order of Matrices = $\boxed{R \times C}$

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Chapter - 3
MATRICES

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix} \quad \text{order} = 3 \times 3$$

$$\Rightarrow \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \quad \text{order} = 2 \times 2$$

* Identity matrix :- A square matrix in which elements in the diagonal are all 1 and rest are all zero is called Identity matrix.

$$I_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$I_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Exercise 3.1

Q.1) In the matrix $A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & 5/2 & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$

i) order of the matrix.

ii) No. of elements.

iii) write the elements $a_{13}, a_{21}, a_{33}, a_{24}, a_{23}$.

Sol. Given

$$A = \begin{bmatrix} 2 & 5 & 19 & -7 \\ 35 & -2 & \frac{5}{2} & 12 \\ \sqrt{3} & 1 & -5 & 17 \end{bmatrix}$$

i) order of matrix,

In matrix A, 3 rows and 4 column present

order $= R \times C$

$= 3 \times 4$

ii) No. of elements,

In matrix A, no. of elements are $= R \times C$

$= 3 \times 4$

$= 12$

iii) The Elements in matrix A,

$a_{13} = 19$, $a_{24} = 12$

$a_{21} = 35$, $a_{23} = \frac{5}{2}$

$a_{33} = -5$

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Q.2) If a matrix has 24 elements, what are the possible orders it can have? What, if it has 13 elements?

Sol. Possible orders

for matrix A in which 24 Elements present,

$= R \times C$
$= 1 \times 24$
$= 2 \times 12$
$= 3 \times 8$
$= 4 \times 6$
$= 6 \times 4$
$= 8 \times 3$
$= 12 \times 2$
$= 24 \times 1$

The no. of possible orders for matrix A = 8

possible orders for matrix B in which 13 Elements present,

$= 1 \times 13$
$= 13 \times 1$

The no. of possible orders for matrix B = 2.

Q.3) If a matrix has 18 Elements, what are the possible orders it can have? What, if it has 5 elements?

Sol. possible orders for matrix A in which 18 Elements present,

$= 1 \times 18$
$= 2 \times 9$
$= 3 \times 6$
$= 6 \times 3$
$= 9 \times 2$
$= 18 \times 1$

The no. of possible orders for matrix A = 6

Possible orders for matrix B in which 5 Elements present,

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$= 1 \times 5$, The no. of possible ord
$= 5 \times 1$ for matrix $B = 2$

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Q.4) construct 2×2 matrix , $A = [a_{ij}]$, whose elements are given by

i) $a_{ij} = \frac{(i+j)^2}{2}$

Sol. $a_{ij} = \frac{(i+j)^2}{2}$

~~⇒ a.~~

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$, By Given in question.

$\Rightarrow a_{11} = \frac{(1+1)^2}{2} = 2$

$\Rightarrow a_{12} = \frac{(1+2)^2}{2} = \frac{9}{2}$

$\Rightarrow a_{21} = \frac{(2+1)^2}{2} = \frac{9}{2}$

$\Rightarrow a_{22} = \frac{(2+2)^2}{2} = 8$

Therefore matrix $A = \begin{bmatrix} 2 & \frac{9}{2} \\ \frac{9}{2} & 8 \end{bmatrix}$

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Q.6 Find the values of x, y and z from the following eq^n :-

ii) $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

Sol. $\begin{bmatrix} x+y & 2 \\ 5+z & xy \end{bmatrix} = \begin{bmatrix} 6 & 2 \\ 5 & 8 \end{bmatrix}$

on comparing the same position of both matrix

$x+y = 6 \longrightarrow$ (i)

$5+z = 5 \longrightarrow$ (ii)

$xy = 8 \longrightarrow$ (iii)

from (1), $x = 6-y$

put $x = 6-y$ in (3),

$= (6-y)y = 8$

$= 6y - y^2 - 8 = 0$

$= y^2 - 6y + 8 = 0$

$= y^2 - 4y - 2y + 8 = 0$

$= y(y-4) - 2(y-4) = 0$

$= y = 4 \quad , \quad y = 2$

put $y=4$ in (1), $x = 2$

put $y = 2$ in (1) $x = 4$

and from (2), $z = 0$

Hence, value of z is 0 and $y = 4, 2$

$x = 2, 4$

ii) $\begin{bmatrix} 4 & 3 \\ x & 5 \end{bmatrix} = \begin{bmatrix} y & z \\ 1 & 5 \end{bmatrix}$

Sol. on comparing same position of both matrix,

$\Rightarrow \boxed{x = 1}$

$\Rightarrow y = 4$

$\Rightarrow z = 3$

iii) $\begin{bmatrix} x+y+z \\ x+y \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Sol. on comparing same position of both matrix,

$\Rightarrow x+y+z = 9 \rightarrow (1)$

$\Rightarrow x+y = 5 \rightarrow (2)$

$\Rightarrow y+z = 7 \rightarrow (3)$

from (3)

$\Rightarrow y = 7 - z$

put in (2),

$\Rightarrow y + 7 - z = 5$

$\Rightarrow y - z = -2$

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Addition and substraction in matrix

for Addition or substraction, it is compulsory the ord of matrix is same.

Eg:- $\begin{bmatrix} 2 & 4 & 3 \\ 1 & 6 & 9 \end{bmatrix} + \begin{bmatrix} 5 & 7 & 8 \\ 2 & 1 & 3 \end{bmatrix}$

$= \begin{bmatrix} 7 & 11 & 11 \\ 3 & 7 & 12 \end{bmatrix}$

Multiplication :-

Eg:- $4\begin{bmatrix} 2 & 5 \\ 3 & 6 \end{bmatrix}$

$= \begin{bmatrix} 8 & 20 \\ 12 & 24 \end{bmatrix}$

Multiplication of two matrices :-

Multiplication in two matrix is possible when no. of column of 1st matrix is equal to no. of row of 2nd matrix.

$$\boxed{A_{m \times n} \cdot B_{n \times p} = C_{m \times p}}$$

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Multiplication

Eg:- $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

$$= \begin{bmatrix} 2\times1 + 3\times0 + 4\times3 & 2\times-3 + 3\times2 + 4\times0 & 2\times5 + 3\times4 + \\ 3\times1 + 4\times0 + 5\times3 & 3\times-3 + 4\times2 + 5\times0 & 3\times5 + 4\times4 + \\ 4\times1 + 5\times0 + 6\times3 & 4\times-3 + 5\times2 + 6\times0 & 4\times5 + 5\times4 + \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

Exercise 3.1

Q.6) iii) $\begin{bmatrix} x+y+z \\ x+z \\ y+z \end{bmatrix} = \begin{bmatrix} 9 \\ 5 \\ 7 \end{bmatrix}$

Sol. comparing same position of both matrix.

$x+y+z = 9 \longrightarrow (1)$

$x+z = 5 \longrightarrow (2)$

$y+z = 7 \longrightarrow (3)$

~~from (1),~~

~~$\Rightarrow x+y = 9-z$~~

~~put value of $x+y = 9-z$ in eqn (2)~~

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$\Rightarrow 9 - z = 5$

$\Rightarrow -z = -4$

$\Rightarrow z = 4$

put $z = 4$ in eq^n (3)

$\Rightarrow y + 4 = 7$

$\Rightarrow y = 3$

put ~~$z = 4$ and~~ $y = 3$ in eq^n (2),

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 2$

Hence, $x = 2$, $y = 3$ and $z = 4$

from (1) and (2)

$\Rightarrow 5 + y = 9$

$\Rightarrow y = 4$

put $y = 4$ in (3),

$\Rightarrow 4 + z = 7$

$\Rightarrow z = 3$

put $z = 3$ in (2)

$\Rightarrow x + 3 = 5$

$\Rightarrow x = 2$

$\Rightarrow$ Hence, $x = 2$, $y = 4$ and $z = 3$.

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Q.4 Construct 2x2 matrix, $A = [a_{ij}]$, whose elements are -

ii) $a_{ij} = \frac{i}{j}$

Sol. Given,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$\Rightarrow a_{11} = \frac{1}{1} = 1$

$\Rightarrow a_{12} = \frac{1}{2} = 0.5$ or $1/2$

$\Rightarrow a_{21} = \frac{2}{1} = 2$

$\Rightarrow a_{22} = \frac{2}{2} = 1$

$$A = \begin{bmatrix} 1 & 1/2 \\ 2 & 1 \end{bmatrix}$$

iii) $a_{ij} = \frac{(i+2j)^2}{2}$

Sol. Given,

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$\Rightarrow a_{11} = \frac{\{1+2(1)\}^2}{2} = \frac{9}{2}$

$\Rightarrow a_{12} = \frac{\{1+2(2)\}^2}{2} = \frac{25}{2}$

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$\Rightarrow a_{21} = \frac{\{2+2(1)\}^2}{2} = 8$

$\Rightarrow a_{22} = \frac{\{2+2(2)\}^2}{2} = 18$

$$A = \begin{bmatrix} \frac{9}{2} & \frac{25}{2} \\ 8 & 18 \end{bmatrix}$$

Q.5 Construct a 3x4 matrices, whose elements are given by -

i) $a_{ij} = \frac{1}{2}\left|-3i + j\right|$

Sol. Given,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$\Rightarrow a_{11} = \frac{1}{2}\left|-3(1)+1\right| = 1$

$\Rightarrow a_{12} = \frac{1}{2}\left|-3(1)+2\right| = \frac{1}{2}$

$\Rightarrow a_{13} = \frac{1}{2}\left|-3(1)+3\right| = 0$

$\Rightarrow a_{14} = \frac{1}{2}\left|-3(1)+4\right| = \frac{1}{2}$

$\Rightarrow a_{21} = \frac{1}{2}\left|-3(2)+1\right| = +\frac{5}{2}$

$\Rightarrow a_{22} = \frac{1}{2}\left|-3(2)+2\right| = 2$

$\Rightarrow a_{23} = \frac{1}{2}\left|-3(2)+3\right| = \frac{3}{2}$

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$\Rightarrow a_{24} = \frac{1}{2}\left|-3(2)+4\right| = 1$

$\Rightarrow a_{31} = \frac{1}{2}\left|-3(3)+1\right| = 4$

$\Rightarrow a_{32} = \frac{1}{2}\left|-3(3)+2\right| = \frac{7}{2}$

$\Rightarrow a_{33} = \frac{1}{2}\left|-3(3)+3\right| = 3$

$\Rightarrow a_{34} = \frac{1}{2}\left|-3(3)+4\right| = \frac{5}{2}$

$$A = \begin{bmatrix} 1 & \frac{1}{2} & 0 & \frac{1}{2} \\ \frac{5}{2} & 2 & \frac{3}{2} & 1 \\ 4 & \frac{7}{2} & 3 & \frac{5}{2} \end{bmatrix}$$

ii) $a_{ij} = 2i - j$

Sol. Given,

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & a_{14} \\ a_{21} & a_{22} & a_{23} & a_{24} \\ a_{31} & a_{32} & a_{33} & a_{34} \end{bmatrix}$$

$\Rightarrow a_{11} = 2(1) - 1 = 1$

$\Rightarrow a_{12} = 2(1) - 2 = 0$

$\Rightarrow a_{13} = 2(1) - 3 = -1$

$\Rightarrow a_{14} = 2(1) - 4 = -2$

$\Rightarrow a_{21} = 2(2) - 1 = 3$

$\Rightarrow a_{22} = 2(2) - 2 = 2$

$\Rightarrow a_{23} = 2(2) - 3 = 1$

$\Rightarrow a_{24} = 2(2) - 4 = 0$

$\Rightarrow a_{31} = 2(3) - 1 = 5$

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$\Rightarrow a_{32} = 2(3) - 2 = 4$

$\Rightarrow a_{33} = 2(3) - 3 = 3$

$\Rightarrow a_{34} = 2(3) - 4 = 2$

$$\Rightarrow A = \begin{bmatrix} 1 & 0 & -1 & -2 \\ 3 & 2 & 1 & 0 \\ 5 & 4 & 3 & 2 \end{bmatrix}$$

Q.7) find the value of a, b, c and d from the eqⁿ.

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

Sol:

$$\begin{bmatrix} a-b & 2a+c \\ 2a-b & 3c+d \end{bmatrix} = \begin{bmatrix} -1 & 5 \\ 0 & 13 \end{bmatrix}$$

on comparing same position of both matrix,

$\Rightarrow a-b = -1 \longrightarrow (1)$

$\Rightarrow 2a+c = 5 \longrightarrow (2)$

$\Rightarrow 2a-b = 0 \longrightarrow (3)$

$\Rightarrow 3c+d = 13 \longrightarrow (4)$

from (1),

$\Rightarrow -b = -1 - a$

$\Rightarrow b = 1 + a$

$\Rightarrow b = a + 1$

put $b = a+1$ in eqⁿ (3),

$\Rightarrow 2a - (a+1) = 0$

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$\Rightarrow a - 1 = 0$

$\Rightarrow a = 1$

put $a = 1$ in eqn ①,

$\Rightarrow 1 - b = -1$

$\Rightarrow -b = -2$

$\Rightarrow b = 2$

put $a = 1$ in eqn ②

$\Rightarrow 2(1) + c = 5$

$\Rightarrow c = 3$

put $c = 3$ in eqn ④,

$\Rightarrow 3(3) + d = 13$

$\Rightarrow d = 4$

Hence, $a = 1$, $b = 2$, $c = 3$, $d = 4$

Q.8. $A = [a_{ij}]_{m \times n}$ is a square matrix if

a) $m < n$, b) $m > n$, c) $m = n$ ✓, d) None

Ans. option c) $m = n$.

