Chapter - 2

Inverse Trigonometric Functions

1. $\sin(A+B) = \sin A\cos B + \cos A\sin B$
2. $\sin(A-B) = \sin A\cos B - \cos A\sin B$
3. $\cos(A+B) = \cos A\cos B - \sin A\sin B$
4. $\cos(A-B) = \cos A\cos B + \sin A\sin B$.
5. $\tan(A+B) = \dfrac{\tan A + \tan B}{1 - \tan A\tan B}$.
6. $\tan(A-B) = \dfrac{\tan A - \tan B}{1 + \tan A\tan B}$.
7. $\sin^2 A - \sin^2 B = \sin(A+B)\sin(A-B)$
8. $\cos^2 B - \cos^2 A = \sin(A+B)\sin(A-B)$
9. $\cos^2 A - \sin^2 B = \cos(A+B)\cos(A-B)$
10. $\cos^2 B - \sin^2 A = \cos(A+B)\cos(A-B)$
11. $\cos 2A = \cos^2 A - \sin^2 A$

    $\cos 2A = 1 - 2\sin^2 A$

    $\cos 2A = 2\cos^2 A - 1$

    $\cos 2A = \dfrac{1 - \tan^2 A}{1 + \tan^2 A}$
12. $\sin 2A = 2\sin A\cos A$

    $\sin 2A = \dfrac{2\tan A}{1 + \tan^2 A}$
13. $\tan 2A = \dfrac{2\tan A}{1 - \tan^2 A}$
14. $\sin 3A = 3\sin A - 4\sin^3 A$
15. $\cos 3A = 4\cos^3 A - 3\cos A$
16. $\tan 3A = \dfrac{3\tan A - \tan^3 A}{1 - 3\tan^2 A}$

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(17) $\sin A + \sin B = 2\sin\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

(18) $\sin A - \sin B = 2\cos\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

(19) $\cos A + \cos B = 2\cos\left(\frac{A+B}{2}\right)\cos\left(\frac{A-B}{2}\right)$

(20) $\cos A - \cos B = -2\sin\left(\frac{A+B}{2}\right)\sin\left(\frac{A-B}{2}\right)$

(21) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(22) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(23) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

(24) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(25) $\cot(A+B) = \frac{\cot A\cot B - 1}{\cot A + \cot B}$

(26) $\cot(A-B) = \frac{\cot A\cot B + 1}{\cot A - \cot B}$

(27) $2\sin A\cos B = \sin(A+B) + \sin(A-B)$

(28) $2\cos A\sin B = \sin(A+B) - \sin(A-B)$

(29) $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

(30) $2\cos A\cos B = \cos(A+B) + \cos(A-B)$

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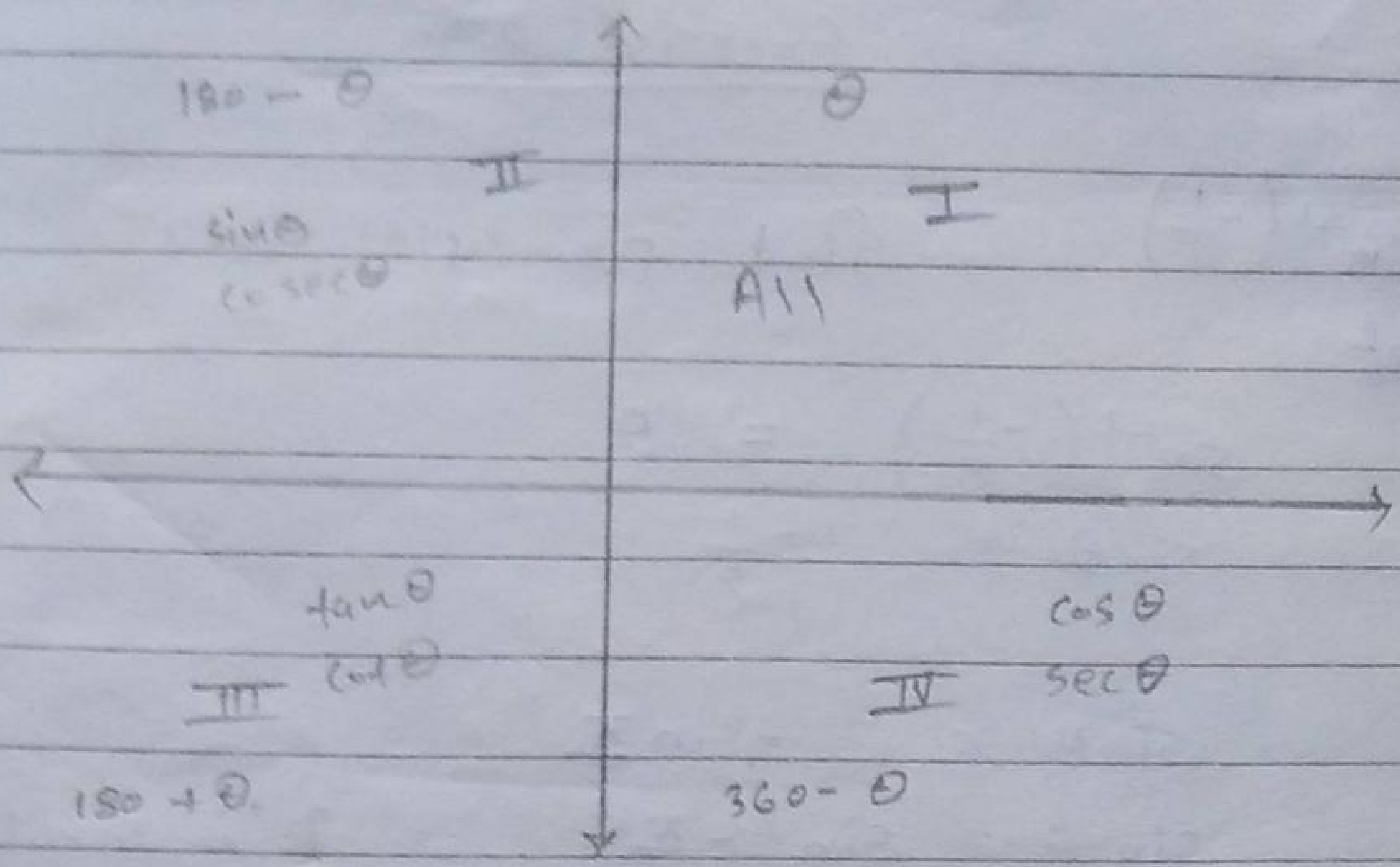

Inverse Trigonometry

$\Rightarrow$ If, $\sin\theta = x$

$\theta = \sin^{-1}x$

$\Rightarrow$ If, $\sin\frac{\pi^c}{6} = \frac{1}{2}$

then $\sin^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{6}$

* $\tan^{-1}(1) = \pi/4$
* $\cos^{-1}(1/2) = \pi/3$

	Function	Domain	Range
1)	$\sin^{-1}$	$[-1, 1]$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
2)	$\cos^{-1}$	$[-1, 1]$	$[0, \pi]$
3)	$\tan^{-1}$	R	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

C.W.

Exercise 2.1

(1) $\sin^{-1}\left(-\frac{1}{2}\right)$, find principle value.

Sol. let

$$\sin^{-1}\left(-\frac{1}{2}\right) = \theta$$

$$\because \quad \sin\theta = \frac{-1}{2}$$

$$\Rightarrow \quad \sin\theta = -\sin\frac{\pi}{6}$$

$$\Rightarrow \quad \sin\theta = \sin\left(-\frac{\pi}{6}\right) \qquad \left\{\because \sin(-\theta) = -\sin\theta\right\}$$

$$\theta = \frac{\pi}{6}$$

$\Rightarrow$ $\sin\theta$ is negative in 3^{rd} or 4^{th} quadrant

for 3^{rd} quadrant $= \pi + \theta$

$$= \pi + \frac{\pi}{6}$$

$$= \frac{7\pi}{6}$$

for 4^{th} quadrant $= -\theta$

$$= -\frac{\pi}{6}$$

Range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

then $\frac{7\pi}{6}$ not comes between $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ therefore we ignore $\frac{7\pi}{6}$ and $-\frac{\pi}{6}$ comes in the range of $\sin^{-1}$.

principle Value $= -\pi/6$ Ans.

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② $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

Sol. let, $\cos^{-1}\left(\frac{\sqrt{3}}{2}\right) = \theta$

$\because \cos\theta = \frac{\sqrt{3}}{2}$

$\cos\theta = \cos\frac{\pi}{6}$

$\theta = \frac{\pi}{6}$

$\cos\theta$ is positive in 1^{st} and 4^{th} quadrant.

for 4^{th} quadrant $= (-\theta)$

$= -\pi/6$

for 1^{st} quadrant $= \theta$

$= \frac{\pi}{6}$

Range of $\cos^{-1}$ is $[0, \pi]$ then we take 1^{st} quadrant.

principle value $= \frac{\pi}{6}$

③ $\operatorname{cosec}^{-1}(2)$

Sol. let, $\operatorname{cosec}^{-1}(2) = \theta$

$\because \operatorname{cosec}\theta = 2$

$\left\{\because \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right\}$

$\operatorname{cosec}\theta = \operatorname{cosec}\frac{\pi}{6} \Rightarrow \frac{1}{\sin\theta} = \frac{1}{\sin\frac{\pi}{6}} \Rightarrow \sin\frac{\pi}{6} = \sin\theta$

$\Rightarrow \theta = \frac{\pi}{6}$

for 1^{st} quadrant $= \theta$

$= \frac{\pi}{6}$

for 2^{nd} quadrant $= \frac{\pi}{2} + \theta$

$= \frac{\pi}{2} + \frac{\pi}{6} = \frac{3\pi}{4}$

Range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ therefore

$\Rightarrow$ principle value $= \pi/6$

Hold.

(4) $\tan^{-1}(-\sqrt{3})$

Sol. let, $\tan^{-1}(-\sqrt{3}) = \theta$ $\{\because \tan(-\theta) =$

$\because \tan\theta = -\sqrt{3}$

$\tan\theta = -\tan\frac{\pi}{3} \Rightarrow \tan\theta = \tan\left(-\frac{\pi}{3}\right)$

$\theta = \cancel{4}\,\pi/3$

$\tan\theta$ is negative in 2^{nd} and 4^{th} quadrant.

for 2^{nd} quadrant $= \frac{\pi}{2} + \theta$

$= \frac{\pi}{2} + \frac{\pi}{3}$

$= \frac{5\pi}{6}$

for 4^{th} quadrant $= -\theta$

$= -\frac{\pi}{3}$

Range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ therefore

$\Rightarrow$ principle value $= -\frac{\pi}{3}$

Constant.

