Chapter - 3.

Motion In a straight line

1) Mechanique :- The branch of physics which deals with the study of the motion of objects or bodies and the equilibrium of the object under several forces is called mechanique.

There are 2 types of mechanique :-

i) Statics
ii) Dynamics

i) Statics :- The study of object at rest Position.

ii) Dynamics :- The study of object ~~at~~ motion.

Dynamics have 2 Types :-

i) Kinematics :- Study of motion without knowing the cause of motion.

ii) Dynamics Proper :- Study of motion with the cause of motion.

Mechanique

- Statics (study of obj. at rest)
- Dynamics (study of obj. at motion).
  - Kinematics
  - Dynamics Proper

2) <u>Scalar Quantity</u> :- The physical quantity which have magnitude only but no direction are called scalar quantity.

Eg:- speed, distance.

3) <u>Vector Quantity</u> :- The physical quantity which have magnitude as well as direction are known as vector quantity.

Eg:- Displacement, Velocity.

Point Object ⇒ When the dimension of any object is very less in comparison of ~~the obje~~ distance travelled by the object then the object is said to be point object.

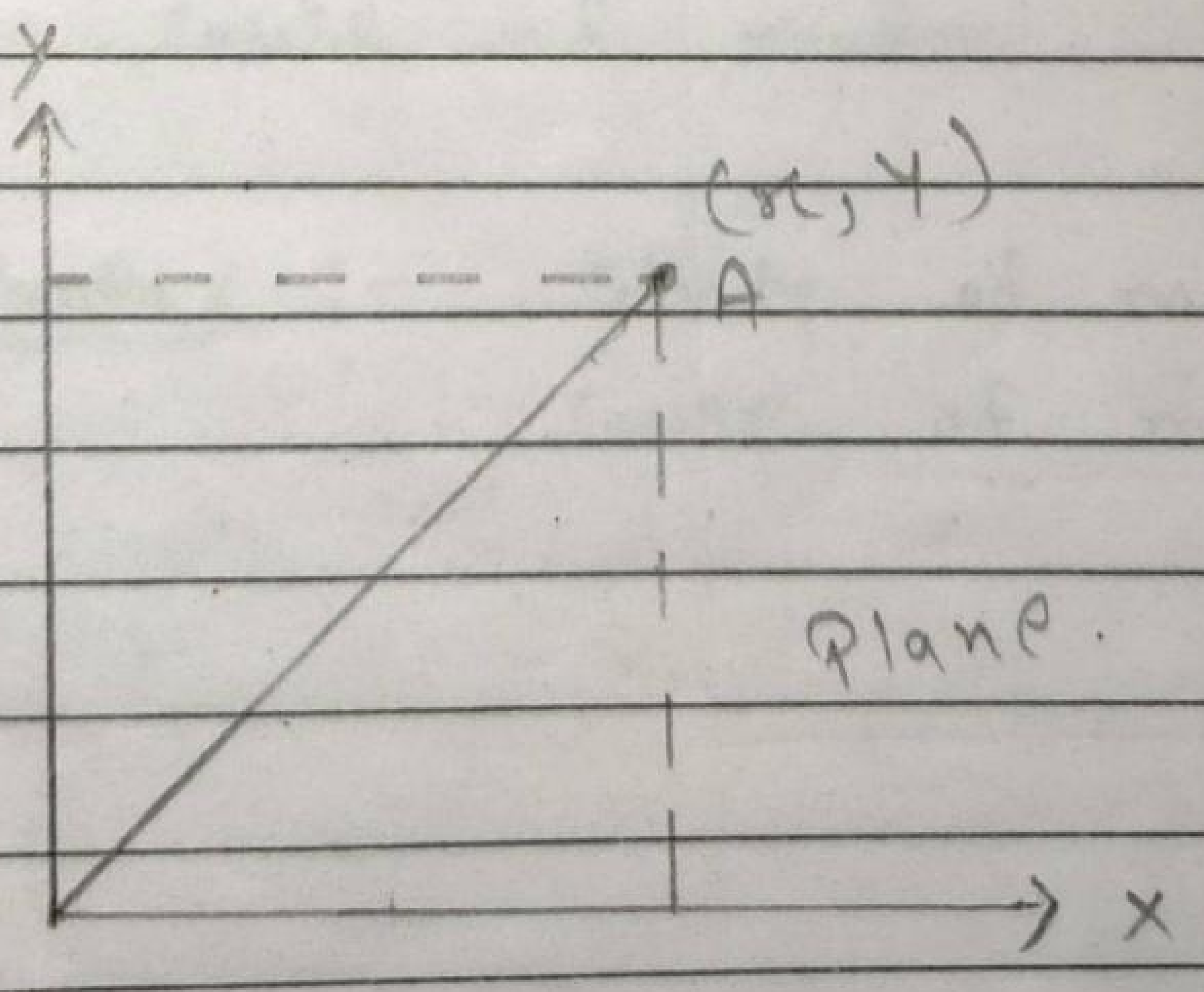
Types of Motion ⇒

i) One dimensional Motion ⇒ When the body moves in one direction then the motion is known as one dimensional motion.

Eg:- A car is moving in straight road.

ii) Two dimensional Motion ⇒ If the body moves in a plane then the motion of the body is known as Two dimensional Motion. In two dimensional motion we need 2 axis and in one dimensional motion only 1 axis required.

