

## Chapter -

### CIRCULAR MOTION

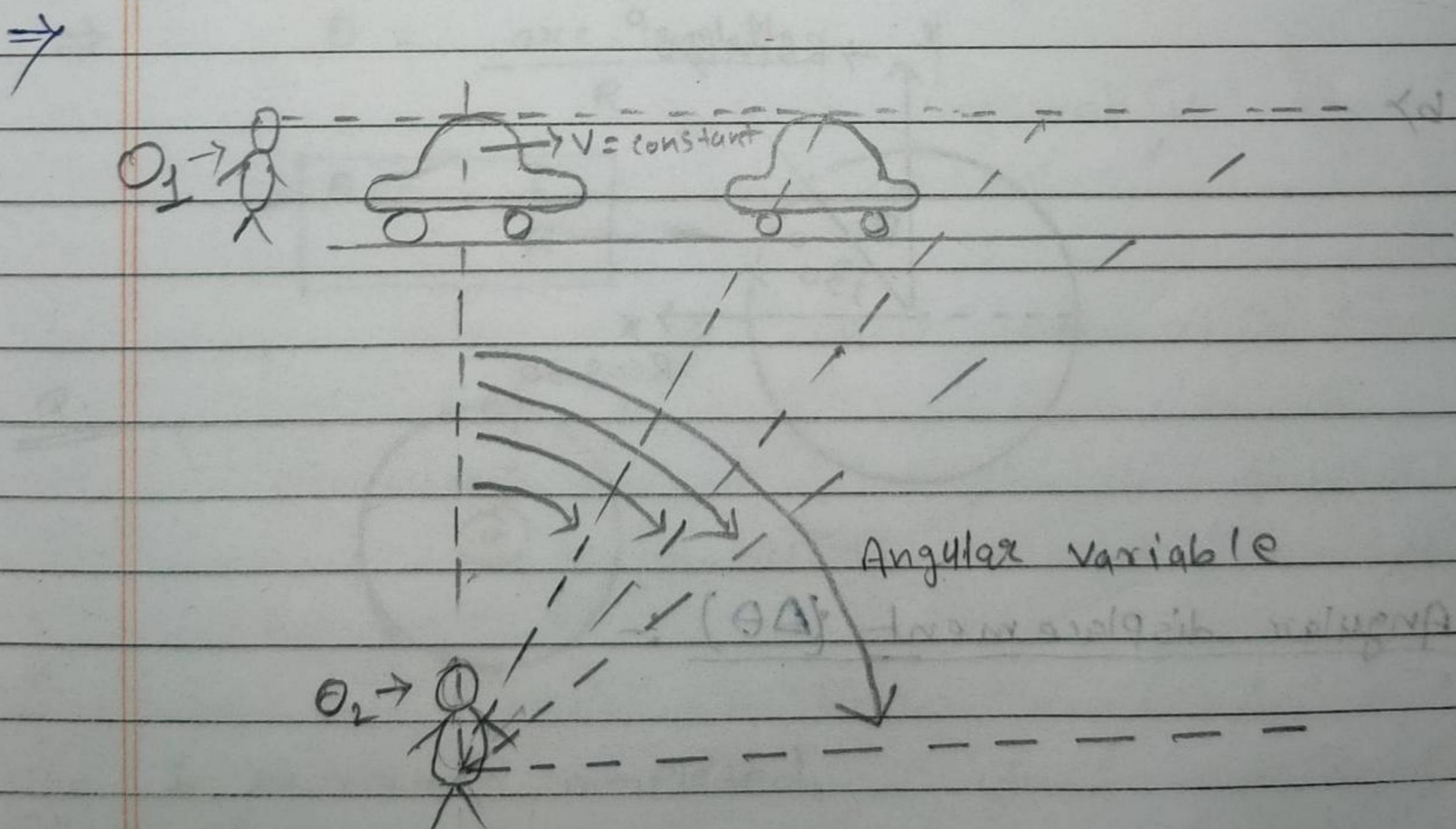
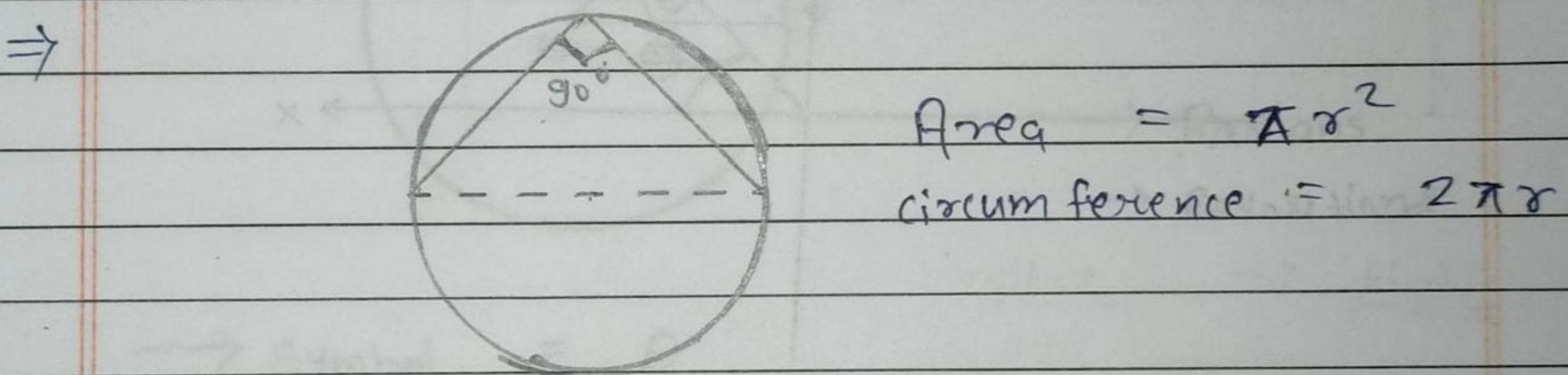
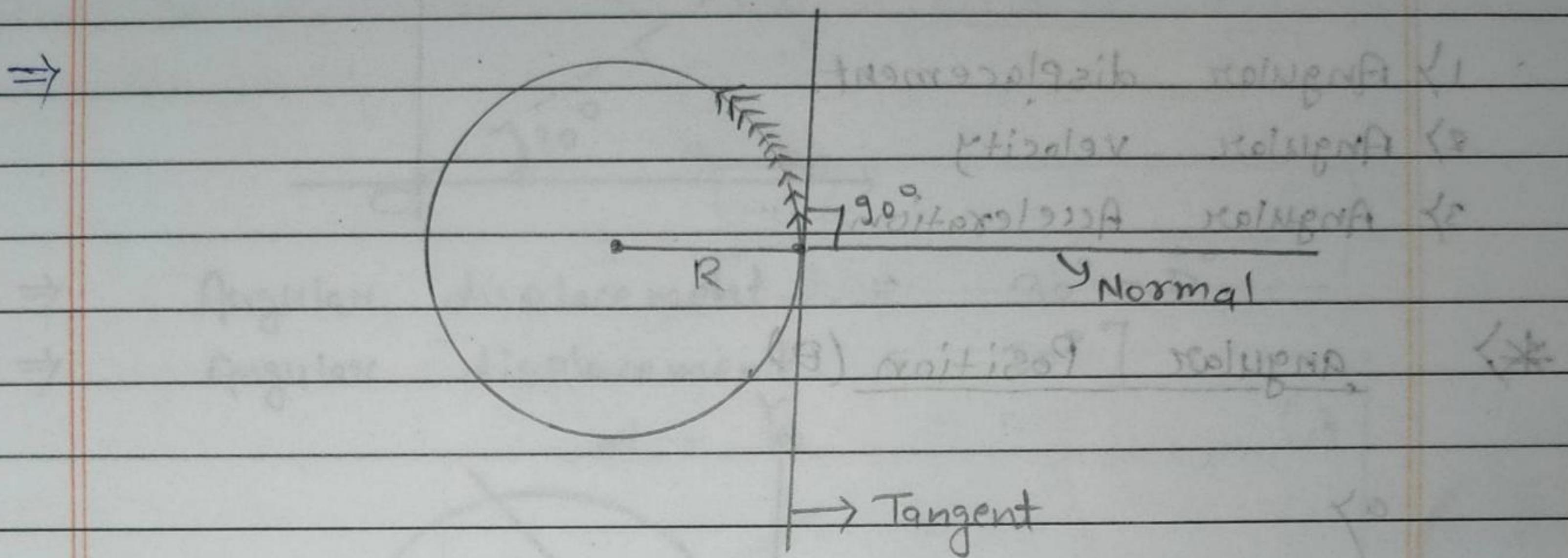


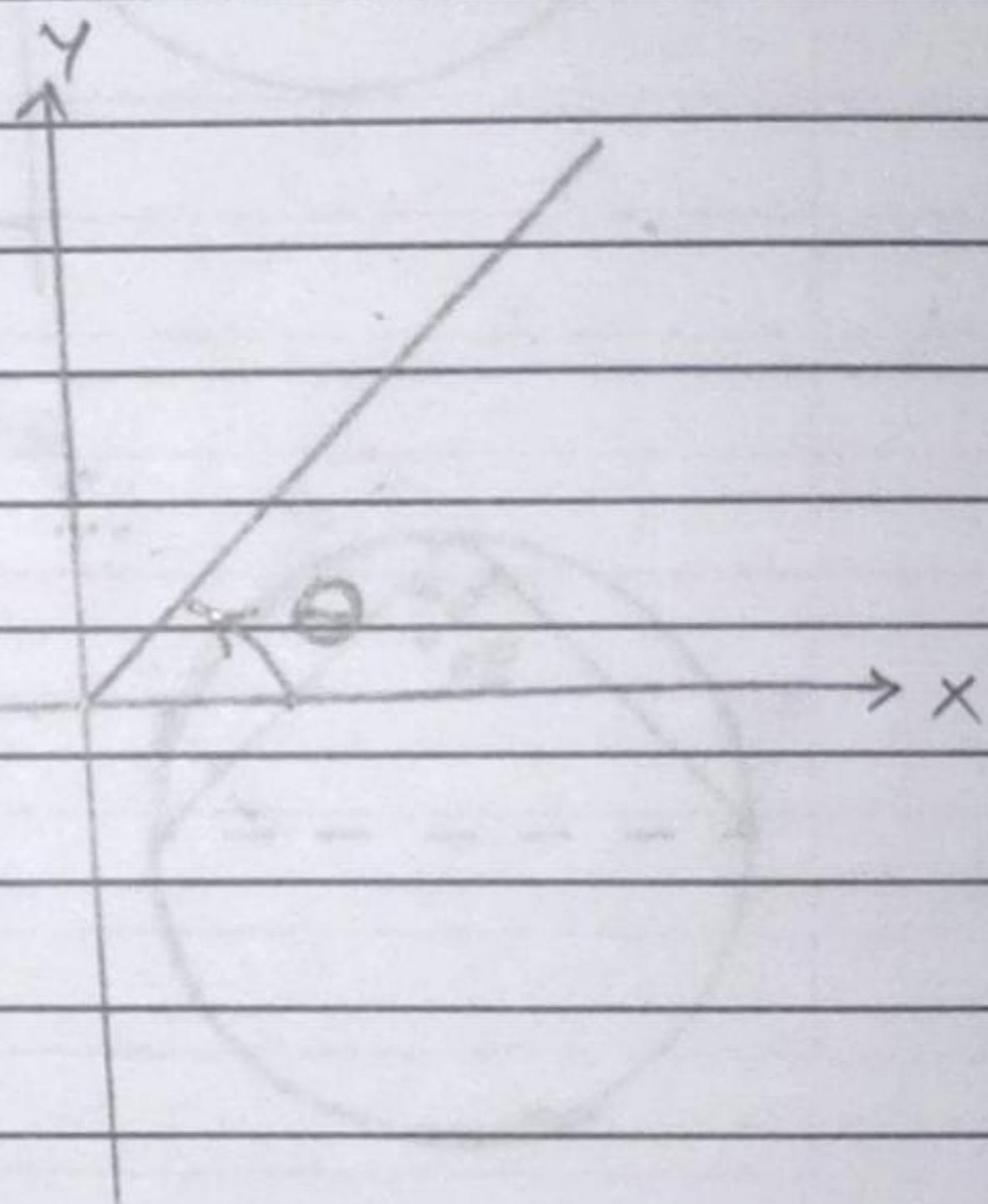
Fig. 1

## Angular Variable :-

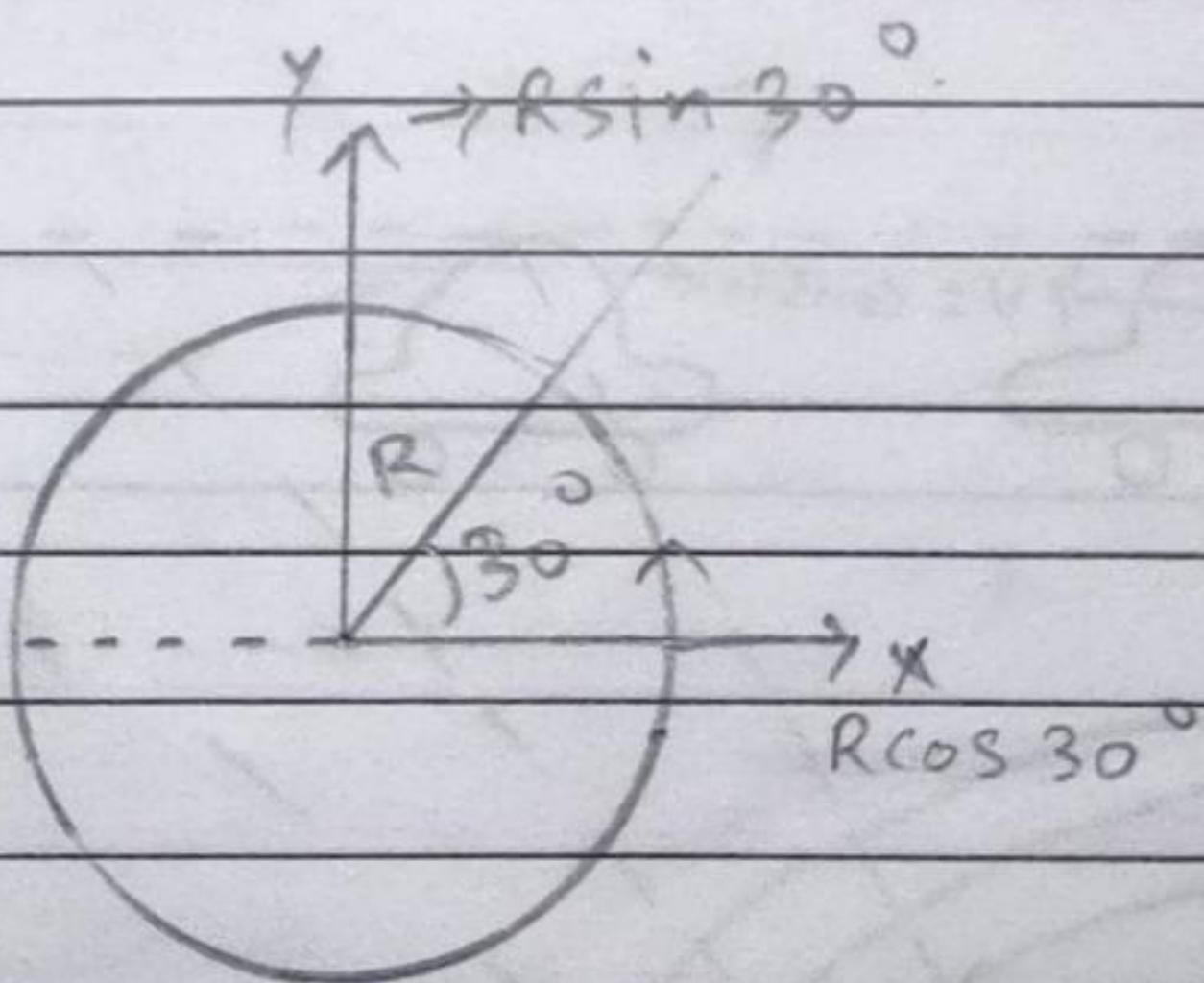
- 1) Angular displacement
- 2) Angular velocity
- 3) Angular Acceleration

\* angular Position ( $\theta$ )

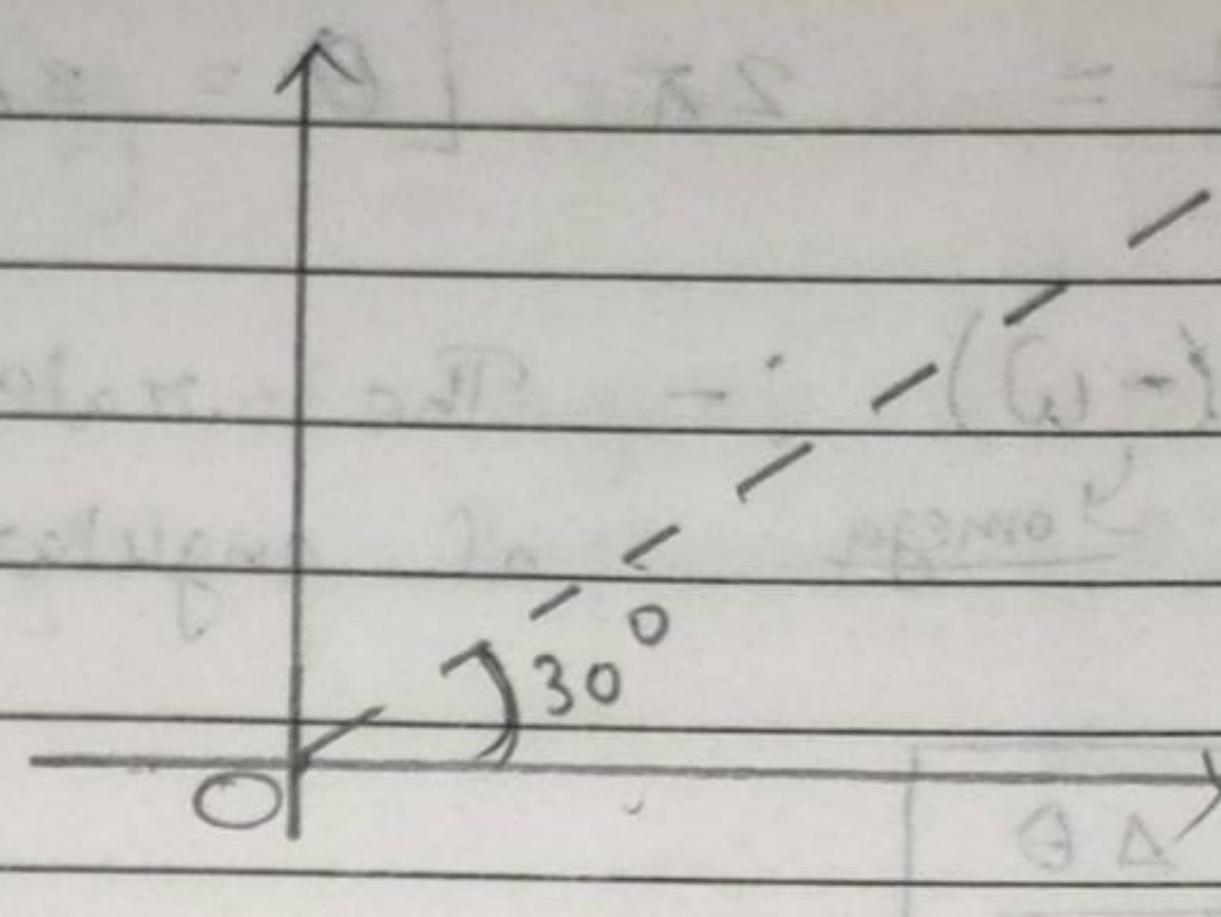
a)



b)

