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Chapter-5
Continuity And Differentiability

undefined Value :- $\frac{0}{0}$, $\frac{1}{0}$, ∞, $\sqrt{-ve}$, 0^0.

formula of limits :-

① $\lim_{x \to a} \frac{x^n - a^n}{x - a} = na^{n-1}$

② $\lim_{\theta \to 0} \frac{\sin\theta}{\theta} = 1$

③ $\lim_{\theta \to 0} \cos\theta = 1$

④ $\lim_{\theta \to 0} \frac{\tan\theta}{\theta} = 1$

⑤ $\lim_{x \to 0} \frac{e^x - 1}{x} = 1$

⑥ $\lim_{x \to 0} \frac{a^x - 1}{x} = \log_e a$

⑦ $\lim_{x \to 0} \sin\frac{1}{x}$ = A finite value.

Continuity :-

(i) a function f(x) is called continous at $x = c$ if

$$\lim_{x \to c} f(x) = f(c)$$

(ii) a function f(x) is continous at $x = c$ when

$$L.H.L = f(c) = R.H.L$$

where,

$$L.H.L = \lim_{x \to c^-} f(x)$$

$$= \lim_{x \to 0} f(c-x)$$

$$R.H.L = \lim_{x \to c^+} f(x)$$

$$= \lim_{x \to 0} f(c+x)$$

Exercise 5.1

① prove that the function $f(x) = 5x-3$ is continous at $x=0$, at $x=-3$ and at $x=5$.

Sol.

$$f(x) = 5x-3$$

at $x=0$,

$$f(0) = 5(0)-3 = -3$$

$$\Rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 0} (5x-3)$$

$$= 5(0)-3$$

$$= -3.$$

Here,

$$\lim_{x \to 0} f(x) = f(0)$$

Here, $f(x)$ is continous at $x=0$.

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at $x = -3$,

$f(-3) = 5(-3) - 3$
$= -15 - 3$
$= -18.$

$\Rightarrow \lim_{x \to -3} f(x) = \lim_{x \to -3} (5x - 3)$

$= (5(-3) - 3)$

$\Rightarrow \lim_{x \to -3} f(x) = -18$

$\Rightarrow \lim_{x \to -3} f(x) = f(-3)$

Here, $f(x)$ is continous at $x = -3$.

at $x = 5^-$,

$f(5^-) = 5(5) - 3$
$= 22$

$\Rightarrow \lim_{x \to 0} f(x) = \lim_{x \to 5} (5x - 3)$

$= 5(5) - 3$
$= 25 - 3$
$= 22$

Here, $f(x)$ is continous.

3) Examine the following function for continuity.

(a) $f(x) = \dfrac{x^2 - 25}{x + 5}$, $x \neq -5$

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Sol. $f(x) = \frac{x^2-25}{x+5}$

at $x = c$,

$\Rightarrow f(c) = \frac{c^2-25}{c+5}$

$= \frac{\cancel{(c+5)}(c-5)}{\cancel{(c+5)}}$

$\Rightarrow f(c) = c-5$

$\Rightarrow \lim_{x \to c} f(x) = \lim_{x \to c} \frac{x^2-25}{x+5}$

$= \lim_{x \to c} \frac{(x+5)(x-5)}{(x+5)}$

$= \lim_{x \to c} (x-5)$

$\Rightarrow \lim_{x \to c} f(x) = c-5$

Here, $\lim_{x \to c} f(x) = f(c)$ therefor $f(x)$ is continous.

(a) $f(x) = x-5$

Sol. $f(x) = x-5$

at $x = c$,

$\Rightarrow f(c) = c-5$

$\Rightarrow \lim_{x \to c} f(x) = \lim_{x \to c} (x-5)$

$\Rightarrow \lim_{x \to c} f(x) = c-5$

$\Rightarrow \lim_{x \to c} f(x) = f(c)$

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Here, f(x) is continous.

(b) $f(x) = \frac{1}{x-5}, x \neq 5$

Sol. $f(x) = \frac{1}{x-5}$.

at $x = c$,

$f(c) = \frac{1}{c-5}$.

$\Rightarrow \lim_{x\to c} f(x) = \lim_{x\to c} \left(\frac{1}{x-5}\right)$

$\Rightarrow \lim_{x\to c} f(x) = \frac{1}{c-5}$

$\Rightarrow \lim_{x\to c} f(x) = f(c)$

Here, f(x) is continous.

(d) $f(x) = |x-5|$

Sol. $f(x) = |x-5|$

at $x = c$,

$\Rightarrow f(c) = |c-5|$

$\Rightarrow \lim_{x\to c} f(x) = \lim_{x\to c} |x-5|$

$= \cancel{\lim} \; |c-5|$

$\Rightarrow \lim_{x\to c} f(x) = f(c)$

Here, f(x) is continous.

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(3) Examine the continuity of the function $f(x) = 2x^2 - 1$ at $x = 3$.

Sol.

$f(x) = 2x^2 - 1$

at $x = 3$,

$f(3) = 2(3)^2 - 1$

$f(3) = 2(9) - 1$

$f(3) = 18 - 1$

$f(3) = 17$

$\Rightarrow \lim_{x \to 3} f(x) = \lim_{x \to 3} (2x^2 - 1)$

$= \{2(3)^2 - 1\}$

$= \{2(9) - 1\}$

$= 18 - 1$

$= 17$

$\Rightarrow \lim_{x \to 3} f(x) = f(3)$

Here, $f(x)$ is continous.

(4) Prove that the function $f(x) = x^n$ is continous at $x = n$, where n is positive integer.

Sol.

$\Rightarrow f(x) = x^n$

f is defined at all positive integers, n, it value is n^n.

then,

$\lim_{x \to n} f(n) = \lim_{x \to n} (x^n) = n^n$

$\therefore \lim_{x \to n} f(x) = f(n)$

Hence f is continous.

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(5) Is the function f defined by

$$f(x) = \begin{cases} x, & \text{if } x \le 1 \\ 5, & \text{if } x > 1 \end{cases}$$

continous at $x = 0$ At $x = 1$? and at $x = 2$?

Now,

Find all points of discontinuity of f, where f is defined by {from Q.6 to Q.12}.

(6) $$f(x) = \begin{cases} 2x+3, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$$

Sol. $$f(x) = \begin{cases} 2x+3, & x \le 2 \\ 2x-3, & x > 2 \end{cases}$$

at $x = 2$,

$$f(2) = 2(2) + 3$$
$$= 7$$

L.H.L. $= \lim_{x \to 2^-} f(x)$

$= \lim_{x \to 0} f(2-x)$

$= \lim_{x \to 0} 2(2-x) + 3$

$= 4 + 3$

$= 7$

R.H.L $= \lim_{x \to 2^+} f(x)$

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$$= \lim_{x \to 0} f(2+x)$$

$$= \lim_{x \to 0} \{2(2+x) - 3\}$$

$$= 2(2+0) - 3$$

$$= 4 - 3$$

$$\text{R.H.L.} = 1$$

Here,

$$\lim_{x \to 2^-} f(x) = f(2) \neq \lim_{x \to 2^+} f(x)$$

$\therefore$ $f(x)$ is discontinous at $x = 2$.

(8) $f(x) = \begin{cases} \frac{|x|}{x} & , \text{ if } x \neq 0 \\ 0 & , \text{ if } x = 0 \end{cases}$

Sol. $f(x) = \begin{cases} \frac{|x|}{x} & , \text{ if } x \neq 0 \\ 0 & , \text{ if } x = 0 \end{cases}$

at $x = 0$,

$$f(0) = 0$$

$$\text{L.H.L} = \lim_{x \to 0^-} f(x)$$

$$= \lim_{x \to 0} f(0 - x)$$

$$= \lim_{x \to 0} f(-x)$$

$$= \lim_{x \to 0} \frac{|-x|}{-x}$$

$$= \lim_{x \to 0} \frac{x}{-x}$$

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$$= \lim_{x \to 0} (-1)$$

$\Rightarrow$ L.H.L. $= -1$

$$\text{R.H.L} = \lim_{x \to 0^+} f(x)$$

$$= \lim_{x \to 0} f(0+x)$$

$$= \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} \frac{|x|}{x}$$

$$= \lim_{x \to 0} \frac{x}{x}$$

$$= \lim_{x \to 0} 1$$

$\Rightarrow$ R.H.L. $= 1$

Here,

$$\lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x) \neq f(0)$$

(7) $f(x) = \begin{cases} |x|+3, & x \leq -3, \\ -2x, & -3 < x < 3 \\ 6x+2, & x \geq 3 \end{cases}$

Sol.

$$f(x) = \begin{cases} |x|+3, & x \leq -3. \\ -2x, & -3 < x < 3. \\ 6x+2, & x \geq 3 \end{cases}$$

at $x = -3$,

$f(-3) = |-3| + 3 = -(-3) + 3 = 3+3 = 6.$

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Rough
-3 -2
-4 -1
-5 0

$\underline{\text{L.H.L.}} = \lim_{x \to -3^-} f(x)$

$= \lim_{x \to 0} f(-3-x)$

$= \lim_{x \to 0} \{|-3-x| + 3\}$

$= \lim_{x \to 0} \{-(-3-x) + 3\}$

$= \lim_{x \to 0} \{(3+x) + 3\}$

$= \lim_{x \to 0} \{6 + x\}$

$= 6 + 0 = 6$

$\underline{\text{R.H.L.}} = \lim_{x \to -3^+} f(x)$

$= \lim_{x \to 0} \cancel{f(3+x)}\ f(-3+x)$

$= \lim_{x \to 0} \{-2(-3+x)\}$

$= \lim_{x \to 0} \{6 + 2x\}$

$= \cancel{\lim_{x \to 0}} \{\ 6 + 2(0)$

R.H.L. $= 6$

<u>here</u>

~~f(-3) = L.H.S = R~~

$f(-3) = \lim_{x \to -3^-} f(x) = \lim_{x \to -3^+} f(x)$

$\therefore$ $f(x)$ is continous. where $f(x) = \begin{cases} |x| + 3, & x \le -3 \\ -2x & -3 < x < 3 \end{cases}$

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Now,
at $x = 3$

$f(x) = 6x + 2$
$f(3) = 6(3) + 2$
$= 20$

L.H.L. $= \lim_{x \to 3^-} f(x)$

$= \lim_{x \to 0} f(3-x)$

$= \lim_{x \to 0} -2(3-x)$

$= \lim_{x \to 0} \{-6 + x\}$

$= -6 + 0$

L.H.L. $= -6.$

R.H.L. $= \lim_{x \to 3^+} f(x)$

$= \lim_{x \to 0} f(3+x)$

$= \lim_{x \to 0} \{6(3+x) + 2\}$

$= \lim_{x \to 0} \{18 + 6x + 2\}$

$= 20 + 6(0)$

R.H.L. $= 20$

$\lim_{x \to 3^-} f(x) \neq \lim_{x \to 3^+} = f(3)$

$\therefore$ f(x) is discontinous at $x = 3$ where $f(x) = \begin{cases} -2x, & x < 3 \\ 6x+2, & x \geq 3 \end{cases}$

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$$\therefore f(x) = \begin{cases} 4x + 3 & , \quad 3 \le x \\ -2x & , \quad -3 < x < 3 \\ 6x + 2 & , \quad x \ge 3 \end{cases}$$

Sol. 5

$$f(x) = \begin{cases} x & , \quad x \le 1 \\ 5 & , \quad x > 1 \end{cases}$$

at $x = 0$.

$f(0) = 0$

<u>L.H.L</u> $= \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0} f(0-x)$

$= \lim_{x \to 0} f(-x)$

$= \lim_{x \to 0} (-x) = -0 = 0.$

<u>R.H.L</u> $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0} f(0+x)$

$= \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} x$

$= 0$

$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

$\therefore$ $f(x)$ is continuous at $x = 0$.

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Now, at $x = 1$,

$f(1) = 1$

R.H.L. $= \lim\limits_{x \to 0^+} f(x) = \lim\limits_{x \to 1^+} f(x)$

$= \lim\limits_{x \to 0} f(0+x) = \lim\limits_{x \to 0} f(1+x)$

$= \lim\limits_{x \to 0} f(x) = \lim\limits_{x \to 0} 5$

$= \lim\limits_{x \to 0} x \quad = 5$

$= 0.$

L.H.L. $= \lim\limits_{x \to 0^-} f(x) = \lim\limits_{x \to 1^-} f(x)$

$= \lim\limits_{x \to 0} f(0-x) = \lim\limits_{x \to 0} f(1-x)$

$= \lim\limits_{x \to 0} f(-x) = \lim\limits_{x \to 0} 1-x$

$= \lim\limits_{x \to 0} (-x) = 1-0$

$= 1$

$= -0$

$= 0.$

$\Rightarrow f(1) \neq \lim\limits_{x \to 1^+} f(x)$

$\Rightarrow f(1) = \lim\limits_{x \to 1^-} f(x)$

$\Rightarrow f(1) = \lim\limits_{x \to 1^-} f(x) \neq \lim\limits_{x \to 1^+} f(x)$

$\therefore$ $f(x)$ is discontinous at $x = 1$.

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Now, at $x=2$,

$\Rightarrow f(2) = 5$.

L.H.L. $= \lim_{x \to 2^-} f(x)$

$= \lim_{x \to 0} f(2-x)$

$= \lim_{x \to 0} 5$

$= 5$

$\Rightarrow$ R.H.L. $= \lim_{x \to 2^+} f(x)$

$= \lim_{x \to 0} f(2+x)$

$= \lim_{x \to 0} 5$

$= 5$

$\Rightarrow f(2) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$

$\therefore$ $f(x)$ is continous at $x=2$.

(9) $f(x) = \begin{cases} \frac{(x)}{|x|} & , x < 0 \\ -1 & , x \geq 0. \end{cases}$

Sol.

$f(x) = \begin{cases} \frac{x}{|x|} & , x < 0 \\ -1 & , x \geq 0 \end{cases}$

at $x = 0$

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$f(0) = -1$

R.H.L. $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0} f(0+x)$

$= \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} -1$

$= -1$

L.H.L. $= \cancel{\lim_{x \to 0^+}} \lim_{x \to 0^-} f(x)$

$= \lim_{x \to 0} f(0-x)$

$= \lim_{x \to 0} f(-x)$

$= \lim_{x \to 0} \frac{-x}{|-x|}$

$= \lim_{x \to 0} \frac{-x}{-(-x)}$

$= \lim_{x \to 0} -1$

$= -1.$

$f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$\therefore$ $f(x)$ is continous at $x = 0$.

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(10) $f(x) = \begin{cases} x+1 & , \quad x \geq 1 \\ x^2+1 & , \quad x < 1. \end{cases}$

Sol. $f(x) = \begin{cases} x+1 & , \quad x \geq 1 \\ x^2+1 & , \quad x < 1 \end{cases}$

at $x = 1.$,

$\Rightarrow f(1) = 1+1 = 2$

L.H.L. $= \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 0} f(1-x)$

$= \lim_{x \to 0} \{(1-x)^2+1\}$

$= \lim_{x \to 0} \{1+x^2-2x+1\}$

$= \lim_{x \to 0} \{x^2-2x+2\}$

$= (0)^2 - 2(0) + 2$

$= 2$

R.H.L. $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 0} f(1+x)$

$= \lim_{x \to 0} \{(1+x)+1\}$

$= \lim_{x \to 0} (2+x)$

$= 2+0 = 2$

$f(1) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\therefore$ $f(x)$ is continous at $x = 1$

1) $f(x) = \begin{cases} \cancel{x} \; x^3 - 3 & , x \leq 2 \\ x^2 + 1 & , x > 2 \end{cases}$

Sol $f(x) = \begin{cases} x^3 - 3 & , x \leq 2 \\ x^2 + 1 & , x > 2 \end{cases}$

at $x = 2$,

$f(2) = 2^3 - 3 = 8 - 3 = 5$

$L.H.L = \lim_{x \to 2^-} f(x)$

$= \lim_{x \to 0} f(2 - x)$

$= \lim_{x \to 0} \{(2-x)^3 - 3\}$

$= \lim_{x \to 0} \{(2-x)^2 (2-x) - 3\}$

$= \lim_{x \to 0} \{(4 + x^2 - 4x)(2-x) - 3\}$

$= \lim_{x \to 0} \{(8 + 2x^2 - 8x - 4x - x^3 + 4x^2) - 3\}$

$= \lim_{x \to 0} \{(-x^3 + 6x^2 - 12x + 5)\}$

$= (-0^3 + 6(0)^2 - 12(0) + 5)$

$= 5$

$R.H.L. = \lim_{x \to 2^+} f(x)$

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$$= \lim_{x \to 0} f(2+x)$$

$$= \lim_{x \to 0} (2+x)^2 + 1$$

$$= \lim_{x \to 0} (5 + x^2 + 4x)$$

$$= 5 + (0)^2 + 4(0)$$

$$= 5$$

$$\Rightarrow f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$\therefore f(x)$ is continous at $x = 2$.

(12) $$f(x) = \begin{cases} x^{10} - 1 & , x \leq 1 \\ x^2 & , x > 1. \end{cases}$$

Sol. $$f(x) = \begin{cases} x^{10} - 1 & , x \leq 1 \\ x^2 & , x > 1 \end{cases}$$

at $x = 1$,

$$\Rightarrow f(1) = (1)^{10} - 1$$
$$= 0$$

$$\text{L.H.L} = \lim_{x \to 1^-} f(x)$$

$$= \lim_{x \to 0} f(1 - x)$$

$$= \lim_{x \to 0} \{(1-x)^{10} - 1\}$$

$$\cancel{= \lim_{x \to 0} \{(1)^{10} - 1\}} = \lim_{x \to 0} \{(1 + (-x))^{10} - 1\}$$

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let $a = 1$ and $b = (-x)$

$$= \lim_{x\to 0}\left\{\left(a^{10} + {}^{10}C_1 a^9 b + {}^{10}C_2 a^8 b^2 + {}^{10}C_3 a^7 b^3 + {}^{10}C_4 a^6 b^4 + {}^{10}C_5 a^5 b^5 + {}^{10}C_6 a^4 b^6 + {}^{10}C_7 a^3 b^7 + {}^{10}C_8 a^2 b^8 + {}^{10}C_9 a b^9 + {}^{10}C_{10} b^{10}\right) - 1\right\}$$

$$= \lim_{x\to 0}\left\{\left(a^{10} + \frac{10!}{9!\,1!} a^9 b + \frac{10!}{8!\,2!} a^8 b^2 + \frac{10!}{7!\,3!} a^7 b^3 + \frac{10!}{6!\,4!} a^6 b^4 + \frac{10!}{5!\,5!} a^5 b^5 + \frac{10!}{4!\,6!} a^4 b^6 + \frac{10!}{3!\,7!} a^3 b^7 + \frac{10!}{2!\,8!} a^2 b^8 + \frac{10!}{1!\,9!} a b^9 + \frac{10!}{0!\,10!} b^{10} a^0\right) - 1\right\}$$

$$= \lim_{x\to 0}\left\{\left(a^{10} + 10 a^9 b + 45 a^8 b^2 + 120 a^7 b^3 + 210 a^6 b^4 + 252 a^5 b^5 + 210 a^4 b^6 + 120 a^3 b^7 + 45 a^2 b^8 + 10 a b^9 + b^{10}\right) - 1\right\}$$

$$= \lim_{x\to 0}\left\{\left(1^{10} + 10(1)^9(-x) + 45(1)^8(-x)^2 + 120(1)^7(-x)^3 + 210(1)^6(-x)^4 + 252(1)^5(-x)^5 + 210(1)^4(-x)^6 + 120(1)^3(-x)^7 + 45(1)^2(-x)^8 + 10(1)(-x)^9 + (-x)^{10}\right) - 1\right\}$$

$$= \left\{\left(1 + 10(1)(-0) + 45(-0) + 120(-0) + 210(1)(-0)^4 + 252(-0) + 210(-0) + 120(-0) + 45(-0) + 10(-0) + (0)^{10}\right) - 1\right\}$$

$$= \left\{(1) - 1\right\}$$

$$= 0$$

R.H.L. $= \lim_{x\to 1^+} f(x) = \lim_{x\to 0} f(1+x)$

$$= \lim_{x\to 0}(1+x)^2 = \lim_{x\to 0}(1 + x^2 + 2x) = (1 + 0^2 + 2(0))$$

$= 1$

$\Rightarrow f(1) = \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore$ f(x) is discontinous at $x = 1$.

(13) Is the function defined by

$$f(x) = \begin{cases} x+5 , & x \leq 1 \\ x-5 , & x > 1 \end{cases}$$

a continous function?

Sol

$$f(x) = \begin{cases} x+5 , & x \leq 1. \\ x-5 , & x > 1 \end{cases}$$

at $x = 1$,

$f(1) = 1 + 5 = 6$.

$L.H.L = \lim_{x \to 1^-} f(x)$

$= \lim_{x \to 0} f(1 - x)$

$= \lim_{x \to 0} (1 - x) - 5$

$= (1 - 0) - 5$

$= 1 - 5$

$= -4$

$R.H.L = \lim_{x \to 1^+} f(x) = \lim_{x \to 0} f(1 + x)$

$= \lim_{x \to 0} (1 + x) - 5$.

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$= 1 - 5$
$= -4.$

$\Rightarrow f(1) \neq \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\therefore f(x)$ is discontious. It is not continuous.

Q. → Discuss the continuity of the function f, where f is defined by

$$f(x) = \begin{cases} 3, & 0 \le x \le 1 \\ 4, & 1 < x < 3 \\ 5, & 3 \le x \le 10 \end{cases}$$

Sol

$$f(x) = \begin{cases} 3, & 0 \le x \le 1 \\ 4, & 1 < x < 3 \\ 5, & 3 \le x \le 10 \end{cases}$$

at $x = 1$,

$f(1) = 3$

$$\text{L.H.L.} = \lim_{x \to 1^-} f(x) = \lim_{x \to 0} f(1-x)$$

$$= \lim_{x \to 0} 3$$

$$= 3.$$

$$\text{R.H.L.} = \lim_{x \to 1^+} f(x) = \lim_{x \to 0} f(1+x)$$

$$= 4$$

~~L.H.L~~ $f(1) = \lim_{x \to 1^-} f(x) \neq \lim_{x \to 1^+} f(x)$

$\therefore f(x)$ is disontinous at ~~for~~ $x = 1$

Now,

at $x=3$,

$\Rightarrow f(3) = 5$

$\Rightarrow$ L.H.L $= \lim_{x \to 3^-} f(x) = \lim_{x \to 0} f(3-x)$

$= \lim_{x \to 0} 4$

$= 4$

R.H.L $= \lim_{x \to 3^+} f(x) = \lim_{x \to 0} f(3+x)$

$= \lim_{x \to 0} 5$

$= 5$

$f(3) = \lim_{x \to 3^+} f(x) \neq \lim_{x \to 3^-} f(x)$

$\therefore f(x)$ is discontinous at $x = 3$.

15 $f(x) = \begin{cases} 2x, & x<0 \\ 0, & 0 \le x \le 1 \\ 4x, & x>1 \end{cases}$

Sol.

$f(x) = \begin{cases} 2x, & x<0 \\ 0, & 0 \le x \le 1 \\ 4x, & x>1 \end{cases}$

at $x = 0$

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$f(0) = 0$

L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(0-x)$

$= \lim_{x \to 0} f(-x)$

$= \lim_{x \to 0} 2x$

$= 2(0)$

$= 0.$

R.H.L $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(0+x) = \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} 0.$

$= 0.$

$\Rightarrow f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$\therefore$ $f(x)$ is continuous at $x = 0$.

Now,

at $x = 1$,

$\Rightarrow f(1) = 0$

L.H.L $= \lim_{x \to 1^-} f(x) = \lim_{x \to 0} f(1-x)$

$= \lim_{x \to 0} 0$

$= 0.$

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R.H.L $= \lim_{x \to 1^+} f(x)$

$= \lim_{x \to 0} f(1+x)$

$= \lim_{x \to 0} 4x.$

$= \cancel{\lim} \; 4(0)$

$= 0.$

$\Rightarrow f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

$\therefore$ f(x) is continuous at $x = 1$.

(16) $f(x) = \begin{cases} -2, & x \le -1 \\ 2x, & -1 < x \le 1 \\ 2, & x > 1 \end{cases}$

Sol

$f(x) = \begin{cases} -2, & x \le -1 \\ 2x, & -1 < x \le 1 \\ 2, & x > 1 \end{cases}$

at $x = -1$,

$\Rightarrow f(-1) = -2$

$\Rightarrow$ L.H.L $= \lim_{x \to -1^-} f(x) = \lim_{x \to 0} f(-1-x)$

$= \lim_{x \to 0} -2$

$= -2$

R.H.L $= \lim_{x \to -1^+} f(x) = \lim_{x \to 0} f(-1+x)$

$= \lim_{x \to 0} 2(-1+x)$

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$$= \lim_{x \to 0} -2+2x$$

$$= -2+2(0)$$

$$= -2$$

$\Rightarrow f(-1) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\therefore$ $f(x)$ is continous at $x = -1$

Now,

at $x = 1$,

$\Rightarrow f(1) = 2(1) = 2$.

L.H.L $= \lim_{x \to 1^-} f(x) = \lim_{x \to 0} f(1-x)$

$$= \lim_{x \to 0} 2(1-x)$$

$$= \lim_{x \to 0} 2-2x$$

$$= 2-2(0)$$

$$= 2.$$

R.H.L. $= \lim_{x \to 1^+} f(x) = \lim_{x \to 0} f(1+x)$

$$= \lim_{x \to 0} 2$$

$$= 2.$$

$\Rightarrow f(1) = \lim_{x \to 1^+} f(x) = \lim_{x \to 1^-} f(x)$

$\therefore$ $f(x)$ is continous at $x = 1$.

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(17) Find the relationship b/w a and b so that the function f defined by

$$f(x) = \begin{cases} ax+1 , & x \le 3 \\ bx+3 , & x > 3. \end{cases}$$

Sol.

$$f(x) = \begin{cases} ax+1 , & x \le 3 \\ bx+3 , & x > 3. \end{cases}$$

$\Rightarrow \cancel{f(x)}\ f(3) = 3a+1$

$$\text{L.H.L} = \lim_{x \to 3^-} f(x)$$

$$= \lim_{x \to 0} f(3-x)$$

$$= \lim_{x \to 0} \{a(3-x)+1\}$$

$$= \lim_{x \to 0} \{\cancel{3a}\ 3a - ax + 1\}$$

$$= 3a - a(0) + 1$$

$$= 3a + 1.$$

$$\text{R.H.L} = \lim_{x \to 3^+} f(x) = \lim_{x \to 0} f(3+x)$$

$$= \lim_{x \to 0} \{b(3+x)+3\}$$

$$= \lim_{x \to 0} (3b + bx + 3)$$

$$= 3b + b(0) + 3$$

$$= 3b + 3$$

$\because$ f(x) is continous at $x = 3$ therefore

$$\Rightarrow \lim_{x\to 3^-} f(x) = f(3) = \lim_{x\to 3^+} f(x)$$

$$\Rightarrow 3a + 1 = 3a + 1 = 3b + 3.$$

$$\therefore 3a + 1 = 3b + 3$$

$$\Rightarrow 3a - 3b = 2$$

$$\Rightarrow a - b = \frac{2}{3}$$

$$\Rightarrow a = b + \frac{2}{3}$$

(24) Determine if f defined by

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & , x \neq 0 \\ 0 & , x = 0 \end{cases}$$

is a continous function?

Sol.

$$f(x) = \begin{cases} x^2 \sin\frac{1}{x} & , x \neq 0 \\ 0 & x = 0 \end{cases}$$

$$\Rightarrow f(0) = 0.$$

$$\text{L.H.L} = \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(0-x)$$

$$= \lim_{x\to 0} f(-x)$$

$$= \lim_{x\to 0} \left\{ (-x)^2 \sin\frac{1}{-x} \right\}$$

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$$= \lim_{x \to 0} x^2 \sin \frac{1}{-x}.$$

$$= -(0)^2 \times \text{A finite value}$$

$$= 0.$$

R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(0+x)$

$$= \lim_{x \to 0} f(x)$$

$$= \lim_{x \to 0} x^2 \sin \frac{1}{x}$$

$$= (0)^2 \times \text{A finite value}$$

$$= 0$$

$\Rightarrow f(0) = \lim_{x \to 0^+} f(x) = \lim_{x \to 0^-} f(x)$

$\therefore$ f(x) is continous at $x = 0$.

Q.26. find the value of k.

$$f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2} \end{cases}$$

at $x = \frac{\pi}{2}$.

Sol. $f(x) = \begin{cases} \dfrac{k\cos x}{\pi - 2x}, & x \neq \dfrac{\pi}{2} \\ 3, & x = \dfrac{\pi}{2} \end{cases}$

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$\Rightarrow f\left(\frac{\pi}{2}\right) = 3.$

L.H.L. $= \lim_{h \to \frac{\pi}{2}^-} f(h)$

$= \lim_{h \to 0} f\left(\frac{\pi}{2} - h\right)$

$= \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} - h\right)}{\pi - 2\left(\frac{\pi}{2} - h\right)}$

$= \lim_{h \to 0} \frac{k\sin h}{2h.}$

$= \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h}$

$= \frac{k}{2} \times 1$

$= \frac{k}{2.}$

R.H.L $= \lim_{h \to \frac{\pi}{2}^+} f(h)$

$= \lim_{h \to 0} f\left(\frac{\pi}{2} + h\right)$

$= \lim_{h \to 0} \frac{k\cos\left(\frac{\pi}{2} + h\right)}{\pi - 2\left(\frac{\pi}{2} + h\right)}$

$= \lim_{h \to 0} \frac{k(-\sin h)}{\pi - \pi - 2h.}$

$= \lim_{h \to 0} \frac{+k\sin h}{+2h}$

$= \frac{k}{2} \lim_{h \to 0} \frac{\sin h}{h}$

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$$= \frac{K}{2} \times 1 \quad = \frac{K}{2}$$

$\because$ Given function is continous at $x = \frac{\pi}{2}$.

$$\therefore \quad f(x) = f\left(\frac{\pi}{2}\right) = \lim_{x \to \frac{\pi}{2}^+} f(x)$$

$$\Rightarrow \quad \frac{K}{2} = 3 = \frac{K}{2}$$

$$= \quad K = 6$$

Hence value of $K = 6$.

(18) for what value of λ is the function defined by

$$f(x) = \begin{cases} \lambda(x^2 - 2x) & , \quad x \leq 0 \\ 4x + 1 & , \quad x > 0. \end{cases}$$

continous at $x = 0$? what about continuity at $x = 1$?

Sol.

$$f(x) = \begin{cases} \lambda(x^2 - 2x) & , \quad x \leq 0 \\ 4x + 1 & , \quad x > 0 \end{cases}$$

at $x = 0$,

$$\rightarrow f(0) = \lambda(0^2 - 2(0))$$
$$\Rightarrow f(0) = \lambda(0 - 0)$$
$$\rightarrow f(0) = 0.$$

$$\text{L.H.L.} = \lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(-x)$$

$$\cancel{= \lim_{x \to 0} \lambda(x^2 - 2(x))}$$

$$= \lim_{x \to 0} \lambda\{(-x)^2 - 2(-x)\}$$

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$= \lim_{x \to 0} \lambda(x^2 + 2x)$

$= \lambda(0^2 + 2(0))$

$= 0$

<u>R.H.L.</u> $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(0+x)$

$= \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} (4x+1)$

$= \cancel{\lim} \; 4(0)+1$

$= 1$

$\Rightarrow f(0) = \lim_{x \to 0^-} f(x) \neq \lim_{x \to 0^+} f(x)$

Hence $f(x)$ is discontinous at $x=0$, no value of λ is present at $x=0$.

