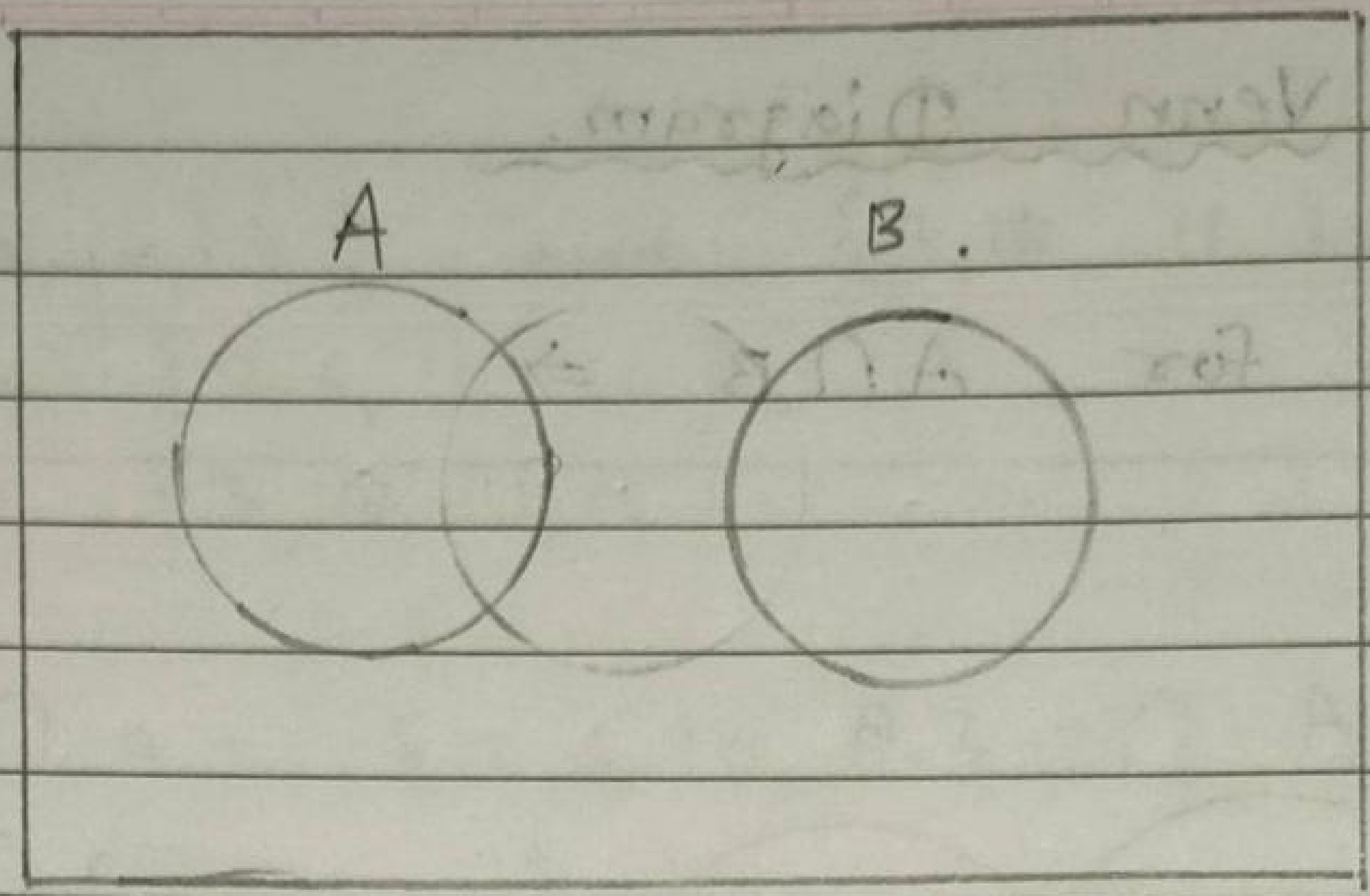
Lining area represent the $A \cap B$

2) venn Diagram for $A \cup B$ ⇒



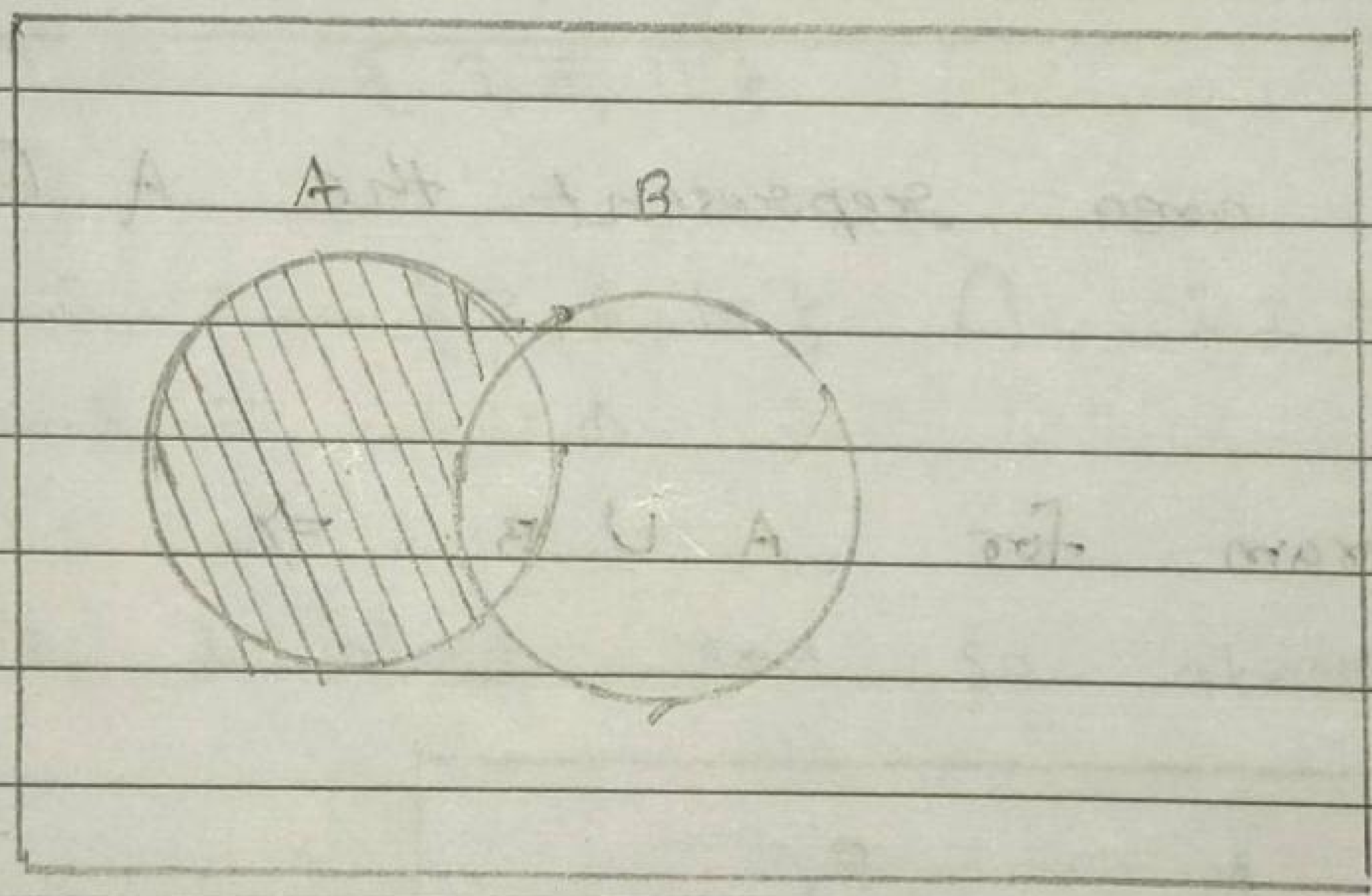
Lining area represent the $A \cup B$.