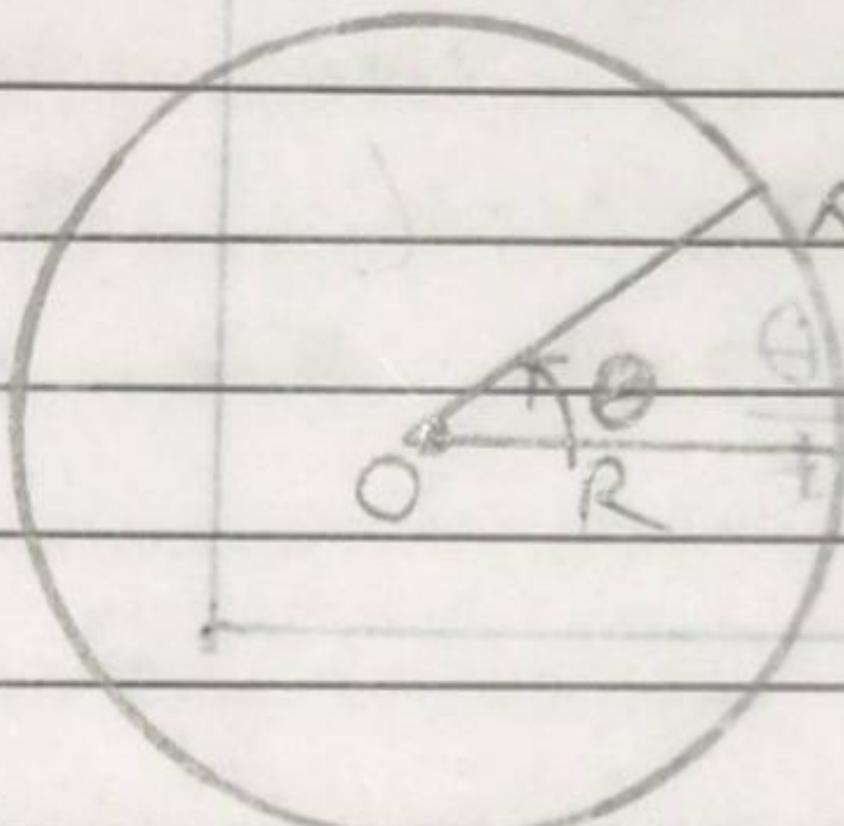


1) Angular displacement ( $\Delta\theta$ ) :-



$\Rightarrow$  Angular displacement  $= 30^\circ - 0^\circ$

$\Rightarrow$  Angular displacement  $= 30^\circ$



arc length  $= l$ .

monoplane (u)

$\rightarrow$  Radians

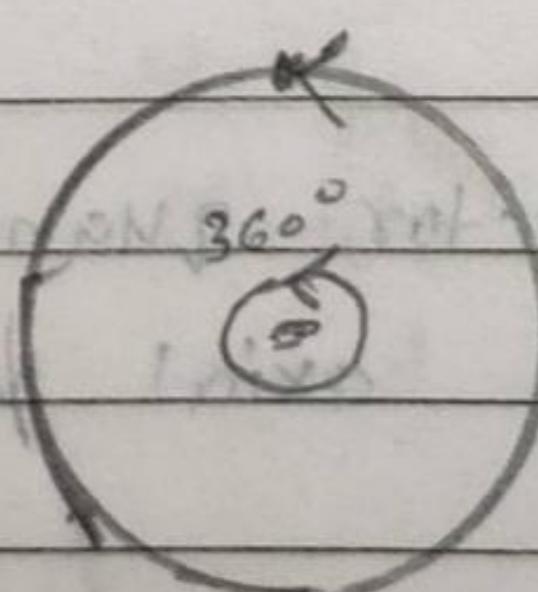
$\rightarrow$  Revolutions

$\rightarrow$  Symbol  $= \theta$

$$\Rightarrow \theta = \frac{\text{arc length}}{R}$$

$$\boxed{\theta = \frac{l}{R}}$$

(opposite G)



1 revolution completed, if ant. goes back to its

initial pos. in 1 sec. and starts moving in a straight

$$\text{Angular displacement} = 2\pi [\theta = 2\pi]$$

2) Angular Velocity ( $\omega$ ) :- The rate of change of angular displacement.

$$\omega_{\text{average}} = \frac{\Delta\theta}{\Delta t}$$

$$\omega_{\text{instantaneous}} = \lim_{\Delta t \rightarrow 0} \frac{\Delta\theta}{\Delta t}$$

$$\omega_{\text{instantaneous}} = \frac{d\theta}{dt}$$

Unit :- radian

sec

$\theta = \text{radian}^2$

= radian/sec.

$\rightarrow \omega$  (omega)  
 vector quantity

$\rightarrow \theta$  = axial vector

\* Angular velocity is a vector quantity.  
 \* This is because  $\theta$  is an axial vector.

\* For direction curve the fingers of your right hand along the direction of motion of a particle in a circular path then the thumb

will going to give you direction of this vector.

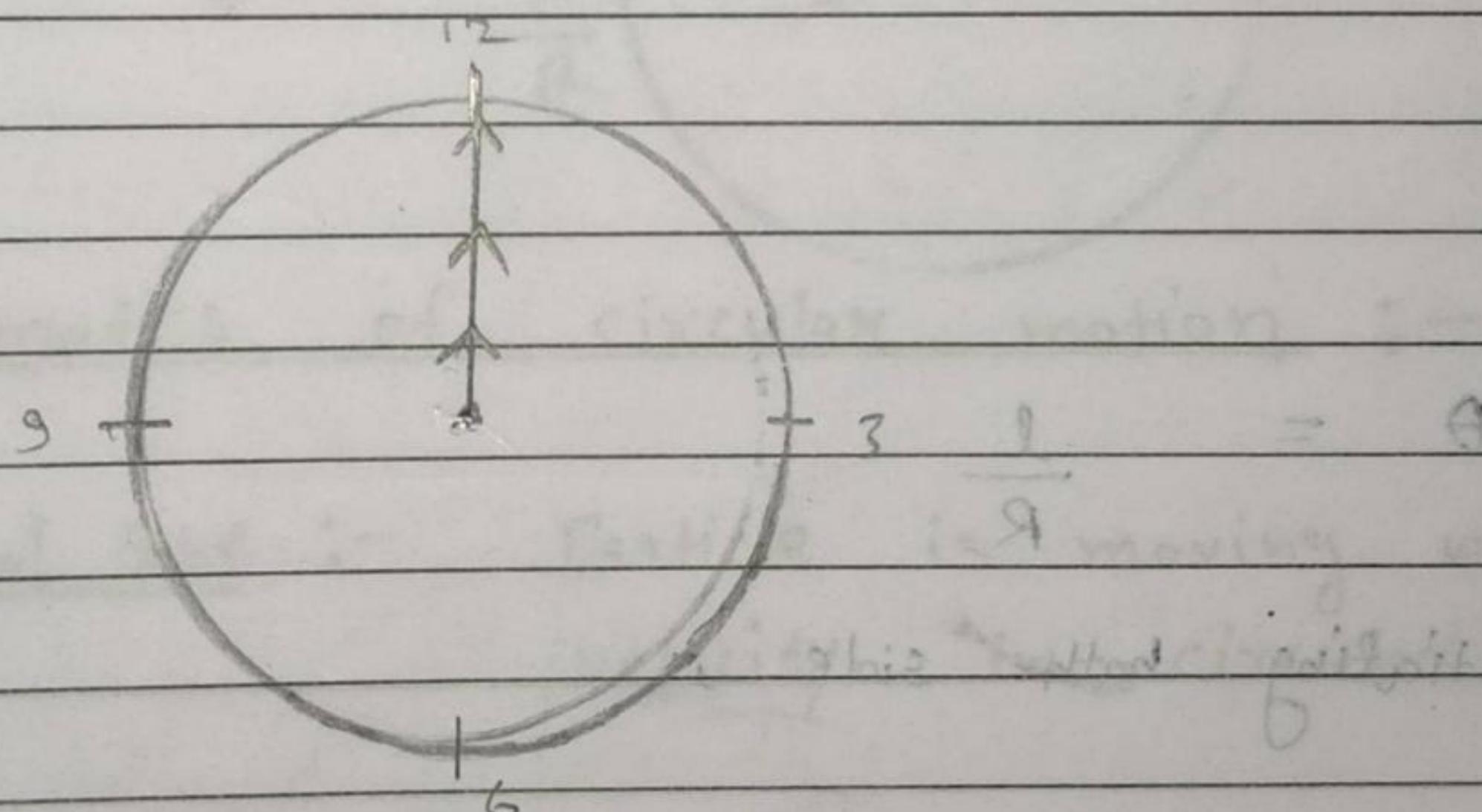
\* Direction of angular velocity ( $\omega$ ) along the direction of angular displacement ( $\theta$ ).

Q. From Fig 1., the rate of change of angular displacement vector linked with observer (2) -

- a) remains same
- b) increases with time
- c) decreases with time
- d) None

Ans.  $\Rightarrow$  option C.

Q.



Find Hour, Minute and second hand revolution.

$$\text{For Hour hand} = 2\pi$$

$$\text{Minute hand} = 12 \times 2\pi = 24\pi$$

$$\text{second hand} = 60 \times 12 \times 2\pi = 1440\pi$$

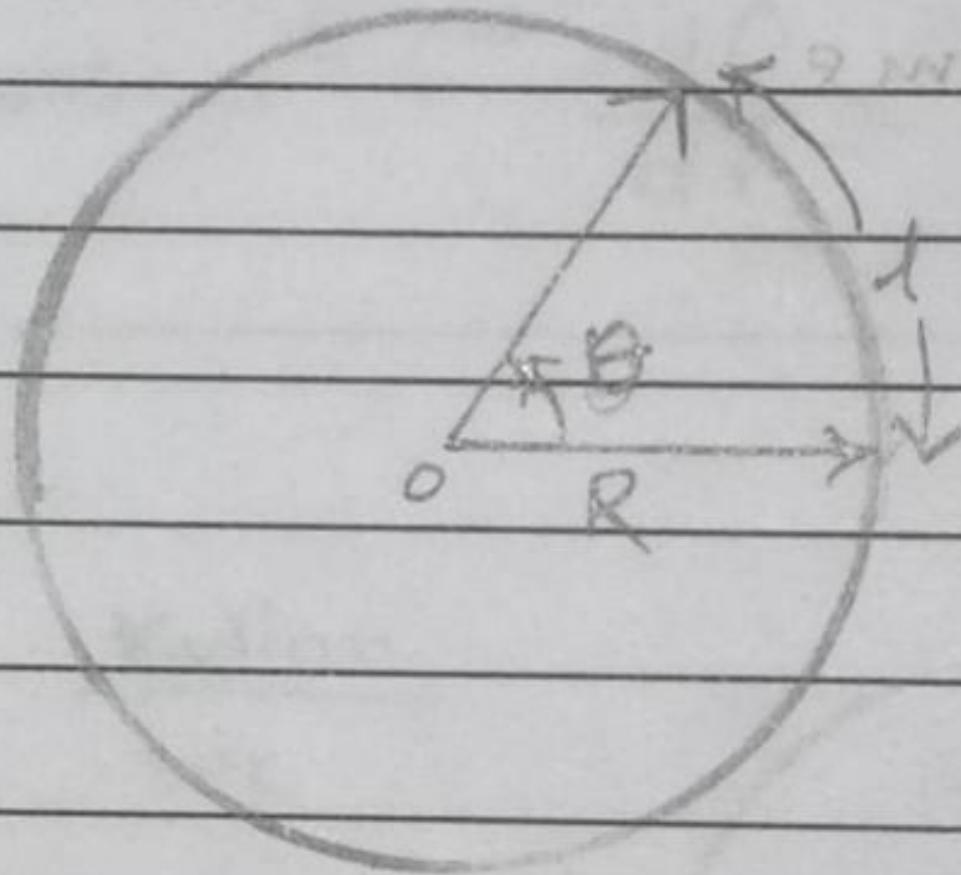
3) Angular Acceleration :- The rate of change of angular displacement.  
 $\{\alpha\}$

$$\alpha = \frac{d\omega}{dt}$$

[Unit is rad./sec<sup>2</sup>]

\* Angular acceleration is vector quantity.

# Relationship between linear variable and angular variable :-



$$\theta = \frac{l}{R}$$

Differentiating both sides,

$$\Rightarrow \frac{d\theta}{dt} = \frac{d}{dt} \left( \frac{l}{R} \right)$$

$$\Rightarrow \frac{d\theta}{dt} = \frac{1}{R} \left( \frac{dl}{dt} \right)$$

$$\Rightarrow \omega = \frac{1}{R} v$$

$$\Rightarrow \boxed{\omega = \frac{v}{R}} \quad \text{or} \quad \boxed{\vec{v} = R\vec{\omega}}$$

# Relationship between angular variable

and  $\alpha = \frac{d\omega}{dt}$

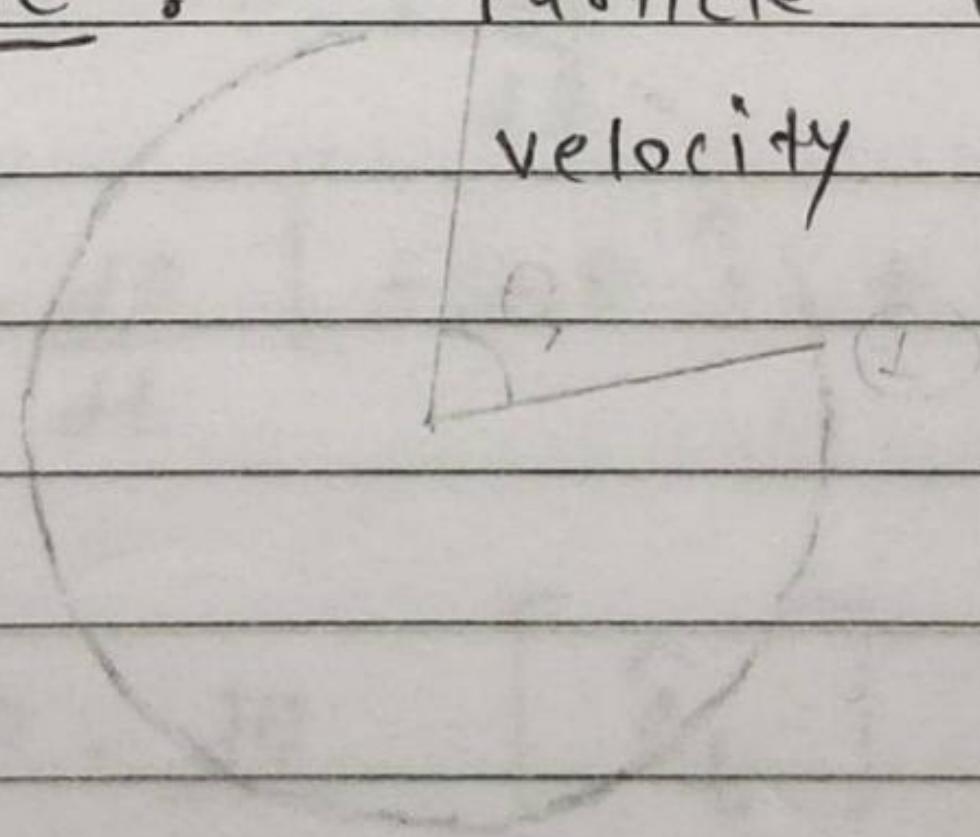
$$\Rightarrow \alpha = \frac{d}{dt} \times \frac{v}{R}$$

$$\Rightarrow \alpha = \frac{dv}{dt} \times \frac{1}{R}$$

$$\Rightarrow \alpha = \frac{a}{R}$$

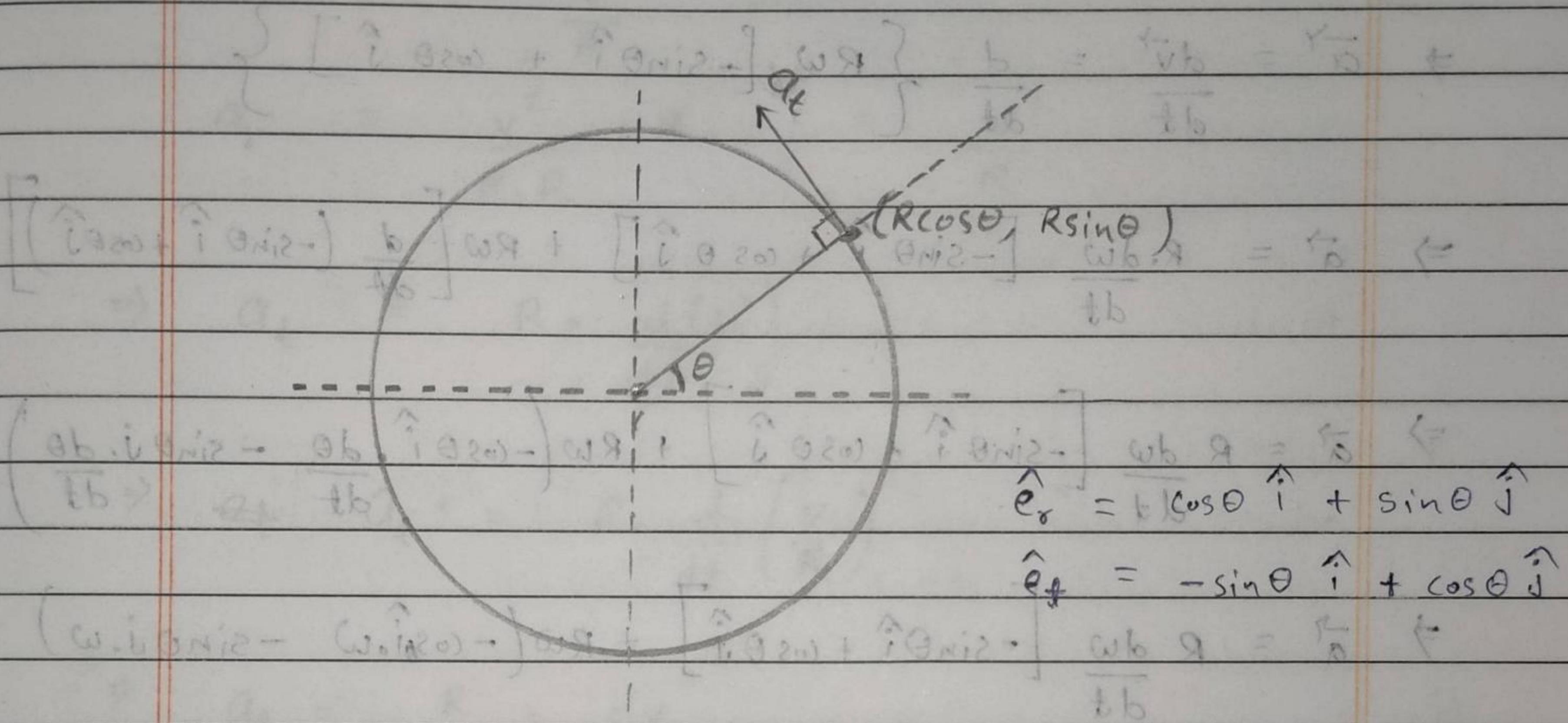
# Kinematics of circular motion :-

General Case :- Particle is moving with variable linear velocity in circular Path.



