Chapter-14
Oscillation

1) # Periodic Motion :- A motion which repeats in a periodic interval of time is known as periodic motion.

2) # Oscilatory Motion :- when a body moves periodically to and fro, or back & fourth about a fixed point then this motion is called oscillatory or vibratory motion.

3) # Time Period :- The time taken by the object to complete one cycle is known as Time period.

4) # Frequency (ν {new}) :-

ν → new

$$\boxed{\nu = \frac{1}{T}}$$

→ It is also relation b/w frequency and Time Period.

where,

ν → frequency → This sign is called "Neu".

T → Time Period.

S.I unit of frequency is Hertz (Hz).

Amplitude :- Maximum displacement from Mean Position is called Amplitude.

→ denoted by "A"

Displacement :- The distance Measure from Mean Position.

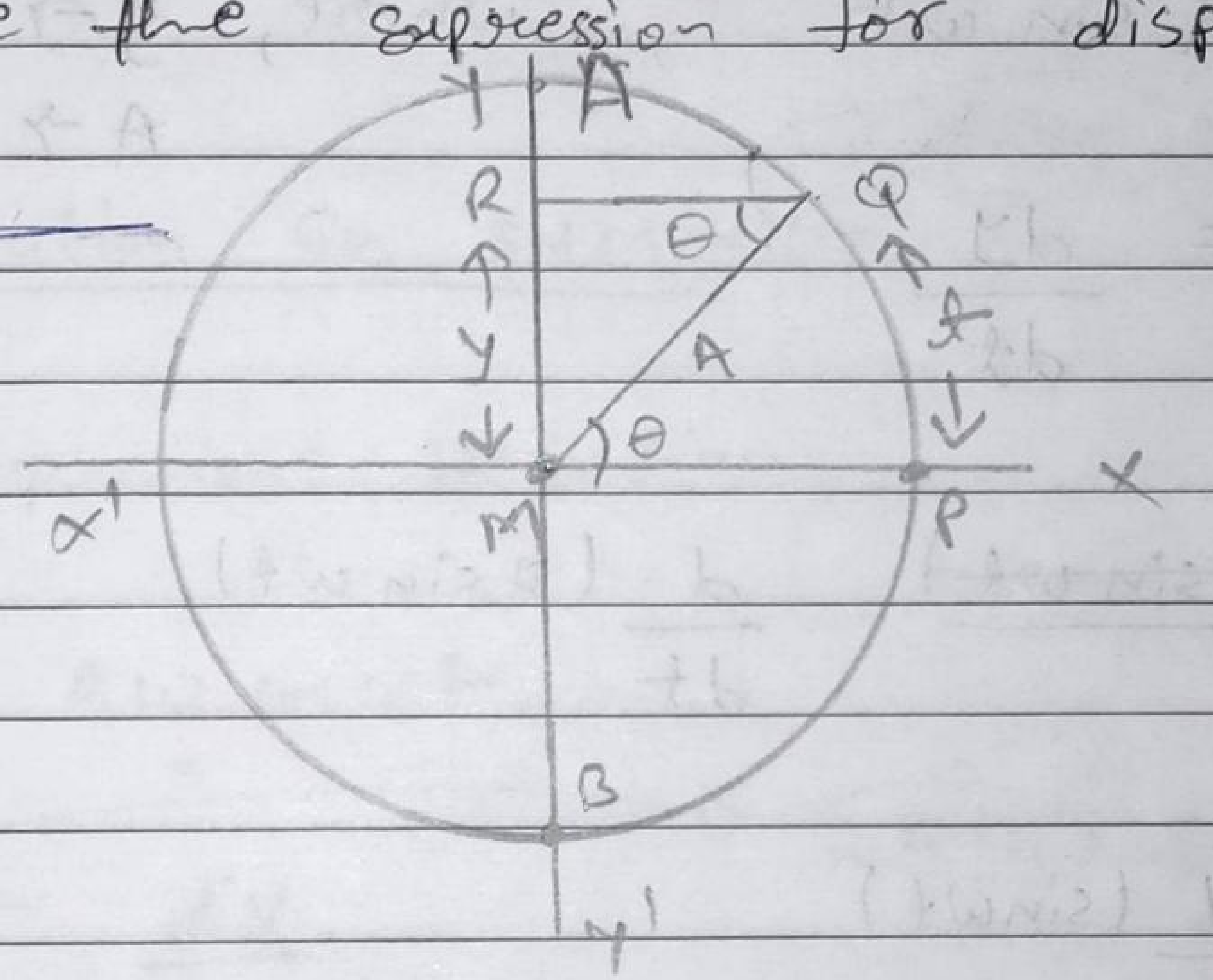
→ denoted by "y"

