$Y \rightarrow$ Range
$(x, y) \rightarrow$ order Pair.

Chapter - 1
Relations and functions

Equivalance Relation :-

A relation R in a set A is called

i) reflexive, if $(a, a) \in R$, for every $a \in A$
ii) symmetric, if $(a_1, a_2) \in R$ implies that (a_2, a_1), for all ~~iii)~~ $a_1, a_2 \in A$
iii) transitive, if $(a_1, a_2) \in R$ and $(a_2, a_3) \in R$ implies that $(a_1, a_3) \in R$, for all $a_1, a_2, a_3 \in A$.

A relation R in a set A is said to be an equivalance relation if R is reflexive, symmetric and transitive.

Exercise 1.1

(1) Determine the following relation are reflexive, symmetric and transitive

i) Relation R in the set $A = \{1, 2, 3, \ldots, 14\}$ defined as
$$R = \{(x, y) : 3x - y = 0\}$$

Sol. $R = \{(x, y) : 3x - y = 0\}$
$$3x - y = 0$$
$$y = 3x$$

When, $x = 1$, $y = 3$
$x = 2$, $y = 6$
$x = 3$, $y = 9$
$x = 4$, $y = 12$

$$R = \{(1, 3), (2, 6), (3, 9), (4, 12)\}$$

$y = 3x$
$z = 3y$
$z = 3(x)$
$(z) = 9x$

Rough.

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~~i) It is not reflexive because if $x \neq 3x$. then $x \neq 3y$.~~

~~ii) It is not symmetric because~~

i) It is not reflexive because $x \neq 3x$

ii) It is not symmetric because $y = 3x$ then

$$x = \frac{y}{3} \neq 3y$$

iii) It is not transitive because $y = 3x$ & $z = 3y$ then $z = 3(3x) = 9x \neq 3x$.

ii) Relation R in the set N of Natural no. defined as

$$R = \{(x, y) : y = x + 5 \text{ and } x < 4\}$$

Sol. $y = x + 5$

When, $x = 1$, $y = 6$
$x = 2$, $y = 7$
$x = 3$, $y = 8$

$N = \{(1,6), (2,7), (3,8)\}$

i) It is not reflexive because $y = x + 5$ and $x \neq x + 5$.

ii) It is not symmetric because $y = x + 5$ then $x = y - 5 \neq y + 5$

iii) It is not transitive because $y = x + 5$ & $z = y + 5$ then $z = (x + 5) + 5 \neq x + 10$ $x + 10$

iii) Relation R in the set $A = \{1, 2, 3, 4, 5, 6\}$ as

$$R = \{(x, y) : y \text{ is divisible by } x\}$$

Sol. It is not transitive because of $y = 3x$ and $z = 3y$

then $z = 3(3x)$

$z = 9x$

$9x \neq 3x$.

Hence it is not transitive.

(v) Relation R in the set A of human beings in a town at a particular time given by

(a) $R = \{(x,y) : x \text{ and } y \text{ work at the same place}\}$

Ans. i) yes, R is reflexive because it is true 'x and x work at same place.

ii) R is symmetric because if x and y work at the same place then y and x also work at the same place.

iii) R is transitive because

if x and y work at same place and y and z work at place then it can be said x and z also work at same place.

(2) Show that the relation R in the set R of real no. defined as

$R = \{(a,b) : a \leq b^2\}$ is neither reflexive nor symmetric nor transitive.

Sol. R is not reflexive because if $a = \frac{1}{2}$ then

$$\frac{1}{2} \nleq \left(\frac{1}{2}\right)^2$$

$$\frac{1}{2} \nleq \frac{1}{4}$$

$\cancel{2 \leq 2^2}$

$\cancel{2 \leq 2^2}$

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R is not symmetric because when we taking ~~a=2~~ $a = 2$, $b = 5$ then

$$2 \leq (5)^2$$
$$5 \not\leq (2)^2$$

R is not transitive because if $a \leq b^2$ & $b \leq c^2$ then it is not compulsory that

$$a \leq c^2$$

$a = 2$, $b = -2$, $c = -1$

$$2 \leq (-2)^2 \quad \& \quad (-2) \leq (-1)^2$$

but $2 \not\leq (-1)^2$

(3) check whether the relation R defined in the set $\{1, 2, 3, 4, 5, 6\}$ as $R = \{(a, b) : b = a + 1\}$ is reflexive, symmetric or transitive.

Sol.

let $A = \{1, 2, 3, 4, 5, 6\}$
and Given, $R = \{(a, b) : b = a + 1\}$

R is not reflexive because $b = a + 1$ then $a \neq b + 1$
if we take $a = 2$ ~~and $b = 1$~~ then

~~$1 \neq 1 + 1$~~ $2 \neq 2 + 1$
but ~~$1 \neq 1 + 1$~~ but $2 \neq 2 + 1$

R is not symmetric because if we take $a = 2$ and $b = 3$ then

$$3 = 2 + 1$$
but $2 \neq 3 + 1$

R is not transitive because ~~when we have a = b~~
$b = a+1$ and $c = b+1 =$ ~~then~~ $(a+1)+1 \neq$

(4) Show that the relation R in R defined as
$R = \{(a,b) : a \leq b\}$, is reflexive and
transitive but not symmetric

Sol.

$R = \{(a,b) : a \leq b\}$
Set R is real number.

R is reflexive because if $a = 1$ then

$$1 \leq 1$$

R is not symmetric because if $a = 1$ and
$b = 2$ then

$$1 \leq 2$$

but $2 \nleq 1$

R is transitive because if $a = 1$, $b = 2$, c
then

$$1 \leq 2$$
$$2 \leq 3$$
$$1 \leq 3$$ Hence Proved.

(5) check wheather the relation R in R defined
by $R = \{(a,b) : a \leq b^3\}$ is reflexive, symmetric
or transitive.

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Sol. $R = \{(a,b) : a \leq b^3\}$
Let A is the set of all real no.

R is not reflexive because if $a = 1/3$ then

$$\frac{1}{3} \nleq \left(\frac{1}{3}\right)^3$$

$$\frac{1}{3} \nleq \frac{1}{27}$$

R is not symmetric because if $a = 1$, $b = 3$

$$1 \leq 3^3$$
$$1 \leq 27$$
$$3 \nleq 1$$

R is not transitive because if ~~$a \leq b^3 \in R$~~
~~then it is $b \leq c^3 \in R$ then~~
$a = 3$, $b = \frac{3}{2}$ and $c = \frac{6}{5}$ then

$$3 < \left(\frac{3}{2}\right)^3$$

and $\frac{3}{2} < \left(\frac{6}{5}\right)^3$

But $3 \nleq \left(\frac{6}{5}\right)^3$

$\therefore$ R is not transitive.

R is neither reflexive, neither transitive and nor symmetric.

(6) show that the relation R in the set $\{1,2,3\}$ given by $R = \{(1,2),(2,1)\}$ is symmetric but

reflexive nor transitive.

Sol. → $R = \{1, 2, 3\}$
→ $R = \{(1,2), (2,1)\}$

R is symmetric because $(1,2) \in R$ then also $(2,1) \in R$.

R is not reflexive because $(1,1) \notin R$ then also $(2,2) \notin R$

R is not transitive because if $(1,2) \in R$ and $(2,1) \in R$ but $(1,1) \notin R$.

(7) Show that the relation R in the set A of all the books in a library of a college given by $R = \{(x,y) : x$ and y have same no. of pages$\}$ is an equivalence relation.

Sol. $A = \{$set of all the books in library$\}$

$R = \{(x,y) :$ x and y ~~are~~ have the same no. of pages$\}$

R is reflexive because x and y have same no. of pages $x = x$.

R is symmetric because if $(x,y) \in A$ then also $(y,x) \in A$.

R is transitive because if $(x,y) \in R$, $(y,z) \in R$ then also $(x,z) \in R$

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If is an equivalent relation.

(8) Show that the relation R in the set $A = \{1,2,3,4,5\}$ given by $R = \{(a,b) : |a-b| \text{ is even}\}$, is an equivalence relation. Show that all the elements of $\{1,3,5\}$ are related to each other and all the elements of $\{2,4\}$ are related to each other. But no element $\{1,3,5\}$ is related to any element of $\{2,4\}$.

Sol:

$A = \{1,2,3,4,5\}$

$R = \{(a,b) : |a-b| \text{ is even}\}$

$R = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5), (4,4), (4,2), (5,1), (5,3), (5,5)\}$

Here R is reflexive because $|a-a| = 0$ and 0 is a even.

$\because$ R is symmetric because if $(1,1) \in R$ then $(1,1)$ also belongs to R.

$\because$ R is transitive because if $(1,5) \in R$ & $(5,1) \in R$ the $(1,1) \in R$.

Hence it is an equivalence relation.

All the element of $\{1,3,5\}$ are related to each other because there subsets are odd and the modulus $|a-b|$ is always even.

$R = \{(1,1), (1,3), (1,5), (2,2), (2,4), (3,1), (3,3), (3,5),$

$(4,2), (4,4), (5,1), (5,3), (5,5)\}$

All the elements of $\{2,4\}$ are related to each other because subsets are even and the modulus of $|a-b|$ is always even.

But set $\{1,3,5\}$ and $\{2,4\}$ are not related to each other because the modulus of $|a-b|$ odd.

(9)

Sol. (i) $R = \{(a,b) : |a-b|$ is a multiple of $4\}$

$\therefore$ R is reflexive because $|a-a| = 0$ and is multiple of all the nos.

R is symmetric because if $|a-b| = 0$ then

$\Rightarrow |-||b-a| = 0$

$\Rightarrow |b-a| = |a-b|$

R is the transitive because $|a-b|$ is multiple of 4 and $|b-c|$ is multiple of 4 then $|a-c|$ is also multiple of 4.

Hence it is an equivalence relation.

(ii) $R = \{(a,b) : a = b\}$

R is a reflexive because if $(a,b) \in R$ then $(a,a) \in R$, $\boxed{a = a}$

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R is symmetric because if $(a,b) \in R$ then $(b,a) \in R$ $(a=b)$

R is transitive because if $(a,b) \in R$ & $(b,c) \in R$ then $(a,c) \in R$ because of $(a=b)$ is in relation.