(5) $\cos^{-1}\left(\frac{-1}{2}\right)$

Sol. let, $\cos^{-1}\left(\frac{-1}{2}\right) = \theta$

$\because \cos\theta = \frac{-1}{2}$

$\cos\theta = -\cos\frac{\pi}{3}$

$\cos\theta = \cos\left(\pi - \frac{\pi}{3}\right)$ $\{\because -\cos\theta = \cos(\pi-\theta)$

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$\Rightarrow \cos\theta = \cos\frac{2\pi}{3}$

Principle

$\Rightarrow$ sol. $\theta = \frac{2\pi}{3}$, range of $\cos^{-1}$ is $[0, \pi]$

$\cos\theta$ is negative in 2^{nd} and 3^{rd} quadrant.

for 2^{nd} quadrant $= \frac{\pi}{2} + \theta$

$= \frac{\pi}{2} + \frac{2\pi}{3}$

$= \frac{7\pi}{6}$

for 3^{rd} quadrant $= \pi + \theta$

$= \pi + \frac{2\pi}{3}$

$= \frac{5\pi}{3}$

(6) $\tan^{-1}(-1)$

Sol. let $\tan^{-1}(-1) = \theta$

$\therefore \tan\theta = -1$

$\tan\theta = -\tan\left(\frac{\pi}{4}\right)$

$\tan\theta = \tan\left(-\frac{\pi}{4}\right)$ $\{\because -\tan\theta = \tan(-\theta)\}$

$\theta = \frac{\pi}{4}$

$\tan\theta$ is negative in 2^{nd} and 4^{th} quadrant.

for 2^{nd} quadrant $= \frac{\pi}{2} + \theta$

$= \frac{\pi}{2} + \frac{\pi}{4}$

$= \frac{3\pi}{4}$

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for 4^{th} quadrant $= -\theta$

$= -\dfrac{\pi}{4}$

Range of $\tan^{-1}$ is $\left(-\dfrac{\pi}{2}, \dfrac{\pi}{2}\right)$ therefore

$\Rightarrow$ principle value $= -\dfrac{\pi}{4}$ Ans.

(7) $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right)$

Sol. let, $\sec^{-1}\left(\dfrac{2}{\sqrt{3}}\right) = \theta$

$\therefore \sec\theta = \dfrac{2}{\sqrt{3}}$

$\sec\theta = = \sec\dfrac{\pi}{6}$

$\dfrac{1}{\cos\theta} = \dfrac{1}{\cos\frac{\pi}{6}}$ $\left\{\because \sec\theta = \dfrac{1}{\cos\theta}\right.$

$\cos\theta = \cos\dfrac{\pi}{6}$

$\theta = \dfrac{\pi}{6}$

$\sec\theta$ is positive in 1^{st} and 4^{th} quadrant.

for 1^{st} quadrant $= \theta$

$= \dfrac{\pi}{6}$

for 4^{th} quadrant $= -\theta$

$= -\dfrac{\pi}{6}$

Range of $\cos^{-1}$ is $[0, \pi]$ therefore

principle value $= \dfrac{\pi}{6}$

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(8) $\cot^{-1}(\sqrt{3})$

Sol. let $\cot^{-1}(\sqrt{3}) = \theta$

$\therefore \cot\theta = \sqrt{3}$

$\Rightarrow \cot\theta = \cot\frac{\pi}{6}$

$\Rightarrow \frac{1}{\tan\theta} = \frac{1}{\tan\frac{\pi}{6}}$ $\left\{\because \tan\theta = \frac{1}{\cot\theta}\right\}$

$\Rightarrow \tan\theta = \tan\frac{\pi}{6}$

$\Rightarrow \theta = \frac{\pi}{6}$

$\cot\theta$ or $\tan\theta$ is positive in 1^{st} and 3^{rd} quadrant.

for 1^{st} quadrant $= \theta$

$= \frac{\pi}{6}$

for 3^{rd} quadrant $= \pi + \theta$

$= \pi + \frac{\pi}{6}$

$= \frac{7\pi}{6}$

Rough

$\frac{7 \times 180}{6}$ 30

210

Range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$ then principle value $= \frac{\pi}{6}$

(9) $\cos^{-1}(-1/\sqrt{2})$

Sol. let $\cos^{-1}(-1/\sqrt{2}) = \theta$

$\therefore \cos\theta = -1/\sqrt{2}$

$\cos\theta = -\cos\frac{\pi}{4}$

$\cos\theta = \cos(\pi - \pi/4)$ $\{\because -\cos\theta = \cos(\pi-\theta)\}$

$\cos\theta = \cos\frac{3\pi}{4}$

Principle value $\theta = \frac{3\pi}{4}$, because range of $\cos^{-1}$ is $[0, \pi]$

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(10) $\operatorname{cosec}^{-1}(-\sqrt{2})$

Sol: let $\operatorname{cosec}^{-1}(-\sqrt{2}) = \theta$

$\because \operatorname{cosec}\theta = -\sqrt{2}$

$\operatorname{cosec}\theta = -\operatorname{cosec}\frac{\pi}{4}$

$\operatorname{cosec}\theta = \operatorname{cosec}(-\pi/4)$ $\{\because -\operatorname{cosec}\theta = \operatorname{cosec}(-\theta)$

$\theta = \pi/4$

$\operatorname{cosec}\theta$ is negative in 3^{rd} and 4^{th} quadrant.

for 3^{rd} quadrant $= \pi + \theta$

$= \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

for 4^{th} quadrant $= -\theta$

$= -\pi/4$

range of $\sin^{-1}\theta$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ therefore

principle value $= \frac{-\pi}{4}$ Ans.

(5) $\cos^{-1}\left(-\frac{1}{2}\right)$

Sol. let $\cos^{-1}\left(-\frac{1}{2}\right) = \theta$

$\Rightarrow \because \cos\theta = -\frac{1}{2}$

$\Rightarrow \cos\theta = -\cos\frac{\pi}{3}$

$\Rightarrow \cos\theta = \cos\left(\pi - \frac{\pi}{3}\right)$ $\{\because -\cos\theta = \cos(-\theta)\}$

$\Rightarrow \cos\theta = \cos\frac{2\pi}{3}$ $\rightarrow$ Range of $\cos^{-1}$ is $[0, \pi]$

$\Rightarrow$ principle value $\theta = \frac{2\pi}{3}$

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(14) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to -

Sol. let, $\tan^{-1}\sqrt{3} = \alpha$

$\because \tan\alpha = \sqrt{3}$

$\tan\alpha = \tan\frac{\pi}{3}$

Range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\because$ Principle value $(\alpha) = \frac{\pi}{3}$

Now let,

$\sec^{-1}(-2) = \beta$

$\sec\beta = -2$

$\therefore \cos\beta = -\frac{1}{2}$

$\cos\beta = -\cos\frac{\pi}{3}$

$\cos\beta = \cos\left(\pi - \frac{\pi}{3}\right)$

$\cos\beta = \cos\frac{2\pi}{3}$

$\therefore \beta = \frac{2\pi}{3}$, Principle value,

Range of ~~cos β~~ $\cos^{-1}$ is $[0, \pi]$

Now,

$= \tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \frac{2\pi}{3}$

$= \frac{\pi - 2\pi}{3}$

$= -\frac{\pi}{3}$ Hence option B is correct.

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(10) $cosec^{-1}(-\sqrt{2})$

Sol: let $cosec^{-1}(-\sqrt{2}) = \theta$

$\because cosec\theta = -\sqrt{2}$

$cosec\theta = -cosec\frac{\pi}{4}$

$cosec\theta = cosec(-\pi/4)$ $\{\because -cosec\theta = cosec(-\theta)\}$

$\theta = \pi/4$

$cosec\theta$ is negative in 3^{rd} and 4^{th} quadrant.

for 3^{rd} quadrant $= \pi + \theta$

$= \pi + \frac{\pi}{4}$

$= \frac{5\pi}{4}$

for 4^{th} quadrant $= -\theta$

$= -\pi/4$

range of $sin^{-1}\theta$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$ therefore

principle value $= -\frac{\pi}{4}$ Ans.

(5) $cos^{-1}\left(-\frac{1}{2}\right)$

Sol. let $cos^{-1}\left(-\frac{1}{2}\right) = \theta$

$\Rightarrow \because cos\theta = -\frac{1}{2}$

$\Rightarrow cos\theta = -cos\frac{\pi}{3}$

$\Rightarrow cos\theta = cos\left(\pi - \frac{\pi}{3}\right)$ $\{\because -cos\theta = cos(-\theta)\}$

$\Rightarrow cos\theta = cos\frac{2\pi}{3}$ $\rightarrow$ Range of cos^{-1} is $[0, \pi]$

$\Rightarrow$ Principle value $\theta = \frac{2\pi}{3}$

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(14) $\tan^{-1}\sqrt{3} - \sec^{-1}(-2)$ is equal to -

Sol. let, $\tan^{-1}\sqrt{3} = \alpha$

$\therefore \tan\alpha = \sqrt{3}$

$\tan\alpha = \tan\frac{\pi}{3}$

Range of $\tan^{-1}$ is $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore$ Principle value $(\alpha) = \frac{\pi}{3}$

Now let,

$\sec^{-1}(-2) = \beta$

$\sec\beta = -2$

$\therefore \cos\beta = -\frac{1}{2}$

$\cos\beta = -\cos\frac{\pi}{3}$

$\cos\beta = \cos\left(\pi - \frac{\pi}{3}\right)$

$\cos\beta = \cos\frac{2\pi}{3}$

$\therefore \beta = \frac{2\pi}{3}$, Principle value,

Range of ~~cos~~ $\cos^{-1}$ is $[0, \pi]$

Now,

$= \tan^{-1}\sqrt{3} - \sec^{-1}(-2)$

$= \frac{\pi}{3} - \frac{2\pi}{3}$

$= \frac{\pi - 2\pi}{3}$

$= -\frac{\pi}{3}$ Hence option B is correct.

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cont.

(11) $\tan^{-1}(1) + \cos^{-1}\left(-\frac{1}{2}\right) + \sin^{-1}\left(\frac{1}{2}\right)$

Sol. let, $\tan^{-1}(1) = \theta$

$\therefore \tan\theta = 1$

$\tan\theta = \tan\frac{\pi}{4}$ [Range of $\tan^{-1}$ $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\therefore \theta = \frac{\pi}{4}$

Now, $\cos^{-1}\left(-\frac{1}{2}\right) = \alpha$

$\therefore \cos\alpha = -\frac{1}{2}$

$\cos\alpha = -\cos\frac{\pi}{3}$

$\cos\alpha = \cos(\pi - \pi/3)$

$\cos\alpha = \cos(2\pi/3)$

$\therefore \alpha = \frac{2\pi}{3}$

Range of $\cos^{-1}$ is $[0, \pi]$

Now, $\sin^{-1}\left(\frac{1}{2}\right) = \beta$

$\therefore \sin\beta = \frac{1}{2}$

$\sin\beta = \sin\frac{\pi}{6}$

Range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$, then

$\therefore \beta = \frac{\pi}{6}$

Now,

$= \tan^{-1}(1) + \cos^{-1}(-1/2) + \sin^{-1}(1/2)$

$= \frac{\pi}{4} + \frac{2\pi}{3} + \frac{\pi}{6}$

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$$= \frac{11\pi}{12} + \frac{\pi}{6}$$

$$= \frac{66\pi + 12\pi}{72}$$

$$= \frac{88\pi}{72}$$

$$= \frac{11\pi}{9}$$

(12) $\cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

Sol: let $\cos^{-1}\left(\frac{1}{2}\right) = \alpha$

$\therefore \cos\alpha = \frac{1}{2}$

$\cos\alpha = \cos\frac{\pi}{3}$

$\alpha = \frac{\pi}{3}$, since Range of $\cos^{-1}$ is $[0, \pi]$

Now,

$\sin^{-1}\left(\frac{1}{2}\right) = \beta$

$\therefore \sin\beta = \frac{1}{2}$

$\sin\beta = \sin\frac{\pi}{6}$

$\beta = \frac{\pi}{6}$, Range of $\sin^{-1}$ is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

Now,

$= \cos^{-1}\left(\frac{1}{2}\right) + 2\sin^{-1}\left(\frac{1}{2}\right)$

$= \frac{\pi}{3} + \cancel{2}\left(\frac{\pi}{\cancel{6}_3}\right)$

$= \frac{2\pi}{3}$ Ans.

