Chapter - 6

Work, Energy & Power

Product of vector

Vector Product (cross Product)

Scalar Product (dot Product)

Scalar product (dot Product) →

$$\boxed{\begin{aligned} \vec{a}\cdot\vec{b} &= ab\cos\theta \\ \cos\theta &= \frac{\vec{a}\cdot\vec{b}}{a.b} \end{aligned}}$$

Q.1 Find the angle betn $\vec{a}$ & $\vec{b}$.

$$\vec{a} = \hat{i} + \hat{j} + \hat{k}$$
$$\vec{b} = -2\hat{i} - 2\hat{j} - 2\hat{k}$$

Ans.

$$a = |\vec{a}| = \sqrt{(1)^2 + (1)^2 + (1)^2} = \sqrt{3} = 1.73$$

$$b = |\vec{b}| = \sqrt{(-2)^2 + (-2)^2 + (-2)^2} = \sqrt{4\times3} = 2\sqrt{3}$$

$$\begin{aligned} \vec{a}\cdot\vec{b} &= \{1\times(-2)\} + \{1\times(-2)\} + \{1\times(-2)\} \\ &= \{-2\} + \{-2\} + \{-2\} \\ &= -2-2-2 \\ &= -6 \end{aligned}$$

$$\cos\theta = \frac{\vec{a}\cdot\vec{b}}{a.b} = \frac{-6}{\sqrt{3}\cdot2\sqrt{3}} = -1$$

$\Rightarrow \cos\theta = -1$

$\Rightarrow \cos\theta = \cos 180^\circ$

$\Rightarrow \theta = 180^\circ$

Example 6.1)

Find the angle bet^n $\vec{F}$ and $\vec{d}$.

$$\vec{F} = 3\hat{i} + 4\hat{j} - 5\hat{k}$$
$$\vec{d} = 5\hat{i} + 4\hat{j} + 3\hat{k}$$

Ans. $\Rightarrow F = |\vec{F}| = \sqrt{(3)^2 + (4)^2 + (-5)^2} = \sqrt{50} = 5\sqrt{2}$

$\Rightarrow d = |\vec{d}| = \sqrt{(5)^2 + (4)^2 + (3)^2} = \sqrt{50} = 5\sqrt{2}$

$\Rightarrow \vec{F} \cdot \vec{d} = (3 \times 5) + (4 \times 4) + (-5 \times 3)$
$= 16$

$\Rightarrow \cos\theta = \dfrac{\vec{F} \cdot \vec{d}}{F \cdot d} = \dfrac{16}{5\sqrt{2} \cdot 5\sqrt{2}} = \dfrac{16}{50}$

$\Rightarrow \cos\theta = 0.32$

$\cos\theta = \cos 71^\circ$

$\theta = 71^\circ$

or

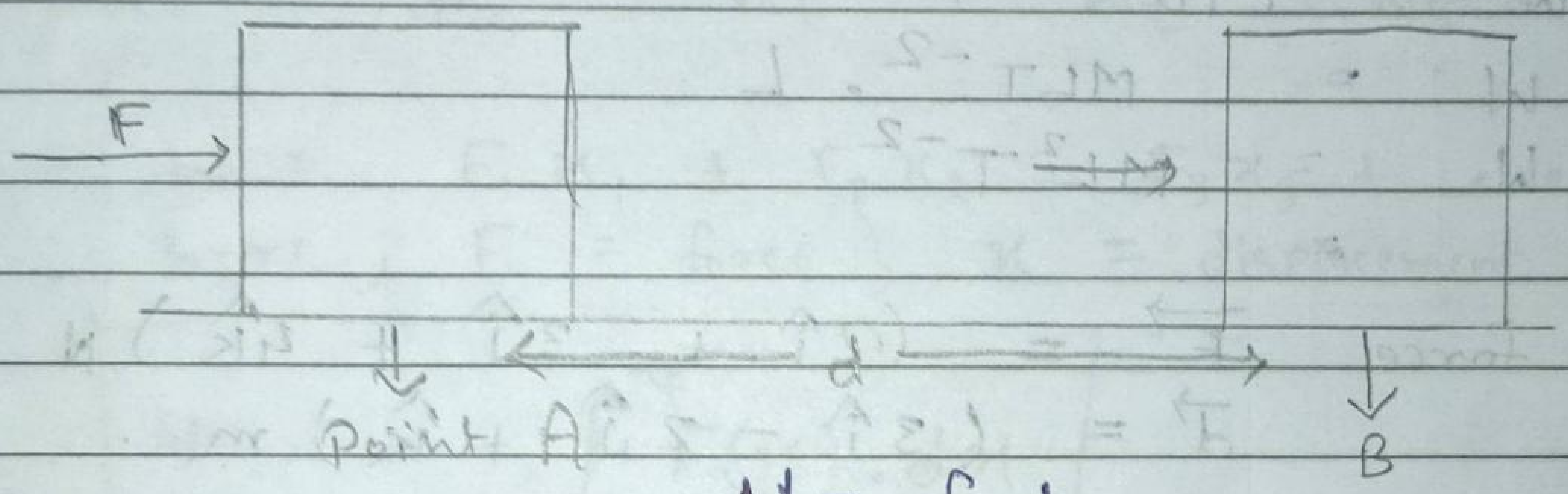
$\cos\theta = 0.32$

$\theta = \cos^{-1} 0.32$

(Work done by constant force)