<u>Now, at $x = 1$</u>,

$\Rightarrow f(1) = 4(1)+1 = 4+1 = 5$

L.H.L. $= \lim_{x \to 1^-} f(x) = \lim_{x \to 0} f(1-x)$

$= \lim_{x \to 0} \{4(1-x)+1\}$

$= 4(1-0)+1$

$= 4+1$

$= 5$

<u>R.H.L.</u> $= \lim_{x \to 1^+} f(x) = \lim_{x \to 0} f(1+x) = \lim_{x \to 0} 4(1+x)+1$

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$= 4(1+0)+1$
$= 5$

$\Rightarrow f(1) = \lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

Here, f(x) is continous at $x = 1$.
Therefore for any value of λ

(19) Show that the function defined by $g(x) = x - [x]$ is discontinous at all integral points. Here $[x]$ denotes the greatest integer less than or equal to x.

Sol. Given,

$\Rightarrow g(x) = x - [x]$, $[x] \to$ G.I.F $\le x$

let c is a positive integer, $c > 0$, $c \in Z^+$

at $x = c$,

$\Rightarrow g(c) = c - [c]$
$= c - c$
$= 0$.

L.H.L. $= \lim_{x \to c^-} g(x)$

$= \cancel{\lim_{x \to 0} g(c-x)}$

$= \lim_{x \to c^-} \{x - [x]\}$

$= \lim_{x \to c^-} (x) - \lim_{x \to c^-} [x]$

$\left\{ \because \lim_{x \to y} (a-b) = \lim_{x \to y} a - \lim_{x \to y} b \right\}$

$= c - (c-1)$
$= 1$

Rough

$g(c-x)$

$(c-x) - [c-x]$

$c - x -$

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$$\underline{L.H.L.} = \lim_{x \to c^-} g(x)$$

$$= \lim_{x \to 0} g(c-x)$$

$$= \lim_{x \to 0} (c-x) - [c-x]$$

$$= \lim_{x \to 0} (c-x) - (c-1)$$

$$= \cancel{c} - 0 - \cancel{c} + 1$$

$$= 1$$

$$\underline{R.H.L.} = \lim_{x \to c^+} g(x)$$

$$= \lim_{x \to 0} g(c+x)$$

$$= \lim_{x \to 0} (c+x) - [c+x]$$

$$= \lim_{x \to 0} c + x - c$$

$$= c + 0 - c$$

$$= 0$$

for $c > 0$, $c \in z^+$,

$$g(c) = \lim_{x \to c^+} g(x) \neq \lim_{x \to c^-} g(x)$$

Here, $g(x)$ is discontinous at c or $x > 0$.

20) Is the function defined by $f(x) = x^2 - \sin x + 5$ continous at $x = \pi$

Sol. $f(x) = x^2 + \sin x + 5$

at $x = \pi$,

$$f(\pi) = \pi^2 + \sin \pi + 5$$

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$= \pi^2 + 0 + 5$

$= \pi^2 + 5.$

<u>L.H.L.</u> $= \lim_{x \to \pi^-} f(x)$

$= \lim_{x \to 0} f(\pi - x)$

$= \lim_{x \to 0} (\pi - x)^2 + \sin(\pi - x) + 5$

$= (\pi - 0)^2 + \sin(\pi - 0) + 5$

$= \pi^2 + \sin\pi + 5$

$= \pi^2 + 0 + 5$

$= \pi^2 + 5.$

<u>R.H.L.</u> $= \lim_{x \to \pi^+} f(x) = \lim_{x \to \pi} f(\pi + x)$

$= \lim_{x \to 0} (\pi + x)^2 + \sin(\pi + x) + 5$

$= (\pi + 0)^2 + \sin(\pi + 0) + 5$

$= \pi^2 + \sin\pi + 5$

$= \pi^2 + 0 + 5$

$= \pi^2 + 5.$

$\Rightarrow f(\pi) = \lim_{x \to \pi^-} f(x) = \lim_{x \to \pi^+} f(x)$

Here, $f(x)$ is continous at $x = \pi$.

~~21~~

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Theorem 1:- suppose f and g be two real functions continous at a real no. c, Then

(i) $f+g$ is continous at $x=c$
(ii) $f-g$ is continous at $x=c$
(iii) $f \cdot g$ is continous at $x=c$
(iv) f/g is continous at $x=c$.

Q1) discuss the continuity of the following :-

(a) $f(x) = \sin x + \cos x$ (b) $f(x) = \sin x - \cos x$
(c) $f(x) = \sin x \cdot \cos x$.

Sol. ~~for~~ $\sin x$,

let $g(x) = \sin x$.
let $c \in R$.
at $x = c$,

$\Rightarrow g(c) = \sin c$.

L.H.L. $= \lim_{x \to c^-} g(x)$

$= \lim_{x \to 0} g(c-x)$

$= \lim_{x \to 0} \sin(c-x)$

$= \cancel{\lim} \sin(c-0)$

$= \sin c$.

R.H.L. $= \lim_{x \to c^+} g(x) = \lim_{x \to 0} g(c+x)$

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$= \lim_{x \to 0} \sin(c+x)$

$= \sin(c+0)$

$= \sin c$

$\Rightarrow g(c) = \lim_{x \to c^-} g(c) = \lim_{x \to c^+} g(c)$

Here $g(x)$ is continuous at $x = c$, $c \in R$.

Now,
let $K(x) = \cos x$.

at $x = c$,

$\Rightarrow K(c) = \cos c$

L.H.L $= \lim_{x \to c^-} K(x)$

$= \lim_{x \to 0} K(c-x)$

$= \lim_{x \to 0} \cos(c-x)$

$= \cos(c-0)$

$= \cos c$

R.H.L. $= \lim_{x \to c^+} K(x)$

$= \lim_{x \to 0} K(c+x)$

$= \lim_{x \to 0} \cos(c+x)$

$= \cos(c+0)$

$= \cos c$

$\therefore k(c) = \lim_{x \to c^-} k(x) = \lim_{x \to c^+} k(x)$

Here, $k(c)$ is continous at $x = c$, $c \in R$

Now, Here $\sin x$ and $\cos x$ both are continous then by theorem.

(i) $\sin x + \cos x$ (v) $f \circ g$ is also continous.
(ii) $\sin x - \cos x$
(iii) $\sin x \cdot \cos x$
(iv) $\dfrac{\sin x}{\cos x}$

are also continous.

Q. Discuss the continuity of the cosine, cosecant, secant and cotangent functions.

Sol. since $f(x) = \sin x$ is continous function
and $g(x) = \cos x$ is continous function.

(i) for cosine, $\rightarrow$ $g(x) = \cos x$ is continous.

(iv) for cotangent $\rightarrow$
By theorem, if $f(x)$ & $g(x)$ is continous then

$\dfrac{f(x)}{g(x)}$ is also continous, $g(x) \neq 0$.

$$\frac{f(x)}{g(x)} = \frac{\sin x}{\cos x}.$$

$$\frac{f(x)}{g(x)} = \tan x.$$

$\sec \frac{3\pi}{2} \Rightarrow$ $\frac{3\pi}{2} \to 270^\circ \to 180^\circ + 90^\circ \to 90^\circ + 90^\circ + 90^\circ$

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$\Rightarrow \cot x = \frac{g(x)}{f(x)}, \quad f(x) \neq 0.$

$\Rightarrow \cot x = \frac{\cos x}{\sin x}, \longrightarrow$ where $x \neq n\pi$

~~$\frac{g(x)}{f(x)}$ is also continuous. therefore cot x is also continuous except $x \neq n\pi$~~ $x = n\pi$

Here, $\cot x$ is continous except $x = n\pi$

<u>for secant,</u>

$\Rightarrow \cos x = \frac{1}{\sec x}$

$\Rightarrow \sec x = \frac{1}{\cos x}$ where $x \neq 0$

$\Rightarrow \sec x, \quad x \neq (2k+1)\frac{\pi}{2}, \quad k \in \mathbb{Z}$ is continous.

<u>for cosec,</u>

$\Rightarrow \operatorname{cosec} x = \frac{1}{\sin x} \quad ; \quad x \neq 0.$

$\Rightarrow \operatorname{cosec} x, \quad x \neq n\pi$ is continous.

(23) find all points of discontinuity of f, where,

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0 \\ x+1, & x \geq 0 \end{cases}$$

<u>Sol.</u>

$$f(x) = \begin{cases} \frac{\sin x}{x}, & x < 0. \\ x+1, & x \geq 0 \end{cases}$$

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at $x=0$,

$f(0) = 0+1$
$= 1.$

L.H.L $= \lim_{x\to 0^-} f(x) = \lim_{x\to 0} f(-x)$

$= \lim_{x\to 0} \frac{\sin(-x)}{-x}$

$= \lim_{x\to 0} \frac{-\sin x}{-x}$

$= \lim_{x\to 0} \frac{\sin x}{x}$

$= 1$ $\left\{ \because \lim_{\theta\to 0} \frac{\sin\theta}{\theta} = 1 \right\}$

R.H.L. $= \lim_{x\to 0^+} f(x) = \lim_{x\to 0} f(0+x)$

$= \lim_{x\to 0} f(x)$

$= \lim_{x\to 0} (x+1)$

$= 0+1$

$= 1.$

$\Rightarrow f(0) = \cancel{f(+} \lim_{x\to 0^-} f(x) = \lim_{x\to 0^+} f(x)$

Here, $f(x)$ is continous at $x=0$.
Hence there is no discontinous point.

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(25) Examine the continuity of f, where f defined by

$$f(x) = \begin{cases} \sin x - \cos x, & x \neq 0 \\ -1, & x = 0 \end{cases}$$

Sol.

$$f(x) = \begin{cases} \sin x - \cos x, & x \neq 0 \\ -1, & x = 0 \end{cases}$$

at $x = 0$,

$\Rightarrow f(0) = -1.$

L.H.L. $= \lim_{x \to 0^-} f(x) = \lim_{x \to 0} f(-x)$

$= \lim_{x \to 0} \{\sin(-x) - \cos(-x)\}$

$= \lim_{x \to 0} \{-\sin x - \cos x\}$

$= -\sin 0^\circ - \cos 0^\circ$

$= -0 - 1$

$= -1.$

R.H.L. $= \lim_{x \to 0^+} f(x) = \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} \{\sin x - \cos x\}$

$= \sin 0^\circ - \cos 0^\circ$

$= 0 - 1$

$= -1.$

$\Rightarrow f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

Here, $f(x)$ is continous at $x = 0$.

check continuity from Q.26 to Q.29.

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$$f(x) = \begin{cases} kx^2 & , \quad x \leq 2 \\ 3 & , \quad x > 2 \end{cases} \quad \text{at } x = 2.$$

Sol. at $x = 2$,

$$f(2) = k(2)^2$$
$$= 4k.$$

$$\text{L.H.L.} = \lim_{x \to 2^-} f(x) = \lim_{x \to 0} f(2-x)$$
$$= \lim_{x \to 0} k(2-x)^2$$
$$= k(2-0)^2$$
$$= 4k.$$

$$\text{R.H.L.} = \lim_{x \to 2^+} f(x) = \lim_{x \to 0} f(2+x)$$
$$= \lim_{x \to 0} 3$$
$$= 3.$$

$$\cancel{\Rightarrow f(2) = \lim_{x \to 2^-} f(x) \neq \lim_{x \to 2^+} f(x)}$$

Given,
function $f(x)$ is contirous at $x = 2$.
therefore

$$\Rightarrow f(2) = \lim_{x \to 2^-} f(x) = \lim_{x \to 2^+} f(x)$$

$$\Rightarrow 4k = 4k = 3.$$

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$\therefore 4k = 3$

$\Rightarrow k = \frac{3}{4}$. Ans.

(28) $f(x) = \begin{cases} kx+1, & x \leq \pi \\ \cos x, & x > \pi \end{cases}$ at $x = \pi$.

At $x = \pi$,

$f(\pi) = k(\pi) + 1$

$= k\pi + 1$

L.H.L. $= \lim_{x \to \pi^-} f(x)$

$= \lim_{x \to 0} f(\pi - x)$

$= \lim_{x \to 0} k(\pi - x) + 1$

$= k(\pi - 0) + 1$

$= k\pi + 1$

R.H.L. $= \lim_{x \to \pi^+} f(x)$

$= \lim_{x \to 0} f(\pi + x)$

$= \lim_{x \to 0} \cos(\pi + x)$

$= \cos(\pi + 0)$

$= \cos \pi$

$= -1$

Given, function is continous at $x = \pi$.
therefore,

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$\Rightarrow f(\pi) = \lim_{x\to\pi^-} f(x) = \lim_{x\to\pi^+} f(x)$

$\Rightarrow k\pi + 1 = k\pi + 1 = -1$

$\therefore k\pi + 1 = -1$

$k\pi = -2$

$\Rightarrow k = \frac{-2}{\pi}$

$f(x) = \begin{cases} kx+1, & x \le 5 \\ 3x-5, & x > 5 \end{cases}$ at $x = 5^-$.

at $x = 5^-$,

$f(5^-) = k(5)+1$
$= 5k+1$

L.H.L. $= \lim_{x\to 5^-} f(x)$

$= \lim_{x\to 0} f(5-x)$

$= \lim_{x\to 0} k(5-x)+1$

$= k(5-0)+1$

$= 5k+1$

R.H.L. $= \lim_{x\to 5^+} f(x)$

$= \lim_{x\to 0} f(5+x)$

$= \lim_{x\to 0} \{3(5+x) - 5\}$

$= 15-5 = 10.$

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Given, function is continous at $x=5$.

$\Rightarrow f(5) = \lim_{x \to 5^-} f(x) = \lim_{x \to 5^+} f(x)$

$\Rightarrow \cancel{k x + 1}\ 5k+1 = 5k+1 = 10$

$\Rightarrow \therefore 5k+1 = 10$

$\Rightarrow k = \frac{9}{5}$ Ans

(30) find value of a and b,

$$f(x) = \begin{cases} 5 & , x \le 2 \\ ax+b & , 2 < x < 10 \\ 21 & , x \ge 10 \end{cases}$$ is a continouss function.

Sol. at $x=2$,

$f(2) = 5$

L.H.L. $= \lim_{x \to 0} f(2-x)$

$= \lim_{x \to 0} \cancel{\{a(2-x)+b\}} \{5\}$

$\cancel{= a(5-0)+b} = 5$

$\cancel{= 5a+b}$

R.H.L. $= \lim_{x \to 2^+} f(x)$

$= \lim_{x \to 0} f(2+x)$

$= \lim_{x \to 0} a(2+x)+5$

$= 2a+5$

$\because f(x)$ is continouss.

$\Rightarrow f(2) = \lim_{x \to 2^+} f(x) = \lim_{x \to 2^-} f(x)$.

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$5 = 2a + b = 5$

$\therefore \quad 2a + b = 5 \longrightarrow (1)$

$\boxed{\cancel{a = \frac{1}{2}}}$

Now, at $x = 10$,

$f(10) = 21$

L.H.L. $= \lim_{x \to 0} f(10 - x) = \lim_{x \to 0} \{a(10 - x) + b\}$

$= a(10 - 0) + b = 10a + b$

R.H.L. $= \lim_{x \to 0} f(10 + x) = \lim_{x \to 0} (21)$

$= 21$

$\because f(x)$ is continous at $x = 10$,

then, $f(10) = \lim_{x \to 10^+} f(x) = \lim_{x \to 10^-} f(x)$

$\Rightarrow \quad 21 = 10a + b = 21$

$\therefore \quad 10a + b = 21 \longrightarrow (2)$

from (1), $b = 5 - 2a$

put $b = 5 - 2a$ in (2),

$\Rightarrow 10a + (5 - 2a) = 21$

$\Rightarrow 8a + 5 = 21$

$\Rightarrow a = 2$

put $a = 2$ in eq^n (1),

$\Rightarrow 2(2) + b = 5$

$\Rightarrow b = 1$

therefor,

$a = 2$ and $b = 1$. Ans.

12 → 70
4 → 90

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(31) show that the function defined by $f(x) = \cos(x^2)$ is a continous function.

Sol.

$f(x) = \cos(x^2)$

let, $g(x) = x^2$ and $h(x) = \cos x$.

$$hog(x) = h\{g(x)\}$$
$$= h(x^2)$$
$$= \cos(x^2)$$
$$= f(x)$$

let c is an real point

$\Rightarrow$ $g(c) = c^2$

$\Rightarrow$ $\lim_{x \to c} g(x) = \lim_{x \to c} x^2 = c^2$

$\Rightarrow$ $\lim_{x \to c} g(x) = g(c)$

$\therefore$ $g(x)$ is continous at $x = c$.

Now,

$h(c) = \cos c$.

$\Rightarrow$ $\lim_{x \to c} h(x) = \lim_{x \to c} \cos x$
$= \cos c$.

$\Rightarrow$ $\lim_{x \to c} h(x) = h(c)$

$\therefore$ $h(x)$ is continous at $x = c$.

since $g(x)$ is continous at $x = c$ and $h(x)$ is continous at $x = c$ therefor $hog(x)$ ~~or~~ $f(x)$ is continous at $x = c$.

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find all the points of discontinuity of f defined by $f(x) = |x| - |x+1|$

$f(x) = |x| - |x+1|$

If $x < -1$

$f(x) = -x - \{-(x+1)\}$
$f(x) = -x + x + 1$
$f(x) = 1$

If $-1 \leq x < 0$,

$\Rightarrow f(x) = -x - (x+1)$
$= -2x - 1$

If $x \geq 0$,

$f(x) = x - (x+1)$
$= -1$

$$f(x) = \begin{cases} 1, & \text{if } x < -1 \\ -2x-1, & \text{if } -1 \leq x < 0 \\ -1, & \text{if } x \geq 0 \end{cases}$$

at $x = -1$,

$\Rightarrow f(-1) = -2(-1) - 1$
$= 1.$

L.H.L. $= \lim_{x \to -1^-} f(x) = \lim_{x \to 0} f(-1-x)$

$$= \lim_{x \to 0} (1)$$

$$= 1.$$

R.H.L. $= \lim_{x \to -1^+} f(x)$

$$= \lim_{x \to 0} f(-1+x)$$

$$= \lim_{x \to 0} \{-2(-1+x) - 1\}$$

$$= \lim_{x \to 0} \{2 - 2x - 1\}$$

$$= \lim_{x \to 0} (1 - 2x\}$$

$$= 1 - 2(0)$$

$$= 1.$$

at $x = -1$, $f(x)$ is continous, because

$\Rightarrow f(-1) = \lim_{x \to -1^+} f(x) = \lim_{x \to -1^-} f(x)$,

Now, at $x = 0$.

$f(0) = -1$

L.H.L. $= \lim_{x \to 0^-} f(x)$

$$= \lim_{x \to 0} f(-x)$$

$$= \lim_{x \to 0} \{-2(-x) - 1\} = \lim_{x \to 0} (2x - 1)$$

$$= -1$$

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R.H.L. $= \lim_{x \to 0^+} f(x)$

$= \lim_{x \to 0} f(x)$

$= \lim_{x \to 0} -1$

$= -1$

$f(0) = \lim_{x \to 0^-} f(x) = \lim_{x \to 0^+} f(x)$

Here, $f(x)$ is continous

Therefore $f(x)$ does not have any discontinous point.

(52) show that the function defined by $f(x) = |\cos x|$ is continous function.

S.l. $f(x) = |\cos x|$

let,

~~if~~ $g(x) = |x|$ and $h(x) = \cos x$.

$hog(x) = h\{g(x)\}$

$= h\{|x|\}$

$= \cos|x|$

$= f(x)$

let c is an real ~~no~~ point.

$\Rightarrow g(c) = |c|$

$\Rightarrow \lim_{x \to c} g(x) = \lim_{x \to c} |x| = |c|$

Now,

$\lim_{x \to c} g(x) = g(c)$

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$\therefore$ $g(x)$ is continous at $x = c$.

Again,

$\Rightarrow$ $h(c) = \cos|c|$

$\Rightarrow$ $\lim_{x\to c} h(x) = \lim_{x\to 0} \cos|c| = \cos|c|$

$\Rightarrow$ $\lim_{x\to c} h(x) = g(c)$

$\therefore$ $h(x)$ is continous at $x = c$.

Here, $f(x) = hog(x)$

$h(x)$ and $g(x)$ both are continous therefore $f(x)$ is also continous.

(33) Examine that $\sin|x|$ is a continous function.

Sol.

let $f(x) = \sin|x|$
$h(x) = \sin x$
$g(x) = |x|$

Now,

$\Rightarrow$ $hog(x) = h\{g(x)\}$
$= h\{|x|\}$
$= \sin|x|$
$= f(x)$

let c is an real point.

$\Rightarrow$ $g(c) = |c|$

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$\Rightarrow \lim_{x \to c} g(x) = \lim_{x \to c} |x| = |c|$

$\Rightarrow \lim_{x \to c} g(x) = g(c)$

$\therefore$ $g(x)$ is continous at $x=c$.

now,
$h(c) = \sin|c|$

$\Rightarrow \lim_{x \to c} h(x) = \lim_{x \to c} \sin|x| = \sin|c|$

$\Rightarrow \lim_{x \to c} h(x) = h(c)$

$\therefore$ $h(x)$ is continous at $x=c$.

If $h(x)$ and $g(x)$ then $h(x)+g(x)$, $h(x)-g(x)$, $h(x)\cdot g(x)$, $h(x)/g(x)$ and also $hog(x)$ is continous.

$hog(x) = f(x)$,

$f(x)$ is continous.

Practice

$\sin(x^2+5)$

$\Rightarrow \cos(x^2+5)\cdot 2x + 0$

$= 0 + 2x\cos(x^2+5)$

$\sin(x^2+5)$

$= \cos(x^2+5)\cdot 2x + 0$

$= 2x\cdot\cos(x^2+5)$

Ans

$\frac{d}{dx}$ Practice

① $\sin(\cos(x^2))$

$= \frac{d}{dx}\sin\underbrace{(\cos(x^2))}_{t}$

$=$

$\cos t \cdot -\sin m \cdot 2x$

$-2x\cos t\sin m$

$= \sin(\cos(x^2))$

$= \cos(\cos(x^2))\cdot -\sin x^2\cdot 2x$

$= -2x\sin x^2\cos(\cos(x^2))$

Ans.

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Formulas:-

(1) First Principle :-

$$\lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$= \frac{dy}{dx}.$$

Standard Formulas :-

1. $\frac{d}{dx} x^n = nx^{n-1}$
2. $\frac{d}{dx} a^x = a^x \log a.$
3. $\frac{d}{dx} e^x = e^x$
4. $\frac{d}{dx} \log x = \frac{1}{x}.$
5. $\frac{d}{dx} \sin x = \cos x.$
6. $\frac{d}{dx} \cos x = -\sin x.$
7. $\frac{d}{dx} \tan x = \sec^2 x.$
8. $\frac{d}{dx} \cot x = -\operatorname{cosec}^2 x.$
9. $\frac{d}{dx} \sec x = \sec x \cdot \tan x.$
10. $\frac{d}{dx} \operatorname{cosec} x = -\operatorname{cosec} x \cdot \cot x.$

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Rules of Differentiation:-

1. $\frac{d}{dx}\{f(x)+g(x)\} = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$

2. $\frac{d}{dx}\{f(x)-g(x)\} = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$

3. $\frac{d}{dx}\{f(x).g(x)\} = f(x)\frac{d}{dx}g(x) + g(x)\frac{d}{dx}f(x)$

4. $\frac{d}{dx}\left\{\frac{f(x)}{g(x)}\right\} = \frac{g(x)\frac{d}{dx}f(x) - f(x)\frac{d}{dx}g(x)}{\{g(x)\}^2}$

5. $\frac{d}{dx}c = 0$, where $c \rightarrow$ constant

6. $\frac{d}{dx}\{c.f(x)\} = c\frac{d}{dx}f(x)$, where $c \rightarrow$ constant

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- Differentiation :-

(1) $\sin^3 x = f(x)$

Sol. D.w.r. to x

$\Rightarrow \frac{d}{dx}(\sin^3 x) = f'(x)$

$\Rightarrow 3\sin^2 x \cdot \cos x = f'(x)$

$\Rightarrow \boxed{f' = 3\sin^2 x \cdot \cos x}$

(2) $\sin x^3 = f(x)$

Sol. D.w.r. to x.

$\Rightarrow f'(x) = \frac{d}{dx} \sin x^3$

$\Rightarrow f'(x) = \cos x^3 \cdot 3x^2$

$\Rightarrow \boxed{f'(x) = 3x^2 \cos x^3}$

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Differentiability :-

a function $f(x)$ is differentiable at $x = c$, when

$\rightarrow$ $\boxed{Lf'(c) = Rf'(c)}$, where, $Lf'(c) \rightarrow$ left hand derivative (L.H.D)

$Rf'(c) \rightarrow$ Right hand derivative (R.H.D)

Here,

$$Lf'(c) = \lim_{h \to 0} \frac{f(c-h) - f(c)}{-h}$$

$$Rf'(c) = \lim_{h \to 0} \frac{f(c+h) - f(c)}{h}$$

Note :-

(i) If a function $f(x)$ is differentiable at $x = c$ then it is continous also at the same point.

(ii) but if it is continous at $x = c$ then it is not necessary that it is differentiable also.

Q. Show that the function $f(x) = \begin{cases} x-1 & , x < 2 \\ 2x-3 & , x \geq 2 \end{cases}$

if not differentiable at $x = 2$.

Sol.

$$f(x) = \begin{cases} x-1 & , x < 2 \\ 2x-3 & , x \geq 2 \end{cases}$$

at $x = 2$,

$\Rightarrow$ $Lf'(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$

$= \lim_{h \to 0} \frac{(2-h-1) - \{2(2)-3\}}{-h}$

$= \lim_{h \to 0} \frac{(1-h) - 1}{-h}$

$= \lim_{h \to 0} \frac{-\cancel{h}}{-\cancel{h}}$

$= \lim_{h \to 0} 1$

$= 1.$

$\Rightarrow$ $Rf'(2) = \lim_{h \to 0} \frac{f(2+h) - f(2)}{h}$

$= \lim_{h \to 0} \frac{\{2(2+h)-3\} - \{2(2)-3\}}{h}.$

$= \lim_{h \to 0} \frac{\{4+2h-3\} - 1}{h}$

$= \lim_{h \to 0} \frac{2\cancel{h}}{\cancel{h}}$

$= \lim_{h \to 0} 2$

$= 2$

$\Rightarrow$ Here,

$Lf'(2) \neq Rf'(2)$

Hence $f(x)$ is not differentiable at $x = 2$.

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Q. check the differenciability at $x=0$ where

$$f(x) = \begin{cases} 2+x & , x \geq 0 \\ 2-x & , x < 0 \end{cases}$$

$$f(x) = \begin{cases} 2+x & , x \geq 0 \\ 2-x & , x < 0 \end{cases}$$

at $x=0$,

$$\Rightarrow Lf'(0) = \lim_{h \to 0} \frac{f(0-h) - f(0)}{-h}$$

$$= \lim_{h \to 0} \frac{f(-h) - \{2-0\}}{-h}$$

$$= \lim_{h \to 0} \frac{\{2-(-h)\} - 2}{-h}$$

$$= \lim_{h \to 0} \frac{2+h-2}{-h}$$

$$= \lim_{h \to 0} -1$$

$$= -1.$$

$$\Rightarrow Rf'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

$$= \lim_{h \to 0} \frac{f(h) - \{2-0\}}{h}$$

$$= \lim_{h \to 0} \frac{2+h-2}{h}$$

$$= \lim_{h \to 0} \frac{h}{h}$$

$$= 1$$

$$\Rightarrow Lf'(0) \neq Rf'(0)$$

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$\therefore$ $f(x)$ is not differentiable at $x = 0$

Exercise 5.2

(9) Prove that the function f given by
$$f(x) = |x-1|, \quad x \in R$$
is not differentiable at $x = 1$.

Sol. $f(x) = |x-1|$,
at $x = 1$,

$$\Rightarrow Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$$
$$= \lim_{h \to 0} \frac{|1-h-1| - |1-1|}{-h}$$
$$= \lim_{h \to 0} \frac{|-h| - 0}{-h}$$
$$= \lim_{h \to 0} (-1)$$
$$= -1$$

$$Rf'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}$$
$$= \lim_{h \to 0} \frac{|1+h-1| - |1-1|}{h}$$
$$= \lim_{h \to 0} \frac{|h| - 0}{h}$$
$$= \lim_{h \to 0} 1$$
$$= 1$$

$\Rightarrow Lf'(1) \neq Rf'(1)$
$\therefore$ $f(x)$ is not differentiable at $x = 1$.

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Q) prove that the greatest integer function defined by
$f(x) = [x]$, $0 < x < 3$.
is not differentiable at $x=1$ and $x=2$.

$f(x) = [x]$, $0 < x < 3$

at $x=1$,

$Lf'(1) = \lim_{h \to 0} \frac{f(1-h) - f(1)}{-h}$

$= \lim_{h \to 0} \frac{[1-h] - [1]}{-h}$

$= \lim_{h \to 0} \frac{0 - 1}{-h}$

$= \lim_{h \to 0} \frac{-1}{-h} = \lim_{h \to 0} \frac{1}{h}$

$= \frac{1}{0}$

= Not defined.

f(x) at x = 1 is not differenciable.

at $x = 2$,

~~$f(x) = [x]$~~
~~$f(2) = [2]$~~ $= 2$

$Lf'(2) = \lim_{h \to 0} \frac{f(2-h) - f(2)}{-h}$

$= \lim_{h \to 0} \frac{[2-h] - [2]}{-h} = \lim_{h \to 0} \frac{1-2}{-h} = \lim_{h \to 0} \frac{1}{h}$

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$= \frac{1}{0}$

= not defined.

f(x) is not differenciable at $x = 2$.

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Ex.5.2. ~~Practice~~

(I) Differenciate following w.r.t x.

① $\sin(x^2+5)$

Sol. $f(x) = \sin(x^2+5)$

$\Rightarrow \frac{d}{dx}(f(x)) = \frac{d}{dx}\{\sin(x^2+5)\}$

$= \cos(x^2+5) \cdot \frac{d}{dx}(x^2+5)$

$= \cos(x^2+5) \cdot 2x + 0$

$= 2x \cdot \cos(x^2+5)$

② $\cos(\sin x)$

Sol. $\frac{d}{dx} f(x) = \frac{d}{dx}\{\cos(\sin x)\}$

$= -\sin(\sin x) \cdot \frac{d}{dx} \sin x$

$= -\sin(\sin x) \cdot \cos x$

$= -\cos x \sin(\sin x)$

③ $\sin(ax+b)$

Sol. $\frac{d}{dx} f(x) = \frac{d}{dx}\{\sin(ax+b)\}$

$= \cos(ax+b) \cdot \frac{d}{dx}(ax+b)$

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$= \cos(ax+b) \cdot (a+0)$
$= a\cos(ax+b)$

$\sec(\tan(\sqrt{x}))$

Sol. $\frac{d}{dx} f(x) = \frac{d}{dx}\left(\sec\left\{\tan(\sqrt{x})\right\}\right)$

$= \sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \frac{d}{dx}\tan\sqrt{x}$

$= \sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2} x^{\frac{1}{2}-1}$

$= \sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x} \cdot \frac{1}{2} x^{-\frac{1}{2}}$

$= \frac{\sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x}}{2x^{1/2}}$

$= \frac{\sec(\tan\sqrt{x}) \cdot \tan(\tan\sqrt{x}) \cdot \sec^2\sqrt{x}}{2\sqrt{x}}$ Ans.

