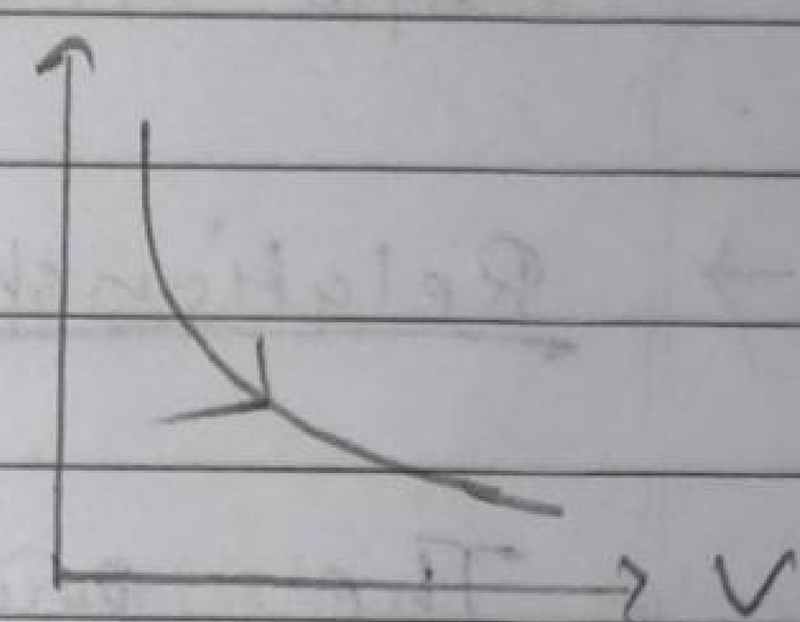
Chapter - 13

① write the boyel's law?
② write the charle's law.
③ what do you mean by ideal gas, derive the expression for it.

Ans ① AT constant temperature, pressure is inversely proportional to volume, this is boyel's law.

$$\boxed{P \propto \frac{1}{V}}$$

Graph :- i)

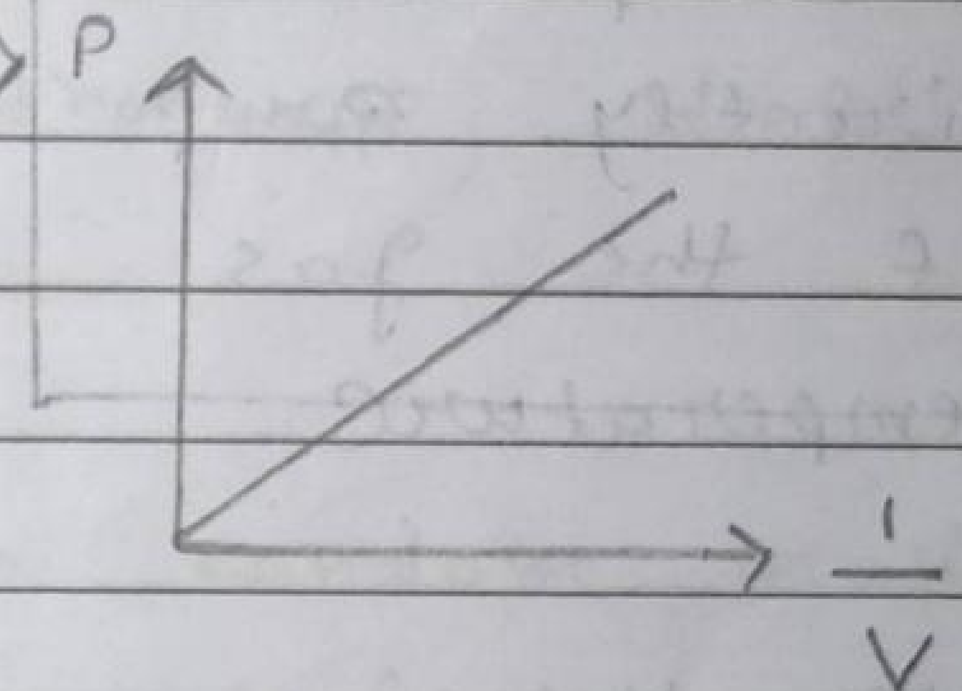


$\Rightarrow \quad P = \frac{K}{V}$

$\Rightarrow \quad PV = K$

$\Rightarrow \quad \boxed{P_1 V_1 = P_2 V_2}$

ii)