3) venn Diagram for Disjoint set ⇒



~~Lin~~

4> venn Diagram for A - B (Difference of sets) =>

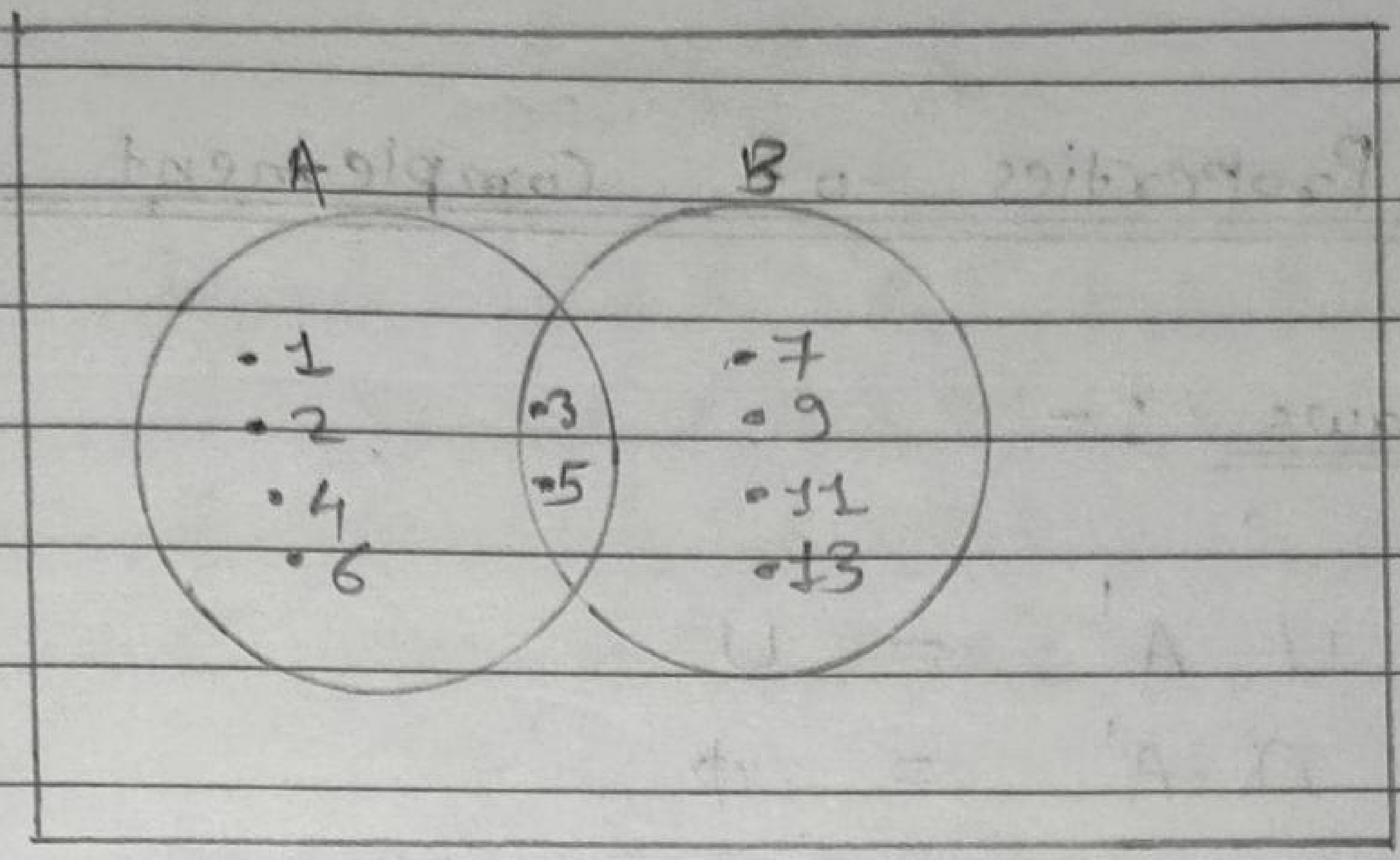


Lining area shows A - B.

<u>Example</u> -> By using venn diagram show which elements belongs to $A \cap B$ if -> A

$A = \{1,2,3,4,5,6\}$
$B = \{3,5,7,9,11,13\}$

Ans.



$A \cap B = \{3, 5\}$

Complement of a set

Let U be the universal set and A a subset of U. Then the complement of A is the set of all elements of U which are not the elements of A.

$$\boxed{A' = \{x : x \in U \text{ and } x \notin A\}}$$

Exam. $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$

$A = \{1, 3, 5, 7\}$

$A' = U - A$

$A' = \{2, 4, 6, 8, 9, 10\}$

Some Properties of Complement sets

1. **Complement laws :-**

i) $A \cup A' = U$
ii) $A \cap A' = \phi$

2. **De Morgan's law :-**

i) $(A \cup B)' = A' \cap B'$
ii) $(A \cap B)' = A' \cup B'$

3. **Law of Double complement :-**

i) $(A')' = A$

4. **Laws of empty set and Universal set :-**

i) $\phi' = U$
ii) $U' = \phi$

Exercise - 1.5

Q.1. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, \cancel{10}\}$
$A = \{1, 2, 3, 4\}$
$B = \{2, 4, 6, 8\}$
$C = \{3, 4, 5, 6\}$

i) $A' = \{5, 6, 7, 8, 9\}$

ii) $B' = \{1, 3, 5, 7, 9\}$

iii) $(A \cup C)' =$

Sol.
$A \cup C = \{1, 2, 3, 4\} \cup \{3, 4, 5, 6\}$
$A \cup C = \{1, 2, 3, 4, 5, 6\}$
$(A \cup C)' = \{7, 8, 9\}$

(iv) $(A \cup B)' =$

Sol.
$A \cup B = \{1, 2, 3, 4\} \cup \{2, 4, 6, 8\}$
$A \cup B = \{1, 2, 3, 4, 6, 8\}$
$(A \cup B)' = \{5, 7, 9\}$

v) $(A')' =$

Sol.
$A' = \{5, 6, 7, 8, 9\}$
$(A')' = \{1, 2, 3, 4\}$
$(A')' = A$

Que.2. If $U = \{a, b, c, d, e, f, g, h,\}$ find the Complements of the following Sets:-

i) $A = \{a, b, c\}$

Sol. $A' = \{d, e, f, i, g, h\}$

(ii) $B = \{d, e, f, g\}$

Sol. $B' = \{a, b, c, h\}$

iii) $C = \{a, c, e, g\}$

Sol. $C' = \{b, d, f, h\}$

iv) $D = \{f, g, h, a\}$

Sol. $D' = \{b, c, d, e\}$

Que. 3. Taking the set of natural numbers as the Universal set, write down the complements of the following sets :-