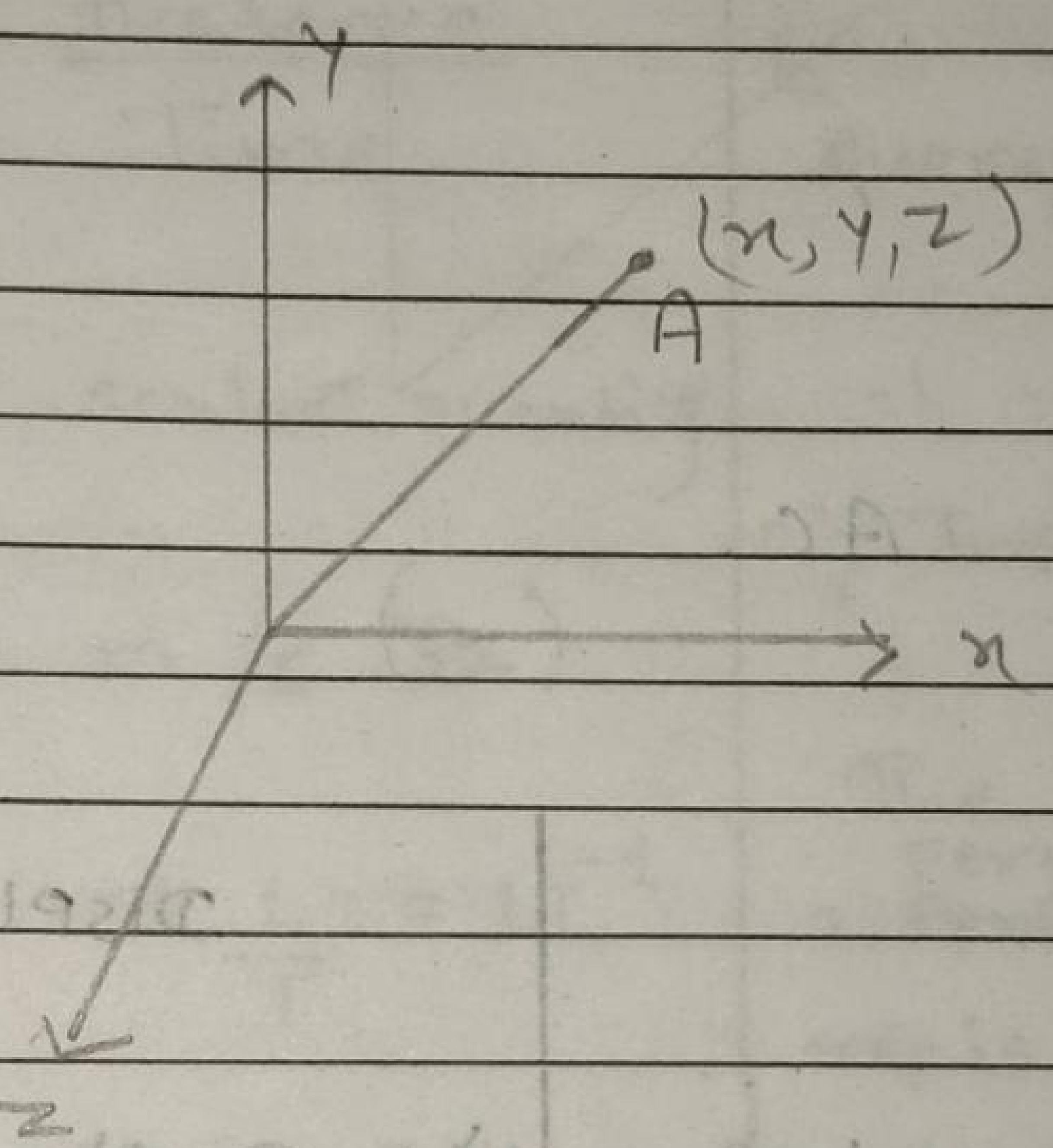
Eg:- ~~Moti~~



Eg:- Motion of any insect.

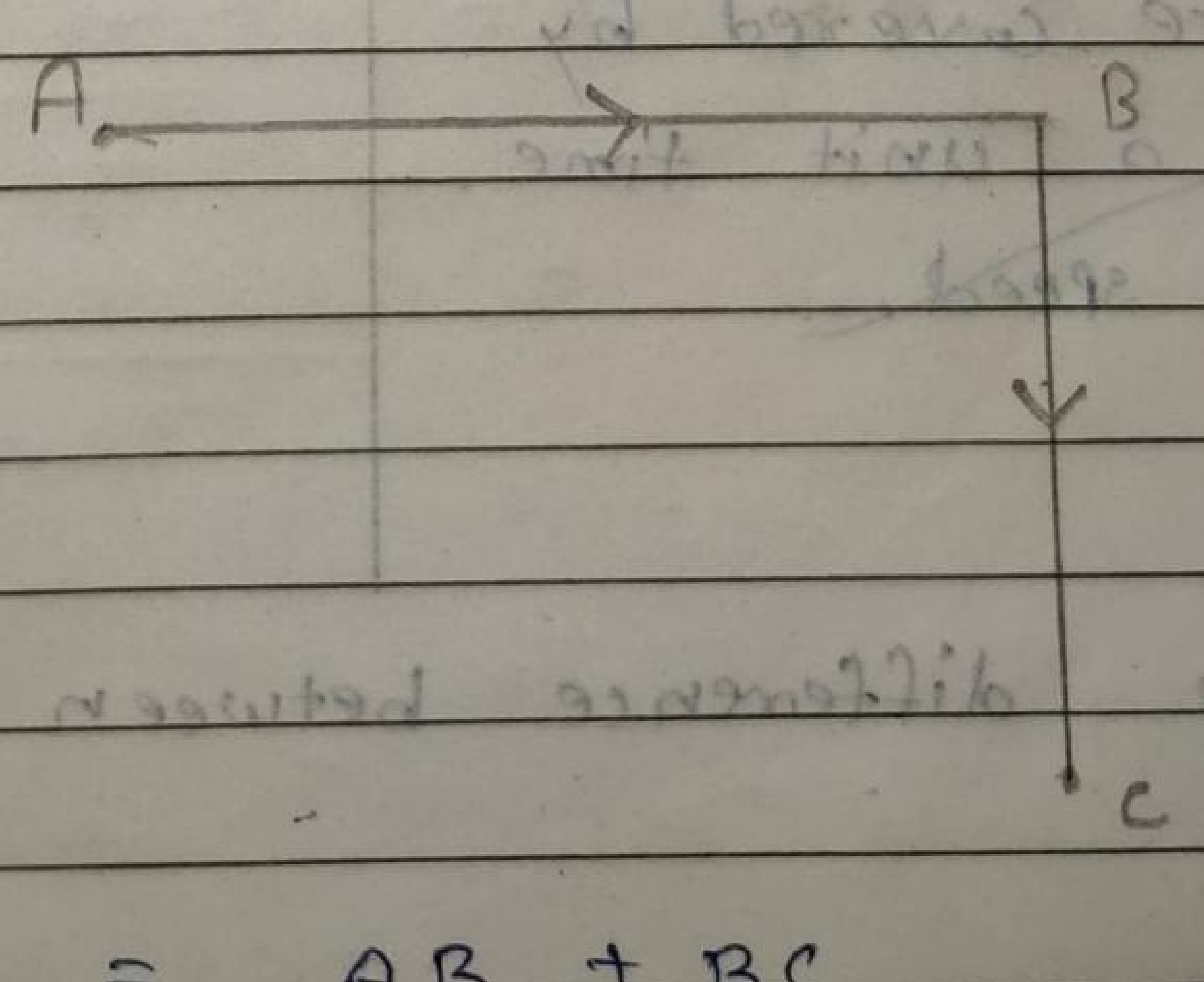
iii) Three Dimensional Motion => If the body in a space then the motion of the body is known as 3D Motion. We need 3 Dimension.