Q. What do you mean by Simple Harmonic Motion?

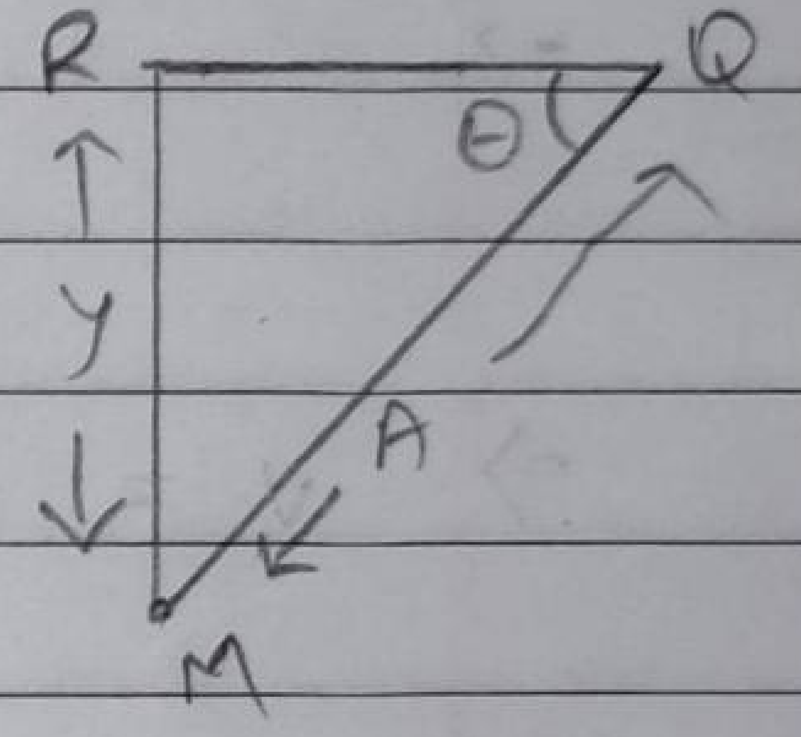
Ans: When a body perform oscillatory motion in a straight line on both sides of ~~the~~ it's equilibrium position then such a motion of the body is called SHM.

Q. derive the expression for displacement in the SHM.

Ans - it is ← oscillatory motion.



→



let MQ = A

In ΔMQR,

$\Rightarrow \quad \sin\theta = \frac{y}{A}$

$\Rightarrow \quad A\sin\theta = y$

Since $\frac{\theta}{t} = \omega$

then $\theta = \omega t$.

So,

$\Rightarrow \quad \boxed{A\sin\omega t = y}$

Q. Derive the expression for velocity in SHM.

Ans. we know that,

$y = A \sin \omega t$, where, $y \rightarrow$ displacement

we know, $A \rightarrow$ Amplitude

$\Rightarrow V = \frac{dy}{dt}$

$\Rightarrow V = \cancel{\frac{d(A\sin\omega t)}{}} \frac{d}{dt}(A \sin \omega t)$

$\Rightarrow V = A \frac{d}{dt}(\sin \omega t)$

$\Rightarrow V = A(\omega \cos \omega t)$

$\Rightarrow V = \cancel{A\omega(\cos\omega t)} \; A\omega \cos \omega t$

$\because \cos\theta = \sqrt{1-\sin^2\theta}$

$\cos \omega t = \sqrt{1-\sin^2 \omega t}$

$\Rightarrow V = A\omega \sqrt{1-\sin^2 \omega t}$

$\Rightarrow V = A\omega \sqrt{1-\left(\frac{y}{A}\right)^2}$ $\left[\because \sin \omega t = \frac{y}{A}\right]$