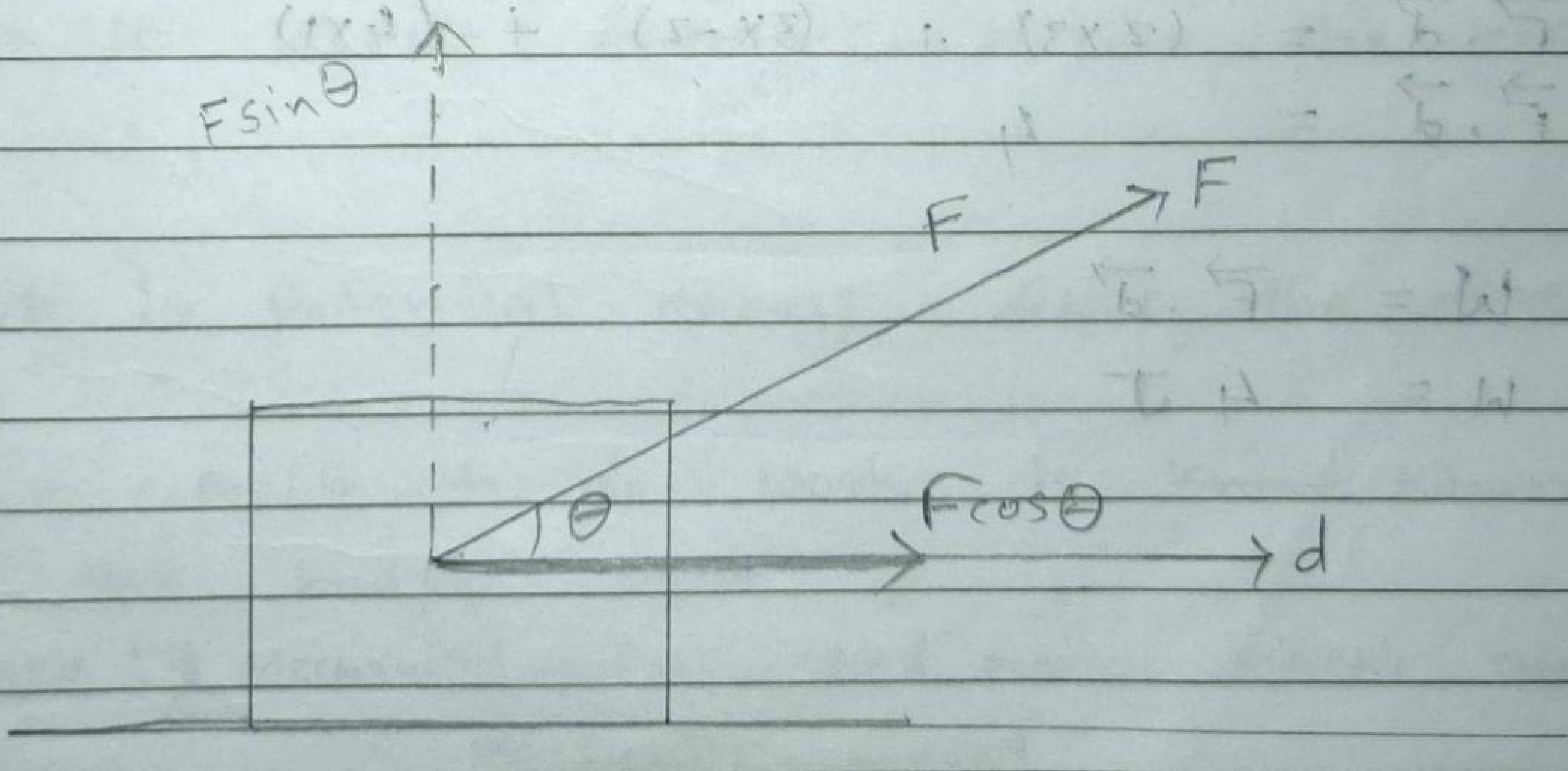
Work :- The magnitude of work done by the force is given by the product of the force and the displacement of the body along the direction of force

work is equal to force x displacement of the body along the direction of force.



$$W = f \cdot d.$$

⇒ Work = (force) x (displacement of the body along the direction of force.)



$$W = F\cos\theta \cdot d$$

$$W = Fd\cos\theta$$

$$W = \vec{F} \cdot \vec{d}$$

since, $\vec{a} \cdot \vec{b} = ab\cos\theta$

Work is a scalar quantity.
The SI unit of work is Joule (J)
The CGS unit of work is "erg"

$$\boxed{1 \text{ Joule} = 10^7 \text{ erg}}$$

<u>Dimension of Work</u> :-

$W = F.d$
$W = MLT^{-2}. L$
$W = ML^2T^{-2}$

<u>Q.2</u> A force $\vec{F} = (2\hat{i} + 3\hat{j} + 4\hat{k})$ N
$\vec{d} = (3\hat{i} - 2\hat{j} + \hat{k})$ m.

<u>Ans.</u>

$F = |\vec{F}| = \sqrt{(2)^2 + (3)^2 + (4)^2} = \sqrt{29}$
$d = |\vec{d}| = \sqrt{(3)^2 + (-2)^2 + (1)^2} = \sqrt{14}$

$\vec{F}.\vec{d} = (2 \times 3) + (3 \times -2) + (4 \times 1)$
$\vec{F}.\vec{d} = 4$

$W = \vec{F}.\vec{d}$
$W = 4$ J

Force ⇒

Work

Work done by Constant force | Work done by Variable force

Work done by variable force ⇒

Total Work (W) = $W_1 + W_2 + W_3 + \dots$ upto 'n'

⇒ $W = F_1 x_1 + F_2 x_2 + F_3 x_3 + \dots$

here, F = force, x = displacement

$$\boxed{W = \int_{x_i}^{x_f} F(x) \cdot dx}$$

Q.3 What is energy?

Q.4 What is kinetic energy. derive the expression for it.

Q. 5 What is Potential energy. derive the expression for it.

Ans.3) The capacity to do work is ~~Know~~ known as energy of the body.

Example :- Mechanical energy, Wind energy, Kinetic energy etc.

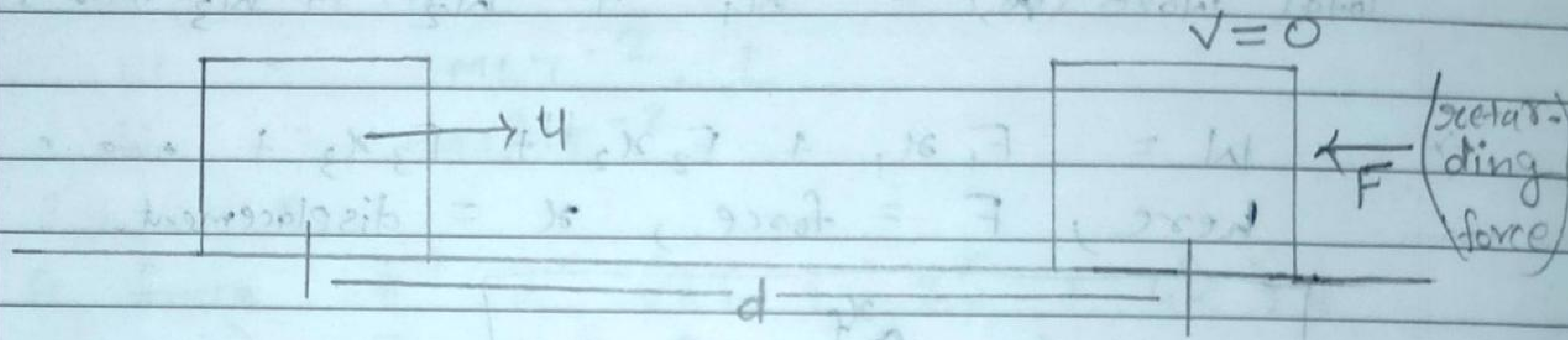
⇒ Mechanical energy

Kinetic energy | Potential energy

Ans. 4 The energy ~~due to~~ possesed by a body by virtue of its ~~fixed~~ motion is known as kinetic energy.

The energy of any body due to its motion is known as kinetic energy.