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(13) If $\sin^{-1} x = y$ then

Ans. $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$

Formula of Inverse Trigonometry Function

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(1)

(a) $\sin^{-1} \sin x = x$

(b) $\cos^{-1} \cos x = x$

(c) $\tan^{-1} \tan x = x$

(2)

(a) $\sin^{-1}\left(\frac{1}{x}\right) = \operatorname{cosec}^{-1} x$

(b) $\cos^{-1}\left(\frac{1}{x}\right) = \sec^{-1} x$

(c) $\tan^{-1}\left(\frac{1}{x}\right) = \cot^{-1} x$

(3)

(a) $\sin^{-1}(-x) = -\sin^{-1} x$

(b) $\cos^{-1}(-x) = \pi - \cos^{-1} x$

(c) $\tan^{-1}(-x) = -\tan^{-1} x$

(d) $\cot^{-1}(-x) = \pi - \cot^{-1} x$

(e) $\sec^{-1}(-x) = \pi - \sec^{-1} x$

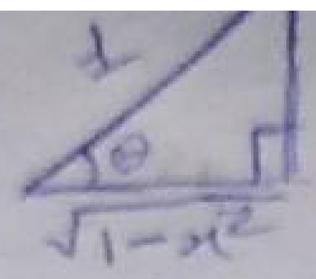
(f) $\operatorname{cosec}^{-1}(-x) = -\operatorname{cosec}^{-1} x$

(4) Conversion property :-

(a) $\sin^{-1} x = \cos^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

(b) $\cos^{-1} x = \sin^{-1}\sqrt{1-x^2} = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)$

(c) $\tan^{-1} x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right) = \cos^{-1}\left(\frac{1}{\sqrt{1+x^2}}\right) = \sec^{-1}\sqrt{1+x^2}$



$\Rightarrow \theta = \sin^{-1}x, \quad \theta = \cos^{-1}\sqrt{1-x^2}, \quad \theta = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$

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(5) (a) $\sin^{-1}x + \cos^{-1}x = \pi/2$

(b) $\tan^{-1}x + \cot^{-1}x = \pi/2$

(c) $\sec^{-1}x + \operatorname{cosec}^{-1}x = \pi/2$

(6) $\tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)$

(7) $\tan^{-1}x - \tan^{-1}y = \tan^{-1}\left(\frac{x-y}{1+xy}\right)$

(8) $\sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$

(9) $\sin^{-1}x - \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} - y\sqrt{1-x^2}\}$

(10) $\cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$

(11) $\cos^{-1}x - \cos^{-1}y = \cos^{-1}\{xy + \sqrt{1-x^2}\sqrt{1-y^2}\}$

(12) $\cot^{-1}x + \cot^{-1}y = \cot^{-1}\left(\frac{xy-1}{x+y}\right)$

(13) $\cot^{-1}x - \cot^{-1}y = \cot^{-1}\left(\frac{xy+1}{y-x}\right)$

(14) (a) $2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)$

(b) $2\tan^{-1}x = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

(c) $2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)$

(15) $2\sin^{-1}x = \sin^{-1}(2x\sqrt{1-x^2})$

(16) $2\cos^{-1}x = \cos^{-1}(2x^2-1)$

(17) $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

(18) $3\cos^{-1}x = \cos^{-1}(4x^3-3x)$

(19) $3\tan^{-1}x = \tan^{-1}\left(\frac{3x-x^3}{1-3x^2}\right)$

(20) $2\cot^{-1}x = \cot^{-1}\left(\frac{x^2-1}{2x}\right)$

C.W.

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Proof of, $\sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

$\Rightarrow \because \sin(A+B) = \sin A \cos B + \cos A \sin B$

let,

$\sin A = x$, $\sin B = y$

$\because \cos^2 A = 1 - \sin^2 A$ | $\cos B = \sqrt{1-y^2}$

$\cos A = \sqrt{1-x^2}$

$\Rightarrow \sin(A+B) = x\sqrt{1-y^2} + y\sqrt{1-x^2}$

$\Rightarrow A+B = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\}$

$\Rightarrow \sin^{-1}x + \sin^{-1}y = \sin^{-1}(x\sqrt{1-y^2} + y\sqrt{1-x^2})$

proof of $3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

$\Rightarrow$ let $\sin\theta = x$ and $\theta = \sin^{-1}x$

$\because \sin 3\theta = 3x - 4x^3$

$\Rightarrow 3\theta = \sin^{-1}(3x-4x^3)$

$\Rightarrow 3\sin^{-1}x = \sin^{-1}(3x-4x^3)$

Hence proved

proof of $\cos^{-1}x + \cos^{-1}y = \cos^{-1}(xy - \sqrt{1-x^2}\sqrt{1-y^2})$

$\Rightarrow \cos(A+B) = \cos A \cos B - \sin A \sin B$

let, $\cos A = x$, $\cos B = y$

$\because \sin^2 A = 1 - \cos^2 A$ | $\because \sin^2 B = 1 - \cos^2 B$

$\sin A = \sqrt{1-x^2}$ | $\sin B = \sqrt{1-y^2}$

Now,

$\Rightarrow \cos(A+B) = xy - \sqrt{1-x^2}\sqrt{1-y^2}$

$\Rightarrow A+B = \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$

$\Rightarrow \cos^{-1}x + \cos^{-1}y = \cos^{-1}\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\}$

1) prove that $\sin^{-1}x + \cos^{-1}x = \pi/2$

$\because \sin\left(\frac{\pi}{2} - \theta\right) = \cos\theta$

let $\cos\theta = x$ (or) $\theta = \cos^{-1}x$

$\therefore \sin\left(\frac{\pi}{2} - \theta\right) = x$

$\Rightarrow \frac{\pi}{2} - \theta = \sin^{-1}x.$

$\Rightarrow \frac{\pi}{2} - \cos^{-1}x = \sin^{-1}x$

$\Rightarrow \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2}$

2) prove that $\tan^{-1}x + \cot^{-1}x = \pi/2$

$\because \tan\left(\frac{\pi}{2} - \theta\right) = \cot\theta$

let $\cot\theta = x$ (or) $\theta = \cot^{-1}x$

$\therefore \tan\left(\frac{\pi}{2} - \theta\right) = x$

$\Rightarrow \frac{\pi}{2} - \theta = \tan^{-1}x$

$\Rightarrow \frac{\pi}{2} - \cot^{-1}x = \tan^{-1}x$

$\Rightarrow \cot^{-1}x + \tan^{-1}x = \frac{\pi}{2}$

3) Conversion property :-

let, $\tan^{-1}x = \theta$

$\Rightarrow \tan\theta = x = \frac{x}{1}$

$\because \tan\theta = \frac{P}{B}$

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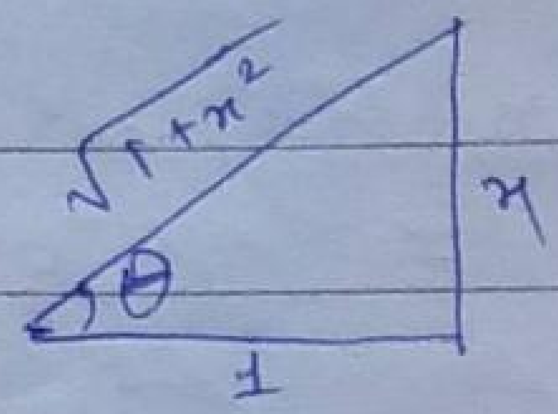
$\Rightarrow H^2 = P^2 + B^2$

$\Rightarrow H^2 = x^2 + 1$

$\Rightarrow H = \sqrt{x^2+1}$



$\Rightarrow \sin\theta = \frac{P}{H} = \frac{x}{\sqrt{1+x^2}}$

$\therefore \theta = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

$\Rightarrow \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)$

4) Prove that $3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

let $\cos\theta = x$ or $\theta = \cos^{-1}x$

$\because \cos 3\theta = 4\cos^3\theta - 3\cos\theta$

$\Rightarrow \cos 3\theta = 4x^3 - 3x$

$\Rightarrow 3\theta = \cos^{-1}(4x^3 - 3x)$

$\Rightarrow 3\cos^{-1}x = \cos^{-1}(4x^3 - 3x)$

Exercise 2.2

Prove that

(4) $2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7} = \tan^{-1}\frac{31}{17}$

Sol.

L.H.S. $= 2\tan^{-1}\frac{1}{2} + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left\{\frac{2\cdot\frac{1}{2}}{1-\left(\frac{1}{2}\right)^2}\right\} + \tan^{-1}\frac{1}{7}$ $\left\{\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right.$

$= \tan^{-1}\left(\frac{4}{3}\right) + \tan^{-1}\frac{1}{7}$

$= \tan^{-1}\left(\frac{\frac{4}{3}+\frac{1}{7}}{1-\frac{4}{3}\times\frac{1}{7}}\right)$ $\left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right.$

$= \tan^{-1}\left(\frac{31}{17}\right)$

Hence Proved.

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Prove that :

(3) $\tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24} = \tan^{-1}\frac{1}{2}$

Sol. L.H.S $= \tan^{-1}\frac{2}{11} + \tan^{-1}\frac{7}{24}$

$$= \tan^{-1}\left(\frac{\frac{2}{11}+\frac{7}{24}}{1-\frac{2}{11}\cdot\frac{7}{24}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{48+77}{264}}{1-\frac{14}{264}}\right)$$

$\left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$

$$= \tan^{-1}\left(\frac{\frac{125}{264}}{\frac{250}{264}}\right)$$

$$= \tan^{-1}\left(\frac{125}{250}\right)$$

$$= \tan^{-1}\left(\frac{1}{2}\right)$$ Ans.

Home-Work

Prove that :-

(1) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$

Sol. L.H.S. $= 2\tan^{-1}\left(\frac{1}{5}\right) + \tan^{-1}\left(\frac{1}{8}\right)$

$$= \tan^{-1}\left(\frac{2\cdot\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$\left\{\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right.$

$$= \tan^{-1}\left(\frac{\frac{2}{5}}{\frac{24}{25}}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

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$$= \tan^{-1}\left(\frac{10}{0\times 24}\right) + \tan^{-1}\frac{1}{8}$$

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\cdot\frac{1}{8}}\right) \quad \left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$$= \tan^{-1}\left(\frac{\frac{40+12}{96}}{1-\frac{5}{96}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{52}{96}}{\frac{91}{96}}\right) = \tan^{-1}\left(\frac{52}{91}\right)$$

$$= \cancel{\tan^{-1}\left(\frac{4}{19}\right)} = \tan^{-1}\left(\frac{4}{7}\right)$$

L.H.S = R.H.S
Hence Proved.

2) $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Sol. L.H.S. $= \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{8}$.