Q.9. Which of the given value of x and y make the following pair of matrices equal

$$\begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix}, \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

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a) $x = \frac{-1}{3}$, $y = 7$, B) Not possible to find

✓ c) $y = 7$, $x = \frac{-2}{3}$, D) $x = \frac{-1}{3}$, $y = -\frac{2}{3}$.

Sol: Case :- let both matrices are equal, then

$$\Rightarrow \begin{bmatrix} 3x+7 & 5 \\ y+1 & 2-3x \end{bmatrix} = \begin{bmatrix} 0 & y-2 \\ 8 & 4 \end{bmatrix}$$

comparing same position of matrices.

$\Rightarrow 3x+7 = 0 \longrightarrow (1)$

$\Rightarrow y+1 = 8 \longrightarrow (2)$

$\Rightarrow 2-3x = 4 \longrightarrow (3)$

from (3),

$-3x = 4 - 2$

$x = -\frac{2}{3}$

from (2),

$y = 7$

$x = -\frac{2}{3}$, $y = 7$.

Option c is correct.

5) The no. of all possible matrices of order 3x3 with each entry 0 or 1 is

a) 27 b) 18 c) 81 d) 512

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Exercise 3.2

Q.1) let $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$, $B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$, $C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

find each of the following :-

(i) $A + B$, (ii) $A - B$, (iii) $3A - C$

(iv) AB (v) BA

Sol. Given $A = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$

$B = \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$

$C = \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$

(i) $A+B$.

$$A+B = \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$A+B = \begin{bmatrix} 3 & 7 \\ 1 & 7 \end{bmatrix}$$

ii) $A - B$

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 5 & -3 \end{bmatrix}$$

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(iii) $3A - C$

$$= 3\begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 12 \\ 9 & 6 \end{bmatrix} - \begin{bmatrix} -2 & 5 \\ 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 8 & 7 \\ 6 & 2 \end{bmatrix}$$

iv) AB

$$= \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix} \cdot \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2\times1 + 4\times-2 & 2\times3 + 4\times5 \\ 3\times1 + 2\times-2 & 3\times3 + 2\times5 \end{bmatrix}$$

$$= \begin{bmatrix} -6 & 26 \\ -1 & 19 \end{bmatrix}$$

v) BA

$$= \begin{bmatrix} 1 & 3 \\ -2 & 5 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 \\ 3 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 1\times2 + 3\times3 & 1\times4 + 3\times2 \\ -2\times2 + 5\times3 & -2\times4 + 5\times2 \end{bmatrix}$$

$$= \begin{bmatrix} 11 & 10 \\ 11 & 2 \end{bmatrix}$$

Q.2. Compute the following

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} + \begin{bmatrix} a & b \\ b & a \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 2a & 2b \\ 0 & 2a \end{bmatrix}$$

(ii) $\begin{bmatrix} a^2+b^2 & b^2+c^2 \\ a^2+c^2 & a^2+b^2 \end{bmatrix} + \begin{bmatrix} 2ab & 2bc \\ -2ac & -2ab \end{bmatrix}$

Sol.

$$= \begin{bmatrix} a^2+b^2+2ab & b^2+c^2+2bc \\ a^2+c^2-2ac & a^2+b^2-2ab \end{bmatrix}$$

$$= \begin{bmatrix} (a+b)^2 & (b+c)^2 \\ (a-c)^2 & (a-b)^2 \end{bmatrix}$$

(iii) $\begin{bmatrix} -1 & 4 & -6 \\ 8 & 5 & 16 \\ 2 & 8 & 05 \end{bmatrix} + \begin{bmatrix} 12 & 7 & 6 \\ 8 & 0 & 5 \\ 3 & 2 & 4 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 11 & 11 & 0 \\ 16 & 5 & 21 \\ 5 & 10 & 9 \end{bmatrix}$$

iv) $\begin{bmatrix} \cos^2 x & \sin^2 x \\ \sin^2 x & \cos^2 x \end{bmatrix} + \begin{bmatrix} \sin^2 x & \cos^2 x \\ \cos^2 x & \sin^2 x \end{bmatrix}$

Sol.

$$= \begin{bmatrix} \cos^2 x + \sin^2 x & \sin^2 x + \cos^2 x \\ \sin^2 x + \cos^2 x & \cos^2 x + \sin^2 x \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}$$

Q.3. Compute the indicated products.

(i) $\begin{bmatrix} a & b \\ -b & a \end{bmatrix} \cdot \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$

Sol.

$$= \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \cdot \begin{bmatrix} a & -b \\ b & a \end{bmatrix}$$

$$= \begin{bmatrix} a \times a + b \times b & a \times -b + b \times a \\ -b \times a + ab & -b \times -b + a \times a \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & -ab + ab \\ -ab + ab & b^2 + a^2 \end{bmatrix}$$

$$= \begin{bmatrix} a^2 + b^2 & 0 \\ 0 & a^2 + b^2 \end{bmatrix}$$

ii) $\begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} \begin{bmatrix} 2 & 3 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 2 & 1 \times 3 & 1 \times 4 \\ 2 \times 2 & 2 \times 3 & 2 \times 4 \\ 3 \times 2 & 3 \times 3 & 3 \times 4 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 4 \\ 4 & 6 & 8 \\ 6 & 9 & 12 \end{bmatrix}$$

iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 \times 1 + -2 \times 3 & 1 \times 2 + -2 \times 2 & 1 \times 3 + -2 \times 1 \\ 2 \times 1 + 3 \times 3 & 2 \times 2 + 3 \times 2 & 2 \times 3 + 3 \times 1 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -2 & 1 \\ 11 & 10 & 9 \end{bmatrix}$$

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iii) $\begin{bmatrix} 1 & -2 \\ 2 & 3 \end{bmatrix} \cdot \begin{bmatrix} 1 & 2 & 3 \\ 2 & 3 & 1 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 1\times1 + -2\times2 & 1\times2 + -2\times3 & 1\times3 + -2\times1 \\ 2\times1 + 3\times2 & 2\times2 + 3\times3 & 2\times3 + 3\times1 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & -4 & 1 \\ 8 & 13 & 9 \end{bmatrix}$$

iv) $\begin{bmatrix} 2 & 3 & 4 \\ 3 & 4 & 5 \\ 4 & 5 & 6 \end{bmatrix} \cdot \begin{bmatrix} 1 & -3 & 5 \\ 0 & 2 & 4 \\ 3 & 0 & 5 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 2\times1 + 3\times0 + 4\times3 & 2\times-3 + 3\times2 + 4\times0 & 2\times5 + 3\times4 + 4\times5 \\ 3\times1 + 4\times0 + 5\times3 & 3\times-3 + 4\times2 + 5\times0 & 3\times5 + 4\times4 + 5\times5 \\ 4\times1 + 5\times0 + 6\times3 & 4\times-3 + 5\times2 + 6\times0 & 4\times5 + 5\times4 + 6\times5 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & 0 & 42 \\ 18 & -1 & 56 \\ 22 & -2 & 70 \end{bmatrix}$$

v) $\begin{bmatrix} 2 & 1 \\ 3 & 2 \\ -1 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 1 \\ -1 & 2 & 1 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 2\times1 + 1\times-1 & 2\times0 + 1\times2 & 2\times1 + 1\times1 \\ 3\times1 + 2\times-1 & 3\times0 + 2\times2 & 3\times1 + 2\times1 \\ -1\times1 + 1\times-1 & -1\times0 + 1\times2 & -1\times1 + 1\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 5 \\ -2 & 2 & 0 \end{bmatrix}$$

vi) $\begin{bmatrix} 3 & -1 & 3 \\ -1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 2 & -3 \\ 1 & 0 \\ 3 & 1 \end{bmatrix}$

Sol.

$$= \begin{bmatrix} 3\times2 + -1\times1 + 3\times3 & 3\times-3 + -1\times0 + 3\times1 \\ -1\times2 + 0\times1 + 2\times3 & -1\times-3 + 0\times0 + 2\times1 \end{bmatrix}$$

$$= \begin{bmatrix} 14 & -6 \\ 4 & 5 \end{bmatrix}$$

6) simplify $\cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$

Sol.

$$= \cos\theta \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix} + \sin\theta \begin{bmatrix} \sin\theta & -\cos\theta \\ \cos\theta & \sin\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta & \sin\theta\cos\theta \\ -\sin\theta\cos\theta & \cos^2\theta \end{bmatrix} + \begin{bmatrix} \sin^2\theta & -\sin\theta\cos\theta \\ \sin\theta\cos\theta & \sin^2\theta \end{bmatrix}$$

$$= \begin{bmatrix} \cos^2\theta + \sin^2\theta & \cancel{\sin\theta\cos\theta} - \cancel{\sin\theta\cos\theta} \\ \cancel{-\sin\theta\cos\theta} + \cancel{\sin\theta\cos\theta} & \cos^2\theta + \sin^2\theta \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = I$$

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9) ii) $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$ and $3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix}$

Sol. $2x + 3y = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} \longrightarrow (1)$

$3x + 2y = \begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} \longrightarrow (2)$

Multiply eq^n (1) by (3) and eq^n (2) by 2 and substract them.

$\Rightarrow \cancel{6x} + 9y = 3\begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix}$

$\cancel{6x} + 4y = 2\begin{bmatrix} 2 & -2 \\ -1 & 5 \end{bmatrix} = \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$

$- \quad -$

$\Rightarrow 5y = \begin{bmatrix} 6 & 9 \\ 12 & 0 \end{bmatrix} - \begin{bmatrix} 4 & -4 \\ -2 & 10 \end{bmatrix}$

$\Rightarrow 5y = \begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$

$\Rightarrow y = \frac{1}{5}\begin{bmatrix} 2 & 13 \\ 14 & -10 \end{bmatrix}$

$\Rightarrow y = \begin{bmatrix} \frac{2}{5} & \frac{13}{5} \\ \frac{14}{5} & -2 \end{bmatrix}$

put $y = \begin{bmatrix} 2/5 & 13/5 \\ 14/5 & -2 \end{bmatrix}$ in eq^n (1)

$$\Rightarrow 2X + 3\begin{bmatrix} \frac{2}{5} & \frac{19}{5} \\ \frac{14}{5} & -2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2X + \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} 2 & 3 \\ 4 & 0 \end{bmatrix} - \begin{bmatrix} \frac{6}{5} & \frac{39}{5} \\ \frac{42}{5} & -6 \end{bmatrix}$$

$$\Rightarrow 2X = \begin{bmatrix} \frac{4}{5} & -\frac{24}{5} \\ -\frac{22}{5} & -6 \end{bmatrix}$$

$$\Rightarrow X = \frac{1}{2}\begin{bmatrix} 4/5 & -24/5 \\ -22/5 & -6 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} \frac{4}{10} & -\frac{24}{10} \\ -\frac{22}{10} & -3 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} \frac{2}{5} & -\frac{12}{5} \\ -\frac{11}{5} & -3 \end{bmatrix}$$ Ans.

(13) If $F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$, show that $F(x) \cdot F(y) = F($

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Sol:

$$F(x) = \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(y) = \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$F(x+y) = \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$F(x) \cdot F(y) = F(x+y)$$

L.H.S. $= F(x) \cdot F(y)$

$$= \begin{bmatrix} \cos x & -\sin x & 0 \\ \sin x & \cos x & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos y & -\sin y & 0 \\ \sin y & \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos x \cos y + -\sin x \sin y + 0.0 & \cos x \cdot -\sin y + -\sin x \cdot \cos y + 0.0 & \cos x \cdot 0 + -\sin x \cdot 0 + 0 \cdot 1 \\ \sin x \cdot \cos y + \cos x \cdot \sin y + 0.0 & \sin x \cdot -\sin y + \cos x \cdot \cos y + 0.0 & \sin x \cdot 0 + \cos x \cdot 0 + 0 \cdot 1 \\ 0 \cdot \cos y + 0 \cdot \sin y + 1 \cdot 0 & 0 \cdot -\sin y + 0 \cdot \cos y + 1 \cdot 0 & 0 \cdot 0 + 0 \cdot 0 + 1 \cdot 1 \end{bmatrix}$$

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$$= \begin{bmatrix} \cos x \cos y - \sin x \sin y & -\cos x \sin y - \sin x \cos y & 0 \\ \sin x \cos y + \cos x \sin y & -\sin x \sin y + \cos x \cos y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \cos(x+y) & -(\cos x \sin y + \sin x \cos y) & 0 \\ \sin(x+y) & \cos x \cos y - \sin x \sin y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because \cos(x+y) = \cos x \cos y - \sin x \sin y$

$\sin(x+y) = \sin x \cos y + \cos x \sin y$

$$= \begin{bmatrix} \cos(x+y) & -\sin(x+y) & 0 \\ \sin(x+y) & \cos(x+y) & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$\because \sin(x+y) = \sin x \cos y + \cos x \sin y$

$\cos(x+y) = \cos x \cos y - \sin x \sin y$

L.H.S = R.H.S

Hence proved.