Eg.- Motion of flying bird. or fish and aeroplane.



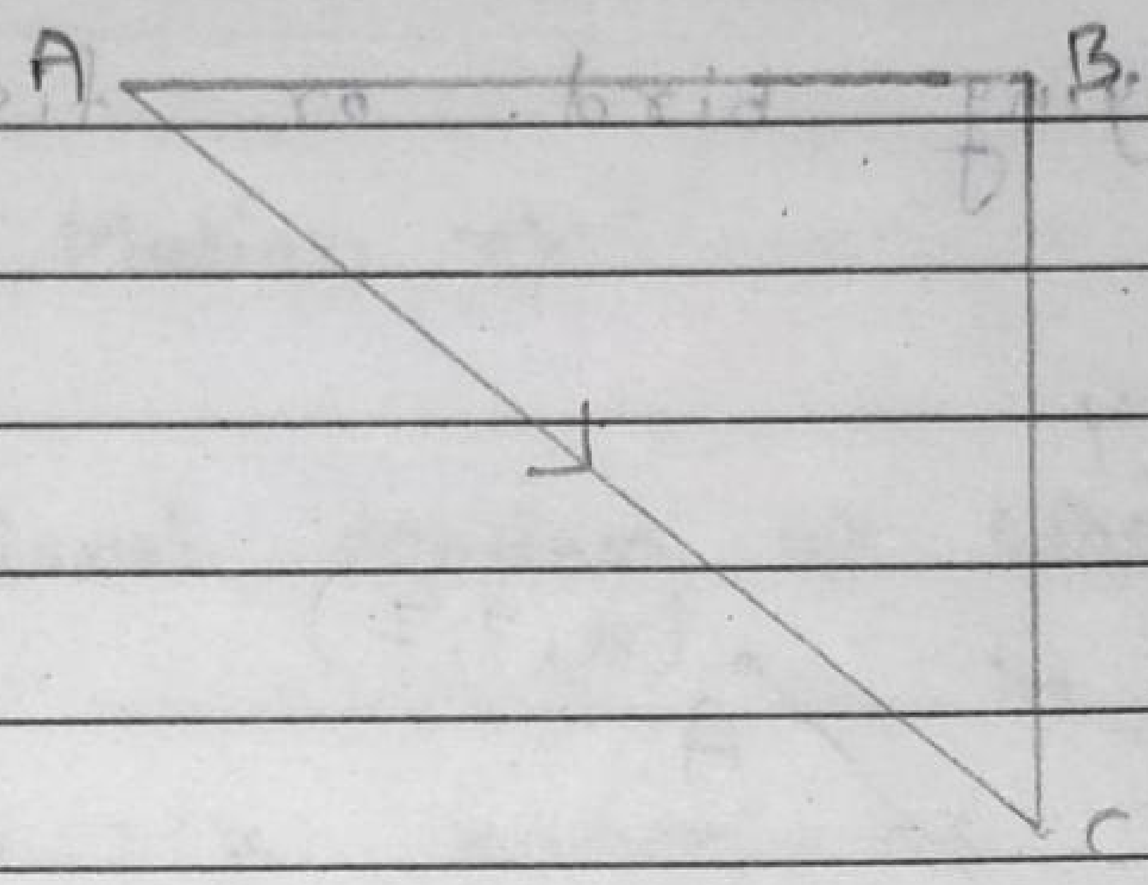
Q.1 Write the difference between Distance and Displacement.

Ans. Distance :- The total Path lenghth covered by any object is known as Distance.



Distance = AB + BC

Displacement ⇒ The shortest distance between intial Point and Final Point is Known as Displacement.



Displacement = AC.

Distance	Displacement
2) Distance is a scalar Quantity.	2) Displacement is a vector Quantity.
3) Distance will be Positive (+ve).	3) Displacement will be Positive, Negative and zero.
4) ~~The distance covered by object in a unit time is called speed.~~	

Q.2 Write the difference between speed & velocity.

Ans. (i)

Speed	Types of speed ~~Velocity~~
1) The distance covered by any object in a unit time is known as speed.	1) Average speed ⇒ The ratio of Total distance and total time is known as Average speed.
2) speed = $\frac{\text{Distance}}{\text{Time}}$	Average speed = $\frac{\text{Total distance}}{\text{Total time}}$
3) speed is scalar quantity.	2) Instataneous speed ⇒
4) unit = m/s (SI)	The speed of an object at a perticular instant ~~perticular distance~~ of time ~~of~~ is known as Instantaneous speed.
5) Dimension ⇒ $\frac{L}{T} = LT^{-1}$	$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$
6) speed is always positive.	Here, V = instataneous speed, Δx = distance, Δt = time

② Velocity →

1) The displacement ~~trav~~ covered by any object is known as velocity.

2) Velocity is vector quantity.

3) $\text{velocity} = \dfrac{\text{displacement}}{\text{time}}$

4) SI unit = m/s

5) dimensions $\Rightarrow \dfrac{L^1}{T^1} = LT^{-1}$

6) It may be positive, zero and Negative.

Types of velocity :-

i) Average velocity → Average velocity is defined as the change in Position or displacement divided by the time interval.

$$\text{Average velocity } (V_{av}) = \frac{x_2 - x_1}{t_2 - t_1}$$

$$= \frac{\text{change in Position}}{\text{Time interval}}$$

here,

x_2 = final Position at time t_2

x_1 = initial Position at time t_1

ii) Instataneous velocity → The velocity ~~it~~ it in instant is known instataneous velocity.

$$\text{Instataneous velocity} = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

0 x 0

Differential Calculus

* Some important Formulas :-

1) $\frac{d}{dx} x^n = n x^{n-1}$

2) $\frac{d}{dx}(c) = 0$ where c = constant.