The set of element related to 1 in each case is $\{1\}$.

$\Rightarrow f(x) = (2x+3)$ → after solving then Ans. is Range.
↳ domain

$\Rightarrow f(5) = 13$, where $5 \rightarrow$ domain
$13 \rightarrow$ Range

$f : A \rightarrow B$ → set of co-
set of Domain

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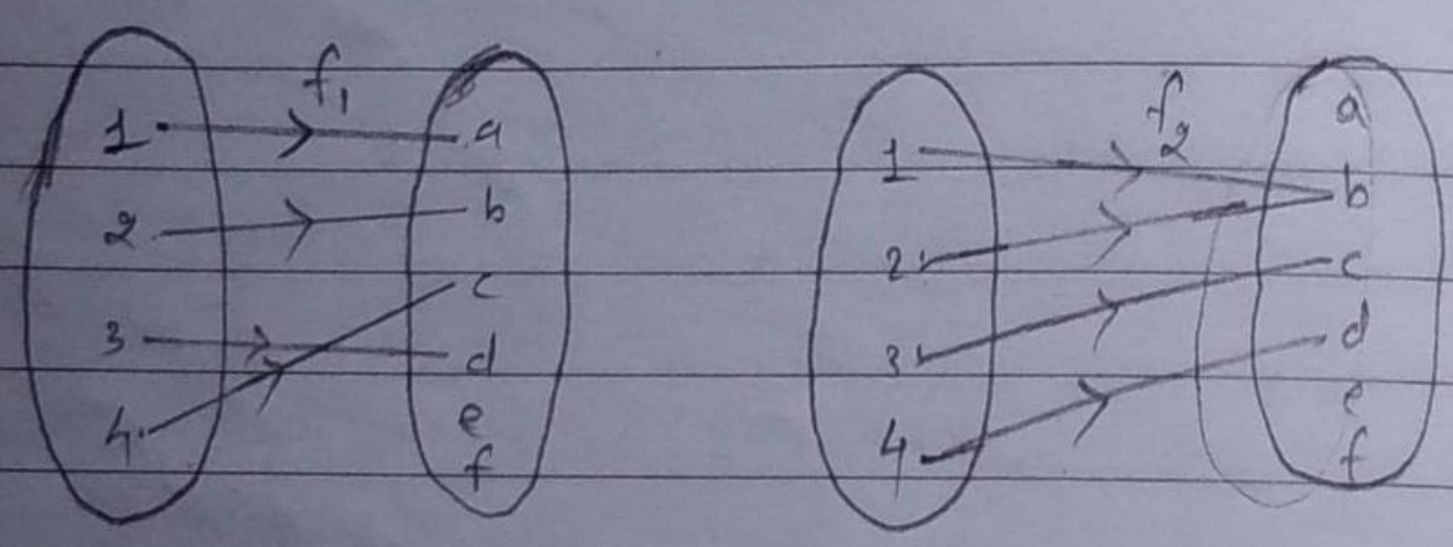
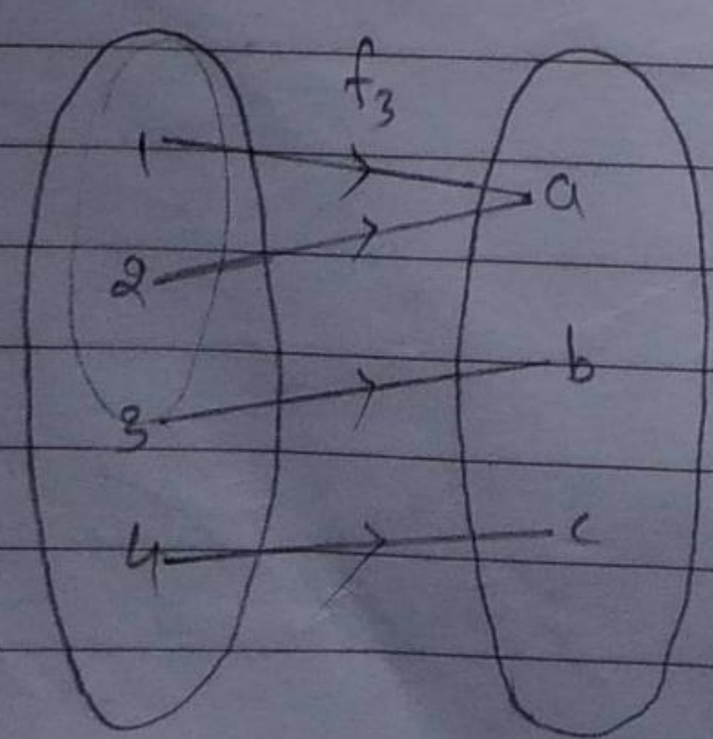
Types of function:-

i) <u>One one Function</u>:- A function $f : X \rightarrow Y$ is defined to be one-one (or injective), the images of distinct elements of X under f are distinct, i.e., for every $x_1, x_2 \in X$, $f(x_1) = f(x_2)$ implies $x_1 = x_2$. otherwise, f is called many-one.



(i) (ii)

ii) <u>Onto Function</u>:- A function $f : X \rightarrow Y$ is said to be onto (or surjective), if every element of Y is the image of some element of X under f, i.e., for every $y \in Y$ there exists an element x in X such that $f(x) = y$.



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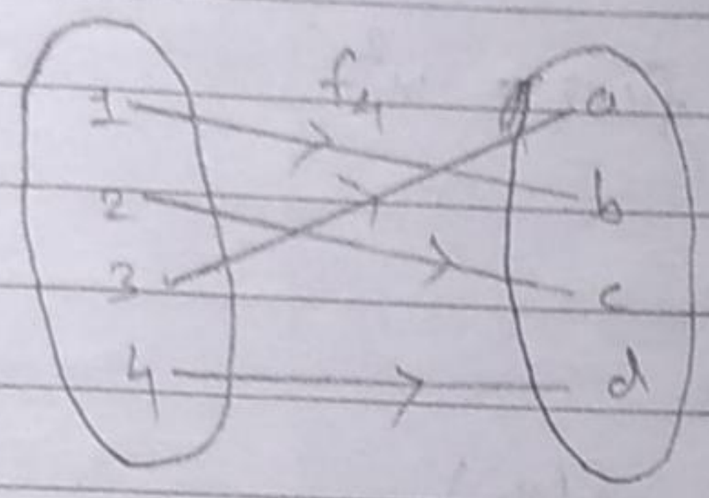
iii) <u>Bijective Function</u> :- A function $f: X \to Y$ is said to be one-one and onto (or bijective), if f is both one-one and onto.



$f: A \to B$.

<u>For one-one</u>

let $x_1, x_2 \in A$
such that,

$$x_1 \neq x_2$$
$$\therefore f(x_1) \neq f(x_2)$$

and if,

$$x_1 = x_2$$
$$f(x_1) = f(x_2)$$

<u>For onto</u>

$f: A \to B$.

<u>Given</u> $y = f(x)$

$\Rightarrow$ $x = f(y) \in A$

Exercise 1.2

Q(2) check the injectivity and surjectivity of the following functio

(i) $f: N \longrightarrow N$ Given by $f(x) = x^2$

Sol. $f: N \longrightarrow N$

$f(x) = x^2$

let, $x_1, x_2 \in N$

such that, $f(x_1) \neq f(x_2)$

$\Rightarrow$ $x_1^2 \neq x_2^2$

$\Rightarrow$ $x_1 \neq x_2$

$\therefore$ $f(x)$ is one-one.

(ii) $f: Z \rightarrow Z$ given by $f(x) = x^2$

Sol. $f: Z \rightarrow Z$

$f(x) = x^2$

let, $x_1, x_2 \in Z$

such that, ~~$f(x_1) \neq f(x_2)$~~ $f(x_1) = f(x_2)$

$\Rightarrow$ ~~$(x_1) \neq (x_2)$~~ $x_1 = x_2$

~~$\Rightarrow$~~

let $x_1 = -2$, and $x = 2$

then,

$\Rightarrow f(x_1) = f(x_2) \Rightarrow x_1^2 = x_2^2$

but $x_1 \neq x_2$

It is not one-one

Now, let $x = -1$ and $y = +1$

$\Rightarrow$ $f(x) = y$

$\Rightarrow$ $f(-1) = \cancel{+2}\ 1$

$\Rightarrow$ $(-1)^2 = 1$

$\Rightarrow$ ~~$4 \neq 2$~~

$\Rightarrow$ $1 = 1$

$\Rightarrow f(y) = x$

$\Rightarrow f(1) = -1$

$\Rightarrow f(1)^2 = -1$

$\Rightarrow 1 \neq -1$

It is in form of,

$\Rightarrow f(x) = y$

$\Rightarrow f(y) \neq x$

therefore it is not onto

(iii) $f: R \longrightarrow R$ given by $f(x) = x^2$

Sol. $f: R \longrightarrow R$

$f(x) = x^2$

let, $x_1, x_2 \in R$

such that, $f(x_1) = f(x_2)$

$\Rightarrow x_1 = x_2$

let $x_1 = -1$

$x_2 = 1$

then,

$f(x_1) = f(x_2)$

but $x_1 \neq x_2$

therefore It is not one-one.

Now,

$f(x) = y$

$f(-1) = 1$ It is true

but $f(1) \neq -1 \in R$

therefore it is not onto.

(iv) $f: N \rightarrow N$ given by $f(x) = x^3$

Sol. $f: N \rightarrow N$

$f(x) = x^3$

let $x_1, x_2 \in N$

$f(-1) = 1$
$-1 = 1$

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$\Rightarrow \quad f(x_1) = f(x_2)$

$\Rightarrow \quad x_1 = x_2$

but when we take $x_1 = +1$, $x_2 = 1$

then,

$\Rightarrow \quad f(+1) = f(1)$

$\Rightarrow \quad 1 = 1$

therefore it is one-one.

Now,

$f(x) = y \in N$

suppose $x = 2$ and $y = 8$

then,

$f(2) = 8$

but $f(y) \neq x$

but $f(8) \neq 2 \in N$

therefore it is not onto.