Magnitude and direction

both varying :-



$$\vec{r} = R \cos\theta \hat{i} + R \sin\theta \hat{j}$$

$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt} [R \cos\theta \hat{i} + R \sin\theta \hat{j}]$$

$$\vec{v} = R \cdot (-\sin\theta) \cdot \frac{d\theta}{dt} \hat{i} + R \cos\theta \cdot \frac{d\theta}{dt} \hat{j}$$

$$\vec{v} = R \cdot \frac{d\theta}{dt} [-\sin\theta \hat{i} + \cos\theta \hat{j}]$$

$$\vec{v} = R \cdot \frac{d\theta}{dt} [\hat{e}_t]$$

$$[v = R\omega] \hat{e}_t \quad [\omega = \text{omega}]$$

$$\Rightarrow \vec{v} = RW \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right]$$

$$\Rightarrow \vec{a}^T = \frac{d\vec{v}}{dt} = \frac{d}{dt} \left\{ RW \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right] \right\}$$

$$\Rightarrow \vec{a} = R \frac{d\omega}{dt} \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right] + RW \left[ \frac{d}{dt} \left( -\sin\theta \hat{i} + \cos\theta \hat{j} \right) \right]$$

$$\Rightarrow \vec{a} = R \frac{d\omega}{dt} \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right] + RW \left( -\cos\theta \hat{i} \cdot \frac{d\theta}{dt} - \sin\theta \hat{j} \cdot \frac{d\theta}{dt} \right)$$

$$\Rightarrow \vec{a} = R \frac{d\omega}{dt} \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right] + RW \left( -\cos\theta \omega - \sin\theta j \cdot \omega \right)$$

$$\Rightarrow \vec{a} = R \frac{d\omega}{dt} \left[ -\sin\theta \hat{i} + \cos\theta \hat{j} \right] - RW^2 \left( \cos\theta \hat{i} + \sin\theta \hat{j} \right)$$

$$\Rightarrow \vec{a} = R \frac{d\omega}{dt} \hat{e}_t - \omega^2 R \hat{e}_r$$

$\hat{e}_t$  = tangential acceleration  
 $\alpha$  = angular acceleration,

$$\Rightarrow \hat{e}_t \cdot a_t = R \alpha$$

$$\Rightarrow \alpha = \frac{a_t}{R}$$

$$a_r = \text{radial acceleration} = \omega^2 R$$

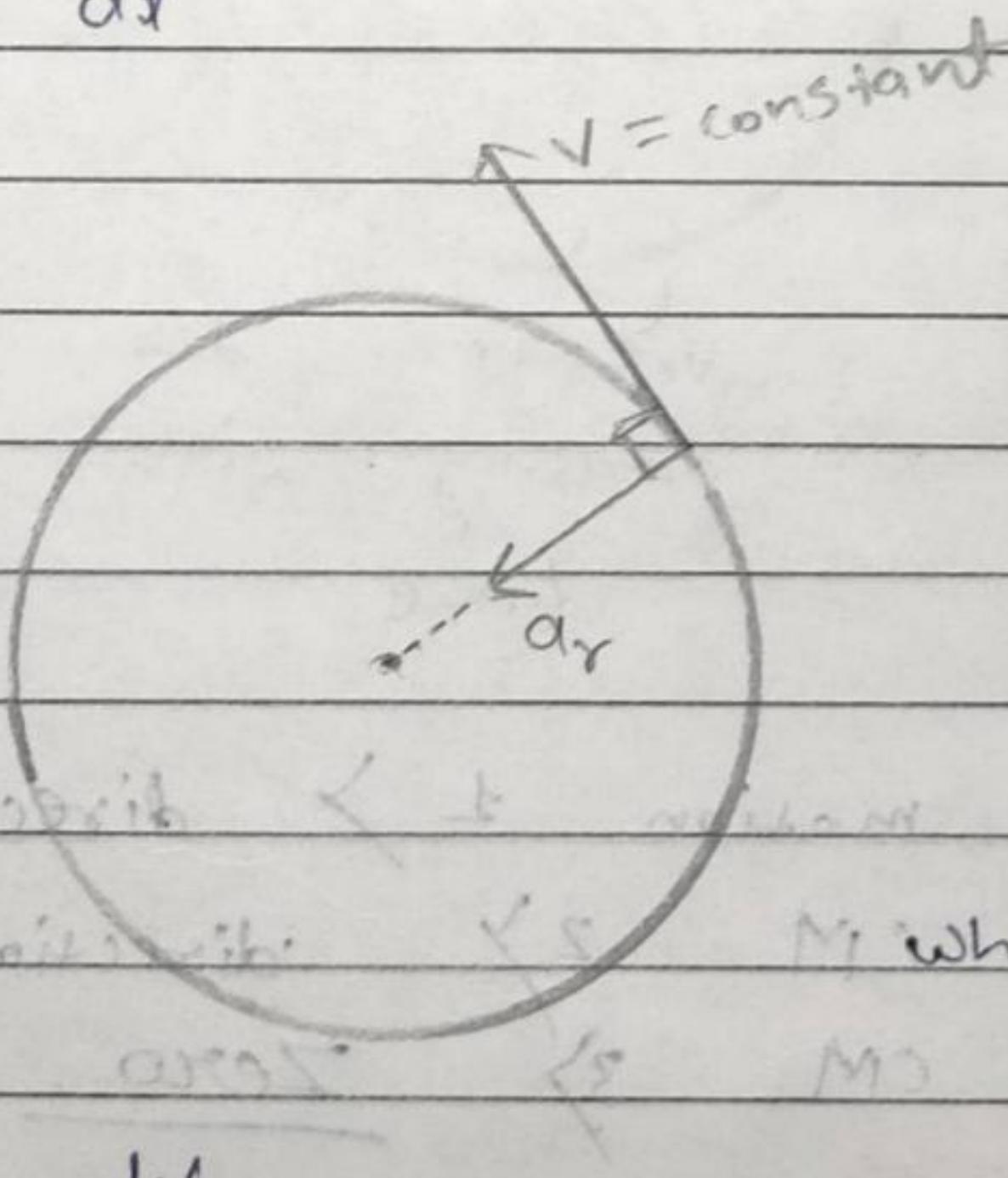
$$a_r = \frac{v^2}{R} \times R \Rightarrow \frac{v^2}{R}$$

$$\Rightarrow a_t = R \cdot \frac{d(\omega)}{dt}$$

$$\Rightarrow a_t = R \cdot \frac{d}{dt} \left( \frac{v}{R} \right)$$

$$\Rightarrow a_t = \frac{R}{R} \cdot \frac{dv}{dt}$$

$$\Rightarrow a_t = \frac{dv}{dt}$$



$$\Rightarrow a_t = \frac{dv}{dt}$$

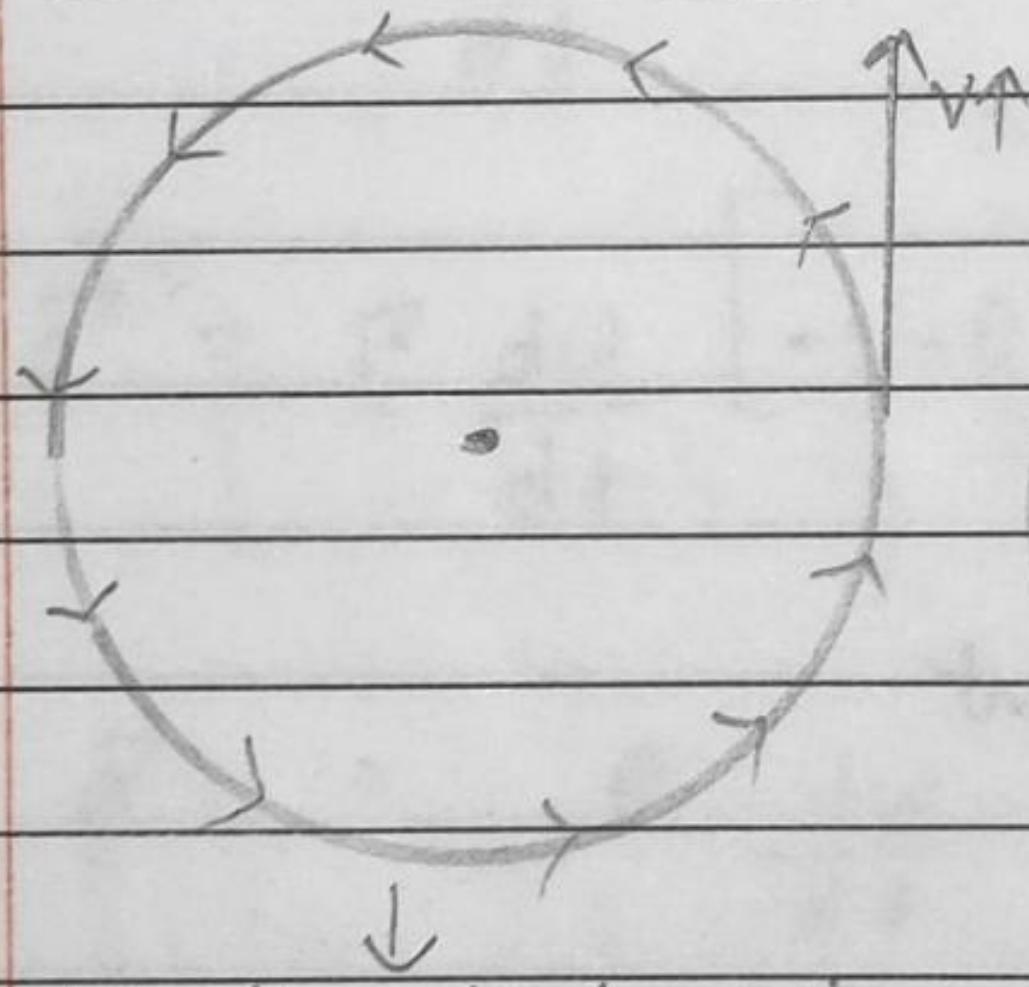
$$\Rightarrow \boxed{\frac{dv}{dt} = 0}$$

\* In circular motion  $\&$  acceleration are present, one is along radial direction and other is along tangential direction. Radial direction acceleration is also known as centripetal acceleration. Centripetal or Radial acceleration help us to change the direction of a particle in circular motion.

\* Another acceleration is tangential acceleration whose direction is along the  $\frac{\text{direction of velocity}}$  or opposite to direction of velocity.

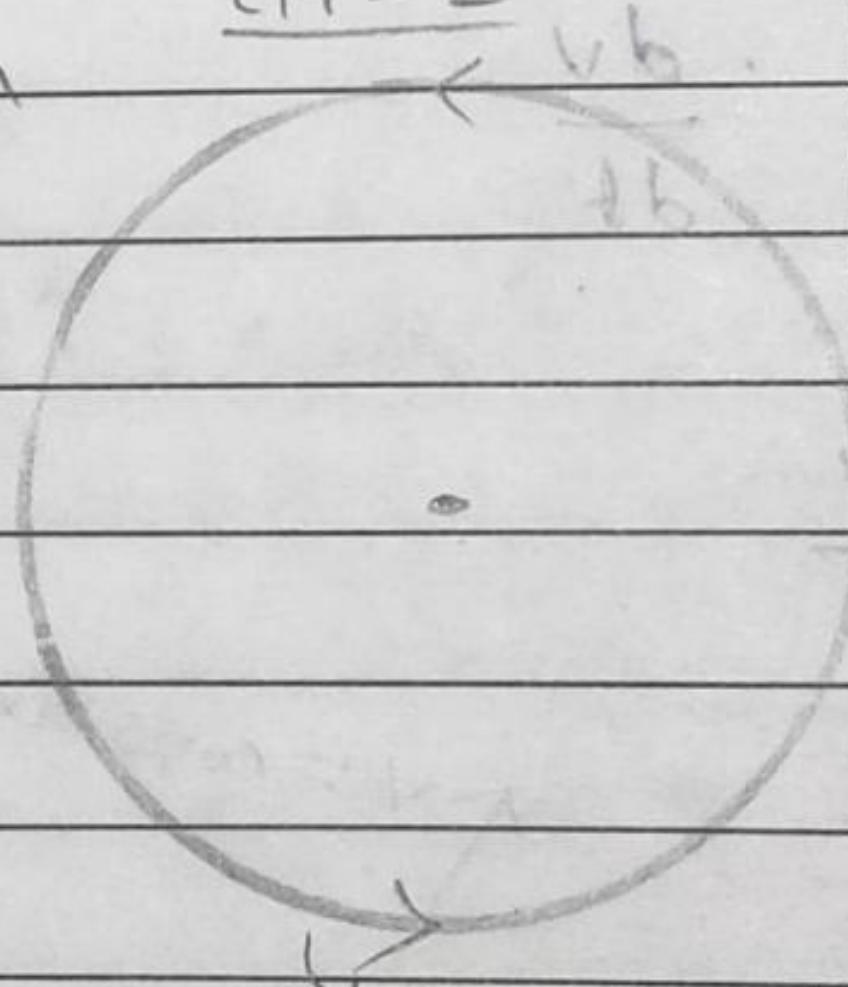
circular motion = 1

Q.



velocity is increasing.  
with time.

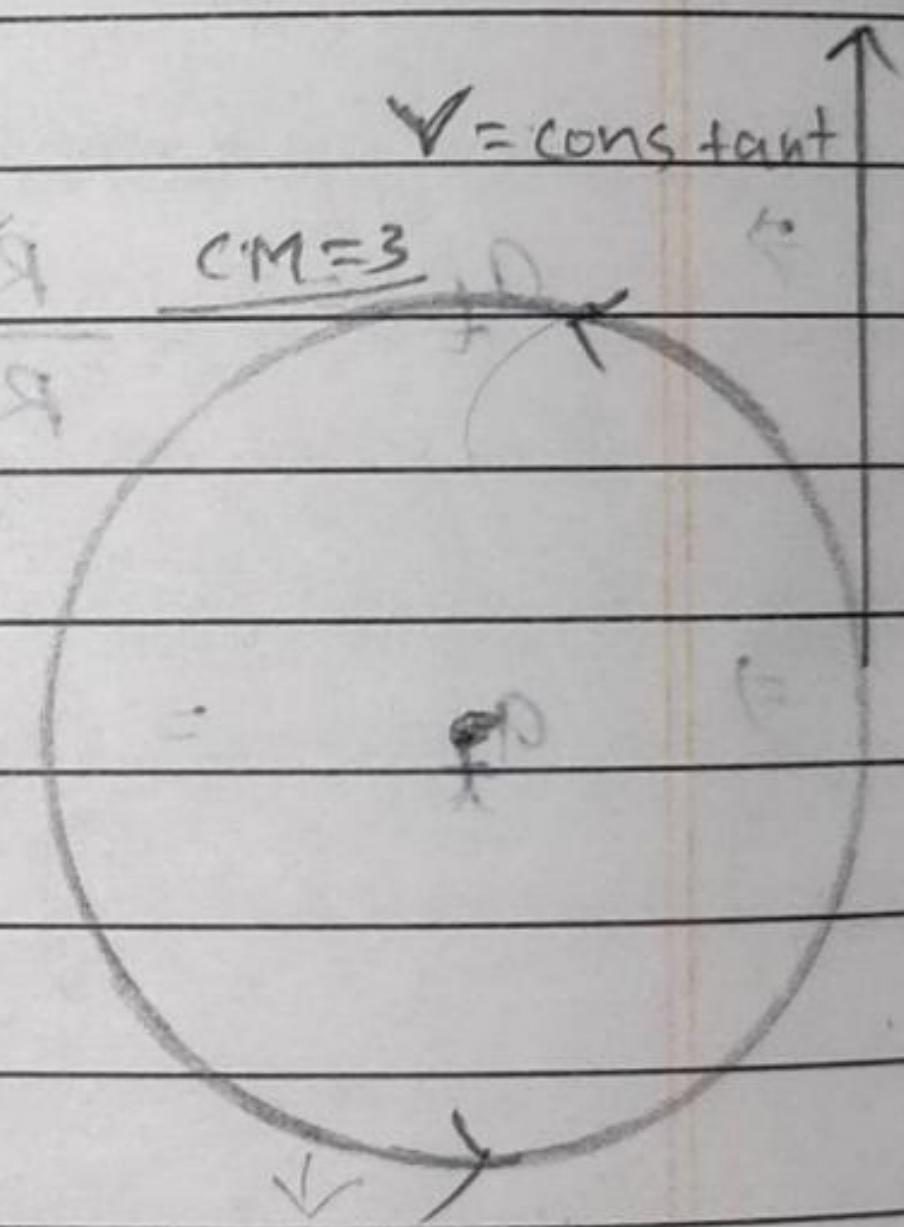
CM = 2



velocity is de-  
creasing with  
time

$\checkmark = \text{constant}$

CM = 3



Ans.