3) $\frac{d}{dx}(\sin x) = \cos x$

4) $\frac{d}{dx}(\cos x) = -\sin x$

Q. x^3

Sol. We know that,

$\Rightarrow \frac{d}{dx} x^n = n x^{n-1}$

$$\Rightarrow \frac{d}{dx} x^3 = 3x^{3-1}$$

$$\Rightarrow \frac{d}{dx} x^3 = 3x^2$$

Q. $\sqrt{x}$

Ans. We know that,

$$\Rightarrow \frac{d}{dx} x^n = nx^{n-1}$$

$$\Rightarrow \frac{d}{dx} \sqrt{x} = n\sqrt{x}^{\,n-1}$$

$$\Rightarrow \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} x^{\frac{1}{2}-1}$$

$$\Rightarrow \frac{d}{dx} x^{1/2} = \frac{1}{2} x^{-\frac{1}{2}}$$

$$\Rightarrow \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} \times \frac{1}{x^{\frac{1}{2}}}$$

$$\Rightarrow \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2} \times \frac{1}{\sqrt{x}}$$

$$\Rightarrow \frac{d}{dx} x^{\frac{1}{2}} = \frac{1}{2\sqrt{x}}.$$

Integral Calculas

* Some important Formulas :-

1) $\int x^n \, dx = \dfrac{x^{n+1}}{n+1}$

2) $\int \sin x \, dx = -\cos x$

3) $\int \cos x \, dx = \sin x$

4) $\int e^x \, dx = \cancel{ex} \; e^x$

5) $\int dx = x$

or

$\int dt = t$

Q. $\int x^5 \, dx$

Sol. We know that,

$$\int x^n \, dx = \frac{x^{n+1}}{n+1}$$

Given,

$$\int x^5 \, dx$$

$$n = 5$$

A.T.Q.

$$\int x^5 \, dx = \frac{x^{5+1}}{5+1}$$

$$\int x^5 \, dx = \frac{x^6}{6}$$

Acceleration

The rate of change of velocity is known as acceleration. It is denoted by "a".

i) ⇒ $a = \frac{\text{change in velocity}}{\text{Time}}$

⇒ $a = \frac{\text{final velocity} - \text{initial velocity}}{\text{Time interval}}$

⇒ $a = \frac{v - u}{t_2 - t_1}$

⇒ $a = \frac{\Delta v}{\Delta t}$

ii) Unit $= \frac{m/s}{s}$

⇒ unit $= \frac{m}{s^2}$

⇒ unit of acceleration $= m s^{-2}$

iii) Dimension $= [L T^{-2}]$

Types of Acceleration:

1) Positive Acceleration ($a \Rightarrow$ positive) $\rightarrow$

$$a = \frac{v-u}{t} \quad \text{if} \quad v > u$$

Eg. $v = 10, \quad u = 6, \quad t = 2$

$$a = \frac{10-6}{2} = \frac{4}{2} = 2 \text{ ('+'ve)}$$

2) Negative Acceleration :- ($a \Rightarrow$ Negative) $\rightarrow$

Negative acceleration is also known as retardation.

$$a = \frac{v-u}{t} \qquad u > v$$

Eg. $v = 6, \quad u = 10, \quad t = 2$

$$a = \frac{v-u}{t} = \frac{6-10}{2} = \frac{-4}{2} = -2 \text{ ('-'ve)}$$

3) Zero acceleration :-

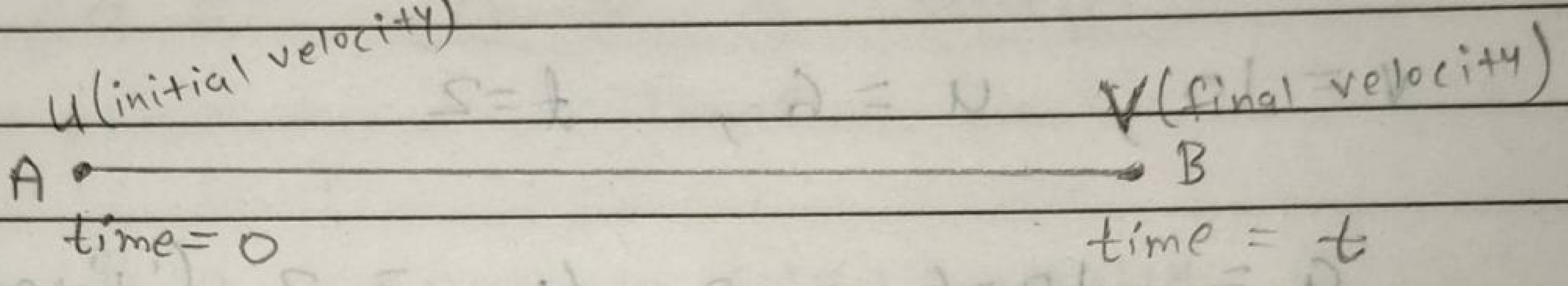
$\Rightarrow v = u$

$\Rightarrow a = \frac{v-u}{t} = \frac{0}{t} = 0$

M. IMP.

Eg. 3.3 obtain eq^n of motion for constant acceleration using method of calculus. (Integration method).

Ans. Derivation of eq^n of motion by calculus method (integration method)



→by integration method

a) First eq^n of motion ⇒

$$\Rightarrow \quad a = \frac{dV}{dt} \qquad \left[\because a = \frac{\Delta V}{\Delta t}\right]$$

$$\Rightarrow \quad a\,dt = dv$$

Integrating Both side,

$$\Rightarrow \quad \int a\,dt = \int dv$$

$$\Rightarrow \quad \int_0^t a\,dt = \int_u^v dv$$

$$\Rightarrow \quad a\int_0^t dt = \int_u^v dv$$

$$\Rightarrow \quad a\,[t]_0^t = [v]_u^v \qquad \left[\because \int dx = x,\ \int dt = t,\ \int dv = v\right]$$