Q.4. IF $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$ and $C = \begin{bmatrix} 4 & 1 \\ 0 & 3 \\ 1 & -2 \end{bmatrix}$

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Q.4. if $A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$, $C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$

then compute $(A+B)$ and $(B-C)$ Also verify that

$A + (B-C) = (A+B) - C$

Sol. Given,

$$A = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$C = \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix}$$

$$(A+B) = \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix}$$

$$(B-C) = \begin{bmatrix} 3 & -1 & 2 \\ 4 & 2 & 5 \\ 2 & 0 & 3 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$(B-C) = \begin{bmatrix} -1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

Now,

$$\cancel{(A+B)}\ A + (B - C) = (A+B) - C$$

L.H.S. $= A + (B - C)$

$$= \begin{bmatrix} 1 & 2 & -3 \\ 5 & 0 & 2 \\ 1 & -1 & 1 \end{bmatrix} + \begin{bmatrix} +1 & -2 & 0 \\ 4 & -1 & 3 \\ 1 & 2 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

R.H.S. $= (A+B) - C$

$$= \begin{bmatrix} 4 & 1 & -1 \\ 9 & 2 & 7 \\ 3 & -1 & 4 \end{bmatrix} - \begin{bmatrix} 4 & 1 & 2 \\ 0 & 3 & 2 \\ 1 & -2 & 3 \end{bmatrix}$$

$$\text{R.H.S.} = \begin{bmatrix} 0 & 0 & -3 \\ 9 & -1 & 5 \\ 2 & 1 & 1 \end{bmatrix}$$

L.H.S = R.H.S

Hence proved.

Q.5 If

$$A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} \text{ and } B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

then compute $3A - 5B$.

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Sol.

$$A = \begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix}$$

$$B = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

A.T.Q.

$$= 3A - 5B.$$

$$= 3\begin{bmatrix} \frac{2}{3} & 1 & \frac{5}{3} \\ \frac{1}{3} & \frac{2}{3} & \frac{4}{3} \\ \frac{7}{3} & 2 & \frac{2}{3} \end{bmatrix} - 5\begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 1 \\ \frac{1}{5} & \frac{2}{5} & \frac{4}{5} \\ \frac{7}{5} & \frac{6}{5} & \frac{2}{5} \end{bmatrix}$$

$$= \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix} - \begin{bmatrix} 2 & 3 & 5 \\ 1 & 2 & 4 \\ 7 & 6 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

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Q.7) i) Find x and y

$$x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \quad \text{and} \quad x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

Sol.

$$x + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} \longrightarrow (1)$$

$$x - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix} \longrightarrow (2)$$

substract eqn (1) − (2)

$$\cancel{x} + y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\cancel{x} - y = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$(-)$ $(+)$ $(-)$

$$2y = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$$

$$\Rightarrow 2y = \begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow y = \frac{1}{2}\begin{bmatrix} 4 & 0 \\ 2 & 2 \end{bmatrix}$$

$$\Rightarrow y = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

put y in eqn (1)

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$$\Rightarrow X + \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 7 & 0 \\ 2 & 5 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix}$$

$$\Rightarrow X = \begin{bmatrix} 5 & 0 \\ 1 & 4 \end{bmatrix}$$

8) Find x if $Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$ and $2x + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$

Sol.

$$\Rightarrow Y = \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} \longrightarrow (1)$$

$$\Rightarrow 2x + Y = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} \longrightarrow (2)$$

from (1) & (2)

$$\Rightarrow 2x + \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix}$$

$$\Rightarrow 2x = \begin{bmatrix} 1 & 0 \\ -3 & 2 \end{bmatrix} - \begin{bmatrix} 3 & 2 \\ 1 & 4 \end{bmatrix}$$

$$\Rightarrow x = \frac{1}{2} \begin{bmatrix} -2 & -2 \\ -4 & -2 \end{bmatrix}$$

$$\Rightarrow x = \begin{bmatrix} -1 & -1 \\ -2 & -1 \end{bmatrix}$$

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9) find x and y if $2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

Sol.

$\Rightarrow 2\begin{bmatrix} 1 & 3 \\ 0 & x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2 & 6 \\ 0 & 2x \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 2+y & 6 \\ 1 & 2x+2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

on comparing both matrix,

$\Rightarrow 2+y = 5 \longrightarrow (1)$

$\Rightarrow 2x+2 = 8 \longrightarrow (2)$

from (1),

$\Rightarrow \boxed{y = 3}$

from (2)

$\Rightarrow x+1 = 4$

$\Rightarrow \boxed{x = 3}$

10) solve eqn for x, y, z and t, if $2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + \begin{bmatrix} y & 0 \\ 1 & 2 \end{bmatrix} = \begin{bmatrix} 5 & 6 \\ 1 & 8 \end{bmatrix}$

$\Rightarrow 2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$

Sol:

$$\Rightarrow 2\begin{bmatrix} x & z \\ y & t \end{bmatrix} + 3\begin{bmatrix} 1 & -1 \\ 0 & 2 \end{bmatrix} = 3\begin{bmatrix} 3 & 5 \\ 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x & 2z \\ 2y & 2t \end{bmatrix} + \begin{bmatrix} 3 & -3 \\ 0 & 6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x+3 & 2z-3 \\ 2y & 2t+6 \end{bmatrix} = \begin{bmatrix} 9 & 15 \\ 12 & 18 \end{bmatrix}$$

on comparing both matrix,

$\Rightarrow 2x+3 = 9 \rightarrow (1)$

$\Rightarrow 2z-3 = 15 \rightarrow (2)$

$\Rightarrow 2y = 12 \rightarrow (3)$

$\Rightarrow 2t+6 = 18 \rightarrow (4)$

from (1)

$\Rightarrow x = 3$

from (2)

$\Rightarrow z = 9$

from (3)

$\Rightarrow y = 6$

from (4)

$\Rightarrow t = 6$ Ans.

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11) If

$$x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$, value of x and y?

Sol.

$$\Rightarrow x \begin{bmatrix} 2 \\ 3 \end{bmatrix} + y \begin{bmatrix} -1 \\ 1 \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x \\ 3x \end{bmatrix} + \begin{bmatrix} -y \\ y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x - y \\ 3x + y \end{bmatrix} = \begin{bmatrix} 10 \\ 5 \end{bmatrix}$$

on comparing both matrices,

$\Rightarrow 2x - y = 10 \rightarrow (1)$

$\Rightarrow 3x + y = 5 \rightarrow (2)$

from (1)

$$\Rightarrow x = \frac{10 + y}{2}$$

put x in eq^n (2)

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$\Rightarrow 3\left(\frac{10+y}{2}\right) + y = 5$

$\Rightarrow \frac{30+3y}{2} + y = 5$

$\Rightarrow \cancel{y = -\frac{5}{2}}$ $\Rightarrow 30 + 5y = 10$

$\Rightarrow y = -4$

put y in eqn ①,

$\Rightarrow 2x - (-4) = 10$

$\Rightarrow 2x + 4 = 10$

$\Rightarrow x = \frac{\cancel{15}\,6}{\cancel{2}\,2}$

$\Rightarrow x = \frac{6}{2}$

$\Rightarrow x = 3$ Ans.

12) Given,

$$3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

Value of x, y, z & w.

Sol.

$$\Rightarrow 3\begin{bmatrix} x & y \\ z & w \end{bmatrix} = \begin{bmatrix} x & 6 \\ -1 & 2w \end{bmatrix} + \begin{bmatrix} 4 & x+y \\ z+w & 3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 3x & 3y \\ 3z & 3w \end{bmatrix} = \begin{bmatrix} 4+x & x+y+6 \\ z+w-1 & 2w+3 \end{bmatrix}$$

on comparing both matrices.

$\Rightarrow 3x = 4 + x \longrightarrow (1)$

$\Rightarrow x = 2 \longrightarrow (1)$

$\Rightarrow 3y = x + y + 6$

$\Rightarrow \cancel{2y +} \; 2y - x - 6 = 0 \longrightarrow (2)$

$\Rightarrow 3z = z + w - 1$

$\Rightarrow 2z - w + 1 = 0 \longrightarrow (3)$

$\Rightarrow 3w = 2w + 3$

$\Rightarrow w = 3 \longrightarrow (4)$

from (1) and (2),

$\Rightarrow 2y - 2 - 6 = 0$

$\Rightarrow y = 4 \longrightarrow (5)$

from (4) and (3)

$\Rightarrow 2z - 3 + 1 = 0$

$\Rightarrow z = 1$

$\Rightarrow x = 2$

$\Rightarrow y = 4$

$\Rightarrow z = 1$

$\Rightarrow w = 3$

Ans.

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12) show that

i) $\begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \neq \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

Sol. L.H.S. $= \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix} \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix}$

$= \begin{bmatrix} 5\times2 + -1\times3 & 5\times1 + -1\times4 \\ 6\times2 + 7\times3 & 6\times1 + 7\times4 \end{bmatrix}$

$= \begin{bmatrix} 7 & 1 \\ 33 & 34 \end{bmatrix}$

R.H.S. $= \begin{bmatrix} 2 & 1 \\ 3 & 4 \end{bmatrix} \begin{bmatrix} 5 & -1 \\ 6 & 7 \end{bmatrix}$

$= \begin{bmatrix} 2\times5 + 1\times6 & 2\times-1 + 1\times7 \\ 3\times5 + 4\times6 & 3\times-1 + 4\times7 \end{bmatrix}$

$= \begin{bmatrix} 16 & 5 \\ 43 & 25 \end{bmatrix}$

L.H.S $\neq$ R.H.S.

ii) $\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \neq \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

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Sol. L.H.S. $= \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix}$

$$= \begin{bmatrix} 1\cdot-1+2\cdot0+3\cdot2 & 1\cdot1+2\cdot-1+3\cdot3 & 1\cdot0+2\cdot1+3\cdot4 \\ 0\cdot-1+1\cdot0+0\cdot2 & 0\cdot1+1\cdot-1+0\cdot3 & 0\cdot0+1\cdot1+0\cdot4 \\ 1\cdot-1+1\cdot0+0\cdot2 & 1\cdot1+1\cdot-1+0\cdot3 & 1\cdot0+1\cdot1+0\cdot4 \end{bmatrix}$$

$$= \begin{bmatrix} 5 & 8 & 14 \\ 0 & -1 & 1 \\ -1 & 0 & 1 \end{bmatrix}$$

R.H.S $= \begin{bmatrix} -1 & 1 & 0 \\ 0 & -1 & 1 \\ 2 & 3 & 4 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}$

$$= \begin{bmatrix} -1\cdot1+1\cdot0+0\cdot1 & -1\cdot2+1\cdot1+0\cdot1 & -1\cdot3+1\cdot0+0\cdot0 \\ 0\cdot1+-1\cdot1+1\cdot0 & 0\cdot2+-1\cdot1+1\cdot1 & 0\cdot3+-1\cdot0+1\cdot0 \\ 2\cdot1+3\cdot0+4\cdot1 & 2\cdot2+3\cdot1+4\cdot1 & 2\cdot3+3\cdot0+4\cdot0 \end{bmatrix}$$

$$= \begin{bmatrix} -1 & -1 & -3 \\ -1 & 0 & 0 \\ 6 & 11 & 6 \end{bmatrix}$$

L.H.S $\neq$ R.H.S

Hence proved.

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16) If $A = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$, prove that $A^3 - 6A^2 + 7A + 2I = 0$

Sol.

$$A^2 = \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1\cdot1+0\cdot0+2\cdot2 & 1\cdot0+0\cdot2+2\cdot0 & 1\cdot2+0\cdot1+2\cdot3 \\ 0\cdot1+2\cdot0+1\cdot0 & 0\cdot0+2\cdot2+1\cdot0 & 0\cdot2+2\cdot1+1\cdot3 \\ 2\cdot1+0\cdot0+3\cdot0 & 2\cdot0+0\cdot2+3\cdot0 & 2\cdot2+0\cdot1+3\cdot3 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 5\cdot1+0\cdot0+8\cdot2 & 5\cdot0+0\cdot2+8\cdot0 & 5\cdot2+0\cdot1+8\cdot3 \\ 2\cdot1+4\cdot0+5\cdot2 & 2\cdot0+4\cdot2+5\cdot0 & 2\cdot2+4\cdot1+5\cdot3 \\ 8\cdot1+0\cdot0+13\cdot2 & 8\cdot0+0\cdot2+13\cdot0 & 8\cdot2+0\cdot1+13\cdot3 \end{bmatrix}$$

$$\Rightarrow A^3 = \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix}$$

and $I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

L.H.S =

Now, $A^3 - 6A^2 + 7A + 2I$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 5 & 23 \\ 34 & 0 & 55 \end{bmatrix} - 6\begin{bmatrix} 5 & 0 & 8 \\ 2 & 4 & 5 \\ 8 & 0 & 13 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} + 2\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 21 & 0 & 34 \\ 12 & 8 & 23 \\ 34 & 0 & 55 \end{bmatrix} - \begin{bmatrix} 30 & 0 & 48 \\ 12 & 24 & 30 \\ 48 & 0 & 78 \end{bmatrix} + \begin{bmatrix} 7 & 0 & 14 \\ 0 & 14 & 7 \\ 14 & 0 & 21 \end{bmatrix} + \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9 & 0 & -14 \\ 0 & -16 & -7 \\ -14 & 0 & -23 \end{bmatrix} + \begin{bmatrix} 9 & 0 & 14 \\ 0 & 16 & 7 \\ 14 & 0 & 23 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$= O$$

L.H.S = R.H.S

Hence proved.