(V) $f: Z \to Z$ given by $f(x) = x^3$

Sol. $f: Z \to Z$

$f(x) = x^3$

let $x_1, x_2 \in R$.

$\Rightarrow \quad f(x_1) \neq f(x_2)$

$\Rightarrow \quad x_1 \neq x_2$ then it is one-one

so, let $x_1 = -1$

$x_2 = 1$

then $f(-1) \neq f(1)$

$-1 \neq 1$

therefore it is one-one

Now,

$f(x) = y \in R$

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$\Rightarrow$ $f(y) = x \in R$ then it is onto
when we take $x = 2$ and $y = 8$
then

$$f(2) = 8 \in R$$

but $f(8) \neq 2 \in R$

Hence it is not onto.

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(3) prove that the Greatest Integer Function $f: R \to R$, given by $f(x) = [x]$, is neither one-one nor onto, where $[x]$ denotes the greatest integer less ~~than~~ than or equal to x.

Sol. $f: R \to R$
$f(x) = [x]$

$f(1.3) = [1.3] = 1$
$f(1.4) = [1.4] = 1$
$f(1.9) = [1.9] = 1$
~~therefore f is not one one.~~
we see image of this no. is one therefore f is not one-one

All the images of x is integer but no pre-image of decimals no.s in it's Domain. therefor f is not onto.

(7) In each of the following cases, state wheather the function is one-one, onto or bijective. Justify your answers.

(i) $f: R \to R$ defined by $f(x) = 3 - 4x$
(ii) $f: R \to R$ defined by $f(x) = 1 + x^2$

Sol. i) $f: R \to R$
$f(x) = 3 - 4x$

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let, $x_1, x_2 \in R$
such that,

$$f(x_1) = f(x_2)$$
$$\Rightarrow \therefore \cancel{3} - 4x_1 = \cancel{3} - 4x_2$$
$$\Rightarrow -4x_1 = -4x_2$$
$$\Rightarrow x_1 = x_2$$

therefore f is one-one.

let, $y = 3 - 4x$
$\Rightarrow 4x = 3 - y$
$\Rightarrow x = \frac{3-y}{4} \in R$

therefore f is onto.

$\Rightarrow f(1) = 3 - 4(1) = -1$
$\Rightarrow f(-1) = 3 - 4(-1) = 7$
$\Rightarrow f(5) = 3 - 4(5) = -17$
$\Rightarrow f\left(\frac{1}{2}\right) = 3 - 4\left(\frac{1}{2}\right) = 1$
etc.

~~$\Rightarrow f(10)$~~
$\Rightarrow y = 3 - 4x$
when, $x = 15$, $y = -57$
$x = 10$, $y = -37$
$x = 20$, $y = -77$
$x = -20$, $y = 83$

(ii) $f : R \to R$
$f(x) = 1 + x^2$

let $x_1, x_2 \in R$
$$f(x_1) = f(x_2)$$
$$\cancel{1} + x_1^2 = \cancel{1} + x_2^2$$
$$x_1^2 = x_2^2$$
$$\therefore x_1 = \pm x_2$$

$\Rightarrow f(-1) = 1 + (-1)^2 = 2$
$\Rightarrow f(1) = 1 + (1)^2 = 2$
$\Rightarrow f(2) = 1 + (2)^2 = 5$
$\Rightarrow f(-2) = 1 + (-2)^2 = 5$

let $y = 1 + x^2$
$x^2 = y - \cancel{1}$
$x = \sqrt{y-1} \notin R$

therefore it is not onto, and also not one-one.

Given

Q. (4) Modulas function $f: R \to R$, Given by $f(x) = |x|$ then show is neither one-one nor onto. where $|x| = x$, if x is Positive or 0 and $|x|$ is $-x$, if x is negative.

$$\boxed{f(x) = \begin{cases} 1 & , x > 0 \\ 0 & , x = 0 \\ -1 & , x < 0 \end{cases}}$$

Q. 5.

Sol. $f: R \to R$

$$f(x) = |x| = \begin{cases} x & , x > 0 \\ -x & , x < 0 \end{cases}$$

$$f(x) = \begin{cases} 1 & x > 0 \\ 0 & x = 0 \\ -1 & x < 0 \end{cases}$$

when we take $f(2)$ then we get 1 as Range and also $f(3)$ the we get 1 as Range.

$\therefore$ $f(2) = f(3)$

$2 \neq 3$.

therefore it is not one-one

Now,

$f: R \to R$.

Real no. (R) contain all the no. between $-\infty$ to $+\infty$ including rational, decimals, naturals no. etc.

Here the Range (R) contains ~~$-\infty$ to $+\infty$~~ only three no.'s -1, 0 and $+1$ and domain contains $-\infty$ to $+\infty$, it means every element of Range

set R is not the image of some element of Domain set R.

Eg. $f(5) = 1$
$f(-5) = -1$
$f(10) = 1$
$f(-17) = -1$
$f(0) = 0$

$f(3) = 1$
$f(-3) = -1$

therefore it is not onto.

Sol. (4) Given,

$$f: R \to R$$

$$f(x) = |x| = \begin{cases} x & x \geq 0 \\ -x & x < 0 \end{cases}$$

let $x_1, x_2 \in R$.

$\Rightarrow f(x_1) = f(x_2)$
$\Rightarrow f(1) = f(-1)$
but $1 \neq -1$

therefore it is not one-one

Rough.

$f(1) = |1| = 1$
$f(-1) = |-1| = -(-1) = 1$

Now,

$-x_1 \in R$.

$\Rightarrow f(-x_1) = y$
$\Rightarrow |-x_1| = y$
$\Rightarrow -(-x_1) = y$
$\Rightarrow x_1 = y$

~~but Range set R contain $-\infty$ to $+\infty$~~

When we take any negative no. as domain then we always get a positive no. but Range set R contains $-\infty$ to $+\infty$, so -ve no.'s of Range set are not image of the some domain therefore it is not onto.

Q. (6) let $A = \{1,2,3\}$, $B = \{4,5,6,7\}$ and let $f = \{(1,4),(2,5),(3,6)\}$ be a function from A to B, show that f is one-one.

Sol.

$f: A \rightarrow B$

$f = \{(1,4),(2,5),(3,6)\}$

$A = \{1,2,3\}$, $B = \{4,5,6,7\}$

Given,

$f(1) = 4$

$f(2) = 5$

$f(3) = 6$

here if we put any value as domain then we always get different value of Range therefore it is one-one.

Q.8. let A and B be sets. show that $f: A \times B \rightarrow B \times A$ such that $f(a,b) = (b,a)$ is bijective function.

Sol.

$f: A \times B \rightarrow B \times A$

$f(a,b) = (b,a)$

let $(a_1, b_1), (a_2, b_2) \in A \times B$.

$\Rightarrow f(a_1, b_1) = f(a_2, b_2)$

$\Rightarrow (b_1, a_1) = (b_2, a_2)$

$\Rightarrow b_1 = b_2$ and $a_1 = a_2$

$\Rightarrow (a_1, b_1) = (a_2, b_2)$

Therefore it is one-one

every elements of set codomain has its pre-image in it's set of domain therefore f is onto.

It is both one-one and onto ~~but~~ so it is bijective.

(9) $f: N \to N$, $f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases}$ $\forall n \in N$

function is bijective or not, justify.

Sol.

$$f(n) = \begin{cases} \frac{n+1}{2}, & \text{if } n \text{ is odd} \\ \frac{n}{2}, & \text{if } n \text{ is even} \end{cases} \quad \forall n \in N.$$

let $n_1 = 1$ and $n_2 = 2$.

then $f(1) \neq f(2)$

$\Rightarrow \frac{1+1}{2} \neq \frac{2}{2}$

$\Rightarrow 1 = \neq 1$

Hence it is not one-one

Now,

$2k+1 \in N$

$\Rightarrow \cancel{f(2k+1) = \frac{2k+1+1}{2}}$

$=$

$2k+1$ is odd then $4k+1$ is also odd and belongs to N.

$\Rightarrow f(4k+1) = \frac{4k+1+1}{2}$

$= 2k+1$

Again,

$2k \in N$ then $4k \in N$

$\Rightarrow f(4k) = \frac{4k}{2}$

$\Rightarrow f(4k) = 2k$.

Hence f is onto.

Date 12/06/21 but not bijective.

Q. 10

Sol. $A = R - \{3\}$

$B = R - \{1\}$

$f: A \rightarrow B$

$f(x) = \left(\frac{x-2}{x-3}\right)$

let $x_1, x_2 \in A$

such that

$f(x_1) = f(x_2)$

$\frac{x_1 - 2}{x_1 - 3} = \frac{x_2 - 2}{x_2 - 3}$

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$\Rightarrow (x_1-2)(x_2-3) = (x_2-2)(x_1-3)$

$\Rightarrow \cancel{x_1x_2} - 3x_1 - 2x_2 + \cancel{6} = \cancel{x_2x_1} - 3x_2 - 2x_1 + \cancel{6}$

$\Rightarrow -3x_1 - 2x_2 = -3x_2 - 2x_1$

$\Rightarrow -3x_1 + 2x_1 - 2x_2 + 3x_2 = 0$

$\Rightarrow -x_1 + x_2 = 0$

$\Rightarrow x_1 = x_2$

Therefore f is one-one.

Let $y = \left(\frac{x-2}{x-3}\right)$

$\Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow yx - 3y = x - 2$

$\Rightarrow -3y + yx + 2 = \cancel{-x}\ x$

$\Rightarrow yx - 3y + 2$

$\Rightarrow y = \frac{x-2}{x-3}$

$\Rightarrow xy - 3y = x - 2$

$\Rightarrow xy - x = 3y - 2$

$\Rightarrow x(y-1) = 3y - 2$

$\Rightarrow x = \frac{3y-2}{y-1} \in A$, for every value of y except $y=1$ there is a pre image.

Therefore f is onto.