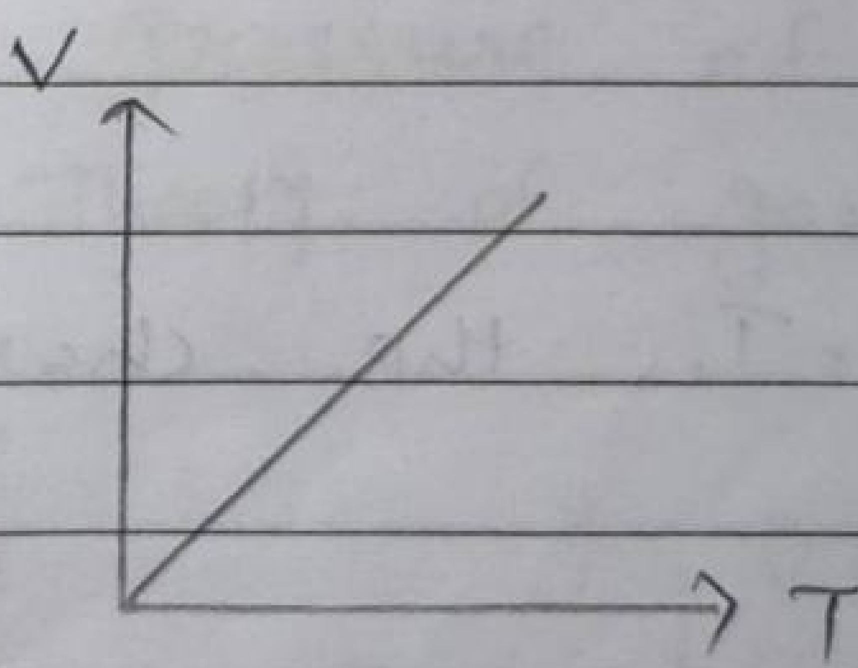


Ans. ② At constant pressure, volume is directly proportional to temperature.

$\Rightarrow \quad PV = nRT$

$\Rightarrow \quad V = \frac{nRT}{P}$

Graph:- i)



$\Rightarrow \quad \boxed{V \propto T}$

$\Rightarrow \quad V = KT$

$\Rightarrow \quad V_1 = K T_1 \longrightarrow$ ①

$V_2 = K T_2 \longrightarrow$ ②

on $\frac{eq^n \text{ (i)}}{eq^n \text{ (ii)}}$

$$\Rightarrow \frac{V_1}{V_2} = \frac{KT_1}{KT_2}$$

$$\Rightarrow \boxed{\frac{V_1}{V_2} = \frac{T_1}{T_2}} \quad \text{or} \quad \Rightarrow \boxed{\frac{V_1}{T_1} = \frac{V_2}{T_2}}$$

Ans (3) Ideal Gas :- An ideal gas is a gas with a very simple relationship b/w pressure, volume and temperature

→ Relationship :- $\boxed{PV = nRT}$

The product of pressure and volume is directly proportional to the no. of moles of the gas and the absolute ~~error~~ temperature.

We know,

A.T. the Boyel's law,

$$V \propto \frac{1}{P} \longrightarrow (1)$$

A.T. the charle's law

$$V \propto T \longrightarrow (2)$$

A.T. Avogadro law,

$$V \propto n \longrightarrow (3)$$

from (1), (2) & (3)

$$V \propto \frac{nT}{P}$$

$\Rightarrow \quad PV \propto nT$

$\Rightarrow \quad \boxed{PV = nRT}$

where, $P \rightarrow$ Pressure
$V \rightarrow$ Volume
$n \rightarrow$ no. of moles
$R \rightarrow$ Universal Gas constant.
$T \rightarrow$ temperature

IMP.