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19) A trust fund has 30,000 that must be invested in two diff. types of bonds. The first bonds pays 5% interest per year. and the 2nd bond pays 7% interest per year, using matrix multiplication, determine how to divide Rs 30,000 among the 2 types of bonds. If the trust fund must obtain an annual total interest of :- a) Rs. 1800 b) Rs. 2000

Sol. a) Let amount invested in 1st type bond = x Rs.
therefore amount invested in 2nd type bond = $(30000 - x)$ Rs.

To obtain Total annual interest = 1800

$$\Rightarrow \begin{bmatrix} x & (30000-x) \end{bmatrix}_{1\times 2} \begin{bmatrix} 5\% \\ 7\% \end{bmatrix}_{2\times 1} = \begin{bmatrix} 1800 \end{bmatrix}_{1\times 1}$$

$$\Rightarrow \begin{bmatrix} x & (30,000-x) \end{bmatrix}_{1\times 2} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}_{2\times 1} = \begin{bmatrix} 1800 \end{bmatrix}_{1\times 1}$$

$$\Rightarrow x \cdot \frac{5}{100} + \frac{7(30,000-x)}{100} = 1800$$

$$\Rightarrow \frac{5x}{100} + \frac{21'0000 - 7x}{100} = 1800$$

$$\Rightarrow -2x + 210000 = 180000$$

$$\Rightarrow -2x = 180000 - 210000$$

$$\Rightarrow x = \frac{-30,000}{-2}$$

$$\Rightarrow x = 15000$$

amount invested in 1st bond $= x = 15000$

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Amount invested on 2nd bond $= 30,000 - x$

$= 30,000 - 15000$

$= 15000$ Rs.

b) Amount invested in 1st bond $= x$ RS.

Amount invested in 2nd bond $= (30,000 - x)$ RS.

To obtain total amount interest $= 2000$

$$\Rightarrow \begin{bmatrix} x & (30000 - x) \end{bmatrix}_{1\times 2} \begin{bmatrix} 5\% \\ 7\% \end{bmatrix}_{2\times 1} = \begin{bmatrix} 2000 \end{bmatrix}_{1\times 1}$$

$$\Rightarrow \begin{bmatrix} x & (30,000 - x) \end{bmatrix}_{1\times 2} \begin{bmatrix} \frac{5}{100} \\ \frac{7}{100} \end{bmatrix}_{2\times 1} = \begin{bmatrix} 2000 \end{bmatrix}_{1\times 1}$$

$$\Rightarrow x \cdot \frac{5}{100} + \frac{7(30,000 - x)}{100} = 2000$$

$$\Rightarrow 5x + 210000 - 7x = 200000$$

$$\Rightarrow -2x = 20,0000 - 210000$$

$$\Rightarrow x = \frac{-10,000}{-2}$$

$$\Rightarrow x = 5000 \text{ RS.}$$

Amount invested in 1st bond $= x = 5000$ RS.

Amount invested in 2nd bond $= 30,000 - x$

$= 30,000 - 5000$

$= 25,000$ RS.

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15) Find $A^2 - 5A + 6I$ if $A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$

Sol. Given,

$$A = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 2\times2 + 0\times2 + 1\times1 & 2\times0 + 0\times1 + 1\times-1 & 2\times1 + 0\times3 + 1\times0 \\ 2\times2 + 1\times2 + 3\times1 & 2\times0 + 1\times1 + 3\times-1 & 2\times1 + 1\times3 + 3\times0 \\ 1\times2 + -1\times2 + 0\times1 & 1\times0 + -1\times1 + 0\times-1 & 1\times1 + -1\times3 + 0\times0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix}$$

$$\Rightarrow I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now,

$$= A^2 - 5A + 6I$$

$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - 5\begin{bmatrix} 2 & 0 & 1 \\ 2 & 1 & 3 \\ 1 & -1 & 0 \end{bmatrix} + 6\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$= \begin{bmatrix} 5 & -1 & 2 \\ 9 & -2 & 5 \\ 0 & -1 & -2 \end{bmatrix} - \begin{bmatrix} 10 & 0 & 5 \\ 10 & 5 & 15 \\ 5 & -5 & 0 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} -5 & -1 & -3 \\ -1 & -7 & -10 \\ -5 & 4 & -2 \end{bmatrix} + \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -1 & -3 \\ -1 & -1 & -10 \\ -5 & 4 & 4 \end{bmatrix}$$

17) If $A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, find k so that $A^2 = kA - 2I$

Sol.

$$A = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 3\times3 + -2\times4 & 3\times-2 + -2\times-2 \\ 4\times3 + -2\times4 & 4\times-2 + -2\times-2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix}$$

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$\Rightarrow \quad I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

Now,

$\Rightarrow \quad A^2 = kA - 2I$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = k \begin{bmatrix} 3 & -2 \\ 4 & -2 \end{bmatrix} - 2 \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k & -2k \\ 4k & -2k \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} 1 & -2 \\ 4 & -4 \end{bmatrix} = \begin{bmatrix} 3k-2 & -2k \\ 4k & -2k-2 \end{bmatrix}$

on comparing both matrix,

$\Rightarrow \quad 1 = 3k - 2$

$\Rightarrow \quad 3k = 3$

$\Rightarrow \quad \boxed{k = 1}$ Ans.

18) If $A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$ and I is the identity matrix of order 2 show that.

$$I + A = (I - A) \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

Sol.

$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

$$\Rightarrow A = \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

<u>L.H.S.</u> $= I + A$

$$= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

<u>R.H.S.</u> $= (I - A)\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$$= \left\{\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} 0 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 0 \end{bmatrix}\right\}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \tan\frac{\alpha}{2} \\ -\tan\frac{\alpha}{2} & 1 \end{bmatrix}\begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \sin\alpha\tan\frac{\alpha}{2} & -\sin\alpha + \cos\alpha\tan\frac{\alpha}{2} \\ -\cos\alpha\tan\frac{\alpha}{2} + \sin\alpha & \sin\alpha\tan\frac{\alpha}{2} + \cos\alpha \end{bmatrix}$$

$$= \begin{bmatrix} \cos\alpha + \frac{\sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & -\sin\alpha + \frac{\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \frac{-\cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \sin\alpha & \frac{\sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} + \cos\alpha \end{bmatrix}$$

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$$= \begin{bmatrix} \dfrac{\cos\alpha\cos\frac{\alpha}{2} + \sin\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{-\sin\alpha\cos\frac{\alpha}{2} + \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{-\cos\alpha\sin\frac{\alpha}{2} + \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\alpha\sin\frac{\alpha}{2} + \cos\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$\cos\left(\cancel{\alpha - \frac{\alpha}{2}}\right)$

$$= \begin{bmatrix} \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\sin\frac{\alpha}{2}\cos\alpha - \sin\alpha\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\alpha\cos\frac{\alpha}{2} - \cos\alpha\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\cos\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & \dfrac{\sin\left(\frac{\alpha}{2} - \alpha\right)}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\left(\alpha - \frac{\alpha}{2}\right)}{\cos\frac{\alpha}{2}} & \dfrac{\cos\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \end{bmatrix}$$

$$= \begin{bmatrix} 1 & \dfrac{-\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} \\ \dfrac{\sin\frac{\alpha}{2}}{\cos\frac{\alpha}{2}} & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & -\tan\frac{\alpha}{2} \\ \tan\frac{\alpha}{2} & 1 \end{bmatrix}$$

L.H.S = R.H.S.

Hence proved.

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20) The bookshop of a particular school has 10 dozen chemistry books, 8 dozen physics books, 10 dozen economics books. Their selling price are Rs. 80, Rs. 60 and Rs. 40 each respectively. Find the total amount the bookshop will receive from selling all the books using matrix algebra.

Sol. In 1 dozen, total no. = 12.

then 10 dozen = $10 \times 12 = 120$

8 dozen = $8 \times 12 = 96$

let total price = x

price of books,

1 chemistry book = 80 RS

1 physics book = 60 RS.

1 economics book = 40 RS.

A.T.Q.

$$\Rightarrow \begin{bmatrix} 120 & 96 & 120 \end{bmatrix}_{1\times3} \begin{bmatrix} 80 \\ 60 \\ 40 \end{bmatrix}_{3\times1} = \begin{bmatrix} x \end{bmatrix}_{1\times1}$$

$\Rightarrow 120 \times 80 + 96 \times 60 + 120 \times 40 = x$

$\Rightarrow 9600 + 5760 + 4800 = x$

$\Rightarrow x = 9600 + 10560$

$\Rightarrow x = 20160$ RS. Ans

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Assume x, y, z, w and p are matrices of order $2 \times n$, $3 \times k$, $2 \times p$, $n \times 3$ and $p \times k$ respectively. choose the correct ans. -

21) The restriction on n, k and p so that $py + wy$ will be defined are.

Ans. A) $k = 3, \quad p = n$

22) If $n = p$ then the order of the matrix $7x - 5z$ is :-

Ans. B) $2 \times n$

Exercise 3.1

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10)

Sol. $= 2^9$ {by using permutation}

$= 512$ Ans.

option D

Transpose of a Matrix :-

Transpose of matrix A is represented by "A'".

Eg:

$$A = \begin{bmatrix} 2 & 3 & 5 \\ 6 & 7 & 9 \end{bmatrix}$$

$$A' = \begin{bmatrix} 2 & 6 \\ 3 & 7 \\ 5 & 9 \end{bmatrix}$$

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Properties of transpose of the matrices:-

i) $(A')' = A$
ii) $(KA)' = KA'$ (where K is constant)
iii) $(A+B)' = A' + B'$
iv) $(A \cdot B)' = B' \cdot A'$

Symmetric matrix :- A square matrix is called symmetric matrix when

$$\boxed{A' = A}$$

Skew symmetric matrix :- A square matrix is called skew symmetric matrix when

$$\boxed{A' = -A}$$

NOTE :- i) In pg no. 86, I have to learn Defination of Theorem 1.

ii) In pg. no. 86, I have to learn Defination of Theorem 2.

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Exercise 3.3

2) If $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$, then verify that

(i) $(A+B)' = A' + B'$ (ii) $(A-B)' = A' - B'$

Sol. Given,

$\Rightarrow$ $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$\Rightarrow$ $A+B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$\Rightarrow$ $(A+B) = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}$

$\Rightarrow$ $(A+B)' = \begin{bmatrix} -5 & 3 & -2 \\ 6 & 9 & 9 \\ -1 & 4 & 2 \end{bmatrix}'$

$\Rightarrow$ $(A+B)' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \longrightarrow (1)$

Theorem-1) For any square matrix A with real no. entries, $A+A'$ is a symmetric matrix and $A-A'$ is a skew symmetric matrix

$\Rightarrow B = A + A' \rightarrow$ symmetric matrix

$\Rightarrow C = A - A' \rightarrow$ skew symmetric matrix

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$$\Rightarrow A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} \qquad B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow A' = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}' \qquad B' = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}'$$

$$\Rightarrow A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} \qquad B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A' + B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} + \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow A' + B' = \begin{bmatrix} -5 & 6 & -1 \\ 3 & 9 & 4 \\ -2 & 9 & 2 \end{bmatrix} \longrightarrow (2)$$

from (1) & (2) $(A+B)' = A' + B'$.

Q) Express the following matrices as the sum of a symmetric and a skew symmetric matrix -

i) $\begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}$

Sol:

$$A = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix}'$$

Theorem-2) Any square matrix can be expressed as the sum of a symmetric and a skew symmetric matrix.

$$\boxed{A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')}$$

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$\Rightarrow A' = \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} + \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} - \begin{bmatrix} 3 & 1 \\ 5 & -1 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

Now, According to Theorem-2,

$\Rightarrow A = \frac{1}{2}(A+A') + \frac{1}{2}(A-A')$

$\Rightarrow \begin{bmatrix} 3 & 5 \\ 1 & -1 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 6 \\ 6 & -2 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 4 \\ -4 & 0 \end{bmatrix}$

ii) $\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

Sol.

$A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}'$

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$\Rightarrow A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} + \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} - \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

$\Rightarrow A - A' = 0$

<u>Now,</u>

$\Rightarrow A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$\Rightarrow \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 12 & -4 & 4 \\ -4 & 6 & -2 \\ 4 & -2 & 6 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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iii) $\begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}$

Sol:- $\Rightarrow A = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix}'$

$$\Rightarrow A' = \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} + \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ -1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A + A' = \begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} - \begin{bmatrix} 3 & -2 & -4 \\ 3 & -2 & -5 \\ 1 & 1 & 2 \end{bmatrix}$$

$$\Rightarrow A - A' = \begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -5 & -6 & 0 \end{bmatrix}$$

Now,

$$\Rightarrow A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$$

$$\Rightarrow \begin{bmatrix} 3 & 3 & -1 \\ -2 & -2 & 1 \\ -4 & -5 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 6 & 1 & -5 \\ 1 & -4 & -4 \\ -5 & -4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 5 & 3 \\ -5 & 0 & 6 \\ -5 & -6 & 0 \end{bmatrix}$$ Ans

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iv) $\begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix}' \Rightarrow A' = \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} + \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} - \begin{bmatrix} 1 & -1 \\ 5 & 2 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$

Now,

$A = \frac{1}{2}(A + A') + \frac{1}{2}(A - A')$

$\Rightarrow \begin{bmatrix} 1 & 5 \\ -1 & 2 \end{bmatrix} = \frac{1}{2}\begin{bmatrix} 2 & 4 \\ 4 & 4 \end{bmatrix} + \frac{1}{2}\begin{bmatrix} 0 & 6 \\ -6 & 0 \end{bmatrix}$ Ans

Q.1: find Transpose of matrix -

iv) $\begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}$ Sol. $\Rightarrow A = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 5 \\ \frac{1}{2} \\ -1 \end{bmatrix}'$

$\Rightarrow A' = \begin{bmatrix} 5 & \frac{1}{2} & -1 \end{bmatrix}$ Ans

ii) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$ Sol. $\Rightarrow A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}' \Rightarrow A' = \begin{bmatrix} 1 & \\ -1 & \end{bmatrix}$

Ans

iii) $\begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} -1 & 5 & 6 \\ \sqrt{3} & 5 & 6 \\ 2 & 3 & -1 \end{bmatrix}'$

$\Rightarrow A' = \begin{bmatrix} -1 & \sqrt{3} & 2 \\ 5 & 5 & 3 \\ 6 & 6 & -1 \end{bmatrix}$ Ans

2) $A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

show that

ii) $(A-B)' = A' - B'$

Sol.