It is one-one and onto therefore it is bijective

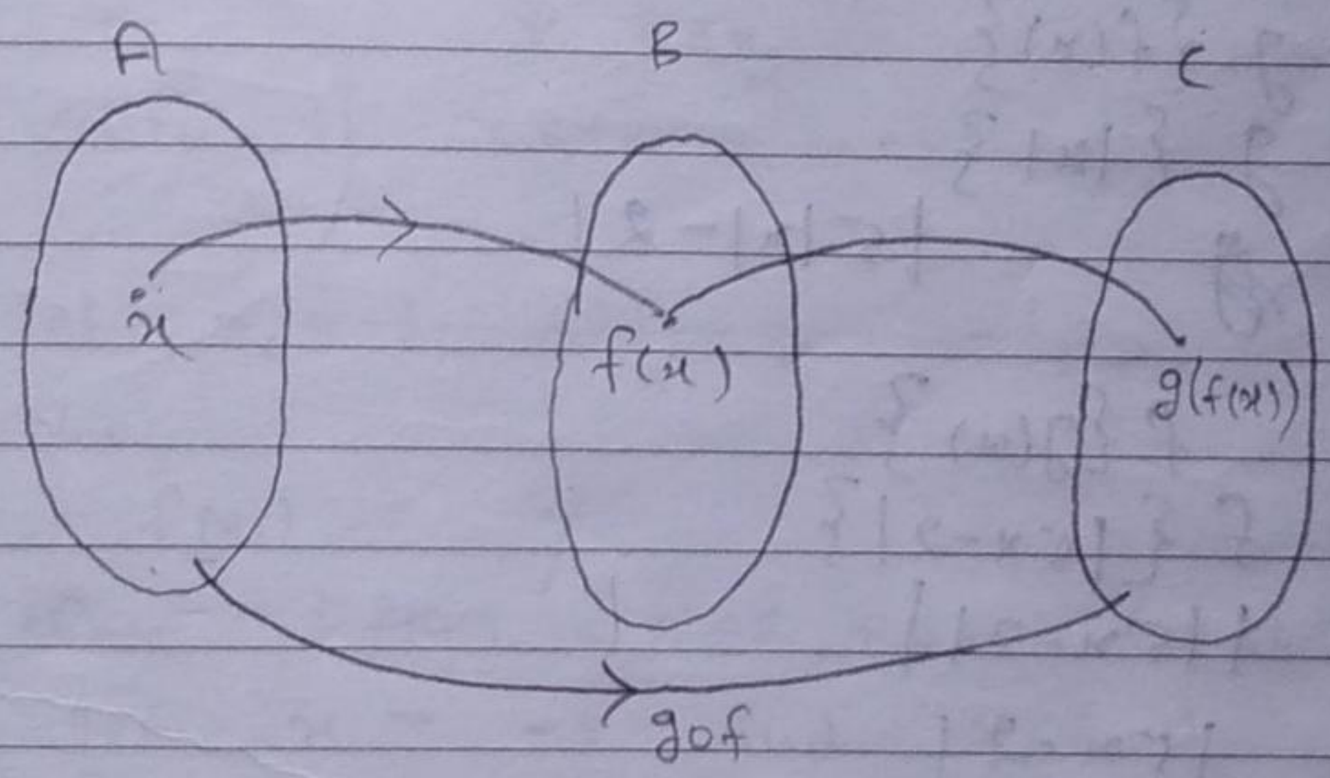
$f(5) = \left(\frac{5-2}{5-3}\right) = \frac{3}{2}$

$f(10) = \left(\frac{10-2}{10-3}\right) = \frac{8}{7}$

Composite of a function :-

Let $f: A \to B$ and $g: B \to C$ be two function. Then the composition of f and g, denoted by gof, is defined as the function $gof: A \to C$ given by

$$gof(x) = g\{f(x)\}, \quad \forall \; x \in A$$



Note :- i) $gof(x) = g\{f(x)\}$
ii) $fog(x) = f\{g(x)\}$

Q. if $f(x) = x^2+5$
$g(x) = \cos x$. then find $fog(x)$ and $gof(x)$

Sol. ~~gof~~ $fog(x) = f(g(x))$
$= f(\cos x)$
$= (\cos x)^2 + 5$

$fog(x) = \cos^2 x + 5$

Now,

$\Rightarrow$ $gof(x) = g\{f(x)\}$

$= g\{x^2+5\}$

$= \cos(x^2+5)$

Exercise -3

Q.3. find gof and fog, if

i) $f(x) = |x|$ and $g(x) = |5x-2|$

Sol.

$gof(x) = g\{f(x)\}$

$= g\{|x|\}$

$= |5|x|-2|$

$fog(x) = f\{g(x)\}$

$= f\{|5x-2|\}$

$= ||5x-2||$

$= |5x-2|$

ii) $f(x) = 8x^3$ and $g(x) = x^{1/3}$

Sol. $gof(x) = g\{f(x)\}$

$= g\{8x^3\}$

$= (8x^3)^{1/3}$

$= ((2x)^3)^{1/3}$

$= (2x)^{3\times\frac{1}{3}} = 2x$ Ans.

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$fog(x) = f\{g(x)\}$
$= f\{x^{1/3}\}$
$= 8(x^{1/3})^3$
$= 8x$ Ans.

Exercise 1.2.

(11) let $f: R \to R$ be defined as $f(x) = x^4$, choose the correct option.

Sol. let $x_1, x_2 \in R$
such that

$\Rightarrow f(x_1) = f(x_2)$
$\Rightarrow x_1^4 = x_2^4$
$\Rightarrow x_1 = \pm x_2$

Hence it is not one - one.

let ~~$x_1 = 1$ and $x_2 = -1$~~

Now,
$\Rightarrow f(x_1) = y$
$\Rightarrow$ ~~then~~ $f(y) = x \in R$, then it is onto
let $x = -1$ and $y = 1$

$\Rightarrow f(x) = y$
$\Rightarrow f(-1) = 1$
$\Rightarrow (-1)^4 = 1$
$\Rightarrow 1 = 1 \in R$

but
$\Rightarrow f(y) = x$
$\Rightarrow f(1) = -1$
$\Rightarrow (1)^4 = -1$
$\Rightarrow 1 \neq -1 \in R$ therefore it is not onto.

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D) It is not one-one neither onto.

(12) Let $f: R \to R$ be defined as $f(x) = 3x$. Choose the correct answer.

Sol. Let $x_1, x_2 \in R$

such that. $f(x_1) = f(x_2)$

$\Rightarrow 3x_1 = 3x_2$

$\Rightarrow \frac{3}{3}x_1 = \frac{3}{3}x_2$

$\Rightarrow x_1 = x_2$

therefore it is one-one.

Let x and $y \in R$.

Now, $f(x) = y$

$\Rightarrow f(y) = x \in R$ then it is onto.

Rough

$3x = y$

$f(y) = 3x$

$3y = 3x$

$x = y$

$\Rightarrow f(x) = y$

$\Rightarrow 3x = y \in R$

$\Rightarrow x = \frac{y}{3} \in R$

$\Rightarrow f\left(\frac{y}{3}\right) = x$

$\Rightarrow 3\left(\frac{y}{3}\right) = x$.

$\Rightarrow y = x \in R$.

therefore it is onto.

A) f is one-one & onto.

Exercise 1.3

① Let $f: \{1,3,4\} \to \{1,2,5\}$ and $g: \{1,2,5\} \to \{1,3\}$ be given by $f = \{(1,2),(3,5),(4,1)\}$ and $g = \{(1,3),(2,3),(5,1)\}$ write down gof

Sol.

$\Rightarrow$ $gof = g\{f(x)\}$

Given,

$f(1) = 2$	$g(1) = 3$
$f(3) = 5$	$g(2) = 3$
$f(4) = 1$	$g(5) = 1$

$\Rightarrow$ $gof(1) = g\{f(1)\} = g\{2\} = 3$

$\Rightarrow$ $gof(3) = g\{f(3)\} = g(5) = 1$

$\Rightarrow$ $gof(4) = g\{f(4)\} = g(1) = 3$

therefore

$$gof = \{(1,3),(3,1),(4,3)\}$$

② Let f, g and h be functions from R to R. show that

$$(f+g)oh = foh + goh$$

$$\text{If } (f.g)oh = (foh)(goh)$$

Sol.

$\Rightarrow$ $(f+g)oh = foh + goh$

$\Rightarrow$ L.H.S. $= (f+g)oh$

$= (f+g)oh(x)$

$= f\{h(x)\} + g\{h(x)\}$

$= x(foh) + (goh)x$

$= x\{foh + goh\}$

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$= x\{oh(f+g)\}$ $\forall x \in R$

$= (f+g)oh$

L.H.S. $= foh + goh$

$\Rightarrow$ L.H.S $=$ R.H.S

Proved.

iv) $(f.g)oh = (foh).(goh)$

$\Rightarrow$ L.H.S $= (f.g)oh$

$= foh . goh.$

$= (f.g)oh(x)$

$= foh(x) . g$

$= f\{h(x)\} . g.$

$\Rightarrow$ L.H.S $= (f.g)oh$

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Sol.

i) $(f+g)oh = foh + goh$

let

$f(x) = x^2$

$g(x) = \sin x$

$h(x) = e^x$

$\Rightarrow f+g = f(x) + g(x)$

$= x^2 + \sin x$

$\Rightarrow (f+g)oh = (f+g)\{h(x)\}$

$= (f+g)\ e^x$

$= (e^x)^2 + \sin e^x$

$= e^{2x} + \sin e^x$

~~R.H.S~~

$\Rightarrow$ $foh = f\{h(x)\}$
$= f(e^x)$
$= (e^x)^2 = e^{2x}$

$\Rightarrow$ $goh = g\{h(x)\}$
$= g(e^x)$
$= \sin e^x$

R.H.S. $= foh + goh$
$= e^{2x} + \sin e^x$
$= (f+g)oh$
L.H.S $=$ R.H.S
Proved.

Invertiable Function:- A function $f: X \to Y$ is defined to be invertible, if there exists a function $g: Y \to X$ such that $gof = I_x$ and $fog = I_y$. The function g is called the inverse of f and is denoted by f^{-1}.

If f is one-one and onto then f must be invertible.