$\frac{\sin(ax+b)}{\cos(cx+d)}$

Sol. $f(x) = \frac{\sin(ax+b)}{\cos(cx+d)}$

$\Rightarrow \frac{d}{dx}\{f(x)\} = \frac{d}{dx}\left\{\frac{\sin(ax+b)}{\cos(cx+d)}\right\}$

$= \frac{\cos(cx+d)\frac{d}{dx}\left\{\sin(ax+b)\right\} - \left\{\sin(ax+b)\right\}\frac{d}{dx}\left\{\cos(cx+d)\right\}}{\left\{\cos(cx+d)\right\}^2}$

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$$= \frac{\cos(cx+d)\cdot\cos(ax+b)\cdot\frac{d}{dx}(ax+b) - \left[\{\sin(ax+b)\}\left\{-\sin(cx+d)\cdot\frac{d}{dx}(cx+d)\right\}\right]}{\{\cos(cx+d)\}^2}$$

$$= \frac{a\cos(cx+d)\cdot\cos(ax+b) - \left[\sin(ax+b)\{c\sin(cx+d)\}\right]}{\{\cos(cx+d)\}^2}$$

$$= \frac{a\cos(cx+d)\cdot\cos(ax+b) - \left[-\sin(ax+b)\cdot c\sin(cx+d)\right]}{\{\cos(cx+d)\}^2}$$

$$= \frac{a\cos(cx+d)\cdot\cos(ax+b) + c\sin(ax+b)\sin(cx+d)}{\{\cos(cx+d)\}^2}$$

$$= \frac{\cancel{a\cos(cx+d)}\cdot\cos(ax+b)}{\cancel{\cos(cx+d)}\cdot\cos(cx+d)} + \frac{c\sin(ax+b)\cdot}{\cos(cx+d)}\cdot\frac{\sin(cx+d)}{\cos(cx+d)}$$

$$= a\cos(ax+b)\cdot\frac{1}{\cos(cx+d)} + c\sin(ax+b)\cdot\tan(cx+d)\cdot\frac{1}{\cos(cx+d)}$$

$$= a\cos(ax+b)\cdot\sec(cx+d) + c\sin(ax+b)\cdot\tan(cx+d)\cdot\sec(cx+d)$$

(8) $\cos\sqrt{x}$

Sol. let $y = \cos\sqrt{x}$

$$\frac{dy}{dx} = \frac{d}{dx}\cos\sqrt{x}$$

$$= -\sin\sqrt{x}\cdot\frac{1}{2\sqrt{x}}$$

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~~Exercise 5.2~~

Differentiation of function of function :-

There are 2 methods :-

(i) substitution method.
(ii) chain rule

$y = \sin x^3$
D.w.r to 'x'

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin x^3$

putting $x^3 = t$.

$= \frac{d}{dt} \sin t \times \frac{dt}{dx}$

$= \cos t \cdot \frac{d}{dx} x^3$

$= \cos x^3 \cdot 3x^2$

$= 3x^2 \cdot \cos x^3$.

Q. $y = \sin^3 x$
D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin^3 x$.

putting $\sin x = t$.

$= \frac{d}{dx} t^3 \cdot \frac{dt}{dx}$.

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$= 3t^2 \cdot \frac{d}{dx} \sin x$

$= 3(\sin x)^2 \cdot \cos x.$

$= 3 \sin^2 x \cdot \cos x.$ Ans.

Q.8 -

$\cos \sqrt{x} = y$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \cos\sqrt{x},$

put $x^{1/2} = t$

$= \frac{d}{dt} \cos t \cdot \frac{dt}{dx}$

$= -\sin t \cdot \frac{d}{dx} \sqrt{x}.$

$= -\sin\sqrt{x} \cdot \frac{1}{2} x^{1/2 - 1}$

$= \frac{-\sin\sqrt{x}}{2 x^{1/2}}$

$= \frac{-\sin\sqrt{x}}{2\sqrt{x}}$ Ans.

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Assignment

Q. D.W.r. to x.

1. $\cos^2 x$
2. $\log(\sin x + 1)$
3. $\sqrt{\cot x}$.
4. $\log \tan \frac{x}{2}$
5. $e^{\tan x}$.
6. $\sin^2(3x+5)$
7. $\sin^2 x^2$
8. $\sin x^\circ$
9. $\log\left(x+\sqrt{x^2+a^2}\right)$
10. $\dfrac{1}{\log(\cos x)}$

Sol. 1 $\cos^2 x = y$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\cos^2 x$

put $\cos x = t$.

$\therefore = \dfrac{d}{dt}t^2 \cdot \dfrac{dt}{dx}$.

$= 2t \cdot \dfrac{d}{dx}\cos x$.

$= 2\cos x \cdot (-\sin x)$

$= -2\sin x \cos x$.

Sol. 2 $y = \log(\sin x + 1)$

$\dfrac{dy}{dx} = \dfrac{d}{dx}\left\{\log(\sin x + 1)\right\}$

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put $1+\sin x = t$.

$$= \frac{d}{dt} \log(t+1)$$

$$= \frac{1}{t \cancel{+1}} \cdot \frac{dt}{dx}.$$

$$= \frac{1}{\sin x + 1} \cdot \frac{d}{dx}(\sin x + 1)$$

$$= \frac{1}{\sin x + 1} \cdot (\cos x)$$

$$= \frac{\cos x}{\sin x + 1}.$$

(3) $\sqrt{\cot x}$.

$\Rightarrow$

$$\frac{dy}{dx} = \frac{d}{dx} \cot^{1/2} x.$$

~~put~~ $\cot x = t$.

$$= \frac{d}{dt} t^{1/2} \cdot \frac{dt}{dx}.$$

$$= \frac{1}{2} t^{1/2 - 1}$$

$$= \frac{1}{2\sqrt{t}} \cdot \frac{d}{dx} \cot x$$

$$= \frac{1}{2\sqrt{\cot x}} \cdot -\operatorname{cosec}^2 x.$$

$$= \frac{-\operatorname{cosec}^2 x}{2\sqrt{\cot x}}.$$

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4) $\log \tan \frac{x}{2}$

$\to \frac{dy}{dx} = \frac{d}{dx}\left\{\log \tan \frac{x}{2}\right\}$

put $\tan \frac{x}{2} = t$

$= \frac{d}{dt}(\log t) \cdot \frac{dt}{dx}$

$= \frac{1}{t} \cdot \frac{d}{dx} \tan \frac{x}{2}$

$= \frac{1}{\tan \frac{x}{2}} \cdot \sec^2 \frac{x}{2} \cdot \frac{d}{dx} \frac{x}{2}$

$= \frac{\sec^2 \frac{x}{2}}{\tan \frac{x}{2}} \cdot \frac{1}{2} \cdot 1 = \frac{\frac{1}{\cos^2 \frac{x}{2}}}{\frac{\sin \frac{x}{2}}{\cos \frac{x}{2}}} \cdot \frac{1}{2}$

$= \frac{1}{2\cos \frac{x}{2}} \cdot \frac{1}{\sin \frac{x}{2}}$

$= \frac{1}{2 \sin \frac{x}{2} \cos \frac{x}{2}}$

$= \frac{1}{\sin 2 \frac{x}{2}}$

$= \frac{1}{\sin x}$

$= \operatorname{cosec} x$

Ans.

5) $y = e^{\tan x}$

$\frac{dy}{dx} = \frac{d}{dx} e^{\tan x}$

put $\tan x = t$

$= \frac{d}{dx} e^t \cdot \frac{dt}{dx}$

$= e^t \cdot \frac{d}{dx} \tan x$

$= e^{\tan x} \cdot \sec^2 x$

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(7) let $y = \sin^2 x^2$

D.W.r. to "x".

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin^2 x^2$

putting $\sin x^2 = t$

$= \frac{d}{dt} t^2 \cdot \frac{dt}{dx}$

$= 2t \cdot \frac{d}{dx}(\sin x^2)$

$= 2\sin x^2 \cdot \frac{d}{dx}(\sin x^2)$

Again putting $x^2 = v$

$= 2\sin x^2 \cdot \frac{d}{dx} \sin v \cdot \frac{dv}{dx}.$

$= 2\sin x^2 \cdot \cos v \cdot \frac{d}{dx} x^2$

$= 2\sin x^2 \cdot \cos x^2 \cdot 2x$

$= \cancel{4x\sin}$

$= \cancel{2s} \sin 2x^2 \cdot 2x \left\{ \because \sin 2A = 2\sin A \cos A \right\}$

$= 2x \cdot \sin 2x^2.$

(6) $\sin^2(3x+5) = y$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\sin^2(3x+5)\right\}$

putting $3x+5 = t$

$= \frac{d}{dt} \sin^2 t \cdot \frac{dt}{dx}.$

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$$= \cos^2(3x+5) \cdot \frac{d}{dx}(3x+5)$$

$$= \cos^2(3x+5) \cdot 3$$

$$= 3\cos^2(3x+5)$$

(3) let $y = \log\left(x + \sqrt{x^2+a^2}\right)$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\log\left(x+\sqrt{x^2+a^2}\right)\right\}$$

putting $x + \sqrt{x^2+a^2} = t$.

$$= \frac{d}{dt}\{\log t\} \cdot \frac{dt}{dx}$$

$$= \frac{1}{t} \cdot \frac{d}{dx}\left\{x + \sqrt{x^2+a^2}\right\}$$

$$= \frac{1}{\left\{x+\sqrt{x^2+a^2}\right\}} \cdot \left\{1 + \frac{d}{dx}\{x^2+a^2\}^{1/2}\right\}$$

putting $x^2+a^2 = v$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left\{\left(1 + \frac{d}{dv}(v)^{1/2}\right) \cdot \left(\frac{dv}{dx}\right)\right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left\{\left(1 + \left(\frac{1}{2}v^{\frac{1}{2}-1}\right)\right) \cdot \left\{\frac{d}{dx}(x^2+a^2)\right\}\right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left\{\left\{1 + \frac{1}{2\sqrt{v}}\right\} \cdot \left(\frac{d}{dx}x^2 + \frac{d}{dx}a^2\right)\right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left\{\left\{1 + \frac{1}{2\sqrt{x^2+a^2}}\right\} \cdot 2x + 0\right\}$$

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$\sqrt{x^2+a^2}$

$\sqrt{x^2+a^2}\left(1+\frac{x}{\sqrt{x^2+a^2}}\right)$

$\left(x+\frac{x^2}{\sqrt{x^2+a^2}}+\frac{a^2}{\sqrt{x^2+a^2}}\right)$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \cdot \left\{ 2x + \frac{2x}{2\sqrt{x^2+a^2}} \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ \frac{4x\sqrt{x^2+a^2}+2x}{2\sqrt{x^2+a^2}} \right\}$$

$$= \frac{4x\sqrt{x^2+a^2}+2x}{(x+\sqrt{x^2+a^2})(2\sqrt{x^2+a^2})}$$

Rough

$\frac{2\sqrt{x^2+a^2}+2x}{2x\sqrt{x^2+a^2}+2(x^2+a^2)}$

$\frac{2\{\sqrt{x^2+a^2}+x\}}{2\{x\sqrt{x^2+a^2}+(x^2+a^2)\}}$

$\frac{1}{x}+\frac{1}{(x^2+a^2)}$

$$= \frac{4x\sqrt{x^2+a^2}+2x}{2x\sqrt{x^2+a^2}+2(x^2+a^2)}$$

$$= \frac{4x\sqrt{x^2+a^2}+2x}{2x\sqrt{x^2+a^2}+2x^2+2a^2}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ 1 + \frac{d}{dv}(v)^{1/2} \cdot \frac{dv}{dx} \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ 1 + \frac{1}{2} v^{1/2-1} \cdot \frac{d}{dx}(x^2+a^2) \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ 1 + \frac{1}{2\sqrt{v}} \cdot 2x + 0. \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ 1 + \frac{2x}{2\sqrt{x^2+a^2}} \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ \frac{2\sqrt{x^2+a^2}+2x}{2\sqrt{x^2+a^2}} \right\}$$

$$= \frac{1}{x+\sqrt{x^2+a^2}} \left\{ 1 + \frac{x}{\sqrt{x^2+a^2}} \right\}$$

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$$= \frac{1}{(x+\sqrt{x^2+a^2})} \left\{ \frac{\sqrt{x^2+a^2}+x}{\sqrt{x^2+a^2}} \right\}$$

$$= \frac{\sqrt{x^2+a^2}+x}{x\sqrt{x^2+a^2}+x^2+a^2}$$

$$= \frac{1}{\sqrt{x^2+a^2}} \quad \underline{\text{Ans.}}$$

(3) let $y = \sin x^\circ$

$\because$ Radian measure $= \frac{\pi}{180^\circ} \times$ degree measure.

$$= \frac{\pi}{180^\circ} \cdot x$$

Radian measure $= \left(\frac{\pi x}{180^\circ}\right)^c$

$$\therefore y = \sin\left(\frac{\pi x}{180^\circ}\right)$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\sin\left(\frac{\pi x}{180^\circ}\right)\right\}$$

putting $t = \frac{\pi x}{180^\circ}$

$$= \frac{d}{dt}\sin t \cdot \frac{dt}{dx}.$$

$$= \cos t \cdot \frac{d}{dx}\left\{\frac{\pi x}{180^\circ}\right\}$$

$$= \cos\left(\frac{\pi x}{180^\circ}\right)^c \cdot \frac{\pi}{180^\circ} \frac{d}{dx}(x)$$

$$= \cos\left(\frac{\pi x}{180^\circ}\right)^c \cdot \frac{\pi}{180^\circ} \cdot 1.$$

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$$= \frac{\pi}{180^\circ} \cos\left(\frac{\pi x}{180^\circ}\right)^c$$

(10) let $y = \dfrac{1}{\log(\cos x)}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\left\{\dfrac{1}{\log(\cos x)}\right\}$

putting $t = \cos x$.

$= \dfrac{d}{dt}\left\{\dfrac{1}{\log t}\right\} \cdot \dfrac{dt}{dx}$

$= \dfrac{1}{\frac{1}{t}} \cdot \dfrac{d}{dx}\cos x$

$= t \cdot \sin x$

$= \cos x \cdot \sin x$ Ans.

$=$

$\Rightarrow y = \{\log(\cos x)\}^{-1}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\{\log(\cos x)\}^{-1}$

putting $\log(\cos x) = t$

$= \dfrac{d}{dt} t^{-1} \cdot \dfrac{dt}{dx}$

$= -t^{-1-1} \cdot \dfrac{d}{dx}\log(\cos x)$

$= -t^{-2} \cdot \dfrac{d}{dx}\{\log(\cos x)\}$

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putting $v = \cos x$.

$= \frac{1}{-t^2} \cdot \frac{d}{dx} \log v \cdot \frac{dv}{dx}$.

$= \frac{1}{-(\log(\cos x))^2} \cdot \frac{1}{V} \cdot \frac{d}{dx} \cos x$.

$= \frac{1}{-\log^2(\cos x)} \cdot \frac{1}{\cos x} \cdot -\sin x$.

$= \frac{\tan x}{\{\log(\cos x)\}^2}$

~~Classwork~~ Ex. 5.2

(7) $2\sqrt{\cot(x^2)} = y$.

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{2\sqrt{\cot(x^2)}\right\}$

putting $x^2 = t$

$= \frac{d}{dt} 2\sqrt{\cot(t)} \cdot \frac{dt}{dx}$.

$= 2\frac{d}{dx}\left(\cot^{1/2}(t)\right) \cdot \frac{d}{dx} x^2$.

$= 2\left(-\operatorname{cosec}^2 t\right)^{1/2} \cdot 2x \cdot \frac{d}{dx} \frac{1}{2}$.

$= -2 \operatorname{cosec} t \cdot 2x$

$= -2 \operatorname{cosec} x^2 \cdot 2x$.

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Modulas function :- $|a| = \begin{cases} a & a > 0 \\ -a & a < 0 \end{cases}$

Eg. $|-5| = -(-5) = 5$

$|5| = 5$

$|10| = 10$

$|-10| = -(-10) = 10.$

signum function :-

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

Greatest Integer function :- always use left side no on the number line

$[1.34] = 1$

$[1.89] = 1$

$[0.54] = 0$

$[-0.5] = -1$

$[-2.5] = -3$

$[3.2] = 3$

$[4] = 4$

$[3.24] = 3.$

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Chapter-5
Continuity And Differentiability

Exercise 5.3.

Find $\frac{dy}{dx}$ in the following

(1) $2x + 3y = \sin x.$

Sol. Differentiate both side with respect to 'x'

$\Rightarrow \frac{d}{dx}(2x+3y) = \frac{d}{dx}\sin x.$

$\Rightarrow \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \cos x.$

$\Rightarrow 2 + 3\frac{dy}{dx} = \cos x.$

$\Rightarrow \frac{dy}{dx} = \frac{\cos x - 2}{3}$ Ans.

(2) $\sin^2 y + \cos xy = \pi.$

Sol.

d.w.r to "x" both side,

$\Rightarrow \frac{d}{dx}\left(\sin^2 y + \cos xy\right) = \frac{d}{dx}\pi$

$\Rightarrow \frac{d}{dx}\sin^2 y + \frac{d}{dx}\cos xy = 0$

$\Rightarrow \left(2\sin y\cos y\frac{dy}{dx}\right) + \left(-x\sin xy\frac{dy}{dx} - y\sin xy\right) = 0$

$\Rightarrow 2\sin y\cos y\frac{dy}{dx} - x\sin xy\frac{dy}{dx} = y\sin xy.$

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$\Rightarrow \frac{dy}{dx}\left[2\sin y\cos y - x\sin yx\right] = y\sin xy$

$\Rightarrow \frac{dy}{dx} = \frac{y\sin xy}{2\sin y\cos y - x\sin yx}$

(9) $y = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

Sol. $y = 2\tan^{-1}x.$

d.w.r.t. 'x' both side,

$\Rightarrow \frac{dy}{dx} = 2\frac{d}{dx}\tan^{-1}x.$

$\Rightarrow \frac{dy}{dx} = 2\left(\frac{1}{1+x^2}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{2}{1+x^2}$

(8) $\sin^2 x + \cos^2 y = 1.$

Sol. d.w.r.t. to "x" both side.

$\Rightarrow \frac{d}{dx}\sin^2 x + \frac{d}{dx}\cos^2 y = \frac{d}{dx}1$

$\Rightarrow \left(2\sin x\cos x\,\cancel{\frac{dy}{dx}}\right) + \left(2\cos y(-\sin y)\frac{dy}{dx}\right) = 0.$

$\Rightarrow 2\sin x\cos x\,\cancel{\frac{dy}{dx}} - 2\cos y\sin y\frac{dy}{dx} = 0.$

$\Rightarrow 2\sin x\cos x - 2\sin y\cos y\frac{dy}{dx} = 0.$

$\therefore \sin 2x - \sin 2y\cdot\frac{dy}{dx} = 0.$

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$\Rightarrow \quad \sin 2y \cdot \frac{dy}{dx} = \sin 2x.$

$\Rightarrow \quad \frac{dy}{dx} = \frac{\sin 2x}{\sin 2y.}$ Ans

(10) $y = \tan^{-1}\left\{\frac{3x - x^3}{1 - 3x^2}\right\}, \quad \frac{-1}{\sqrt{3}} < x < \frac{1}{\sqrt{3}}.$

Sol. $\frac{dy}{dx} = \frac{d}{dx}\left\{\tan^{-1}\left(\frac{3x - x^3}{1 - 3x^2}\right)\right\}$

Putting $x = \tan\theta$. then $\theta = \tan^{-1} x$.

$= \frac{d}{d\theta}\left\{\tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)\right\} \cdot \frac{d\theta}{dx}.$

$= \frac{d}{d\theta}\left\{\tan^{-1}(\tan 3\theta)\right\} \cdot \frac{d}{dx}(\tan^{-1} x)$

$= \frac{d}{d\theta}(3\theta) \cdot \frac{1}{1+x^2}$

$= \frac{d}{d\theta}(3\tan^{-1} x) \cdot \frac{1}{1+x^2}$

$= \frac{3}{1+x^2} \cdot \frac{1}{1+x^2}$

$= \frac{3}{1 + x^4 + 2x^2}$

$\frac{dy}{dx} = \frac{d}{dx}(3\tan^{-1} x)$

$= 3 \frac{d}{dx} \tan^{-1} x.$

$= 3 \cdot \frac{1}{1+x^2} = \frac{3}{1+x^2}$ Ans.

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(11) $y = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$

Sol. $\frac{dy}{dx} = \frac{d}{dx}\left\{\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$

$= \frac{d}{dx}\left\{2\tan^{-1}(x)\right\}$

$= 2 \cdot \frac{1}{1+x^2}$

$= \frac{2}{1+x^2}$ Ans.

(12) $y = \sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)$, $0 < x < 1$

Sol. ~~$\frac{dy}{dx} = \frac{d}{dx}\left\{\sin^{-1}\left(\frac{1-x^2}{1+x^2}\right)\right\}$~~

~~$= \frac{d}{dx}\{\}$~~

$\Rightarrow \sin y = \frac{1-x^2}{1+x^2}$

$\Rightarrow \frac{d}{dx}\sin y = \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$

$\Rightarrow \cos y \frac{dy}{dx} = \frac{\left\{(1+x^2)\frac{d}{dx}(1-x^2)\right\} - \left\{(1-x^2)\frac{d}{dx}(1+x^2)\right\}}{(1+x^2)^2}$

$\Rightarrow \cos y \frac{dy}{dx} = \frac{\left\{(1+x^2)(0-2x)\right\} - \left\{(1-x^2)(0+2x)\right\}}{1+x^4+2x^2}$

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$$\Rightarrow \cos y \frac{dy}{dx} = \frac{-2x-2x^3-\{(2x-2x^3)\}}{1+x^4+2x^2}$$

$$\cos y \frac{dy}{dx} = \frac{-2x-\cancel{2x^3}-2x+\cancel{2x^3}}{1+x^4+2x^2}$$

$$= \frac{-4x}{1+x^4+2x^2} \rightarrow \frac{d}{dx}\left\{\frac{1-x^2}{1+x^2}\right\}.$$

putting $\frac{1-x^2}{1+x^2} = t$,

$$\Rightarrow y = \sin^{-1} t.$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dt}\sin^{-1}t \cdot \frac{dt}{dx}.$$

$$= \frac{1}{\sqrt{1-t^2}} \cdot \frac{d}{dx}\left(\frac{1-x^2}{1+x^2}\right)$$

$$= \frac{1}{\sqrt{1-\left(\frac{1-x^2}{1+x^2}\right)^2}} \cdot \frac{-4x}{1+x^4+2x^2}$$

$$= \frac{1}{\sqrt{\frac{(1+x^2)^2-(1-x^2)^2}{(1+x^2)^2}}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$= \frac{1}{\sqrt{\frac{2x^2 \cdot 2}{(1+x^2)^2}}} \cdot \frac{-4x}{(1+x^2)^2}.$$

$$= \frac{1}{\sqrt{\frac{4x^2}{(1+x^2)^2}}} \cdot \frac{-4x}{(1+x^2)^2}$$

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$$= \frac{1}{\frac{2x}{(1+x^2)}} \cdot \frac{-4x}{(1+x^2)^2}$$

$$= \frac{\cancel{(1+x^2)}}{\cancel{2x}} \cdot \frac{-\cancel{4}^{2}\cancel{x}}{(1+x^2)\cancel{^2}}$$

$$= \frac{-2}{1+x^2} \quad \underline{\text{Ans.}}$$

(13) $y = \cos^{-1}\left(\frac{2x}{1+x^2}\right), \quad 0 < x < 1.$

Sol. $\frac{dy}{dx} = \frac{d}{dx}\left\{\cos^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$

Putting $t = \frac{2x}{1+x^2}$

$$= \frac{d}{dt}(\cos^{-1} t) \cdot \frac{dt}{dx}.$$

$$= \frac{-1}{\sqrt{1-t^2}} \cdot \frac{d}{dx}\left\{\frac{2x}{1+x^2}\right\}$$

$$= \frac{-1}{\sqrt{1-\left(\frac{2x}{1+x^2}\right)^2}} \cdot \frac{(1+x^2)\frac{d}{dx}(2x) - \left\{2x \cdot \frac{d}{dx}(1+x^2)\right\}}{(1+x^2)^2}$$

$$= \frac{-1}{\sqrt{\frac{(1+x^2)^2-(2x)^2}{(1+x^2)^2}}} \cdot \frac{(1+x^2)\cdot 2 - \{2x(0+2x)\}}{(1+x^2)^2}$$

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$\dfrac{(1+x^2+2x)(1+x^2-2x)}{(1+x)^2(1-x)^2}$

$= \dfrac{-1}{\sqrt{\dfrac{(1+x^2+2x)(1+x^2-2x)}{(1+x^2)}}} \cdot \dfrac{2+2x^2-4x^2}{(1+x^2)^2}$

$= \dfrac{-\cancel{(1+x^2)}}{\sqrt{(1+x)^2(1-x)^2}} \cdot \dfrac{2-2x^2}{(1+x^2)^{\cancel{4}}}$

$= \dfrac{-1}{(1+x)(1-x)} \cdot \dfrac{2(1-x^2)}{(1+x^2)}$

$= \dfrac{-1}{\cancel{(1-x^2)}} \cdot \dfrac{2\cancel{(1-x^2)}}{1+x^2}$

$\dfrac{dy}{dx} = \dfrac{-2}{1+x^2}$ Ans.

(14) $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right), \quad \dfrac{-1}{\sqrt{2}} < x < \dfrac{1}{\sqrt{2}}.$

Sol. $\dfrac{dy}{dx} = \dfrac{d}{dx}\left\{\sin^{-1}\left(2x\sqrt{1-x^2}\right)\right\}$

$= \dfrac{d}{dx}\left(2\sin^{-1}x\right)$

$= 2 \cdot \dfrac{1}{\sqrt{1-x^2}}$

$= \dfrac{2}{\sqrt{1-x^2}}$ Ans.

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(15) $y = \sec^{-1}\left(\frac{1}{2x^2-1}\right)$, $0 < x < \frac{1}{\sqrt{2}}$.

Sol. $\frac{dy}{dx} = \frac{d}{dx}\left\{\sec^{-1}\left(\frac{1}{2x^2-1}\right)\right\}$

$= \frac{d}{dx}\left\{\cos^{-1}(2x^2-1)\right\}$

$= \frac{d}{dx}(2\cos^{-1}x)$

$= 2 \cdot \frac{-1}{\sqrt{1-x^2}}$

$= \frac{-2}{\sqrt{1-x^2}}$ Ans.

(2) $2x+3y = \sin y$.

Sol. D.W.R to 'x' both side.

$\Rightarrow \frac{d}{dx}(2x) + \frac{d}{dx}(3y) = \frac{d}{dx}(\sin y)$

$\Rightarrow 2 + 3 \cdot \frac{dy}{dx} = \cos y \cdot \frac{dy}{dx}$.

$\Rightarrow 2 = \cos y \cdot \frac{dy}{dx} - 3\frac{dy}{dx}$.

$\Rightarrow 2 = \frac{dy}{dx}(\cos y - 3)$

$\Rightarrow \frac{dy}{dx} = \frac{2}{(\cos y - 3)}$

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(5). ~~$-\sin(ax+b)$~~ $ax + by^2 = \cos y$

Sol. D.W.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}ax + \frac{d}{dx}by^2 = \frac{d}{dx}(\cos y)$$

$$\Rightarrow a + b\frac{d}{dx}y^2 = -\sin y\frac{dy}{dx}.$$

$$\Rightarrow a + 2by\frac{dy}{dx} = -\sin y\frac{dy}{dx}.$$

$$\Rightarrow a = -\sin y\frac{dy}{dx} - 2by\frac{dy}{dx}.$$

$$\Rightarrow a = \frac{dy}{dx}\left(-\sin y - 2by\right)$$

$$\Rightarrow \frac{dy}{dx} = -\frac{a}{\sin y + 2by}.$$

(6) $xy + y^2 = \tan x + y$

Sol. D.W.r. to "x" Both side,

$$\Rightarrow \frac{d}{dx}(xy) + \frac{d}{dx}(y)^2 = \frac{d}{dx}\tan x + \frac{d}{dx}(y)$$

$$\Rightarrow y + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}$$

$$\Rightarrow y - \sec^2 x = \frac{dy}{dx} - 2y\frac{dy}{dx}$$

$$\Rightarrow y - \sec^2 x = \frac{dy}{dx}(1 - 2y)$$

$$\Rightarrow \frac{dy}{dx} = \left(\frac{y - \sec^2 x}{1 - 2y}\right)$$ Ans.

By product rule,

$$\Rightarrow \left[y \frac{d}{dx}(x) + x \cdot \frac{d}{dx}(y)\right] + 2y \cdot \frac{dy}{dx} = \sec^2 x$$

$$\left[\because \frac{d}{dx} \frac{f(x)}{g(x)} = g(x) \frac{d}{dx} f(x) + f(x) \frac{d}{dx}\right.$$

$$\Rightarrow y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = \sec^2 x + \frac{dy}{dx}.$$

$$\Rightarrow y - \sec^2 x = \frac{dy}{dx} - x\frac{dy}{dx} - 2y\frac{dy}{dx}.$$

$$\Rightarrow \frac{dy}{dx}(1 - x - 2y) = y - \sec^2 x$$

$$\Rightarrow \frac{dy}{dx} = \frac{y - \sec^2 x}{1 - x - 2y}$$

$$= \frac{y - \sec^2 x}{-(-1 + x + 2y)}$$

$$= \frac{\sec^2 x - y}{x + 2y - 1} \quad \text{Ans.}$$

(5) $x^2 + xy + y^2 = 100$

Sol. D.w.r to "x" both side,

$$\Rightarrow \frac{d}{dx}(x^2) + \frac{d}{dx}(xy) + \frac{d}{dx} y^2 = \frac{d}{dx}(100)$$

$$\Rightarrow 2x + \left[y \frac{d}{dx}(x) + x \cdot \frac{dy}{dx}\right] + 2y \cdot \frac{dy}{dx} = 0.$$

$$\Rightarrow 2x + y + x\frac{dy}{dx} + 2y\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx}(x + 2y) = -2x - y$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-(2x+y)}{(x+2y)} \quad \text{Ans.}$$

(6) $x^3 + x^2y + xy^2 + y^3 = 81$

Sol. d.w.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}(x^3) + \frac{d}{dx}(x^2y) + \frac{d}{dx}(xy^2) + \frac{d}{dx}y^3 = \frac{d}{dx}(81)$$

$$\Rightarrow 3x^2 + \left[y \cdot \frac{d}{dx}(x^2) + x^2 \cdot \frac{dy}{dx}\right] + \left[y^2 \frac{d}{dx}(x) + x\frac{d}{dx}y^2\right] + \frac{d}{dx}y^3 = 0$$

$$\Rightarrow 3x^2 + y(2x) + x^2\frac{dy}{dx} + y^2 + x\left(2y\frac{dy}{dx}\right) + \frac{d}{dx}y^3 = 0$$

$$\Rightarrow 3x^2 + 2xy + x^2\frac{dy}{dx} + y^2 + 2xy\frac{dy}{dx} + 3y^2 \cdot \frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 + 2xy + y^2 + x^2\frac{dy}{dx} + 2xy\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$$

$$\Rightarrow 3x^2 + 2xy + y^2 + \left[\frac{dy}{dx}\left(x^2 + 2xy + 3y^2\right)\right] = 0$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3x^2 - 2xy - y^2}{x^2 + 2xy + 3y^2}$$

$$= -\frac{(3x^2 + 2xy + y^2)}{x^2 + 2xy + 3y^2}$$

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Exercise 5.4

D.W.r. to "x"

① $\dfrac{e^x}{\sin x}$

Sol: let,

$$y = \frac{e^x}{\sin x}$$

D.W.r. to "x",

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{e^x}{\sin x}\right)$$

$$= \frac{\sin x \frac{d}{dx} e^x - e^x \frac{d}{dx} \sin x}{(\sin x)^2}$$

$$= \frac{\sin x\, e^x - e^x \cos x}{\sin^2 x}$$

$$= \frac{\cancel{\sin x}\, e^x}{\sin^{\cancel{2}} x} - \frac{e^x \cos x}{\sin^2 x}$$

$$= e^x . \operatorname{cosec} x - \frac{e^x \cot x}{\sin x}$$

⑤ $\log(\cos e^x)$

Sol. let

$$y = \log(\cos e^x)$$

D.W.r. to "x",

$$\frac{dy}{dx} = \frac{d}{dx} \log(\cos e^x)$$

putting $\cos e^x = t$.

$$= \frac{d}{dt} \log t \cdot \frac{dt}{dx}$$

$$= \frac{1}{t} \cdot \frac{d}{dx} \cos e^x$$

$$= \frac{1}{\cos e^x} \cdot \frac{d}{dx} \cos e^x$$

putting $e^x = v$.

$$= \frac{1}{\cos e^x} \cdot \frac{d}{dv} \cos v \cdot \frac{dv}{dx}$$

$$= \frac{1}{\cos e^x} \cdot -\sin v \cdot \frac{d}{dx} e^x$$

$$= -\frac{\sin e^x}{\cos e^x} \cdot e^x$$

$$= -\tan e^x \cdot e^x$$

(7) $\sqrt{e^{\sqrt{x}}}$, $x > 0$.