$A = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix}$ | $B = \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$A' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix}$ | $B' = \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

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$\Rightarrow A - B = \begin{bmatrix} -1 & 2 & 3 \\ 5 & 7 & 9 \\ -2 & 1 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & -5 \\ 1 & 2 & 0 \\ 1 & 3 & 1 \end{bmatrix}$

$\Rightarrow A - B = \begin{bmatrix} 3 & 1 & 8 \\ 4 & 5 & 9 \\ -3 & -2 & 0 \end{bmatrix}$

$\Rightarrow (A-B)' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \longrightarrow (1)$

Now,

$\Rightarrow A' - B' = \begin{bmatrix} -1 & 5 & -2 \\ 2 & 7 & 1 \\ 3 & 9 & 1 \end{bmatrix} - \begin{bmatrix} -4 & 1 & 1 \\ 1 & 2 & 3 \\ -5 & 0 & 1 \end{bmatrix}$

$\Rightarrow A' - B' = \begin{bmatrix} 3 & 4 & -3 \\ 1 & 5 & -2 \\ 8 & 9 & 0 \end{bmatrix} \longrightarrow (2)$

from eqn (1) & (2) $(A-B)' = A' - B'$.

Q.3. $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix}$ & $B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$, then verify

i) $(A+B)' = A' + B'$ ii) $(A-B)' = A' - B'$

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Sol: i) $A' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} \Rightarrow A = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix}$

$\Rightarrow B = \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix} \Rightarrow B' = \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A+B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow A+B = \begin{bmatrix} 2 & 1 & 1 \\ 5 & 4 & 4 \end{bmatrix}$

$\Rightarrow (A+B)' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \longrightarrow (1)$

$\Rightarrow A' + B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} + \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A' + B' = \begin{bmatrix} 2 & 5 \\ 1 & 4 \\ 1 & 4 \end{bmatrix} \longrightarrow (2)$

from eqn (1) & (2) $(A+B)' = A' + B'$.

ii) $(A-B)' = A' - B'$

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$\Rightarrow A - B = \begin{bmatrix} 3 & -1 & 0 \\ 4 & 2 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 2 & 1 \\ 1 & 2 & 3 \end{bmatrix}$

$\Rightarrow A - B = \begin{bmatrix} 4 & -3 & -1 \\ 3 & 0 & -2 \end{bmatrix}$

$\Rightarrow (A-B)' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \longrightarrow (1)$

$\Rightarrow A' - B' = \begin{bmatrix} 3 & 4 \\ -1 & 2 \\ 0 & 1 \end{bmatrix} - \begin{bmatrix} -1 & 1 \\ 2 & 2 \\ 1 & 3 \end{bmatrix}$

$\Rightarrow A' - B' = \begin{bmatrix} 4 & 3 \\ -3 & 0 \\ -1 & -2 \end{bmatrix} \longrightarrow (2)$

from (1) & (2) $(A-B)' = A' - B'$.

(4) If $A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$,

find $A + 2B$.

Sol.

$A' = \begin{bmatrix} -2 & 3 \\ 1 & 2 \end{bmatrix}$

$A = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix}$

$B = \begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

$2B = 2\begin{bmatrix} -1 & 0 \\ 1 & 2 \end{bmatrix}$

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$\Rightarrow 2B = \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow A + 2B = \begin{bmatrix} -2 & 1 \\ 3 & 2 \end{bmatrix} + \begin{bmatrix} -2 & 0 \\ 2 & 4 \end{bmatrix}$

$\Rightarrow A + 2B = \begin{bmatrix} -4 & 1 \\ 5 & 6 \end{bmatrix}$

5) For the matrice A and B, verify that $(AB)' = B'A'$,

i) $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix}_{3\times 1}$, $B = \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}_{1\times 3}$

$\Rightarrow B' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}$

$\Rightarrow A' = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 1 \\ -4 \\ 3 \end{bmatrix} \cdot \begin{bmatrix} -1 & 2 & 1 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} 1.-1 & 1.2 & 1.1 \\ -4.-1 & -4.2 & -4.1 \\ 3.-1 & 3.2 & 3.1 \end{bmatrix}$

$\Rightarrow AB = \begin{bmatrix} -1 & 2 & 1 \\ 4 & -8 & -4 \\ -3 & 6 & 3 \end{bmatrix}$

$\Rightarrow (AB)' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \longrightarrow (1)$

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=> Now,

=> $B'A' = \begin{bmatrix} -1 \\ 2 \\ 1 \end{bmatrix}_{3\times1} \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}_{1\times3}$.

=> $B'A' = \begin{bmatrix} -1.1 & -1.-4 & -1.3 \\ 2.1 & 2.-4 & 2.3 \\ 1.1 & 1.-4 & 1.3 \end{bmatrix}$

=> $B'A' = \begin{bmatrix} -1 & 4 & -3 \\ 2 & -8 & 6 \\ 1 & -4 & 3 \end{bmatrix} \longrightarrow (2)$

from (1) & (2) $(AB)' = B'A'$

(ii) $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ | $B = \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}$

=> $A' = \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}$ | $B' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}$

=> $AB = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}_{3\times1} \begin{bmatrix} 1 & 5 & 7 \end{bmatrix}_{1\times3}$

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$\Rightarrow AB = \begin{bmatrix} 0.3 & 0.5 & 0.7 \\ 1.1 & 1.5 & 1.7 \\ 2.1 & 2.5 & 2.7 \end{bmatrix}_{3\times3}$

$\Rightarrow AB = \begin{bmatrix} 0 & 0 & 0 \\ 1 & 5 & 7 \\ 2 & 10 & 14 \end{bmatrix}$

$\Rightarrow (AB)' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \longrightarrow (1)$

$\Rightarrow B'A' = \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}_{3\times1} \begin{bmatrix} 0 & 1 & 2 \end{bmatrix}_{1\times3}$

$\Rightarrow B'A' = \begin{bmatrix} 1.0 & 1.1 & 1.2 \\ 5.0 & 5.1 & 5.2 \\ 7.0 & 7.1 & 7.2 \end{bmatrix}_{3\times3}$

$\Rightarrow B'A' = \begin{bmatrix} 0 & 1 & 2 \\ 0 & 5 & 10 \\ 0 & 7 & 14 \end{bmatrix} \longrightarrow (2)$

from (1) & (2) $(AB)' = B'A'$

6) If (i) $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$ then verify that $A'A = I$

Sol. $\Rightarrow$ $A = \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow$ $A' = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow$ $A'A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix} \begin{bmatrix} \cos\alpha & \sin\alpha \\ -\sin\alpha & \cos\alpha \end{bmatrix}$

$\Rightarrow$ $A'A = \cos\alpha \cdot \cos\alpha + (-\sin\alpha)\sin\alpha \quad -\sin\alpha$

$\Rightarrow$ $A'A = \begin{bmatrix} \cos\alpha \cdot \cos\alpha + -\sin\alpha \cdot -\sin\alpha & \cos\alpha \cdot \sin\alpha + -\sin\alpha \cos\alpha \\ \sin\alpha \cdot \cos\alpha + \cos\alpha \cdot -\sin\alpha & \sin\alpha \cdot \sin\alpha + \cos\alpha \cdot \cos\alpha \end{bmatrix}$

$\Rightarrow$ $A'A = \begin{bmatrix} \cos^2\alpha + \sin^2\alpha & \sin\alpha\cos\alpha - \sin\alpha\cos\alpha \\ \sin\alpha\cos\alpha - \sin\alpha\cos\alpha & \sin^2\alpha + \cos^2\alpha \end{bmatrix}$

$\Rightarrow$ $A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow (1)$

$\Rightarrow$ $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow (2)$

from (1) & (2) $A'A = I$

ii) If $A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$ then verify that $A'A = I$

Sol.

$A = \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix} \Rightarrow A' = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix}$

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$$\Rightarrow A'A = \begin{bmatrix} \sin\alpha & -\cos\alpha \\ \cos\alpha & \sin\alpha \end{bmatrix} \cdot \begin{bmatrix} \sin\alpha & \cos\alpha \\ -\cos\alpha & \sin\alpha \end{bmatrix}$$

$$\Rightarrow A'A = \begin{bmatrix} \sin\alpha \cdot \sin\alpha + -\cos\alpha \cdot -\cos\alpha & \sin\alpha \cdot \cos\alpha + -\cos\alpha \cdot \sin\alpha \\ \cos\alpha \cdot \sin\alpha + \sin\alpha \cdot -\cos\alpha & \cos\alpha \cdot \cos\alpha + \sin\alpha \cdot \sin\alpha \end{bmatrix}$$

$$\Rightarrow A'A = \begin{bmatrix} \sin^2\alpha + \cos^2\alpha & 0 \\ 0 & \cos^2\alpha + \sin^2\alpha \end{bmatrix}$$

$$\Rightarrow A'A = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow (1)$$

$$\Rightarrow I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \longrightarrow (2)$$

from (1) & (2) $A'A = I$.

7) i) show that the matrix $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$ is a symmetric matrix.

Sol. $A = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$

$$\Rightarrow A' = \begin{bmatrix} 1 & -1 & 5 \\ -1 & 2 & 1 \\ 5 & 1 & 3 \end{bmatrix}$$

$\because A = A'$ Hence it is symmetric matrix.

ii) Show that, $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$ is a skew symmetric matrix.

Sol. $A = \begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$\Rightarrow A' = \begin{bmatrix} 0 & -1 & 1 \\ 1 & 0 & -1 \\ -1 & 1 & 0 \end{bmatrix}$

$\Rightarrow A' = -\begin{bmatrix} 0 & 1 & -1 \\ -1 & 0 & 1 \\ 1 & -1 & 0 \end{bmatrix}$

$\Rightarrow A' = -A$

Hence it is skew symmetric matrix.

8) $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix}$, verify

if $(A+A')$ is a symmetric matrix

Sol. $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} + \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow A + A' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \longrightarrow (1)$

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$\rightarrow (A+A')' = \begin{bmatrix} 2 & 11 \\ 11 & 14 \end{bmatrix} \rightarrow (2)$

$\rightarrow$ from (1) & (2),

$A+A' = (A+A')'$

Hence it is symmetric matrix

ii) $(A-A')$ is a skew symmetric matrix.

Sol. $A = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 1 & 5 \\ 6 & 7 \end{bmatrix} - \begin{bmatrix} 1 & 6 \\ 5 & 7 \end{bmatrix}$

$\Rightarrow A - A' = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow (1)$

$\Rightarrow (A-A')' = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}$

$\Rightarrow (A-A')' = -\begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix} \rightarrow (2)$

from (1) & (2)

$\Rightarrow (A-A')' = -(A-A')$

Hence it is skew symmetric matrix.

9) find $\frac{1}{2}(A+A')$ and $\frac{1}{2}(A-A')$, when $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$

Sol. $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} \Rightarrow A' = \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$

$$\Rightarrow A + A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} + \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

Sol

$$\Rightarrow A + A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \frac{1}{2}\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A + A') = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$$

and,

$$\Rightarrow A - A' = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix} - \begin{bmatrix} 0 & -a & -b \\ a & 0 & -c \\ b & c & 0 \end{bmatrix}$$

$$\Rightarrow (A - A') = \begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

$$\Rightarrow \frac{1}{2}(A - A') = \frac{1}{2}\begin{bmatrix} 0 & 2a & 2b \\ -2a & 0 & 2c \\ -2b & -2c & 0 \end{bmatrix}$$

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$$\Rightarrow \quad \frac{1}{2}(A - A') = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$$

11) if A, B are symmetric matrices of same order, than AB − BA is

a) skew symmetric matrix

✓ b) symmetric matrix

c) zero matrix

d) Identity matrix,

Sol. $\Rightarrow (AB - BA)' = (AB)' - (BA)'$

$= B'A' - A'B'$

$= BA - AB$

$= -(AB - BA)$

skew symmetric matrix

As $\begin{cases} A' = A \\ B' = B \end{cases}$ A & B are symmetric matrix

Ans. b) skew symmetric matrix.