$U = \{1, 2, 3, 4, 5, 6, 7, 8 \ldots\ldots\}$

(i) $\{x : x$ is an even natural no.$\}$

Sol. Let $A = \{x : x$ is an even natural no.$\}$

$A' = \{x : x$ is an odd natural no.$\}$

(ii) $\{x : x$ is an odd natural no.$\}$

Sol. Let $B = \{x : x$ is an odd natural no.$\}$

$B' = \{x : x$ is an even natural no.$\}$

(iii) $\{x : x$ is a positive multiple of $3\}$

Sol. Let $C = \{x : x$ is a positive multiple of $3\}$

$C' = \{x : x$ is a natural no. and is not multiple of $3.\}$

(iv) $\{x : x$ is a Prime no.$\}$

Sol. $D = \{x : x$ is a Prime no.$\}$

$D' = \{ x = 1$ and x is composite natural no. $\}$

(v) $\{x : x$ is a natural no. and divisible by 3 and 5$\}$

Sol. let $E = \{ x : x$ is a natural no. and divisible by 3 and 5$\}$

$E' = \{x : x$ is a natural no. not divisible by 3 and 5$\}$

(vi) $\{x : x$ is a Perfect square$\}$

Sol. let $F = \{x : x$ is a Perfect square$\}$

$F' = \{x : x$ is ~~a Perfe~~ natural no. and not a Perfect square$\}$

(vii) $\{x : x$ is a Perfect cube$\}$

Sol. let $G = \{x : x$ is a Perfect cube$\}$

$G' = \{x : x$ is a natural no. and not a Perfect cube$\}$

(viii) $\{x : x + 5 = 8\}$

Sol. let $H = \{x : x + 5 = 8\}$

$H' = \{x : x \in N$ and $x \neq 3\}$

(ix) $\{x : 2x + 5 = 9\}$

Sol. let $M = \{x : 2x + 5 = 9\}$

$M' = \{x : x \in N$ and $x \neq 2\}$

(x) $\{x : x \geq 7\}$

Sol. let $N = \{x : x \geq 7\}$
$N' = \{x : x \in N \text{ and } x < 7\}$

(xi) $\{x : x \in N \text{ and } 2x + 1 > 10\}$

Sol. let $Z = \{x : x \in N \text{ and } 2x + 1 > 10\}$
$Z' = \{x : x \in N \text{ and } x < 5\}$

Que.4. If $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{2, 4, 6, 8\}$
$B = \{2, 3, 5, 7\}$ Verify that :

(i) $(A \cup B)' = A' \cap B'$

(ii) $(A \cap B)' = A' \cup B'$

Sol. i) Given

$U = \{1, 2, 3, 4, 5, 6, 7, 8, 9\}$
$A = \{2, 4, 6, 8\}$
$B = \{2, 3, 5, 7\}$

$\Rightarrow A \cup B = \{2, 4, 6, 8\} \cup \{2, 3, 5, 7\}$
$\Rightarrow A \cup B = \{2, 3, 4, 5, 6, 7, 8\}$
$\Rightarrow (A \cup B)' = \{1, 9\}$ —— (i)

$\Rightarrow A' = \{1, 3, 5, 7, 9\}$
$\Rightarrow B' = \{1, 4, 6, 8, 9\}$

$\Rightarrow A' \cap B' = \{1, 3, 5, 7, 9\} \cap \{1, 4, 6, 8, 9\}$
$\Rightarrow A' \cap B' = \{1, 9\}$ —— (ii)

from eq^n (i) and (ii),

$\Rightarrow (A \cup B)' = A' \cap B' \rightarrow$ Proved.

ii) $\Rightarrow A \cap B = \{2,4,6,8\} \cap \{2,3,5,7\}$

$\Rightarrow A \cap B = \{2\}$

$\Rightarrow (A \cap B)' = \{1,3,4,5,6,7,8,9\}$ —— (i)

$\Rightarrow A' \cup B' = \{1,3,5,7,9\} \cup \{1,4,6,8,9\}$

$\Rightarrow A' \cup B' = \{1,3,4,5,6,7,8,9\}$ —— (ii)

from eq^n (i) and (ii),

$\Rightarrow (A \cap B)' = A' \cup B' \rightarrow$ Proved.