Sol. let

$$y = \sqrt{e^{\sqrt{x}}}$$

D.W.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \left\{ \sqrt{e^{\sqrt{x}}} \right\}$$

~~putting $e^{\sqrt{x}} = t$~~

$$= \frac{d}{dt} \sqrt{e^t} \cdot \frac{dt}{dx}$$

$$= \frac{d}{dt} e^{\frac{t}{2}} \cdot \frac{d}{dx}(\sqrt{x})$$

Putting $\frac{t}{2} = v$

$$= \frac{d}{dv} e^{v} \cdot \frac{dv}{dt}$$

putting $e^{\sqrt{x}} = t$,

$$= \frac{d}{dt}\sqrt{t} \cdot \frac{dt}{dx}$$

$$= \frac{1}{2\sqrt{t}} \cdot \frac{d}{dx} e^{\sqrt{x}}$$

putting $\sqrt{x} = v$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot \frac{d}{dv} e^{v} \cdot \frac{dv}{dx}$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{v} \cdot \frac{d}{dx}\sqrt{x}$$

$$= \frac{1}{2\sqrt{e^{\sqrt{x}}}} \cdot e^{\sqrt{x}} \cdot \frac{1}{2\sqrt{x}}$$

$$= \frac{e^{\sqrt{x}}}{4\sqrt{x e^{\sqrt{x}}}}$$ Ans.

(10) $\cos(\log x + e^{x})$

Sol. let

$$y = \cos(\log x + e^{x})$$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\cos(\log x + e^{x})\right\}$$

putting $\log x + e^{x} = t$.

$= \frac{d}{dt} \cos t \cdot \frac{dt}{dx}$

$= -\sin t \cdot \frac{d}{dx} (\log x + e^x)$

$= -\sin(\log x + e^x) \cdot \left(\frac{1}{x} + e^x\right)$

~~$= -\frac{e^x \sin(\log x + e^x)}{x}$~~

$= -\left(\frac{1}{x} + e^x\right) \sin(\log x + e^x)$ Ans.

~~Exercise 5.4~~

(2) $e^{\sin^{-1}x}$.

Sol. let

$y = e^{\sin^{-1}x}$.

D.W.R. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} e^{\sin^{-1}x}$.

putting $\sin^{-1}x = t$.

$= \frac{d}{dt} e^t \cdot \frac{dt}{dx}$.

$= e^t \cdot \frac{d}{dx} \sin^{-1}x$.

$= e^{\sin^{-1}x} \cdot \frac{1}{\sqrt{1-x^2}}$

$= \frac{e^{\sin^{-1}x}}{\sqrt{1-x^2}}$

(3). e^{x^3}.

Sol. let

$$y = e^{x^3}$$

d.w.r. to 'x' both side,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} e^{x^3}$$

putting $x^3 = t$.

$$= \frac{d}{dt} e^t \cdot \frac{dt}{dx}$$

$$= e^t \cdot \frac{d}{dx} x^3$$

$$= e^{x^3} \cdot 3x^2$$

$$= 3x^2 e^{x^3} \quad \text{Ans.}$$

(4) $\sin(\tan^{-1} e^{-x})$

Sol. let

$$y = \sin(\tan^{-1} e^{-x})$$

D. w. r. to "x" both side

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{\sin(\tan^{-1} e^{-x})\right\}$$

putting $\tan^{-1} e^{-x} = t$.

$$= \frac{d}{dt} \sin t \cdot \frac{dt}{dx}$$

$$= \cos t \cdot \frac{d}{dx} \tan^{-1} e^{-x}.$$

putting $v = e^{-x}$

$$= \cos(\tan^{-1} e^{-x}) \cdot \frac{d}{dv} \tan^{-1} v \cdot \frac{dv}{dx}$$

$$= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+v^2} \cdot \frac{d}{dx} e^{-x}.$$

$$= \cos(\tan^{-1} e^{-x}) \cdot \frac{1}{1+(e^{-x})^2} \cdot -e^{-x}.$$

$$= \frac{-e^{-x}\cos(\tan^{-1} e^{-x})}{1+e^{-2x}} \quad \text{Ans.}$$

(6) $e^{x} + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$

Sol. let,

$$y = e^{x} + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}$$

D.w.r. to "x" both side,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left\{e^{x} + e^{x^2} + e^{x^3} + e^{x^4} + e^{x^5}\right\}$$

$$= \left[\frac{d}{dx}e^{x} + \frac{d}{dx}e^{x^2} + \frac{d}{dx}e^{x^3} + \frac{d}{dx}e^{x^4} + \frac{d}{dx}e^{x^5}\right]$$

$$= e^{x}$$

putting

$$x^2 = t$$
$$x^3 = v$$
$$x^4 = m$$
$$x^5 = n$$

$$= \left[e^{x} + \left(\frac{d}{dt}e^{t} \cdot \frac{dt}{dx}\right) + \left(\frac{d}{dv}e^{v} \cdot \frac{dv}{dx}\right) + \left(\frac{d}{dm}e^{m} \cdot \frac{dm}{dx}\right) + \left(\frac{d}{dn}e^{n} \cdot \frac{dn}{dx}\right)\right]$$

$$= e^{x} + \left(e^{x^2} \cdot \frac{d}{dx}x^2\right) + \left(e^{x^3} \cdot \frac{d}{dx}x^3\right) + \left(e^{x^4} \cdot \frac{d}{dx}x^4\right) + \left(e^{x^5} \cdot \frac{d}{dx}x^5\right)$$

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$= e^x + (e^{x^2} \cdot 2x) + (e^{x^3} \cdot 3x^2) + (e^{x^4} \cdot 4x^3) + ($

$= e^x + 2x e^{x^2} + 3x^2 e^{x^3} + 4x^3 e^{x^4} + 5$

Ans

(8) $\log(\log x)$

Sol. let,

$y = \log(\log x)$

D.w.r. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\{\log(\log x)\}$

putting $\log x = t$.

$= \frac{d}{dt}\log t \quad \frac{dt}{dx}.$

$= \frac{1}{\log x} \cdot \frac{d}{dx}\log x.$

$= \frac{1}{\log x} \cdot \frac{1}{x}.$

$= \frac{1}{x\log x}, \quad x > 1.$

(9) $\frac{\cos x}{\log x}$

Sol. let, $y = \frac{\cos x}{\log x}$,

D.w.r. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(\frac{\cos x}{\log x}\right)$

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$$= \frac{\log x \frac{d}{dx}(\cos x) - \left\{ \cos x \cdot \frac{d}{dx}(\log x) \right\}}{\{\log x\}^2}$$

$$= \frac{-\log x \sin x - \frac{\cos x}{x}}{\log^2 x}$$

$$= \frac{-\left(\log x \sin x + \frac{\cos x}{x}\right)}{\log^2 x}$$

multiplying "x" in numerator and denominator,

$$= \frac{-x\left(\log x \sin x + \frac{\cos x}{x}\right)}{x \log^2 x}$$

$$= \frac{-(x \log x \sin x + \cos x)}{x \log^2 x} \quad \underline{\text{Ans.}}$$

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Formulas of logarithm:-

① $\log_a a = 1$

② $\log_b a = \dfrac{1}{\log_a b}$

③ $\log_e m.n = \log_e m + \log_e n$

④ $\log_e \dfrac{m}{n} = \log_e m - \log_e n$

⑤ $\log_e m^n = n \log m$

Note:-

i) $\dfrac{d}{dx} x^a = a x^{a-1}$

ii) $\dfrac{d}{dx} a^x = a^x . \log a.$

iii) $\dfrac{d}{dx} a^a = 0$

iv) $\dfrac{d}{dx} x^x =$

By using ~~the~~ logarithsm,

let, $y = x^x$

taking log Both side,

$\Rightarrow \log y = \log x^x$

$\Rightarrow \log y = x \log x.$

D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} x \log x$

$\Rightarrow \frac{d}{dy}(\log y) \cdot \frac{dy}{dx} = \left[x \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx} x\right]$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = x \cdot \frac{1}{x} + \log x \cdot 1$

$\Rightarrow \frac{dy}{dx} = y\left[1 + \log x\right]$

$\Rightarrow \frac{dy}{dx} = x^x\left(1 + \log x\right)$

Q. $y = \sin x^{\cos x}$

Sol. taking log both side,

$\Rightarrow \log y = \log \sin^{\cos x}$

$\Rightarrow \log y = \cos x \log \sin x$

D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx}(\cos x \log \sin x)$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \cos x \frac{d}{dx}(\log \sin x) + \log \sin x \frac{d}{dx} \cos x$

$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \cos x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \, (-\sin x)$

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$$\Rightarrow \frac{dy}{dx} = y\left(\cancel{\tan}\ \cot x \cdot \cos x - \log \sin x \cdot \sin x\right)$$

$$\Rightarrow \frac{dy}{dx} = \sin x^{\cos x}\left(\cot x \cos x - \sin x \log \sin x\right)$$

Exercise 5.5

(2) $\sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Sol. let, $y = \sqrt{\dfrac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$

Taking log both side,

$$\Rightarrow \log y = \log\sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}}$$

$$= \log\left(\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}\right)^{1/2}$$

$$= \frac{1}{2}\log\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}$$

$$= \frac{1}{2}\left[\log(x-1)(x-2) - \log(x-3)(x-4)(x-5)\right]$$

$$\Rightarrow \log y = \frac{1}{2}\left[\log(x-1) + \log(x-2) - \log(x-3) - \log(x-4) - \log(x-5)\right]$$

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D.W.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}\log y = \frac{1}{2}\left[\frac{d}{dx}\log(x-1) + \frac{d}{dx}\log(x-2) - \cancel{\log}\frac{d}{dx}\log(x-3) - \frac{d}{dx}\log(x-4) - \frac{d}{dx}\log(x-5)\right]$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{(x-1)}\cdot\frac{d}{dx}(x-1) + \frac{1}{(x-2)}\frac{d}{dx}(x-2) - \frac{1}{(x-3)}\frac{d}{dx}(x-3) - \frac{1}{(x-4)}\frac{d}{dx}(x-4) - \frac{1}{(x-5)}\frac{d}{dx}(x-5)\right]$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{1}{(x-1)}\cdot 1 + \frac{1}{(x-2)}\cdot 1 - \frac{1}{(x-3)}\cdot 1 - \frac{d}{(x-4)}\cdot 1 - \frac{1}{(x-5)}\cdot 1\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left(\frac{1}{(x-1)} + \frac{1}{(x-2)} - \frac{1}{(x-3)} - \frac{1}{(x-4)} - \frac{1}{(x-5)}\right)$$

$$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{2}\left[\frac{x-2+x-1}{(x-1)(x-2)} - \frac{(x-4)-(x-3)}{(x-3)(x-4)} - \frac{1}{(x-5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-2x-x+2} - \frac{(x-4-x+3)}{x^2-4x-3x+12} - \frac{1}{(x-5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-3x+2} - \frac{(-1)}{x^2-7x+12} - \frac{1}{(x-5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-3x+2} + \frac{(x-5 \quad -x^2+7x-12)}{(x^2-7x+12)(x-5)}\right]$$

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$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-3x+2} + \frac{(-x^2+8x-17)}{(x^2-7x+12)(x-5)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-3x+2} - \frac{x^2-8x+17}{x^3-7x^2+12x-5x^2+35x-60}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x-3}{x^2-3x+2} - \frac{x^2-8x+17}{x^3-12x^2+47x-60}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{(2x-3)(x^3-12x^2+47x-60)-(x^2-3x+2)(x^2-8x+17)}{(x^2-3x+2)(x^3-12x^2+47x-60)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{(2x^4-24x^3+94x^2-120x-3x^3+36x^2-141x+180)-(x^4-8x^3+17x^2-3x^3+24x^2-51x+2x^2-16x+34)}{(x^5-12x^4+47x^3-60x^2-3x^4+36x^3-141x^2+180x+2x^3-24x^2+94x-120)}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{2x^4-27x^3+130x^2-261x+180-x^4+11x^3-43x^2+67x-34}{x^5-15x^4+85x^3-225x^2+274x-120}\right]$$

$$\Rightarrow \frac{dy}{dx} = y \cdot \frac{1}{2}\left[\frac{2x^4-27x^3+130x^2-261x+180-x^4+11x^3-43x^2+67x-34}{x^5-15x^4+85x^3-225x^2+274x-120}\right]$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \cdot \frac{1}{2}\left[\frac{2x^4-27x^3+130x^2-261x+146-x^4+11x^3-43x^2+67x}{x^5-15x^4+85x^3-225x^2+274x-120}\right]$$

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{(x-1)(x-2)}{(x-3)(x-4)(x-5)}} \cdot \frac{1}{2}\left[\frac{x^4-16x^3+87x^2-194x+146}{x^5-15x^4+85x^3-225x^2+274x-120}\right]$$

(3) $(\log x)^{\cos x}$

Sol. let, $y = (\log x)^{\cos x}$

Taking log both side,

$\Rightarrow \log y = \log (\log x)^{\cos x}$

$\Rightarrow \log y = \cos x \cdot \log \log x$

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} \cos x \cdot \log \log x$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \frac{d}{dx} \log \log x + \log \log x \cdot \frac{d}{dx} \cos$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = \cos x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \log \log x \cdot (-\sin x)$

$\Rightarrow \frac{dy}{dx} = y \left(\frac{\cos x}{x \log x} - \sin x \log \log x \right)$

$\Rightarrow \frac{dy}{dx} = (\log x)^{\cos x} \left(\frac{\cos x}{x \log x} - \sin x \log \log x \right)$

(1) $\cos x \cdot \cos 2x \cdot \cos 3x$

Sol. let $y = \cos x \cdot \cos 2x \cdot \cos 3x$

Taking log both side,

$\Rightarrow \log y = \log \cos x \cdot \cos 2x \cdot \cos 3x$

$\Rightarrow \log y = \log \cos x + \log \cos 2x + \log \cos 3x$

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D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\log y = \frac{d}{dx}\log\cos x + \frac{d}{dx}\log\cos 2x + \frac{d}{dx}\log$

$\Rightarrow \frac{1}{y}\cdot\frac{dy}{dx} = \frac{1}{\cos x}(-\sin x).1 + \frac{1}{\cos 2x}(-\sin 2x).2 + \frac{1}{\cos 3x}\cdot(-\sin$

$\Rightarrow \frac{dy}{dx} = y\left[-\tan x - 2\tan 2x - 3\tan 3x\right]$

$\Rightarrow \frac{dy}{dx} = (\cos x.\cos 2x.\cos 3x)(-\tan x - 2\tan 2x - 3\tan 3x)$

$\Rightarrow \frac{dy}{dx} = \cos x.\cos 2x.\cos 3x\left[-(\tan x + 2\tan 2x + 3\tan\right.$

$\Rightarrow \frac{dy}{dx} = -\cos x.\cos 2x.\cos 3x\,(\tan x + 2\tan x + 3\tan$

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(4) $x^x - 2^{\sin x}$

Sol. let, $y = x^x - 2^{\sin x}$.

D.W.r. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left(x^x - 2^{\sin x}\right)$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}x^x - \frac{d}{dx}2^{\sin x}$

let, $u = x^x$ | ~~and $v = 2^{\sin x}$~~

$\Rightarrow \log u = \log x^x$

$\Rightarrow \log u = x\log x$

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$$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx} x \log x.$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \left[x \frac{d}{dx} \log x + \log x \frac{d}{dx} x. \right]$$

$$\Rightarrow \frac{du}{dx} = u\left(x \cdot \frac{1}{x} + \log x \cdot 1 \right)$$

$$\Rightarrow \frac{du}{dx} = u(1 + \log x)$$

$$\Rightarrow \frac{du}{dx} = x^x(1 + \log x)$$

Now, let $v = 2^{\sin x}$

$$\Rightarrow \log v = \log 2^{\sin x}.$$

$$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} (\sin x \cdot \log 2)$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \left(\sin x \cdot \frac{d}{dx} \log 2. + \log 2 \frac{d}{dx} \sin x. \right)$$

$$\Rightarrow \frac{dv}{dx} = v\left(\sin x \cdot 0 + \log 2 \cdot \cos x \right)$$

$$\Rightarrow \frac{dv}{dx} = 2^{\sin x} \cdot \log 2 \cdot \cos x.$$

Now,

$$\frac{dy}{dx} = \frac{du}{dx} - \frac{dv}{dx}$$

$$= x^x(1 + \log x) - \left(2^{\sin x} \cdot \log 2 \cdot \cos x \right)$$

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(9) $x^{\sin x} + (\sin x)^{\cos x}$.

Sol.

let $Y = x^{\sin x} + (\sin x)^{\cos x}$.

and $u = x^{\sin x}$.

$\Rightarrow \log u = \log x^{\sin x}$.

~~D.W.r to "x" both side,~~

~~$\Rightarrow \frac{d}{dx}(u) = \frac{d}{dx}(x^{\sin x})$.~~

$\Rightarrow$ $\cancel{\frac{d}{dx}}$ $\log u = \sin x \log x$.

D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx}\sin x \log x$.

$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = \sin x \frac{d}{dx}\log x + \log x \frac{d}{dx}\sin x$

$\Rightarrow \frac{du}{dx} = u\left(\sin x \cdot \frac{1}{x} + \log x \cdot (-\cos x)\right)$

$\Rightarrow \frac{du}{dx} = x^{\sin x}\left(\frac{\sin x}{x} - \cos x \log x\right) \longrightarrow (1)$

Now, let $V = (\sin x)^{\cos x}$.

$\Rightarrow \log V = \log(\sin x)^{\cos x}$

D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\log V = \frac{d}{dx}.\cos x \log \sin x$.

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$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \cos x \frac{d}{dx} \log \sin x + \log \sin x \frac{d}{dx} \cos x$

$\Rightarrow \frac{dv}{dx} = v\left[\cos x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \, (-\sin x)\right]$

$\Rightarrow \frac{dv}{dx} = \sin x^{\cos x}\left[\cot x \cos x - \sin x \log \sin x\right]$

Now,

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{du}$

$= x^{\sin x}\left(\frac{\sin x}{x} - \cos x \log x\right) + \sin^{\cos x}\left[\cot x \cos x - \sin x \log \sin x\right]$

Ans.

(10) $x^{x\cos x} + \frac{x^2+1}{x^2-1}$

Sol. let, $y = x^{x\cos x} + \frac{x^2+1}{x^2-1}$

and $u = x^{x\cos x}$

$\Rightarrow \log u = \log x^{x\cos x}$

$\Rightarrow \log u = x\cos x \log x.$

D.W.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log u = \frac{d}{dx}\left[(x\cos x)(\log x)\right]$

$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x\cos x \frac{d}{dx} \log x + \log x \frac{d}{dx}(x\cos x)$

$$\Rightarrow \frac{du}{dx} = u\left[\frac{x\cos x}{x} + \log x \cdot \left(x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}(x)\right)\right]$$

$$= u\left[\cos x + \log x\left(x(-\sin x) + \cos x\right)\right]$$

$$= x^{x\cos x}\left[\cos x \mp \log x(\cancel{x}\sin x + \cos x)\right] \rightarrow (1)$$

$$=$$

Now;

$$v = \frac{x^2+1}{x^2-1}$$

$$\Rightarrow \frac{dv}{dx} = \frac{d}{dx}\left(\frac{x^2+1}{x^2-1}\right)$$

$$= \frac{(x^2-1)\frac{d}{dx}(x^2+1) - (x^2+1)\frac{d}{dx}(x^2-1)}{(x^2-1)^2}$$

$$\frac{dv}{dx} = \frac{(x^2-1)2x - (x^2+1)2x}{(x^2-1)^2} \rightarrow (2)$$

Now,

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}.$$

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left(\cos x + \log x(x\sin x + \cos x)\right) + \frac{(x^2-1)2x-(x^2+1)}{(x^2-1)^2}$$

$$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left(\cos x - x\log x\sin x + \log x\cos x\right) + \frac{2x^3-2x-2x^3-2}{(x^2-1)^2}$$

$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left(\cos x + \log x \cos x - x \log x \sin x\right) + \frac{-4x}{(x^2-1)^2}$

$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left[\cos x(1 + \log x) - x \log x \sin x\right] - \frac{4x}{(x^2-1)^2}$

$\Rightarrow \frac{dy}{dx} = x^{x\cos x}\left[\cos x(1+\log x) - x\log x \sin x\right] - \frac{4x}{(x^2-1)^2}$ Ans

(7) $(\log x)^x + (x)^{\log x}$

Sol. let, $y = (\log x)^x + (x)^{\log x}$

and $u = (\log x)^x$.

$\Rightarrow \log u = \log(\log x)^x$

$\Rightarrow \log u = x \cdot \log \log x$.

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx} x \cdot \log\log x$.

$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = x \cdot \frac{d}{dx}\log\log x + \log\log x \frac{d}{dx} x$.

$\Rightarrow \frac{du}{dx} = u\left[x \cdot \frac{1}{\log x}\cdot\frac{1}{x} + \log\log x \cdot 1\right]$

$\Rightarrow \frac{du}{dx} = (\log x)^x\left[\frac{1}{\log x} + \log\log x\right] \longrightarrow (1)$

Now, let $v = (x)^{\log x}$.

$\Rightarrow \log v = \log (x)^{\log x}$.

$\Rightarrow \log v = \log x \cdot \log x$.

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\log v = \frac{d}{dx}\log x \cdot \log x$.

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \log x \cdot \frac{d}{dx}\log x + \log x \frac{d}{dx}\log x$$

$$\Rightarrow \frac{dv}{dx} = v\left(\frac{\log x}{x} + \frac{\log x}{x}\right)$$

$$\Rightarrow \frac{dv}{dx} = (x)^{\log x}\left(\frac{2\log x}{x}\right) \longrightarrow (2)$$

Now,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

from (1) and (2),

$$\frac{dy}{dx} = (\log x)^{x}\left[\frac{1}{\log x} + \log\log x\right] + (x)^{\log x}\left(\frac{2\log x}{x}\right) \quad \text{Ans}$$

(8) $(\sin x)^{x} + \sin^{-1}\sqrt{x}$

Sol. let $y = \sin x^{x} + \sin^{-1}\sqrt{x}$

and $u = \sin x^{x}$

$\Rightarrow \log u = \log \sin x^{x}$

$\Rightarrow \log u = x \log \sin x$

D.w.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx} x \cdot \log \sin x$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x \cdot \frac{1}{\sin x} \cdot \cos x + \log \sin x \cdot 1\right]$$

$\Rightarrow \frac{du}{dx} = (\sin x)^x \left(x\cot x + \log \sin x \right) \longrightarrow (1)$

Now, let $v = \sin^{-1}\sqrt{x}$

D.w.r to "x" both side,

$\Rightarrow \frac{dv}{dx} = \frac{d}{dx} \sin^{-1}\sqrt{x}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{d}{dx}\sqrt{x}$

$\Rightarrow \frac{dv}{dx} = \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{x}} \longrightarrow (2)$

Now,

$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

from (1) and (2),

$\frac{dy}{dx} = (\sin x)^x (x\cot x + \log \sin x) + \frac{1}{\sqrt{1-x^2}} \cdot \frac{1}{2\sqrt{x}}$

Ans.

(5) $(x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$.

Sol. let $y = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

~~D.w.r.~~ Taking log both side,

$\Rightarrow \log y = \log (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4$

$\Rightarrow \log y = \log (x+3)^2 + \log (x+4)^3 + \log (x+5)^4$.

$\Rightarrow \log y = 2\log (x+3) + 3\log (x+4) + 4\log (x+5)$

D.w.r. to "x" by both side,

$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} 2\log (x+3) + \frac{d}{dx} 3\log (x+4) + \frac{d}{dx} \log (x+5$

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$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \left(2 \cdot \frac{1}{(x+3)} \cdot 1\right) + \left(3 \cdot \frac{1}{(x+4)} \cdot 1\right) + \left(4 \cdot \frac{1}{(x+5)} \cdot 1\right)$$

$$\Rightarrow \frac{dy}{dx} = y\left[\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2 \cdot (x+4)^3 \cdot (x+5)^4 \left(\frac{2}{x+3} + \frac{3}{x+4} + \frac{4}{x+5}\right)$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^2 (x+4)^3 (x+5)^4 \left(\frac{2(x+4) + 3(x+3)}{(x+3)(x+4)} + \frac{4}{(x+5)}\right) \text{ Ans}$$

$$\Rightarrow \frac{dy}{dx} = (x+3)^{\cancel{2}} (x+4)^{\cancel{3}\,2} (x+5)^{\cancel{4}\,3} \left(\frac{(2x+8+3x+9)(x+5) + 4\{(x+3)(x+4)\}}{\cancel{(x+3)}\ \cancel{(x+4)}\ \cancel{(x+5)}}\right)$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \left\{\frac{(5x+17)(x+5) + 4(x^2+7x+12)}{1}\right\}$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3 \left[5x^2+25x+17x+85+4x^2+28x+(12)4\right]$$

$$\Rightarrow \frac{dy}{dx} = (x+3)(x+4)^2(x+5)^3\left(9x^2+70x+133\right) \text{ Ans}$$

(6) $\left(x+\frac{1}{x}\right)^x + (x)^{\left(1+\frac{1}{x}\right)}$

Sol let $y = \left(x+\frac{1}{x}\right)^x + (x)^{\left(1+\frac{1}{x}\right)}$

and $u = \left(x+\frac{1}{x}\right)^x$.

$$\Rightarrow \log u = \log\left(x+\frac{1}{x}\right)^x$$

$$\Rightarrow \log u = x \cdot \log\left(x+\frac{1}{x}\right)$$

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log y = \frac{d}{dx} x \log\left(x+\frac{1}{x}\right)$

$\Rightarrow \frac{1}{y} \cdot \frac{dy}{dx} = x \frac{d}{dx} \log\left(x+\frac{1}{x}\right) + \log\left(x+\frac{1}{x}\right) \frac{d}{dx}(x)$

$\Rightarrow \frac{dy}{dx} = y\left[x \cdot \frac{1}{\left(1+\frac{1}{x}\right)} \cdot \left(\frac{-1}{x^2}\right) + \log\left(1+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{-x}{\left(\frac{x+1}{x}\right) x^2} + \log\left(1+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{-x}{\frac{1}{x}\left(x^3+x^2\right)} + \log\left(1+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{-x^2}{x^3+x^2} + \log\left(1+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\frac{-1}{(x+1)} + \log\left(1+\frac{1}{x}\right)\right]$

$\Rightarrow \frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x \left[\log\left(1+\frac{1}{x}\right) - \frac{1}{(x+1)}\right]$

Now,

let $v = (x)^{\left(1+\frac{1}{x}\right)}$

$\Rightarrow \log v = \left(\cancel{x}1+\frac{1}{x}\right) \log x.$

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx} \left(\cancel{x}1+\frac{1}{x}\right) \log x.$

$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = \left(x+\frac{1}{x}\right) \cdot \frac{d}{dx} \log x + \log x \cdot \frac{d}{dx}\left(\cancel{x}1+\frac{1}{x}\right)$

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$$\Rightarrow \frac{dv}{dx} = v\left[\left(1+\frac{1}{x}\right)\cdot\frac{1}{x} + \log x \cdot \left(\frac{-1}{x^2}\right)\right]$$

$$\Rightarrow \frac{dv}{dx} = (x)^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1}{x^2} - \frac{\log x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x)^{\left(1+\frac{1}{x}\right)}\left[\frac{x+1-\log x}{x^2}\right] \longrightarrow (2)$$

Now again

$$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx} x\log\left(x+\frac{1}{x}\right)$$

$$\Rightarrow \frac{1}{u}\cdot\frac{du}{dx} = x\frac{d}{dx}\log\left(x+\frac{1}{x}\right) + \log\left(x+\frac{1}{x}\right)\frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x\cdot\frac{1}{\left(x+\frac{1}{x}\right)}\cdot\left\{1+\left(\frac{-1}{x^2}\right)\right\} + \log\left(x+\frac{1}{x}\right)\cdot 1\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x}{x+\frac{1}{x}}\cdot\left(1-\frac{1}{x^2}\right) + \log\left(x+\frac{1}{x}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{\left(x-\frac{1}{x}\right)}{\left(x+\frac{1}{x}\right)} + \log\left(x+\frac{1}{x}\right)\right]$$

$$\Rightarrow \frac{du}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{\frac{x^2-1}{\cancel{x}}}{\frac{x^2+1}{\cancel{x}}} + \log\left(x+\frac{1}{x}\right)\right]$$

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^x\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] \longrightarrow (1)$$

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Now,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

from ① and ②,

$$\frac{dy}{dx} = \left(x+\frac{1}{x}\right)^{x}\left[\frac{x^2-1}{x^2+1} + \log\left(x+\frac{1}{x}\right)\right] + (x)^{\left(1+\frac{1}{x}\right)}\left(\frac{x+1-\log x}{x^2}\right)$$

Ans.

11) $(x\cos x)^{x} + (x\sin x)^{1/x}$

Sol. let $y = (x\cos x)^{x} + (x\sin x)^{1/x}$.

and $u = (x\cos x)^{x}$

$\Rightarrow$ ~~$\log u = x\log x\cos x$~~

~~D.w.r.t. to "x" both side,~~

$\Rightarrow$ ~~$\frac{1}{u}\cdot\frac{du}{dx} = x\cdot\frac{d}{dx}\log x\cos x + \log x\cos x\frac{d}{dx}(x)$~~

$\Rightarrow$ ~~$\frac{du}{dx} = u\left[x\left\{\log x\cdot(-\sin x) + \frac{\cos x}{x}\right\} + \log x\cos x \cdot 1\right]$~~

$\Rightarrow$ ~~$\frac{du}{dx} = (x\cos x)^{x}\left[-x\log x\sin x + \cos x + \log x\cos x\right] \longrightarrow ①$~~

Now let $v = (x\sin x)^{1/x}$.