12) If $A = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$ then $A + A' = I$, if the value of α is

(A) $\frac{\pi}{6}$ ✓ b) $\frac{\pi}{3}$ c) π d) $\frac{3\pi}{2}$

Ans. b) $\frac{\pi}{3}$

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Note :-

i) if $A \cdot B = I$ then B is said to be inverse of A that is

$$B = A^{-1}$$

ii) $A = A \cdot I$ or $A = I \cdot A$

Elementry operation :-

i) The interchange of any two rows or two columns. symbolically the interchange of ith and j^{th} rows is denoted by $R_i \longleftrightarrow R_j$ and interchange of ith and j^{th} column is denoted by $C_i \longleftrightarrow C_j$

ii) The multiplication of the elements of any row or column by a non-zero number. symbolically, the multiplication of each element of the ith row by K, where $K \neq 0$ is denoted by $R_i \longleftrightarrow KC_i$

The corresponding column operation is denoted by $C_i \longleftrightarrow KC_i$

iii) The addition to the elements of any row or column, the corresponding elements of any other row or column multiplied by any non-zero no., symbolically, the addition to the elements of ith row, the corresponding elements of jth row multiplied by K is denoted by $R_i \longrightarrow R_i + KR_j$

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⇒ NOTE :- i) when we use elementry operation by row, then

$$A = IA$$

ii) In order to use elementry column operation we write

$$\boxed{A = AI}$$

iii) If any matrix having all elements 0 (zero) in any row or any column then A^{-1} does not exist.

Exercise 3.4

3) find the inverse $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix}$

In order to use elementry row operation we may write,

$$\Rightarrow \quad A = IA$$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

$$\Rightarrow I = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 7 & -3 \\ -2 & 1 \end{bmatrix}$$ Ans.

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→ Using Elementry transformations, find the inverse of each of the matrices if it exist in Exercise 1 to 17.

1) $\begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix}$

In order to use elementry row operation, we may write

$$\Rightarrow A = IA$$

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$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 5 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$$

By process $R_2 \to \frac{1}{5} R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

By process $R_1 \to R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{3}{5} & +\frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} \frac{3}{5} & \frac{1}{5} \\ -\frac{2}{5} & \frac{1}{5} \end{bmatrix}$$

4) $\begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix}$

In order to use elementary row operation,

$$A = IA$$

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$$\Rightarrow \begin{bmatrix} 2 & 3 \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_1 \to R_1 - \frac{1}{5} R_2$

$$\Rightarrow \begin{bmatrix} 1 & \frac{8}{5} \\ 5 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{5} \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 5R_1$

$$\Rightarrow \begin{bmatrix} 1 & \frac{8}{5} \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} 1 & -\frac{1}{5} \\ -5 & 2 \end{bmatrix} A$$

By process $R_1 \to R_1 - (-\frac{8}{5} R_2)$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -7 & -\frac{1}{5} + \frac{16}{5} \\ -5 & 2 \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix} A$$

Rough

$\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$ $\begin{matrix} -7 & 3 \\ 5 & -2 \end{matrix}$

By process $R_1 \to \cancel{R_2 - R_1}$ $R_1 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -12 & 5 \\ -5 & 2 \end{bmatrix} A$$

$\Rightarrow$ ~~By process $R_2 \to R_1 - R_2$~~

$\Rightarrow$ ~~$\begin{matrix} 1 & 0 \\ 0 & -1 \end{matrix}$~~

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ -5 & 2 \end{bmatrix} A$

- By process $R_2 \to -1 R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix} A$

$\Rightarrow \therefore A^{-1} = \begin{bmatrix} -7 & 3 \\ 5 & -2 \end{bmatrix}$

6) $\begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix}$

In order to use elementry row transformation,

$\Rightarrow A = IA$

$\Rightarrow \begin{bmatrix} 2 & 5 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

By process $R_1 \to R_1 - R_2$

$\Rightarrow \begin{bmatrix} 1 & 2 \\ 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$

$\Rightarrow$ By process $R_2 \to R_2 - R_1$

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$$\Rightarrow \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} A$$

By process $R_1 \to R_1 - 2R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 3 & -5 \\ -1 & 2 \end{bmatrix}$$

7) $\begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}$

In order to use elementry row tranformation,

$$\Rightarrow A = IA$$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_2 \to R_2 - R_1$

$$\Rightarrow \begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - R_2$

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$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -1 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 2 & -1 \\ -5 & 3 \end{bmatrix}$$

9) $\begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix}$

In order to use elementry row transformation,

$$\Rightarrow A = IA$$

$$\Rightarrow \begin{bmatrix} 3 & 10 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 2 & 7 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 2R_1$

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$$\Rightarrow \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -2 & 3 \end{bmatrix} A$$

By process $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 7 & -10 \\ -2 & 3 \end{bmatrix}$$

10) $\begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix}$

In order to use elementory row operation,

$$\Rightarrow A = IA$$

$$\Rightarrow \begin{bmatrix} 3 & -1 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - \left(-\frac{1}{2} R_2\right)$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ -4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + 4R_1$

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$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 4 & 3 \end{bmatrix} A$$

By process $R_2 \to \frac{1}{2} R_2$.

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 3/2 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 1 & \frac{1}{2} \\ 2 & 3/2 \end{bmatrix}$$

12) $\begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix}$

In order to use elementry row operation,

$$\Rightarrow A = IA$$

$$\Rightarrow \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_1 \to \frac{1}{6} R_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \end{bmatrix} A$$

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$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - \frac{1}{2} R_2$

$$\Rightarrow \begin{bmatrix} 0 & 0 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & \frac{1}{2} \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$

$$\Rightarrow \begin{bmatrix} 6 & -3 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + \frac{1}{3} R_1$

$$\Rightarrow \begin{bmatrix} 6 & -3 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

By process $R_1 \to \frac{1}{3} R_1$

$$\Rightarrow \begin{bmatrix} 2 & -1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{3} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

By process $R_1 \to \frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ \frac{1}{3} & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + (-2 R_1)$

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$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_1 \rightarrow -2R_1$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_2 \rightarrow R_2 - R_1$

$$\Rightarrow \begin{bmatrix} -2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} -\frac{1}{3} & 0 \\ -\frac{1}{3} & -1 \end{bmatrix} A$$

By process $R_1 \rightarrow -\frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & -\frac{1}{2} \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} \frac{1}{6} & 0 \\ -\frac{1}{3} & -1 \end{bmatrix}$$

In this L.H.S matrix 2nd row has only zero (0) elements theirefore it is clear that inverse of matrix is does not exists.

$\Rightarrow \therefore A^{-1}$ = does not exist

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13 $\begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix}$

In order to use elementary row operation,

$\Rightarrow A = IA$

$\Rightarrow \begin{bmatrix} 2 & -3 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

By process $R_1 \to R_1 + R_2$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} A$

By process $R_2 \to R_2 + R_1$

$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & 2 \end{bmatrix} A$

By process $R_1 \to \cancel{R_1 - R_2}\ R_1 + R_2$

$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix} A$

$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 2 & 3 \\ 1 & 2 \end{bmatrix}$

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(ii) $\begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix}$

In order to use elementary row operation,

$\Rightarrow A = IA$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$

By process $R_2 \to R_2 - 2R_1$

$\Rightarrow \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ -2 & 1 \end{bmatrix} A$

$\Rightarrow A^{-1}$ = does not exist

because elements of 2nd row are zero in L.H.S matrix.

(iii) $\begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix}$

In order to use elementry column operation we write,

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$\Rightarrow \quad A = AI$

$$\Rightarrow \begin{bmatrix} 2 & -6 \\ 1 & -2 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By process $C_1 \to \frac{1}{2} C_1$

$$\Rightarrow \begin{bmatrix} 1 & -6 \\ \frac{1}{2} & -2 \end{bmatrix} = A \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & 1 \end{bmatrix}$$

By process $C_2 \to -\frac{1}{2} C_2$

$$\Rightarrow \begin{bmatrix} 1 & 3 \\ \frac{1}{2} & 1 \end{bmatrix} = A \begin{bmatrix} \frac{1}{2} & 0 \\ 0 & -\frac{1}{2} \end{bmatrix}$$

By process $C_2 \to C_2 - 3C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ \frac{1}{2} & -\frac{1}{2} \end{bmatrix} = A \begin{bmatrix} \frac{1}{2} & -\frac{3}{2} \\ 0 & -\frac{1}{2} \end{bmatrix}$$

By process $C_1 \to C_1 + C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & -1/2 \end{bmatrix} = A \begin{bmatrix} -1 & -3/2 \\ -1/2 & -1/2 \end{bmatrix}$$

By process $C_2 \to -2 C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} -1 & 3 \\ -1/2 & 1 \end{bmatrix}$$

$= 2 + \frac{-}{2}$

$= \frac{4+9}{2}$ $\quad \frac{5}{2}$

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15) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

In order to use elimentry row operation,

$\Rightarrow \quad A = IA$

$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

By process $R_2 \to R_2 - R_1$

$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$

By process $R_3 \to R_3 - \frac{3}{2} R_1$

$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 5 & 0 \\ 0 & \frac{5}{2} & -\frac{5}{2} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -\frac{3}{2} & -\frac{3}{2} & -\frac{1}{2} \end{bmatrix} A$

By process $R_3 \to -\frac{2}{5} R_3$

& $R_2 \to \frac{1}{5} R_2$

Rough

$1 - \frac{3}{2}$

Calculation mistake in this page

$1 - \frac{6}{5}$ $\quad \frac{12}{5}$

$-\frac{1}{5}$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/5 & \frac{1}{5} & 0 \\ 3/5 & 3/5 & \frac{1}{5} \end{bmatrix} A$$

By process $R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ +\frac{2}{5} & \frac{4}{5} & \frac{1}{5} \end{bmatrix} A$$

Rough

$1 + \frac{3}{5}$ $\quad 1 - \frac{3}{5}$

$\frac{2}{5} - \frac{6}{5}$ $\quad -\frac{3}{5}$

$\frac{2}{5} + \frac{3}{5}$ $\quad \frac{3}{5} - \frac{3 \cdot 1}{5}$

1 $\quad$ 0

$0 - 3 \cdot 0$

0

By process $R_1 \to R_1 + 3R_2$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{2}{5} & \frac{3}{5} & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ +\frac{2}{5} & \frac{4}{5} & \frac{1}{5} \end{bmatrix} A$$

By process $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 2 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ +2/5 & 4/5 & 1/5 \end{bmatrix} A$$

By process $R_1 \to \frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{1}{5} & \frac{1}{5} & 0 \\ +\frac{2}{5} & 4/5 & 1/5 \end{bmatrix} A$$

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15) $\begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

Sol. Let $A = \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix}$

In order to use elimentory row operation,

$\Rightarrow \quad A = IA$

$$\Rightarrow \begin{bmatrix} 2 & -3 & 3 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_1 - R_3$

$$\Rightarrow \begin{bmatrix} -1 & -1 & 1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_1 \to -1 R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 2 & 2 & 3 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 3 & -2 & 2 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ & & \\ 0 & 0 & 1 \end{bmatrix} A$$

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By process $R_3 \to R_3 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 5 \\ 0 & -5 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 2 & 1 & -2 \\ 3 & 0 & -2 \end{bmatrix} A$$

By process $R_2 \longleftrightarrow R_3$ {interchange each other}

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & -5 & 5 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ 3 & 0 & -2 \\ 2 & 1 & -2 \end{bmatrix} A$$

By process $R_2 = -\frac{1}{5} R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -1 & 0 & 1 \\ -3/5 & 0 & 2/5 \\ 2 & 1 & -2 \end{bmatrix} A$$

By process $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 5 \end{bmatrix} = \begin{bmatrix} -\frac{2}{5} & 0 & \frac{3}{5} \\ -3/5 & 0 & 2/5 \\ 2 & 1 & -2 \end{bmatrix} A$$

By process $R_3 \to \frac{1}{5} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/5 & 0 & 3/5 \\ -3/5 & 0 & 2/5 \\ 2/5 & 1/5 & -2/5 \end{bmatrix} A$$

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By process $R_2 \to R_2 + R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2/5 & 0 & 3/5 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} -2/5 & 0 & 3/5 \\ -1/5 & 1/5 & 0 \\ 2/5 & 1/5 & -2/5 \end{bmatrix}$$ Ans.

Remark

$-\frac{3}{5} + \frac{2}{5}$

$-\frac{1}{5}$

$0 + \frac{1}{5}$

$\frac{2}{5} + \frac{3}{5}$

$\frac{2-10}{5} = -\frac{8}{5}$

2) $\begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix}$

In order to use elimentory column operation,

$\Rightarrow A = AI$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 1 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By process $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

By process $C_2 \to C_2 - 2C_1$

$$\Rightarrow \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$$

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By process $C_2 \to C_2 + C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$ Ans.

5) $\begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

Sol: Let $A = \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix}$

In order to use elimentory column method,

$$\Rightarrow A = AI$$

$$\Rightarrow \begin{bmatrix} 2 & 1 \\ 7 & 4 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

By process $C_1 \to C_1 - C_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = A \begin{bmatrix} 1 & 0 \\ -1 & 1 \end{bmatrix}$$

By process $C_2 \to C_2 - C_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix} = A \begin{bmatrix} 1 & -1 \\ -1 & 2 \end{bmatrix}$$

By process $C_1 \to C_1 - 3C_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = A \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 4 & -1 \\ -7 & 2 \end{bmatrix}$$ Ans.