$\Rightarrow \quad a\,[t-0] = [v-u]$

$\Rightarrow \quad a\,[t] = v-u$

$\Rightarrow \quad at = v-u$

$\Rightarrow \quad at + u = v$

$\Rightarrow \quad \boxed{v = at + u}$

b) **Second eq^n of motion ~~by~~ integration method** $\Rightarrow$

$\Rightarrow$ as we know that,

$$v = \frac{dx}{dt}$$

time = 0   time = t

velocity = u velocity = v

displacement = 0   displacement = S

We know that,

$\Rightarrow$ velocity $= \dfrac{\Delta \text{ displacement}}{\Delta \text{ time}}$

$$\Rightarrow \quad v = \frac{ds}{dt}$$

$\Rightarrow \quad v\,dt = ds$

Integrating both side,

$$\Rightarrow \int v\,dt = \int ds$$

$$\Rightarrow \int_0^t v\,dt = \int_0^s ds$$

$$\Rightarrow \int_0^t (u + at)\,dt = \int_0^s ds$$

$$\Rightarrow \int_0^t u\,dt + at\,dt = \int_0^s ds$$

$$\cancel{\Rightarrow u\int_0^t dt + a\int_0^t t\,dt = \int_0^s ds}$$

$$\Rightarrow u\int_0^t dt + a\int_0^t t^1\,dt = \int_0^s ds$$

$$\left[\begin{array}{l} \because \int dx = x,\ \int dt = t,\ \int ds = s, \\ \int x^n\,dx = \frac{x^{n+1}}{n+1},\ \Rightarrow \int t^1\,dt = \frac{t^{1+1}}{1+1} \\ \Rightarrow \int t^1\,dt = \frac{t^2}{2} \end{array}\right]$$

$$\Rightarrow u\,[t]_0^t + a\left[\frac{t^2}{2}\right]_0^t = [s]_0^s$$

$$\Rightarrow u\,[t-0] + a\left[\frac{t^2}{2} - \frac{(0)^2}{2}\right] = [s-0]$$

$$\Rightarrow ut + \frac{at^2}{2} = s$$

$$\Rightarrow s = ut + \frac{at^2}{2}$$

$$\Rightarrow \boxed{s = ut + \frac{1}{2}at^2}$$

c) **Third eq^n of motion** $\Rightarrow$

We know that,

$$\Rightarrow \quad a = \frac{dv}{dt}$$

$$\Rightarrow \quad a = \frac{dv}{ds} \cdot \frac{ds}{dt}$$

$$\Rightarrow \quad a = \frac{dv}{ds} \cdot v$$

$$\Rightarrow \quad a\,ds = v \cdot dv$$

Integrating both side,

$$\Rightarrow \quad \int a\,ds = \int v \cdot dv$$

$$\Rightarrow \quad \int_0^s a\,ds = \int_u^v v \cdot dv$$

$$\Rightarrow \quad a\int_0^s ds = \int_u^v v \cdot dv$$

$$\because \quad \int dx = x, \qquad \int ds = s$$

$$\int x^n\,dx = \frac{x^{n+1}}{n+1}$$

$$\int v^1 \cdot dv = \frac{v^{1+1}}{1+1}$$

$$\int v^1 \cdot dv = \frac{v^2}{2}$$

$$\Rightarrow a\left[s\right]_0^s = \left[\frac{v^2}{2}\right]_u^v$$

$$\Rightarrow a\left[s-0\right] = \left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$\Rightarrow as = \frac{v^2-u^2}{2}$$