Q. Write the assumptions of Kinetic Theory of gases.

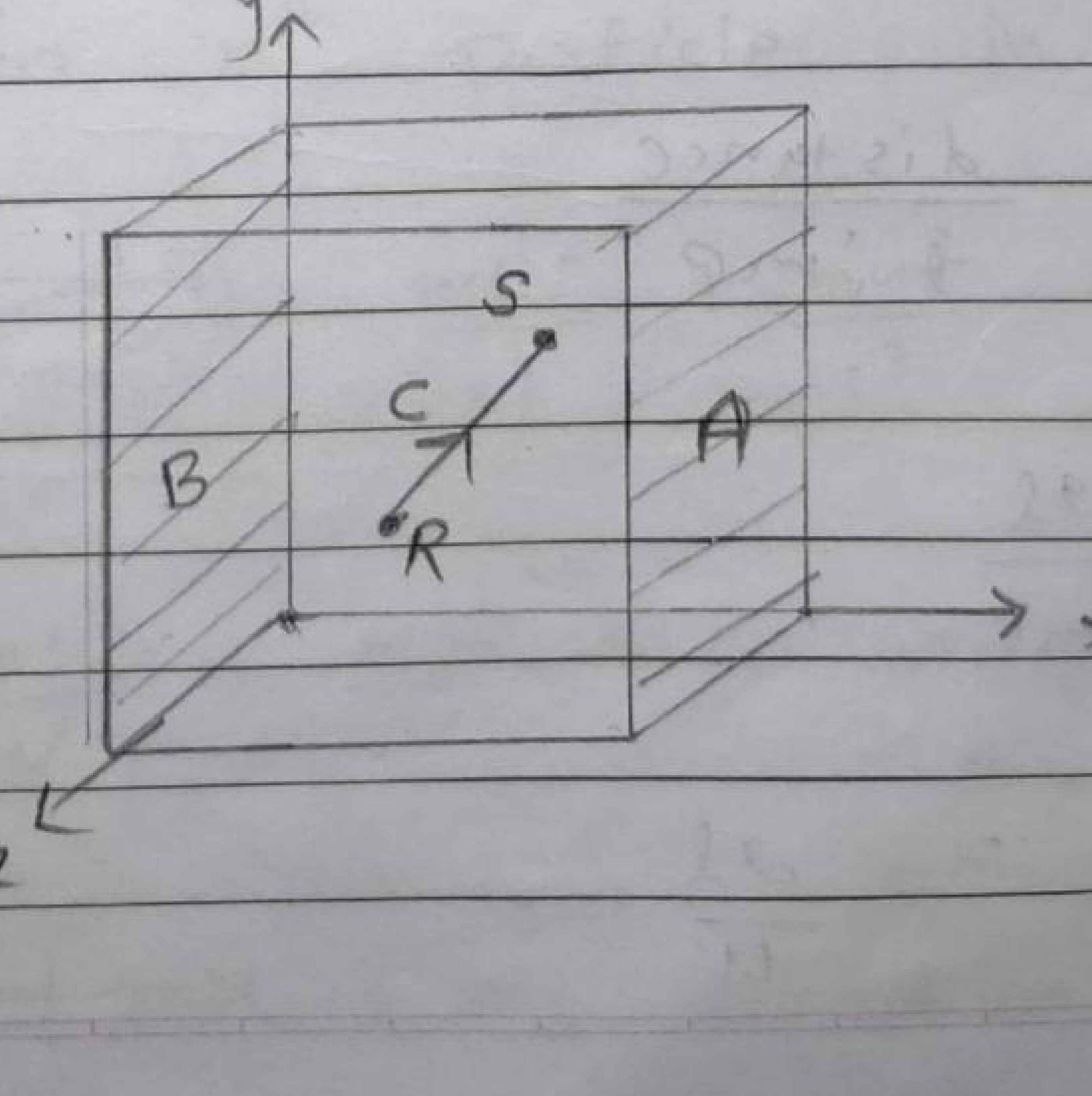
Ans.

i) Every gas is composed of minute (small) Particle is molecule.
ii) Gas Particle → sphere in shape, identical in nature.