In circular motion  $1 \rightarrow$  direction is positive.

$2 \rightarrow$  direction is negative  
 $3 \rightarrow$  zero.

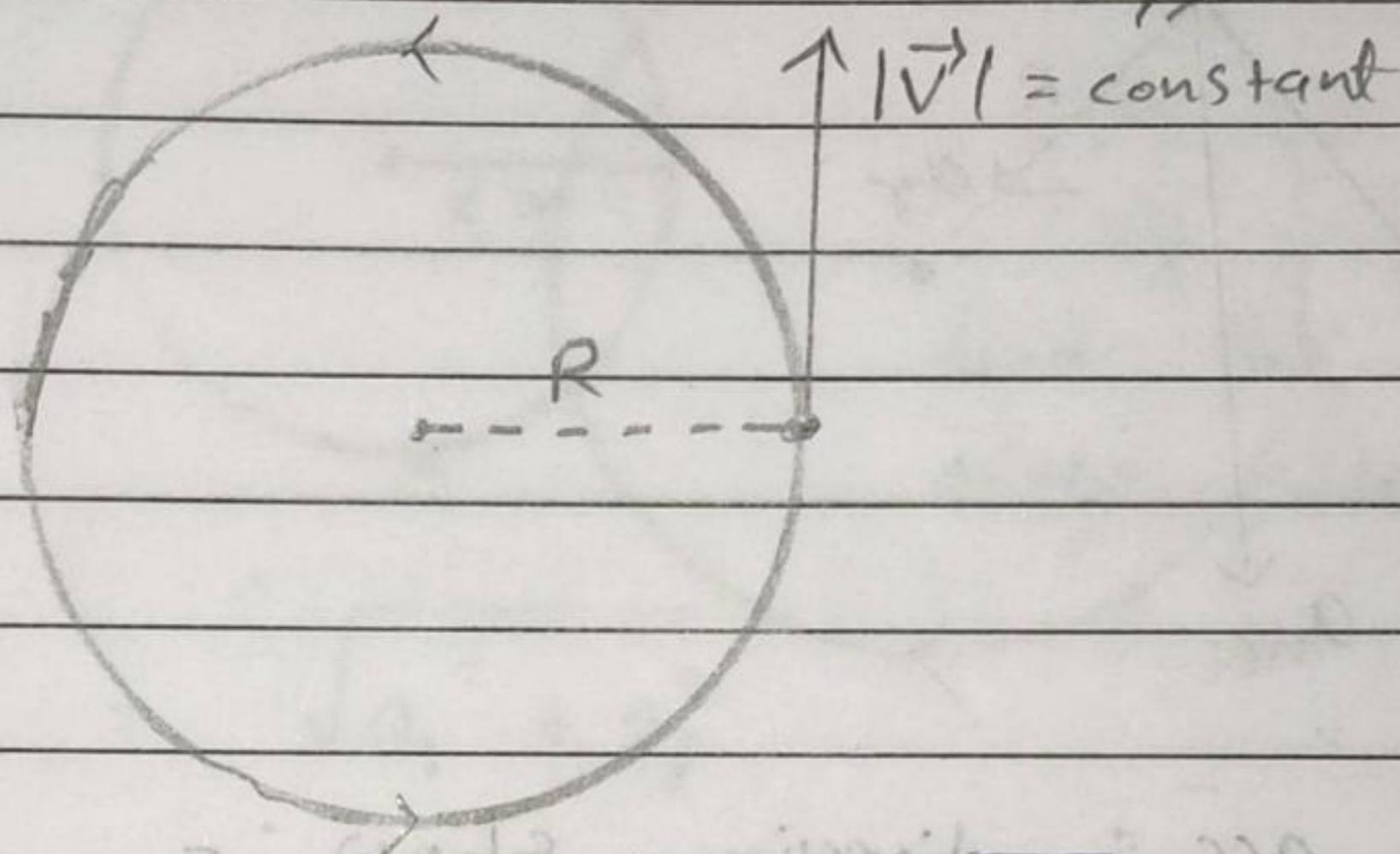
Q. Particle is performing circular motion if radius  $R$  with constant speed.

find  $a_{\text{net}}$ ?

$$a_x = ?$$

$$a_t = ?$$

Uniform circular motion.



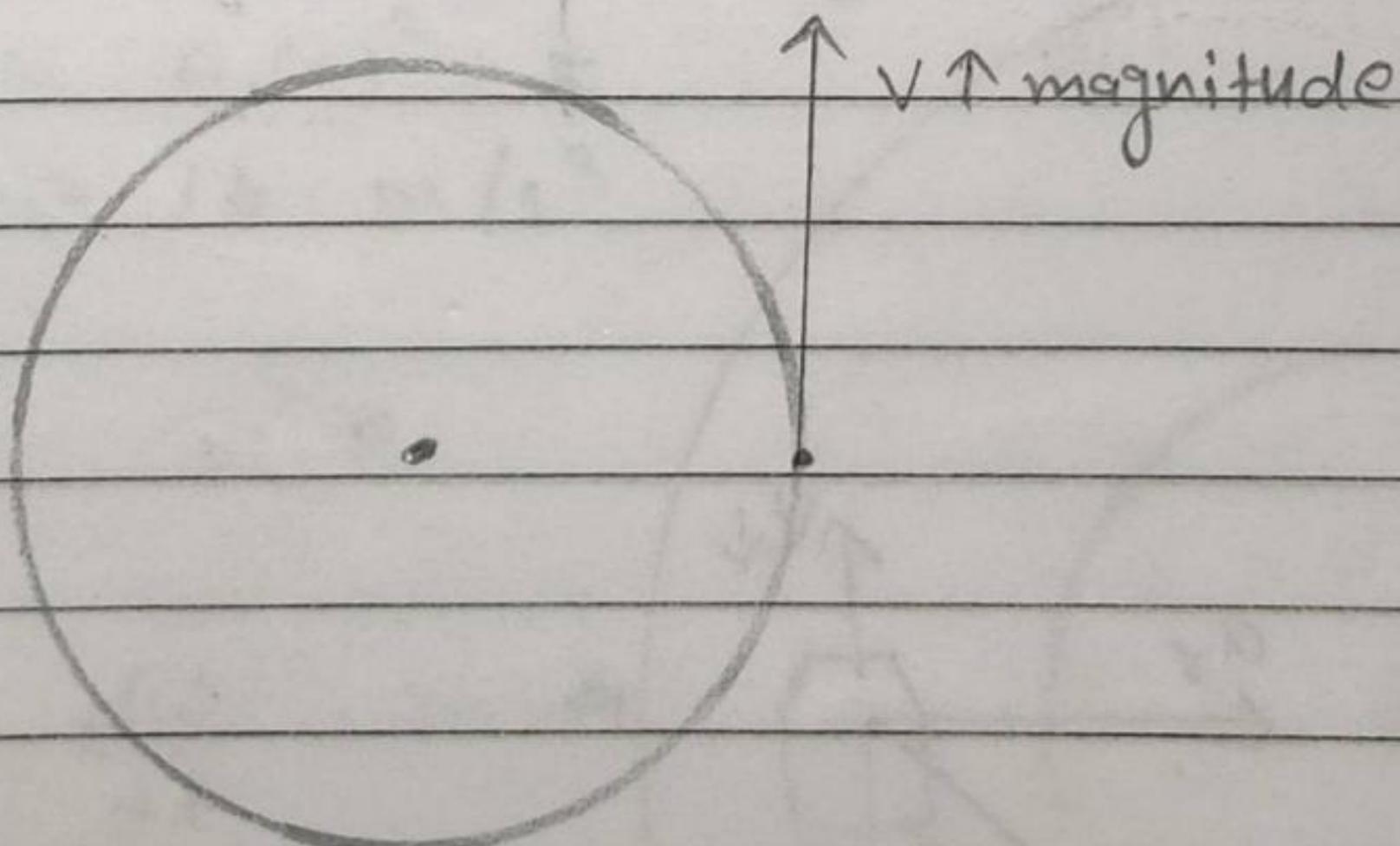
Ans

$$a_{\text{net}} = \sqrt{a_x^2 + a_t^2} = \omega^2 R = \frac{v^2}{R} \times R = \frac{v^2}{R}$$

$$a_x = \omega^2 R$$

$$a_t = \frac{dv}{dt} = 0$$

Q.



Ans.

$$a_{\text{net}} = \sqrt{a_x^2 + a_t^2}$$

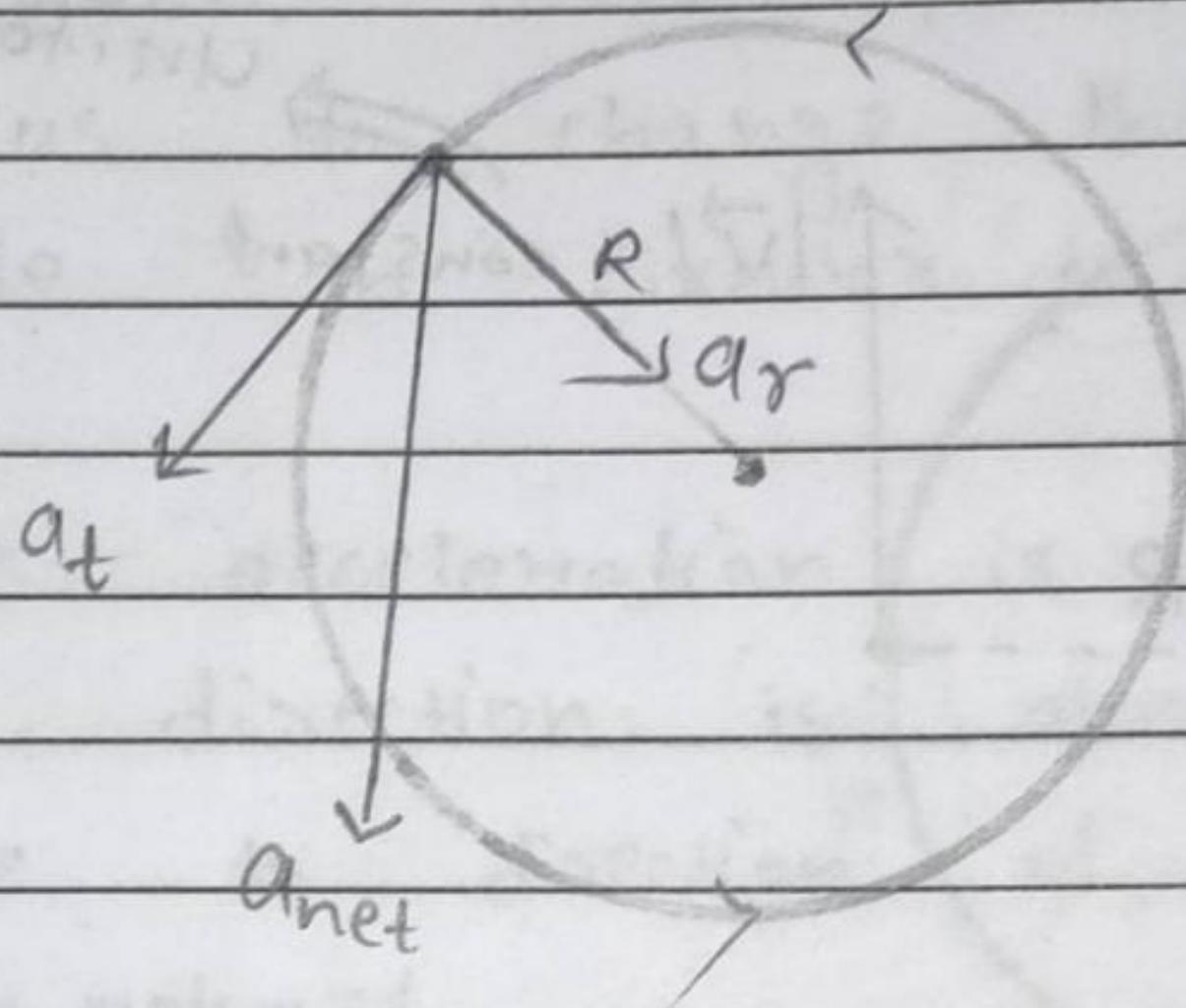
$$a_x = \omega^2 R = \frac{v^2}{R}$$

$$a_t = \frac{dv}{dt}$$

$$a_t = \frac{d\omega \cdot R}{dt} = \alpha R$$

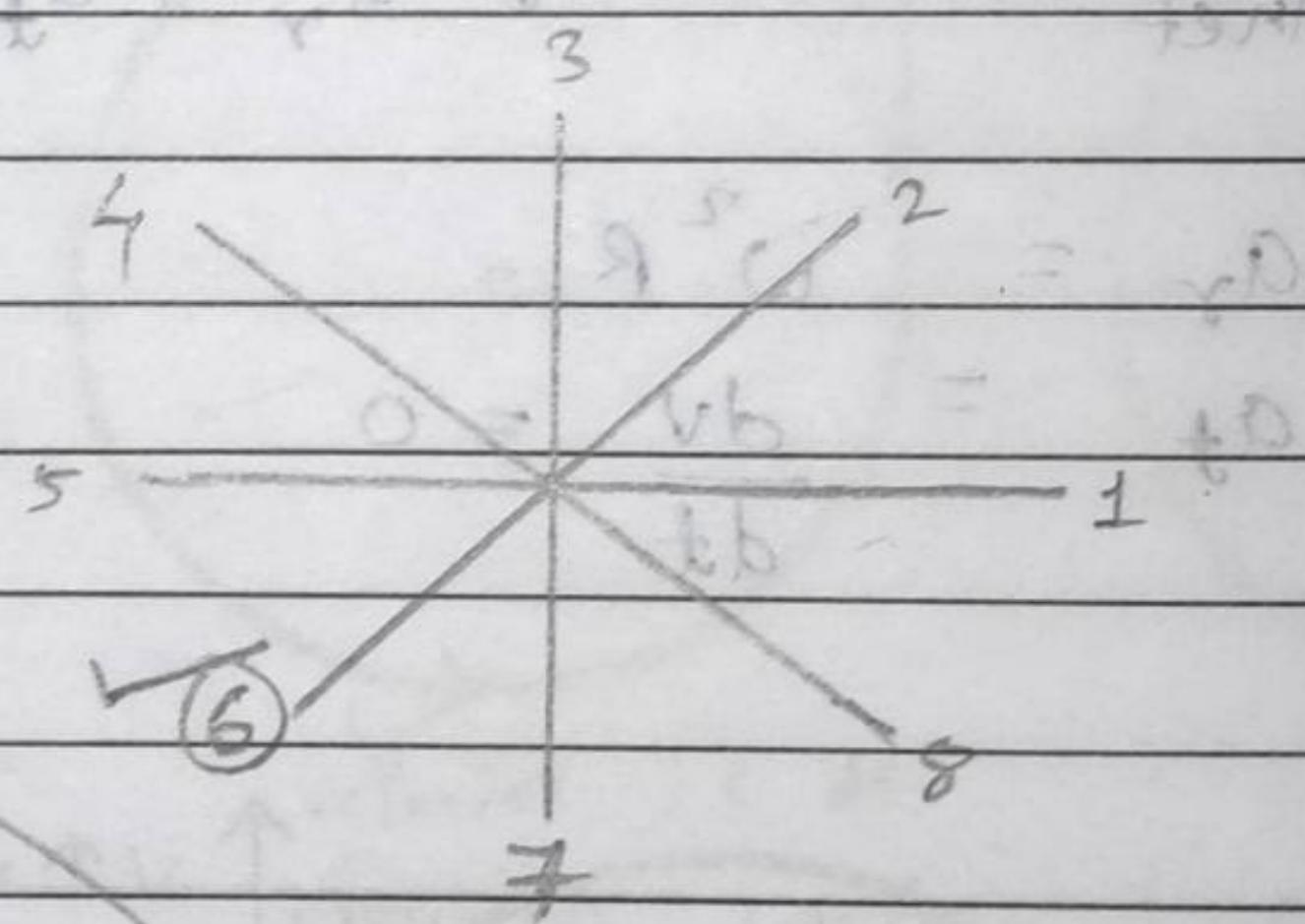
$\rightarrow$  Magnitude of velocity if  
 \* In Uniform circular Motion is fixed  
 Magnitude of velocity changes but in both of  
 the cases direction of velocity changes.