Que. 6 Let U be the set of all triangles in a plane. If A is the set of all triangles with least one angle different from 60°, what is A'?

Sol, A' will be the set of equilateral triangle.

Que. 7. Fill in the blanks to make each of the following ~~to make~~ a true statement :-

i) $A \cup A' = \underline{U}$

ii) $\phi' \cap A = \underline{A}$

iii) $A \cap A' = \phi$

iv) $U' \cap A = \phi$

* **Some important Formula :-**

i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

ii) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C)$
$- n(C \cap A) + n(A \cap B \cap C)$

iii) Only $n(A) = n(A) - n(A \cap B)$

iv) $n(A' \cap B') = n(U) - n(A \cup B)$

v) Only $n(B) = n(B) - n(A \cap B)$

Exercise 1.6

Que. 1. If X and Y are two sets such that $n(X) = 17$, $n(Y) = 23$ and $n(X \cup Y) = 38$, find $n(X \cap Y)$

Sol. Given

$\Rightarrow n(X) = 17$

$\Rightarrow n(Y) = 23$

$\Rightarrow n(X \cup Y) = 38$

$\Rightarrow n(X \cap Y) = ?$

We know that,

$\Rightarrow n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 38 = 17 + 23 - n(X \cap Y)$

$\Rightarrow 38 - 40 = -n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 2$

Que. 2. If X and Y are two sets such that X ∪ Y has 18 elements, X has 8 elements and Y has 15 elements how many elements does X ∩ Y have?

Sol. Given,

$\Rightarrow n(X) = 8$

$\Rightarrow n(Y) = 15$

$\Rightarrow n(X \cup Y) = 18$

$\Rightarrow n(X \cap Y) = ?$

We know that,

$\Rightarrow n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 18 = 8 + 15 - n(X \cap Y)$

$\Rightarrow 18 - 23 = -n(X \cap Y)$

$\Rightarrow n(X \cap Y) = 5$

Que. 3. In a group of 400 People, 250 can speak Hindi and 200 can speak English. How many People can speak both Hindi and English?

Sol. Given Let A be the set of people who speak Hindi, and E be the set of People who speak English

$\Rightarrow \therefore n(A \cup B) = 400,$

$\Rightarrow n(A) = 250$

$\Rightarrow n(B) = 200$

$\Rightarrow n(A \cap B) = ?$

We know that,

$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 400 = 250 + 200 - n(A \cap B)$

$\Rightarrow \quad -50 = -n(A \cap B)$

$\Rightarrow \quad n(A \cap B) = 50.$

Que. 4. If S and T are two sets such that S has 21 elements, T has 32 elements and $S \cap T$ has 11 elements, how many element does $S \cup T$?

Sol. Given

$\Rightarrow \quad n(S) = 21$

$\Rightarrow \quad n(T) = 32$

$\Rightarrow \quad n(S \cap T) = 11$

$\Rightarrow \quad n(S \cup T) = ?$

We know that,

$\Rightarrow \quad n(S \cup T) = n(S) + n(T) - n(S \cap T)$

$\Rightarrow \quad n(S \cup T) = 21 + 32 - 11$

$\Rightarrow \quad n(S \cup T) = 42.$

Hence, The set $n(S \cup T)$ has 42 elements.

Que. 5. If X and Y are two sets such that X has 40 elements, $X \cup Y$ has 60 elements and $X \cap Y$ has 10 elements, how many elements does Y have?

Sol. Given

$\Rightarrow \quad n(X) = 40$

$\Rightarrow \quad n(Y) = ?$

$\Rightarrow \quad n(X \cup Y) = 60$

$\Rightarrow \quad n(X \cap Y) = 10$

We know that,

$\Rightarrow \quad n(X \cup Y) = n(X) + n(Y) - n(X \cap Y)$

$\Rightarrow 60 = 40 + n(Y) - 10$
$\Rightarrow n(Y) = 30$
Hence, The set Y has 30 elements.

Que. 6 In a group of 70 People, 37 like coffee, 52 like tea and each Person likes at least one of the two drinks. How many People like both coffee and Tea.

Sol. Let set A represent for coffee, and B represent for tea.

Given
$\Rightarrow n(A \cup B) = 70$
$\Rightarrow n(A \cap B) = ?$
$\Rightarrow n(A) = 37$
$\Rightarrow n(B) = 52$

We know that,

$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$
$\Rightarrow 70 = 37 + 52 - n(A \cap B)$
$\Rightarrow -19 = -n(A \cap B)$
$\Rightarrow n(A \cap B) = 19$

Thus, 19 People like both coffee and tea.

Que. 7. In a group of 65 People, 40 like cricket, 10 like both cricket and tennis. how many like tennis only and not cricket? How many like tennis

Sol. Let set A represent people who like cricket, and B represent people who like both cricket and tennis.