$$\Rightarrow 2as = v^2 - u^2$$

$$\Rightarrow 2as + u^2 = v^2$$

$$\Rightarrow \boxed{v^2 = u^2 + 2as}$$

Q. Derive the eq^n of motion by Graphical method. (Uniformly acceleration motion)

A ———————— B

Time = 0   Time = t

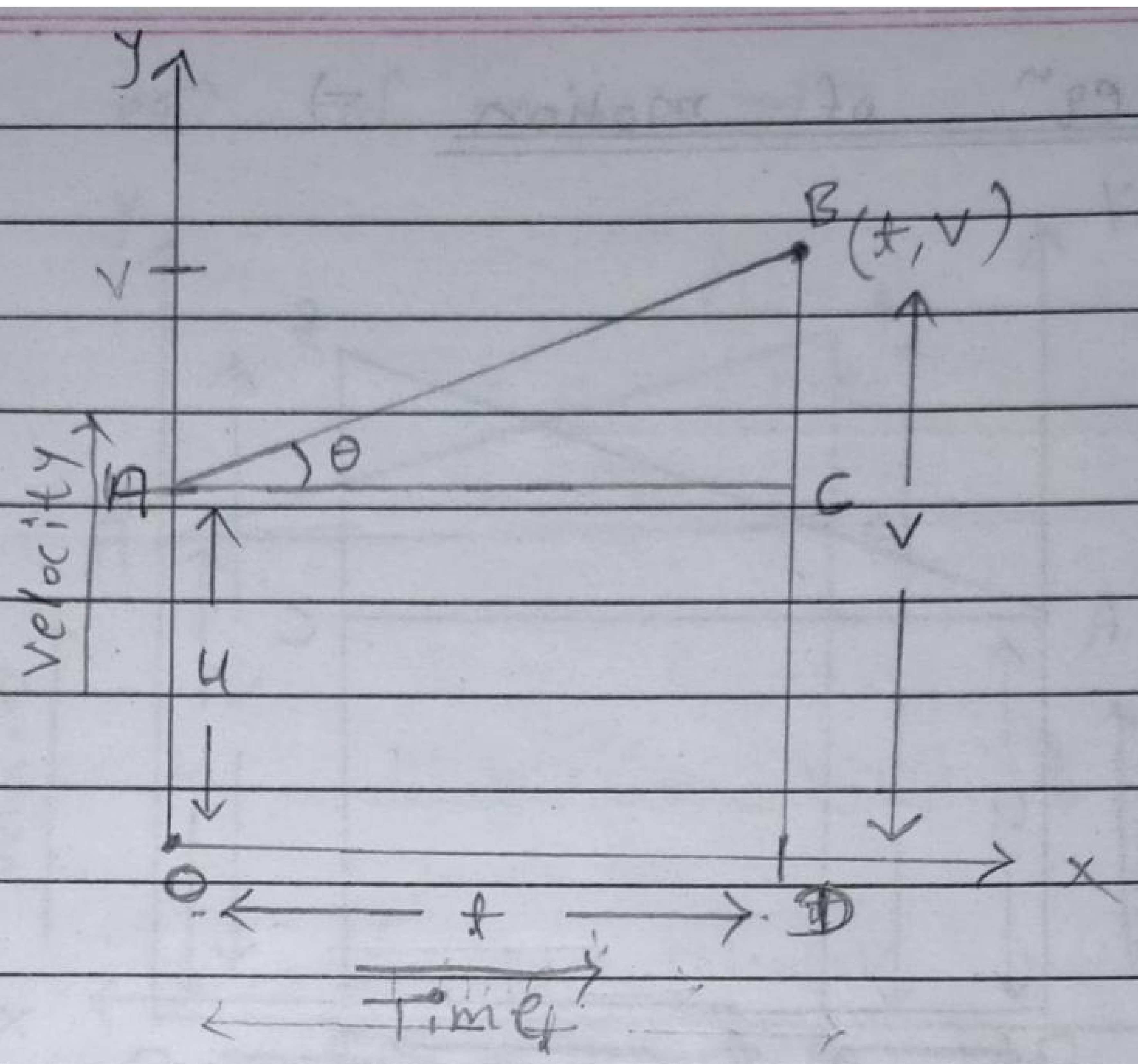
velocity = u velocity = v

for making graph,

time = x co-ordinate, velocity = y co-ordinate.

Point A = $(0, u)$

Point B = (t, v)