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{3}}{1-\frac{1}{5}\cdot\frac{1}{3}}\right) + \tan^{-1}\left(\frac{\frac{1}{7}+\frac{1}{8}}{1-\frac{1}{7}\cdot\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{8}{15}}{\frac{14}{15}}\right) + \tan^{-1}\left(\frac{\frac{15}{56}}{\frac{55}{56}}\right)$$

$$= \tan^{-1}\left(\frac{4}{7}\right) + \tan^{-1}\left(\frac{3}{11}\right)$$

$$= \tan^{-1}\left(\frac{\frac{4}{7}+\frac{3}{11}}{1-\frac{4}{7}\cdot\frac{3}{11}}\right)$$

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$$= \tan^{-1}\left(\frac{\frac{44+21}{77}}{\frac{65}{77}}\right)$$

$$= \tan^{-1}\left(\frac{65}{65}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4}$$

L.H.S = R.H.S

Hence Proved.

7) $2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8} = \tan^{-1}\frac{4}{7}$

Sol: L.H.S. $= 2\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{8}$

$$= \tan^{-1}\left(\frac{2\cdot\frac{1}{5}}{1-\left(\frac{1}{5}\right)^2}\right) + \tan^{-1}\frac{1}{8} \quad \left\{\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right\}$$

$$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{1}{8}\right)$$

$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{1}{8}}{1-\frac{5}{12}\cdot\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{40+12}{96}}{1-\frac{5}{96}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{52}{96}}{\frac{91}{96}}\right)$$

$$= \tan^{-1}\left(\frac{52}{96}\right) = \tan^{-1}\left(\frac{4}{7}\right)$$

Hence Proved.

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Exercise 2.2

Simplify :-

(5) $\tan^{-1} \frac{\sqrt{1+x^2}-1}{x}$, $x \neq 0$

Sol. Putting $x = \tan\theta$

$$= \tan^{-1}\left(\frac{\sqrt{1+\tan^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sqrt{\sec^2\theta}-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\sec\theta-1}{\tan\theta}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1}{\cos\theta}-1}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{1-\cos\theta}{\cos\theta}}{\frac{\sin\theta}{\cos\theta}}\right)$$

$$= \tan^{-1}\left(\frac{1-\cos\theta}{\sin\theta}\right)$$

$$= \tan^{-1}\left(\frac{\cancel{2}\sin^{\cancel{2}}\frac{\theta}{2}}{\cancel{2}\cancel{\sin\frac{\theta}{2}}\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\frac{\sin\frac{\theta}{2}}{\cos\frac{\theta}{2}}\right)$$

$$= \tan^{-1}\left(\tan\frac{\theta}{2}\right)$$

$$= \frac{\theta}{2}$$

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(8) $\tan^{-1}\left(\frac{\cos x - \sin x}{\cos x + \sin x}\right)$, $0 < x < \pi$

Sol.

Dividing Numerator and denominator by $\cos x$.

$$= \tan^{-1}\left(\frac{\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}}{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}\right)$$

$$= \tan^{-1}\left(\frac{1 - \tan x}{1 + \tan x}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + (1)(\tan x)}\right)$$

$$= \tan^{-1}\left(\frac{\tan\frac{\pi}{4} - \tan x}{1 + \tan\frac{\pi}{4}\ \tan x}\right)$$

$$\because \tan(A-B) = \frac{\tan A - \tan B}{1 + \tan A\ \tan B}$$

$$= \tan^{-1}\left(\tan\left(\frac{\pi}{4} - x\right)\right)$$

$$= \frac{\pi}{4} - x$$

11) $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

$$= \tan^{-1}\left[2\cos\left(2\cdot\frac{\pi}{6}\right)\right]$$

$$= \tan^{-1}\left[2\cos\left(\frac{\pi}{3}\right)\right]$$

$$= \tan^{-1}\left[2\cdot\frac{1}{2}\right]$$

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$= \tan^{-1}\left[1\right]$

$= \tan^{-1}\left[\tan\frac{\pi}{4}\right]$

$= \frac{\pi}{4}$

15) $\tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$, find x.

Sol.

$\Rightarrow \tan^{-1}\frac{x-1}{x-2} + \tan^{-1}\frac{x+1}{x+2} = \frac{\pi}{4}$

$$\Rightarrow \tan^{-1}\left\{\frac{\frac{x-1}{x-2} + \frac{x+1}{x+2}}{1 - \left(\frac{x-1}{x-2}\right)\left(\frac{x+1}{x+2}\right)}\right\} = \frac{\pi}{4}$$

$$\Rightarrow \frac{\frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2)}}{\frac{(x-2)(x+2) - (x-1)(x+1)}{(x-2)(x+2)}} = \tan\frac{\pi}{4}$$

$$\Rightarrow \frac{(x-1)(x+2) + (x+1)(x-2)}{(x-2)(x+2) - (x-1)(x+1)} = 1$$

$$\Rightarrow \frac{x^2+2x-x-2+x^2-2x+x-2}{x^2-4-(x^2-1)} = 1$$

$\Rightarrow 2x^2 - 4 = x^2 - 4 - x^2 + 1$

$\Rightarrow 2x^2 - x^2 + x^2 - 1 = 0$

$\Rightarrow x^2 = \frac{1}{2}$

$\Rightarrow x = \pm\frac{1}{\sqrt{2}}$ Ans.

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Home Work.

Prove that:

1) $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Sol. L.H.S $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$= \cos^{-1}\left\{\frac{4}{5}\cdot\frac{12}{13} - \sqrt{1-\left(\frac{12}{13}\right)^2}\cdot\sqrt{1-\left(\frac{4}{5}\right)^2}\right\}$

$\left\{\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}\right\}$

$= \cos^{-1}\left\{\frac{48}{65} - \frac{5}{13}\cdot\frac{3}{5}\right\}$

$= \cos^{-1}\left\{\frac{48-15}{65}\right\}$

$= \cos^{-1}\left\{\frac{33}{65}\right\}$

L.H.S = R.H.S

Hence proved.

2) $\cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5} = \tan^{-1}\frac{27}{11}$

Sol. L.H.S $= \cos^{-1}\frac{4}{5} + \tan^{-1}\frac{3}{5}$

$= \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{4}{5}\right)^2}}{\frac{4}{5}}\right) + \tan^{-1}\frac{3}{5}$ $\left\{\because \cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right.$

$= \tan^{-1}\left(\frac{\frac{3}{5}}{\frac{4}{5}}\right) + \tan^{-1}\frac{3}{5}$

$= \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{3}{5}$

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$$= \tan^{-1}\left(\frac{\frac{3}{4}+\frac{3}{5}}{1-\frac{3}{4}\cdot\frac{3}{5}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{27}{20}}{\frac{11}{20}}\right)$$

$$= \tan^{-1}\frac{27}{11}$$

L.H.S = R.H.S

Hence Proved.

3) $\sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3} = \frac{\pi}{2}$

Sol. L.H.S $= \sin^{-1}\frac{4}{5} + 2\tan^{-1}\frac{1}{3}$

$$= \sin^{-1}\frac{4}{5} + \sin^{-1}\left(\frac{2\cdot\frac{1}{3}}{1+\left(\frac{1}{3}\right)^2}\right)$$

$$\left\{\because 2\tan^{-1}x = \sin^{-1}\left(\frac{2x}{1+x^2}\right)\right\}$$

$$= \sin^{-1}\frac{4}{5} + \sin^{-1}\left(\frac{\frac{2}{3}}{\frac{10}{9}}\right)$$

$$= \sin^{-1}\frac{4}{5} + \sin^{-1}\frac{3}{5}$$

$$= \sin^{-1}\left\{\frac{4}{5}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{4}{5}\right)^2}\right\}$$

$$\left\{\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right.$$

$$= \sin^{-1}\left\{\frac{16}{25} + \frac{9}{25}\right\}$$

$$= \sin^{-1}\{1\}$$

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$= \sin^{-1}\{\sin\frac{\pi}{2}\}$

$= \frac{\pi}{2}$

L.H.S = R.H.S

Hence proved.

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Exercise 2.2.

6) $\tan^{-1}\frac{1}{\sqrt{x^2-1}}$, $|x| > 1$

Sol: $= \tan^{-1}\frac{1}{\sqrt{x^2-1}}$

putting $x = \operatorname{cosec}\theta$ and $\theta = \operatorname{cosec}^{-1}x$

$= \tan^{-1}\frac{1}{\sqrt{\operatorname{cosec}^2\theta - 1}}$

$= \tan^{-1}\frac{1}{\cot\theta}$

$= \tan^{-1}(\tan\theta)$

$= \theta$

$= \operatorname{cosec}^{-1}x$

$\because \operatorname{cosec}^{-1}x + \sec^{-1}x = \frac{\pi}{2}$

$\operatorname{cosec}^{-1}x = \frac{\pi}{2} - \sec^{-1}x$

$= \frac{\pi}{2} - \sec^{-1}x$ Ans.

7) $\tan^{-1}\left(\sqrt{\frac{1-\cos x}{1+\cos x}}\right)$, $x < \pi$

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Sol. $= \tan^{-1}\left(\sqrt{\dfrac{1-\cos x}{1+\cos x}}\right)$

$= \tan^{-1}\left\{\sqrt{\dfrac{2\sin^2\frac{x}{2}}{2\cos^2\frac{x}{2}}}\right\}$

$= \tan^{-1}\left(\dfrac{\sin\frac{x}{2}}{\cos\frac{x}{2}}\right)$

$= \tan^{-1}\left(\tan\frac{x}{2}\right)$

$= \dfrac{x}{2}$

9) $\tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}$, $|x| < a$

Sol. $= \tan^{-1}\dfrac{x}{\sqrt{a^2-x^2}}$

~~let~~ put $x = a\sin\theta$ and $\theta = \sin^{-1}\left(\frac{x}{a}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{\sqrt{a^2-a^2\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{a\sin\theta}{a\sqrt{1-\sin^2\theta}}\right)$

$= \tan^{-1}\left(\dfrac{\sin\theta}{\cos\theta}\right)$

$= \tan^{-1}(\tan\theta)$

$= \theta$

$= \sin^{-1}\dfrac{x}{a}$

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11) Find Value of $\tan^{-1}\left[2\cos\left(2\sin^{-1}\frac{1}{2}\right)\right]$

Sol. $= \tan^{-1}\left[2\cos\left(2\sin^{-1}\left(\sin\frac{\pi}{6}\right)\right)\right]$

$= \tan^{-1}\left[2\cos\left(2\cdot\frac{\pi}{6}\right)\right]$

$= \tan^{-1}\left[2\cos\frac{\pi}{3}\right]$

$= \tan^{-1}\left[2\cdot\frac{1}{2}\right]$

$= \tan^{-1}[1]$

$= \tan^{-1}\left[\tan\frac{\pi}{4}\right]$

$= \frac{\pi}{4}$

12) $\cot(\tan^{-1}a + \cot^{-1}a)$

$= \cot(\tan^{-1}a + \cot^{-1}a)$

$= \cot\left(\frac{\pi}{2}\right)$

$= 0$

13) $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$, $|x| < 1$, $y > 0$ and $xy < 1$

Sol. put $x = \tan\theta$

or $\theta = \tan^{-1}x$

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$$= \tan \frac{1}{2}\left[\sin^{-1}\left(\frac{2\tan\theta}{1+\tan^2\theta}\right) + \cos^{-1}\frac{1-y^2}{1+y^2}\right]$$

$$= \tan \frac{1}{2}\left[\sin^{-1}(\sin 2\theta) + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$= \tan \frac{1}{2}\left[2\theta + \cos^{-1}\left(\frac{1-y^2}{1+y^2}\right)\right]$$

$$\cancel{= \tan \frac{1}{2}}\left[\right.$$

put, $y = \tan\phi$ or $\phi = \tan^{-1}y$

$$= \tan \frac{1}{2}\left[2\theta + \cos^{-1}\left(\frac{1-\tan^2\phi}{1+\tan^2\phi}\right)\right]$$

$$= \tan \frac{1}{2}\left[2\theta + \cos^{-1}(\cos 2\phi)\right]$$

$$= \tan \frac{1}{2}\left[2\theta + 2\phi\right]$$

$$= \tan(\theta + \phi)$$

$$= \tan(\tan^{-1}x + \tan^{-1}y)$$

$$= \tan\left[\tan^{-1}\left(\frac{x+y}{1-xy}\right)\right]$$

$$= \frac{x+y}{1-xy}$$

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14) $\sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$, find value of x.