Derivation :-



We know that,

$\Rightarrow W = F.d$
$\Rightarrow W = ma.d \quad [\because F = ma]$ —— (i)

from 3rd eqn of motion,

$\Rightarrow v^2 = u^2 + 2as$
$\Rightarrow (0)^2 = u^2 + 2(-a)d$
$\Rightarrow 2ad = u^2$
$\Rightarrow ad = \frac{u^2}{2}$ —— (ii)

from eqn (i), ~~and (ii)~~,

$\Rightarrow W = mad$
$\Rightarrow W = \frac{mu^2}{2} \quad$ [from eqn (ii)]

$\Rightarrow KE = \frac{mu^2}{2}$

$$KE = \frac{1}{2} \times m \times u^2$$

$$KE = \frac{1}{2} \times mass \times (velocity)^2.$$

$$\boxed{KE = \frac{1}{2} mv^2}$$

Ans. 5) The energy Possesed by a body by virtue of its Position in a field or due to deformation is Called its Potential energy.

There are 3 different types of Potential energy :-

i) Gravitational Potential energy
ii) Elastic Potential energy
iii) Electrostatic Potential energy → 12^{th} class

i) Gravitational Potential energy :- The energy Possesed by a body due to its height above the surface of the earth is called its Gravitational Potential energy.

⇒ Work done = (Force) x (height from earth surface)

$$W = (mg) \times (h)$$

$$\boxed{PE = mgh}$$

Q. 6 Write the Work energy theorem and prove it for —

i) constant force

ii) variable force

Ans. **Work energy Theorem:-**

According to the work energy theorem the work done by a force is equal to the change in kinetic energy (K.E).

work done is always equal to change in kinetic energy.

$\Rightarrow$ W.D = change in KE

$$\Rightarrow \boxed{W = K_f - K_i}$$

i) for constant force $\Rightarrow$

Work done = (force) (displacement) $\cos\theta$

$W = F \cdot d \cos\theta$

if $\theta = 0$

$W = F \cdot d \cos 0^\circ$

$$\boxed{W = F \cdot d}$$

from 2nd law of Newton,

$W = ma \cdot d \quad [\because F = ma]$

from 3rd eq^n of motion,

$\Rightarrow v^2 = u^2 + 2aS$

$\Rightarrow v^2 - u^2 = 2aS$

$(d = S)$

$\Rightarrow v^2 - u^2 = 2ad$

$\Rightarrow \dfrac{v^2 - u^2}{2} = ad$

and,

$$W = mad$$

$$W = m\left(\frac{v^2 - u^2}{2}\right)$$

$$W = \frac{mv^2}{2} - \frac{mu^2}{2}$$

$$\boxed{W = K_f - K_i}$$

Hence Proved.

ii) For variable force :-

$$W = FS\cos\theta$$

if $\theta = 0°$

$$W = FS\cos 0°$$

$$\boxed{W = FS}$$

for small work,

$$dW = F.ds$$

$$dW = ma.ds$$

$$dW = m\frac{dv}{dt}.ds \quad \left[\because a = \frac{dv}{dt}\right]$$

$$dW = m.dv.\frac{ds}{dt}$$

$$dW = \cancel{m.dv.dv} \quad mv.dv \quad \left[v = \frac{ds}{dt}\right]$$

Integrating both side,

$$\Rightarrow \int_0^W dW = \int_u^v mv\, dv$$

$$\Rightarrow [W]_0^W = m\int_u^v v.dv$$

$$\Rightarrow [W]_0^W = m\left[\frac{v^2}{2}\right]_u^v \quad \text{since } \int x^n dx = \frac{x^{n+1}}{n+1}$$