(a) 1^{st} eq^n of motion $\Rightarrow$

$\Rightarrow$ (a) = slope of v – t graph

$\Rightarrow$ $a = \tan\theta$

We know that,

$$\tan\theta = \frac{P}{B}$$

$$\Rightarrow a = \frac{BC}{AC}$$

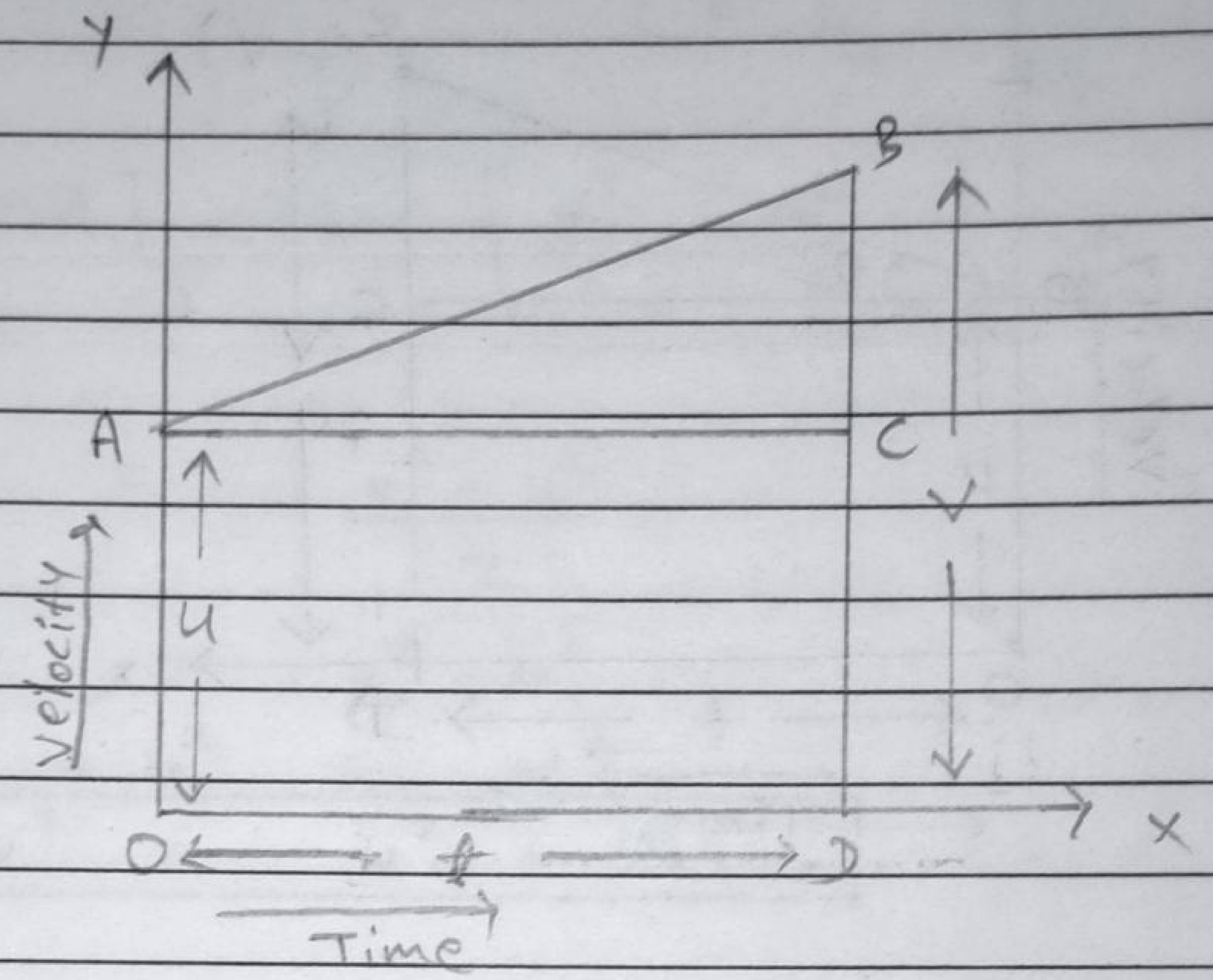
$$\Rightarrow a = \frac{BD - CD}{AC}$$

$$\Rightarrow a = \frac{v - u}{t}$$

$$\Rightarrow at = v - u$$

$$\Rightarrow \boxed{v = u + at}$$

b) **Second eqⁿ of motion** ⇒



⇒ S = Area of Δ ABC + Area of □ AODC

⇒ $S = \frac{1}{2} t (v-u) \cancel{t} + t \times u$

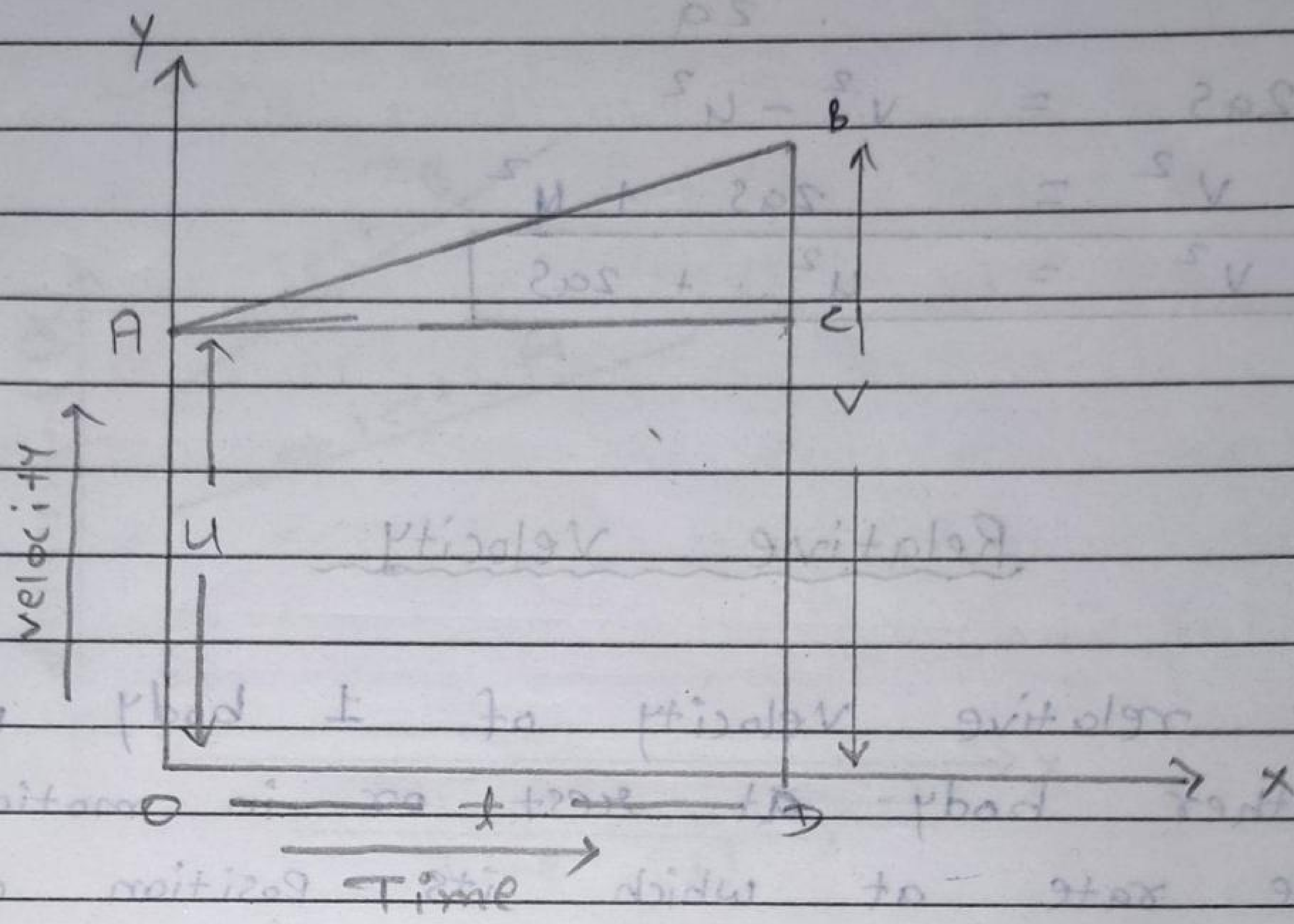
⇒ ∵ $v = u + at$ (first eqⁿ of motion)

⇒ $v - u = at$

⇒ $S = \frac{1}{2} t (at) \cancel{t} + ut$

⇒ $\boxed{S = \frac{1}{2} at^2 + ut}$

c) Third eq^n of motion ⇒



$\Rightarrow \quad S = \text{Area of } ABDO$

$\Rightarrow \quad S = \frac{1}{2}(AO + BD) \times OD$

$\Rightarrow \quad S = \frac{1}{2}(u + v) \times t$

$\Rightarrow \quad \because \quad v = u + at$ (from 1st eq^n of motion)