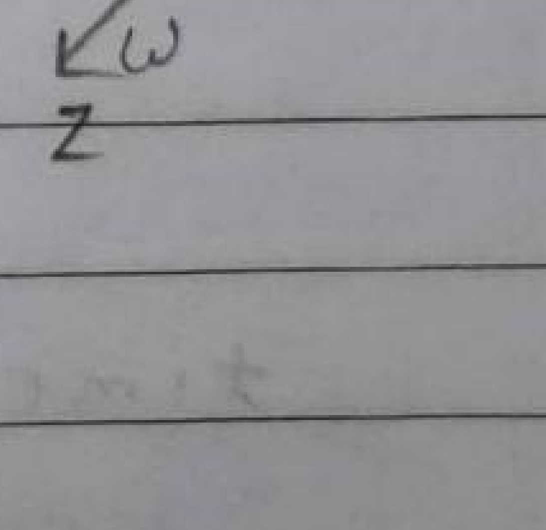
iii) Gas particle is always in Random motion.
iv) collision b/w 2 gas particle is elastic collision.

Q. Derive the expression for Pressure of a gas on the basis of Kinetic Theory of gases.

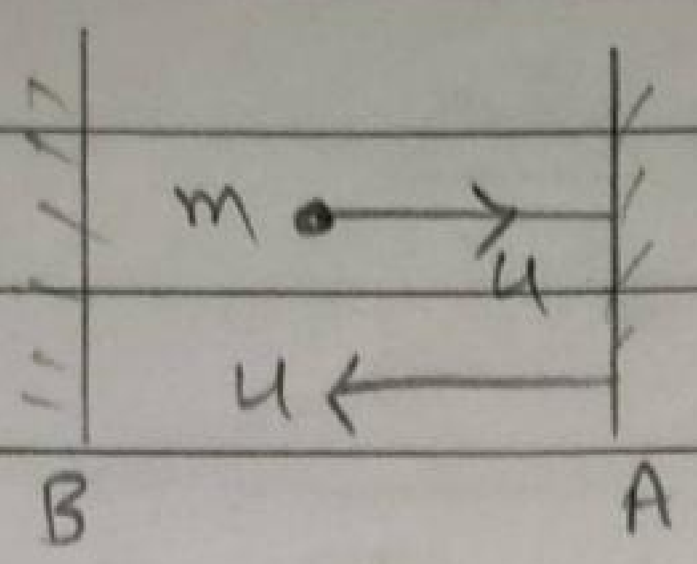
Ans. P



$\Rightarrow$



→



we consider x axis only,

$P_1 = mu$

$P_2 = m(-u)$

$P_2 = -mu$

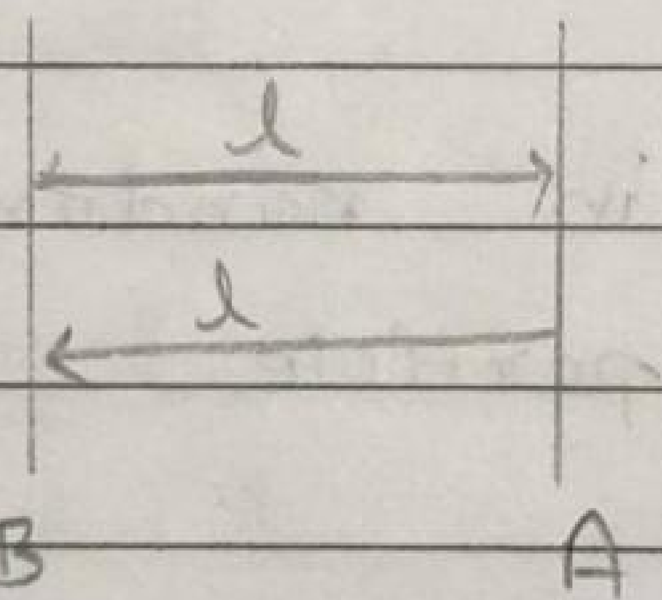
$\Delta P = P_2 - P_1$

$\Delta P = -mu - mu$

$\Delta P = -2mu$

on considering only magnitude,

$\Delta P = 2mu$



Total length $= l + l$

$= 2l$.