$\Rightarrow V = \frac{\cancel{A}\omega}{\cancel{A}} \sqrt{A^2 - y^2}$

$\Rightarrow \boxed{V = \omega \sqrt{A^2 - y^2}}$

Case - I → If $y = 0$

$\Rightarrow V = \omega\sqrt{A^2 - 0^2}$

$\boxed{V = \omega A}$

→ Case II → If $y = A$

$\Rightarrow V = \omega\sqrt{A^2 - A^2}$

$\Rightarrow V = 0$

alpha

Acceleration In SHM (α):-

from previous derivation,

$\Rightarrow V = A\omega \cos\omega t$

$\Rightarrow \alpha = \frac{dV}{dt}$ alpha → Acceleration

$\Rightarrow \alpha = \frac{d}{dt}(A\omega \cos\omega t)$

$\Rightarrow \alpha = A\omega \frac{d}{dt}(\cos\omega t)$

$\Rightarrow \alpha = A\omega[-\omega \sin\omega t]$

$\Rightarrow \alpha = -A\omega^2 \sin\omega t$

$\Rightarrow \alpha = -A\omega^2 \frac{y}{A}$ $\left[\because \sin\omega t = \frac{y}{A}\right]$

$\Rightarrow \boxed{\alpha = -\omega^2 y}$

Q. What do you mean by phase.

Ans - ~~When a particle vibrates,~~

The Phase of vibrating Particle at any instant indicates the position and direction of motion of the Particle at that instant.

Q. Derive the Expression for Time Period & frequency in SHM.

Ans. $\alpha = -\omega^2 y$

Taking only magnitude,

$\Rightarrow \alpha = \omega^2 y$

$\Rightarrow \omega = \sqrt{\frac{\alpha}{y}}$

and $\omega = \frac{2\pi}{T}$

$\therefore \frac{2\pi}{\omega} = T$

$\Rightarrow T = \frac{2\pi}{\sqrt{\frac{\alpha}{y}}}$

$\Rightarrow T = 2\pi\sqrt{\frac{y}{\alpha}}$

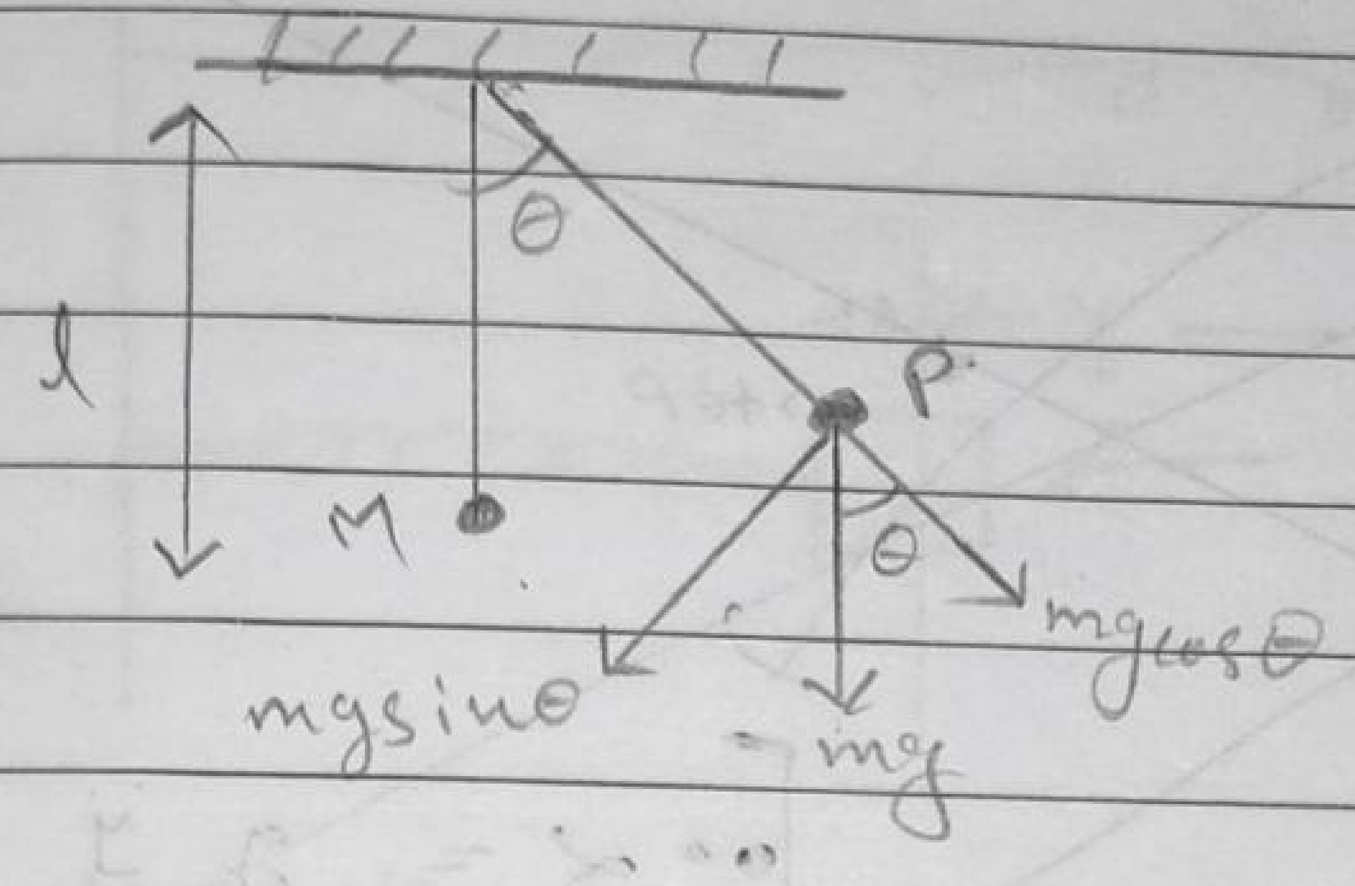
$\Rightarrow T = 2\pi\sqrt{\frac{\text{displacement}}{\text{acceleration}}}$