$\Rightarrow \log v = \frac{1}{x}\log x\sin x$

D.w.r.x to "x" both side,

$\Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \frac{1}{x}\cdot\frac{d}{dx}\log x\sin x + \log x\sin x\frac{d}{dx}\frac{1}{x}$

$\Rightarrow \frac{dv}{dx} = v\left[\frac{1}{x}\cdot\frac{1}{x\sin x}\,\cancel{\cos x}\,\frac{d}{dx}x\sin x + \log x\sin x\left(\frac{-1}{x^2}\right)\right]$

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$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/x}\left[\frac{1}{x^2\sin x}\left(x\cos x+\sin x\right) - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x\cos x+\sin x}{x^2\sin x} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x\cos x}{x^2\sin x} + \frac{\sin x}{x^2\sin x} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{\cot x}{x} + \frac{1}{x^2} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x^2\cot x + x}{x^3} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x^2\cot x}{x^3} + \frac{x}{x^3} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x\cot x + 1}{x^2} - \frac{\log x\sin x}{x^2}\right]$$

$$\Rightarrow \frac{dv}{dx} = (x\sin x)^{1/2}\left[\frac{x\cot x + 1 - \log x\sin x}{x^2}\right]$$

and $u = (x\cos x)^x$

$\Rightarrow \log u = x\log x\cos x,$

$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx}\{(x)(\log x\cos x)\}$

$\Rightarrow \cancel{\frac{1}{u}\cdot\frac{du}{dx} = x\cdot\frac{d}{dx}\log x\cos x + \log x\cos x\frac{d}{dx}(x)}$

$= x\cdot\left[\log x\frac{d}{dx}\cos x + \cos x\frac{d}{dx}\log x\right] + \log x\cos x$

$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \cdot \frac{d}{dx} \log\cos x + \log\cos x \frac{d}{dx}(x)$

$= x \cdot \frac{1}{\cos x} \frac{d}{dx} \cos x + \log\cos x \cdot 1$

$= -\frac{\sin x}{\cos x} + \log\cos x$

And $u = (x\cos x)^x$

$\Rightarrow \log u = \log(x\cos x)^x$

$\Rightarrow \log u = x \log(x\cos x)$

D. w.r.t. to "x",

$$\Rightarrow \frac{d}{dx}(\log u) = \frac{d}{dx}\left[x \log(x\cos x)\right]$$

$$\Rightarrow \frac{1}{u} \cdot \frac{du}{dx} = x \frac{d}{dx} \log(x\cos x) + \log(x\cos x) \frac{d}{dx}(x)$$

$$\Rightarrow \frac{du}{dx} = u\left[x \cdot \frac{1}{x\cos x} \cdot \frac{d}{dx}(x\cos x) + \log(x\cos x) \cdot 1\right]$$

$$= u\left[\frac{1}{\cos x}\left(-x\sin x + \cos x\right) + \log(x\cos x)\right]$$

$$= u\left(-\frac{x\sin x}{\cos x} + \frac{\cos x}{\cos x} + \log(x\cos x)\right)$$

$$= u(-x\tan x + 1 + \log(x\cos x))$$

$$\frac{du}{dx} = u(1 - x\tan x + \log(x\cos x)).$$

Now,

$$\frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$$

$$= u\left[1 - x\tan x + \log(x\cos x)\right] + (x\sin x)^{\frac{1}{2}}\left[\frac{x\cot x + 1}{\ldots} - \frac{\log x\sin x}{x^2}\right]$$

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$\Rightarrow \frac{dy}{dx} = (x\cos x)^x \left(1 - x\tan x + \cancel{\log\cos x} + \log(x\cos x)\right) + (x\sin x)^{1/2} \left(\frac{x\cot x + 1 - \log x\sin x}{x^2}\right)$ Ans.

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(12) $x^y + y^x = 1$.

Sol. D. w.r. to "x",

$\Rightarrow \frac{d}{dx}\{x^y + y^x\} = \frac{d}{dx}(1)$

$\Rightarrow \frac{d}{dx}x^y + \frac{d}{dx}y^x = 0$.

$\Rightarrow$ let, $u = x^y$

$\Rightarrow \log u = \log x^y$

D. w.r. to "x"

$\Rightarrow \frac{d}{dx}\log u = \frac{d}{dx}\log x^y$

$\Rightarrow \frac{1}{u}\cdot\frac{dy}{dx} = \frac{d}{dx}(y\log x)$

$\Rightarrow \frac{dy}{dx} = u\left[y\frac{d}{dx}\log x + \log x\frac{d}{dx}(y)\right]$

$= u\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$

$= x^y\left(\frac{y}{x} + \log x\frac{dy}{dx}\right)$

$= \frac{yx^y}{x} + x^y\log x\frac{dy}{dx}$

$\Rightarrow \frac{dy}{dx} = yx^{y-1} + x^y\log x\frac{dy}{dx}$

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let $v = y^x$

$\Rightarrow \log v = \log y^x$

$\Rightarrow \log v = x \log y$

D.w.r. to x

$\Rightarrow \frac{d}{dx} \log v = \frac{d}{dx}(x \log y)$

$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = x \frac{d}{dx}(\log y) + \log y \frac{d}{dx}(x)$

$\Rightarrow \frac{dv}{dx} = v \left[\frac{x}{y} \cdot \frac{dy}{dx} + \log y \right]$

$\Rightarrow \frac{dv}{dx} = y^x \left(\frac{x}{y} \frac{dy}{dx} + \log y \right)$

$\Rightarrow \frac{dv}{dx} = \frac{xy^x}{y} \frac{dy}{dx} + \log y \cdot y^x$

Now,

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx} = 0$

$\Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$

$\Rightarrow \left(yx^{y-1} + x^y \log x \frac{dy}{dx} \right) + \left(xy^{x-1} \frac{dy}{dx} + y^x \log y \right) = 0$

$\Rightarrow yx^{y-1} + x^y \log x \frac{dy}{dx} + xy^{x-1} \frac{dy}{dx} + y^x \log y = 0$

$\Rightarrow x^y \log x \frac{dy}{dx} + xy^{x-1} \frac{dy}{dx} = -y^x \log y - yx^{y-1}$

$\Rightarrow \frac{dy}{dx} \left(x^y \log x + xy^{x-1} \right) = -\left[y^x \log y + yx^{y-1} \right]$

$\Rightarrow \frac{dy}{dx} = \frac{-\left[y^x \log y + yx^{y-1} \right]}{x^y \log x + xy^{x-1}}$ Ans.

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Symbol of differentiation :-

y	y	$f(x)$	y
$\Rightarrow \frac{dy}{dx}$	y_1	$f'(x)$	$D'(y)$
$\Rightarrow \frac{d^2y}{dx^2}$	y_2	$f''(x)$	$D^2(y)$
$\Rightarrow \frac{d^3y}{dx^3}$	y_3	$f'''(x)$	$D^3(y)$
$\Rightarrow \frac{d^4y}{dx^4}$	y_4	$f''''(x)$	$D^4(y)$
$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\vdots$	$\vdots$	$\vdots$	$\vdots$

Exercise 5.7

(1) $y = x^2+3x+2$

D.W.r. to "x" Both side

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^2+3x+2)$$

$$\Rightarrow \frac{dy}{dx} = 2x+3.$$

Again D.W.r. to "x"

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}(2x+3)$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2\times1+0$$

$$\Rightarrow \frac{d^2y}{dx^2} = 2 \quad \text{Ans.}$$

(6) $e^x \sin 5x = y$

Sol.

$y = e^x \sin 5x$

D.W.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^x \sin 5x)$$

$$= \sin 5x \cdot e^x \cdot \cos 5x \cdot \frac{d}{dx}(5x)$$

$$= e^x \sin 5x + e^x \cos 5x \cdot 5$$

$$= e^x(\sin 5x + 5\cos 5x)$$

Again D.W.r. to "x"

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$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left\{e^x(\sin 5x + 5\cos 5x)\right\}$$

$$= \frac{d}{dx}\left(5e^x\cos 5x + e^x\sin 5x\right)$$

$$= \frac{d}{dx}\left(5e^x\cos 5x\right) + \frac{d}{dx}\left(e^x\sin 5x\right)$$

$$= 5\left(e^x\frac{d}{dx}\cos 5x + \cos 5x\frac{d}{dx}(e^x)\right) + \left(e^x\frac{d}{dx}\sin 5x + \sin 5x\frac{d}{dx}e^x\right)$$

$$= 5\left(e^x.(-\sin 5x).5 + \cos 5x.e^x\right) + \left(e^x.\cos 5x.5 + \sin 5x.e^x\right)$$

$$\cancel{= 5\left(-5e^x\sin 5x + e^x\cos 5x\right) + \left(5e^x\cos 5x + e^x\sin 5x\right)}$$

$$\cancel{= 5\left\{e^x(-5\sin 5x + \cos 5x) + e^x(5\cos 5x + \sin 5x)\right.}$$

$$\cancel{= 5\left[e^x\left\{-5\sin 5x + \cos 5x + 5\cos 5x + \sin 5x\right\}\right]}$$

$$\cancel{= 5\left[e^x\left\{6\cos 5x + 4\sin 5x\right\}\right]}$$

$$= \left(-25e^x\sin 5x + 5e^x\cos 5x\right) + \left(e^x\cos 5x.5 + e^x\sin 5x\right)$$

$$= e^x\left(-25\sin 5x + 5\cos 5x\right) + e^x\left(5\cos 5x + \sin 5x\right)$$

$$= e^x\left[5\cos 5x - 25\sin 5x + 5\cos 5x + \sin 5x\right]$$

$$= e^x\left[10\cos 5x - 24\sin 5x\right]$$

$$= 2e^x\left(5\cos 5x - 12\sin 5x\right) \quad \underline{\text{Ans}}$$

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(10) $\sin(\log x) = y$

Sol. D. w.r. to "x",

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\{\sin(\log x)\}$$

$$= \cos(\log x) \cdot \frac{1}{x}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos(\log x)}{x}.$$

Again D.w.r. to "x",

$$\Rightarrow \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left\{\frac{\cos(\log x)}{x}\right\}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{x \cdot \frac{d}{dx}\cos(\log x) - \cos(\log x)\frac{d}{dx}(x)}{x^2}$$

$$= \frac{x \cdot (-\sin(\log x)) \cdot \frac{1}{x} - \cos(\log x) \cdot 1}{x^2}$$

$$= \frac{-\sin(\log x) - \cos(\log x)}{x^2}$$

$$= -\frac{[\sin(\log x) + \cos(\log x)]}{x^2}$$ Ans.

(2) $x^{20} = y$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^{20})$$

$$\Rightarrow \frac{dy}{dx} = 20x^{19}$$

Again D.w.r. to "x"

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(20x^{19})$$

$$\Rightarrow \frac{d^2y}{dx^2} = 20\left[19x^{18}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = 380x^{18}$$

(3) $y = x.\cos x$

Sol D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x.\cos x)$$

$$= x\frac{d}{dx}(\cos x) + \cos x \frac{d}{dx}(x)$$

$$= -x\sin x + \cos x.$$

$$= \cos x - x\sin x,$$

Again D.w.r. to "x",

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\{\cos x - x\sin x\}$$

$$= \frac{d}{dx}\cos x - \left[\frac{d}{dx}\{x\sin x\}\right]$$

$$= -\sin x - \left[x\frac{d}{dx}\sin x + \sin x\frac{d}{dx}(x)\right]$$

$$= -\sin x - \left[x\cos x + \sin x\right]$$

$$= -\sin x - x\cos x - \sin x.$$

$$= -2\sin x - x\cos x.$$

(4) $\log x = y$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\log x)$$

$$= \frac{1}{x}.$$

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Again D.W.r. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x}\right)$

$= \frac{d}{dx}(x^{-1})$

$= -x^{-1-1}$

$= -x^{-2}$

$\frac{d^2y}{dx^2} = -\frac{1}{x^2}$

(5) $y = x^3 \cdot \log x$.

Sol. D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(x^3 \cdot \log x)$

$= x^3 \frac{d}{dx}(\log x) + \log x \frac{d}{dx}(x^3)$

$= x^3 \cdot \frac{1}{x} + \log x \cdot 3x^2$.

$= x^2 + 3x^2 \log x$.

Again D.W.r. to "x"

$\frac{d^2y}{dx^2} = \frac{d}{dx}(x^2 + 3x^2 \log x)$

$= \frac{d}{dx}(x^2) + 3\frac{d}{dx}(x^2 \log x)$

$= 2x + 3\left[x^2 \cdot \frac{1}{x} + \log x \cdot 2x\right]$

$= 2x + 3(x + 2x \log x)$

$= 2x + 3x + 6x \log x$.

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$= 5x + 6x \log x$

$= x(5 + 6\log x)$ Ans.

(7) $y = e^{6x}\cos 3x$.

D.w.r.t. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(e^{6x}\cos 3x)$

$= e^{6x}\frac{d}{dx}\cos 3x + \cos 3x \frac{d}{dx}e^{6x}$

$= e^{6x}(-\sin 3x)\frac{d}{dx}(3x) + \cos 3x \cdot e^{6x}\cdot 6$

$= -e^{6x}\sin 3x \cdot 3 + 6\cos 3x \cdot e^{6x}$

$= -3e^{6x}\sin 3x + 6e^{6x}\cos 3x$

Again D.w.r.t. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(-3e^{6x}\sin 3x + 6e^{6x}\cos 3x)$

$= \frac{d}{dx}(-3e^{6x}\sin 3x) + \frac{d}{dx}(6e^{6x}\cos 3x)$

$= -3\left[e^{6x}\frac{d}{dx}\sin 3x + \sin 3x\frac{d}{dx}e^{6x}\right] + 6\left[e^{6x}\frac{d}{dx}\cos 3x + \cos 3x\frac{d}{dx}e^{6x}\right]$

$= -3\left[e^{6x}\cos 3x \cdot 3 + \sin 3x\, e^{6x}\cdot 6\right] + 6\left[e^{6x}(-\sin 3x)\cdot 3 + \cos 3x\, e^{6x}\cdot 6\right]$

$= -3(3e^{6x}\cos 3x + 6e^{6x}\sin 3x) + 6(-3e^{6x}\sin 3x + 6e^{6x}\cos 3x)$

$= -9e^{6x}\cos 3x - 18e^{6x}\sin 3x + (-18e^{6x}\sin 3x + 36e^{6x}\cos 3x)$

$= -9e^{6x}\cos 3x - 18e^{6x}\sin 3x - 18e^{6x}\sin 3x + 36e^{6x}\cos 3x$

$= e^{6x}27\cos 3x - 36e^{6x}\sin 3x$

$\Rightarrow \frac{d^2y}{dx^2} = 9e^{6x}(3\cos 3x - 4\sin 3x)$ Ans.

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(8) $y = \tan^{-1}x$.

D.W.R. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)$

$\Rightarrow \frac{dy}{dx} = \frac{1}{1+x^2}$

Again D.W.R. to "x",

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{1+x^2}\right)$

$= \frac{(1+x^2)\frac{d}{dx}(1) - 1\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$= \frac{(1+x^2)(0) - 2x}{(1+x^2)^2}$

$= \frac{0-2x}{(1+x^2)^2}$

$\boxed{\Rightarrow \frac{d^2y}{dx^2} = \frac{-2x}{(1+x^2)^2}}$ Ans.

Again D.W.R. to "x"

$\Rightarrow \frac{d^3y}{dx^3} = \frac{d}{dx} \frac{-2x}{(1+x^2)^2}$

$= -1\left[\frac{(1+x^2)^2\frac{d}{dx}(2x) - (2x)\frac{d}{dx}(1+x^2)^2}{((1+x^2)^2)^2}\right]$

$= -\left[\frac{(1+x^2)^2 \cdot 2 - 2x \cdot 2(1+x^2) \cdot 2x}{(1+x^2)^4}\right]$

$\Rightarrow \frac{d^3y}{dx^3} = -\left[\frac{2(1+x^2)^2 - 8x^2(1+x^2)}{(1+x^2)^4}\right]$ Ans.

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(i) $y = \log(\log x)$

D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log(\log x)\right]$

$= \frac{1}{\log x} \cdot \frac{1}{x}$

$\Rightarrow \frac{dy}{dx} = \frac{1}{x\log x}$

Again D.w.r. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{1}{x\log x}\right)$

$= \frac{x\log x \frac{d}{dx}(1) - 1\frac{d}{dx}(x\log x)}{(x\log x)^2}$

$= \frac{x\log x(0) - \left[x\frac{d}{dx}\log x + \log x\frac{d}{dx}(x)\right]}{(x\log x)^2}$

$= \frac{-\left[x \cdot \frac{1}{x} + \log x\right]}{(x\log x)^2}$

$= \frac{-1-\log x}{(x\log x)^2}$ Ans.

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(13) If $y = 3\cos(\log x) + 4\sin(\log x)$, show that $x^2y_2 + xy_1 + y = 0$

Sol. Given,

$\Rightarrow \quad y = 3\cos(\log x) + 4\sin(\log x)$

D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\{3\cos(\log x) + 4\sin(\log x)\}$

$\Rightarrow y_1 = 3\frac{d}{dx}\cos(\log x) + 4\frac{d}{dx}\sin(\log x)$

$\Rightarrow y_1 = 3\left[-\sin(\log x)\frac{d}{dx}\log x\right] + 4\cos(\log x)\frac{d}{dx}\log x$

$\Rightarrow y_1 = -\frac{3\sin(\log x)}{x} + 4\frac{\cos(\log x)}{x}$

$\Rightarrow y_1 = \frac{1}{x}\left[-3\sin(\log x) + 4\cos(\log x)\right]$

$\Rightarrow xy_1 = -3\sin(\log x) + 4\cos(\log x)$

Again D.W.r. to "x"

$\Rightarrow \frac{d}{dx}(x \cdot y_1) = \frac{d}{dx}\left[-3\sin(\log x) + 4\cos(\log x)\right]$

$\Rightarrow x\frac{d}{dx}y_1 + y_1 * \frac{d}{dx}(x) = -3\frac{d}{dx}\sin(\log x) + 4\frac{d}{dx}\cos(\log x)$

$\Rightarrow xy_2 + y_1 \cdot 1 = -3\frac{\cos(\log x)}{x} + 4\left\{-\frac{\sin(\log x)}{x}\right\}$

$\Rightarrow xy_2 + y_1 = -\frac{3\cos(\log x)}{x} - \frac{4\sin(\log x)}{x}$

$\Rightarrow xy_2 + y_1 = -\frac{1}{x}\left(+3\cos(\log x) + 4\sin(\log x)\right)$

$\Rightarrow x^2y_2 + xy_1 = -\left(3\cos(\log x) + 4\sin(\log x)\right)$

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$\Rightarrow x^2y_2 + xy_1 = -y$

$\Rightarrow x^2y_2 + xy_1 + y = 0$

Proved.

(17) If $y = (\tan^{-1}x)^2$, show that $(x^2+1)^2y_2 + 2x(x^2+1)y_1 = 2$

Sol.

Given.

$$y = (\tan^{-1}x)^2 \longrightarrow (1)$$

D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)^2$

$\Rightarrow y_1 = 2\tan^{-1}x \frac{d}{dx}\tan^{-1}x$

$\Rightarrow y_1 = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$

$\Rightarrow y_1 = \frac{2\tan^{-1}x}{1+x^2} \longrightarrow (2)$

$\Rightarrow (1+x^2)y_1 = 2\tan^{-1}x.$

Again D.W.r. to "x" both side, we have

$\Rightarrow \frac{d}{dx}[(1+x^2)y_1] = \frac{d}{dx}2\tan^{-1}x$

$\Rightarrow (1+x^2)\frac{d}{dx}y_1 + y_1\frac{d}{dx}(1+x^2) = 2 \cdot \frac{1}{1+x^2}$

$\Rightarrow (1+x^2)y_2 + y_1 2x = \frac{2}{1+x^2}$

$\Rightarrow (1+x^2)^2y_2 + 2xy_1(1+x^2) = 2$

$\Rightarrow (1+x^2)^2y_2 + 2x(1+x^2)y_1 = 2.$

Proved.

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~~Again restart~~

(17) ~~$y = \tan$~~

Second Method :- $y = (\tan^{-1}x)^2$

D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\tan^{-1}x)^2$

$\Rightarrow y_1 = 2\tan^{-1}x \cdot \frac{1}{1+x^2}$

$\Rightarrow y_1 = \left(\frac{2\tan^{-1}x}{1+x^2}\right) \longrightarrow (1)$

Again D.W.r. to "x" both side,

$\Rightarrow \frac{dy_1}{dx} = \frac{d}{dx}\left(\frac{2\tan^{-1}x}{1+x^2}\right)$

$\Rightarrow y_2 = \frac{(1+x^2)\frac{d}{dx}(2\tan^{-1}x) - (2\tan^{-1}x)\frac{d}{dx}(1+x^2)}{(1+x^2)^2}$

$\Rightarrow (1+x^2)^2 y_2 = 2(1+x^2)\frac{1}{1+x^2} - 2\tan^{-1}x \cdot 2x$

$\Rightarrow y_2 = \frac{\frac{2(1+x^2)}{(1+x^2)} - 2\tan^{-1}x \cdot 2x}{(1+x^2)^2}$

$\Rightarrow y_2 = \frac{2 - 2\tan^{-1}x \cdot 2x}{(1+x^2)^2} \longrightarrow (2)$

~~Putting value of y_2, y_1,~~

Now,

$\Rightarrow (x^2+1)^2 y_2 + 2x(x^2+1)y_1 = 2$

then, L.H.S $= (x^2+1)^2 y_2 + 2x(x^2+1)$

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putting value of y_1 and y_2

$$\cancel{(x^2+1)^2}\left[\frac{2-2\tan^{-1}x\cdot 2x}{\cancel{(1+x^2)^2}}\right]+2x\cancel{(x^2+1)}\left[\frac{2\tan^{-1}x}{\cancel{1+x^2}}\right]$$

$$= 2-\cancel{2\tan^{-1}x\, 2x}+\cancel{2x\cdot 2\tan^{-1}x}$$

$$= 2$$

(14) If $y = Ae^{mx} + Be^{nx}$, show that $\frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny = 0$

Sol. Given,

$$y = Ae^{mx} + Be^{nx}$$

D.W.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}Ae^{mx} + \frac{d}{dx}Be^{nx}$$

$\cancel{\text{let } u = Ae^{mx}}$

$\Rightarrow \cancel{\log u = \log Ae^{mx}}$

$\Rightarrow \cancel{\log u = mx\log Ae}$

$$\Rightarrow \frac{dy}{dx} = A\frac{d}{dx}e^{mx} + B\frac{d}{dx}e^{nx}$$

$$\cancel{= Ae^{mx}\frac{d}{dx}(mx) + Be^{nx}\frac{d}{dx}nx}$$

$$\cancel{= mAe^{mx} + nBe^{nx}}$$

$$= Ae^{mx}\frac{d}{dx}(mx) + Be^{nx}\frac{d}{dx}(nx)$$

$$= mAe^{mx} + nBe^{nx}$$

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Again D.W.r. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left(Ame^{mx} + Bne^{nx}\right)$

$= \frac{d}{dx}(Ame^{mx}) + \frac{d}{dx}(Bne^{nx})$

$= Am\frac{d}{dx}e^{mx} + Bn\frac{d}{dx}e^{nx}$

$= Ame^{mx}\frac{d}{dx}(mx) + Bne^{nx}\frac{d}{dx}(nx)$

$\Rightarrow \frac{d^2y}{dx^2} = Am^2e^{mx} + Bn^2e^{nx}$

Now, L.H.S. $= \frac{d^2y}{dx^2} - (m+n)\frac{dy}{dx} + mny$

$= Am^2e^{mx} + Bn^2e^{nx} - (m+n)(Ame^{mx} + Bne^{nx}) + mn(Ae^{mx} + Be^{nx})$

$= \cancel{Am^2e^{mx}} + \cancel{Bn^2e^{nx}} - \cancel{Am^2e^{mx}} - \cancel{Bmne^{nx}} - \cancel{Amne^{mx}} - \cancel{Bn^2e^{nx}} + \cancel{Amne^{mx}} + \cancel{Bmne^{nx}}$

$= 0$

Proved.

(11) If $y = 5\cos x - 3\sin x$, Prove that $\frac{d^2y}{dx^2} + y = 0$

Sol. Given,

$y = 5\cos x - 3\sin x$

D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}5\cos x - \frac{d}{dx}3\sin x$

$\Rightarrow y_1 = -5\sin x - 3\cos x$

$\Rightarrow y_1 = -(5\sin x + 3\cos x)$

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Again D.w.r. to "x"

$$\Rightarrow \frac{d y_1}{dx} = -\frac{d}{dx}(5\sin x + 3\cos x)$$

$$\Rightarrow \frac{d y_1}{dx} = -\left[\frac{d}{dx} 5\sin x + 3\frac{d}{dx}\cos x\right]$$

$$\Rightarrow \frac{d y_1}{dx} = -\left[5\cos x - 3\sin x\right]$$

$$\Rightarrow \frac{d^2 y}{dx^2} = -5\cos x + 3\sin x$$

Now, $\quad L.H.S. = \frac{d^2y}{dx^2} + y$

$$= (-5\cos x + 3\sin x) + (5\cos x - 3\sin x)$$

$$= 0$$

L.H.S = R.H.S

Proved.

(12) If $y = \cos^{-1}x$, find $\frac{d^2y}{dx^2}$ in terms of y alone

Sol. Given, $y = \cos^{-1}x$

D.W.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\cos^{-1}x$$

$$\frac{dy}{dx} = \frac{-1}{\sqrt{1-x^2}}$$

$$\left[\because \cos^{-1}x = y \Rightarrow x = \cancel{\cos^{-1}y}\ \cos y\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{(1-x^2)^{1/2}} = -(1-x^2)^{-1/2}$$

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Again D. w.r. to "x"

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\left\{-(1-x^2)^{-1/2}\right\}$$

$$= -\left(-\frac{1}{2}\right)(1-x^2)^{-1/2-1} \cdot \frac{d}{dx}(1-x^2)$$

$$= \frac{1}{2}(1-x^2)^{-3/2}(-2x)$$

$$= \frac{(1-x^2)^{-3/2}(-\cancel{2}x)}{\cancel{2}}$$

$$= \frac{-x}{(1-x^2)^{3/2}}$$

$$\frac{d^2y}{dx^2} = \frac{-x}{\sqrt{(1-x^2)^3}}$$

Now, $x = \cos y \quad \{\because y = \cos^{-1}x\}$

$$\frac{dy^2}{dx^2} = \frac{-\cos y}{\sqrt{(1-\cos^2 y)^3}}$$

$$= \frac{-\cos y}{\sqrt{(\sin^2 y)^3}}$$

$$= \frac{-\cos y}{\sqrt{(\sin y \cdot \sin y)^3}}$$

$$= \frac{-\cos y}{\sin^3 y}$$

$$= -\frac{\cos y}{\sin y} \cdot \frac{1}{\sin^2 y}$$

$$\frac{d^2y}{dx^2} = -\cot y \cdot \operatorname{cosec}^2 y$$ Ans.

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(15) If $y = 500e^{7x} + 600e^{-7x}$, show that $\cancel{\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2}$

$$\frac{d^2y}{dx^2} = 49y$$

Sol Given, $y = 500e^{7x} + 600e^{-7x}$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\{500e^{7x} + 600e^{-7x}\}$$

$$= 500\frac{d}{dx}e^{7x} + 600\frac{d}{dx}e^{-7x}$$

$$= 7(500e^{7x}) + (600e^{-7x})(-7)$$

$$= 3500e^{7x} - 4200e^{-7x}$$

Again D.w.r. to "x"

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\{3500e^{7x} - 4200e^{-7x}\}$$

$$\Rightarrow y_2 = 3500\frac{d}{dx}e^{7x} - 4200\frac{d}{dx}e^{-7x}$$

$$\Rightarrow y_2 = 7(3500e^{7x}) - (-7)(4200e^{-7x})$$

$$\Rightarrow \cancel{\frac{d^2y}{dx^2} = 24500e^{7x} + 29400e^{-7x}}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 7\times7\times500e^{7x} + 7\times7\times600e^{-7x}$$

$$\Rightarrow \frac{d^2y}{dx^2} = 49\times500e^{7x} + 49\times600e^{-7x}$$

$$= 49\left[500e^{7x} + 600e^{-7x}\right]$$

$$= 49y$$

Proved.

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(16) If $e^y(x+1) = 1$, show that $\frac{d^2y}{dx^2} = \left(\frac{dy}{dx}\right)^2$

Sol. Given,

$\Rightarrow e^y(x+1) = 1$

$\Rightarrow e^y = \frac{1}{(x+1)}$

$\Rightarrow \log e^y = \log\left(\frac{1}{(x+1)}\right)$

~~$\Rightarrow y \log e = \log 1 - \log(x+1)$~~

$\Rightarrow y = \log\left(\frac{1}{(x+1)}\right)$

D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\log \frac{1}{(x+1)}\right]$

$= \frac{1}{\frac{1}{(x+1)}} \cdot \frac{d}{dx}\left(\frac{1}{(x+1)}\right)$

$= (x+1)\left[\frac{(x+1)\frac{d}{dx}(1) - (1)\frac{d}{dx}(x+1)}{(x+1)^2}\right]$

$= \cancel{(x+1)}\left[\frac{0 - 1}{(x+1)\cancel{^2}}\right]$

$\frac{dy}{dx} = \frac{-1}{(x+1)}$

Again D.w.r. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\frac{-1}{(x+1)}$

$= -\left[\frac{(x+1)\frac{d}{dx}(1) - (1)\frac{d}{dx}(x+1)}{(x+1)^2}\right]$

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$$= -\left[\frac{0-1}{(x+1)^2}\right]$$

$$= -\left(\frac{-1}{(x+1)^2}\right)$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1}{(x+1)^2} \quad \text{~~Ans.~~} \longrightarrow (1)$$

Now, $= \left(\frac{dy}{dx}\right)^2$

$$= \left[\frac{-1}{(x+1)}\right]^2$$

$$= \frac{(-1)^2}{[(x+1)]^2}$$

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{1}{(x+1)^2} \longrightarrow (2)$$

from (1) and (2),

$$\Rightarrow \frac{d^2y}{dx} = \left(\frac{dy}{dx}\right)^2$$

Hence Proved.

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(12)

Sol.

$$y = \cos^{-1}x$$

$$x = \cos y$$

D.w.r. to "x"

$$\Rightarrow \frac{dx}{dx} = \frac{d}{dx}\cos y$$

$$\Rightarrow 1 = -\sin y \frac{dy}{dx}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{\sin y}$$

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$\Rightarrow \frac{dy}{dx} = -\csc y$

Again D.W.R. to "x",

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}(-\csc y)$

$= -[\csc y \cdot \cot y] \frac{dy}{dx}$

$= \csc y \cot y (-\csc y)$

$\Rightarrow \frac{d^2y}{dx^2} = -\csc^2 y \cot y$ Ans

Miscellaneous.

①

Sol. Let $y = (3x^2 - 9x + 5)^9$

D.W.R. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(3x^2 - 9x + 5)^9$

putting $t = 3x^2 - 9x + 5$

$\Rightarrow \frac{dy}{dx} = \frac{d}{dt}(t^9) \cdot \frac{dt}{dx}$

$= 9t^8 \cdot \frac{d}{dx}(3x^2 - 9x + 5)$

$= 9(3x^2 - 9x + 5)^8 [6x - 9 + 0]$

$=$

$= 9(3x^2 - 9x + 5)^8 (6x - 9)$

$= 27(3x^2 - 9x + 5)^8 (2x - 3)$

(6)

Sol. Let $y = \cot^{-1}\left[\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right]$

$\Rightarrow y = \cot^{-1}\left[\dfrac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}+\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}-\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}\right]$

$= \cot^{-1}\left[\dfrac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right]$

$= \cot^{-1}\left[\dfrac{\cos\frac{x}{2}+\sin\frac{x}{2}+\cos\frac{x}{2}-\sin\frac{x}{2}}{\cos\frac{x}{2}+\sin\frac{x}{2}-\cos\frac{x}{2}+\sin\frac{x}{2}}\right]$

$= \cot^{-1}\left[\dfrac{2\cos\frac{x}{2}}{2\sin\frac{x}{2}}\right]$

$= \cot^{-1}\left[\cot\frac{x}{2}\right]$

$y = \dfrac{x}{2}$

D.W.R. to "x"

$\Rightarrow \dfrac{dy}{dx} = \dfrac{d}{dx}\left(\dfrac{x}{2}\right)$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}\dfrac{dx}{dx}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{1}{2}$ Ans.