8) $\begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

Sol: let $A = \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix}$

In order to use elimentory ~~column~~ row operation,

$$\Rightarrow A = IA$$

$$\Rightarrow \begin{bmatrix} 4 & 5 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 3 & 4 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_2 \to R_2 - 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ -3 & 4 \end{bmatrix} A$$

By process $R_1 \to R_1 - R_2$

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$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix} A$$

$$\Rightarrow \therefore A^{-1} = \begin{bmatrix} 4 & -5 \\ -3 & 4 \end{bmatrix}$$ Ans.

(c) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$ wrong

Sol. $\det A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

In order to use elementary row operation,

$$\Rightarrow A = I A$$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_3 \to R_3 - 2R_1$

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$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

$\Rightarrow$ By process $R_2 \to R_2 + 8R_3$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

By process $R_1 \to R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

By process $R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

By process $R_3 \to \frac{1}{25} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

By process $R_2 \to R_2 - 21 R_3$

Rough

$\frac{180}{5}$ $\quad -13 + \frac{63}{5}$ $\quad 8 - \frac{25}{9} \cdot 21$ $\quad \frac{528}{72}$ $\quad \frac{453}{9}$

$\frac{20}{5}$ $\quad \frac{-9}{5}$ $\quad \frac{72-525}{9}$ $\Rightarrow$ $\frac{453}{9}$

$\frac{200-189}{25}$ $\quad \frac{11}{25}$

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$\frac{200-195}{5}$

1

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -\frac{2}{5} & -\frac{21}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$-24 + 65$

$\frac{65 \cdot 25}{5}$

$\frac{-216+625}{9}$

$625 - 216$

409

$\Rightarrow$ By process $R_1 \to R_1 + 65 R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{13}{5} & \frac{117}{5} \\ -\frac{2}{5} & -\frac{21}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & \frac{13}{5} & \frac{117}{5} \\ -\frac{2}{5} & -\frac{21}{25} & \frac{11}{25} \\ \frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & 0 & \frac{409}{9} \\ -\frac{2}{5} & 0 & \frac{151}{3} \\ -\frac{3}{5} & 0 & \frac{25}{9} \end{bmatrix} \qquad \begin{bmatrix} 1 & \frac{13}{5} & \frac{117}{5} \\ -\frac{2}{5} & -\frac{21}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

---

17) $\begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 2 & 0 & -1 \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix}$

In order to use Elementary ~~column~~ row operation,

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$\Rightarrow \quad A = IA$

$$\Rightarrow \begin{bmatrix} 2 & 0 & -\frac{1}{2} \\ 5 & 1 & 0 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_1 \to \frac{1}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & 5/2 \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -5/2 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 - 5R_1$,

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 1 & 3 \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_3 \to R_3 - R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -\frac{1}{2} \\ 0 & 1 & \frac{5}{2} \\ 0 & 0 & \frac{1}{2} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} & 0 & 0 \\ -\frac{5}{2} & 1 & 0 \\ 5 & -2 & 2 \end{bmatrix} A$$

By process $R_1 \to R_1 + \frac{1}{2} R_3$

$R_2 \to R_2 - \frac{5}{2} R_1$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix} A$$

$$\Rightarrow \therefore \quad A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

16) $\begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

Sol. let $A = \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix}$

In order to use elementry row operation

$\Rightarrow \quad A = IA$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + 3R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 2 & 5 & 0 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} A$$

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By process $R_3 \to R_3 - 2R_1$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 9 & -11 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 1 & 0 \\ -2 & 0 & 1 \end{bmatrix} A$$

By process $R_2 \to R_2 + 8R_3$

$$\Rightarrow \begin{bmatrix} 1 & 3 & -2 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

By process $R_1 - 3R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & -1 & 4 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -2 & 0 & 1 \end{bmatrix} A$$

By process $R_3 \to R_3 + R_2$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & 0 & 25 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -15 & 1 & 9 \end{bmatrix} A$$

By process $R_3 \to \frac{1}{25} R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 21 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -13 & 1 & 8 \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$1 - \frac{21}{25}$ $\quad -\frac{63+63}{25}$ $\quad -\frac{2}{25}$ $\quad \frac{4}{25}$

$\frac{4}{25}$

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By process $R_2 \to R_2 - 21R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & -65 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 40 & 0 & -24 \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

By process $R_1 \to R_1 + 65R_3$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & & \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix} A$$

$$\Rightarrow A^{-1} = \begin{bmatrix} 1 & -\frac{2}{5} & -\frac{3}{5} \\ -\frac{2}{5} & \frac{4}{25} & \frac{11}{25} \\ -\frac{3}{5} & \frac{1}{25} & \frac{9}{25} \end{bmatrix}$$

C.W.

Miscellaneous Exercise

3) If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then prove that

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}.$$

where n is any positive integer.

Sol. Given,

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

To prove

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

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for $n = 1$

$$A = \begin{bmatrix} 1+2\times1 & -4\times1 \\ 1 & 1-2\times1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

it is true for $n=1$

let it is true for $n=k$.

Now,

$$A^k = \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix}$$

Now,

$$A^{k+1} = k^k \cdot A$$

$$= \begin{bmatrix} 1+2k & -4k \\ k & 1-2k \end{bmatrix} \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$$

$$= \begin{bmatrix} (1+2k)3 + -4k(1) & (1+2k)-4 + (-4k)-1 \\ (k)(3) + (1-2k)(1) & k.-4 + 1-2k.-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+6k-4k & -4+8k-4k \\ 3k-2k+1 & -4k+2k-1 \end{bmatrix}$$

$$= \begin{bmatrix} 3+2k & -4-4k \\ k+1 & -1-2k \end{bmatrix}$$

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$$\Rightarrow A^{k+1} = \begin{bmatrix} 1+2(k+1) & -4(k+1) \\ k+1 & 1-2(k+1) \end{bmatrix}$$

$\therefore$ it is true for $n = k+1$.

Hence if it true for $n=k$ then it is also true for $n=k+1$ ~~by meth~~

By mathematical induction

$$A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$$

4) If A and B are symmetric matrix, prove that $AB - BA$ is a skew symmetric matrix.

Sol. Given,

A and B are symmetric matrix

Given,

$A' = A$

$B' = B$

Now,

$$(AB - BA)' = (AB)' - (BA)'$$
$$= B'A' - A'B'$$
$$= BA - AB$$
$$(AB - BA)' = -(AB - BA)$$

$\therefore$ $AB - BA$ is skew symmetric matrix

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a) Find x, if $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

Sol. $\begin{bmatrix} x & -5 & -1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 2 \\ 0 & 2 & 1 \\ 2 & 0 & 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x\cdot 1 + -5\cdot 0 + -1\cdot 2 & x\cdot 0 + -5\cdot 2 + -1\cdot 0 & x\cdot 2 + -5\cdot 1 + -1\cdot 3 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x-2 & -10 & 2x-8 \end{bmatrix} \begin{bmatrix} x \\ 4 \\ 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} (x-2)(x) - 10(4) + (2x-8)1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x^2 - \cancel{2x} - 40 + \cancel{2x} - 8 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} x^2 - 48 \end{bmatrix} = \begin{bmatrix} 0 \end{bmatrix}$

$\Rightarrow x^2 - 48 = 0$

$\Rightarrow x^2 = 48$

$\Rightarrow x = \sqrt{48}$

$\Rightarrow x = 4\sqrt{3}$

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H.W.

Exercise 3.4

(18) Matrice A and B are inverse of each other only if :-

Ans. D) $AB = BA = I$

Miscellaneous

1) let $A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$, show that, ~~$aI + bA$~~

$(aI + bA)^n = a^n I + na^{n-1} bA$, where I is the identity matrix of order 2 and $n \in N$.

Sol. Given,

$$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \text{ and } I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

To prove,

$$\Rightarrow (aI + bA)^n = a^n I + na^{n-1} bA$$

for $n = 1$,

$$\Rightarrow (aI + bA) = a^1 I + 1a^{1-1} bA$$

$$= aI + 1(bA)$$

$$= aI + bA$$

It is true for $n = 1$.

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Let it is true for $n = k$.

Now,

$$(aI + bA)^k = a^k I + k a^{k-1} bA \longrightarrow (1)$$

Now,

$$\Rightarrow (aI + bA)^{k+1} = \cancel{a^{k+1} I + (k+1) a^{k+1-1} bA (aI+bA)^k (aI+b}$$

$$= (a^k I + k a^{k-1} bA)(aI + bA)$$

$(\because$ from $(1))$

$$= a^k \cdot aI + k \cdot a^{k-1} bA \cdot aI + a^k \cdot I \cdot bA + k a^{k-1} bA \cdot bA$$

$$= a^{k+1} I + k a^k bAI + a^k bAI + k a^{k-1} b^2 A^2$$

$$= a^{k+1} I + a^k k bAI + a^k bAI + k a^{k-1} b^2 A^2$$

$$= a^{k+1} I + a^k bA(k+1) + k a^{k-1} A^2 b^2 \longrightarrow (2)$$

$$\because A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0.0 + 1.0 & 0.1 + 1.0 \\ 0.0 + 0.0 & 0.1 + 0.0 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

$$\Rightarrow A^2 = 0 \longrightarrow (3)$$

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from (2) and (3)

$\Rightarrow (aI + bA)^{k+1} = a^{k+1} I + a^k bA(k+1) + ka^{k-1}(0) b^2$

$\Rightarrow (aI + bA)^{k+1} = a^{k+1} I + a^k bA(k+1)$

$\Rightarrow (aI + bA)^{k+1} = a^{k+1} I + (k+1) a^k bA.$

It is also true for $n = k+1$.

Hence it is true for $n = k$ ~~and~~ then also true for $n = k+1$.

By mathematical induction,

$(aI + bA)^n = a^n I + n a^n bA$ and

$A = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}$.

$\rightarrow n \in \mathbb{N}$.

2) If $A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$, prove that

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

Sol. Given,

$$A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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To prove,

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix}$$

for $n = 1$

$$\Rightarrow A = \begin{bmatrix} 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \\ 3^{1-1} & 3^{1-1} & 3^{1-1} \end{bmatrix}$$

$$\Rightarrow A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

It is true for $n = 1$.

Now,

for $n = k$,

$$\Rightarrow A^k = \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \longrightarrow (1)$$

Now,

for $n = k+1$

$$\Rightarrow A^{k+1} = A^k \cdot A$$

$$= \begin{bmatrix} 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \\ 3^{k-1} & 3^{k-1} & 3^{k-1} \end{bmatrix} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix} \{\text{from } (1)\}$$

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$$= \begin{bmatrix} 3^{(k-1)}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 \\ 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 \\ 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 & 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 + 3^{k-1}\cdot 1 \end{bmatrix}$$

$$= \begin{bmatrix} 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \\ 3\cdot 3^{k-1} & 3\cdot 3^{k-1} & 3\cdot 3^{k-1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{k-1+1} & 3^{k-1+1} & 3^{k-1+1} \\ 3^{k-1+1} & 3^{k-1+1} & 3^{k-1+1} \\ 3^{k-1+1} & 3^{k-1+1} & 3^{k-1+1} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{k} & 3^{k} & 3^{k} \\ 3^{k} & 3^{k} & 3^{k} \\ 3^{k} & 3^{k} & 3^{k} \end{bmatrix}$$

$$= \begin{bmatrix} 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \\ 3^{(k+1)-1} & 3^{(k+1)-1} & 3^{(k+1)-1} \end{bmatrix}$$

It is also true for $n = k+1$

Hence it is true for $n = k$ then it is ~~also~~ true for $n = k+1$

by mathematical induction,

$$A^n = \begin{bmatrix} 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \\ 3^{n-1} & 3^{n-1} & 3^{n-1} \end{bmatrix} \quad \text{where, } A = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

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Q5 show that the matrix $B'AB$ is a symmetric or skew symmetric ~~assuming~~ ~~assuming~~ according as A is symmetric or skew symmetric.

Sol. let A is symmetric.

$$A' = A$$

Now,

$$(B'AB)' = (B'(AB))'$$
$$= (AB)'(B')'$$
$$= B'A' \cdot B$$
$$= B'AB \qquad \{\because A \text{ is symmetric } A' = A\}$$

$$(B'AB)' = B'AB$$

Hence it is symmetric matrix.

if A is skew symmetric,
then $A' = -A$.

Now,

$$(B'AB)' = (B'(AB))'$$
$$= (AB)'(B')'$$
$$= B'A'B$$
$$= B'(-A)B \qquad \{A \text{ is skew symmetric matrix } A' = -A\}$$

$$(B'AB)' = -B'AB$$

Hence, it is skew symmetric matrix.

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(6) find the values of x, y, z if the matrix

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$ satisfy the eq^n $A'A = I$

Sol:

Given,

$$A = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}$$

$$A' = \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix}'$$

$$A' = \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix}$$

and $$I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

A.T.Q.