$$\Rightarrow [W-0] = m\left[\frac{v^2}{2} - \frac{u^2}{2}\right]$$

$$\Rightarrow W = \frac{mv^2}{2} - \frac{mu^2}{2}$$

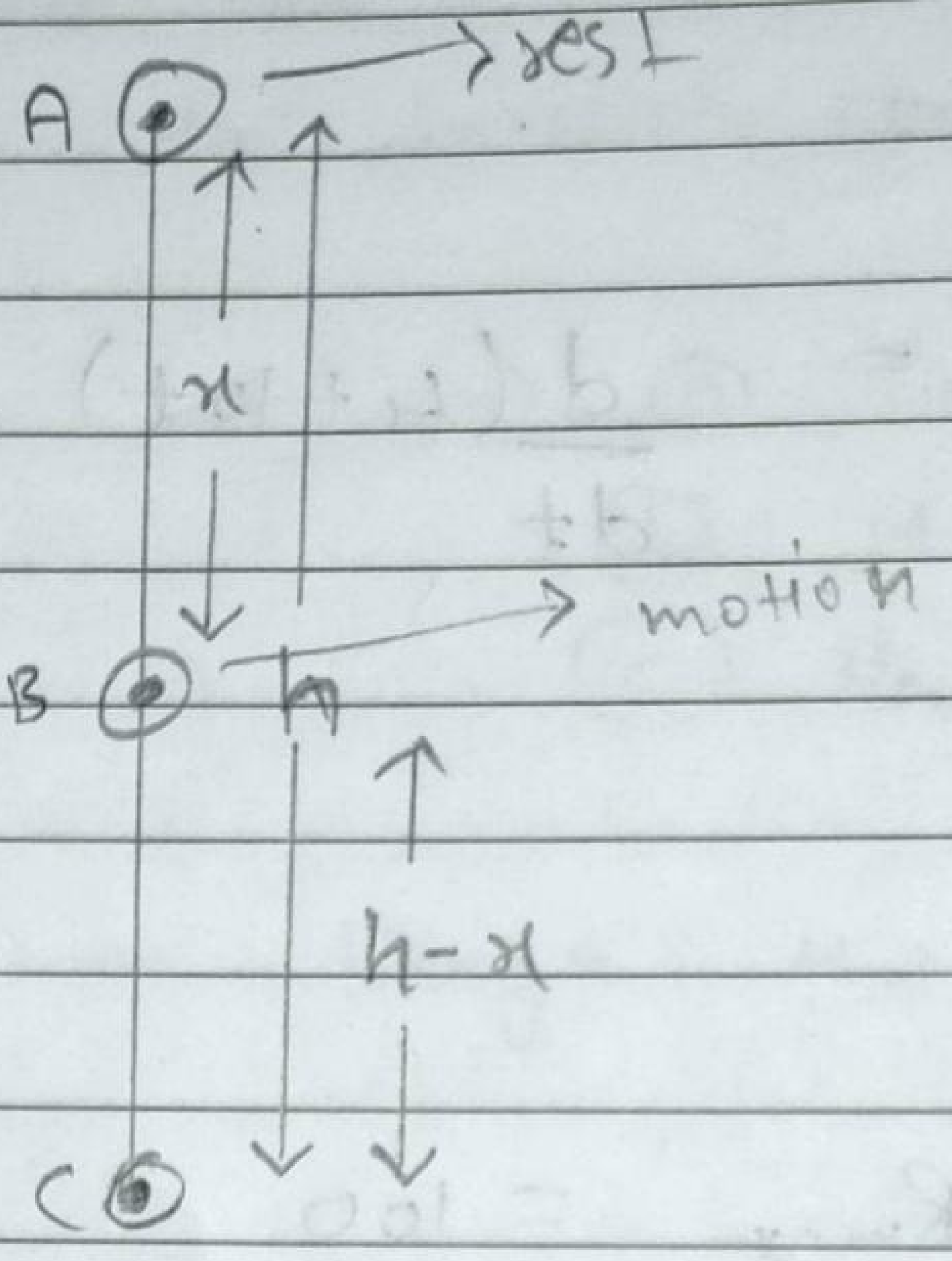
$$\Rightarrow \boxed{W = K_f - K_i}$$

Q. 7 Write the law of conservation of mechanical energy and prove it.

or

State law of conservation of energy Prove that energy of free falling body remains constant at during the motion.

Ans. 7 Conservation of Mechanical energy :-
of a freely falling body



At Point A,

$\Rightarrow$ $K.E = \frac{1}{2} mv^2$

$\Rightarrow$ $V = 0$

$\Rightarrow$ $K.E = \frac{1}{2} m(0)^2$

$\Rightarrow$ $K.E = 0$

$\Rightarrow$ $P.E = mg \times \text{height}$

$\Rightarrow$ $P.E = mgh$

$\Rightarrow$ $T_A = KE + PE$

$\Rightarrow$ $T_A = 0 + mgh$

$\Rightarrow$ $\boxed{T_A = mgh}$

At Point B,

$\Rightarrow KE = \frac{1}{2} mv^2$

$\Rightarrow v = ?$

$\Rightarrow v^2 = u^2 + 2as$ [third eqⁿ of motion]

$\Rightarrow v^2 = (0)^2 + 2gx$

$\Rightarrow v^2 = 2gx$

$\Rightarrow KE = \frac{1}{2} mv^2$

$\Rightarrow KE = \frac{1}{2} m(2gx)$

$\Rightarrow KE = mgx$

$\Rightarrow PE = mg \times \text{height}$

$\Rightarrow PE = mg(h-x) = mgh - mgx$

$\Rightarrow T_B = KE + PE$

$\Rightarrow T_B = mgx + mgh - mgx$

$\Rightarrow T_B = \cancel{mg(x+g} \quad mg(x+h-x)$

$\Rightarrow T_B = mgh$

At Point C,

$\Rightarrow KE = \frac{1}{2} mv^2$

$\Rightarrow v = ?$

$\Rightarrow v^2 = u^2 + 2as$

$\Rightarrow \quad v^2 = 0 + 2gh$

$\Rightarrow \quad KE = \frac{1}{2} mv^2$

$\Rightarrow \quad KE = \frac{1}{2} m (2gh)$

$\Rightarrow \quad KE = mgh$

$\Rightarrow \quad PE = mg \times \text{height}$

$\Rightarrow \quad PE = mg\ (0)$

$\Rightarrow \quad PE = 0$

$\Rightarrow \quad T_C = KE + PE$

$\Rightarrow \quad T_C = mgh + 0$

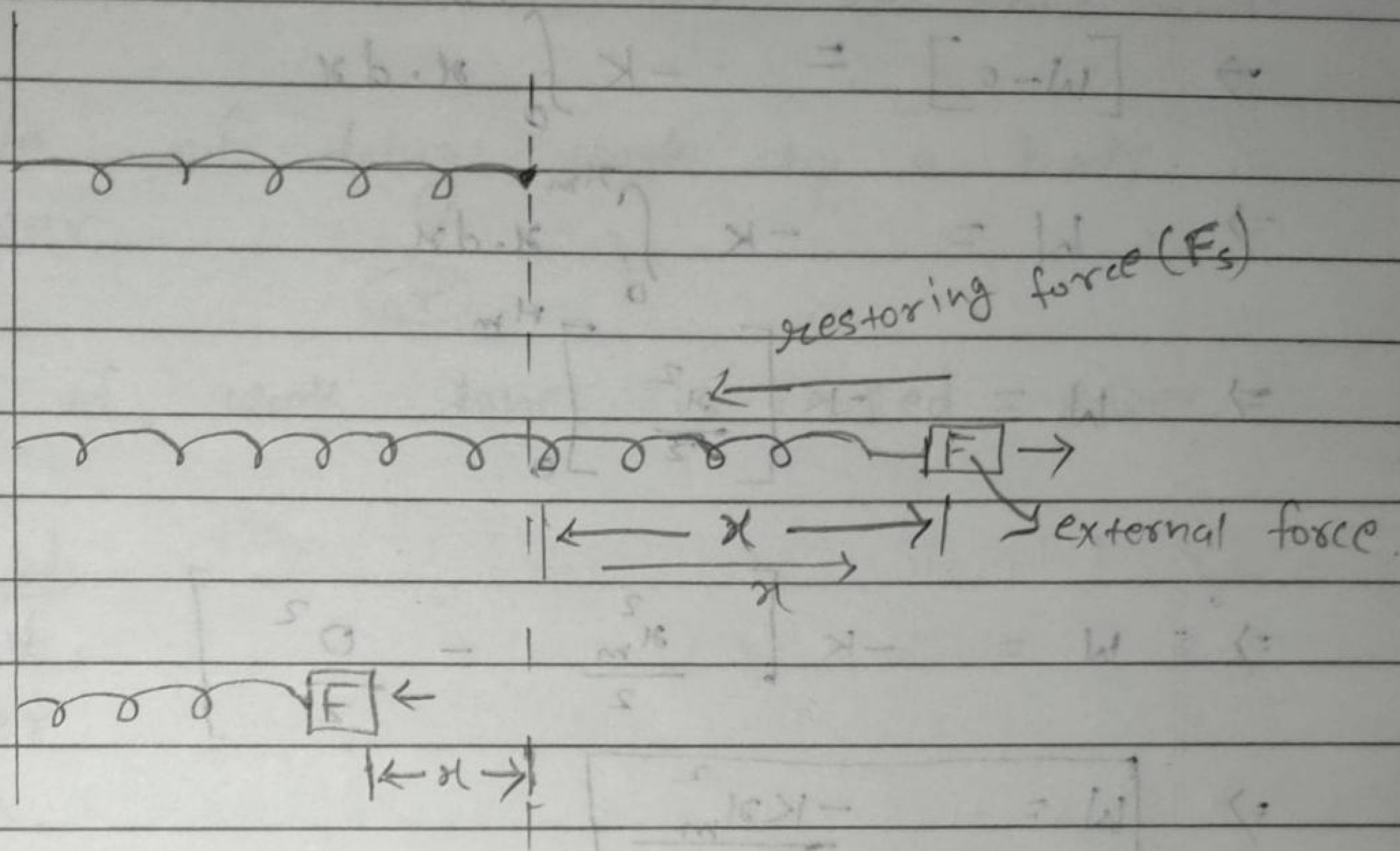
$\Rightarrow \quad T_C = mgh$

$\Rightarrow \quad T_A = T_B = T_C$

Hence proved.