Q. 7

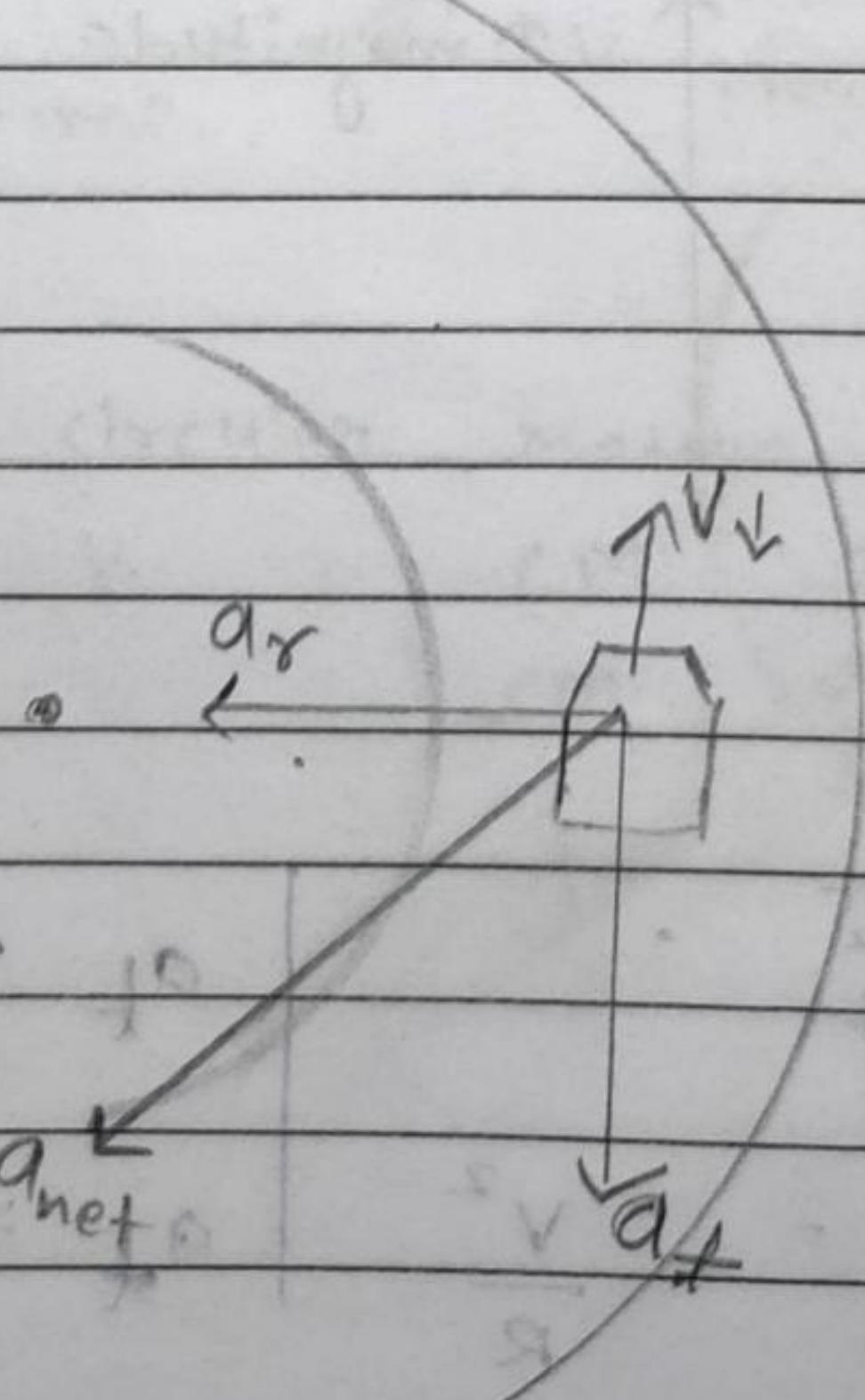


$a_{net}$  acc. direction show :-

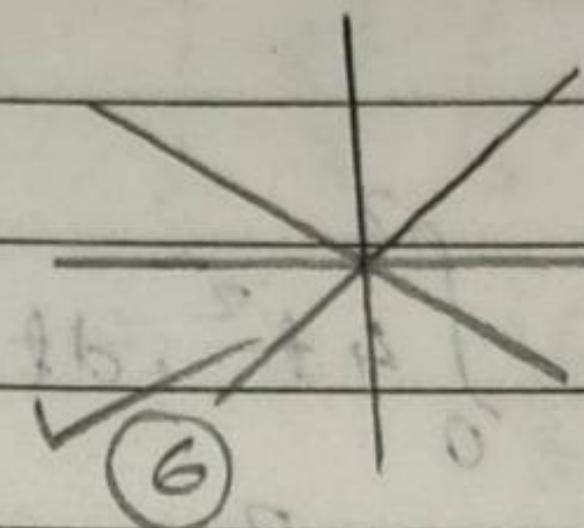
if  $v \uparrow$



Q.



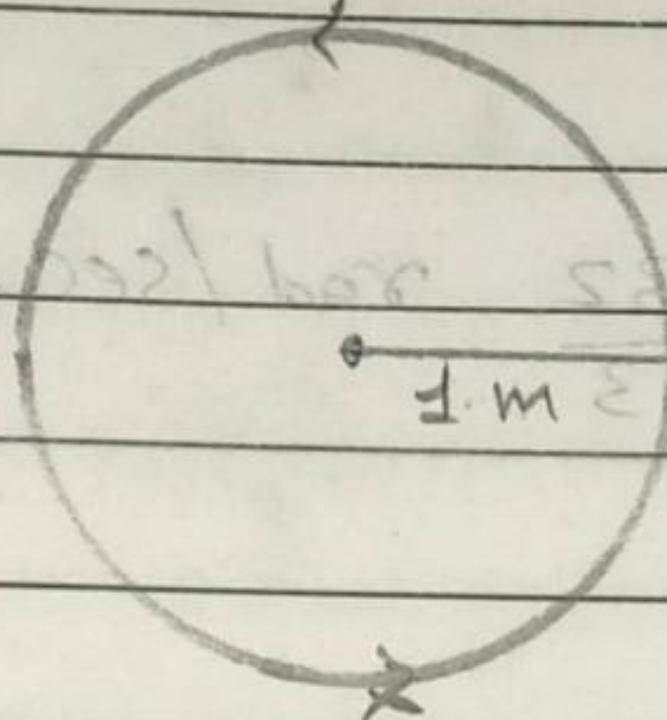
$$a_{net} =$$



$$\checkmark \quad (6) \quad = \{ \text{Ans} \}$$

$$\left[ \frac{\omega}{\theta} \right] \text{A} = \alpha$$

Q.



$$\alpha = f(t)$$

$$\frac{d\omega}{dt} = 4t^3$$

$$\omega_0 = 0 \text{ rad/sec.}$$

find the value of  $a_{net}$  after 2 sec.

Ans.

$$a_{net} = \sqrt{a_r^2 + a_t^2}$$

$$a_r = \omega^2 R \text{ or } \frac{V^2}{R}$$

$$a_t = \alpha \cdot R \text{ or } R \cdot \frac{d\omega}{dt}$$

$$\Rightarrow a_t = \alpha \cdot R$$

$$\Rightarrow a_t = 4t^2 \cdot 1$$

$$\Rightarrow a_t = 4(2)^2 \cdot 1$$

$$\Rightarrow a_t = 16 \text{ m/s}^2$$

$$\Rightarrow a_r = \omega^2 R$$

$$\Rightarrow \alpha = \frac{d\omega}{dt} = 4$$

$$\Rightarrow \frac{d\omega}{dt} = 4t^2$$

$$\Rightarrow d\omega = 4t^2 \cdot dt$$

Integrating,

$$\Rightarrow \int_0^{\omega} d\omega = \int_0^2 4t^2 dt$$

$$\Rightarrow \omega = 4 \left[ \frac{t^3}{3} \right]_0^2$$

$$(t) ? = \infty$$

$$\Rightarrow \omega = 4 \times \frac{8}{3} = \frac{32}{3} \text{ rad/sec.}$$

$$\Rightarrow a_x = \omega^2 R$$

$$a_x = \left( \frac{32}{3} \right)^2 \cdot 1 = \frac{1024}{9}$$

Q.  $\alpha = 2t^2$ ,  $r = 1 \text{ m}$ ,  $t = 2 \text{ sec.}$

find  $a_{net} = ?$

Sol.  $a_{net} = \sqrt{a_x^2 + a_t^2}$

$$\Rightarrow a_x = \omega^2 R \text{ or } \frac{v^2}{R}$$

$$\Rightarrow a_t = \alpha \cdot R \text{ or } R \cdot \frac{d\omega}{dt}$$

$$\Rightarrow a_t = \alpha \cdot R$$

$$\Rightarrow \frac{d\omega}{dt} = 2t^2$$

$$\Rightarrow \int_0^{\omega} d\omega = \int_0^2 2t^2 dt + A$$

$$\Rightarrow \omega = 2 \left[ \frac{t^3}{3} \right]_0^2$$

$$\Rightarrow \omega = 2 \times \frac{8}{3} = \frac{16}{3} \text{ rad./sec.}$$

$$\Rightarrow a_x = \omega^2 R$$

$$\Rightarrow a_x = \left( \frac{16}{3} \right)^2 \cdot 1 = \frac{256}{9} \text{ m/sec}^2 = \theta \text{ rad.}$$

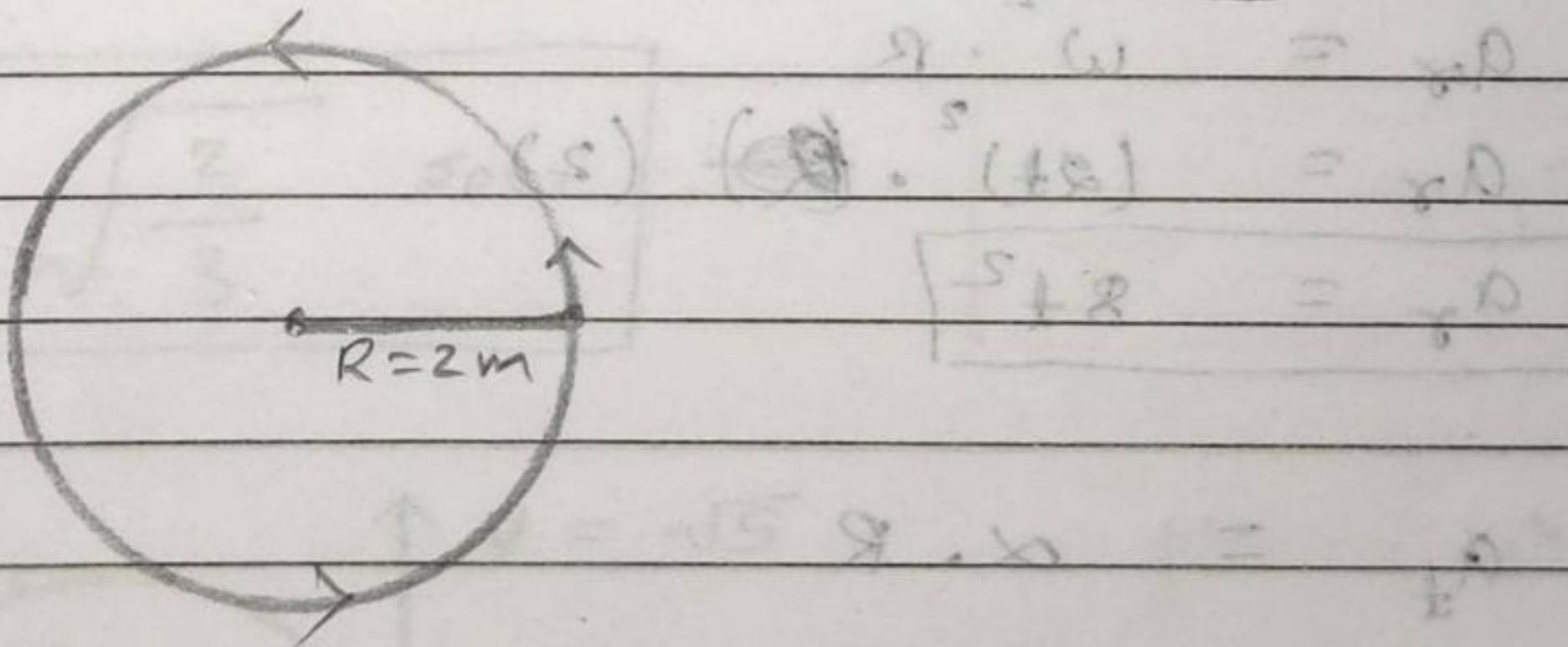
$$\Rightarrow a_y = \frac{256}{9} \cdot \frac{1}{R} = \frac{256}{9} \text{ m/sec}^2 = \theta \text{ rad.}$$

$$a_y = \frac{256}{9} \cdot \frac{1}{8} = \frac{32}{9} \text{ m/sec}^2 = \theta \text{ rad.}$$

rotating motion w.r.t. center of circle  $\omega = \theta/t$

Q. Find the angle b/w  $a_x$  and  $a_y$  after 2 sec.

$$[\omega = 2t], [\omega_i = 0 \text{ rad./sec.}], t = 2 \text{ sec.}$$



$$\text{Ans. } a_x = \omega^2 R$$

$$a_x = (2t)^2 \cdot 2$$

$$a_x = 4t^2 \cdot 2$$

$$a_x = 8t^2$$

$$a_x = 8 \times (2)^2$$

$$a_x = 32$$

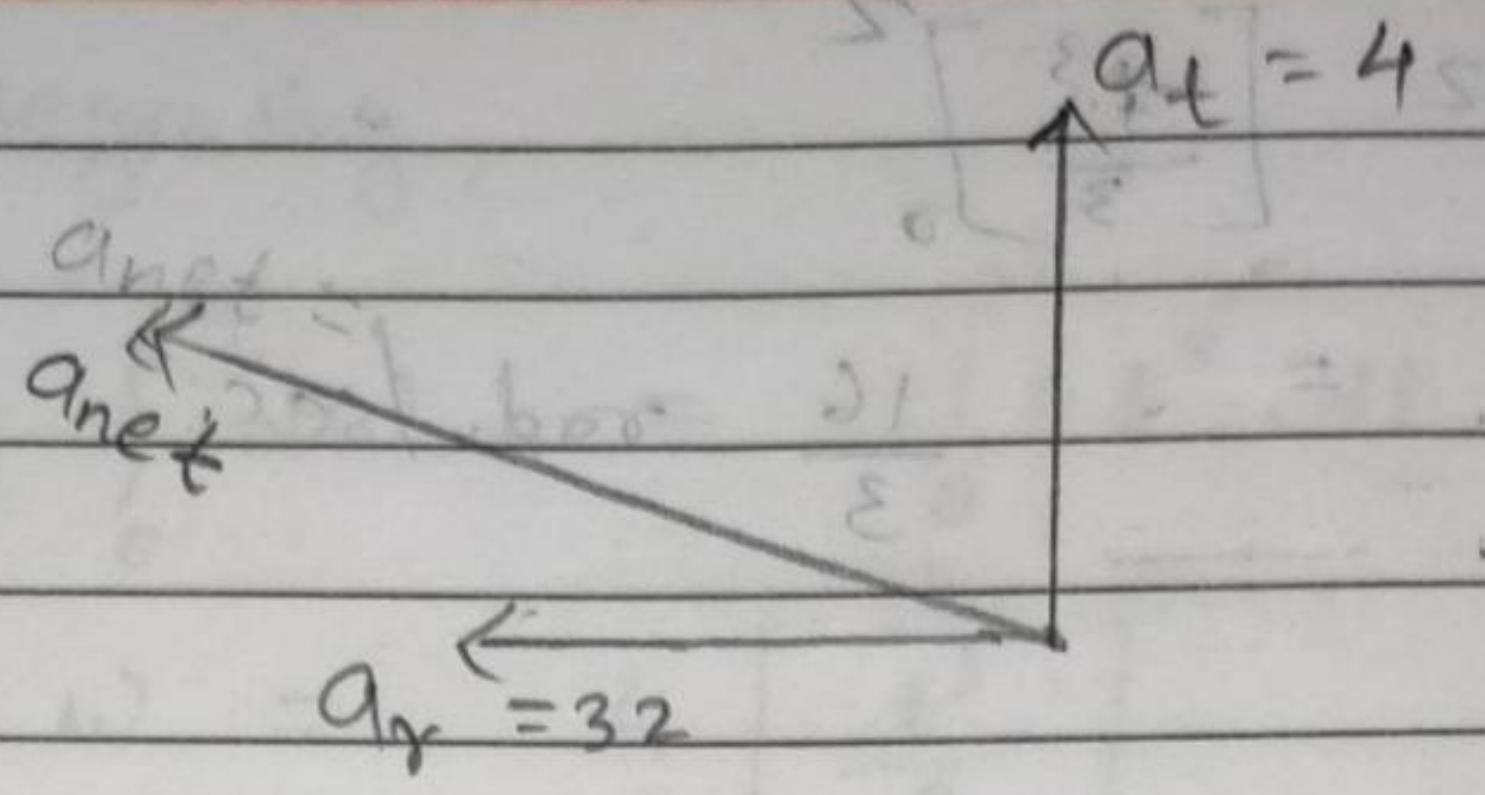
$$a_t = \alpha \cdot R$$

$$\omega = 2t$$

$$\alpha = \frac{d\omega}{dt}$$

$$\alpha = \frac{d(2t)}{dt}$$

$$\alpha = 2$$



$$\tan \theta = \frac{4}{32}$$

$$\tan \theta = \frac{1}{8}$$

$$\theta = \tan^{-1} \frac{1}{8}$$

Q.)  $\omega = 2t$ , particle is performing circular motion radius of 2m find time 't' when angle b/w  $a_x$  and  $a_{net}$  is  $37^\circ$