Eg.- $f(x) = 2x + 3$.
let
$y = 2x + 3$
$\Rightarrow x = \frac{y-3}{2}$
$\Rightarrow g(y) = \frac{y-3}{2}$
$\Rightarrow$ or $f^{-1}(y) = \frac{y-3}{2}$

and $\boxed{f^{-1}(x) = \frac{x-3}{2}}$

Exercise 1.3.

(8)

Sol. $f: R_+ \to [4, \infty)$
$f(x) = x^2+4 \longrightarrow (1)$

let
$y = x^2+4$
$\Rightarrow x^2 = y-4$
$\Rightarrow x = \sqrt{y-4}$

$\therefore g(y) = \sqrt{y-4} \longrightarrow (2)$

$\Rightarrow gof(x) = g\{f(x)\}$
$= g\{x^2+4\}$
$= \sqrt{x^2+4-4}$
$= \sqrt{x^2}$
$= x = I_x$

$\Rightarrow fog(y) = f\{g(y)\}$
$= f\{\sqrt{y-4}\}$
$= (\sqrt{y-4})^2 + 4$
$= y$
$= I_y$

Hence
$gof(x) = I_x$ & $fog(y) = I_y$
$\therefore f$ is invertible function.

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Now,

$\because \quad g(y) = \sqrt{y-4}$

$\therefore \quad f^{-1}(y) = \sqrt{y-4}$

Q.2. ii) $(f.g)oh = foh . goh$ Prove this

Sol. let, $f(x) = x^2$

$g(x) = \sin x$

$h(x) = e^x$

$\Rightarrow \quad f.g = f(x) . g(x)$

$\Rightarrow \quad f.g = x^2 . \sin x$

$\Rightarrow \quad (f.g)oh = (f.g)\{h(x)\}$

$= \cancel{x^2} \, (f.g) \; e^x$

$= (e^x)^2 . \sin e^x$

$= e^{2x} . \sin e^x$

$\Rightarrow \quad foh = f\{h(x)\}$

$= f\{e^x\}$

$= (e^x)^2$

$= e^{2x}$

$\Rightarrow \quad goh = g\{h(x)\}$

$= g\{e^x\}$

$= \sin e^x$

$R.H.S = (foh)(goh)$

$= e^{2x} . \sin e^x$

$= L.H.S$

$L.H.S = R.H.S$

Hence Proved.

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(4) If $f(x) = \frac{(4x+3)}{(6x-4)}$, $x \neq \frac{2}{3}$, show that $fof(x) = x$, $\forall x \neq \frac{2}{3}$ what is the inverse of x?

Sol.

$$f(x) = \frac{4x+3}{6x-4}$$

let,

$$\Rightarrow \quad y = \frac{4x+3}{6x-4}$$

$$\Rightarrow \quad 6xy - 4y = 4x+3$$

$$\Rightarrow \quad 6xy - 4x = 3+4y$$

$$\Rightarrow \quad 2x(3y-2) = 3+4y$$

$$\Rightarrow \quad 2x = \frac{3+4y}{3y-2}$$

$$\Rightarrow \quad x = \frac{3+4y}{6y-4}$$

$$\Rightarrow \quad g(y) = \frac{3+4y}{6y-4}$$

$$\therefore \quad f^{-1}(y) = \frac{3+4y}{6y-4}$$

$$\Rightarrow \quad f^{-1}(x) = \frac{3+4x}{6x-4}$$ Ans

(5) state with reason wheather following function have inverse ~~or~~

i) $f: \{1,2,3,4\} \longrightarrow \{10\}$ with,
$f = \{(1,10), (2,10), (3,10), (4,10)\}$

Sol.

$$f = \{(1,10), (2,10), (3,10), (4,10)\}$$

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$\Rightarrow f(1) = 10$ | $\Rightarrow f(3) = 10$
$\Rightarrow f(2) = 10$ | $\Rightarrow f(4) = 10$

Hence it is not one-one but it is onto.

Therefore f does not have inverse.

ii) $g : \{5, 6, 7, 8\} \rightarrow \{1, 2, 3, 4\}$ with
$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

Sol.

$g = \{(5, 4), (6, 3), (7, 4), (8, 2)\}$

By defination,

$g(5) = 4$ and also $g(7) = 4$
Hence it is not one-one, it is only onto.

Therefore g does not have inverse.

iii) $h : \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$ with
$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$

Sol.

$h = \{(2, 7), (3, 9), (4, 11), (5, 13)\}$
$h : \{2, 3, 4, 5\} \rightarrow \{7, 9, 11, 13\}$

$\Rightarrow h(2) = 7$ | $\Rightarrow h(4) = 11$
$\Rightarrow h(3) = 9$ | $\Rightarrow h(5) = 13$

It has distinct value for each function therefore h is one-one

Every element of range set present in domain set therefore it is onto.
Hence h has inverse.

(6) show that $f: [-1, 1] \to R$, given by $f(x) = \frac{x}{x+2}$ is one-one. Find the inverse of function $f: [-1, 1] \to$ range f

Sol.

$f: [-1, 1] \to R$

$f(x) = \frac{x}{x+2}$.

let $x_1, x_2 \in R$

such that,

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow \frac{x_1}{x_1+2} = \frac{x_2}{x_2+2}$

$\Rightarrow \cancel{x_1x_2} + 2x_1 = \cancel{x_1x_2} + 2x_2$

$\Rightarrow 2x_1 = 2x_2$

$\Rightarrow x_1 = x_2$

Hence it is one-one.

$f: [-1, 1] \to$ range f is onto

therefore f has inverse.

$\Rightarrow f(x) = \frac{x}{x+2}$

let $y = \frac{x}{x+2}$

$\Rightarrow yx + 2y = x$

$\Rightarrow yx - x = -2y$

$\Rightarrow x(y-1) = -2y$

$\Rightarrow x = \frac{-2y}{y-1}$

$g(y) = \frac{-2y}{y-1}$

$\Rightarrow f^{-1}(y) = \frac{-2y}{y-1}$

$\Rightarrow f^{-1}(x) = \frac{-2x}{x-1}$ Ans

(4) Consider $f: R \to R$ given by $f(x) = 4x+3$. Show that f is invertible. Find the inverse of f.

Sol.

$f: R \to R$

$\Rightarrow f(x) = 4x+3$

Let $x_1, x_2 \in R$

such that,

$\Rightarrow f(x_1) = f(x_2)$

$\Rightarrow 4x_1 + 3 = 4x_2 + 3$

$\Rightarrow 4x_1 = 4x_2$

$\Rightarrow \boxed{x_1 = x_2}$

Hence it is one-one.

Let

$\Rightarrow f(x) = y$

Then for this function,

$\Rightarrow f(y) = x$

Hence it is onto therefor $f(x)$ is invertible.

$\Rightarrow f(x) = 4x+3$

let,

$y = 4x+3$

$\Rightarrow y-3 = 4x$

$\Rightarrow x = \frac{y-3}{4}$

$\Rightarrow g(y) = \frac{y-3}{4}$

or

$\Rightarrow \quad f^{-1}(y) = \dfrac{y-3}{4}$

$\Rightarrow \quad f^{-1}(x) = \dfrac{x-3}{4}$ Ans

(13) If $f: R \to R$ be given by $f(x) = (3-x^3)^{1/3}$, then $fof(x)$ is

Sol. $f: R \to R$
$f(x) = (3-x^3)^{1/3}$

$\Rightarrow \quad fof(x) = f\{f(x)\}$
$\cancel{= f\{\sqrt[3]{(3-x^3)}\}}$

$= f\{(3-x^3)^{1/3}\}$
$= f\{\sqrt[3]{3-x^3}\}$
$= \left\{3-(\sqrt[3]{3-x^3})^3\right\}^{1/3}$
$= \{3-(3-x^3)\}^{1/3}$
$= \{3-3+x^3\}^{1/3}$
$= (x^3)^{1/3}$
$= x^{3\times\frac{1}{3}}$
$= x$
$= x$ Ans.

(14) Let $f: R - \{\frac{4}{3}\} \to R$ be a function defined as

$f(x) = \dfrac{4x}{3x+4}$. The inverse of f is the map g:

g: Range $f \to R - \{-\frac{4}{3}\}$ given by

Sol. $f: R - \left\{-\frac{4}{3}\right\} \rightarrow R$

$\Rightarrow f(x) = \frac{4x}{3x+4}$

let,

$\Rightarrow y = \frac{4x}{3x+4}$

$\Rightarrow 3xy + 4y = 4x$

$\Rightarrow 3xy - 4x = -4y$

$\Rightarrow x(3y-4) = -4y$

$\Rightarrow x = \frac{-4y}{3y-4}$

$\Rightarrow x = \frac{4y}{4-3y}$

$\Rightarrow g(y) = \frac{4y}{4-3y}$ Ans. option B.

$\Rightarrow f^{-1}(y) = \frac{4y}{4-3y}$

$\Rightarrow f^{-1}(x) = \frac{4x}{4-3x}$

(9) Consider $f: R \rightarrow [-5, \infty)$ given by $f(x) = 9x^2 + 6x - 5$. show that f is invertible with $f^{-1}(y) = \left(\frac{\sqrt{y+6}) - 1}{3}\right)$

Sol. $f(x) = 9x^2 + 6x - 5$

let,

$\Rightarrow y = 9x^2 + 6x - 5$

$\Rightarrow y = 9x^2 + 6x + 1 - 5 - 1$

$\Rightarrow y = (3x)^2 + 2(3x)(1) + (1)^2 - 6$

$\Rightarrow y = (3x+1)^2 - 6$ $\{\because (a+b)^2 = a^2 + 2ab + b^2\}$

$\Rightarrow y + 6 = (3x+1)^2$

$\Rightarrow (y+6)^{1/2} = (3x+1)^{1/2 \times 2}$

$\Rightarrow (y+6)^{1/2} = 3x+1$

$\Rightarrow \sqrt{y+6} - 1 = 3x$

$\Rightarrow x = \dfrac{\sqrt{y+6} - 1}{3}$

$\Rightarrow g(y) = \dfrac{\sqrt{y+6} - 1}{3}$

$\Rightarrow f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$

Hence $f(x)$ is invertible with $f^{-1}(y) = \dfrac{\sqrt{y+6} - 1}{3}$

$$f^{-1}(y) = \frac{\sqrt{y+6} - 1}{3}$$

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(10) let $f: X \to Y$ be an invertible function. show that f has unique inverse.
(HINT: suppose g_1 and g_2 are two inverses of f. $\forall y$ $f \circ g_1(y) = I_y(y) = f \circ g_2(y)$. Use one-one ness of f

Sol. since $f: X \to Y$ is invertible function.
so, $g \circ f(x) = I_x$ & $f \circ g(y) = I_y$

therefore f is one-one and onto

suppose g_1 and g_2 are two inverse function of f
then ~~$f \circ g_1(y) = I_{y_1}$~~ $f \circ g_1(y) = I_y$
$\Rightarrow$ ~~$f \circ g_2(y_2)$~~ $f \circ g_2(y) = I_y$

I_y be a unique image for even function f because f is one-one and onto

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Therefore, $g_1(y) = g_2(y)$
Hence f has unique inverse.