$\Rightarrow$ $A'A = I$

$$\Rightarrow \begin{bmatrix} 0 & x & x \\ 2y & y & -y \\ z & -z & z \end{bmatrix} \cdot \begin{bmatrix} 0 & 2y & z \\ x & y & -z \\ x & -y & z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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$$\Rightarrow \begin{bmatrix} 0\cdot 0 + x\cdot x + x\cdot x & 0\cdot 2y + x\cdot y + x\cdot -y & 0\cdot z + x\cdot -z + x\cdot z \\ 2y\cdot 0 + y\cdot x + -y\cdot x & 2y\cdot 2y + y\cdot y + -y\cdot -y & 2y\cdot z + y\cdot -z + -y\cdot z \\ z\cdot 0 + z\cdot x + z\cdot x & z\cdot 2y + -z\cdot y + z\cdot -y & z\cdot z + -z\cdot -z + z\cdot z \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} 2x^2 & 0 & 0 \\ 0 & 6y^2 & 0 \\ 0 & 0 & 3z^2 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

on comparing both matrix,

$\Rightarrow 2x^2 = 1 \rightarrow (1)$

$\Rightarrow 6y^2 = 1 \rightarrow (2)$

$\Rightarrow 3z^2 = 1 \rightarrow (3)$

from (1),

$\Rightarrow x = \pm \frac{1}{\sqrt{2}}$

from (2),

$\Rightarrow y = \pm \frac{1}{\sqrt{6}}$

from (3),

$\Rightarrow z = \pm \frac{1}{\sqrt{3}}$

(4) for what value of x, $\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$

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Sol:

$$\begin{bmatrix} 1 & 2 & 1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 0 & 2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1\cdot1+2\cdot2+1\cdot1 & 1\cdot2+2\cdot0+1\cdot0 & 1\cdot0+2\cdot1+1\cdot2 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4 & 2 & 4 \end{bmatrix} \begin{bmatrix} 0 \\ 2 \\ x \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 4\cdot0+2\cdot2+4\cdot x \end{bmatrix}_{1\times1} = \begin{bmatrix} 0 \end{bmatrix}_{1\times1}$$

$$\Rightarrow 4+4x = 0$$

$$\Rightarrow x = \frac{-4}{4}$$

$$\Rightarrow x = -1$$

8) If $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, show that $A^2 - 5A + 7I = 0$

Sol: Given,

$$I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

$$A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$$

$$A^2 = \begin{bmatrix} 3\cdot3+1\cdot-1 & 3\cdot1+1\cdot2 \\ -1\cdot3+2\cdot-1 & -1\cdot1+2\cdot2 \end{bmatrix}$$

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$\Rightarrow A^2 = \begin{bmatrix} 8 & 5 \\ -5 & +3 \end{bmatrix}$

Now,

$\Rightarrow A^2 - 5A + 7I = 0$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & +3 \end{bmatrix} - 5\begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix} + 7\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 8 & 5 \\ -5 & +3 \end{bmatrix} - \begin{bmatrix} 15 & 5 \\ -5 & 10 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} -7 & 0 \\ 0 & -7 \end{bmatrix} + \begin{bmatrix} 7 & 0 \\ 0 & 7 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = 0$

$\Rightarrow \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$

L.H.S = R.H.S.

Hence proved.

Q.No.

11) Find the matrix X so that $X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

Sol. Given,

$\Rightarrow X\begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$

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order of required matrix is 2×2 because
⇒ no. of row in L.H.S matrix is 2 and no. of row in R.H.S matrix is 2.

let $X = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$

$$\Rightarrow \begin{bmatrix} a & b \\ c & d \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a\cdot1+b\cdot4 & a\cdot2+b\cdot5 & a\cdot3+b\cdot6 \\ c\cdot1+d\cdot4 & c\cdot2+d\cdot5 & c\cdot3+d\cdot6 \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} a+4b & 2a+5b & 3a+6b \\ c+4d & 2c+5d & 3c+6d \end{bmatrix} = \begin{bmatrix} -7 & -8 & -9 \\ 2 & 4 & 6 \end{bmatrix}$$

→

$\Rightarrow a + 4b = -7 \rightarrow (1)$
$\Rightarrow c + 4d = 2 \rightarrow (2)$
$\Rightarrow 2a + 5b = -8 \rightarrow (3)$
$\Rightarrow 2c + 5d = 4 \rightarrow (4)$

from (1)
$\Rightarrow a = -7 - 4b$
put in eqⁿ (3),

$\Rightarrow 2(-7-4b) + 5b = -8$
$\Rightarrow -14 - 8b + 5b + 8 = 0$
$\Rightarrow -6 - 3b = 0$
$\Rightarrow -3b = 6$

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$\Rightarrow \boxed{b = -2}$

put in (i),

$\Rightarrow a + 4(-2) = -7$

$\Rightarrow a = -7 + 8$

$\boxed{a = 1}$

from (2),

$\Rightarrow c = 2 - 4d$

put in (4),

$\Rightarrow 2(2-4d) + 5d = 4$

$\Rightarrow 4 - 8d + 5d = 4$

$\Rightarrow -3d = 0$

$\Rightarrow \boxed{d = 0}$

put in (2)

$\Rightarrow c + 4(0) = 2$

$\Rightarrow \boxed{c = 2}$

$a = 1$
$b = -2$
$c = 2$
$d = 0$

Hence the required matrix

$$\Rightarrow X = \begin{bmatrix} a & b \\ c & d \end{bmatrix},$$

~~Hence~~

$$\Rightarrow X = \begin{bmatrix} 1 & -2 \\ 2 & 0 \end{bmatrix}$$

12) If A and B are square matrices of the same order such that $AB = BA$, then prove ~~that~~ by induction method tha $AB^n = B^nA$. Further, prove that $(AB)^n = A^nB^n$ for all $n \in N$

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Sol. Given, $AB = BA \longrightarrow (1)$

To prove, $AB^n = B^n A$

for $n=1$,

$$AB^1 = B^1 A$$
$$AB = BA$$

Hence it is true for $n=1$

Let it is true for $n=k$.

$$A \cdot B^k = B^k \cdot A \longrightarrow (2)$$

Now,

$$A \cdot B^{k+1} = A \cdot B^k \cdot B$$
$$= B^k \cdot A \cdot B \quad (\text{from } (2))$$
$$= B^k \cdot B \cdot A \quad (\text{from } (1))$$
$$A \cdot B^{k+1} = B^{k+1} \cdot A$$

Hence it is true for $n = k+1$

$\therefore$ by induction method,

$\Rightarrow A \cdot B^n = B^n \cdot A$

Now,
~~for n~~

To prove $(AB)^n = A^n B^n$

for $n=1$,

$(AB) = AB$

it is true for $n=1$,

Let it is true for $n=k$

$\Rightarrow (AB)^k = A^k B^k \longrightarrow (3)$

Now,

$\Rightarrow (AB)^{k+1} = (AB)^k (AB)$

$= (A^k B^k)(AB) \quad (\text{by } (3))$

$= A^k (B^k \cdot A) B$

$= A^k (A B^k) B \quad (\text{By } (2))$

$= A^k \cdot A \; B^k \cdot B$

$= A^{k+1} B^{k+1}$

It is true for $n = k+1$

By induction method,

$(AB)^n = A^n B^n$

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15) If A is square matrix such that $A^2 = A$, then $(I+A)^3 - 7A$ is equal to,

Sol.

Given, $A^2 = A \longrightarrow (1)$

Now, $= (I+A)^3 - 7A$

$= I^3 + 3I^2A + 3IA^2 + A^3 - 7A$

$= I + 3.I.A + 3IA + A^2.A - 7A$ (from (1))

$= I + 3A + 3A + A.A - 7A$ (from (1))

$= I + 6A + A^2 - 7A$

$= I + 6A + A - 7A$ (from (1))

$= I$ Ans.

H.W.

10) A manufacturer produces three products x, y, z, which he sells in two markets. Annual sells are indicated below

Market	Product		
I	10,000	2000	18,000
II	6,000	20,000	8000

a) if unit sale price of x, y and z are Rs. 2.50, Rs. 1.50 and Rs 1.00, respectively, find the total revenue in each market with the help of matrix algebra

b) If the unit costs of the above three commodities are Rs. 2.00, Rs 1.00 and 50 paise respectively. Find the gross profit.

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Sol. Given,

Market	Product		
I	10000	2000	18000
II	6000	20000	8000

a) Given

for ~~part~~ Market I,

unit price → 2.50 1.50 1

product → 10,000 2000 18,000

⇒ let total revenue = x

⇒ Total revenue =

$$\Rightarrow \begin{bmatrix} 10,000 & 2000 & 18000 \end{bmatrix}_{1\times3} \begin{bmatrix} 2.50 \\ 1.50 \\ 1 \end{bmatrix}_{3\times1} = \begin{bmatrix} x \end{bmatrix}_{1\times1}$$

$$\Rightarrow \begin{bmatrix} 10,000 \cdot 2.50 + 2000 \cdot 1.50 + 18000 \cdot 1 \end{bmatrix}_{1\times1} = \begin{bmatrix} x \end{bmatrix}_{1\times1}$$

$$\Rightarrow 25000 + 3000 + 18000 = x$$

$$\Rightarrow \boxed{x = 46000 \text{ Rs.}}$$

~~b) Given,~~ for Market II,

unit price → 2.50 1.50 1

Market → 6000 20,000 8000

let Total revenue = Y

Total revenue (Y) =

$$\Rightarrow [y]_{1\times1} = \begin{bmatrix} 6000 & 20{,}000 & 8000 \end{bmatrix}_{1\times3} \begin{bmatrix} 2.50 \\ 1.50 \\ 1 \end{bmatrix}_{3\times1}$$

$$\Rightarrow [y]_{1\times1} = \begin{bmatrix} 6000 \cdot 2.50 + 20{,}000 \cdot 1.50 + 8000 \cdot 1 \end{bmatrix}_{1\times1}$$

$$\Rightarrow y = 15000 + 30000 + 8000$$

$$\Rightarrow y = 53000 \text{ Rs.}$$

b) Given,

for Market I,

Market → 10000 2000 18000

unit cost → 2.00 1.0 0.5

Total cost (z) =

$$\Rightarrow [z]_{1\times1} = \begin{bmatrix} 10000 & 2000 & 18000 \end{bmatrix}_{1\times3} \begin{bmatrix} 2.0 \\ 1.0 \\ 0.5 \end{bmatrix}_{3\times1}$$

$$\Rightarrow [z]_{1\times1} = \begin{bmatrix} 10000 \cdot 2 + 2000 \cdot 1 + 18000 \cdot 0.5 \end{bmatrix}_{1\times1}$$

$$\Rightarrow z = 31000 \text{ RS.}$$

$\Rightarrow$ Total revenue of Market I = 46000 RS.

$\Rightarrow$ Gross profit = 46000 − 31000

= 15000 RS.

for Market - II

Market → 6000 20,000 8000

unit cost → 2.0 1.0 0.5

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Total cost (M) =

$$\Rightarrow [M]_{1\times1} = \begin{bmatrix} 6000 & 20000 & 8000 \end{bmatrix}_{1\times3} \begin{bmatrix} 2.0 \\ 1.0 \\ 0.5 \end{bmatrix}_{3\times1}$$

$$\Rightarrow [M]_{1\times1} = \begin{bmatrix} 6000\cdot2 + 2000\cdot1 + 8000\cdot0.5 \end{bmatrix}_{1\times1}$$

$\Rightarrow M = 18000$ Rs.

$\Rightarrow$ Total revenue of Market II = 53000

$\Rightarrow$ Gross profit = 53000 − 18000

= 35000 Rs.

13) If $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$ is such that $A^2 = I$, then

Sol.

Given; $A = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$, $I = A^2$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix} \begin{bmatrix} \alpha & \beta \\ \gamma & -\alpha \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha\cdot\alpha + \beta\cdot\gamma & \alpha\cdot\beta + \beta\cdot-\alpha \\ \gamma\cdot\alpha + -\alpha\cdot\gamma & \gamma\cdot\beta + -\alpha\cdot-\alpha \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & \beta\alpha - \alpha\beta \\ \alpha\gamma - \alpha\gamma & \gamma\beta + \alpha^2 \end{bmatrix}$$

$$\Rightarrow A^2 = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix}$$

$\cancel{\therefore A^2}$

$$\Rightarrow I = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix}$$

{ Given $\because I = A^2$ }

$$\Rightarrow \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}_{2\times 2} = \begin{bmatrix} \alpha^2 + \beta\gamma & 0 \\ 0 & \alpha^2 + \beta\gamma \end{bmatrix}_{2\times 2}$$

on comparing both matrix, I get

$\Rightarrow \alpha^2 + \beta\gamma = 1$

$\Rightarrow \alpha^2 + \beta\gamma - 1 = 0$

multiply (-1) both side

$\Rightarrow -\alpha^2 - \beta\gamma + 1 = 0$

$\Rightarrow 1 - \alpha^2 - \beta\gamma = 0$ Ans. option C

14) If the matrix A is both symmetric and skew symmetric then

Sol. Given,

$A' = A$

$A' = -A$

for option A

$$= \begin{bmatrix} m & n & p \\ Q & R & S \\ T & U & V \end{bmatrix}$$

$$A' = \begin{bmatrix} m & Q & T \\ n & R & U \\ P & S & V \end{bmatrix}$$

$A' \neq A$ (or) $A' \neq -A$) This is false.

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=> for option B.

Zero matrix (0) = $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

A

=> $A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

=> $A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

=> $\boxed{A' = A}$

multiplying (-1) in both matrix

=> $-A' = -\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

=> $-A' = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

=> $-A' = A$

=> $\boxed{A' = -A}$

~~Hence~~ it satisfy $A' = A$ and $A' = -A$, therefore zero matrix is correct answer.

option B.