→ Speed $= \dfrac{\text{distance}}{\text{time}}$

→ $u = \dfrac{2l}{\text{time}}$

→ time $= \dfrac{2l}{u}$

$$\Rightarrow F = \frac{\Delta p}{\Delta t}$$

$$\Rightarrow F = \frac{2mu}{\frac{2l}{u}}$$

$$\Rightarrow F = \frac{mu^2}{l}$$

$$\Rightarrow \text{Pressure} = \frac{F}{A} = \frac{\frac{mu^2}{l}}{A} = \frac{\frac{mu^2}{l}}{l^2}$$

$$\Rightarrow \text{Pressure} = \frac{mu^2}{l \cdot l^2}$$

$$\Rightarrow \boxed{\text{Pressure} = \frac{mu^2}{l^3}}$$ → This is the pressure for 1 particle in x-direction.

Pressure for n^{th} particle in x-direction.

$$P_x = \cancel{\frac{m}{V}} \left[\frac{mu_1^2}{l^3} + \frac{mu_2^2}{l^3} + \cdots \frac{u_n^2}{l^3} \right]$$

$$P_x = \frac{mu_1^2}{V} + \frac{mu_2^2}{V} + \cdots + \frac{u_n^2}{V}$$

$$P_x = \frac{m}{V} \left[u_1^2 + u_2^2 + \cdots + u_n^2 \right]$$

Pressure for n particle in y - direction :-

$$P_y = \frac{m}{V}\left[v_1^2 + v_2^2 + \dots v_n^2\right]$$

Pressur for n particle in z- direction :-

$$P_z = \frac{m}{V}\left[\omega_1^2 + \omega_2^2 + \dots + \omega_n^2\right]$$

$$\Rightarrow \quad P_x = P_y = P_z = P$$

$$\Rightarrow \quad P_x + P_y + P_z = P + P + P$$

$$\Rightarrow \quad P_x + P_y + P_z = 3P$$

$$\Rightarrow \quad 3P = \left\{\frac{m}{V}\left(u_1^2 + u_2^2 + \dots + u_n^2\right) + \frac{m}{V}\left(v_1^2 + v_2^2 + \dots v_n^2\right) + \frac{m}{V}\left(\omega_1^2 + \omega_2^2 + \dots + \omega_n^2\right)\right\}$$

$$\Rightarrow \quad P = \frac{1}{3}\left[\frac{m}{V}\left\{\left(u_1^2 + v_1^2 + \omega_1^2\right) + \left(u_2^2 + v_2^2 + \omega_2^2\right) + \dots + \left(u_n^2 + v_n^2 + \omega_n^2\right)\right\}\right]$$

$$\Rightarrow \quad P = \frac{m}{3V}\left\{c_1^2 + c_2^2 + \dots c_n^2\right]$$

$$\because \quad \bar{c}^2 = \frac{c_1^2 + c_2^2 + \dots + c_n^2}{n}$$

here, $\bar{c}$ is the mean square root velocity

$$\Rightarrow \quad \bar{c}^2 \cdot n = c_1^2 + c_2^2 + \dots + c_n^2$$

$$\Rightarrow \quad P = \frac{m N \bar{c}^2}{3V}$$

$$\boxed{P = \frac{mN\bar{c}^2}{3V}}$$

where, Avogadro no. — N

$P \to$ pressure

$n \to$ no. of Particle ~~molecule~~

$\bar{c}^2 \to$ mean square root velocity.

$V \to$ Volume

Kinetic Interpretation of tem. :-

We know,

$$P = \frac{mN\bar{c}^2}{3V}$$

Ideal gas eq^n,

$$PV = nRT$$

$$PV = RT \quad \text{(for 1 mole)} \to (1)$$

$$\Rightarrow P = \frac{mN\bar{c}^2}{3V}$$

$$\Rightarrow PV = \frac{mN\bar{c}^2}{3} \longrightarrow (2)$$

from (1) & (2),

$$\Rightarrow \frac{mN\bar{c}^2}{3} = RT$$

$$\Rightarrow \bar{c} = \sqrt{\frac{3RT}{mN}}$$

let $mn = M$,

$$\Rightarrow \bar{c} = \sqrt{\frac{3RT}{M}} \longrightarrow \left(\text{here } \frac{3T}{M} \text{ is constant}\right)$$