$\because \nu = \frac{1}{T}$

$\Rightarrow \nu = \frac{1}{2\pi\sqrt{\frac{y}{\alpha}}}$

$\Rightarrow \boxed{\nu = \frac{1}{2\pi}\sqrt{\frac{\alpha}{y}}}$ where,

Simple Pendulam :- when a heavy object of mass 'm' hanging from a string of length 'L' and fixed at a pivoted point P when it can displaced to an initial angle and redkeased, the pendulam will swing back and forth with periodic motion is called simple pendulam.



$F = -mg\sin\theta$ $\left[\because \; \overrightarrow{y} \; \overleftarrow{F}\right]$

where, $\sin\theta$ = small quantity

then we take $\boxed{\sin\theta = \theta}$

$F = -mg\theta$

$\theta = \frac{y}{l}$ $\left[\text{since, } \theta = \frac{\text{arc}}{\text{radius}}\right]$

$\Rightarrow F = -mg\frac{y}{l} \longrightarrow ①$

and $F = m\alpha \longrightarrow ②$ [where α is acceleration]

$\Rightarrow m\alpha = -mg\frac{y}{l}$

$\Rightarrow \alpha = -\frac{g}{l} \cdot y$

where, $\frac{-g}{l}$ is equal to constant.

$\boxed{\alpha \propto -y}$

It means the motion of simple Pendulam is SHM.