$\Rightarrow \quad v - u = at$

$\Rightarrow \quad \frac{v-u}{a} = t$

$\Rightarrow \quad S = \frac{1}{2}(u+v) \times \left(\frac{v-u}{a}\right)$

$\Rightarrow \quad \underline{S} = \frac{1}{2}(v+u)\left(\frac{v-u}{a}\right)$

$\Rightarrow \quad S = \frac{(v+u)(v-u)}{2a}$

$\Rightarrow \quad S = \dfrac{v^2 - u^2}{2a}$

$\Rightarrow \quad 2aS = v^2 - u^2$

$\Rightarrow \quad v^2 = 2aS + u^2$

$\Rightarrow \quad \boxed{v^2 = u^2 + 2aS}$

Relative Velocity

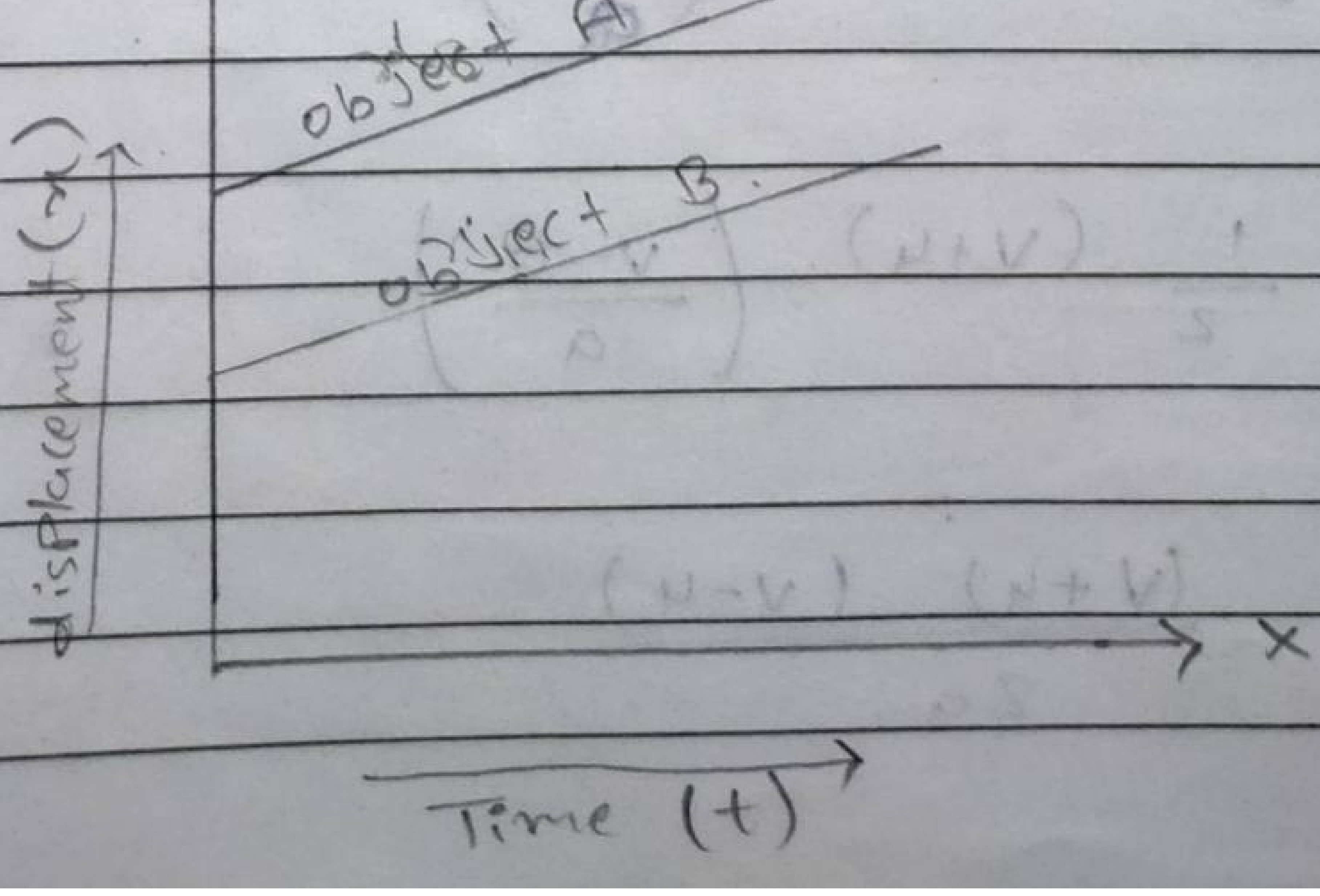
The relative velocity of 1 body with respect to another body at rest or in motion is the time rate at which its position changes relative to the other body.

Let there are two object A and B having the velocity V_A and V_B respectively.

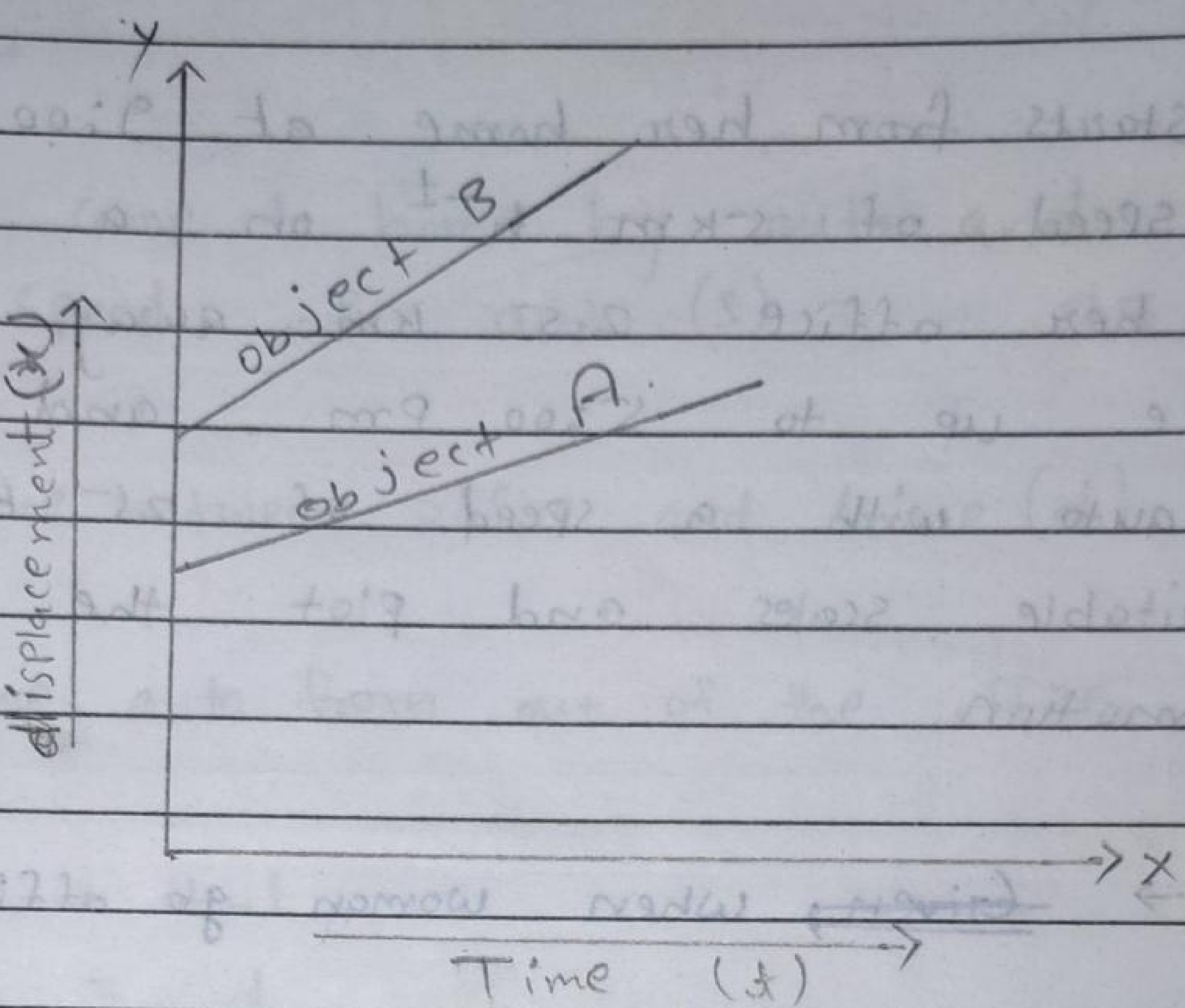
So, The relative velocity A with respect ~~to~~ B $(V_{AB}) = V_A - V_B$

The relative velocity B with respect to A $(V_{BA}) = V_B - V_A$

Case I :- when $V_A = V_B$



Case II :- when, $V_B > V_A$



Case III :- when $V_A > V_B$