Sol. $\Rightarrow \sin\left(\sin^{-1}\frac{1}{5} + \cos^{-1}x\right) = 1$

$\because \sin(A+B) = \sin A\cos B + \cos A\sin B.$

$\Rightarrow \sin\left(\sin^{-1}\frac{1}{5}\right)\cos(\cos^{-1}x) + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin(\cos^{-1}x) = 1$

$\Rightarrow \frac{1}{5}\cdot x + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin(\cos^{-1}x) = 1$

$\Rightarrow \frac{x}{5} + \cos\left(\sin^{-1}\frac{1}{5}\right)\sin(\cos^{-1}x) = 1 \longrightarrow (1)$

let $\sin^{-1}\frac{1}{5} = \theta \longrightarrow (2)$

$\Rightarrow$ or $\sin\theta = \frac{1}{5}$

$\because \sin^2\theta + \cos^2\theta = 1$

$\cos\theta = \sqrt{1-\sin^2\theta}$

$\cos\theta = \sqrt{1-\left(\frac{1}{5}\right)^2}$

$\cos\theta = \sqrt{\frac{24}{25}}$

$\Rightarrow \cos\theta = \frac{2\sqrt{6}}{5}$ or $\theta = \cos^{-1}\frac{2\sqrt{6}}{5} \longrightarrow (3)$

from (2) & (3)

$\Rightarrow \sin^{-1}\frac{1}{5} = \cos^{-1}\frac{2\sqrt{6}}{5} \longrightarrow (4)$

Now, let, $\cos^{-1}x = \phi \longrightarrow (5)$

or $\cos\phi = x$

$\because \sin^2\phi + \cos^2\phi = 1$

$\Rightarrow \sin\phi = \sqrt{1-\cos^2\phi}$

$\Rightarrow \sin\phi = \sqrt{1-x^2} \longrightarrow (6)$

$\Rightarrow \sin^{-1}\sqrt{1-x^2} = \phi$

from (5) and (6),

$\Rightarrow \cos^{-1}x = \sin^{-1}\left(\sqrt{1-x^2}\right) \longrightarrow (7)$

from (1), (4) and (7),

$\Rightarrow \frac{x}{5} + \cos\left(\cos^{-1}\left(\frac{2\sqrt{6}}{5}\right)\right) \cdot \sin\left(\sin^{-1}\sqrt{1-x^2}\right) = 1$

$\Rightarrow \frac{x}{5} + \frac{2\sqrt{6}}{5} \cdot \sqrt{1-x^2} = 1$

$\Rightarrow x + 2\sqrt{6} \cdot \sqrt{1-x^2} = 5$

$\Rightarrow 2\sqrt{6}\sqrt{1-x^2} = 5-x$

$\Rightarrow \left(2\sqrt{6}\sqrt{1-x^2}\right)^2 = (5-x)^2$

$\Rightarrow (24)(1-x^2) = 25+x^2-10x$

$\Rightarrow 24-24x^2 = 25+x^2-10x$

$\Rightarrow -25x^2+10x-1 = 0$

$\Rightarrow 25x^2-10x+1 = 0$

$\Rightarrow (5x-1)^2 = 0$

$\Rightarrow 5x-1 = 0$

$\Rightarrow \boxed{x = \frac{1}{5}}$ Ans.

10) $\tan^{-1}\left(\frac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a>0$; $-\frac{a}{\sqrt{3}} \le x \le \frac{a}{\sqrt{3}}$

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Sol. $= \tan^{-1}\left(\dfrac{3a^2x - x^3}{a^3 - 3ax^2}\right)$, $a>0$; $\dfrac{-a}{\sqrt{3}} \le x \le \dfrac{a}{\sqrt{3}}$

~~let~~ put $x = a\tan\theta$ and $\theta = \tan^{-1}\left(\dfrac{x}{a}\right)$

$$= \tan^{-1}\left(\frac{3a^2(a\tan\theta) - a^3\tan^3\theta}{a^3 - 3a(a^2\tan^2\theta)}\right)$$

$$= \tan^{-1}\left(\frac{3a^3\tan\theta - a^3\tan^3\theta}{a^3 - 3a^3\tan^2\theta}\right)$$

$$= \tan^{-1}\left(\frac{a^3}{a^3}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)\right)$$

$$= \tan^{-1}\left(\frac{3\tan\theta - \tan^3\theta}{1 - 3\tan^2\theta}\right)$$

$$= \tan^{-1}(\tan 3\theta)$$

$$= 3\theta$$

$$= 3\tan^{-1}\left(\frac{x}{a}\right)$$ Ans.

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16) $\sin^{-1}\left(\sin\frac{2\pi}{3}\right)$

$= \sin^{-1}\left(\sin\left(\pi - \frac{\pi}{3}\right)\right) \quad \left\{\because \sin(\pi-\theta) = \sin\theta\right\}$

$= \cancel{\sin^{-1}} \sin^{-1}\left(\sin\frac{\pi}{3}\right)$

$= \frac{\pi}{3}$ Ans.

18) $\tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

Sol.

$= \tan\left(\sin^{-1}\frac{3}{5} + \cot^{-1}\frac{3}{2}\right)$

$= \tan\left(\tan^{-1}\left\{\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\right\} + \tan^{-1}\frac{2}{3}\right)$

$\because \sin^{-1}x = \tan^{-1}\frac{x}{\sqrt{1-x^2}}$

$\because \cot^{-1}x = \tan^{-1}\left(\frac{1}{x}\right)$

$= \tan\left(\tan^{-1}\left(\frac{3}{4}\right) + \tan^{-1}\frac{2}{3}\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{3}{4}+\frac{2}{3}}{1-\frac{3}{4}\cdot\frac{2}{3}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{\frac{17}{12}}{\frac{6}{12}}\right)\right)$

$= \tan\left(\tan^{-1}\left(\frac{17}{6}\right)\right)$

$= \frac{17}{6}$ Ans.

$\sin^{-1} = \tan^{-1} \frac{x}{\sqrt{1-x^2}}$

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Miscellaneous

Prove

4) $\sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5} = \tan^{-1}\frac{77}{36}$

Sol. L.H.S. $= \sin^{-1}\frac{8}{17} + \sin^{-1}\frac{3}{5}$

$$= \sin^{-1}\left\{\frac{8}{17}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{8}{17}\right)^2}\right\}$$

$$= \sin^{-1}\left\{\frac{8}{17}\sqrt{\frac{16}{25}} + \frac{3}{5}\sqrt{\frac{289-64}{289}}\right\}$$

$$= \sin^{-1}\left\{\left(\frac{8}{17}\right)\left(\frac{4}{5}\right) + \left(\frac{3}{5}\right)\frac{\sqrt{225}}{\sqrt{289}}\right\}$$

$$= \sin^{-1}\left\{\frac{32}{85} + \frac{45}{85}\right\}$$

$$= \sin^{-1}\left\{\frac{77}{85}\right\}$$

now,

$$\because \sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{77}{85}}{\sqrt{1-\left(\frac{77}{85}\right)^2}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{77}{85}}{\sqrt{\frac{(85)^2-(77)^2}{(85)^2}}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{77}{85}}{\frac{\sqrt{(85)^2-(77)^2}}{85}}\right)$$

$$= \tan^{-1}\left(\frac{77}{\sqrt{(85+77)(85-77)}}\right)$$

$85 + 77 = 162$

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$$= \tan^{-1}\left(\frac{77}{\sqrt{162 \times 8}}\right)$$

$$= \tan^{-1}\left(\frac{77}{\sqrt{81 \times 16}}\right)$$

$$= \tan^{-1}\left(\frac{77}{36}\right)$$ ~~Ans.~~

L.H.S = R.H.S

Hence proved.

Exercise 2.2.

Prev. 17) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

Sol.

$$= \tan^{-1}\left(\tan\frac{3\pi}{4}\right)$$

$$= \tan^{-1}\left(\cancel{-}\tan\left(\cancel{-}\frac{3\pi}{4}\right)\right)$$

$$= \tan^{-1}\left(-\tan\left(\pi - \frac{\pi}{4}\right)\right)$$

$$\left\{ \because \tan(\pi - \theta) = -\tan\theta \quad -\tan\theta = \tan(-\theta) \right\}$$

$$= \tan^{-1}\left\{-\tan\frac{\pi}{4}\right\}$$

$$= \tan^{-1}\left\{\tan\left(-\frac{\pi}{4}\right)\right\}$$

$$= -\frac{\pi}{4}$$

$$-\frac{\pi}{4} \in \left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$$

19) $\cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

Sol. $= \cos^{-1}\left(\cos\frac{7\pi}{6}\right)$

~~$= \cos^{-1}\left(\cos\left(2\pi - \frac{\pi}{4}\right)\right)$~~

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$= \cos^{-1}\left(\cos \frac{7\pi}{6}\right)$

$= \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{3}\right)\right)$

$= \cos^{-1}\left(\cos \frac{\pi}{3}\right)$

$= \frac{\pi}{3}$

$= \cos^{-1}\left(\cos \frac{7\pi}{6}\right)$

$= \cos^{-1}\left\{\cos\left(-\frac{7\pi}{6}\right)\right\}$

$= \cos^{-1}\left\{\cos\left(2\pi + \left(-\frac{7\pi}{6}\right)\right)\right\}$ $\left\{\because \cos(2\pi + x) = \cos x\right\}$

$= \cos^{-1}\left\{\cos\left(2\pi - \frac{7\pi}{6}\right)\right\}$

$= \cos^{-1}\left\{\cos\left(\frac{12\pi - 7\pi}{6}\right)\right\}$

$= \cos^{-1}\left\{\cos\left(\frac{5\pi}{6}\right)\right\}$

$= \frac{5\pi}{6}$ <u>Ans.</u>, <u>option B.</u>

20) $\sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\frac{-1}{2}\right)\right\}$

<u>Sol.</u>

$= \sin\left\{\frac{\pi}{3} - \sin^{-1}\left(\sin\left(-\frac{\pi}{6}\right)\right)\right\}$

$= \sin\left\{\frac{\pi}{3} + \frac{\pi}{6}\right\}$

$= \sin \frac{\pi}{2}$

$= 1$ <u>Ans.</u>, <u>option D</u>

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Miscellaneous.