Q.8. Derive an expression for Potential energy of a spring.

Ans. The work done in compressing or streching a spring is stored in it in the form of Potential energy.



In ideal spring follow the Hooke's law. According to this law the restoring force F is directly proportional to the change in lenghth of the spring (x).

$$F_s \propto -x$$
$$F_s = -Kx$$

where. K is spring constant.

$\Rightarrow$ $W = F \cdot d$

$\Rightarrow$ $W = F_s \cdot x$

$\Rightarrow$ $dW = F_s \cdot dx$

Integrating both side,

$\Rightarrow$ $\int_0^W dW = \int_0^{x_m} F_s \cdot dx$

$\Rightarrow$ $[w]_0^w = \int_0^{x_m} (-Kx)\, dx$

$$\Rightarrow [W - 0] = -K\int_0^{x_m} x \cdot dx$$

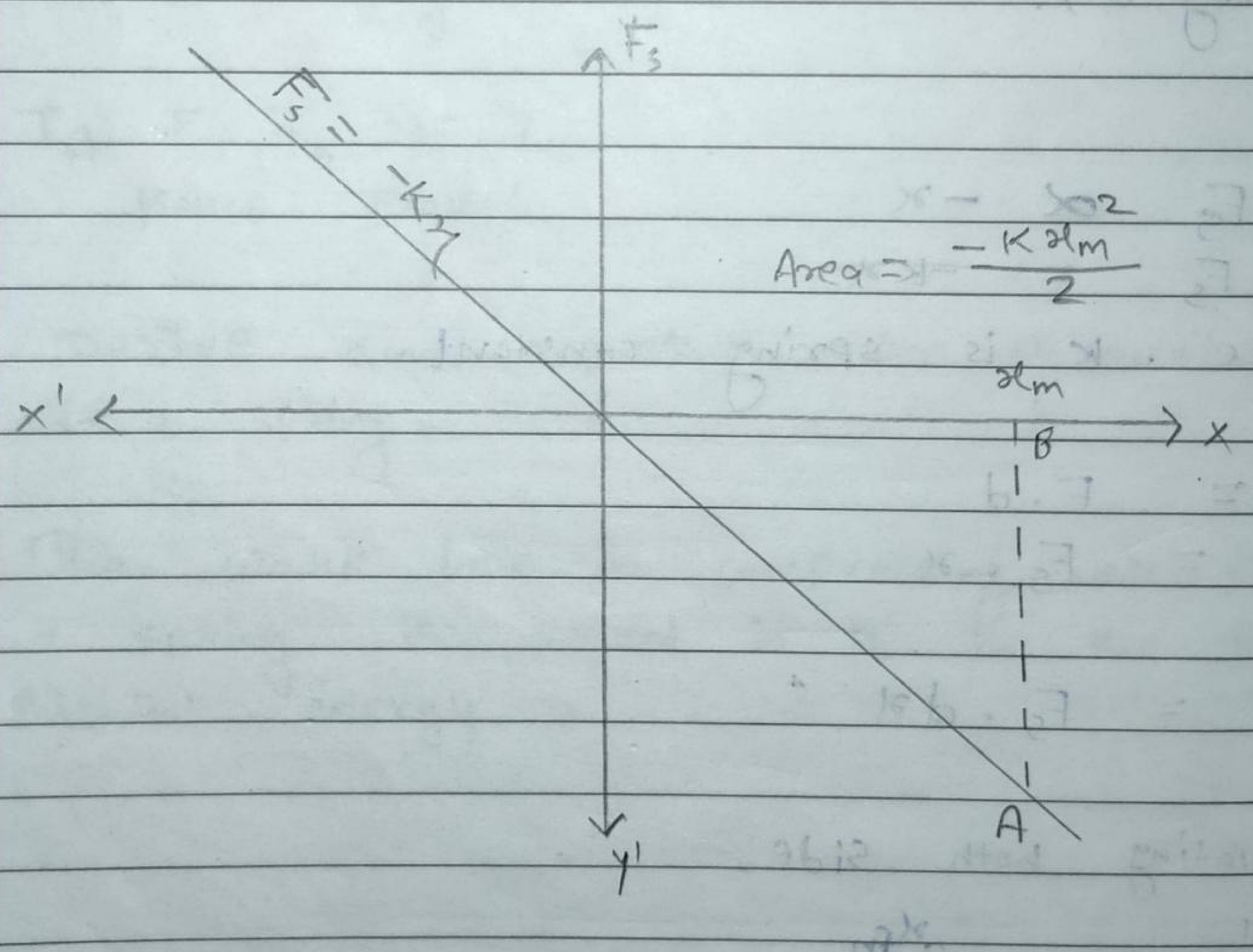
$$\Rightarrow W = -K\int_0^{x_m} x \cdot dx$$

$$\Rightarrow W = -K\left[\frac{x^2}{2}\right]_0^{x_m}$$

$$\Rightarrow W = -K\left[\frac{x_m^2}{2} - \frac{0^2}{2}\right]$$

$$\Rightarrow \boxed{W = \frac{-Kx_m^2}{2}}$$

$$\Rightarrow \boxed{\text{Potential energy } (U) = \frac{Kx_m^2}{2}}$$



Q.9) What is power, write it's SI unit and dimension.

Ans. The rate of doing work by a body is called its Power.

or

The rate of work done is called Power.

$$P = \frac{W}{t}$$

P = Power
W = Work
t = time

Power

SI Unit ⇒ watt (W)