(12) let $f: X \to Y$ be an invertible function. show that the inverse of f^{-1} is f, i.e., $(f^{-1})^{-1} = f$.

Sol:

since $f: X \to Y$ is an invertible function
therefore f is one-one and onto

let, $g: Y \to X$ is inverse function of f.
that is,

$$g = f^{-1}$$

We know, $g \circ f = I$

$\Rightarrow$ $g \circ f = I \Rightarrow f^{-1} \circ f = I$
and also, $g \circ g^{-1} = I$

and,

$$f^{-1} \circ (f^{-1})^{-1} = I$$

$\Rightarrow$ $f \circ \{f^{-1} \circ (f^{-1})^{-1}\} = f \circ I$
$\Rightarrow$ $(f \circ f^{-1}) \circ (f^{-1})^{-1} = f$ $\{\because$ assosiative law$\}$
$\Rightarrow$ $I \circ (f^{-1})^{-1} = f$
$\Rightarrow$ $(f^{-1})^{-1} = f$

$\Rightarrow$ $\therefore (f^{-1})^{-1} = f$
Hence proved.

Binary operation :-

A binary operation $*$ on a set A is a function $* : A \times A \to A$. We denote $*(a,b)$ by $a*b$

⇒ Commutative law :- A binary operation $*$ on the set X is called commutative, if $a*b = b*a$, for every $a, b \in X$

⇒ Associative law :- A binary operation $* : A \times A \to A$ is said to be associative if $(a*b)*c = a*(b*c)$, $\forall\, a,b,c \in A$

⇒ Identity law :- Given a binary operation $* : A \times A \to A$, an element $e \in A$, if it exists, is called identity for the operation $*$, if

$$a*e = a = e*a, \ \forall\, a \in A.$$

Invertable operation :-

Given a binary operation $* : A \times A \to A$ with the identity element e in A, an element $a \in A$ is said to be invertible with respect to the operation $*$, if there exists an element b in A such that $a*b = e = b*a$ and b is called the inverse of a and is denoted by a^{-1}.

Exercise 1.3

Q.11 Consider $f: \{1,2,3\} \rightarrow \{a,b,c\}$ given by $f(1)=a$, $f(2)=b$ and $f(3)=c$. Find f^{-1} and show that $(f^{-1})^{-1} = f$

Sol. $f: \{1,2,3\} \rightarrow \{a,b,c\}$

$f(1) = a$

$f(2) = b$

$f(3) = c$

let, $f^{-1} = g$.

$g: \{a,b,c\} \rightarrow \{1,2,3\}$ then $g(a) = 1$, $g(c) = 3$.
$g(b) = 2$

$\Rightarrow fog(a) = f\{g(a)\}$
$= f\{1\}$
$= a$

$\Rightarrow fog(b) = f\{g(b)\}$
$= f\{2\}$
$= b$.

$\Rightarrow fog(c) = f\{g(c)\}$
$= f\{3\}$
$= c$

$\therefore fog = I_y$

Now,

$\Rightarrow gof(1) = g\{f(1)\}$
$= g\{a\}$
$= 1$

$\Rightarrow gof(2) = g\{f(2)\}$
$= g(b) = 2$

$\Rightarrow gof(3) = g\{f(3)\}$
$= g\{c\}$
$= 3.$

$\therefore gof = I_x$

Hence f^{-1} exist.

let, $h = f$
then
$h : \{1,2,3\} \rightarrow \{a,b,c\}$
so,
$h(1) = a$
$h(2) = b$
$h(3) = c$

$\Rightarrow goh(1) = g\{h(1)\}$
$= g\{a\}$
$= 1$

$\Rightarrow goh(2) = g\{h(2)\}$
$= g\{b\}$
$= 2$

$\Rightarrow goh(3) = g\{h(3)\}$
$= g\{c\}$
$= 3.$

$\therefore goh = I_x$

and,
$\Rightarrow hog(a) = h\{g(a)\}$
$= h\{1\}$
$= a$

$\Rightarrow hog(b) = h\{g(b)\}$

$= h(2)$
$= b$

$\Rightarrow goh(c) = g\{h(c)\}$
$= g\{3\}$
$= c$

$\therefore goh = I_y$

Hence g^{-1} exist.

$\Rightarrow g^{-1} = h$
$\Rightarrow (f^{-1})^{-1} = f$ $\{\because f^{-1} = g$ and $h = f\}$
Hence proved.

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Exercise 1.4.

① check this are Binary operation or not if not then give justification.

(i) on z^+, define $*$ by $a * b = a - b$

Sol.

$a * b = a - b$.
If $a > b$
then, $a * b = a - b \in z^+$
but if $a < b$
then,
$a * b = a - b \notin z^+$

for example,
if $a = 5$, $b = 1$
then $a * b = 5 - 1 = 4 \in z^+$
and if $a = 1$, $b = 5$

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then $a * b = a - b = 1 - 5 = -4 \notin Z^+$

Therefore given operation $*$ is not a binary operation.

Q.2 Determine wheather $*$ is binary, commutative or associative.

(i) on Z, define $a * b = a - b$

Sol. for commutative we have to prove $a * b = b * a$

Now,

$a * b = b * a$

and, $b * a = b - a$

Here

$a - b \neq b - a$

$\therefore a * b \neq b * a$, Hence it is not defined as commutative law.

for associative law we have to prove

$\Rightarrow a * (b * c) = (a * b) * c$

Now,

L.H.S $= a * (b * c)$

$= a * (b - c)$

$= a - (b - c)$

$= a - b + c$

R.H.S $= (a * b) * c$

$= (a - b) * c$

$= (a - b) - c$

$= a - b - c$

L.H.S $\neq$ R.H.S

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Hence it is not defined as associative law.

~~for identity law,~~

(iv) on Q, define $a * b = \frac{ab}{2}$

Sol. for commutative we have to prove $a * b = b * a$,

Now,

$$a * b = \frac{ab}{2}$$

and $$b * a = \frac{ba}{2}$$

Here,

$$\frac{ab}{2} = \frac{ba}{2}$$

$$a * b = b * a$$

Hence it is defined as commutative law,

for associative law we have to prove $a*(b*c) = (a*b)*c$

Now,

$$L.H.S = a*(b*c)$$

$$= a * \left(\frac{bc}{2}\right)$$

$$= \frac{a\frac{bc}{2}}{2}$$

$$= \frac{abc}{4}$$

$$R.H.S = (a*b)*c$$
$$= \left(\frac{ab}{2}\right)*c$$
$$= \frac{\frac{abc}{2}}{2}$$
$$= \frac{abc}{4}$$

L.H.S = R.H.S

Hence it is defined as associative law.

Q.3. Consider the binary operation $\wedge$ on the set $\{1,2,3,4,5\}$ defined by $a \wedge b = \min\{a,b\}$. Write the operation table of the operation $\wedge$.

Sol. $a \wedge b = \min\{a,b\}$

$\wedge$	1	2	3	4	5
1	1	1	1	1	1
2	1	2	2	2	2
3	1	2	3	3	3
4	1	2	3	4	4
5	1	2	3	4	5

(1) iii) on Z^+, define $*$ by $a*b = ab$.

Sol.

$a*b = ab$,

if we put any value of a, b from Z^+ then we always get ab as result therefore it is binary operation.

for example,
if $a = 5$, $b = 2$

$\Rightarrow$ $a * b = ab = (5)(2) = 10 \in Z^+$

if $a = 2$ and $b = 7$

$\Rightarrow$ $a * b = ab = (2)(7) = 14 \in Z^+$

if $a = 7$ and $b = 14$

$a * b = ab = (7)(14)$
$= 98 \in Z^+$

(iii) on R, define $*$ by $a * b = ab^2$

Sol.

$a * b = ab^2$

yes, it is binary operation because when we take any value of a, b from R set then ~~we always get~~ the resultant value $a * b$ is always belong to R set.

for example,
if $a = 5$, $b = 4$

then,

$a * b = ab^2 = (5)(4)^2 = 5 \times 16 = 80 \in R$

if $a = 2$ and $b = 3$

then,

$a * b = ab^2 = (2)(3)^2 = 2 \times 9 = 18 \in R$

if $a = \frac{1}{4}$ and $b = 2$

then,

$a * b = ab^2 = \left(\frac{1}{4}\right)(2)^2 = \frac{1}{4} \times 4 = 1 \in R$

(iv) on Z^+, define $*$ by $a * b = |a-b|$

Sol.

$$a * b = |a-b|$$

If $a > b$

then,

$$a * b = |a-b|$$
$$= |a-b| \in Z^+$$

If $a < b$

then

$$a * b = |a-b| \in Z^+$$

Therefore given operation is binary operation.

for example,

if $a = 5$ and $b = 4$

then

$$a * b = |a-b| = |5-4| = |1| = 1 \in Z^+$$

~~but~~ if $a = 4$ and $b = 5$

then,

$$a * b = |a-b| = |4-5| = |(-1)| = -(-1) = 1 \in Z^+$$

(v) on Z^+, define $*$ by $a * b = a$

Sol.

$$a * b = a.$$

yes, it is binary operation because we take element (a, b) from Z^+ then according to operation we get same first element as result then it is clear that result also belongs to Z^+.