1) $\cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

Sol. $= \cos^{-1}\left(\cos\frac{13\pi}{6}\right)$

$= \cos^{-1}\left(\cos\left(2\pi + \frac{\pi}{6}\right)\right) \quad \left\{\because \cos(2\pi+\theta) = \cos\theta\right\}$

$= \cos^{-1}\left(\cos\frac{\pi}{6}\right)$

$= \frac{\pi}{6}$ Ans.

2) $\tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

Sol. $= \tan^{-1}\left(\tan\frac{7\pi}{6}\right)$

$= \tan^{-1}\left(\tan\left(\pi + \frac{\pi}{6}\right)\right) \quad \left\{\because \tan(\pi+\theta) = \tan\theta\right\}$

$= \tan^{-1}\left(\tan\frac{\pi}{6}\right)$

$= \frac{\pi}{6}$ Ans.

Prove that

3) $2\sin^{-1}\frac{3}{5} = \tan^{-1}\frac{24}{7}$

Sol. L.H.S $= 2\sin^{-1}\frac{3}{5}$

$= 2 \cdot \tan^{-1}\left(\frac{\frac{3}{5}}{\sqrt{1-\left(\frac{3}{5}\right)^2}}\right) \quad \left\{\because \sin^{-1}(x) = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right)\right\}$

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$= 2\tan^{-1}\left(\frac{3}{4}\right)$

$= \tan^{-1}\left(\frac{2\cdot\frac{3}{4}}{1-\left(\frac{3}{4}\right)^2}\right)$ $\left\{\because 2\tan^{-1}(x) = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right.$

$= \tan^{-1}\left(\frac{\frac{3}{2}}{\frac{7}{16}}\right)$

$= \tan^{-1}\left(\frac{24}{7}\right)$

L.H.S = R.H.S

Hence proved.

5) $\cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13} = \cos^{-1}\frac{33}{65}$

Sol. L.H.S. $= \cos^{-1}\frac{4}{5} + \cos^{-1}\frac{12}{13}$

$= \cos^{-1}\left\{\frac{4}{5}\cdot\frac{12}{13} - \sqrt{1-\left(\frac{4}{5}\right)^2}\sqrt{1-\left(\frac{12}{13}\right)^2}\right\}$

$\left\{\because \cos^{-1}x + \cos^{-1}y = \cos^{-1}\left\{xy - \sqrt{1-x^2}\sqrt{1-y^2}\right\}\right.$

$= \cos^{-1}\left\{\frac{48}{65} - \frac{3}{5}\cdot\frac{5}{13}\right\}$

$= \cos^{-1}\left(\frac{48-15}{65}\right)$

$= \cos^{-1}\frac{33}{65}$

L.H.S = R.H.S

Hence proved.

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6) $\cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5} = \sin^{-1}\frac{56}{65}$

Sol. $L.H.S = \cos^{-1}\frac{12}{13} + \sin^{-1}\frac{3}{5}$

$= \sin^{-1}\left(\sqrt{1-\left(\frac{12}{13}\right)^2}\right) + \sin^{-1}\frac{3}{5}$

$= \sin^{-1}\left(\frac{5}{13}\right) + \sin^{-1}\left(\frac{3}{5}\right)$

$= \sin^{-1}\left\{\frac{5}{13}\sqrt{1-\left(\frac{3}{5}\right)^2} + \frac{3}{5}\sqrt{1-\left(\frac{5}{13}\right)^2}\right\}$

$\left\{\because \sin^{-1}x + \sin^{-1}y = \sin^{-1}\left\{x\sqrt{1-y^2} + y\sqrt{1-x^2}\right\}\right\}$

$= \sin^{-1}\left\{\frac{20}{65} + \frac{36}{65}\right\}$

$= \sin^{-1}\left(\frac{56}{65}\right)$

$L.H.S = R.H.S$

Hence Proved.

7) $\tan^{-1}\frac{33}{16} = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

Sol. $R.H.S. = \sin^{-1}\frac{5}{13} + \cos^{-1}\frac{3}{5}$

$= \tan^{-1}\left(\frac{\frac{5}{13}}{\sqrt{1-\left(\frac{5}{13}\right)^2}}\right) + \tan^{-1}\left(\frac{\sqrt{1-\left(\frac{3}{5}\right)^2}}{\frac{3}{5}}\right)$

$\left\{\because \sin^{-1}x = \tan^{-1}\left(\frac{x}{\sqrt{1-x^2}}\right),\quad \cos^{-1}x = \tan^{-1}\left(\frac{\sqrt{1-x^2}}{x}\right)\right\}$

$= \tan^{-1}\left(\frac{5}{12}\right) + \tan^{-1}\left(\frac{4}{3}\right)$

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$$= \tan^{-1}\left(\frac{\frac{5}{12}+\frac{4}{3}}{1-\frac{5}{12}\cdot\frac{4}{3}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{63}{36}}{\frac{16}{36}}\right)$$

$$= \tan^{-1}\left(\frac{63}{16}\right)$$

R.H.S = L.H.S

Hence Proved

5) $\tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8} = \frac{\pi}{4}$

Sol. L.H.S $= \tan^{-1}\frac{1}{5} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{8}$

$$= \tan^{-1}\left(\frac{\frac{1}{5}+\frac{1}{7}}{1-\frac{1}{5}\cdot\frac{1}{7}}\right) + \tan^{-1}\left(\frac{\frac{1}{3}+\frac{1}{8}}{1-\frac{1}{3}\cdot\frac{1}{8}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{12}{35}}{\frac{34}{35}}\right) + \tan^{-1}\left(\frac{\frac{11}{24}}{\frac{23}{24}}\right)$$

$$= \tan^{-1}\left(\frac{12}{34}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{6}{17}\right) + \tan^{-1}\left(\frac{11}{23}\right)$$

$$= \tan^{-1}\left(\frac{\frac{6}{17}+\frac{11}{23}}{1-\frac{6}{17}\cdot\frac{11}{23}}\right)$$

$$= \tan^{-1}\left(\frac{\frac{138+187}{391}}{\frac{391-66}{391}}\right)$$

$$= \tan^{-1}\left(\frac{325}{325}\right)$$

Rough.

17×11
187
138
177
315

Rough

23×17 17×11
169 177
$23\times$ 23×6
391 138

$138+$ 391
$+147$ 66
315 325

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$= \tan^{-1}(1)$

$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$= \frac{\pi}{4}$

L.H.S = R.H.S

Hence Proved.

Exercise 2.2.

17) $\tan^{-1}\left(\tan\frac{3\pi}{4}\right)$

sol. $= \tan^{-1}\tan\left(\pi - \frac{\pi}{4}\right)$ $\qquad \because \tan(\pi-\theta) = -\tan\theta$

$= \tan^{-1}\left(-\tan\frac{\pi}{4}\right)$

$= \tan^{-1}\left(\tan\left(-\frac{\pi}{4}\right)\right)$ $\qquad \because -\tan\theta = \tan(-\theta)$

$= -\frac{\pi}{4}$ Ans.

21) $\tan^{-1}\sqrt{3} - \cot^{-1}(-\sqrt{3})$

sol. $= \tan^{-1}\sqrt{3} - \{\pi - \cot^{-1}\sqrt{3})$

$\{\because \cot^{-1}(-x) = \pi - \cot^{-1}x\}$

$= \tan^{-1}\sqrt{3} + \cot^{-1}\sqrt{3} - \pi$

$= \frac{\pi}{2} - \pi$ $\qquad \left\{\because \tan^{-1}x + \cot^{-1}x = \frac{\pi}{2}\right\}$

$= -\frac{\pi}{2}$ Ans. , option B.

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Miscellaneous

Prove that:-

Q.10) $\cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right) = \dfrac{x}{2}$

Sol. L.H.S $= \cot^{-1}\left(\dfrac{\sqrt{1+\sin x}+\sqrt{1-\sin x}}{\sqrt{1+\sin x}-\sqrt{1-\sin x}}\right)$

$$= \cot^{-1}\left\{\frac{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}+\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}{\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}+2\sin\frac{x}{2}\cos\frac{x}{2}}-\sqrt{\cos^2\frac{x}{2}+\sin^2\frac{x}{2}-2\sin\frac{x}{2}\cos\frac{x}{2}}}\right\}$$

$$\left\{\begin{array}{l}\because \sin^2\theta+\cos^2\theta = 1 \\ \text{and} \quad \sin\theta = 2\sin\frac{\theta}{2}\cos\frac{\theta}{2}\end{array}\right.$$

$$= \cot^{-1}\left\{\frac{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}+\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}{\sqrt{\left(\cos\frac{x}{2}+\sin\frac{x}{2}\right)^2}-\sqrt{\left(\cos\frac{x}{2}-\sin\frac{x}{2}\right)^2}}\right\}$$

$$\left\{\begin{array}{l}\because a^2+b^2-2ab = \cancel{(a+b)^2}\,(a-b)^2 \\ a^2+b^2+2ab = (a+b)^2\end{array}\right.$$

$$= \cot^{-1}\left\{\frac{\cos\frac{x}{2}+\cancel{\sin\frac{x}{2}}+\cos\frac{x}{2}-\cancel{\sin\frac{x}{2}}}{\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}-\cancel{\cos\frac{x}{2}}+\sin\frac{x}{2}}\right\}$$

$$= \cot^{-1}\left\{\frac{\cancel{2}\cos\frac{x}{2}}{\cancel{2}\sin\frac{x}{2}}\right\}$$

$$= \cot^{-1}\left\{\cot\frac{x}{2}\right\}$$

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$$= \frac{x}{2}$$

L.H.S = R.H.S

Hence Proved.

iii) $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$

Sol. $\tan^{-1}\frac{1-x}{1+x} = \frac{1}{2}\tan^{-1}x$

put $x = \tan\theta$

$$\Rightarrow \tan^{-1}\left(\frac{1-\tan\theta}{1+\tan\theta}\right) = \frac{1}{2}\tan^{-1}\tan\theta$$

$$\Rightarrow \tan^{-1}\left\{\frac{\tan\frac{\pi}{4}-\tan\theta}{1+\tan\frac{\pi}{4}\tan\theta}\right\} = \frac{1}{2}\cdot\theta$$

$$\Rightarrow \tan^{-1}\tan\left(\frac{\pi}{4}-\theta\right) = \frac{1}{2}\cdot\theta$$

$$\Rightarrow \frac{\pi}{4}-\theta = \frac{1}{2}\cdot\theta$$

$$\Rightarrow \frac{\pi}{4} = \frac{1}{2}\theta+\theta$$

$$\Rightarrow \frac{\pi}{4} = \frac{3\theta}{2}$$

$$\Rightarrow \theta = \frac{\pi}{6}$$

$$\Rightarrow \tan^{-1}x = \frac{\pi}{6} \qquad \left\{\because x = \tan\theta,\ \theta = \tan^{-1}x\right\}$$

$$\Rightarrow x = \tan\frac{\pi}{6}$$

$$\Rightarrow x = \frac{1}{\sqrt{3}}$$

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Prove

11) $\tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right) = \dfrac{\pi}{4} - \dfrac{1}{2}\cos^{-1}x$