$$P = \frac{W}{t} = \frac{J}{s} \Rightarrow \text{watt}$$

Dimension ⇒ $\frac{[ML^2T^{-2}]}{[T]} = [ML^2T^{-3}]$

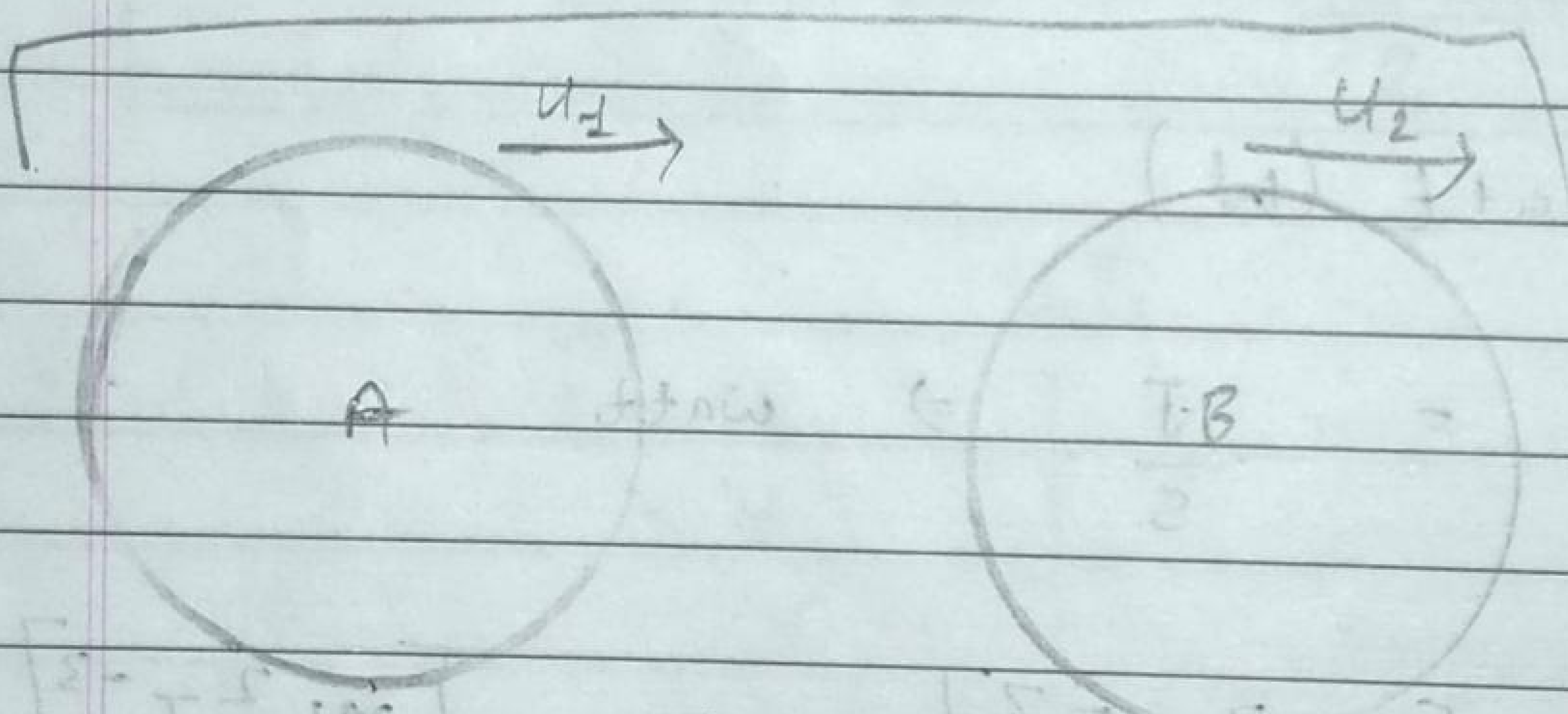
Collision :- The striking of one particle with the other or an interaction between them is called collision.

There are 2 types of collision -

i) Elastic Collision
ii) Inelastic Collision

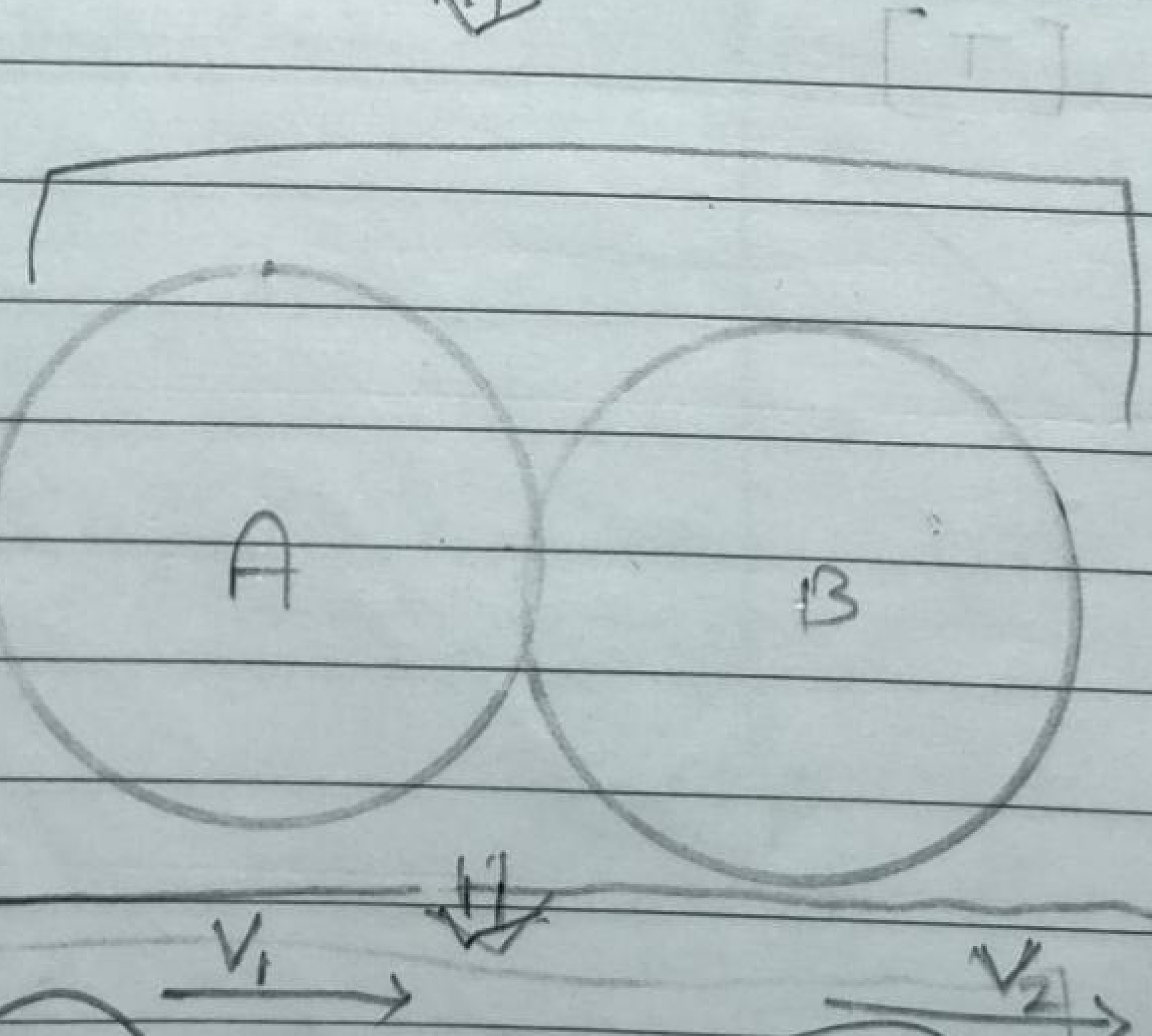
i) Elastic collision :- In an elastic collision both momentum as well as kinetic energy of the system remains conserve.