Given,

$\Rightarrow n(A) = 40$

$\Rightarrow n(A \cup B) = 65$

$\Rightarrow n(A \cap B) = 10$

$\Rightarrow n(B) = ?$

We know that,

$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow 65 = 40 + n(B) - 10$

$\Rightarrow 35 = n(B)$

$\Rightarrow n(B) = 35.$

Therefore, 35 people like tennis.

Que. 8: In a committee, 50 people speak French, 20 speak spanish and 10 speak both spanish and French. How many speak at least one of these two languages?

Sol. Let set A denote people who speak French, B denote people who speak spanish, C denote people who speak both spanish and French.

Given.

$\Rightarrow n(A \cap B) = 10$

$\Rightarrow n(A) = 50$

$\Rightarrow n(B) = 20$

$\Rightarrow n(A \cup B) = ?$

We know that,

$\Rightarrow n(A \cup B) = n(A) + n(B) - n(A \cap B)$

$\Rightarrow n(A \cup B) = 50 + 20 - 10$

$\Rightarrow n(A \cup B) = 60.$

Miscellaneus Exercise.

Que. 1. Decide, among the following sets ~~determine~~ ~~one example~~ and another.

$A = \{x : x \in R$ and x satisfy $x^2 - 8x + 12 = 0\}$
$A = \{2, 6\}$
$B = \{2, 4, 6\}$
$C = \{2, 4, 6, 8 \ldots\}$
$D = \{6\}$

Sol. $D \subset A$, $D \subset B$, $D \subset C$, $A \subset B$, $A \subset C$

Que. 2. In each of the following, ~~false~~, give an example.

(i) If $x \in A$ and $A \in B$, then $x \in B$.
Ans. suppose two sets A and B are there.

$A = \{1, 2\}$
$B = \{1, \{1, 2\}, \{3\}\}$

Here
$2 \in A$ and $2 \notin B$ & $A \in B$.

Hence, Above statement is False.

(ii) If $A \subset B$ and $B \in C$ then $A \in C$.
Ans let, $A = \cancel{\{1, 2\}}$ $\{2\}$
$B = \cancel{\{1, \{1, 2\}, 3\}}$ $\{0, 2\}$
$C = \{1, \{0, 2\}, 3\}$

Here,

$$A \subset B \quad , \quad B \in C$$

but $A \notin C$

Hence, The above statement is false.

(iii) If $A \subset B$ and $B \subset C$, then $A \subset C$?

Ans. Let $x \in A$

$x \in B \quad (\because A \subset B)$

$x \in C \quad (\because B \subset C)$

$\therefore A \subset C.$

(iv) If $A \not\subset B$ and $B \not\subset C$, then $A \not\subset C$.

Ans. Let $A = \{1, 2\}$

$B = \{0, 6, 8\}$

$C = \{1, 2, 6, 9\}$

Here,

$A \not\subset B$ and $B \not\subset C$ but $A \subset C$.

Hence above statement is false.

(v) If $x \in A$ and $A \not\subset B$, then $x \in B$.

Ans. Let $A = \{3, 5, 7\}$

$B = \{2, 4, 6, 8\}$

Here, $A \not\subset B$ and $x \notin B$.

Hence, Above statement is false

(vi) If $A \subset B$ and $x \notin B$, then $x \notin A$.

Ans. let $A = \{2, 4\}$
$B = \{0, 2, 4, 6\}$

and. $A \subset B$ and $x \notin B$.
suppose. $x \in B$,
Then, $x \in A$ which is a contradiction as $x \notin A$.

$\therefore \quad x \notin A$.

Que. 3. Let A, B and C be the sets ——— Show that $B = C$.

Sol. let A, B and C be the sets such that $A \cup B = A \cup C$ and $A \cap B = A \cap C$.

let $x \in B$

$\Rightarrow \quad x \in A \cup B \quad [\because B \subset A \cup B]$
$\Rightarrow \quad x \in A \cup C \quad [\because A \cup B = A \cup C]$
$\Rightarrow \quad x \in A$ or $x \in C$

Case 1 :- $x \in A$
$x \in B$
$\therefore \quad x \in A \cap B$.

$\Rightarrow \quad x \in A \cap C \quad [\because A \cap B = A \cap C]$
$\therefore \quad x \in A, \quad x \in C$
$\therefore \quad x \in C$
$\therefore \quad B \subset C$
Similarly we can show that $C \subset B$.
$\therefore \quad B \subset C$,

Que. 4. Show that the following 4 conditions are equivalent.

i) $A \subset B$, ii) $A - B = \phi$
iii) $A \cup B = B$, iv) $A \cap B = A$

Sol: i) $A \subset B \Rightarrow$ All the elements of set A belongs to set B.
$\Rightarrow A - B = \phi$ (i) $\Leftrightarrow$ (ii)

ii) $A - B = \phi \Leftrightarrow$ All the elements of set A belongs to set B.
$\Leftrightarrow A \cup B = B$
= (ii) $\Leftrightarrow$ (iii)

iii) $A \cup B = B \Leftrightarrow$ All the elements of set A belongs to set B.

iv) $A \cap B = A$ (iii) $\Longleftrightarrow$ (iv)
Hence, all the given relations are equivalent.