Sol. L.H.S $= \tan^{-1}\left(\dfrac{\sqrt{1+x}-\sqrt{1-x}}{\sqrt{1+x}+\sqrt{1-x}}\right)$

put $x = \cos 2\theta$ or $2\theta = \cos^{-1}x \Rightarrow \theta = \frac{1}{2}\cos^{-1}$

$= \tan^{-1}\left\{\dfrac{\sqrt{1+\cos 2\theta} - \sqrt{1-\cos 2\theta}}{\sqrt{1+\cos 2\theta} + \sqrt{1-\cos 2\theta}}\right\}$

$= \tan^{-1}\left\{\dfrac{\sqrt{2\cos^2\theta} - \sqrt{2\sin^2\theta}}{\sqrt{2\cos^2\theta} + \sqrt{2\sin^2\theta}}\right\}$

$= \tan^{-1}\left\{\dfrac{\sqrt{2}\cos\theta - \sqrt{2}\sin\theta}{\sqrt{2}\cos\theta + \sqrt{2}\sin\theta}\right\}$

$= \tan^{-1}\left\{\dfrac{\sqrt{2}(\cos\theta - \sin\theta)}{\sqrt{2}(\cos\theta + \sin\theta)}\right\}$

$= \tan^{-1}\left\{\dfrac{\cos\theta - \sin\theta}{\cos\theta + \sin\theta}\right\}$

on dividing $\cos\theta$ on numerator and denominat

$= \tan^{-1}\left\{\dfrac{\frac{\cos\theta}{\cos\theta} - \frac{\sin\theta}{\cos\theta}}{\frac{\cos\theta}{\cos\theta} + \frac{\sin\theta}{\cos\theta}}\right\}$

$= \tan^{-1}\left\{\dfrac{1 - \tan\theta}{1 + \tan\theta}\right\}$

$= \tan^{-1}\left\{\dfrac{\tan\frac{\pi}{4} - \tan\theta}{1 + \tan\frac{\pi}{4}\tan\theta}\right\}$

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$$= \tan^{-1}\left\{\tan\left(\frac{\pi}{4}-\theta\right)\right\}$$

$$= \frac{\pi}{4} - \theta$$

$$= \frac{\pi}{4} - \frac{1}{2}\cos^{-1}x$$

$$= \frac{2\pi - 4\cos^{-1}x}{8} \quad \text{Ans.}$$

L.H.S = R.H.S

Hence proved

Prove that

9) $\tan^{-1}\sqrt{x} = \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$, $x \in [0,1]$

Sol. R.H.S $= \frac{1}{2}\cos^{-1}\left(\frac{1-x}{1+x}\right)$

Put $x = \tan^2\theta$ or $\theta = \tan^{-1}\sqrt{x}$

$$= \frac{1}{2}\cos^{-1}\left(\frac{1-\tan^2\theta}{1+\tan^2\theta}\right)$$

$$= \frac{1}{2}\cos^{-1}(\cos 2\theta) \quad \left\{\because \cos 2\theta = \frac{1-\tan^2\theta}{1+\tan^2\theta}\right\}$$

$$= \frac{1}{2}\cdot 2\theta$$

$$= \theta$$

$$= \tan^{-1}\sqrt{x}$$

L.H.S = R.H.S

Hence proved.

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Prove that

12) $\frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3} = \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

Sol. $\underline{L.H.S} = \frac{9\pi}{8} - \frac{9}{4}\sin^{-1}\frac{1}{3}$

$= \frac{9}{4}\left(\frac{\pi}{2} - \sin^{-1}\frac{1}{3}\right)$

$= \frac{9}{4}\left(\cos^{-1}\frac{1}{3}\right)$ $\left\{\because \cos^{-1}x + \sin^{-1}x = \frac{\pi}{2} \Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x\right.$

$= \frac{9}{4}\left(\sin^{-1}\sqrt{1-\left(\frac{1}{3}\right)^2}\right)$ $\left\{\because \cos^{-1}x = \sin^{-1}\sqrt{1-x^2}\right.$

$= \frac{9}{4}\left(\sin^{-1}\frac{\sqrt{8}}{3}\right)$

$= \frac{9}{4}\sin^{-1}\frac{\sqrt{2\times2\times2}}{3}$

$= \frac{9}{4} \quad \frac{9}{4}\sin^{-1}\frac{2\sqrt{2}}{3}$

L.H.S = R.H.S

Hence proved.

13) Solve.

$2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

Sol. $\Rightarrow 2\tan^{-1}(\cos x) = \tan^{-1}(2\operatorname{cosec} x)$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{1-\cos^2 x}\right) = \tan^{-1}(2\operatorname{cosec} x)$ $\left\{\because 2\tan^{-1}x = \tan^{-1}\left(\frac{2x}{1-x^2}\right)\right.$

$\Rightarrow \tan^{-1}\left(\frac{2\cos x}{\sin^2 x}\right) = \tan^{-1}\left(2\cdot\frac{1}{\sin x}\right)$ $\left\{\because 1-\cos^2\theta = \sin^2\theta,\ \operatorname{cosec}\theta = \frac{1}{\sin\theta}\right.$

$\Rightarrow \frac{2\cos x}{\sin^2 x} = \frac{2}{\sin x}$

$\Rightarrow \frac{\cos x}{\sin x} = 1$

$\Rightarrow \tan x = 1$

Rough

$-5x = 20$
$x = -4$

$x = -20$
$x = -4$

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$\Rightarrow x = \tan^{-1}(1)$

$\Rightarrow x = \tan^{-1}\left(\tan\frac{\pi}{4}\right)$

$\Rightarrow x = \frac{\pi}{4}$

15) $\sin(\tan^{-1}x)$

Sol. $= \sin(\tan^{-1}x)$

$= \sin\left(\sin^{-1}\frac{x}{\sqrt{1+x^2}}\right)$ $\left\{\because \tan^{-1}x = \sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right\}$

$= \sin\left\{\sin^{-1}\left(\frac{x}{\sqrt{1+x^2}}\right)\right\}$

$= \frac{x}{\sqrt{1+x^2}}$ Ans., option D

16) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Sol. $\Rightarrow \sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow -2\sin^{-1}x = \frac{\pi}{2} - \sin^{-1}(1-x)$

$\Rightarrow -2\sin^{-1}x = \cos^{-1}(1-x)$ $\left\{\because \sin^{-1}x + \cos^{-1}x = \frac{\pi}{2} \Rightarrow \cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x\right\}$

$\Rightarrow 2\sin^{-1}x = -\cos^{-1}(1-x)$

$\Rightarrow 2\left(\cos^{-1}\sqrt{1-x^2}\right) = -\cos^{-1}(1-x)$

$\Rightarrow 2\cos^{-1}\sqrt{1-x^2} + \cos^{-1}(1-x) = 0$

$\Rightarrow \cos^{-1}\left(2\left(\sqrt{1-x^2}\right)^2 - 1\right) + \cos^{-1}(1-x) = 0$ $\left\{\because 2\cos^{-1}x = \cos^{-1}(2x^2-1)\right\}$

$\Rightarrow \cos^{-1}\left(2(1-x^2)-1\right) + \cos^{-1}(1-x) = 0$

$\Rightarrow \cos^{-1}\left(-x^2-1\right) + \cos^{-1}(1-x) = 0$

$\Rightarrow \cos^{-1}\left(-(x^2+1)\right) + \cos^{-1}(1-x) = 0$

$\Rightarrow \pi - \cos^{-1}(x^2+1) + \cos^{-1}(1-x) = 0$

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$\Rightarrow -\cos^{-1}(x^2+1) + \cos^{-1}(1-x) = -\pi$

$\Rightarrow \cos^{-1}(x^2+1) - \cos^{-1}(1-x) = \pi$

$\Rightarrow \cos^{-1}\left\{(x^2+1)(1-x) + \sqrt{1-(x^2+1)^2}\,\sqrt{1-(1-x)^2}\right\} = \pi$

$\left\{\because \cos^{-1}x - \cos^{-1}y = \cos^{-1}\{xy+\sqrt{1-x^2}\sqrt{1-y^2}\}\right\}$

$\Rightarrow \cos^{-1}\left\{x^2-x^3+1-x+\sqrt{1-(x^4+1+2x^2)}\,\sqrt{1-(1+x^2-2x)}\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{x^2-x^3+1-x+\sqrt{1-x^4-1-2x^2}\,\sqrt{1-1-x^2-2x}\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{x^2-x^3+1-x+\sqrt{-x^4-2x^2}\,\sqrt{-x^2-2x}\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{x^2-x^3+1-x+\sqrt{-(x^4+2x^2)}\,\sqrt{-(x^2+2x)}\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{-x^3+x^2-x+1+\sqrt{-\{(x^2)^2+(\sqrt{2}x)^2+2\sqrt{2}x^2\}}\,\sqrt{-\{(x)^2+(\sqrt{2x})^2+2x\sqrt{2x}\}}\right.$

$\left.-2\sqrt{2}x^2-2\sqrt{2x}\cdot x\right\}$

$\Rightarrow \cos^{-1}\left\{-x^3+x^2-x+1+\sqrt{-(x^2+\sqrt{2}x)^2}\,\sqrt{-(x+\sqrt{2x})^2}-2\sqrt{2}(x^2-\sqrt{x}\cdot x)\right\} =$

$\Rightarrow \cos^{-1}\left\{-x^3+x^2-x+1+(-(x^2+\sqrt{2}x))(-(x+\sqrt{2x}))-2\sqrt{2}x(x-\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{-x^3+x^2-x+1+(-x^2-\sqrt{2}x)(-x-\sqrt{2}x)-2\sqrt{2}x(x-\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{(-x^3+x^2-x+1)+(-x^2-\sqrt{2}x)-x-\sqrt{2}x(x^2-\sqrt{2}x)-2\sqrt{2}x(x-\sqrt{x})\right\} =$

$\Rightarrow \cos^{-1}\left\{(-x^3+x^2-x+1)+(x^3+\sqrt{2}x^2)-\sqrt{2}x^2+2x^2-2\sqrt{2}x(x-\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{-\cancel{x^3}+x^2-x+1+\cancel{x^3}+\cancel{\sqrt{2}x^2}-\cancel{\sqrt{2}x^2}+2x^2-2\sqrt{2}x(x-\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{3x^2-x+1-2\sqrt{2}x(x-\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{3x^2-x+1-2\sqrt{2}x^2+2\sqrt{2}x(\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{3x^2-x+1-2\sqrt{2}x^2+\sqrt{8}x(\sqrt{x})\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{3x^2-2\sqrt{2}x^2-x+1+\sqrt{8x^3}\right\} = \pi$