$$\Rightarrow \quad T = 2\pi\sqrt{\frac{y}{\alpha}} \longrightarrow (1)$$

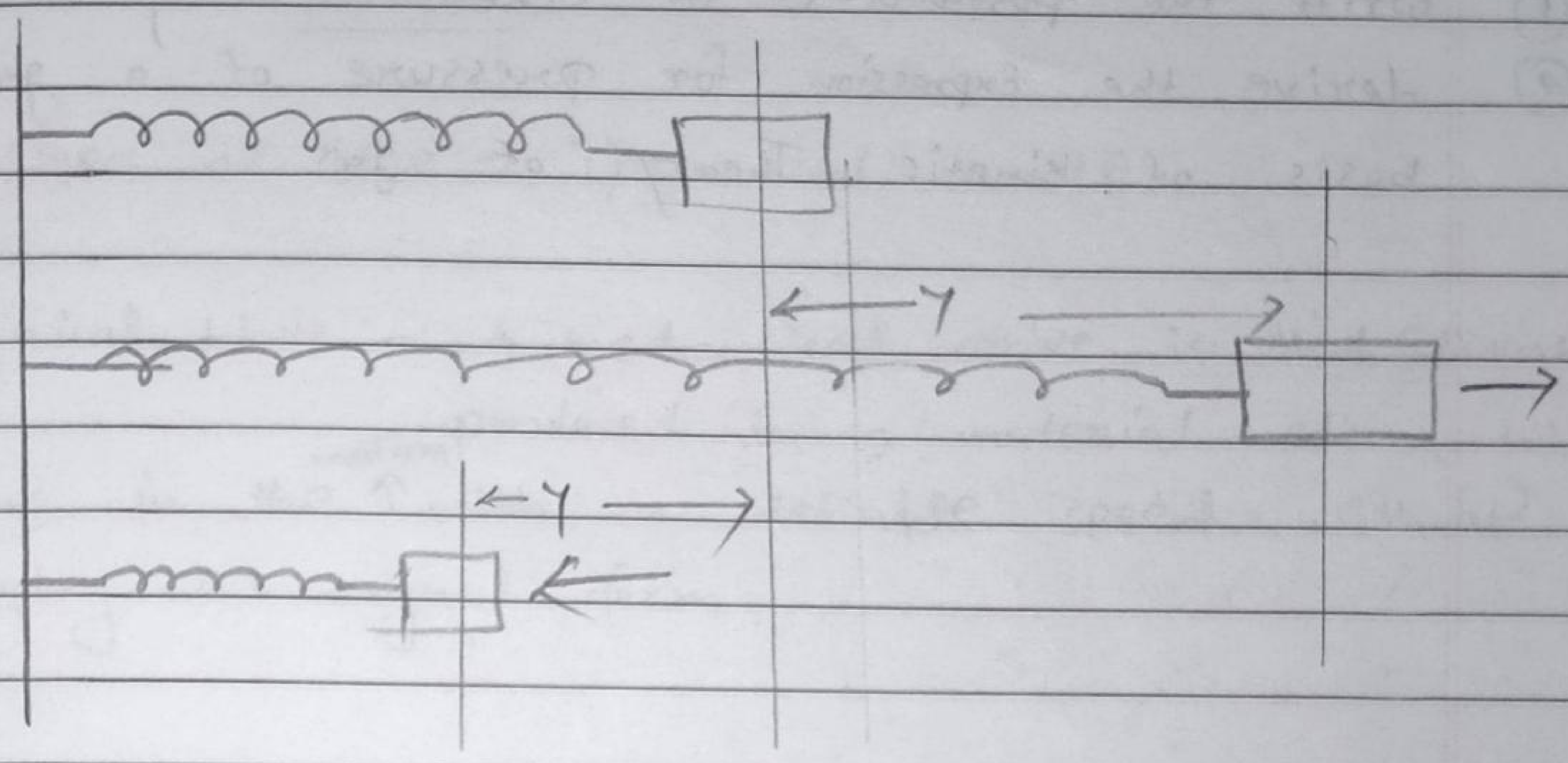
$$\because \quad \alpha = g\frac{y}{l}$$

$$\Rightarrow \quad \frac{y}{\alpha} = \frac{l}{g} \longrightarrow (2)$$

from (1) & (2)

$$\Rightarrow \quad \boxed{T = 2\pi\sqrt{\frac{l}{g}}}$$

SHM of a body ~~att~~ attached with spring (Horizontal plane) :-



A.T. Hooke's law,

$$F = -Ky \longrightarrow (1)$$

and,

$$F = m\alpha \longrightarrow (2)$$

Taking only magnitude in eq^n (1),

$$F = Ky \longrightarrow (3)$$

from (2) & (3),

$$\Rightarrow m\alpha = Ky$$

$$\because T = 2\pi\sqrt{\frac{y}{\alpha}}$$

$$\Rightarrow \frac{m}{K} = \frac{y}{\alpha} \longrightarrow (4)$$

$$\Rightarrow T = 2\pi\sqrt{\frac{y}{\alpha}}$$

$$\Rightarrow \boxed{T = 2\pi\sqrt{\frac{m}{K}}} \longleftarrow \text{(from eq}^n\text{ (4))}$$