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for example,
if $a = 5$ and $b = 10$
then

$$a * b = a = 5 \in z^+$$

if $a = 10$ and $b = 20$
then

$$a * b = a = 10 \in z^+$$

if $a = 20$ and $b = 40$
then

$$a * b = a = 20 \in z^+$$

Q2.
(ii) on Q, define $a * b = ab + 1$
Sol.
for commutative law we have to prove $a * b = b * a$.

Now,

$$a * b = ab + 1$$
and $$b * a = ba + 1$$

$$a * b = b * a$$
Hence it is defined as commutative law.

for Associative law we have to prove

$$a * (b * c) = (a * b) * c$$

Now,

$$L.H.S = a * (b * c)$$
$$= a * (bc + 1)$$

2^{2a}

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$= a(bc+1)+1$
$= abc + a + 1$

R.H.S $= (a*b)*c$
$= (ab+1)*c$
$= (ab+1)c + 1$
$= abc + c + 1$

L.H.S $\neq$ R.H.S.
Hence it is ~~does~~ not defined as Associative law.

(iv) on z^+, define $a*b = 2^{ab}$

Sol. for commutative law we have to prove $a*b = b*a$.

Now, $a*b = 2^{ab}$
and $b*a = 2^{ba}$

$a*b = b*a$
Hence it is defined as commutative law.

for Associative law we have to prove

$a*(b*c) = (a*b)*c$

L.H.S $= a*(b*c)$
$= a*(2^{bc})$
$= 2^{(a)(2^{bc})}$
$= 2^{(a)(2bc)}$
$= 2^{2abc}$.

Rough
$a=2$, $b=\frac{1}{2}$, $c=4$.
$2*(\frac{1}{2}*4)$
$2*(2^{2})$
$2*(4)$
2^{8}
$=256$ 2^{8}

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R.H.S. $= (a * b) * c$

$= (2^{ab}) * c$

$= 2^{(2ab)(c)}$

$= 2^{2abc}$

L.H.S $= a * (b * c)$

$= a * (2^{bc})$

$= 2^{a \cdot 2^{bc}}$

$= 2^{(2^{bc}) \cdot a}$

$= 2^{2bca}$

R.H.S $= (a * b) * c$

$= (2^{ab}) * c$

$= 2^{2^{ab} \cdot c}$

$= 2^{2ab \cdot c}$

L.H.S $=$ R.H.S

let $a = 1$, $b = 2$, $c = 3$,

then,

L.H.S $= 1 * (2 * 3)$

$= 1 * (2^6)$

$= 1 * (64)$

$= 2^{64}$

R.H.S $= (1 * 2) * 3$

$= (2^2) * 3$

$= (4) * 3$

$= 2^{12}$

L.H.S $\neq$ R.H.S

Hence it is not defined as associative law.

Check: $a = 1$; $b = \frac{1}{2}$, $c = 2$.

① $(a * b) * c = a * (b * c)$

$(\sqrt{2}) * c = a * (2^1)$

$2^{\sqrt{2} \cdot 2} = 2^{1 \cdot 2}$

$2^{2\sqrt{2}} = 4$

Justification, $a = 1$, $b = 2$, $c = 3$.

$= a * b * c$

$= a * 2^{2 \cdot 3}$

$\Rightarrow a * 2^6$

$= a * 64$

$= 2^{64}$

$(a * b) * c$

$4 * c$

$2^{4 \times 3}$ 2^{12}

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(v) on z^+, define $a * b = a^b$

Sol.

$$a * b = a^b$$

for commutative law we have to prove $a * b = b * a$

$\Rightarrow a * b = a^b$

$\Rightarrow b * a = b^a$

$\Rightarrow a * b \neq b * a$

Hence it is not defined as commutative law.

for associative law we have to prove.

$$a * (b * c) = (a * b) * c$$

Now,

L.H.S $= a * (b * c)$

$= a * (b^c)$

$= a^{b^c}$

$= a^{bc}$

R.H.S $= (a * b) * c$

$= (a^b) * c$

$= a^{b^c}$

$= a^{bc}$

let $a = 4$, $b = 2$ and $c = 3$.

then.

L.H.S $= a * (b * c)$

$= 4 * (2 * 3)$

$= 4 * (2^3)$

$= 4 * (8)$

$= 4^8$

$= 4^8 = (4 \times 4 \times 4 \times 4)(4 \times 4 \times 4 \times 4)$

$= 256 \times 256 = 65536$

R.H.S $= (a * b) * c$

$= (4 * 2) * 3$

$= (4^2) * 3$

$= (16) * 3$

$= 16^3$

$= 64 \times 16$

$= 1024$

L.H.S. = R.H.S

Hence it ~~does not~~ f is not defined as associative law.

(vi) on $R - \{-1\}$, define $a * b = \frac{a}{b+1}$

Sol. for ~~associative~~ commutative law we have to prove $a * b = b * a$.

$a * b = \frac{a}{b+1}$

$b * a = \frac{b}{a+1}$

$\frac{a}{b+1} \neq \frac{b}{a+1}$

$a * b \neq b * a$

Hence it does not follow ~~asso~~ commutative law.

for associative law we have to prove.

$a * (b * c) = (a * b) * c$

Now,

L.H.S $= a * (b * c)$

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$$= a * \left(\frac{b}{c+1}\right)$$

$$= \frac{a}{\frac{b}{c+1}+1}$$

$$= \frac{a(c+1)}{b+c+1}$$

$$= \frac{ac+a}{b+c+1}$$

$$R.H.S = (a * b) * c$$

$$= \frac{a}{b+1} * c$$

$$= \frac{\frac{(a)^1}{(b+1)^1}}{c+1}$$

$$= \frac{(a)^1 (b+1)^{-1}}{c+1}$$

L.H.S $\neq$ R.H.S

Hence it is not defined as Associative law.

(4) Consider a binary operation $*$ on the set $\{1,2,3,4,5\}$ given by the following multiplication (table 1.2).

(i) compute $(2*3)*4$ and $2*(3*4)$

(ii) Is $*$ commutative?

(iii) compute $(2*3)*(4*5)$

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Sol.

*	1	2	3	4	5
1	1	1	1	1	1
2	1	2	1	2	1
3	1	1	3	1	1
4	1	2	1	4	1
5	1	1	1	1	5

(i) $(2 * 3) * 4$
$= (1) * 4$
$= 1$ Ans

(ii) for commutative we have to prove
$\Rightarrow a * b = b * a$
It is true Hence it is commutative.

(iii) $(2 * 3) * (4 * 5)$
$= 1 * 1$
$= 1$ Ans

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(6) Let * be the binary operation on N given by $a * b =$ L.C.M. of a and b. Find.

(i) $5 * 7$, $20 * 16$.

Sol. $5 * 7 = 35$
$20 * 16 = 80$

Rough

80	2	20, 16
	2	10, 8
	2	5, 4
	2	5, 2
	5	5, 1
		1, 1

(ii) is * commutative.

Sol. for commutative we have to prove
$a * b = b * a$

yes, it is commutative.

Eg. :-

(i) $2 * 5 = 10$ & $5 * 2 = 10$.
(ii) $16 * 20 = 80$ & $20 * 16 = 80$.

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(iii) Is * associative?

Sol. suppose, $a = 2$, $b = 5$, $c = 3$.

$$L.H.S = a * (b * c)$$
$$= 2 * (5 * 3)$$
$$= 2 * (15)$$
$$= 30$$

$$R.H.S = (a * b) * c$$
$$= (2 * 5) * 3$$
$$= (10) * 3$$
$$= 30.$$

suppose $a = 1$, $b = 2$, $c = 3$

$$L.H.S = a * (b * c)$$
$$= 1 * (2 * 3)$$
$$= 1 * 6$$
$$= 6$$

$$R.H.S = (a * b) * c$$
$$= (1 * 2) * 3$$
$$= (2) * 3$$
$$= 6$$

L.H.S = R.H.S

Hence it is associative.

(iv) find the identity of the * in N

Sol.

$1 * 1 = 1$
$2 * 1 = 2$
$3 * 1 = 3$
$4 * 1 = 4$

Here 1 is identity on N.

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(v) which element of N are invertible for the operation.

Sol.

for invertible defined as $a * b = 1$
than only for $a = 1$ and $b = 1$
we get the L.C.M 1

Hence the invertible of this operation is 1.

(9) Let * be a binary operation on the set Q of of rational no.'s as follows :-

ii) $a * b = a^2 + b^2$

Sol.

for commutative law,

$$a * b = b * a.$$

Now,

$$a * b = a^2 + b^2$$
$$b * a = b^2 + a^2$$

Here $a^2 + b^2 = b^2 + a^2$

$$a + b = b * a$$

Hence it is defined as commutative law.

Now, for associative law,

$$a * (b * c) = (a * b) * c.$$

L.H.S $= a * (b * c)$
$= a * (b^2 + c^2)$
$= a^2 + (b^2 + c^2)^2$
$= a^2 + b^4 + c^4 + 2b^2c^2$

R.H.S $= (a * b) * c$

$= (a^2+b^2) * c$

$= (a^2+b^2)^2 + (c)^2$

$= a^4 + b^4 + 2a^2b^2 + c^2$

$= a^4 + b^4 + c^2 + 2a^2b^2$

L.H.S $\neq$ R.H.S.

Hence it does not follow Associative law.

(i) $a * b = a - b$.

Sol. for commutative law,

$a * b = b * a$.

$a * b = a - b$

$b * a = b - a$.

$a - b \neq b - a$

$a * b \neq b * a$.

Hence $*$ is not commutative.

for Associative,

$a * (b * c) = (a * b) * c$

L.H.S $= a * (b * c)$

$= a * (b - c)$

$= a - (b - c)$

$= a - b + c$.

R.H.S $= (a * b) * c$

$= (a - b) * c$

$= a - b - c$

L.H.S $\neq$ R.H.S.

Hence $*$ is not associative.