$$\text{Ans.} \Rightarrow a_x = \omega^2 \cdot R$$

$$\Rightarrow a_x = (2t)^2 \cdot (2) \quad (2)$$

$$\Rightarrow a_x = 8t^2$$

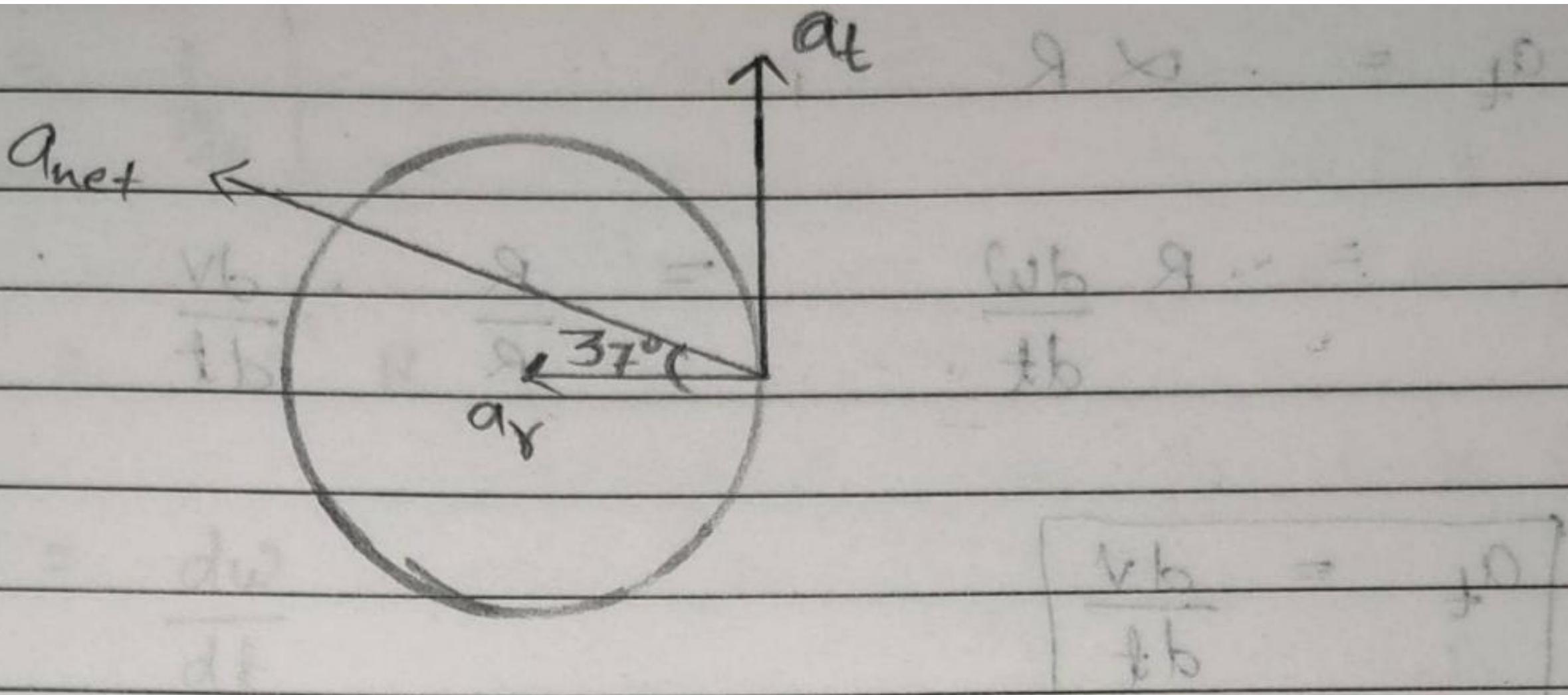
$$\Rightarrow a_t = \alpha \cdot R$$

$$\Rightarrow \frac{d\omega}{dt} = \alpha = 2$$

$$\alpha = 2$$

$$\Rightarrow a_t = 16 \cdot 2 \cdot 2$$

$$\Rightarrow a_t = 4$$



$$\tan \theta = \frac{a_t}{a_r}$$

$$\Rightarrow \tan 37^\circ = \frac{4}{8t^2}$$

$$\Rightarrow \frac{3}{4} = \frac{4}{8t^2}$$

$$\Rightarrow 8t^2 = \frac{16}{3}$$

$$\Rightarrow t = \sqrt{\frac{2}{3}} \text{ sec.}$$

$$\frac{vb}{tb} = 3$$

$$\frac{vb}{tb} = 10$$

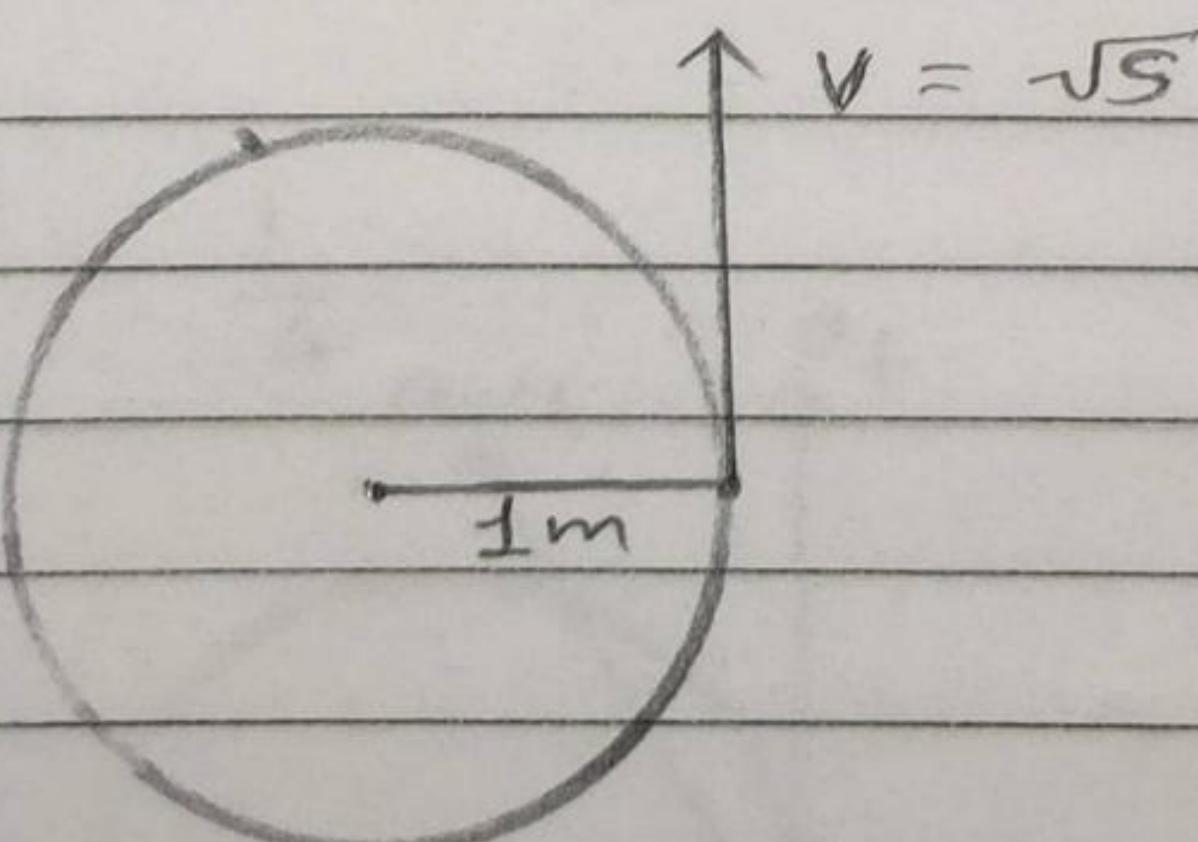
$$\frac{vb}{2b} v = 10$$

$$\frac{(2\pi)b}{2b} v = 10$$

$$\frac{t \cdot 2\pi}{2\pi} = 10$$

$$t = 10$$

Q. Y



find  $\theta$  b/w  $a_r$  and  
a<sub>net</sub> in 1 sec.

Sol.

$$a_r = \omega^2 \cdot R$$

$$a_r = \frac{v^2}{R^2} \cdot R = \frac{v^2}{R}$$

$$a_t = \alpha R$$

$$= R \frac{d\omega}{dt} = \frac{R}{R} \cdot \frac{dv}{dt}$$

$$\boxed{a_t = \frac{dv}{dt}}$$

$$\Rightarrow a_t = v \frac{dv}{ds}$$

$$\Rightarrow a_t = v \frac{d(-\sqrt{s})}{ds}$$

$$\Rightarrow a_t = -\sqrt{s}, \frac{1}{2\sqrt{s}}$$

$$\Rightarrow \boxed{a_t = \frac{1}{2}}$$

$$-\alpha \cdot R = a_t$$

$$\Rightarrow a_t = -\alpha \cdot R$$

$$\Rightarrow \alpha \cdot R = \frac{1}{2}$$

$$\Rightarrow \alpha \cdot \frac{1}{2} = \frac{1}{2}$$

$$\Rightarrow \boxed{\alpha = \frac{1}{2}}$$

$$\Rightarrow a_r = \omega^2 R$$

$$\Rightarrow \alpha = \frac{d\omega}{dt}$$

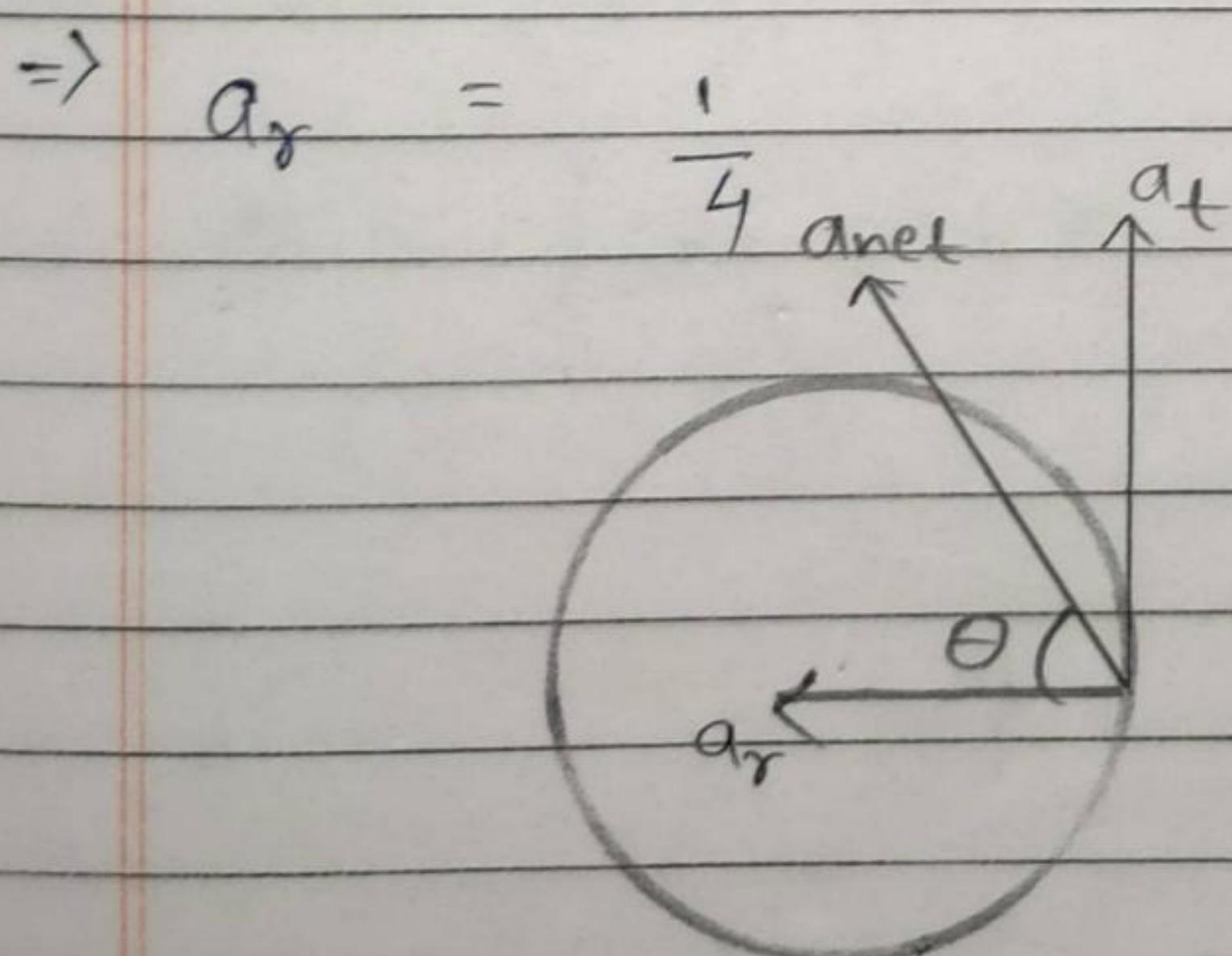
$$\Rightarrow \frac{d\omega}{dt} = \frac{1}{2}$$

$$\Rightarrow \int_0^\omega d\omega = \int_0^1 \frac{1}{2} dt$$

$$\Rightarrow \omega = \frac{1}{2} \cdot 1$$

$$\Rightarrow \boxed{\omega = \frac{1}{2}}$$

$$\Rightarrow a_r = \left(\frac{1}{2}\right)^2 \cdot (1)$$



$$\tan \theta = \frac{\frac{1}{2}}{\frac{1}{4}}$$

$$\tan \theta = 2$$

$$\boxed{\theta = \tan^{-1} 2}$$

Q. In circular motion, a particle is performing circular motion. Radius = 'r' whose centripetal acceleration is given as  $a_c = kr^2 t^2$ . Find the value of

i) Speed of this particle.

Sol.

$$a_c = \frac{v^2}{r}$$

$$kr^2 t^2 = \frac{v^2}{r}$$

$$kr^2 t^2 = v^2$$

$$v = \sqrt{kr \cdot t}$$

ii)  $a_t = ?$

Sol.

$$a_t = \frac{dv}{dt}$$

$$a_t = \frac{d}{dt}(\sqrt{kr} \cdot t)$$

$$a_t = \sqrt{kr} \cdot 1$$

$$a_t = \sqrt{kr}$$

#	Linear Variable	Angular Variable
i)	$\vec{r}$ or $\vec{s}$	$\theta$
ii)	$\vec{v}$	$\omega$
iii)	$\vec{a}$	$\alpha$
iv)	$t$	

\* initial angular velocity =  $\omega_0 / \omega_i$

\* final angular velocity =  $\omega / \omega_f$

$$\textcircled{1} \quad v = u + at$$

$$\omega = \omega_0 + \alpha t$$

$$\textcircled{2} \quad v^2 - u^2 = 2as$$

$$w^2 - \omega_0^2 = 2\alpha\theta \rightarrow \alpha = \text{constant}$$

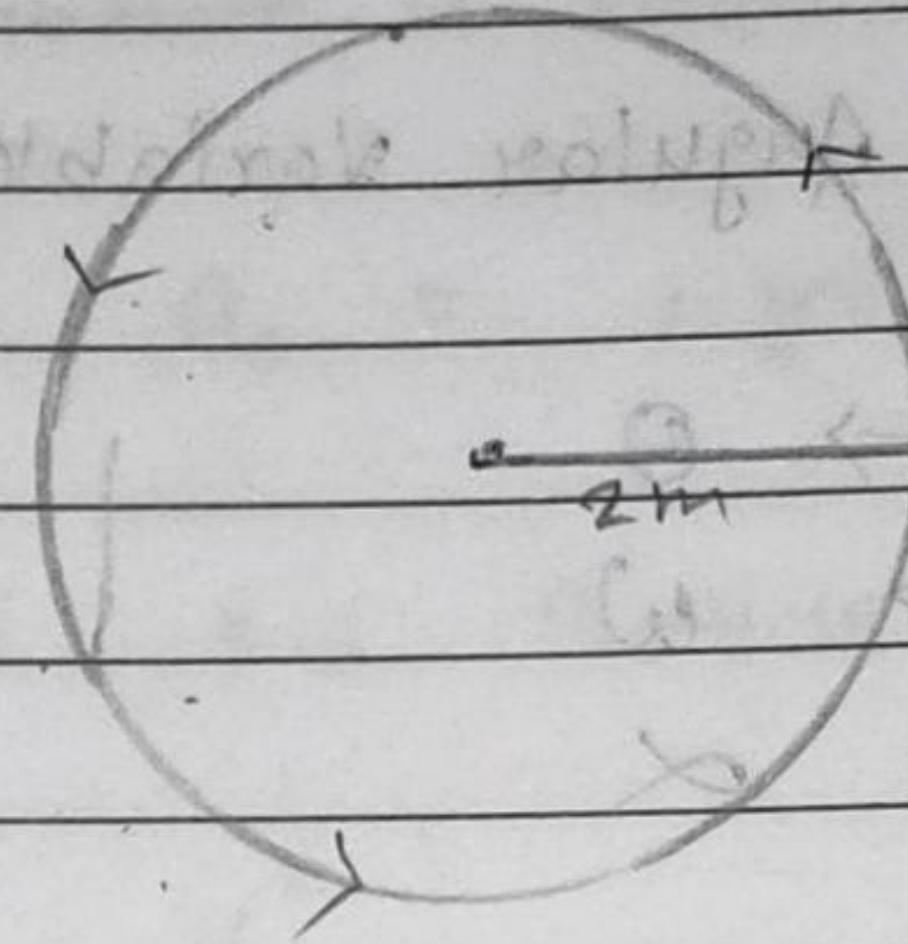
$$\textcircled{3} \quad s = ut + \frac{1}{2}at^2$$

$$\theta = \omega_0 t + \frac{1}{2}\alpha t^2$$

$$4) \quad s = \left( \frac{u+v}{2} \right) t$$

$$\theta = \left( \frac{\omega_0 + \omega}{2} \right) t$$

Q.



$$\omega_0 = 0 \text{ rad/sec.}$$

$$a_t = \alpha m/s^2$$

~~initial angular velocity~~  
~~initial angular position~~

find the time taken by the particle to achieve angular velocity of 4 rad/sec.

$$\theta + \omega t = \omega \quad (1)$$

Ans.

$$\omega_0 = 0$$

$$t = ?$$

$$\omega = 4$$

$$\alpha = ? \quad \theta + \omega t = \omega \quad (2)$$

$$\Rightarrow \omega = \omega_0 + \alpha t \quad \frac{\pi}{2} + \omega t = \omega \quad (3)$$

$$\Rightarrow a_t = \alpha R$$

$$\Rightarrow 2 = \alpha \cdot 2$$

$$\Rightarrow \alpha = 1 \text{ rad/sec}^2$$

$$+ (v + \omega) = 2 \quad (A)$$

$$+ (wt + \omega) = \omega \quad (B)$$

Q. A particle is performing circular motion of radius 5cm whose tendencial acceleration is  $\alpha \text{ m/s}^2$ . Particle angular velocity increases from 10 rad./sec to 20 rad./sec. during some time interval. Find out the time taken for this angular velocity to achieve and no. of revolution completed by this time.