i



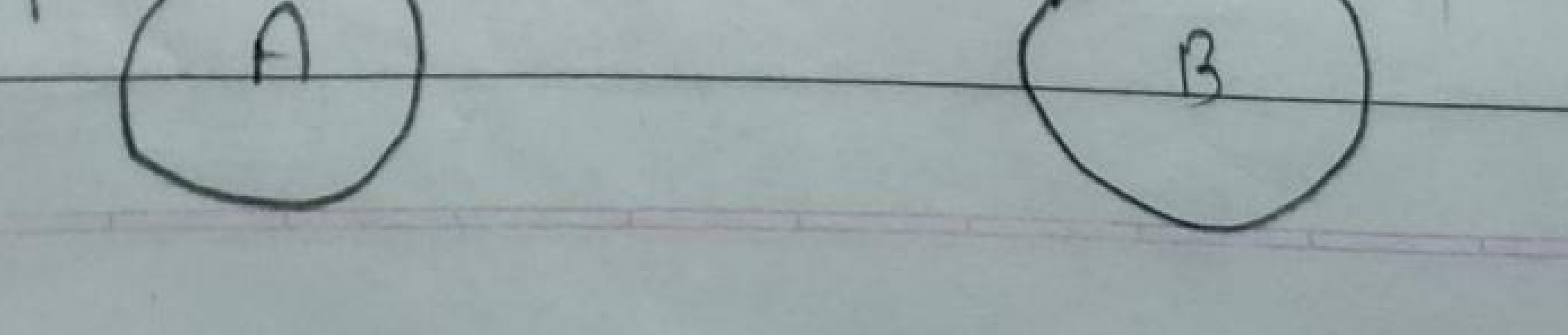
→ Before collision

⇓



→ during collision

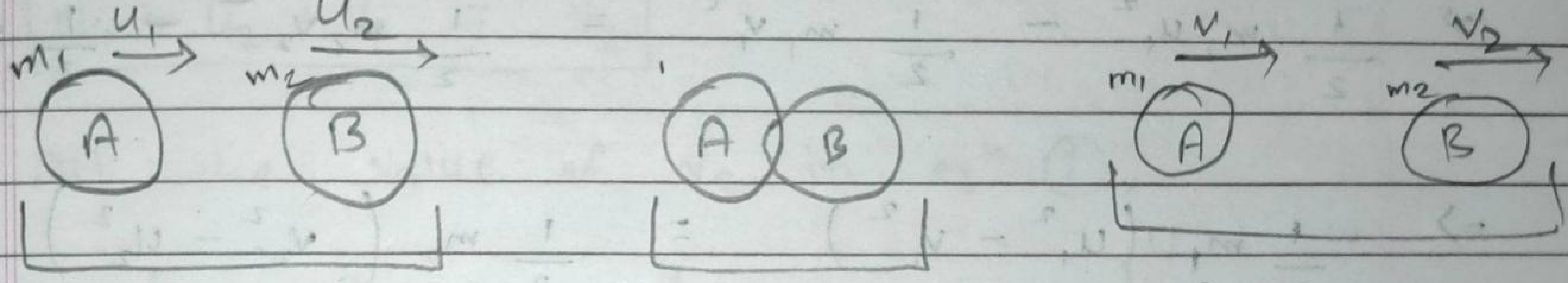
⇓



→ after collison

i) One dimensional elastic collision :-

In the case of elastic collision in 1D, 2 bodies move initially along the same straight line



Before collision   During collision   after collision.

Before collision momentum

$$= m_1 u_1 + m_2 u_2$$

After collision momentum

$$= m_1 v_1 + m_2 v_2$$

$\Rightarrow$ Momentum Before collision = Momentum after collision

$$\Rightarrow m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2$$

$$\Rightarrow m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

$$\Rightarrow m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \longrightarrow (i)$$

$\Rightarrow$ Kinetic energy Before collision

$$= \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2$$

Kinetic energy after collision

$$= \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$\Rightarrow$ Kinetic energy Before collision = K.E after collision

$$\Rightarrow \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

$$\Rightarrow \frac{1}{2} m_1 u_1^2 - \frac{1}{2} m_1 v_1^2 = \frac{1}{2} m_2 v_2^2 - \frac{1}{2} m_2 u_2^2$$

$$\Rightarrow \frac{1}{2} m_1 (u_1^2 - v_1^2) = \frac{1}{2} m_2 (v_2^2 - u_2^2)$$

$$\Rightarrow m_1 (u_1^2 - v_1^2) = m_2 (v_2^2 - u_2^2) \quad \text{— (ii)}$$

Eq^n (2) $\div$ eq^n (i)

$$\Rightarrow \frac{m_1 (u_1^2 - v_1^2)}{m_1 (u_1 - v_1)} = \frac{m_2 (v_2^2 - u_2^2)}{m_2 (v_2 - u_2)}$$

$$\Rightarrow \frac{(u_1 + v_1)(u_1 - v_1)}{(u_1 - v_1)} = \frac{(v_2 + u_2)(v_2 - u_2)}{(v_2 - u_2)}$$

$$\Rightarrow u_1 + v_1 = v_2 + u_2$$

$$\Rightarrow \boxed{u_1 - u_2 = v_2 - v_1} \longrightarrow \text{(iii)}$$

$\Rightarrow$ Relative velocity befor collision (approach) = Relative velocity after collision (raciding) from eqⁿ (3),

$$\Rightarrow \boxed{u_1 - u_2 + v_1 = v_2}$$

Put the value of v_2 in eqⁿ (i),

$$\Rightarrow m_1(u_1 - v_1) = m_2(u_1 - u_2 + v_1 - u_2)$$
$$\Rightarrow m_1u_1 - m_1v_1 = m_2u_1 - m_2u_2 + m_2v_1 - m_2u_2$$

$$\Rightarrow m_1u_1 - m_1v_1 = m_2u_1 - 2m_2u_2 + m_2v_1$$

$$\Rightarrow m_1u_1 - m_2u_1 + 2m_2u_2 = m_2v_1 + m_1v_1$$

$$\Rightarrow u_1(m_1 - m_2) + 2m_2u_2 = v_1(m_2 + m_1)$$

$$\Rightarrow \frac{u_1(m_1 - m_2)}{m_2 + m_1} + \frac{2m_2u_2}{m_2 + m_1} = v_1$$

$$\Rightarrow \boxed{v_1 = \frac{u_1(m_1 - m_2)}{m_2 + m_1} + \frac{2m_2u_2}{m_2 + m_1}}$$

⇒ For v_2,

from eqⁿ (iii),

$$\Rightarrow u_1 - u_2 = v_2 - v_1$$
$$\Rightarrow \boxed{v_1 = u_2 - u_1 + v_2}$$