Que. 5. show that if $A \subset B$, then $C-B \subset C-A$.

Sol. Let,

$$A \subset B$$

To Represent :- $C-B \subset C-A$

let $x \in \cancel{C-B}\ C-B$

$\Rightarrow$ $x \in C$ and $x \notin B$

$\Rightarrow$ $x \in C$ and $x \notin A$ $[A \subset B]$

$\Rightarrow$ $x \in C-A$

$\therefore$ $C-B \subset C-A$

Que. 6. Assume that $P(A) = P(B)$. show that $A = B$.

Sol. let x be an element of set A or $x \in A$. Then their will be sure a set X such that $x \in X$.

$\therefore$ $x \subset A$

$\Rightarrow$ $x \in P(A)$

$\Rightarrow$ $x \in P(B)$ $\{P(A) = P(B)\}$

$\Rightarrow$ $x \subset B$ $\{P(A) = P(B)\}$

$\Rightarrow$ $x \in B$

$\therefore$ $x \in A$ and $x \in B$

$\Rightarrow$ $A \subset B$ —— (i)

similarly

$B \subset A$ —— (ii)

from eq^n (i) and (ii),

$\therefore$ $A = B$.

Que. 4. Is it true that for any sets ——————— ——————— your answer.

Sol. let $A = \{0, 1\}$,
$B = \{1, 2\}$

$\therefore A \cup B = \{0, 1, 2\}$
$\Rightarrow P(A) = \{\phi, \{0\}, \{1\}, \{0, 1\}\}$
$\Rightarrow P(B) = \{\phi, \{1\}, \{2\}, \{1, 2\}\}$
$\Rightarrow P(A \cup B) = \{\phi, \{0\}, \{1\}, \{2\}, \{0, 1\}, \{1, 2\}, \{0, 2\}, \{0, 1, 2\}\}$

$\Rightarrow P(A) \cup P(B) = \{\phi, \{0\}, \{1\}, \{0, 1\}, \{2\}, \{1, 2\}\}$

$\therefore P(A) \cup P(B) \neq P(A \cup B)$

Que. 10. Show that $A \cap B = A \cap C$ need not imply $B = C$.

Sol. Let,
$A = \{0, 1\}$
$B = \{0, 2, 3\}$
$C = \{0, 4, 5\}$

According to question,

$\Rightarrow A \cap B = \{0, 1\} \cap \{0, 2, 3\}$
$\Rightarrow A \cap B = \{0\}$ —— (i)

and,
$\Rightarrow A \cap C = \{0, 1\} \cap \{0, 4, 5\}$
$\Rightarrow A \cap C = \{0\}$ —— (ii)

from eqⁿ (i) and (ii),

$\Rightarrow A \cap B = A \cap C = \{0\}$
$\Rightarrow B \neq C \quad [2 \in B \text{ and } 2 \notin C]$

Que. 11 Let A and B be sets. If $A \cap X = B \cap X = \phi$ and ~~$A \cap X$~~ $A \cup X = B \cup X$ for some set X, show that $A = B$.

~~Sol.~~ (Hints $A = A \cap (A \cup X)$, $B = B \cap (B \cup X)$ and use distributive law.)

Sol. Let A and B be two sets such that $A \cap X = B \cap X = \phi$ and $A \cup X = B \cup X$ for some set X.

To show : $A = B$

According to Question,

$\Rightarrow A = A \cap (A \cup X) = A \cap (B \cup X) \quad [\because A \cup X = B \cup X]$

$= (A \cap B) \cup (A \cap X) \quad [\because \text{Distributive law}]$
$= (A \cap B) \cup \phi \quad [\because A \cap X = \phi]$
$= A \cap B$ ———— (1)

Now,
$\Rightarrow B = B \cap (B \cup X)$
$\Rightarrow B = B \cap (A \cup X) \quad [\because B \cup X = A \cup X]$
$\Rightarrow B = (B \cap A) \cup (B \cap X) \quad [\text{Distributive law}]$
$\Rightarrow B = (B \cap A) \cup \phi \quad [\because B \cap X = \phi]$
$\Rightarrow B = B \cap A$
$\Rightarrow B = A \cap B$ ———— (2)

from eqⁿ ① and ②,

$\Rightarrow$ $A = B$

Que. 12 Find sets A, B and C such that $A \cap B$, $B \cap C$ and $A \cap C$ are non empty sets and $A \cap B \cap C = \phi$