$$\Rightarrow \boxed{\bar{c} \propto \sqrt{T}}$$

where, T is absolute tem.,

$\bar{c} \to$ root mean square velocity

for this Kinetic Energy —

$$\Rightarrow \frac{mN\bar{c}^2}{3V} = RT$$

$$\Rightarrow \frac{2}{2} \cdot \frac{mN\bar{c}^2}{3V}$$

$$\Rightarrow \frac{1}{2} m\bar{c}^2 = \frac{3}{2} \frac{RT}{N}$$

$$\Rightarrow \frac{1}{2} m\bar{c}^2 = \frac{3}{2} \frac{R}{N} \cdot T$$

$$\Rightarrow \text{Energy } (E) = \frac{3}{2} \cdot \frac{R}{N} \cdot T$$

$$\Rightarrow \boxed{\text{Energy } (E) = \frac{3}{2} KT} \qquad \left[\because \frac{R}{N} = K\right]$$

here, n → Avogadro no.

Boltzmann constant

What do you mean by degree of freedon?

Ans. The degree of freedom of a system are defined as the total no. of co-ordinate or independent quantities required to describe the configuration of the system completly.

~~form~~ formula for degree of freedom = 3N − K

Calculation of Degree of Freedom :-

① for Monoatomic gas :-

$$f = 3N - K$$

here, $K = 0$, $N = 1$.

$$f = 3(1) - 0$$

$$\boxed{f = 3}$$

② for Diatomic gas molecule :-

$$f = 3N - K$$

here, $K = 1$, $N = 2$

$$f = 3(2) - 1$$

$$\boxed{f = 5}$$

Law of Equipartition of Energy :-

It states that for a dynamical system in thermal Equilibrium the energy of the system is equally distributed among the various degree of freedom and the energy associated with each degree of freedom per molecule is $\frac{1}{2} KT$, where 'K' is the boltzmann constant.

Q. Derive the expression for specific heat of monoatomic gases

Ans. K.E. for per molecule,

$$= 3 \times \frac{1}{2} KT$$

$$= \frac{3}{2} KT$$

K.E for per mole,

$$\Rightarrow E = N \cdot \frac{3}{2} KT$$

$$\Rightarrow E = \frac{3}{2} NKT.$$

$$\Rightarrow E = \frac{3}{2} RT$$

[Since, $K = \frac{R}{N}$

$\Rightarrow R = KN$]

for 1 mole gas at constant volume,

Note:—

$C_V \rightarrow$ specific heat capacity at constant Volume

$C_P \rightarrow$ specific heat capacity at constant Pressure

$$\Rightarrow \Delta Q = C_V \Delta T$$

$$\Rightarrow \Delta E = C_V \Delta T$$

$$\Rightarrow C_V = \frac{\Delta E}{\Delta T}$$

$$\Rightarrow \boxed{C_V = \frac{dE}{dT}}$$

$$\Rightarrow C_V = \frac{d}{dT}\left(\frac{3}{2} RT\right)$$

$$\Rightarrow C_V = \frac{3}{2} R \frac{d(T)}{dT}$$

$$\Rightarrow C_V = \frac{3}{2} R$$

Mayer's formula,

$$\Rightarrow C_p - C_v = R$$

$$\Rightarrow C_p - \frac{3}{2}R = R$$

$$\Rightarrow \boxed{C_p = \frac{5}{2}R}$$

and $\gamma = \frac{C_p}{C_v}$

$$= \frac{\frac{5}{2}R}{\frac{3}{2}R}$$

$$= \frac{5}{3}$$

$$\boxed{\gamma = 1.67}$$

Q. what do you mean by "mean free Path"?

Ans. The distance travelled by the molecule b/w two sussesive collision is called free Path.

The average (mean) of the free Path of a molecule is called mean free Path. It is denoted by "λ".

$$\lambda = \frac{1}{\sqrt{2}\, n \pi d^2}$$

where, $n \rightarrow$ molecular density

$d \rightarrow$ diameter of molecule

$\lambda \rightarrow$ mean free Path.