Aus -

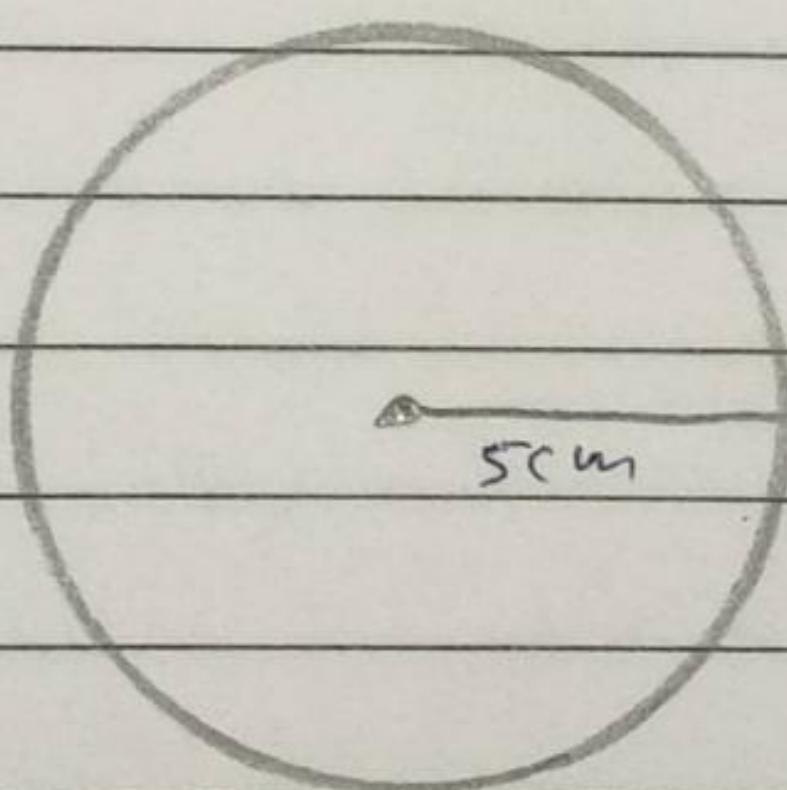
$$r = 5 \text{ cm}, \omega_i = 10 \text{ rad/s}$$

$$t = ?, \omega_f = 20 \text{ rad/s}$$

$$\Rightarrow a_t = \alpha R$$

$$\Rightarrow 2 = \alpha \cdot 5 \times 10^{-2}$$

$$\Rightarrow \alpha = \frac{2 \times 10^2}{5} \text{ rad/sec}^2 = 40 \text{ rad/sec}^2$$



$$\Rightarrow \omega = \omega_0 + \alpha t$$

$$\Rightarrow 20 = 10 + 40 \times t$$

$$\Rightarrow t = \frac{10}{40}$$

$$\Rightarrow t = \boxed{\frac{1}{4} \text{ sec.}}$$

$$\Rightarrow \theta = \omega_0 t + \frac{1}{2} \alpha t^2$$

$$\Rightarrow \theta = 10 \times \frac{1}{4} + \frac{1}{2} \times 10 \times \frac{1}{4} \times \frac{1}{4}$$

$$\Rightarrow \theta = \frac{15}{2} + \frac{5}{4}$$

$$\Rightarrow \theta = \frac{15}{4} \text{ rad.}$$

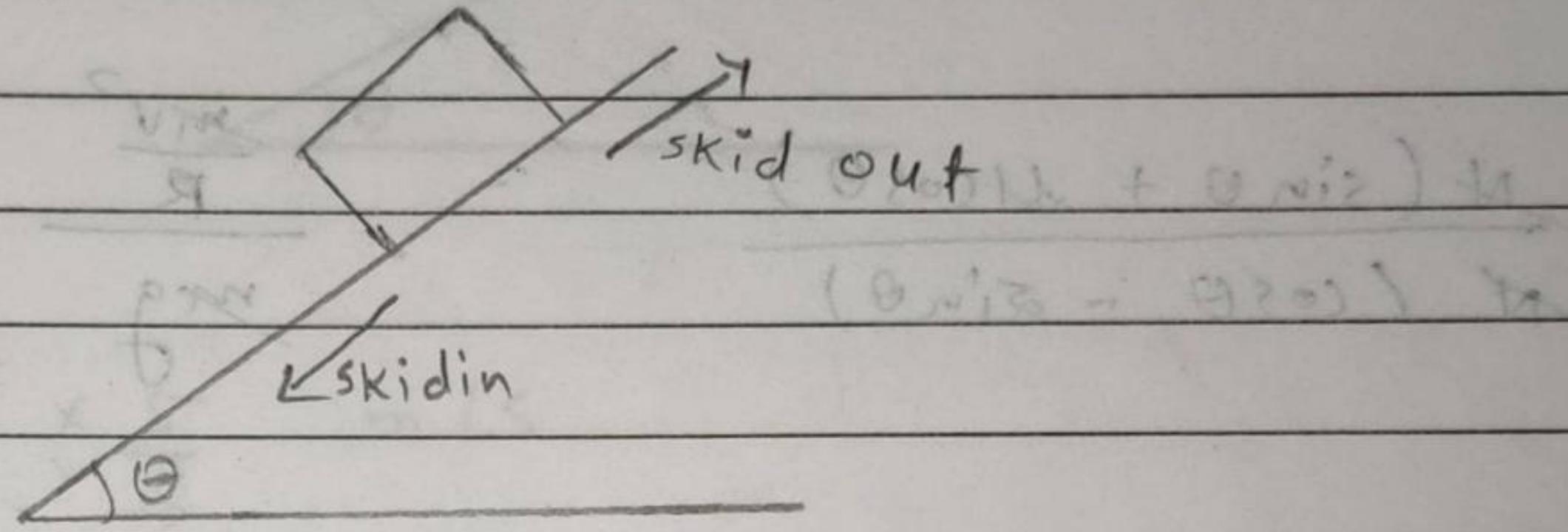
$$\Rightarrow 2\pi = 1 \text{ revolution}$$

$$\Rightarrow \frac{15}{4} = \frac{1}{2\pi} \times \frac{15}{4}$$

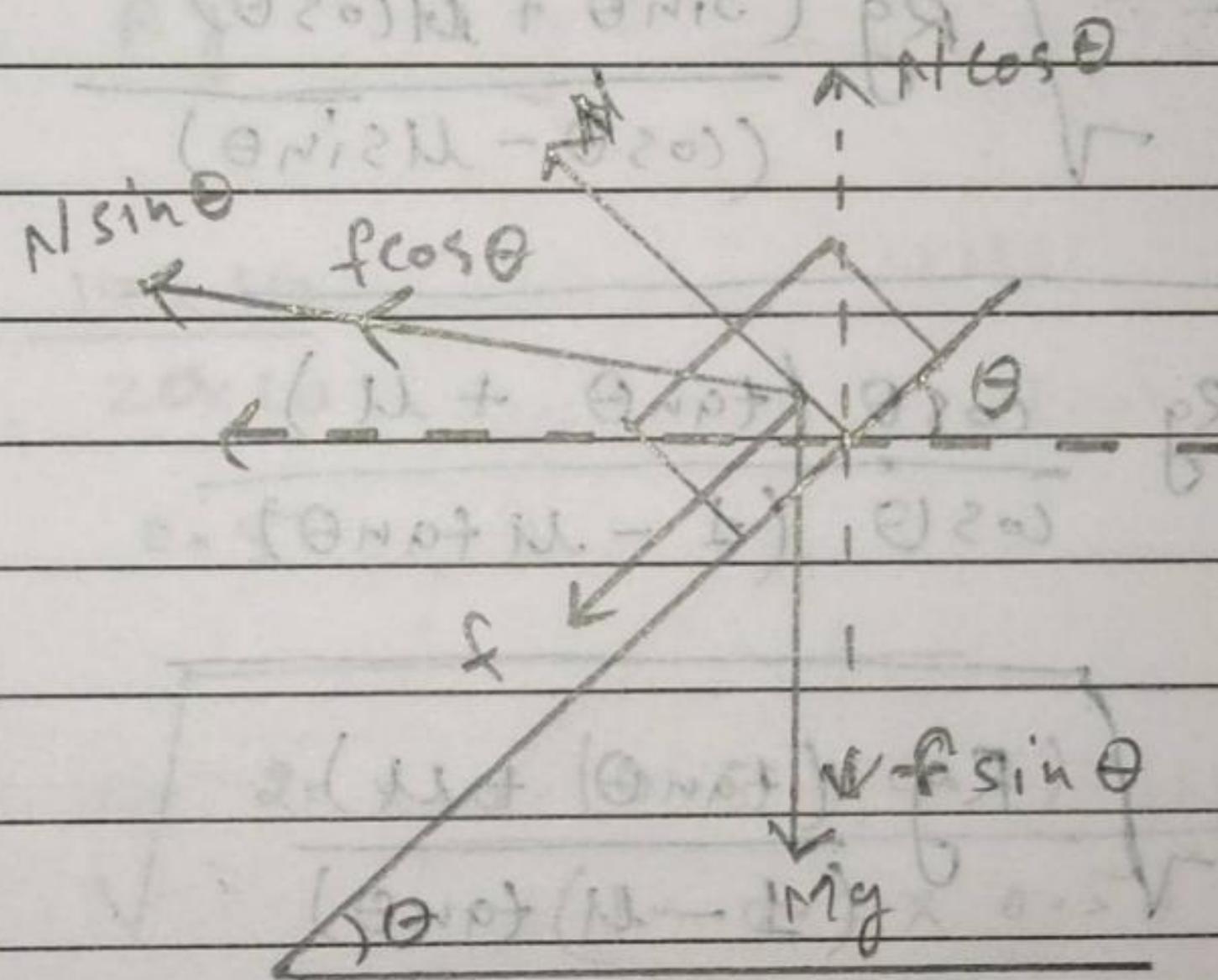
$$\Rightarrow \frac{15}{4} = \frac{1}{2 \times 3.14} \times \frac{15}{4}$$

$$\Rightarrow \frac{15}{4} = 3.2$$

b) Banking of road with friction :-



maximum velocity with which vehicle can take a turn so it does not skid out.



$$\Rightarrow N \sin \theta + f \cos \theta = \frac{mv^2}{R}$$

$$\Rightarrow N \cos \theta = mg + f \sin \theta$$

$$f = \mu N$$

$$N \sin \theta + \mu N \cos \theta = \frac{mv^2}{R} \quad \text{--- (i)}$$

$$N \cos \theta = mg + \mu N \sin \theta$$

$$N \cos \theta - \mu N \sin \theta = mg \quad \text{--- (2)}$$

$\perp \div 2$

$$\Rightarrow \frac{N(\sin \theta + \mu \cos \theta)}{N(\cos \theta - \mu \sin \theta)} = \frac{\frac{mv^2}{R}}{mg}$$

$$\Rightarrow \frac{v^2}{Rg} = \frac{\sin \theta + \mu \cos \theta}{\cos \theta - \mu \sin \theta}$$

$$\Rightarrow v_{max} = \sqrt{Rg \frac{(\sin \theta + \mu \cos \theta)}{(\cos \theta - \mu \sin \theta)}}$$

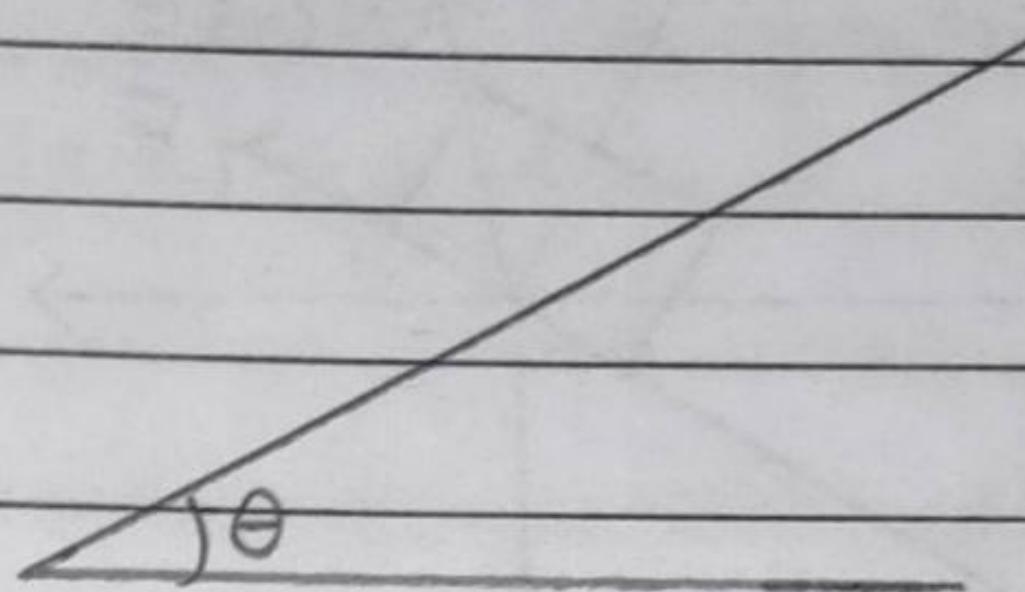
$$\Rightarrow v_{max} = \sqrt{Rg \frac{\cos \theta (\tan \theta + \mu)}{\cos \theta (1 - \mu \tan \theta)}}$$

$$\Rightarrow v_{max} = \sqrt{Rg \frac{(\tan \theta + \mu)}{(1 - \mu \tan \theta)}}$$

Q. The radius of a circular track is  $20 \text{ m}$  which is <sup>safe</sup> band for a turning for a vehicle of  $36 \text{ KM/h}$  if the coefficient of friction b/w the road and tier is  $0.2$  then find out the minimum and maximum speed at which vehicle can take a safe turn.

Ans.

$$\mu = 0.2$$



$$\Rightarrow v = \frac{36^2 \times 5}{18} \text{ m/s}$$

$$\Rightarrow v = 10 \text{ m/s}$$

$$\Rightarrow \tan \theta = \frac{v^2}{R g}$$

$$\Rightarrow \tan \theta = \frac{10 \times 10}{20 \times 10}$$

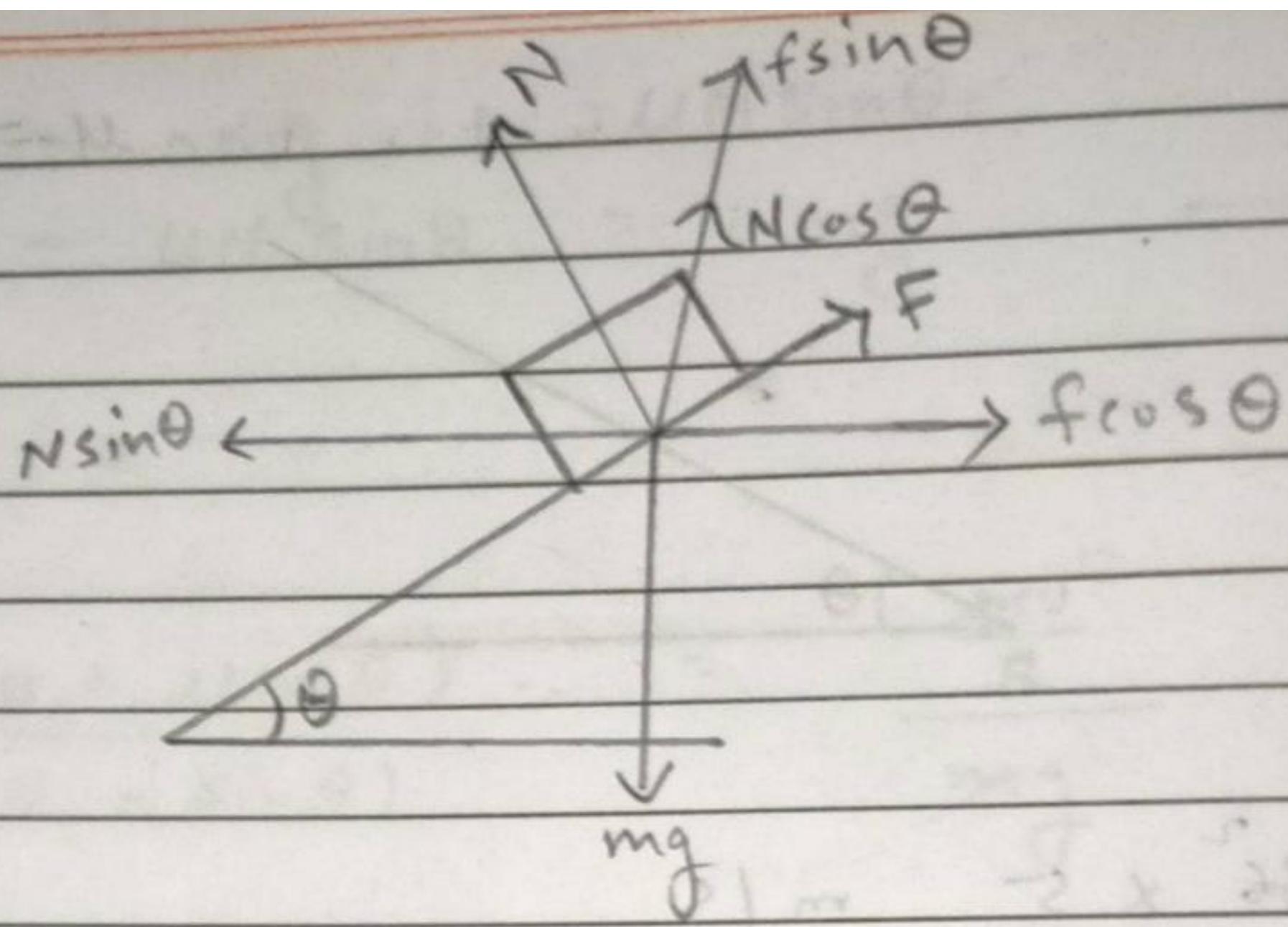
$$\Rightarrow \tan \theta = 0.5$$

$$\Rightarrow v_{max} = \sqrt{\frac{20 \times 10 (0.5 + 0.2)}{(1 - 0.2 \times 0.5)}}$$

$$= \sqrt{\frac{200 (0.7)}{(1 - 0.1)}}$$

$$= \sqrt{\frac{140}{0.9}}$$

$$v_{max} = 15 \text{ m/s}$$



$$\Rightarrow N \sin \theta - f \cos \theta = \frac{mv_{\min}^2}{R}$$

$$\Rightarrow N \cos \theta + f \sin \theta = mg$$

$$f = \mu N$$

$$\Rightarrow N \sin \theta - \mu N \cos \theta = \frac{mv_{\min}^2}{R} \quad \textcircled{1}$$