Sol. Let,

$\Rightarrow$ $A = \{0, 1\}$
$\Rightarrow$ $B = \{1, 2\}$
$\Rightarrow$ $C = \{2, 0\}$

According to question,

$\Rightarrow$ $A \cap B = \{0, 1\} \cap \{1, 2\}$
$\Rightarrow$ $A \cap B = \{1\}$

and

$\Rightarrow$ $B \cap C = \{1, 2\} \cap \{0, 2\}$
$\Rightarrow$ $B \cap C = \{2\}$

and

$\Rightarrow$ $A \cap C = \{0, 1\} \cap \{2, 0\}$
$\Rightarrow$ $A \cap C = \{0\}$

$\therefore$ $A \cap B$, $B \cap C$ and $A \cap C$ are non empty sets.

but

$\Rightarrow$ $A \cap B \cap C = \{0, 1\} \cap \{1, 2\} \cap \{0, 2\}$

$\Rightarrow A \cap B \cap C = \{1\} \cap \{0, 2\}$

$\Rightarrow A \cap B \cap C = \phi$

Que.13. In a survey of 600 students in a School, 150 students were found to be taking tea and 225 taking coffee. 100 were taking both tea and coffee. Find how many students were taking neither tea nor coffee?

Sol. Let A be the set of all students who took part in the survey.

B be the set of students who take tea.

C be the set of students who take coffee

~~D be the set of students~~

Given,

$\Rightarrow \quad n(A) = 600, \; n(B) = 150, \; n(C) = 225,$

$n(B \cap C) = 100$

~~According to question,~~

~~We know that,~~

$\Rightarrow \cancel{n(A \cup B) = n(A) + n(B) - n(A \cap B)}$

To find : $n(B' \cap C')$

$$\Rightarrow n(B' \cap C') = \cancel{n(B \cap C)'} \; n(B \cup C)'$$

$$= n(A) - n(B \cup C)$$

$$= n(A) - [n(B) + n(C) - n(B \cap C)]$$

$$= 600 - [150 + 225 - 100]$$

$$= 600 - [275]$$

$$n(B' \cap C') = 325.$$

Hence, 325 students were taking neither tea nor coffee.

Que. 14. In a group of students, 100 students know Hindi, 50 know Eng. and 25 know both. Each of the students knows either Hindi or English. How many students are their in group?

Sol. Let,

A be the set of all students.
B be the set of students who know Hindi.
C be the set of students who know English.

$\therefore$ $B \cup C = A$

According to question,

$\Rightarrow n(B) = 100$, $n(C) = 50$, $n(B \cap C) = 25$

$\Rightarrow n(B \cup C) = n(B) + n(C) - n(B \cap C)$
$\Rightarrow n(B \cup C) = 100 + 50 - 25$
$\Rightarrow n(B \cup C) = 125$

Hence, there are 125 students in the group.

Que. 15. In a survey of 60 People, It was found that 25 people read newspaper ______________ Newspaper. find:

i) The no. of people who read at least one of the newspaper.

ii) The no. of People who read exactly one newspaper.

Sol: Let,

A be the set of people who read newspaper H.
B be the set of people who read newspaper T.
C be the set of people who read newspaper I.

Given,

$\Rightarrow$ $n(A) = 25$, $n(B) = 26$, $n(C) = 26$,
$n(A \cap B) = 11$,
$n(A \cap C) = 9$
$n(B \cap C) = 8$,
$n(A \cap B \cap C) = 3$.

i) We know that,

$\Rightarrow n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(A \cap C) - n(B \cap C) + n(A \cap B \cap C)$

$\Rightarrow \cancel{3} = 25 + 26 + 26 - 11 - 9 - 8 + \overset{3}{\cancel{n(A \cap B \cap C)}}$
$\Rightarrow \cancel{n(A \cap B \cap C)}$

$\Rightarrow n(A \cup B \cup C) = 52$

ii) Let,

a be the no. of people who read newspaper H and T only.

b be the no. of people who read newspaper I and ~~H~~ only.

c be the no. of people who read newspaper T and I.

d be the no. of people who read newspaper H, T and I all.

According to question,

$\Rightarrow d = n(A \cap B \cap C) = 3$

$\Rightarrow$ now, $n(A \cap B) = a + d$

$\Rightarrow n(B \cap C) = c + d$

$\Rightarrow n(C \cap A) = b + d$

$\therefore (a+d) + (c+d) + (b+d) =$

$= 11 + 8 + 9$

$= 28$

$\Rightarrow a + b + c + d = 28 - 2d$

$= 28 - 6$

$= 22$

Hence, $(52 - 22) = 30$ people read exactly one newspaper.