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(i) $a * b = a + ab$

Sol. for commutative,

$$a * b = b * a$$

$a * b = a + ab$

$b * a = b + ba$

$a + ab \neq b + ba$

$a * b \neq b * a$

Hence $*$ is not commutative.

for Associative,

$$a * (b * c) = (a * b) * c$$

$L.H.S = a * (b * c)$

$= a * (b + bc)$

$= a + a(b + bc)$

$= a + ab + abc$

$R.H.S = (a * b) * c$

$= (a + ab) * c$

$= (a + ab) + (a + ab)c$

$= a + ab + ac + abc$

$L.H.S \neq R.H.S$

Hence $*$ is not Associative.

(ii) $a * b = (a - b)^2$

Sol. for commutative,

$$a * b = b * a$$

$\Rightarrow a * b = (a - b)^2$

$= a^2 + b^2 - 2ab$

$\Rightarrow b * a = (b - a)^2 = b^2 + a^2 - 2ba$

$\Rightarrow \quad a^2+b^2-2ab = b^2+a^2-2ba$

$\Rightarrow \quad a*b = b*a$

Hence $*$ is commutative.

for Associative,

$a*(b*c) = (a*b)*c$

L.H.S $= a*(b*c)$

$= a*(b-c)^2$

let $(b-c)^2 = M$.

$= a*M$

$= (a-M)^2$

$= a^2+M^2-2aM$

R.H.S $= (a*b)*c$

$= (a-b)^2*c$

let $(a-b)^2 = K$.

$= K*c$

$= (K-c)^2$

$= K^2+c^2-2Kc$

$= (a-b)^2+c^2-2(\sqrt{(a-b)^2})c \quad \{\because K=(a-b)^2\}$

$= (a^2+b^2-2ab)+c^2-2(a-b)c$

$= (a^2+b^2-2ab)^2+c^2-(2a-2b)c$

$= (a^2+b^2-2ab)^2+c^2-2ac-2bc$

$\cancel{= a^2+b^2+c^2-2ab-2ac-2bc}$

$\cancel{\text{L.H.S} = a^2+M^2-2aM}$

$\cancel{= a^2+(b-c)}$

$= (a^2+b^2-2ab)^2 + c^2 - 2ac - 2bc$

let $a^2+b^2 = V$,

$= (V-2ab)^2 + c^2 - 2ac - 2bc$

$= V^2 + 4a^2b^2 - 4abV + c^2 - 2ac - 2bc.$

$= (a^2+b^2)^2 + 4a^2b^2 - 4ab(a^2+b^2) + c^2 - 2ac - 2bc$

$= a^4+b^4+2a^2b^2 + 4a^2b^2 - 4a^3b - 4ab^3 + c^2 - 2ac - 2bc$

R.H.S. $= a^4+b^4+6a^2b^2 - 4a^3b - 4ab^3 + c^2 - 2ac - 2bc.$

LHS $= a^2 + m^2 - 2am$

$= a^2 + ((b-c)^2)^2 - 2a((b-c)^2)$

$= a^2 + (b^2+c^2-2bc)^2 - 2a(b^2+c^2-2bc)$

$= a^2 + (l-2bc)^2 - 2ab^2 + 2ac^2 - 4abc$

$\{\because$ let $l = b^2+c^2\}$

$= a^2 + l^2 + 4b^2c^2 - 4bcl - 2ab^2 + 2ac^2 - 4abc$

$= a^2 + (b^2+c^2)^2 + 4b^2c^2 - 4bc(b^2+c^2) - 2ab^2 + 2ac^2 - 4abc$

LHS $= a^2 + b^4 + c^4 + 2b^2c^2 + 4b^2c^2 - 4b^3c - 4bc^3 - 2ab^2 + 2ac^2 - 4abc$

$\Rightarrow a^4+b^4+c^2+6a^2b^2 - 4a^3b - 4ab^3 - 2ac - 2bc \neq a^2+b^4+c^4$
$+ 2b^2c^2 + 4b^2c^2 + 4(-b^3)(c) - 4bc^3 - 2ab^2$
$+ 2ac^2 - 4abc.$

L.H.S $\neq$ R.H.S

Hence $*$ is not associative.

(v) $a * b = \frac{ab}{4}$

3.1 for commutative,

$a*b = b*a$

$a*b = \frac{ab}{4}$

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$b * a = \frac{ba}{4}$

$\Rightarrow \frac{ab}{4} = \frac{ba}{4}$

$\Rightarrow a * b = b * a$

Hence $*$ is commutative.

for Associative,

$a * (b * c) = (a * b) * c$

$L.H.S = a * (b * c)$

$= a * \left(\frac{bc}{4}\right)$

$= \frac{a \frac{bc}{4}}{4}$

$= \frac{abc}{(4)^1 4}$

$= abc$

$R.H.S = (a * b) * c$

$= \left(\frac{ab}{4}\right) * c$

$= \frac{\frac{ab}{4} c}{4}$

$= \frac{abc}{4.(4)^{-1}}$

$= abc$

$L.H.S = R.H.S$

Hence $*$ is Associative.

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(vi) $a * b = ab^2$

Sol. for commutative,

$$a * b = b * a.$$

$\Rightarrow a * b = ab^2$

$\Rightarrow b * a = ba^2$

$\Rightarrow ab^2 \neq ba^2$

$\Rightarrow a * b \neq b * a.$

Hence $*$ is not commutative,

for Associative,

$$a * (b * c) = (a * b) * c$$

$$\begin{aligned} L.H.S &= a * (b * c) \\ &= a * bc^2 \\ &= a(bc^2)^2 \\ &= ab^2c^4 \end{aligned}$$

$$\begin{aligned} R.H.S &= (a * b) * c \\ &= ab^2 * c \\ &= ab^2c^2 \end{aligned}$$

L.H.S = R.H.S

Hence $*$ is not Associative.

(8) let $*$ be the binary operation on N defined by $a * b$ = H.C.F. of a and b. ~~a binary~~ Is $*$ commutative? Is $*$ associative? Does there exist identity for this binary operation on N?

Sol. On N,

$a * b$ = H.C.F of a and b.

for commutative,

$$a * b = b * a.$$

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$\Rightarrow$ $a * b$ = H.C.F of a and b.
$\Rightarrow$ $b * a$ = H.C.F of b and a.
$\Rightarrow$ H.C.F of a and b = H.C.F of b and a.
$\Rightarrow$ $a * b = b * a$
Hence $*$ is commutative.

for Associative,
$a * (b * c) = (a * b) * c$

L.H.S = $a * (b * c)$
= $a *$ (H.C.F. of b and c)
= H.C.F of a and b and c.

R.H.S = $(a * b) * c$
= (H.C.F of a and b) $* c$
= H.C.F of a and b and c.

L.H.S = R.H.S
Hence $*$ is Associative.

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Miscellaneous Exercise.

① Let $f: R \to R$ be defined as $f(x) = 10x + 7$. find the function $g: R \to R$ such that $gof = fog = I_R$.

Sol. $f: R \to R$

$f(x) = 10x + 7$

let $y = 10x + 7$

$\Rightarrow y - 7 = 10x$

$\Rightarrow x = \frac{y-7}{10}$

$\Rightarrow \therefore g(y) = \frac{y-7}{10}$

$\Rightarrow gof(x) = g\{f(x)\}$

$= g(10x + 7)$

$= \frac{10x + 7 - 7}{10}$

$= \frac{10x}{10}$

$= x.$

$\Rightarrow gof(R) = R$

$\Rightarrow$

$\Rightarrow fog(y) = f(g(y))$

$= f\left(\frac{y-7}{10}\right)$

$= 10\left(\frac{y-7}{10}\right) + 7$

$= y$

$f \circ g(R) = R$

$\therefore g \circ f = f \circ g = I_R$

(12)

Sol.

for commutative,

$$a * b = b * a$$

$\Rightarrow$ L.H.S $= a * b$
$= |a-b|$

R.H.S $= b * a$
$= |b-a|$
$= |a-b|$
$= a * b$.

Hence $*$ is commutative.

Now,

$$a \circ b = a$$
& $b \circ a = b$

Here

$$a \circ b \neq b \circ a.$$

Hence this is not commutative.

for associate law,

$$a * (b * c) = (a * b) * c.$$

L.H.S $= a * (b * c)$
$= a * |b-c|$
$= |a - |b-c||$

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$= |a - (b-c)|$
$= |a-b+c|$

R.H.S $= (a * b) * c$
$= |a-b| * c$
$= ||a-b| - c|$
$= |a-b-c|$

L.H.S $\neq$ R.H.S

Now,
$a \circ (b \circ c) = (a \circ b) \circ c.$

L.H.S $= a \circ (b \circ c)$
$= a \circ (b)$
$= a$

R.H.S $= (a \circ b) \circ c$
$= a \circ c$
$= a$

L.H.S = R.H.S

Here $*$ is commutative but not as associative and $\circ$ is not commutative but associative.

We have to prove,

$a * (b \circ c) = (a * b) \circ (a * c)$

L.H.S $= a * (b \circ c)$
$= a * (b)$
$= |a-b|$

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$R.H.S = (a * b) \circ (a * c)$
$= |a-b| \circ |a-c|$
$= |a-b|$
$L.H.S = R.H.S$

Hence $*$ distribute over $\circ$.

Now, $a \circ (b * c) = (a \circ b) * (a \circ c)$

$L.H.S = a \circ (b * c)$
$= a \circ |b-c|$
$= a$

$R.H.S = (a \circ b) * (a \circ c)$
$= a * a$
$= |a-a|$
$= 0$

therefore $\circ$ does not over $*$.

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(14) Define a binary operation * on the set $\{0,1,2,3,4,5\}$ as

$$a * b = \begin{cases} a+b & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

Sol. $\{0,1,2,3,4,5\}$

$$a * b = \begin{cases} a+b & \text{if } a+b < 6 \\ a+b-6 & \text{if } a+b \geq 6 \end{cases}$$

*	0	1	2	3	4	5
0	0	1	2	3	4	5
1	1	2	3	4	5	0
2	2	3	4	5	0	1
3	3	4	5	0	1	2
4	4	5	0	1	2	3
5	5	0	1	2	3	4

Here $a * b$ belongs to $\{0,1,2,3,4,5\}$ therefore it is defined.