$$\Rightarrow N \cos \theta + \mu N \sin \theta = mg \quad \textcircled{2}$$

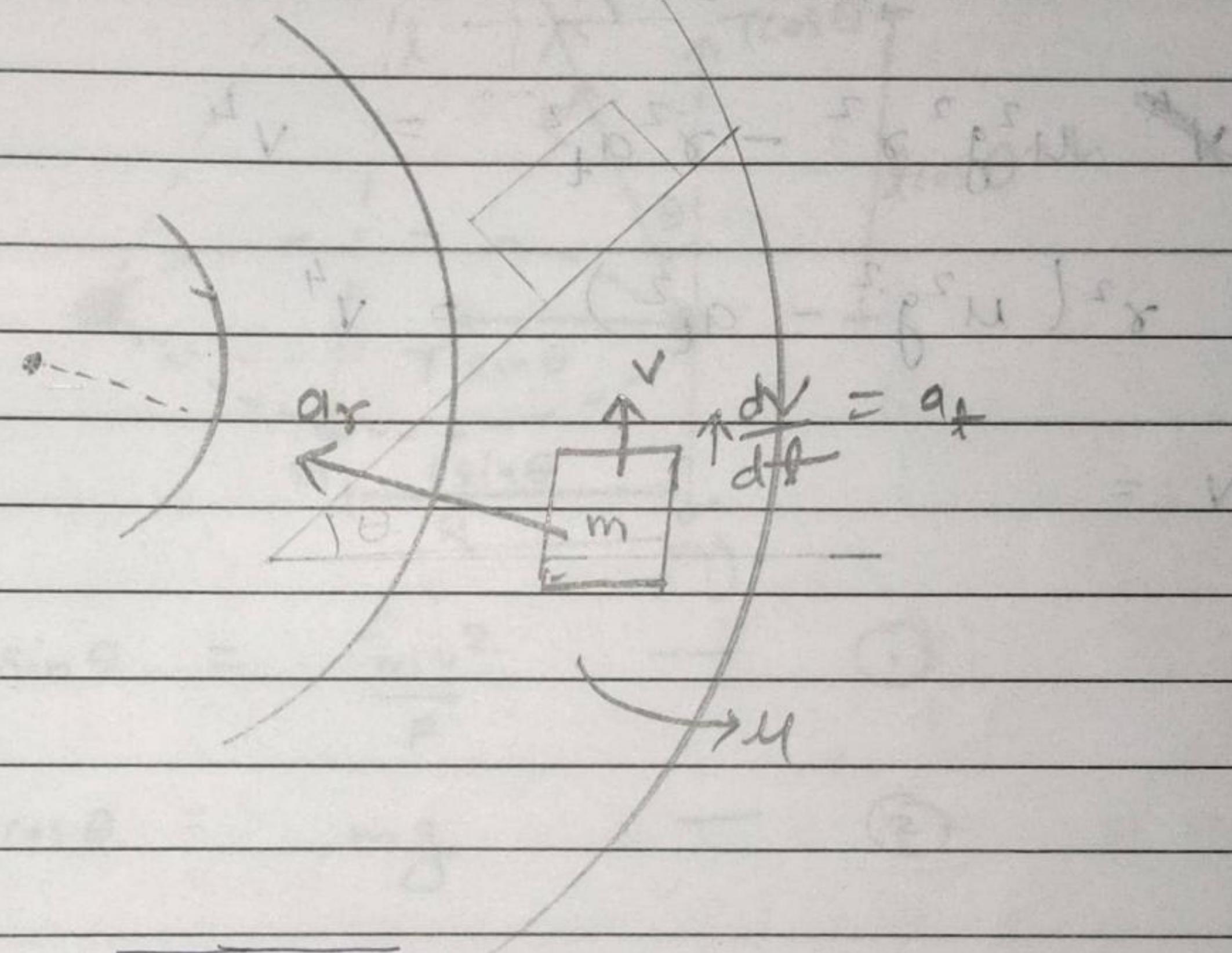
Solving  $\textcircled{1}$  &  $\textcircled{2}$ ,

$$\Rightarrow v_{\min} = \sqrt{\frac{Rg (\tan \theta - \mu)}{(1 + \mu \tan \theta)}}$$

$$\Rightarrow \frac{v_{\max}}{v_{\min}} = \frac{\sqrt{\frac{Rg (\tan \theta + \mu)}{(1 - \mu \tan \theta)}}}{\sqrt{\frac{Rg (\tan \theta - \mu)}{(1 + \mu \tan \theta)}}}$$

Q. A car is going through a circular track of radius  $R$  whose tangential acceleration is increasing, the coefficient friction b/w tier and road is  $\mu$ . Find out the speed at which car can take a safe turn.

Aus.



$$\Rightarrow a_{net} = \sqrt{a_x^2 + a_t^2}$$

$$\Rightarrow a_{net} = \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2}$$

$$\Rightarrow F = m a_{net}$$

$$\Rightarrow f = m a_{net}$$

$$\Rightarrow \mu N = m a_{net}$$

$$\Rightarrow \mu mg = m \sqrt{\left(\frac{v^2}{R}\right)^2 + a_t^2}$$

$$\Rightarrow \mu^2 g^2 = \left(\frac{v^2}{R}\right)^2 + a_t^2$$

$$\Rightarrow u^2 g^2 = \frac{v^4}{\gamma^2} + a_t^2$$

$$\Rightarrow u^2 g^2 = \frac{v^4 + \gamma^2 a_t^2}{\gamma^2}$$

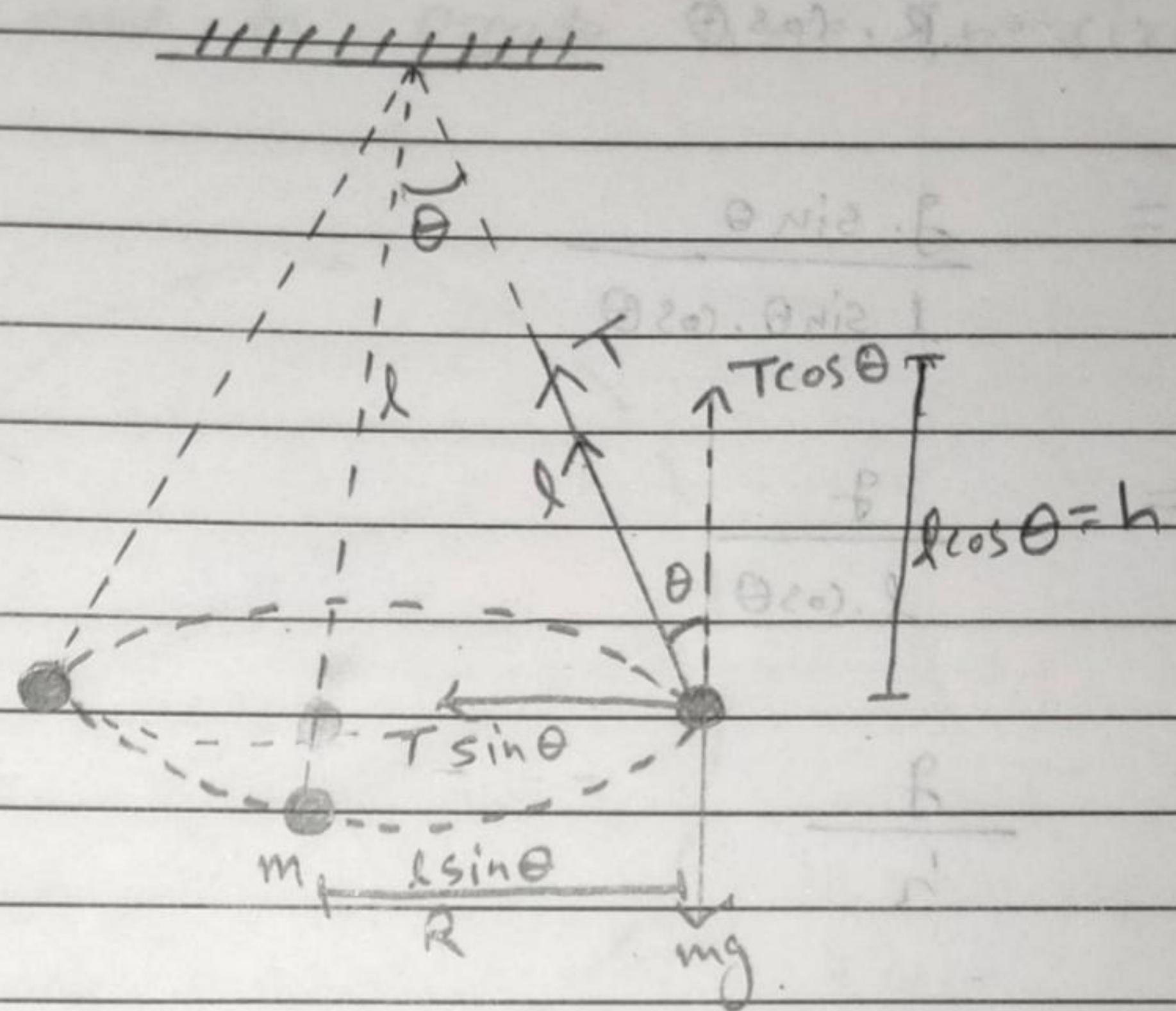
$$\Rightarrow \cancel{u^2 g^2} \gamma^2 - \gamma^2 a_t^2 = v^4$$

$$\Rightarrow \gamma^2 (u^2 g^2 - a_t^2) = v^4$$

$$\Rightarrow v^2 = \gamma \sqrt{u^2 g^2 - a_t^2}$$

Aus.

## # Conical Pendulum :-



$$\Rightarrow T \sin \theta = \frac{mv^2}{R} \quad \text{--- (1)}$$

$$\Rightarrow T \cos \theta = mg \quad \text{--- (2)}$$

$$(2) \div (1)$$

$$\frac{T \cos \theta}{T \sin \theta} = \frac{mg}{\frac{mv^2}{R}}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{mg}{\frac{mv^2}{R}}$$

$$V = R\omega$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{g}{R \cdot R \cdot \omega^2}$$

$$\Rightarrow \frac{\cos \theta}{\sin \theta} = \frac{g}{R \omega^2}$$

$$\Rightarrow \omega^2 = \frac{g \cdot \sin\theta}{R \cdot \cos\theta}$$

$$\Rightarrow \omega^2 = \frac{g \cdot \sin\theta}{l \cdot \sin\theta \cdot \cos\theta}$$

$$\Rightarrow \omega^2 = \frac{g}{l \cdot \cos\theta}$$

$$\Rightarrow \omega^2 = \frac{g}{h}$$

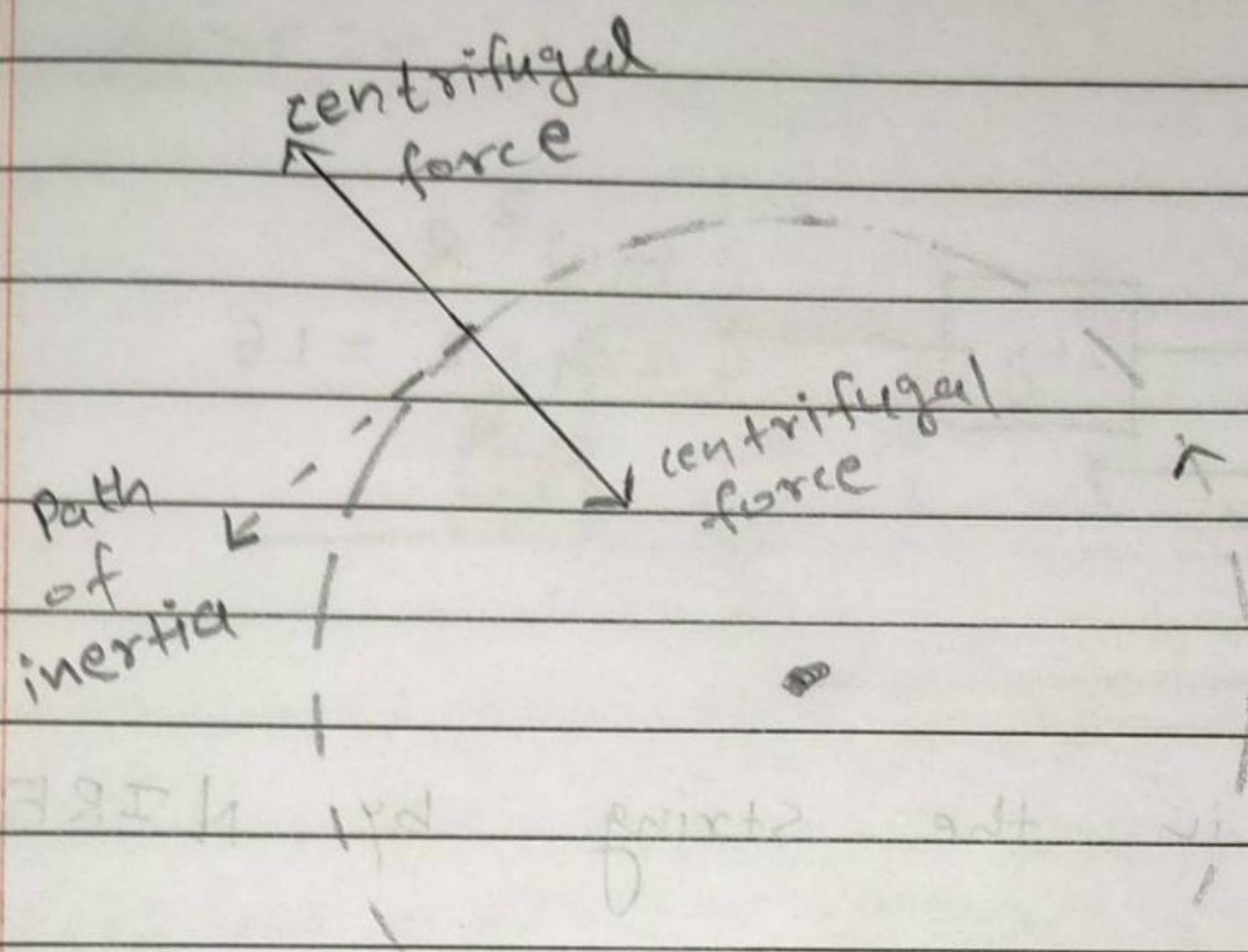
$$\Rightarrow \omega = \sqrt{\frac{g}{h}}$$

$$\Rightarrow T = \frac{2\pi}{\omega}$$

$$\Rightarrow \boxed{T = 2\pi \sqrt{\frac{h}{g}}}$$

# centrifugal /  
centrifetal force :-

→ It's a name to pseudo force in circular motion.



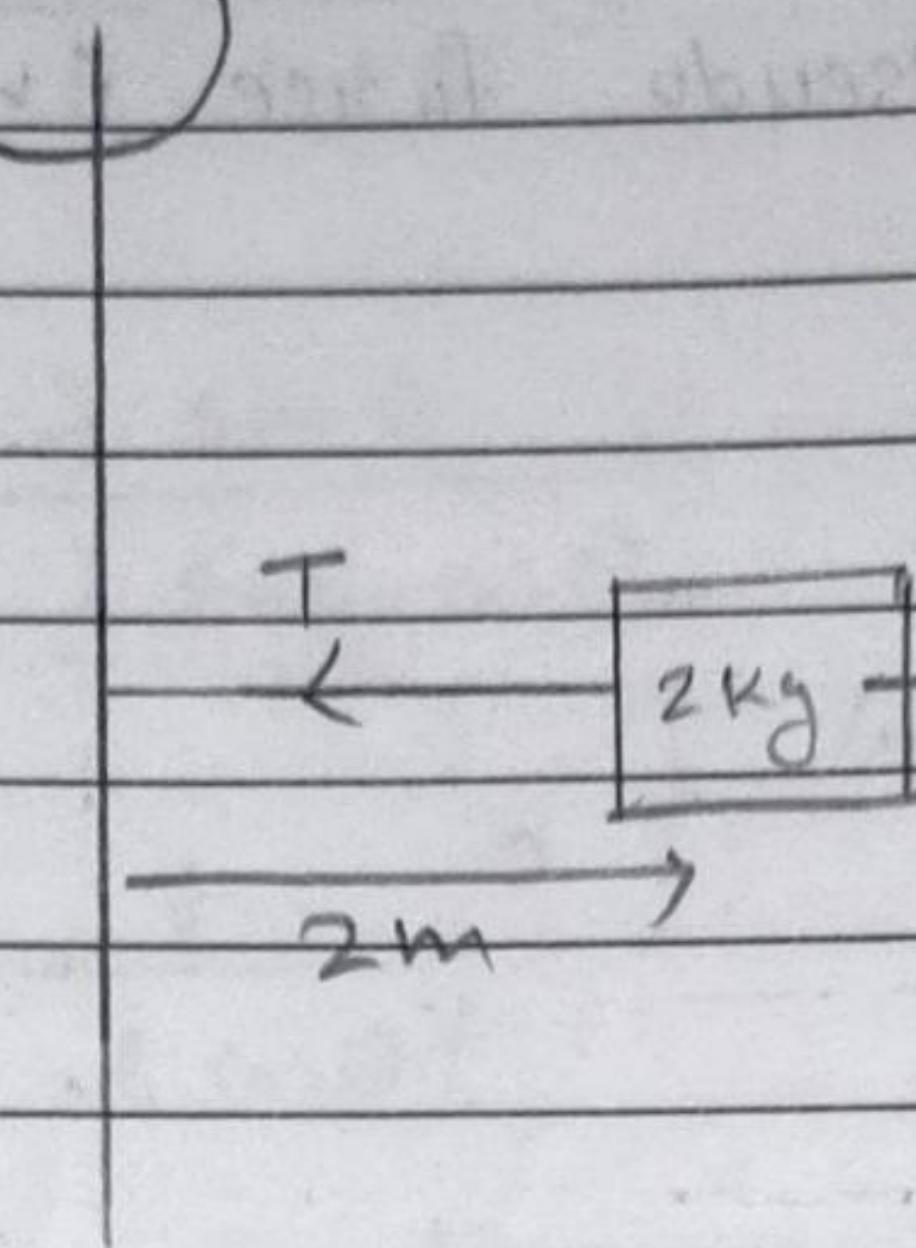
Centrifugal force,  $F = \frac{mv^2}{r}$

$$F = \frac{m(\omega r)^2}{r}$$

$$F = m\omega^2 r$$

Q.

$$\omega = 2 \text{ rad/sec}$$



$$m\omega^2 R \\ 2 \times 4 \times 2 = 16$$

find Tension in the string by N I RF.

$$\Rightarrow T = m\omega^2 R = 2 \times (2)^2 \times 2$$

$$\Rightarrow T = 16 \text{ N}$$