Put the value of v_1 in eq^n (i),

$$\Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2)$$

$$\Rightarrow m_1[u_1 - (u_2 - u_1 + v_2)] = m_2[v_2 - u_2]$$

$$\Rightarrow m_1(u_1 - u_2 + u_1 - v_2) = m_2v_2 - m_2u_2$$

$$\Rightarrow m_1(2u_1 - u_2 - v_2) = m_2v_2 - m_2u_2$$

$$\Rightarrow 2u_1m_1 - m_1u_2 - v_2m_1 = m_2v_2 - m_2u_2$$

$$\Rightarrow 2m_1u_1 - m_1u_2 - m_1v_2 = m_2v_2 - m_2u_2$$

$$\Rightarrow 2m_1u_1 - m_1u_2 - m_1v_2 - m_2v_2 + m_2u_2 = 0$$

$$\cancel{\Rightarrow 2m_1u_1 - m_1(u_2 + v_2) = m_2v_2 - m_2u_2}$$

$$\cancel{\Rightarrow 2m_1u_1 = m_1(u_2 + v_2)}$$

$$\cancel{\Rightarrow 2m_1u_2 - m_1(u_2 + v_2) = m_2(v_2 - u_2)}.$$

$$\Rightarrow 2m_1u_1 - m_1u_2 + m_2u_2 = m_2v_2 + m_1v_2$$

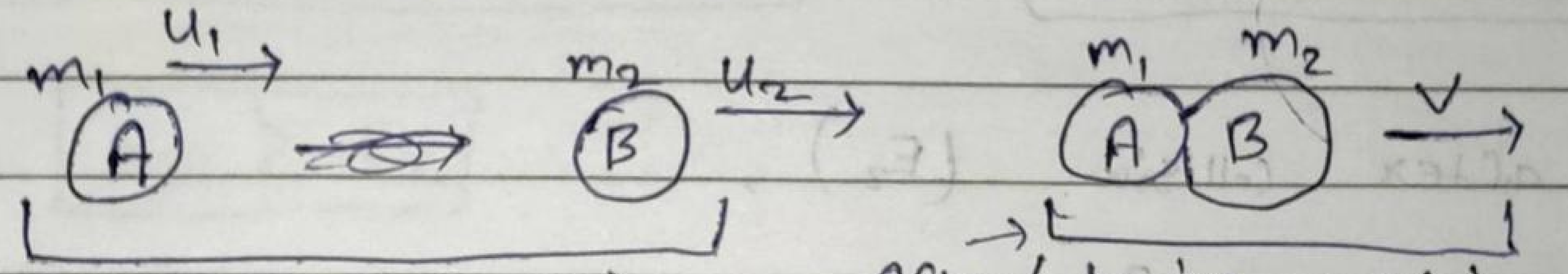
$$\Rightarrow 2m_1u_1 - u_2(m_1 - m_2) = v_2(m_2 + m_1)$$

$$\Rightarrow \frac{2m_1u_1}{m_2 + m_1} - \frac{u_2(m_1 - m_2)}{m_1 + m_2} = v_2$$

$$\Rightarrow \boxed{v_2 = \frac{2m_1u_1}{m_1 + m_2} + \frac{u_2(m_2 - m_1)}{m_1 + m_2}}$$

Q.10. What is Inelastic collision. Prove that in inelastic collision there is a loss of energy. also find the expression for energy loss.

Ans. Define: → In inelastic collision, the momentum of the system is conserved but not Kinetic energy.



Before collision After/during collision

⇒ Momentum before collision = Momentum after collision

⇒ Momentum = mass x velocity

$$m_1u_1 + m_2u_2 = m_1V + m_2V$$

$$\Rightarrow m_1u_1 + m_2u_2 = V(m_1 + m_2)$$

$$\Rightarrow \boxed{V = \frac{m_1u_1 + m_2u_2}{m_1 + m_2}}$$

When,

$$u_2 = 0$$

$$V = \frac{m_1u_1 + 0}{m_1 + m_2}$$

$$\Rightarrow \boxed{V = \frac{m_1u_1}{m_1 + m_2}}$$

KE before collision (E_1)

$\Rightarrow$ $E_1 = \frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 (0)^2$ $[\because u_2 = 0]$

$\Rightarrow$ $\boxed{E_1 = \frac{1}{2} m_1 u_1^2}$

KE after collision (E_2),

$\cancel{E_2 = \frac{1}{2} m_1 u_1^2}$

$\Rightarrow$ $E_2 = \frac{1}{2} m_1 V^2 + \frac{1}{2} m_2 V^2$

$\Rightarrow$ $E_2 = \frac{1}{2} V^2 (m_1 + m_2)$

$\Rightarrow$ $E_2 = \frac{1}{2} \left(\frac{m_1 u_1}{m_1 + m_2}\right)^2 (m_1 + m_2)$

$\Rightarrow$ $E_2 = \frac{1}{2} \left[\frac{m_1^2 u_1^2}{(m_1+m_2)(m_1+m_2)}\right] (m_1+m_2)$

$\Rightarrow$ $E_2 = \frac{1}{2} \frac{m_1^2 u_1^2}{m_1 + m_2}$

when $E_1 > E_2 \rightarrow$ Energy loss.

$\Rightarrow$ $\cancel{\frac{E_1}{E_2} = \frac{\frac{1}{2} m_1 u_1^2}{\frac{1}{2} m_1^2 u_1^2}}$

$\Rightarrow$ $\frac{E_1}{E_2} = \frac{\frac{1}{2} m_1^2 u_1^2}{\frac{1}{2} \frac{m_1^2 u_1^2}{m_1 + m_2}}$

$$\Rightarrow \frac{E_1}{E_2} = \frac{m_1}{\frac{m_1^2}{m_1 + m_2}}$$

$$\Rightarrow \boxed{\frac{E_1}{E_2} = \frac{m_1 + m_2}{m_1}}$$

$$\boxed{E_1 > E_2}$$

we can say that energy is less in inelastic collision.

$\Rightarrow$ Energy loss $(\Delta E) = E_1 - E_2$

$$\Rightarrow \Delta E = \frac{1}{2} m_1 u_1^2 - \frac{1}{2} \frac{m_1^2 u_1^2}{m_1 + m_2}$$

$$\Rightarrow \Delta E = \frac{1}{2} m_1 u_1^2 \left[1 - \frac{m_1}{m_1 + m_2} \right]$$

$$\Rightarrow \Delta E = \frac{1}{2} m_1 u_1^2 \left[\frac{m_1 + m_2 - m_1}{m_1 + m_2} \right]$$

$$\Rightarrow \boxed{\Delta E = \frac{1}{2} \frac{m_1 u_1^2 \cdot m_2}{m_1 + m_2}}$$