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(14) If $x\sqrt{1+y} + y\sqrt{1+x} = 0$ for, $-1<x<1$
Prove that $\frac{dy}{dx} = \frac{-1}{(1+x)^2}$

Sol. Given,

$\Rightarrow x\sqrt{1+y} + y\sqrt{1+x} = 0$

$\Rightarrow x\sqrt{1+y} = -y\sqrt{1+x}$

squaring both side,

$\Rightarrow (x\sqrt{1+y})^2 = (-y\sqrt{1+x})^2$

$\Rightarrow x^2(1+y) = y^2(1+x)$

$\Rightarrow x^2 + x^2y = y^2 + y^2x$

$\Rightarrow x^2 - y^2 + x^2y - y^2x = 0$

$\Rightarrow (x+y)(x-y) + xy(x-y) = 0$

$\Rightarrow (x-y)(x+y+xy) = 0$

$\Rightarrow x-y = 0$ or $x+y+xy = 0$

here,

$x-y \neq 0$ because when $x = y$ then it not possible because range of x is $-1<x<1$

Taking, $x+y+xy = 0$

$\Rightarrow y(1+x) = -x$

$\Rightarrow y = \frac{-x}{(1+x)}$

D.W.r. to "x"

eq^n ① pur ... fir $(y-b)$ common karna hai.

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$$\frac{dy}{dx} = \frac{d}{dx}\frac{-x}{(1+x)}$$

$$= -\left[\frac{(1+x)\frac{dx}{dx} - x\frac{d}{dx}(1+x)}{(1+x)^2}\right]$$

$$= -\left(\frac{(1+x) - x}{(1+x)^2}\right)$$

$$= -\left(\frac{1+x-x}{(1+x)^2}\right)$$

$$= \frac{-1}{(1+x)^2}$$ proved

(15)

Sol. $(x-a)^2 + (y-b)^2 = c^2 \longrightarrow (i)$

D.W.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}\left[(x-a)^2 + (y-b)^2\right] = \frac{d}{dx}c^2$$

$$\Rightarrow \frac{d}{dx}(x-a)^2 + \frac{d}{dx}(y-b)^2 = \cancel{2c \cdot \frac{dc}{dx}} \; 0$$

$$\Rightarrow 2(x-a)\frac{d}{dx}(x-a) + 2(y-b)\frac{d}{dx}(y-b) = \cancel{2c \cdot \frac{dc}{dx}} \; 0$$

$$\Rightarrow 2(x-a)(1-0) + 2(y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow \cancel{2x-2a}\;(x-a) + (y-b)\frac{dy}{dx} = 0 \longrightarrow (2)$$

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Again D.W.R. to "x"

$\Rightarrow \frac{d}{dx}\left[(x-a) + (y-b)\frac{dy}{dx}\right] = \frac{d}{dx}(0)$

$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \frac{dy}{dx}\frac{d}{dx}(y-b) = 0$

$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0$

$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0$

~~from eq^n ②,~~

$\Rightarrow 1 + \cancel{(y-b)}\left(\frac{dy}{dx}\right)^2 = -\left\{(y-b)\frac{d^2y}{dx^2}\right\} \longrightarrow$ ③

from eq^n ②,

$\Rightarrow \cancel{(y-b)}\,(x-a) + (y-b)\frac{dy}{dx} = 0$

$\Rightarrow (x-a) = -(y-b)\frac{dy}{dx} \longrightarrow$ ④

from eq^n ① & ④,

$\Rightarrow \left(-(y-b)\frac{dy}{dx}\right)^2 + (y-b)^2 = c^2$

$\Rightarrow (y-b)^2\left(\frac{dy}{dx}\right)^2 + (y-b)^2 = c^2$

$\Rightarrow (y-b)^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = c^2$

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$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = \frac{c^2}{(y-b)^2} \longrightarrow (5)$$

from eqn (5) and (3),

$$\Rightarrow \frac{c^2}{(y-b)^2} = -\left\{(y-b)\frac{d^2y}{dx^2}\right\}$$

$$\Rightarrow \frac{c^2}{(y-b)^2} = -(y-b)\frac{d^2y}{dx^2} \longrightarrow (6)$$

Taking eqn (5),

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 + 1 = \frac{c^2}{(y-b)^2}$$

doing $3/2$ in the power of both side,

$$\Rightarrow \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^{3/2} = \left[\frac{c^2}{(y-b)^2}\right]^{3/2}$$

$$\Rightarrow \left[\left(\frac{dy}{dx}\right)^2 + 1\right]^{3/2} = \frac{c^3}{(y-b)^3} \longrightarrow (7)$$

Taking eqn (3),

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = -(y-b)\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{\left[1 + \left(\frac{dy}{dx}\right)^2\right]}{(y-b)}$$

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$$\Rightarrow \frac{d^2y}{dx^2} = \frac{1+\left(\frac{dy}{dx}\right)^2}{(b-y)} \longrightarrow (8)$$

Dividing eq^n $\frac{(7)}{(8)}$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\frac{c^3}{(y-b)^3}}{\frac{1+\left(\frac{dy}{dx}\right)^2}{(b-y)}}$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\frac{c^3}{(y-b)^{3/2}}}{\frac{-\left[1+\left(\frac{dy}{dx}\right)^2\right]}{\cancel{(y-b)}}}$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\frac{-c^3}{(y-b)^2}}{\frac{1+\left(\frac{dy}{dx}\right)^2}{1}}$$

Now, Taking eq^n (2),

$$\Rightarrow (x-a) + (y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(x-a)}{(y-b)} \longrightarrow (9)$$

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squaring both side,

$$\Rightarrow \left(\frac{dy}{dx}\right)^2 = \frac{(x-a)^2}{(y-b)^2}$$

Adding 1 both side,

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = 1 + \frac{(x-a)^2}{(y-b)^2}$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = \frac{(y-b)^2 + (x-a)^2}{(y-b)^2} \longrightarrow (10)$$

Now, Again taking eq^n (2),

$$\Rightarrow (x-a) + (y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{dy}{dx} = -\frac{(x-a)}{(y-b)}$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\frac{d}{dx}\left[\frac{(x-a)}{(y-b)}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{(y-b)\frac{d}{dx}(x-a) - (x-a)\frac{d}{dx}(y-b)}{(y-b)^2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{(y-b)\cdot 1 - (x-a)\frac{dy}{dx}}{(y-b)^2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{(y-b) - (x-a)\left\{-\frac{(x-a)}{(y-b)}\right\}}{(y-b)^2}\right]$$

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$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{(y-b) + \frac{(x-a)^2}{(y-b)}}{(y-b)^2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{\frac{(y-b)^2 + (x-a)^2}{(y-b)}}{(y-b)^2}\right]$$

$$\Rightarrow \frac{d^2y}{dx^2} = -\left[\frac{(y-b)^2 + (x-a)^2}{(y-b)^3}\right] \longrightarrow (11)$$

dividing eqⁿ $\frac{(9)}{(10)}$,

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[1+\left\{\frac{(x-a)^2}{(y-b)^2}\right\}\right]^{3/2}}{-\left[\frac{(y-b)^2+(x-a)^2}{(y-b)^3}\right]}$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[\frac{c^2}{(y-b)^2}\right]^{3/2}}{-\left[\frac{(y-b)^2+(x-a)^2}{(y-b)^3}\right]}$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx}} = \frac{\frac{c^3}{\cancel{(y-b)^3}}}{-\left[\frac{(y-b)^2+(x-a)^2}{\cancel{(y-b)^3}}\right]}$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{-c^3}{(y-b)^2+(x-a)^2} \longrightarrow$$

Taking eq^n ⑥,

$$\Rightarrow \frac{c^2}{(y-b)^2} = -(y-b)\frac{d^2y}{dx^2}$$

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{c^2}{-(y-b)^3} \longrightarrow (12)$$

Dividing eq^n $\frac{(10)}{(11)}$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = \frac{\left[\frac{c^2}{(y-b)^2}\right]^{3/2}}{\left[\frac{c^2}{-(y-b)^3}\right]} \quad \left\{\begin{array}{l}\text{from eq}^n \text{ (5)}\\ \text{and eq}^n \text{ (12)}\end{array}\right\}$$

$$= \frac{\frac{-c^3}{(y-b)^3}}{\frac{c^2}{(y-b)^3}}$$

$$= \frac{-c^3}{c^2}$$

$$\frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = -c$$

Which is independent of a and b.

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(10) $x^x + x^a + a^x + a^a$, for some fixed $a > 0$ and $x > 0$

Sol. let, $y = x^x + x^a + a^x + a^a$

D.w.r. to "x" both side,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}x^x + ax^{a-1} + a^x \cdot \log a + 0$$

Now, let $v = x^x$

$$\Rightarrow \log v = \log x^x$$

$$\Rightarrow \log v = x \log x$$

$$\Rightarrow \frac{d}{dx}\log v = \frac{d}{dx} x \log x$$

$$\Rightarrow \frac{1}{v} \cdot \frac{dv}{dx} = x \frac{d}{dx}\log x + \log x \frac{d}{dx}(x)$$

$$\Rightarrow \frac{dv}{dx} = v\left[x \cdot \frac{1}{x} + \log x\right]$$

$$\Rightarrow \frac{dv}{dx} = x^x\left[1 + \log x\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}x^x + ax^{a-1} + a^x \log a.$$

$$\Rightarrow \frac{dy}{dx} = x^x(1+\log x) + ax^{a-1} + a^x \log a.$$

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(15) If $\cos y = x\cos(a+y)$, with $\cos a \neq \pm 1$, prove that
$$\frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}.$$

Sol. ~~let~~, ~~$\cos y = \frac{d}{}$~~

$\Rightarrow \cos y = x\cos(a+y) \longrightarrow (1)$

$\Rightarrow \frac{d}{dx}\cos y = \frac{d}{dx} x\cos(a+y)$

$\Rightarrow -\sin y \frac{dy}{dx} = x\frac{d}{dx}\cos(a+y) + \cos(a+y)\frac{dx}{dx}$

$\Rightarrow -\sin y\frac{dy}{dx} = x\left\{-\sin(a+y)\frac{d}{dx}(a+y)\right\} + \cos(a+y)$

$\Rightarrow -\sin y\frac{dy}{dx} = -x\sin(a+y)\frac{dy}{dx} + \cos(a+y)$

$\Rightarrow x\sin(a+y)\frac{dy}{dx} - \sin y\frac{dy}{dx} = \cos(a+y)$

$\Rightarrow \frac{dy}{dx}\left\{x\sin(a+y) - \sin y\right\} = \cos(a+y)$

$\Rightarrow \frac{dy}{dx}\left\{\frac{\cos y}{\cos(a+y)} \times \sin(a+y) - \sin y\right\} = \cos(a+y)$

{from eq^n (1)}

$\Rightarrow \frac{dy}{dx}\left\{\frac{\sin(a+y)\cos(y) - \sin y\cos(a+y)}{\cos(a+y)}\right\} = \cos(a+y)$

$\Rightarrow \frac{dy}{dx} \times \frac{\sin(a+y-y)}{\cos(a+y)} = \cos(a+y)$

[$\because \sin(a-b) = \sin a\cos b - \cos a\sin b$]

$\Rightarrow \frac{dy}{dx} \times \frac{\sin a}{\cos(a+y)} = \cos(a+y).$

$\Rightarrow \frac{dy}{dx} = \frac{\cos^2(a+y)}{\sin a}$ Proved.

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(22) if $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$, then Prove that $\frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) \\ l & m \\ a & b \end{vmatrix}$

Sol: Let $y = \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$

D. w. r. to "x",

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} \frac{d}{dx}f(x) & \frac{d}{dx}g(x) & \frac{d}{dx}h(x) \\ l & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ \frac{d}{dx}l & \frac{d}{dx}m & \frac{d}{dx}n \\ a & b & c \end{vmatrix}$$

$$+ \begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ \frac{d}{dx}a & \frac{d}{dx}b & \frac{d}{dx}c \end{vmatrix}$$

$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} + \begin{vmatrix} f(x) & g(x) & h(x) \\ 0 & 0 & 0 \\ a & b & c \end{vmatrix} +$$

$$\begin{vmatrix} f(x) & g(x) & h(x) \\ l & m & n \\ 0 & 0 & 0 \end{vmatrix}$$

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$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix} + 0 + 0$$

$$\Rightarrow \frac{dy}{dx} = \begin{vmatrix} f'(x) & g'(x) & h'(x) \\ l & m & n \\ a & b & c \end{vmatrix}$$

Proved.

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second method

$$(x-a)^2 + (y-b)^2 = c^2 \longrightarrow (1)$$

D.w.r. to "x" both side,

$$\Rightarrow \frac{d}{dx}\left\{(x-a)^2 + (y-b)^2\right\} = \frac{d}{dx} c^2$$

$$\Rightarrow 2(x-a) + 2(y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow (x-a) + (y-b)\frac{dy}{dx} = 0 \longrightarrow (2)$$

Again D.w.r. to "x",

$$\Rightarrow \frac{d}{dx}\left[(x-a) + (y-b)\frac{dy}{dx}\right] = \frac{d}{dx}(0)$$

$$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \frac{dy}{dx}\cdot\frac{dy}{dx} = 0.$$

$$\Rightarrow 1 + (y-b)\frac{d^2y}{dx^2} + \left(\frac{dy}{dx}\right)^2 = 0.$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = -(y-b)\frac{d^2y}{dx^2} \longrightarrow (3)$$

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from ②,

$$\Rightarrow (x-a) + (y-b)\frac{dy}{dx} = 0$$

$$\Rightarrow (x-a) = -(y-b)\frac{dy}{dx} \longrightarrow ④$$

from ① and ④,

$$\Rightarrow \left\{-(y-b)\frac{dy}{dx}\right\}^2 + (y-b)^2 = c^2$$

$$\Rightarrow (y-b)^2\left(\frac{dy}{dx}\right)^2 + (y-b)^2 = c^2$$

$$\Rightarrow (y-b)^2\left[\left(\frac{dy}{dx}\right)^2 + 1\right] = c^2$$

$$\Rightarrow 1 + \left(\frac{dy}{dx}\right)^2 = \frac{c^2}{(y-b)^2} \longrightarrow ⑤$$

from ③,

$$\Rightarrow -(y-b) = \frac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}$$

squaring both side,

$$\Rightarrow (y-b)^2 = \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^2}{\left(\frac{d^2y}{dx^2}\right)^2} \longrightarrow ⑥$$

put in ⑤,

$$\Rightarrow \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^2\left\{1+\left(\frac{dy}{dx}\right)^2\right\}}{\left(\frac{d^2y}{dx^2}\right)^2} = c^2$$

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$$\Rightarrow \frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3}{\left(\frac{d^2y}{dx^2}\right)^2} = c^2$$

Taking square root both side,

$$\Rightarrow \sqrt{\frac{\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^3}{\left(\frac{d^2y}{dx^2}\right)^2}} = c$$

$$\Rightarrow \frac{\left[1+\left(\frac{dy}{dx}\right)^2\right]^{3/2}}{\frac{d^2y}{dx^2}} = c$$

which is indepent of a and b.

(2) $\sin^3 x + \cos^6 x$

Sol. let $y = \sin^3 x + \cos^6 x$

D. w. r. to "x" both side,

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin^3 x + \frac{d}{dx}\cos^6 x.$$

$$\cancel{\Rightarrow \frac{dy}{dx} = 3\sin^2 x\,(+\cos x) + 6\cos^5 x \sin x}$$

$$\cancel{\Rightarrow \frac{dy}{dx} = 6\cos x \sin x - 3\sin x \cos x}$$

$$\cancel{\Rightarrow \frac{dy}{dx} = 3\sin x \cos x}$$

$$\Rightarrow \frac{dy}{dx} = 3\sin^2 x \cos x + 6\cos^5 x\,(-\sin x)$$

$= 3\sin^2 x \cos x - 6\cos^5 x \sin x$

$= 3\sin x \cos x \,(\sin x - 2\cos^4 x)$ Ans.

(3) $y = (5x)^{3\cos 2x}$

D.w.r. to "x" both side,

$\Rightarrow \dfrac{dy}{dx} = 5\dfrac{d}{dx}(x)^{3\cos 2x}$

$\Rightarrow y = (5x)^{3\cos 2x}$

using log both side,

$\Rightarrow \log y = \log (5x)^{3\cos 2x}$

$\Rightarrow \log y = 3\cos 2x \log 5x$,

D.w.r. to "x" both side,

$\Rightarrow \dfrac{1}{y}\dfrac{dy}{dx} = \dfrac{d}{dx}\left[3\cos 2x \log 5x\right]$

$\Rightarrow \dfrac{dy}{dx} = y\left[3\left\{\cos 2x \dfrac{d}{dx}\log 5x + \log 5x \dfrac{d}{dx}\cos 2x\right\}\right]$

$= (5x)^{3\cos 2x}\left[3\left\{\dfrac{\cos 2x}{5x}(5) + \log 5x\,(-\sin 2x)(2)\right\}\right]$

$= (5x)^{3\cos 2x}\left[3\left\{\dfrac{\cos 2x}{x} - 2\sin 2x \log 5x\right\}\right]$

$= 3(5x)^{3\cos 2x}\left[\dfrac{\cos 2x}{x} - 2\sin 2x \log 5x\right]$ Ans.

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(4) $y = \sin^{-1}(x\sqrt{x})$

D.W.R. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\sin^{-1}(x\sqrt{x})$

$= \frac{1}{\sqrt{1-(x\sqrt{x})^2}} \frac{d}{dx} x\sqrt{x}.$

$= \frac{1}{\sqrt{1-x^3}} \cdot \frac{d}{dx} x^{3/2}$

$= \frac{1}{\sqrt{1-x^3}} \cdot \frac{3}{2} x^{\frac{3}{2}-1}$

$= \frac{1}{\sqrt{1-x^3}} \cdot \frac{3}{2} x^{1/2}$

$= \frac{3\sqrt{x}}{2\sqrt{1-x^3}}$ Ans

(5) $y = \frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}$

D.W.R. to "x" both side,

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\frac{\cos^{-1}\frac{x}{2}}{\sqrt{2x+7}}\right]$

$= \frac{\sqrt{2x+7}\frac{d}{dx}\cos^{-1}\frac{x}{2} - \cos^{-1}\frac{x}{2}\frac{d}{dx}(2x+7)^{1/2}}{\left\{(2x+7)^{1/2}\right\}^2}$

$= \frac{\sqrt{2x+7}\left(\frac{-1}{\sqrt{1-\frac{x^2}{4}}}\right)\frac{d}{dx}\left(\frac{x}{2}\right) - \cos^{-1}\frac{x}{2}\left(\frac{1}{2}(2x+7)^{-1/2}\right)(2)}{(2x+7)}$

$= \frac{-\sqrt{2x+7}\left(\frac{2}{\sqrt{4-x^2}}\right)\frac{1}{2} - \cos^{-1}\frac{x}{2}(2x+7)^{-1/2}}{(2x+7)}$

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$$= \frac{\frac{-\sqrt{2x+7}}{\sqrt{4-x^2}} - \frac{\cos^{-1}\frac{x}{2}}{} \cdot (2x+7)^{-1/2}}{(2x+7)}$$

$$= -\left[\frac{\sqrt{2x+7}}{\sqrt{4-x^2}\cdot(2x+7)} + \frac{\cos^{-1}\frac{x}{2}\,(2x+7)^{-1/2}}{(2x+7)}\right]$$

$$= -\left[\frac{1}{\sqrt{4-x^2}\,\sqrt{2x+7}} + \frac{\cos^{-1}\frac{x}{2}}{(2x+7)\sqrt{2x+7}}\right]$$

$$= -\left[\frac{1}{\sqrt{4-x^2}\,\sqrt{2x+7}} + \frac{\cos^{-1}\frac{x}{2}}{(2x+7)^{3/2}}\right]$$ Ans.

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(19) Using mathematical induction prove that $\frac{d}{dx}x^n = nx^{n-1}$ for all positive integers n.

Sol.

$$\frac{d}{dx}x^n = nx^{n-1}$$

for $n = 1$,

$$\Rightarrow \frac{d}{dx}x^1 = 1.x^{1-1}$$

$$\Rightarrow \frac{dx}{dx} = x^0$$

$$\Rightarrow 1 = 1$$

Here it is true for $n=1$.

Let it is true for $n=K$

$$\frac{d}{dx}x^K = Kx^{K-1} \longrightarrow (1)$$

Now,

$$\Rightarrow \frac{d}{dx} x^{k+1} = \frac{d}{dx} x^k \cdot x.$$

$$= x^k \frac{d}{dx}(x) + x \frac{d}{dx} x^k$$

$$= x^k \times 1 + x\,(k x^{k-1})$$

[$\because$ from eq^n ①]

$$= x^k + kx^k$$

$$= x^k(1+k)$$

$$= (1+k)\,x^k$$

$$= (k+1)\,x^{(k+1)-1}$$

~~Here~~

If it is true for $n = k$ then it is also true for $n = k+1$.

$\therefore$ By mathematical induction $\frac{d}{dx} x^n = nx^{n-1}$ is true for every positive integer.

(20)

Sol. Given,

$$\Rightarrow \sin(A+B) = \sin A \cos B + \cos A \sin B.$$

D.W.r. to "x" both side,

$$\Rightarrow \frac{d}{dx} \sin(A+B) = \frac{d}{dx} \sin A \cos B + \frac{d}{dx} \cos A \sin B$$

$$\Rightarrow \cos(A+B) \frac{d}{dx}(A+B) = \sin A \frac{d}{dx} \cos B + \cancel{\cos B} \cos B \frac{d}{dx} \sin A + \cos A \frac{d}{dx} \sin B + \sin B \frac{d}{dx} \cos A$$

$$\Rightarrow \cos(A+B)\left(\frac{dA}{dx} + \frac{dB}{dx}\right) = \sin A(-\sin B)\frac{dB}{dx} + \cos B \cos A \frac{dA}{dx} + \cos A \cos B \frac{dB}{dx} + \sin B(-\sin A)\frac{dA}{dx}$$

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$\Rightarrow \cos(A+B)\left(\frac{dA}{dx} + \frac{dB}{dx}\right) = -\sin A \sin B \frac{dB}{dx} + \cos A \cos B \cdot \frac{dB}{dx}$

$+ \cos A \cos B \frac{dA}{dx} - \sin A \sin B \frac{d}{d}$

$\Rightarrow \cos(A+B)\cancel{\left(\frac{dA}{dx} + \frac{dB}{dx}\right)} = (\cos A \cos B - \sin A \sin B)\cancel{\left(\frac{dA}{dx} + \frac{d}{d}\right)}$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B.$

Proved.

(23) If $y = e^{a\cos^{-1}x}$, $-1 \leq x \leq 1$, show that

$$1-x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} - a^2y = 0.$$

Sol.

$y = e^{a\cos^{-1}x} \longrightarrow (1)$

D.W.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} e^{a\cos^{-1}x}$

$= e^{a\cos^{-1}x} \frac{d}{dx} a\cos^{-1}x$

$\Rightarrow \frac{dy}{dx} = e^{a\cos^{-1}x} \times \cancel{\frac{-a}{\cos^{-1}x}} \quad \frac{-a}{\sqrt{1-x^2}}$

$\Rightarrow \frac{dy}{dx} = \cancel{y\left(\frac{-a}{\cos^{-1}x}\right)} \quad y\left(\frac{-a}{\sqrt{1-x^2}}\right)$

$\Rightarrow \sqrt{1-x^2}\frac{dy}{dx} = -ay \longrightarrow (2)$

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Again D.W.r. to "x"

$\Rightarrow \frac{d}{dx}\left\{\sqrt{1-x^2} \cdot \frac{dy}{dx}\right\} = \frac{d}{dx}(-ay)$

$\Rightarrow \sqrt{1-x^2} \cdot \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{d}{dx}\sqrt{1-x^2} = -ay$

$\Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \times \frac{1}{2\sqrt{1-x^2}} \frac{d}{dx}(1-x^2) = \cancel{d} -ay$

$\Rightarrow \sqrt{1-x^2} \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot \frac{1}{\sqrt{1-x^2}} = -ay$

multiplyin $\sqrt{1-x^2}$ on both side,

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} + \frac{dy}{dx} \cdot (-x) = -a\sqrt{1-x^2} \frac{dy}{dx}$

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x\frac{dy}{dx} = -a(-ay)$ $\{\because$ from ②$\}$

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} = a^2y$

$\Rightarrow (1-x^2) \frac{d^2y}{dx^2} - x \frac{dy}{dx} - a^2y = 0$ Proved.

7) $(\log x)^{\log x}$.

Sol. let $y = (\log x)^{\log x}$.

~~D.W.r. to "x"~~

$\Rightarrow \cancel{\frac{dy}{dx} = \frac{d}{d}}$

Taking log both side,

$\Rightarrow \log y = \log x \log\log x$.

D.W.r. to "x"

$\Rightarrow \frac{1}{y} \frac{dy}{dx} = \frac{d}{dx}[\log x \log\log x]$

$$\Rightarrow \frac{dy}{dx} = y\left[\log x \frac{d}{dx}\log\log x + \log\log x \frac{d}{dx}\log x\right]$$

$$\Rightarrow \frac{dy}{dx} = y\left[\log x \cdot \frac{1}{\log x} \cdot \frac{1}{x} + \frac{\log\log x}{x}\right]$$

$$\Rightarrow \frac{dy}{dx} = (\log x)^{\log x}\left[\frac{1+\log\log x}{x}\right] \text{ Ans.}$$

(8) $\cos(a\cos x + b\sin x)$

Sol. let $y = \cos(a\cos x + b\sin x)$

D.w.r. to "x"

$$\rightarrow \frac{dy}{dx} = \frac{d}{dx}\cos(a\cos x + b\sin x)$$

$$= -\sin(a\cos x + b\sin x)\cdot\frac{d}{dx}(a\cos x + b\sin x)$$

$$= -\sin(a\cos x + b\sin x)\left[-a\sin x + b\cos x\right]$$

$$\Rightarrow \frac{dy}{dx} = \sin(a\cos x + b\sin x)\,(a\sin x - b\cos x) \text{ Ans.}$$

(9) $(\sin x - \cos x)^{(\sin x - \cos x)}$

Sol. let $y = (\sin x - \cos x)^{(\sin x - \cos x)}$

Taking log both side,

$$\Rightarrow \log y = \log(\sin x - \cos x)^{(\sin x - \cos x)}$$

$$\Rightarrow \log y = (\sin x - \cos x)\log(\sin x - \cos x)$$

D.w.r. to "x"

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{d}{dx}\left[(\sin x - \cos x)\left(\log(\sin x - \cos x)\right)\right]$$

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$$\Rightarrow \frac{dy}{dx} = y\left[(\sin x-\cos x)\frac{d}{dx}\log(\sin x-\cos x) + \log(\sin x-\cos x)\frac{d}{dx}(\sin x-\cos x)\right]$$

$$\Rightarrow \frac{dy}{dx} = y\left[(\sin x-\cos x)\cdot\frac{1}{(\sin x-\cos x)}\frac{d}{dx}(\sin x-\cos x) + \log(\sin x-\cos x)\{\cos x+\sin x\}\right]$$

$$\Rightarrow \frac{dy}{dx} = y\left[\cos x+\sin x + \log(\sin x-\cos x)(\cos x+\sin x)\right]$$

$$\Rightarrow \frac{dy}{dx} = (\sin x-\cos x)^{(\sin x-\cos x)}\left(\cos x+\sin x + \log(\sin x-\cos x)(\cos x+\sin x)\right)$$

$$\Rightarrow \frac{dy}{dx} = (\sin x-\cos x)^{(\sin x-\cos x)}(\cos x+\sin x)\left(1+\log(\sin x-\cos x)\right)$$

(11) $x^{x^2-3} + (x-3)^{x^2}$

Sol. let $y = (x^{x^2-3}) + (x-3)^{x^2}$

let $u = x^{x^2-3}$

$\Rightarrow \log u = \log x^{x^2-3}$

$\Rightarrow \log u = (x^2-3)\log x.$

D.w.r. to "x"

$$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx}\left[(x^2-3)\log x\right]$$

$$\Rightarrow \frac{du}{dx} = u\left[(x^2-3)\frac{d}{dx}\log x + \log x\frac{d}{dx}(x^2-3)\right]$$

$$\Rightarrow \frac{du}{dx} = (x^{x^2-3})\left[\frac{(x^2-3)}{x} + 2x\log x\right]$$

$$\Rightarrow \frac{du}{dx} = (x^{x^2-3})\left(\frac{x^2-3}{x} + 2x\log x\right)$$

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let $V = (x-3)^{x^2}$

$\Rightarrow \log V = x^2 \log(x-3)$

D.W.r. to "x"

$$\Rightarrow \frac{1}{V}\frac{dV}{dx} = \frac{d}{dx}\left[x^2 \log(x-3)\right]$$

$$\Rightarrow \frac{dV}{dx} = V\left[x^2 \cdot \frac{d}{dx}\log(x-3) + \log(x-3)\frac{d}{dx}x^2\right]$$

$$= V\left[\frac{x^2}{x-3}\frac{d}{dx}(x-3) + \log(x-3)\,2x\right]$$

$$\frac{dV}{dx} = (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log(x-3)\right]$$

Now,

$$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dV}{dx}$$

$$\Rightarrow \frac{dy}{dx} = (x^{x^2-3})\left(\frac{x^2-3}{x} + 2x\log x\right) + (x-3)^{x^2}\left[\frac{x^2}{x-3} + 2x\log\right.$$

Ans.

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Rolle's Theorem :- if $f(x)$ is a real value function such that

(i) $f(x)$ is continous on the close interval $[a,b]$.
(ii) $f(x)$ is differentiable on open interval (a,b).
(iii) $f(a) = f(b)$

then their exict 1 value 'C' of x in the open interval (a,b) is such that

$$f'(c) = 0$$

Q. verify Rolle's theorem for the following function-

$$f(x) = x^2 - 2x - 15, \quad [-3,5]$$

Sol.

To verify Rolle's Theorem

(i) $f(x) = x^2 - 2x - 15$, is a polynomial therefore it is continous on $[-3,5]$

(ii) $f'(x) = \frac{d}{dx}(x^2 - 2x - 15)$

$f'(x) = 2x - 2$, $(-3,5)$

$f'(x) = 2x - 2$ exist for every value of x on $(-3,5)$.

Hence it is differentiable.

(iii) $f(-3) = (-3)^2 - 2(-3) - 15 = 0$

$f(5) = (5)^2 - 2(5) - 15 = 0$

$\therefore f(-3) = f(5)$

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Now we assume a real no. c such that

$\Rightarrow \quad f'(c) = 0$

$\Rightarrow \quad 2c - 2 = 0$

$\Rightarrow \quad c = 1 \in (-3, 5)$

Hence Rolle's theorem is verified.

Exercise 5.8

Q.1

Sol. Given,

$$f(x) = x^2 + 2x - 8 \quad , \quad [-4, 2]$$

To verify Rolle's theorem,

(i) $f(x)$ is a polynomial therefore it is continous on $[-4, 2]$

(ii) $f'(x) = \frac{d}{dx}(x^2 + 2x - 8)$

$f'(x) = 2x + 2$ exist for every value of x on $(-4, 2)$.

(iii) $f(-4) = (-4)^2 + 2(-4) - 8 = 0$

$f(2) = (2)^2 + 2(2) - 8 = 0$

$\therefore \quad f(-4) = f(2)$

Now we assume a real no. c such that,

$\Rightarrow \quad f'(c) = 0$

$\Rightarrow \quad 2c + 2 = 0$

$\Rightarrow \quad c = -1 \in (-4, 2)$

Hence Rolle's Theorem is verified.

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Q. Verify Relle's theorem (i) $f(x) = x^2 - 3x - 18$ on $[-3, 6]$

(ii) $f(x) = 2(x+1)(x-2)$ on $[-1, 2]$

(iii) $f(x) = \sin 2x$ on $[0, \pi/2]$

Sol. (i) $f(x) = x^2 - 3x - 18$, $[-3, 6]$

To verify Rolle's Theorem,

(i) $f(x)$ is a polynomial therefore it is continous on $[-3, 6]$

(ii) $f'(x) = \frac{d}{dx}(x^2 - 3x - 18)$

$\Rightarrow f'(x) = 2x - 3$ exist for every value of x on $(-3, 6)$.