$\Rightarrow \cos^{-1}\left\{3x^2-2\sqrt{2}x^2+\sqrt{8x^3}-x+1\right\} = \pi$

$\Rightarrow 3x^2-2\sqrt{2}x^2+\sqrt{8x^3}-x+1 = \cos\pi$

$\Rightarrow x^2(3-2\sqrt{2})+2\sqrt{2x}\,x^2-x+1 = -1$

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$$\Rightarrow x^2(3-2\sqrt{2}+2\sqrt{2x})-x+1=-1$$

$$\Rightarrow x^2(3-2\sqrt{2}+2\sqrt{2x})-x=0$$

$$\Rightarrow x^2(3-2\sqrt{2}+2\sqrt{2x})=x$$

$$\Rightarrow x(3-2\sqrt{2}+2\sqrt{2x})=1$$

$$\Rightarrow 3-2\sqrt{2}+2\sqrt{2x}=\frac{1}{x}$$

$$\Rightarrow \cancel{3-2\sqrt{2}}\ 3-2(1.41)+2.(1.41)\sqrt{x}=\frac{1}{x}$$

$$\Rightarrow 3-(2.82)+2.82\sqrt{x}=\frac{1}{x}$$

$$\Rightarrow 0.18+2.82(x)^{\frac{1}{2}}=\frac{1}{x}$$

$$\Rightarrow 2.82(x)^{1/2}=\frac{1}{x}-0.18$$

$$\Rightarrow 2.82(x)^{\frac{1}{2}}=\frac{1-0.18x}{x}$$

$$\Rightarrow (x)^{1/2}=\frac{1-0.18x}{2.82x}$$

$$\Rightarrow \sqrt{x^{1/2}}=\sqrt{\frac{1-0.18x}{2.82x}}$$

$$\Rightarrow x^{1/2}=\frac{1-0.18x}{2.82x}$$

$$\Rightarrow x=\left(\frac{1-0.18x}{2.82x}\right)^2$$

$$\Rightarrow x=\frac{1+(0.18x)^2-2(0.18x)(1)}{(2.82x)^2}$$

$$\Rightarrow x=\frac{1+0.032x^2-0.36x}{(2.82x)^2}$$

$$\Rightarrow x=\frac{1+0.032x^2-0.36x}{7.95x^2}$$

$$\Rightarrow x+0.36x=1+0.032x^2/7.95x^2$$

$$\Rightarrow 1.36x=\frac{1+0.032x^2}{7.95x^2}$$

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$\Rightarrow (1.36x)(7.95x^2) = 1 + 0.032x^2$

$\Rightarrow x^3(10.81) = 1 + 0.032x^2$

$\Rightarrow 10.81x^3 - 0.032x^2 - 1 = 0$

$\Rightarrow x^2(10.81x - 0.032) - 1 = 0$

$\Rightarrow \cancel{10.81x - 0.032} = \cancel{\frac{1}{x^2}}$

$\Rightarrow \cancel{10.81x - 0.032} = \cancel{x^{-2}}$

$\Rightarrow$ on dividing 2 both side

$\Rightarrow \frac{10.81x^3}{2} - \frac{0.032x^2}{2} - \frac{1}{2} = \frac{0}{2}$

$\Rightarrow 5.405x^3 - 0.16x^2 - 0.5 = 0$

$\Rightarrow \frac{5.40x^3}{5} - \frac{0.16}{5}x^2 - \frac{0.5}{5} = \frac{0}{5}$

$\Rightarrow 1.8x^3 - 0.03x^2 - 0.1 = 0$

$\Rightarrow \frac{1.8x^3}{1.8} - \frac{0.03}{1.8}x^2 - \frac{0.1}{1.8} = 0$

$\Rightarrow x^3 - 0.016x^2 - 0.05 = 0$

$\Rightarrow$ Here, $0.016x^2$ and 0.05 can be ignorable because this is very less quantity.

$\Rightarrow x^3 = 0$

$\Rightarrow x = \sqrt[3]{0}$

$\Rightarrow x = 0$. Ans., option C,

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17) $\tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

Sol. $= \tan^{-1}\left(\frac{x}{y}\right) - \tan^{-1}\frac{x-y}{x+y}$

$$= \tan^{-1}\left[\frac{\frac{x}{y} - \frac{x-y}{x+y}}{1 + \left(\frac{x}{y}\right)\left(\frac{x-y}{x+y}\right)}\right] \quad \left\{\because \tan^{-1}x + \tan^{-1}y = \tan^{-1}\left(\frac{x+y}{1-xy}\right)\right\}$$

$$= \tan^{-1}\left[\frac{\frac{x(x+y) - y(x-y)}{y(x+y)}}{\frac{y(x+y) + x(x-y)}{y(x+y)}}\right]$$

$$= \tan^{-1}\left[\frac{x^2 + xy - yx + y^2}{yx + y^2 + x^2 - xy}\right]$$

$$= \tan^{-1}\left(\frac{x^2+y^2}{x^2+y^2}\right)$$

$$= \tan^{-1}(1)$$

$$= \tan^{-1}\left(\tan\frac{\pi}{4}\right)$$

$$= \frac{\pi}{4} \quad \text{Ans.} \quad \underline{\text{option } c}$$

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16) $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

Sol. $\Rightarrow \sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$

$\Rightarrow \sin^{-1}(1-x) - \frac{\pi}{2} = 2\sin^{-1}x$

$\Rightarrow \sin^{-1}(1-x) - \sin^{-1}(1) = 2\sin^{-1}x$

$\Rightarrow \sin^{-1}\left\{(1-x)\sqrt{1-1^2} - 1\sqrt{1-(1-x)^2}\right\} = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$\left\{\begin{array}{l} \because \sin^{-1}x - \sin^{-1}y = \sin^{-1}\left(x\sqrt{1-y^2} - y\sqrt{1-x^2}\right) \\ \because 2\sin^{-1}x = \sin^{-1}\left(2x\sqrt{1-x^2}\right) \end{array}\right.$

$\Rightarrow \sin^{-1}\left\{0 - 1\sqrt{1-(1-2x+x^2)}\right\} = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$

$\Rightarrow -1\sqrt{1-(1-2x+x^2)} = 2x\sqrt{1-x^2}$

$\Rightarrow -\sqrt{1-1+2x-x^2} = 2x\sqrt{1-x^2}$

$\Rightarrow -\sqrt{2x-x^2} = 2x\sqrt{1-x^2}$

squaring both side

$\Rightarrow \left(-\sqrt{2x-x^2}\right)^2 = \left(2x\sqrt{1-x^2}\right)^2$

$\Rightarrow 2x - x^2 = 4x^2(1-x^2)$

$\Rightarrow 2x - x^2 = 4x^2 - 4x^4$

$\Rightarrow 4x^2 - 4x^4 - 2x + x^2 = 0$

$\Rightarrow -4x^4 + 4x^2 + x^2 - 2x = 0$

$\Rightarrow -4x^4 + 5x^2 - 2x = 0$

$\Rightarrow 4x^4 - 5x^2 + 2x = 0$

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$\Rightarrow x(4x^3-5x+2)=0$

$\Rightarrow x(4x^3-5x+2)=0$

Either,

$x=0$ or $4x^3-5x+2=0$

$\Rightarrow x=0$, or $4x^3-2x^2+2x^2-x-4x+2=0$

$\Rightarrow x=0$ or $\Rightarrow 2x^2(2x-1)+x(2x-1)-2(2x-1)=0$

$\Rightarrow x=0$ or $\Rightarrow (2x-1)(2x^2+x-2)=0$

$\Rightarrow x=0$ or $\Rightarrow (2x-1)=0$ or $2x^2+x-2=0$

$\Rightarrow x=0$ or $\Rightarrow x=\frac{1}{2}$ or $x=\frac{-1\pm\sqrt{(1)^2-4\times2\times(-2)}}{2\times2}$

$\Rightarrow x=0$ or $x=\frac{1}{2}$ or $x=\frac{-1\pm\sqrt{17}}{4}$

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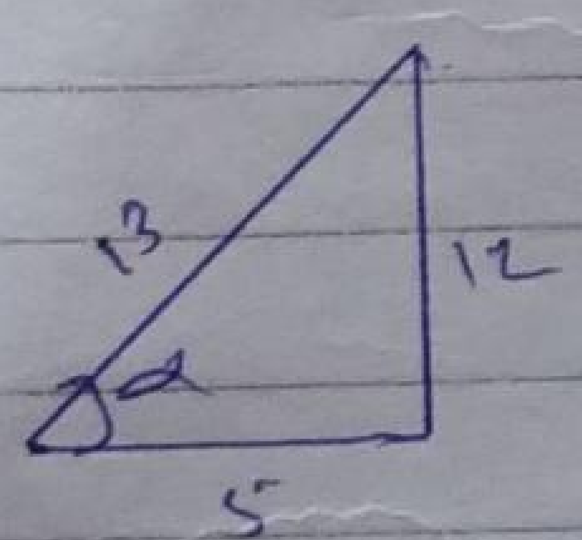
Example - 11) Prove that $\sin^{-1}\frac{12}{13}+\cos^{-1}\frac{4}{5}+\tan^{-1}\frac{63}{16}=\pi$

Sol. $\sin^{-1}\left(\frac{12}{13}\right)=\alpha$

$\sin\alpha=\frac{12}{13}$



$H^2=P^2+B^2$

$(13)^2=(12)^2+B^2$

$B=5$

$\alpha=\tan^{-1}\left(\frac{12}{5}\right)$

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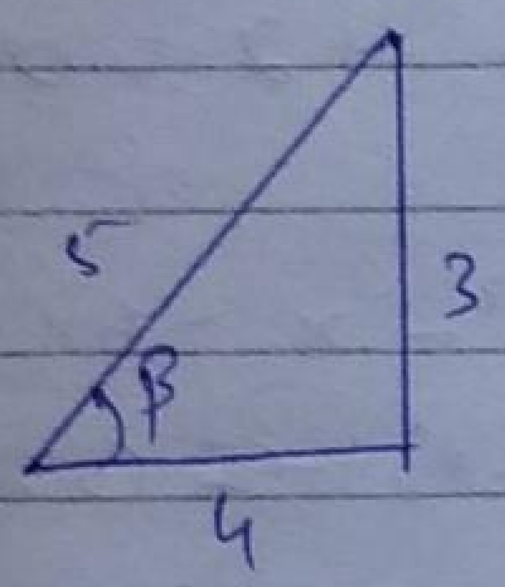
$\Rightarrow \cos^{-1}\frac{4}{5} = \beta$

$\Rightarrow \cos^{-1}\beta = \frac{4}{5}$



$H^2 = P^2 + B^2$

$(5)^2 = P^2 + (4)^2$

$P = 3$

$\tan\beta = 3/4$

$\beta = \tan^{-1}(3/4)$

Now,

$= \tan^{-1}\frac{12}{5} + \tan^{-1}\frac{3}{4} + \tan^{-1}\frac{63}{16}$

$= \tan^{-1}\left(\frac{\frac{12}{5}+\frac{3}{4}}{1-\frac{12}{5}\cdot\frac{3}{4}}\right) + \tan^{-1}\frac{63}{16}$

$= \tan^{-1}\left(-\frac{63}{16}\right) + \tan^{-1}\frac{63}{16}$

$= \pi - \cancel{\tan^{-1}\left(\frac{63}{16}\right) + \tan^{-1}\left(\frac{63}{16}\right)}$

$\because$ let $\tan\theta = x$

$\tan(\pi-\theta) = -\tan\theta$

$\tan(\pi-\theta) = -x$

$\pi - \theta = \tan^{-1}(-x)$

~~$\pi - \theta = \tan^{-1}x$~~

$\pi - \tan^{-1}x = \tan^{-1}(-$

$= \pi$

L.H.S = R.H.S

Hence proved.