(iii) $f(-3) = (-3)^2 - 3(-3) - 18 = 0$

$\Rightarrow f(6) = 6^2 - 3(6) - 18 = 0$

$\therefore f(-3) = f(6)$

Now we assume a real no. c such that

$\Rightarrow f'(c) = 0$

$\Rightarrow 2c - 3 = 0$

$\Rightarrow c = \frac{3}{2} \in (-3, 6)$

Hence Rolle's Theorem verified.

Sol. (ii) Given,

$f(x) = 2(x+1)(x-2)$, $[-1, 2]$

To verify Rolle's Theorem,

(i) $f(x)$ is a polynomial therefore it is continuous on $[-1, 2]$

(ii) $f'(x) = \frac{d}{dx}(2(x+1)(x-2))$

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$\Rightarrow f'(x) = \frac{d}{dx}(2x^2 - 4x + 2x - 4)$

$\Rightarrow f'(x) = \frac{d}{dx}(2x^2 - 2x - 4)$

$\Rightarrow f'(x) = 4x - 2$ exist for every value of x on $(-1, 2)$

(iii) $f(-1) = 2(-1+1)(-1-2) = 0$

$f(2) = 2(2+1)(2-2) = 0.$

$\therefore f(-1) = f(2)$

Now we assume a real no. 'c' such that

$\Rightarrow f'(c) = 0$

$\Rightarrow \cancel{6c^2 - 2 = 0}$ $\qquad 4c - 2 = 0$

$\Rightarrow \cancel{c^2 = \frac{1}{3}}$ $\qquad c = 2/4$

$c = 1/2 \in (-1, 2)$

$\Rightarrow \cancel{c = \pm\frac{1}{\sqrt{3}}}$ $\qquad$ Hence Rolle's Theorem verified

$\Rightarrow \cancel{c = \pm\frac{1}{1.73} \in (-1, 2)}$

$\Rightarrow$ Hence Rolle's Theorem is verified.

Sol. (iii) Given,

$f(x) = \sin 2x$, $\left[0, \frac{\pi}{2}\right]$

To verify Rolle's Theorem,

(i) $f(x) = \sin 2x$ is continous because we already proved $\sin x$ function is continous. on $\left[0, \frac{\pi}{2}\right]$

(ii) $f'(x) = \frac{d}{dx} \sin 2x$

$\Rightarrow f'(x) = \cos 2x \cdot 2$

$\Rightarrow f'(x) = 2\cos 2x$

(iii) $\Rightarrow f(0) = \sin 2(0) = 0$

$\Rightarrow f\left(\frac{\pi}{2}\right) = \sin 2 \cdot \frac{\pi}{2} = 0$

$\therefore f(0) = f\left(\pi/2\right)$

Now we assume a real no. 'c' such that

$\Rightarrow f'(c) = 0$

$\Rightarrow 2\cos 2c = 0$

$\Rightarrow \cos 2c = 0$

$\Rightarrow 2c = \cos^{-1} 0$

$\Rightarrow c = \frac{\cos^{-1} 0}{2}$

$\Rightarrow c = \frac{\frac{\pi}{2}}{2}$

$\Rightarrow c = \frac{\pi}{4} \in \left(0, \frac{\pi}{4}\right)$

Hence Rolle's Theorem is verified.

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Miscellaneous

(2) Find $\frac{dy}{dx}$, if $y = 12(1-\cos t)$, $x = 10(t - \sin t)$,

Sol. Given,

$y = 12(1-\cos t)$

D.w.r.t. to "t"

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt} 12(1-\cos t)$

$= 12(0 + \sin t)$

$\Rightarrow \frac{dy}{dt} = 12\sin t \longrightarrow (1)$

Also,

$x = 10(t - \sin t)$

D.w.r.t. to 't',

$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} 10(t - \sin t)$

$\Rightarrow \frac{dx}{dt} = 10(1 - \cos t) \longrightarrow (2)$

$\Rightarrow \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{12\sin t}{10(1-\cos t)}$

$= \frac{12\sin t}{10\left(2\sin^2\frac{t}{2}\right)}$

$= \frac{12\sin t}{20\sin^2\frac{t}{2}}$

$= \frac{\cancel{12}^{6} \cdot \cancel{2}^{1}\sin\frac{t}{2}\cos\frac{t}{2}}{{}_{5}\cancel{10}\,\cancel{20}\sin^2\frac{t}{2}}$

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$$= \frac{6}{5} \frac{\cancel{\sin\frac{t}{2}} \cos\frac{t}{2}}{\sin^{\cancel{2}}\frac{t}{2}}$$

$$= \frac{6}{5} \frac{\cos\frac{t}{2}}{\sin\frac{t}{2}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{6}{5} \cot\frac{t}{2}$$ Ans

(18) find $\frac{dy}{dx}$, if $y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$, $-1 \le x \le 1$

Sol. Given,

$$y = \sin^{-1}x + \sin^{-1}\sqrt{1-x^2}$$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \cancel{\frac{d}{dx}\sin^{-1}x + \frac{d}{dx}\sin^{-1}\sqrt{1-x^2}}\ \frac{d}{dx}\left(\sin^{-1}x + \sin^{-1}\sqrt{1-x^2}\right)$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\sqrt{1-x^2}}} \frac{d}{dx}\sqrt{1-x^2}$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-\sqrt{1-x^2}}} \cdot \frac{1}{2}(1-x^2)^{\frac{1}{2}-1} \frac{d}{dx}(1-x^2)$$

$$= \frac{1}{\sqrt{1-x^2}} + \frac{1}{2\sqrt{1-\sqrt{1-x^2}}} \frac{1}{\sqrt{1-x^2}} (-2x)$$

$$= \frac{1}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-\sqrt{1-x^2}}\sqrt{1-x^2}}$$

$$= \frac{d}{dx}\cdot\left(\sin^{-1}x + \cos^{-1}x\right) \quad \left[\because \sin^{-1}\sqrt{1-x^2} = \cos^{-1}x\right]$$

$$= \frac{d}{dx}\left(\frac{\pi}{2}\right) \quad \left[\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}\right]$$

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$$\frac{dy}{dx} = 0 \quad \left[\because \frac{d}{dx}(c) = 0\right]$$

(17) If $x = a(\cos t + t\sin t)$ and $y = a(\sin t - t\cos t)$ find $\frac{d^2y}{dx^2}$

Sol. Given,

$$y = a(\sin t - t\cos t)$$

D.w.r. to "t"

$$\rightarrow \frac{dy}{dt} = \frac{d}{dt} a(\sin t - t\cos t)$$

$$= a\left[\cos t - \frac{d}{dt} t\cos t\right]$$

$$= a\left[\cos t - \left\{t\frac{d}{dt}\cos t + \cos t\frac{d}{dt}(t)\right\}\right]$$

$$= a\left[\cos t - \left\{-t\sin t + \cos t\right\}\right]$$

$$= a\left[\cos t + t\sin t - \cos t\right]$$

$$\frac{dy}{dt} = at\sin t \longrightarrow (1)$$

and $x = a(\cos t + t\sin t)$

D.w.r. to 't'

$$\rightarrow \frac{dx}{dt} = \frac{d}{dt} a(\cos t + t\sin t)$$

$$= a\left[-\sin t + \left\{t\cos t + \sin t\right\}\right]$$

$$= a\left[-\sin t + t\cos t + \sin t\right]$$

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$\Rightarrow \dfrac{dx}{dt} = at\cos t \longrightarrow (2).$

Now,

$$\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{at\sin t}{at\cos t}.$$

$$\Rightarrow \frac{dy}{dx} = \tan t.$$

~~Again~~ D.w.r. to "x"

$$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}\tan t.$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t \; \frac{dt}{dx}.$$

Taking eq^n (2),

$$\Rightarrow \frac{dx}{dt} = at\cos t.$$

taking resiprocal both side,

$$\Rightarrow \frac{dt}{dx} = \frac{1}{at\cos t}. \longrightarrow (3).$$

$$\Rightarrow \frac{d^2y}{dx^2} = \sec^2 t \cdot \frac{dt}{dx}$$

$$= \sec^2 t \cdot \frac{1}{at\cos t}. \qquad \{\because \text{from } (3)\}$$

$$= \frac{\sec^2 t}{at} \cdot \sec t.$$

$$\frac{d^2y}{dx^2} = \frac{\sec^3 t}{at} \quad \underline{\text{Ans}}.$$

(15) If $f(x) = |x|^3$, show that $f''(x)$ exist for all real x and find it.

Sol. Given,

$$f(x) = |x|^3$$

case i) $x > 0$.

$$\Rightarrow f(x) = x^3$$

D.w.r. to "x"

$$\Rightarrow \frac{d}{dx} f(x) = \frac{d}{dx} x^3$$

$$\Rightarrow f'(x) = 3x^2$$

Again D.w.r. to "x"

$$\Rightarrow f''(x) = \frac{d}{dx} 3x^2$$

$$\Rightarrow f''(x) = 6x \longrightarrow (1).$$

Case ii) $x < 0$

$$\Rightarrow f(x) = (-x)^3$$

$$\Rightarrow f(x) = -x^3$$

D.w.r to "x"

$$\Rightarrow f'(x) = \frac{d}{dx}(-x^3)$$

$$\Rightarrow f'(x) = -3x^2$$

Again D.w.r. to "x"

$$\Rightarrow f''(x) = \frac{d}{dx}(-3x^2)$$

$$\Rightarrow f''(x) = -6x \longrightarrow (2).$$

from (1) and (2),
$f(x) = |x|^3$ is exist for every real x, that is defined as

$$f(x) = \begin{cases} 6x^2, & \text{if } x \geq 0 \\ -6x^2, & \text{if } x < 0 \end{cases}$$

<u>Exercise - 5.6</u>

find $\frac{dy}{dx}$,

(1) $x = 2at^2$, $y = at^4$

<u>Sol.</u> <u>Given</u>, $y = at^4$
D.w.r. to "t"

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}(at^4)$

$\Rightarrow y_1 = a(4t^3)$

$\Rightarrow \frac{dy}{dt} = 4at^3$

and $x = 2at^2$
D.w.r. to "t",

$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}(2at^2)$

$= 2a(2t)$

$\Rightarrow \frac{dx}{dt} = 4at$

<u>Now</u>,

$$\frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\cancel{4a}t^{\cancel{3}\,2}}{\cancel{4at}} \Rightarrow \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = t^2 \quad \underline{\text{Ans.}}$$

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(2) $x = a\cos\theta$, $y = b\cos\theta$

Sol. Given,

$x = a\cos\theta$.

D.W.r. to "θ",

$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta} a\cos\theta.$

$\Rightarrow \frac{dx}{d\theta} = -a\sin\theta.$

and $y = b\cos\theta$.

$\Rightarrow$ D.W.r. to "θ"

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta} b\cos\theta.$

$\Rightarrow \frac{dy}{d\theta} = -b\sin\theta$

Now,

$\Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a\sin\theta}{-b\sin\theta}$

$\Rightarrow \frac{dy}{dx} = \frac{a}{b}$ Ans.

(3) $x = \sin t$, $y = \cos 2t$

Sol. Given,

$x = \sin t$.

D.W.r. to "t",

$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\sin t.$

$\Rightarrow \frac{dx}{dt} = \cos t.$

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$\Rightarrow \quad y = \cos 2t$

D.w.r. to "t"

$\Rightarrow \dfrac{dy}{dt} = \dfrac{d}{dt}(\cos 2t)$

$\Rightarrow \dfrac{dy}{dt} = -\sin 2t \cdot (2)$

$\Rightarrow \dfrac{dy}{dt} = -2\sin 2t.$

Now,

$\Rightarrow \dfrac{\left(\dfrac{dy}{dt}\right)}{\left(\dfrac{dx}{dt}\right)} = \dfrac{-2\sin 2t}{\cos t}$

$\Rightarrow \dfrac{dy}{dx} = \dfrac{-2(2\sin t \cancel{\cos t})}{\cancel{\cos t}}$

$\Rightarrow \dfrac{dy}{dx} = -4\sin t$ Ans.

(4) $x = 4t \quad , y = 4/t.$

Sol. Given, $x = 4t$

D.w.r. to "t"

$\Rightarrow \dfrac{dx}{dt} = \dfrac{d}{dt}(4t)$

$\Rightarrow \dfrac{dx}{dt} = 4$

and $y = \dfrac{4}{t}$

D.w.r. to 't'

$\Rightarrow \dfrac{dy}{dt} = \dfrac{d}{dt}(4t^{-1})$

$= -4t^{-1-1}$

$\Rightarrow \dfrac{dy}{dt} = -\dfrac{4}{t^2}$

Now,

$$\Rightarrow \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{-4/t^2}{4}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-1}{t^2} \quad \text{Ans.}$$

(5) $x = \cos\theta - \cos 2\theta$, $y = \sin\theta - \sin 2\theta$.

Sol Given,

$$y = \sin\theta - \sin 2\theta.$$

D. W.r to "θ"

$$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta}\sin\theta - \frac{d}{d\theta}\sin 2\theta.$$

$$\Rightarrow \frac{dy}{d\theta} = \cos\theta - 2\cos 2\theta$$

and,

$$x = \cos\theta - \cos 2\theta.$$

D. W.r. to "θ"

$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{dx}\cos\theta - \frac{d}{dx}\cos 2\theta.$$

$$\Rightarrow \frac{dx}{d\theta} = -\sin\theta + 2\sin 2\theta$$

$$\Rightarrow \frac{dx}{d\theta} = 2\sin 2\theta - \sin\theta.$$

Now,

$$\Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \cancel{2\sin}\ \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta}.$$

$$\Rightarrow \frac{dy}{dx} = \frac{\cos\theta - 2\cos 2\theta}{2\sin 2\theta - \sin\theta} \quad \text{Ans.}$$

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(6) $x = a(\theta - \sin\theta)$, $y = a(1+\cos\theta)$

Sol. Given,

$y = a(1+\cos\theta)$

D.W.r. to "θ"

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta} a(1+\cos\theta)$

$= a[0 + (-\sin\theta)]$

$\Rightarrow \frac{dy}{d\theta} = -a\sin\theta$

and, $x = a(\theta - \sin\theta)$

$\Rightarrow$ D.W.r. to "θ"

$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta} a(\theta - \sin\theta)$

$\Rightarrow \frac{dx}{d\theta} = a(1-\cos\theta)$

Now,

$\Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{-a\sin\theta}{a(1-\cos\theta)}$

$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{-\sin\theta}{1-\cos\theta}$

$= \frac{-\sin\theta}{2\sin^2\frac{\theta}{2}}$

$= \frac{-\cancel{2}\sin\cancel{\frac{\theta}{2}}\cos\frac{\theta}{2}}{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}$

$= -\frac{\cos\frac{\theta}{2}}{\sin\frac{\theta}{2}}$

$\Rightarrow \left(\frac{dy}{dx}\right) = -\cot\frac{\theta}{2}$ Ans.

Rough

$1-\cos^2\theta = \sin^2\theta$

$(1+\cos\theta)(1-\cos\theta) = \sin^2\theta$

$1-\cos\theta = \frac{\sin^2\theta}{1+\cos\theta}$

$= \frac{\sin^2\theta}{2\sin^2\frac{\theta}{2}}$

Mean Value Theorem :-

(or)

Leingranj Mean Value Theorem :-

if $f(x)$ be a function such that

(i) $f(x)$ is continous on $[a, b]$
(ii) $f(x)$ is differentiable on ~~(a,b)~~ (a, b).
(iii) $f(a) \neq f(b)$

then their exist atleast one $c \in (a, b)$ such that ~~f(b)~~

$$\Rightarrow \boxed{f'(c) = \frac{f(b) - f(a)}{(b-a)}}$$

Q. verify leingranj mean value theorem for the function $f(x) = 2x^2 - 3x + 1$ on $[1, 3]$.

Sol. To verify leingranj mean value theorem.

(i) $f(x)$ is a polynomial hence $f(x)$ is continous on $[1, 3]$

(ii) $f(x) = 2x^2 - 3x + 1$
D.w.r. to "x"

$$\Rightarrow f'(x) = \frac{d}{dx}(2x^2 - 3x + 1)$$

$$\Rightarrow f'(x) = 4x - 3$$

$f'(x)$ exist for every value of x in $(1, 3)$

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(iii) $f(1) = 2(1)^2 - 3(1) + 1 = 0$
$f(3) = 2(3)^2 - 3(3) + 1 = 10$

$\therefore f(1) \neq f(3)$

~~Now,~~

Therefore there exist atleast one value of C such that ~~f'(c)~~

$$\Rightarrow f'(c) = \frac{f(b) - f(a)}{3 - 1}$$

$$\Rightarrow 4c - 3 = \frac{10 - 0}{2}$$

$$\Rightarrow 4c - 3 = 5$$

$$\Rightarrow c = 2 \in (1, 3)$$

Hence leingraj mean value theorem verified.

Exercise 5.8

(4)

Sol. Given,

$$f(x) = x^2 - 4x - 3, \quad \text{on } [a, b]$$
$$\text{on } [1, 4]$$

To verify leingraj mean value theorem,

(i) $f(x)$ is a polynomial hence $f(x)$ is continous on $[1, 4]$

(ii) $f(x) = x^2 - 4x - 3$
$f'(x) = 2x - 4$

$f'(x)$ exist for every value of x in $(1, 4)$

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(iii) $\Rightarrow$ $f(1) = 1^2 - 4(1) - 3 = -6$
$f(4) = 4^2 - 4(4) - 3 = -3$

$\therefore f(1) \neq f(4)$

Therefore there exist atleast one value of c such that

$\Rightarrow 2c - 4 = \dfrac{-3+6}{\cancel{-3+6}\ 4-1}$

$\Rightarrow 2c = \dfrac{3}{3} + 4$

$c = \dfrac{5}{2} \in (1, 4)$

(2) Examin Rolle's theorem, and can you say that ~~inverse~~ converse of Rolle's theorem,

(i) $f(x) = [x]$, $x \in [5, 9]$

$f(x) = [x]$ is neither ~~not a~~ continous nor differentiable on $[5, 9]$. Hence Rolle's theorem is not applicable.

(iii) $f(x) = x^2 - 1$, $[1, 2]$

Ans. To verify Rolle's theorem,

(i) $f(x)$ is polynomial hence $f(x)$ is continous on $[1, 2]$

(ii) $f(x) = x^2 - 1$
$\Rightarrow f'(x) = 2x$ exist real value of x on $(1, 2)$

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(iii) $f(1) = 1^2 - 1 = 0$
$f(2) = 2^2 - 1 = 3$

$\because f(1) \neq f(2)$
Hence Rolle's theorem not applicable.
converse of Rolle's Theorem:-
let $f'(c) = 0$
$\Rightarrow 2c = 0$
$\Rightarrow c = 0 \notin (1,2)$

(ii) $f(x) = [x]$, $[-2,2]$
To verify Rolle's Theorem,

$\Rightarrow$ ~~$f'(c$~~

$f(x) = [x]$ is neither continous nor differentiable on $[-2,2]$. Hence Rolle's theorem is not applicable.

(3)
Sol.

$f : [-5,5] \longrightarrow R$
Given,
f is differentiable, since we know that if any function is differentiable then that function is also continous.

(i) f is continous
(ii) f is differentiable {$\because$ Given}
(iii) since $f'(x)$ is not vanish anywhere, $f'(c) \neq 0$
By Lagrange mean value theorem,
$f(-5) \neq f(5)$
Proved.

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(5)
Sol. $f(x) = x^3 - 5x^2 - 3x$, $[1, 3]$

To verify Leingraj Mean value Theorem,

(i) ⇒ f(x) is a polynomial hence f(x) is continuous on [1, 3]

(ii) $f(x) = x^3 - 5x^2 - 3x$

$\rightarrow f'(x) = \frac{d}{dx}(x^3 - 5x^2 - 3x)$

$\Rightarrow f'(x) = 3x^2 - 10x - 3$ exist for every real value of "x" in (1, 3).

(iii) $f(1) = 1^3 - 5(1)^2 - 3(1) = -7$

$f(3) = 3^3 - 5(3)^2 - 3(3) = -\cancel{27}$

$\therefore f(1) \neq f(3)$

Therefore there exist atleast one value of c such that

$$\Rightarrow f'(c) = \frac{f(b) - f(a)}{(b-a)}$$

$$\Rightarrow 3c^2 - 10c - 3 = \frac{-\cancel{27} + 7}{\cancel{-45+7}\ 3-1}$$

$$\Rightarrow 3c^2 - 10c = \frac{-20}{2} + 3$$

$$\Rightarrow 3c^2 - 10c = -\cancel{16} - 7$$

$$\Rightarrow \cancel{3c^2 - 10c + 16 = 0}$$

$$\Rightarrow$$

$$\Rightarrow \cancel{c = \frac{-(-10) \pm \sqrt{(-10)^2 - 4(3)(16)}}{2(3)}}$$

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$\Rightarrow \cancel{c = \frac{10 \pm \sqrt{100-192}}{6}}$

$\Rightarrow \cancel{c = \frac{10 \pm \sqrt{-92}}{6}}$

$\Rightarrow \cancel{c = \frac{10 + \sqrt{-92}}{6}}$ (or), $\cancel{c = \frac{10 - \sqrt{-92}}{6}}$

$\Rightarrow 3c^2 - 10c + 7 = 0$

$\Rightarrow 3c^2 - 7c - 3c + 7 = 0$

$\Rightarrow c(3c-7) - 1(3c-7) = 0$

$\Rightarrow (c-1)(3c-7) = 0$

Taking, $c - 1 = 0$ and $3c - 7 = 0$

$c = 1$ and $c = 7/3$.

$c = 1, 7/3$ ~~Ans.~~

$c = 1 \notin (1,3)$ and $c = 7/3 \in (1,3)$

So,

$c = 7/3$ Ans.

According to question,

$\Rightarrow f'(c) = 0$

$\Rightarrow 3c^2 - 10c - 3 = 0$

$\Rightarrow \cancel{-3c^2 + 10c = -3}$

$\Rightarrow \cancel{-(3c^2 - 10c) = -3}$

$\Rightarrow \cancel{3c^2 - 9c - c - 3 = 0}$

$\Rightarrow \cancel{3c(c-3) - 1(}$

$\Rightarrow c = \frac{10 \pm \sqrt{(-10)^2 - 4(3)(-3)}}{2(3)}$

$= \frac{10 \pm \sqrt{100 + 36}}{2(3)}$

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$$\Rightarrow c = \frac{10 \pm \sqrt{136}}{6}$$

$$\Rightarrow c = \frac{10 \pm \sqrt{2\times2\times34}}{2\times3}$$

$$\Rightarrow c = \frac{10 \pm 2\sqrt{34}}{2\times3}$$

$$\Rightarrow c = \frac{5 \pm \sqrt{34}}{3}$$

$$\Rightarrow c = \frac{5 \pm 5.83}{3}$$

Rough

$4\overline{)136}(34$
12
$\overline{16}$

$= \frac{x+y}{2\sqrt{y}}$

$= \frac{34+36}{2\times6}$

$= \frac{70}{12}$ 5.83

Taking +ve sign

$$\Rightarrow c = \frac{10.83}{3}$$

$$\Rightarrow c = 3.61$$

Taking −ve sign

$$\Rightarrow c = \frac{-0.83}{3}$$

$$\Rightarrow c = -0.276$$

Now,

$$c = 3.61, -0.27 \notin (1,3)$$

Exercise 5.6

(4) $x = \frac{\sin^3 t}{\sqrt{\cos 2t}}, \ y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$

Sol. Given,

$$y = \frac{\cos^3 t}{\sqrt{\cos 2t}}$$

D.w.r. to "t",

$$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}\left[\frac{\cos^3 t}{\sqrt{\cos 2t}}\right]$$

$$\Rightarrow \frac{dy}{dt} = \frac{\sqrt{\cos 2t}\,\frac{d}{dx}\cos^3 t - \frac{d}{dx}\sqrt{\cos 2t}\cdot\cos^3 t}{\sqrt{(\cos 2t)^2}}$$

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$$\Rightarrow \frac{dy}{dt} = \frac{\sqrt{\cos 2t}\cdot 3\cos^2 t(-\sin t) - \frac{1}{2\sqrt{\cos 2t}}(-2\sin 2t)(\cos^3 t)}{\cos 2t}$$

$$\Rightarrow \frac{dy}{dt} = \frac{-3\cos 2t\cos^2 t\sin t + \cos^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}} \longrightarrow (1)$$

and $x = \dfrac{\sin^3 t}{\sqrt{\cos 2t}}$

D.w.r. to "t" both side,

$$\Rightarrow \frac{dx}{dt} = \frac{d}{dt}\left(\frac{\sin^3 t}{\sqrt{\cos 2t}}\right)$$

$$= \frac{\sqrt{\cos 2t}\frac{d}{dx}\sin^3 t - \sin^3 t\frac{d}{dx}\sqrt{\cos 2t}}{(\sqrt{\cos 2t})^2}$$

$$= \frac{\sqrt{\cos 2t}(3\sin^2 t)(\cos t) - \sin^3 t\left(\frac{1}{2\sqrt{\cos 2t}}\right)(-2\sin 2t)}{\cos 2t}$$

$$\frac{dx}{dt} = \frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}} \longrightarrow (2)$$

Now,

$$\Rightarrow \frac{\frac{dy}{dt}}{\frac{dx}{dt}} = \frac{\frac{-3\cos 2t\cos^2 t\sin t + \cos^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}}{\frac{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}{\cos 2t\sqrt{\cos 2t}}}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-3\cos 2t\cos^2 t\sin t + \cos^3 t\sin 2t}{3\cos 2t\sin^2 t\cos t + \sin^3 t\sin 2t}$$

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$$= \frac{-3\cos 2t \cos^2 t \sin t + \cos^3 t(2\sin t \cos t)}{3\cos 2t \sin^2 t \cos t + \sin^3 t(2\sin t \cos t)}$$

$$= \frac{\sin t \cos t(-3\cos 2t \cos t + 2\cos^3 t)}{\sin t \cos t(3\cos 2t \sin t + 2\sin^3 t)}$$

$$= \frac{-3(2\cos^2 t - 1)\cos t + 2\cos^3 t}{3(1-2\sin^2 t)\sin t + 2\sin^3 t}$$

$\because \cos 2t = 2\cos^2 t - 1$
$\cos 2t = 1 - 2\sin^2 t$

$$= \frac{-4\cos^3 t + 3\cos t}{3\sin t - 4\sin^3 t}$$

$$= \frac{-\cos 3t}{\sin 3t}$$

$\because \cos 3t = 4\cos^3 t - 3\cos t$
$\because \sin 3t = 3\sin t - 4\sin^3 t$

$\Rightarrow \frac{dy}{dx} = -\cot 3t$ Ans.

(8) $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$, $y = a \sin t$.

Sol. Given,

$y = a \sin t$

D.W.r. to "t"

$\Rightarrow \frac{dy}{dt} = \frac{d}{dt} a \sin t$

$\Rightarrow \frac{dy}{dt} = a\cos t.$

and $x = a\left(\cos t + \log \tan \frac{t}{2}\right)$

D.W.r. to "t",

$\Rightarrow \frac{dx}{dt} = \frac{d}{dt} a\left(\cos t + \log \tan \frac{t}{2}\right)$

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$\Rightarrow \frac{dx}{dt} = a\left(-\sin t + \frac{1}{\tan\frac{t}{2}} \sec^2\frac{t}{2} \cdot \frac{1}{2}\right)$

$= a\left(-\sin t + \frac{1}{2}\cot\frac{t}{2} \cdot \frac{1}{\cos^2\frac{t}{2}}\right)$

$\cancel{= a\left(-\sin t + \frac{1}{2}\frac{\cos^2\frac{t}{2}}{\sin^2\frac{t}{2}} \cdot \frac{1}{\cos^2\frac{t}{2}}\right)}$

$\cancel{\frac{dx}{dt} = a\left(-\sin t + \frac{\operatorname{cosec}^2\frac{t}{2}}{2}\right)}$

$= a\left(-\sin t + \frac{1}{2}\frac{\cos\frac{t}{2}}{\sin\frac{t}{2}} \cdot \frac{1}{\cos^2\frac{t}{2}}\right)$

$= a\left(-\sin t + \cos\frac{t}{2} \frac{1}{2\sin\frac{t}{2}\cos\frac{t}{2}}\right)$

$= a\left(-\sin t + \frac{1}{\sin t}\right)$ $\left\{\because \sin 2t = 2\sin t\cos t\right\}$

$= a\left(\frac{-\sin^2 t + 1}{\sin t}\right)$

$= a\left(\frac{\cos^2 t}{\sin t}\right)$

$\Rightarrow \frac{dx}{dt} = \frac{a\cos^2 t}{\sin t}$

Now,

$\Rightarrow \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{a\cos t}{\frac{a\cos^2 t}{\sin t}}$

$\Rightarrow \left(\frac{dy}{dx}\right) = \frac{\sin t}{\cos t}$

$\Rightarrow \left(\frac{dy}{dx}\right) = \tan t$ Ans.

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(9) $x = a\sec\theta$, $y = b\tan\theta$

Sol. Given,

$y = b\tan\theta$

D. w.r to "θ"

$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta} b\tan\theta.$

$\Rightarrow \frac{dy}{d\theta} = b\sec^2\theta.$

and $x = a\sec\theta.$

D. w.r. to "θ"

$\rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta} a\sec\theta$

$\Rightarrow \frac{dx}{d\theta} = a\sec\theta\tan\theta.$

Now,

$$\Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{b\sec^2\theta}{a\sec\theta\tan\theta.}$$

$$= \frac{b\sec\theta}{a\tan\theta.}$$

$$= \frac{\frac{b}{\cos\theta}}{a\frac{\sin\theta}{\cos\theta}}$$

$$= \frac{b}{a\sin\theta,} \quad \text{Ans.}$$

$$\Rightarrow \frac{dy}{dx} = \frac{b}{a}\operatorname{cosec}\theta \quad \text{Ans.}$$

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(10) $x = a(\cos\theta + \theta\sin\theta)$, $y = a(\sin\theta - \theta\cos\theta)$

Sol. Given,

$$y = a(\sin\theta - \theta\cos\theta)$$

D.W.r. to "θ",

$$\Rightarrow \frac{dy}{d\theta} = \frac{d}{d\theta} a(\sin\theta - \theta\cos\theta)$$

$$= a\left[\cos\theta - \{\theta(-\sin\theta) + \cos\theta\}\right]$$

$$= a\left[\cos\theta + \theta\sin\theta - \cos\theta\right]$$

$$\Rightarrow \frac{dy}{d\theta} = a\theta\sin\theta.$$

and $x = a(\cos\theta + \theta\sin\theta)$

D.W.r. to "θ",

$$\Rightarrow \frac{dx}{d\theta} = \frac{d}{d\theta} a(\cos\theta + \theta\sin\theta)$$

$$= a\left[-\sin\theta + \{\theta\cos\theta + \sin\theta\}\right]$$

$$= a\left[-\sin\theta + \theta\cos\theta + \sin\theta\right]$$

$$\Rightarrow \frac{dx}{d\theta} = a\theta\cos\theta.$$

Now,

$$\Rightarrow \frac{\left(\frac{dy}{d\theta}\right)}{\left(\frac{dx}{d\theta}\right)} = \frac{a\theta\sin\theta}{a\theta\cos\theta}$$

$$\Rightarrow \frac{dy}{dx} = \tan\theta$$ Ans.

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(11) If $x = \sqrt{a^{\sin^{-1}t}}$, $y = \sqrt{a^{\cos^{-1}t}}$, show that

$$\frac{dy}{dx} = \frac{-y}{x}$$

Sol. Given,

$$y = \sqrt{a^{\cos^{-1}t}}$$

D.w.r. to "t" $\frac{1}{2}$

$$\Rightarrow \frac{dy}{dt} = \frac{d}{dt}\left(a^{\cos^{-1}t}\right)$$

$$= \cancel{\frac{1}{2} a \cos}$$

$$= \frac{1}{2}\left(a^{\cos^{-1}t}\right)^{-\frac{1}{2}} \frac{d}{dt}\left(a^{\cos^{-1}t}\right)$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1}t}}} a^{\cos^{-1}t} \log \cos^{-1}t \frac{d}{dx} \cos^{-1}t$$

$$= \frac{1}{2\sqrt{a^{\cos^{-1}t}}} \left(a^{\cos^{-1}t} \log \cos^{-1}t\right) \frac{-1}{\sqrt{1-t^2}}$$

$$= \frac{-a^{\cos^{-1}t} \log \cos^{-1}t}{2\sqrt{a^{\cos^{-1}t}} \sqrt{1-t^2}}$$

$$\Rightarrow y = \sqrt{a^{\cos^{-1}t}}$$

$$\Rightarrow y = a^{\frac{1}{2}\cos^{-1}t}$$

Taking log both side,

$$\Rightarrow \log y = \log a^{\frac{1}{2}\cos^{-1}t}$$

$$\Rightarrow \log y = \frac{1}{2}\cos^{-1}t \log a$$

D.w.r. to "t"

$$\Rightarrow \frac{1}{y}\frac{dy}{dt} = \frac{d}{dt} \frac{1}{2}\cos^{-1}t \log a$$

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$$\Rightarrow \cancel{\frac{dy}{dt} = y\left[\frac{1}{2}\left\{\cos^{-1}t\,\frac{d}{dt}\log a + \log a\,\frac{d}{dt}\cos^{-1}t\right\}\right]}$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dt} = \frac{1}{2}\log a\,\frac{d}{dt}\cos^{-1}t$$

$$\Rightarrow \frac{dy}{dt} = y\left[\frac{-\log a}{2\sqrt{1-t^2}}\right]$$

$$\Rightarrow \frac{dy}{dt} = \sqrt{a^{\cos^{-1}t}}\left(\frac{-\log a}{2\sqrt{1-t^2}}\right)$$

and $x = \sqrt{a^{\sin^{-1}t}}$

$$\Rightarrow \log x = \frac{1}{2}\sin^{-1}t\,\log a.$$

D.W.r. to "t",

$$\Rightarrow \frac{1}{x}\frac{dx}{dt} = \frac{d}{dt}\frac{1}{2}\sin^{-1}t\,\log a$$

$$\Rightarrow \frac{dx}{dt} = x\left[\frac{1}{2}\log a\,\frac{d}{dt}\sin^{-1}t\right]$$

$$\Rightarrow \frac{dx}{dt} = x\left[\frac{1}{2}\log a\,\frac{1}{\sqrt{1-t^2}}\right]$$

$$\Rightarrow \frac{dx}{dt} = \sqrt{a^{\sin^{-1}t}}\left[\frac{\log a}{2\sqrt{1-t^2}}\right]$$

Now,

$$\Rightarrow \frac{\left(\frac{dy}{dt}\right)}{\left(\frac{dx}{dt}\right)} = \frac{\sqrt{a^{\cos^{-1}t}}\left(\frac{-\log a}{2\sqrt{1-t^2}}\right)}{\sqrt{a^{\sin^{-1}t}}\left(\frac{\log a}{2\sqrt{1-t^2}}\right)}$$

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$$\Rightarrow \frac{dy}{dx} = \frac{y\left(\frac{\cancel{-\log t}}{\cancel{\sqrt{1+t^2}}}\right)}{x\left(\frac{\cancel{\log t}}{\cancel{\sqrt{1+t^2}}}\right)}$$

$$\Rightarrow \frac{dy}{dx} = \frac{-y}{x}$$

proved.

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Exercise 5.8.

(6) Examine the applicability of Mean value Theorem (leingraj) for all three function given in the above Question no. 2 in this exercise.

(i) $f(x) = [x]$ for $x \in [5,9]$
(ii) $f(x) = [x]$ for $x \in [-2,2]$
(iii) $f(x) = x^2-1$ for $x \in [1,2]$

Sol.(i) $f(x) = [x]$ is neither continous nor differentiable therefore ~~it~~ ~~doe~~ leingraj mean value theorem is not applicable on $[5,9]$

(ii) $f(x) = [x]$ is neither continous nor differentiable therefore leingraj mean value theorem is not applicable.

(iii) (a) $f(x) = x^2-1$ is a polynomial hence it is continous on $[1,2]$

(b) $f(x) = x^2-1$
$f'(x) = 2x.$

(c) $f(1) = 1^2 - 1 = 0$
$f(2) = 2^2 - 1 = 3$

$\therefore f(1) \neq f(2)$

Hence leingraj mean value theorem is applicable.

Eg.37) find $\frac{dy}{dx}$, if $x^{2/3} + y^{2/3} = a^{2/3}$

Sol. Given

$\Rightarrow x^{2/3} + y^{2/3} = a^{2/3}$

D.w.r. to "x" both side,

$\Rightarrow \frac{d}{dx}\left(x^{2/3} + y^{2/3}\right) = \frac{d}{dx} a^{2/3}$

$\Rightarrow \frac{2}{3} x^{\frac{2}{3}-1} + \frac{2}{3} y^{\frac{2}{3}-1} \cdot \frac{dy}{dx} = 0$

$\Rightarrow \frac{2}{3} x^{-1/3} + \frac{2}{3} y^{-1/3} \frac{dy}{dx} = 0$

$\Rightarrow \frac{2}{3\sqrt[3]{x}} + \frac{2}{3\sqrt[3]{y}} \frac{dy}{dx} = 0$

$\Rightarrow \frac{dy}{dx} \frac{2}{3\sqrt[3]{y}} = -\frac{2}{3\sqrt[3]{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{-\cancel{6}\sqrt[3]{y}}{\cancel{6}\sqrt[3]{x}}$

$\Rightarrow \frac{dy}{dx} = \frac{-\sqrt[3]{y}}{\sqrt[3]{x}}$

$\Rightarrow \frac{dy}{dx} = -\sqrt[3]{\frac{y}{x}}$ Ans.

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Class work.

Questions:-

Eg. - 25, 27, 29 (3^{rd} and 4^{th}), 30, 33, 40, 41

Eg. 25 $\frac{dy}{dx} = ?$

$\Rightarrow y + \sin y = \cos x$.

D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} + \cos y \frac{dy}{dx} = -\sin x$.

$\Rightarrow \frac{dy}{dx}(1 + \cos y) = -\sin x$

$\Rightarrow \frac{dy}{dx} = \frac{-\sin x}{1+\cos y}$

Eg. 27 $f'(x) = ?$

$\Rightarrow f(x) = \tan^{-1}x$

$\Rightarrow f'(x) = \frac{1}{1+x^2}$ $\left[\because \frac{d}{dx}\tan^{-1}x = \frac{1}{1+x^2}\right]$

Eg. 29. (iii) $\cos^{-1}(e^x)$

let $y = \cos^{-1}(e^x)$

D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\cos^{-1}(e^x)$

$= \frac{-1}{\sqrt{1-(e^x)^2}} \frac{d}{dx} e^x$

$= \frac{-1}{\sqrt{1-e^{2x}}}(e^x)$

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$$\Rightarrow \frac{dy}{dx} = \frac{-e^x}{\sqrt{1-e^{2x}}} \quad \underline{\text{Ans.}}$$

(iv) $e^{\cos x}$

let $y = e^{\cos x}$

~~Takin~~

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} e^{\cos x}$$

$$= e^{\cos x} \frac{d}{dx} \cos x$$

$$= -e^{\cos x} \sin x.$$

Eg. 30. let $y = \sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}$

~~D.w.r. to "x"~~

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]^{1/2}$$

$$= \frac{1}{2}\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]^{1/2-1} \frac{d}{dx}\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]$$

$$= \frac{1}{2}\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]^{-1/2} \left[\frac{(3x^2+4x+5)\frac{d}{dx}(x-3)(x^2+4) - (x-3)(x^2+4)\frac{d}{dx}(3x^2+4x+5)}{(3x^2+4x+5)^2}\right]$$

Taking log both side,

$$\Rightarrow \log y = \log\left(\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right)^{1/2}$$

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$$= \frac{1}{2} \log\left[\frac{(x-3)(x^2+4)}{3x^2+4x+5}\right]$$

$$= \frac{1}{2}\left[\log(x-3)(x^2+4) - \log(3x^2+4x+5)\right]$$

$$\Rightarrow \log y = \frac{1}{2}\left[\log(x-3) + \log(x^2+4) - \log(3x^2+4x+5)\right]$$

D.W.r. to "x"

$$\rightarrow \frac{d}{dx}\log y = \frac{d}{dx}\frac{1}{2}\left[\log(x-3) + \log(x^2+4) - \log(3x^2+4x+5)\right]$$

$$\Rightarrow \frac{1}{y}\frac{dy}{dx} = \frac{1}{2}\left[\frac{d}{dx}\log(x-3) + \frac{d}{dx}\log(x^2+4) - \frac{d}{dx}\log(3x^2+4x+5)\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{2}\left[\frac{1}{(x-3)} + \frac{1}{(x^2+4)}(2x) - \frac{6x+4}{3x^2+4x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}}{2}\left[\frac{1}{(x-3)} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$$

$$\Rightarrow \frac{dy}{dx} = \frac{1}{2}\sqrt{\frac{(x-3)(x^2+4)}{3x^2+4x+5}}\left[\frac{1}{(x-3)} + \frac{2x}{x^2+4} - \frac{6x+4}{3x^2+4x+5}\right]$$

Ans.

Eg. 33 $\frac{dy}{dx} = ?$

$$\Rightarrow y^x + x^y + x^x = a^b$$

D.W.r. to "x"

$$\Rightarrow \frac{d}{dx}y^x + \frac{d}{dx}x^y + \frac{d}{dx}x^x = \frac{d}{dx}a^b.$$

$$\Rightarrow \frac{d}{dx}y^x + \frac{d}{dx}x^y + \frac{d}{dx}x^x = 0 \longrightarrow (1)$$

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let, $u = y^x$

Taking log both side,

$\Rightarrow \log u = \log y^x$

$\Rightarrow \log u = x \log y$

D.w.r.to "x"

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{d}{dx} x \log y.$

$\Rightarrow \frac{du}{dx} = u\left[x \frac{d}{dx}\log y + \log y \frac{d}{dx}(x)\right]$

$\Rightarrow \frac{du}{dx} = y^x\left[\frac{x}{y}\frac{dy}{dx} + \log y\right] \longrightarrow (2).$

let $v = x^y$

Taking log both side,

$\Rightarrow \log v = \log x^y$

$\Rightarrow \log v = y \log x.$

D.w.r.to "x"

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \frac{d}{dx}\left[y \log x\right]$

$\Rightarrow \frac{dv}{dx} = v\left[y \frac{d}{dx}\log x + \log x \frac{dy}{dx}\right]$

$\Rightarrow \frac{dv}{dx} = x^y\left[\frac{y}{x} + \log x \frac{dy}{dx}\right] \longrightarrow (3)$

let $M = x^x.$

Taking log both side,

$\Rightarrow \log M = \log x^x$

$\Rightarrow \log M = x \log x$

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Diff. w.r.t. to x

$$\Rightarrow \frac{1}{M}\frac{dM}{dx} = \frac{d}{dx}(x\log x)$$

$$\Rightarrow \frac{dM}{dx} = M\left[x\frac{d}{dx}\log x + \log x\frac{d}{dx}(x)\right]$$

$$\Rightarrow \frac{dM}{dx} = x^x(1+\log x) \longrightarrow (3)$$

~~from~~ Put $\frac{du}{dx}$, $\frac{dv}{dx}$, $\frac{dM}{dx}$ in eqn (1),

$$\Rightarrow y^x\left[\frac{x}{y}\frac{dy}{dx} + \log x\right] + x^y\left[\frac{y}{x} + \log x\frac{dy}{dx}\right] + x^x(1+\log x) = 0$$

$$\Rightarrow \left[xy^{x-1}\frac{dy}{dx} + y^x\log x\right] + \left[yx^{y-1} + x^y\log x\frac{dy}{dx}\right] + x^x + x^x\log x = 0$$

$$\Rightarrow xy^{x-1}\frac{dy}{dx} + y^x\log x + yx^{y-1} + x^y\log x\frac{dy}{dx} + x^x + x^x\log x = 0$$

$$\Rightarrow xy^{x-1}\frac{dy}{dx} + x^y\log x\frac{dy}{dx} + y^x\log x + x^x\log x + yx^{y-1} + x^x = 0$$

$$\Rightarrow \frac{dy}{dx}\left(xy^{x-1} + x^y\log x\right) = -y^x\log x - x^x\log x - yx^{y-1} - x^x$$

$$\Rightarrow \frac{dy}{dx}\left(xy^{x-1} + x^y\log x\right) = -y^x\log x - yx^{y-1} - x^x\log x - x^x$$

$$\Rightarrow \frac{dy}{dx}\left(xy^{x-1} + x^y\log x\right) = -y^x\log x - yx^{y-1} - x^x(\log x + 1)$$

$$\Rightarrow \frac{dy}{dx}\left(xy^{x-1} + x^y\log x\right) = -\left[y^x\log x + yx^{y-1} + x^x(\log x + 1)\right]$$

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$$\Rightarrow \frac{dy}{dx} = \frac{-\left[yx^{y-1} + y^x \log y + x^x(1+\log x)\right]}{xy^{x-1} + x^y \log x}$$ Ans

Eg. 4(a) Prove that $\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$.

$\Rightarrow y = 3e^{2x} + 2e^{3x}$

D.W.R. to "x",

$\Rightarrow \frac{dy}{dx} = 3\frac{d}{dx}e^{2x} + 2\frac{d}{dx}e^{3x}$

$= 2(3e^{2x}) + (2e^{3x})3$

$\Rightarrow \frac{dy}{dx} = 6e^{2x} + 6e^{3x} \longrightarrow (1)$.

Again D.W.R. to "x"

$\Rightarrow \frac{d^2y}{dx^2} = \frac{d}{dx}6e^{2x} + \frac{d}{dx}6e^{3x}$

$\Rightarrow \frac{d^2y}{dx^2} = 12e^{2x} + 18e^{3x} \longrightarrow (2)$.

Now, To Prove,

$\frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y = 0$

Taking L.H.S,

$= \frac{d^2y}{dx^2} - 5\frac{dy}{dx} + 6y$

$= 12e^{2x} + 18e^{3x} - 5(6e^{2x} + 6e^{3x}) + 6(3e^{2x} + 2e^{3x})$

$= 12e^{2x} + 18e^{3x} - 30e^{2x} - 30e^{3x} + 18e^{2x} + 12e^{3x}$

$= (12e^{2x} - 30e^{2x} + 18e^{2x}) + (18e^{3x} - 30e^{3x} + 12e^{3x})$

$= 0 + 0$

$= 0$.

Proved.

Eg (4) Show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$.

$\Rightarrow$ $y = \sin^{-1}x$.

D.W.r. to "x"

$\Rightarrow$ $\frac{dy}{dx} = \frac{d}{dx}\sin^{-1}x$

$\Rightarrow$ $\frac{dy}{dx} = \frac{1}{\sqrt{1-x^2}} \longrightarrow (1)$

Again D.W.r. to "x"

$\Rightarrow$ $\frac{d^2y}{dx^2} = \frac{d}{dx}\frac{1}{\sqrt{1-x^2}}$

$$= \frac{(1-x^2)^{\frac{1}{2}}\frac{d}{dx}(1) - 1\frac{d}{dx}(1-x^2)^{\frac{1}{2}}}{(1-x^2)^2}$$

$$= \frac{-\frac{1}{2}(1-x^2)^{-\frac{1}{2}}\frac{d}{dx}(1-x^2)}{(1-x^2)^2}$$

$$= \frac{-\frac{1}{2\sqrt{1-x^2}}(-2x)}{(1-x^2)^2}$$

$$= \frac{\frac{2x}{2\sqrt{1-x^2}}}{(1-x^2)^2}$$

$\Rightarrow$ $\frac{d^2y}{dx^2} = \frac{\frac{x}{\sqrt{1-x^2}}}{(1-x^2)^2} \longrightarrow (2)$

Now,

L.H.S. $= (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}$.

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$$= \frac{(1-x^2)\left(\frac{x}{\sqrt{1-x^2}}\right) - x\left(\frac{1}{\sqrt{1-x^2}}\right)}{(1-x^2)^2}$$

$$= \frac{(1-x^2)}{(1-x^2)^2}\frac{x}{\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$= \frac{x}{(1-x^2)\sqrt{1-x^2}} - \frac{x}{\sqrt{1-x^2}}$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{dy}{dx} = 1$$

Again D.w.r. to "x"

$$\Rightarrow \frac{d}{dx}\sqrt{1-x^2}\,\frac{dy}{dx} = \frac{d}{dx}(1)$$

$$\Rightarrow \sqrt{1-x^2}\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\,\frac{d}{dx}(1-x^2)^{1/2} = 0.$$

$$\Rightarrow (\sqrt{1-x^2})\,\frac{d^2y}{dx^2} + \frac{dy}{dx}\left\{\frac{1}{\cancel{2}}(1-x^2)^{-1/2}(-\cancel{2}x)\right\} = 0$$

$$\Rightarrow (\sqrt{1-x^2})\,\frac{d^2y}{dx^2} - \frac{dy}{dx}\,\frac{(x)}{\sqrt{1-x^2}} = 0$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - \frac{dy}{dx}(x) = 0. \quad \{\because \text{Taking L.C.M.}\}$$

$$\Rightarrow (1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} = 0$$

Proved.

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Q ## Assignment :-

- D.W.r. to $4x^y$

(1) $\cos^2 x^2$
(2) $\log \tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$
(3) $\sin x^\circ$
(4) prove that every constant function is continuous.
(5) $(\sin x)^{\cos x} + (\cos x)^{\sin x}$ find $\frac{dy}{dx}$. ~~if y=~~
(6) if $y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$
then prove that $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

(7) ~~If $y = e^x +$~~

(7) If $y = e^{x+e^{x+e^{x+\cdots}}}$ Then prove that $\frac{dy}{dx} = \frac{y}{1-y}$

(8) If $y = e^{-kt}\cos(pt+c)$ then prove that
$\frac{d^2y}{dt^2} + k\frac{dy}{dt} + n^2y = 0$ where $n^2 = p^2+k^2$

(9) If $x^y = e^{x-y}$ then prove that
$$\frac{dy}{dx} = \frac{\log x}{(1+\log_e x)^2}$$

(10) If $y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$
then prove that
$$\frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

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Sol.6. Given,

$$y = \sqrt{\sin x + \sqrt{\sin x + \sqrt{\sin x + \cdots}}}$$

$\Rightarrow$ $y = \sqrt{\sin x + y}$

$\Rightarrow$ squaring both side

$\Rightarrow$ $y^2 = \sin x + y$

D.W.r. to "x"

$\Rightarrow$ $2y \frac{dy}{dx} = \cos x + \frac{dy}{dx}$

$\Rightarrow$ $2y \frac{dy}{dx} - \frac{dy}{dx} = \cos x$

$\Rightarrow$ $\frac{dy}{dx}(2y-1) = \cos x$

$\Rightarrow$ $\frac{dy}{dx} = \frac{\cos x}{2y-1}$

Proved

Sol.7. Given,

$\Rightarrow$ $y = e^{x+e^{x+e^{x+\cdots}}}$

$\Rightarrow$ $y = e^{x+y}$

Taking log both side,

$\Rightarrow$ $\log y = \log e^{x+y}$

$\Rightarrow$ $\log y = (x+y)\log e$

$\Rightarrow$ $\log y = x+y$

D.W.r. to "x"

$\Rightarrow$ $\frac{d}{dx}(\log y) = \frac{d}{dx}(x+y)$

$\Rightarrow$ $\frac{1}{y}\frac{dy}{dx} = 1 + \frac{dy}{dx}$

$\Rightarrow$ $\frac{dy}{dx} = y\left(1 + \frac{dy}{dx}\right)$

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$$\Rightarrow \frac{dy}{dx} = y + y\frac{dy}{dx}$$

$$\Rightarrow \frac{dy}{dx} - y\frac{dy}{dx} = y$$

$$\Rightarrow \frac{dy}{dx}(1-y) = y$$

$$\Rightarrow \frac{dy}{dx} = \frac{y}{1-y}$$

Proved.

Sol.(i) Let $y = \cos^2 x^2$

$$\Rightarrow y = (\cos x^2)^2$$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}(\cos x^2)^2$$

$$= 2\cos x^2 \frac{d}{dx}\cos x^2$$

$$= 2\cos x^2(-\sin x^2)\frac{d}{dx}x^2$$

$$\Rightarrow \frac{dy}{dx} = -4x\cos x^2 \sin x^2$$

Sol.(ii) Let $y = \log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$

D.w.r. to "x"

$$\Rightarrow \frac{dy}{dx} = \frac{d}{dx}\log\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\frac{d}{dx}\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

$$= \frac{1}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)\frac{d}{dx}\left(\frac{\pi}{4}+\frac{x}{2}\right)$$

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$$= \frac{\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)} \left(0+\frac{1}{2}\right)$$

$$= \frac{\frac{1}{2}\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$

$$= \frac{\sec^2\left(\frac{\pi}{4}+\frac{x}{2}\right)}{2\tan\left(\frac{\pi}{4}+\frac{x}{2}\right)}$$ Ans.

Sol.3. let $y = \sin x^{\circ}$

$\because \pi^{c} = 180^{\circ}$

$\Rightarrow x^{\circ} = \left(\frac{\pi x}{180}\right)^{c}$

$\Rightarrow y = \sin \frac{\pi x}{180}$

D.w.r. to "x"

$\Rightarrow \frac{dy}{dx} = \frac{d}{dx} \sin \frac{\pi x}{180^{\circ}}$

$= \cos \frac{\pi x}{180^{\circ}} \frac{d}{dx}\left(\frac{\pi x}{180}\right)$

$= \frac{\pi}{180} \cos \frac{\pi x}{180}$ Ans.

Sol.4. let $f(x) = k$

at $x = c$, $\{\because$ let c is a continuous function

$f(c) = c$

~~L.H.L. = $\lim_{x \to c^-} \frac{f(x) - f(x)}{h}$~~

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$= \cancel{\lim_{x \to 0} \frac{f(c-x) - f(x)}{x}}$

$= \cancel{\lim_{x \to 0} \frac{(c-x) - x}{x}}$

L.H.L $= \lim_{h \to c^-} \frac{f(h) - f(c)}{h}$

$= \lim_{h \to 0} \frac{f(c-h) - f(c)}{h}$

$= \lim_{h \to 0} \frac{(c-h) - c}{h}$

$= \lim_{h \to 0} \frac{c - h - c}{h}$

$= -1$

L.H.L. $= \lim_{x \to c^-} f(x)$

$= \lim_{x \to 0} f(c-x)$

$= \lim_{x \to 0} (c-x)$

$= c$

R.H.L $= \lim_{x \to c^+} f(x)$

$= \lim_{x \to 0} f(c+x)$

$= \lim_{x \to 0} (c+x)$

$= c$

L.H.L $=$ R.H.L $= f(c)$

Hence every constant function is continuous.

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Q(5) Given, $(\sin x)^{\cos x} + (\cos x)^{\sin x}$

Let $y = (\sin x)^{\cos x} + (\cos x)^{\sin x}$.

$\Rightarrow u = (\sin x)^{\cos x}$.

Taking log both side,

$\Rightarrow \log u = \log (\sin x)^{\cos x}$

$\Rightarrow \log u = \cos x \log \sin x$.

D.w.r. to "x"

$\Rightarrow \frac{1}{u}\frac{du}{dx} = \cos x \frac{d}{dx}\log \sin x + \log \sin x \frac{d}{dx}\cos x$.

$\Rightarrow \frac{du}{dx} = u\left[\frac{\cos x}{\sin x}\cdot \cos x + \log \sin x(-\sin x)\right]$

$= (\sin x)^{\cos x}\left\{\cot x \cos x - \sin x \log \sin x\right\} \rightarrow (1)$

and $V = (\cos x)^{\sin x}$

Taking log both side,

$\Rightarrow \log v = \log (\cos x)^{\sin x}$

$\Rightarrow \log v = \sin x \log \cos x$.

D.w.r. to "x"

$\Rightarrow \frac{1}{v}\frac{dv}{dx} = \sin x \frac{d}{dx}\log \cos x + \log \cos x \frac{d}{dx}\sin x$.

$\Rightarrow \frac{dv}{dx} = v\left[\frac{\sin x}{\cos x}(-\sin x) + \cos x \log \cos x\right]$

$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}\left\{-\tan x \sin x + \cos x \log \cos x\right\}$

$\Rightarrow \frac{dv}{dx} = (\cos x)^{\sin x}\left\{\cos x \log \sin x - \tan x \sin x\right\} \longrightarrow (2)$

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Now,

$\Rightarrow \frac{dy}{dx} = \frac{du}{dx} + \frac{dv}{dx}$

$\Rightarrow \frac{dy}{dx} = (\sin x)^{\cos x}\left\{\cot x \cos x - \sin x \log \sin x\right\} + (\cos x)^{\sin x}\left\{\cos x \log \sin x - \tan x \sin x\right\}$

Ans.

(9) Given,

$x^y = e^{x-y}$

Taking log both side,

$\Rightarrow \log x^y = \log e^{(x-y)}$

$\Rightarrow y \log x = x-y \log e$

$\Rightarrow y\log x = x-y$

~~$\Rightarrow 2y = x$~~

~~D.w.r. to x~~

~~$\Rightarrow 2\frac{dy}{dx} = \frac{d}{dx}(x)$~~

~~$\Rightarrow \frac{dy}{dx} = \frac{1}{2}$~~

$\Rightarrow y \log x = x - y$

$\Rightarrow y \log x + y = x$

$\Rightarrow y(\log x + 1) = x.$

$\Rightarrow y = \frac{x}{(\log x + 1)}$

$\Rightarrow$ D.w.r. to x

$\Rightarrow \frac{dy}{dx} = \frac{(\log x+1)\frac{dx}{dx} - x\frac{d}{dx}(\log x+1)}{(\log x+1)^2}$

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$$= \frac{(\log x + 1) - 1}{(\log x + 1)^2}$$

$$= \frac{\log x}{(\log x + 1)^2} \quad \text{Proved.}$$

10) Given,

$$y\sqrt{1-x^2} + x\sqrt{1-y^2} = 1$$

Taking $\sin^{-1}$ both side,

$$\Rightarrow \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\} = \sin^{-1} 1$$

$$\Rightarrow \sin^{-1} x + \sin^{-1} y = \frac{\pi}{2}$$

D.W.r. to "x"

$$\Rightarrow \frac{d}{dx}\sin^{-1} x + \frac{d}{dx}\sin^{-1} y = \frac{d}{dx}\left(\frac{\pi}{2}\right)$$

$$\Rightarrow \frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$$

$$\Rightarrow \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = -\frac{1}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = -\frac{\sqrt{1-y^2}}{\sqrt{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} = -\sqrt{\frac{1-y^2}{1-x^2}}$$

$$\Rightarrow \frac{dy}{dx} + \sqrt{\frac{1-y^2}{1-x^2}} = 0$$

Proved.

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(8) Given,

$$y = e^{-kt}\cos(pt+c)$$

Taking log both side,

$\Rightarrow \log y = -kt\log e + \log\cos(pt+c)$

$\Rightarrow \log y = -kt + \log\cos(pt+c)$

D.w.r. to "t"

$\Rightarrow \frac{1}{y}\frac{dy}{dt} = \frac{d}{dt}(-kt) + \frac{d}{dt}\log\cos(pt+c)$

$\Rightarrow \frac{dy}{dt} = y\left[-k + \frac{1}{\cos(pt+c)}(-\sin(pt+c))\,p\right]$

$= e^{-kt}\cos(pt+c)\left[-k - p\tan(pt+c)\right]$

$\Rightarrow \frac{dy}{dt} = -e^{-kt}\cos(pt+c)\{k + p\tan(pt+c)\} \longrightarrow (1)$

Again D.w.r. to "t"

$\Rightarrow \frac{d^2y}{dt^2} = \frac{d}{dt}\left\{-e^{-kt}\cos(pt+c)\left\{k + p\tan(pt+c)\right\}\right\}$

$\Rightarrow \frac{d^2y}{dt^2} = \frac{d}{dt}\,-e^{-kt}\cos(pt+c) + \frac{d}{dt}(k + p\tan(pt+c))$

let $u = -e^{-kt}\cos(pt+c)$

Taking log both side

$\Rightarrow \log u = kt + \log\cos(pt+c)$

D.w.r. to "t"

$\Rightarrow \frac{1}{u}\frac{du}{dt} = k + \frac{1}{\cos(pt+c)}(-\sin(pt+c))\,p$

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$\Rightarrow \frac{du}{dt} = -e^{-kt}\cos(pt+c)\left\{k - p\tan(pt+c)\right\}$

and $v = k + p\tan(pt+c)$

D.w.r. to "t"

$\Rightarrow \frac{dv}{dt} = k + p\sec^2(pt+c)\ (p)$

$= k + \{p\sec(pt+c)\}^2$

Now

$\Rightarrow \frac{d^2y}{dt^2} = \frac{du}{dt} + \frac{dv}{dt}$

$\Rightarrow \frac{d^2y}{dt^2} = -e^{-kt}\cos(pt+c)\left\{k - p\tan(pt+c)\right\} + k + p^2\sec^2(pt+c)$

Taking L.H.S. =

$= \frac{d^2y}{dt^2} + k\frac{dy}{dt} + n^2y$

$= -e^{-kt}\cos(pt+c)\left\{k - p\tan(pt+c)\right\} + k + p^2\sec^2(pt+c)$
$+ k\left\{-e^{-kt}\cos(pt+c)\left(k + p\tan(pt+c)\right)\right\} + (p^2+k^2)$
$\left(e^{-kt}\cos(pt+c)\right)$

$\cancel{= -ke^{-kt}\cos(pt+c) + pe^{-kt}\cos(pt+c)\tan(pt+c) + k + p^2\sec^2}$
$(pt+c) + k\left(e^{-kt}\cos(pt+c)\right)(k) - ke^{-kt}$

$= -ke^{-kt}\cos(pt+c) + pe^{-kt}\sin(pt+c) + k + p^2\sec^2(pt+c)$
$+ k\left\{-ke^{-kt}\cos(pt+c) - pe^{-kt}\sin(pt+c)\right\} +$
$p^2e^{-kt}\cos(pt+c) + k^2e^{-kt}\cos(pt+c)$

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$$= -Ke^{-Kt}\cos(pt+c) + pe^{-Kt}\sin(pt+c) + K + p^2\sec^2(pt+c) \cancel{\phi} - K^2e^{-Kt}\cos(pt+c) - pKe^{-Kt}\sin(pt+c) + p^2e^{-Kt}\cos(pt+c) + K^2e^{-Kt}\cos(pt+c)$$

$$= -Ke^{-Kt}\cos(pt+c) + pe^{-Kt}\sin(pt+c) + K + p^2\sec^2(pt+c) - pKe^{-Kt}\sin(pt+c) + p^2e^{-Kt}\cos(pt+c)$$

$$= -Ke^{-Kt}\cos(pt+c) + K + pe^{-Kt}\sin(pt+c) - pKe^{-Kt}\sin(pt+c) + p^2\sec^2(pt+c) + p^2e^{-Kt}\cos(pt+c)$$

$$= K\left(-e^{-Kt}\cos(pt+c)+1\right) + pe^{-Kt}\left(\sin(pt+c) - K\sin(pt+c)\right) + p^2\left(\sec^2(pt+c) + e^{-Kt}\cos(pt+c)\right)$$

=