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ECE 236 B – HW 4

$\boxed{\text{A 3.37}}$

Let $f: \mathbb{R}^n \to \mathbb{R}$ differentiable & $g: \mathbb{R}^n \to \mathbb{R}$ convex.

(b) For $\hat{x} \in \operatorname{dom} f \cap \operatorname{dom} g$, $\nabla f(\hat{x})^T(y-\hat{x}) + g(y) - g(\hat{x}) \geq 0$ $(\forall\ y \in \operatorname{dom}(g))$.

(a.) Suppose x is locally optimal but (b) does not hold. That is, $\exists\ y \in \operatorname{dom}(g)$ s.t. $\nabla f(x)^T(y-x) + g(y) - g(x) < 0$.
Consider the point $z(t) = ty + (1-t)x$, $t \in [0,1]$.
Since $\operatorname{dom} f$ is an open set, $\exists\ \varepsilon$ s.t. $B_\varepsilon(x)$, a ball with radius ε centered on x, is in $\operatorname{dom} f$. Furthermore, $B_\varepsilon(x) \cap \operatorname{dom}(g)$ is convex, so for sufficiently small t, $z(t) \in \operatorname{dom}(f) \cap \operatorname{dom}(g)$. (More simply, $z(t) \in \operatorname{dom} g$ for $\forall\ t \in [0,1]$ and $z(t) \in \operatorname{dom} f$ for t sufficiently small, so $z(t) \in \operatorname{dom} f \cap \operatorname{dom} g$ for t sufficiently small).

For $\forall\ y$, $\exists\ z(t)$, $t \in [0,1]$ s.t. $\nabla g(x)^T(z-x) < g(z) - g(x)$, by the Mean Value Theorem and since x is a local minimum. Also, $z(t)$ is feasible since g is convex.

Note that $z - x = ty + (1-t)x - x = t(y-x)$,
and $g(z) - g(x) = g(ty + (1-t)x) - g(x)$
$\leq t\,g(y) + (1-t)g(x) - g(x) = t(g(y) - g(x))$,
by the convexity of g.

Thus, $\nabla f(x)^T(y-x) + g(y) - g(x) < 0$

$\Leftrightarrow \frac{1}{t}\nabla f(x)^T(z-x) + \frac{1}{t}(g(z) - g(x)) < 0$

$\Leftrightarrow \nabla f(x)^T(z-x) + g(z) - g(x) < 0$.

②

Thus, $\nabla f(x)^T(z-x) + \nabla g(x)^T(z-x) < 0$.

Now, let $q(\ell) = \ell z + (1-\ell)x$, $\ell \in [0,1]$. Again, $q(\ell)$ is feasible for ℓ sufficiently small. Note that:

$$\frac{d}{d\ell}\left(f(q(\ell)) + g(q(\ell))\right)\Big|_{\ell=0} = \nabla f(x)^T(z-x) + \nabla g(x)^T(z-x) < 0.$$

Thus, for sufficiently small ℓ, we have $f(q(\ell)) + g(q(\ell)) < f(x) + g(x)$, a contradiction.
$\Rightarrow$ (6) is a necessary condition for $\bar{x}$ to be optimal

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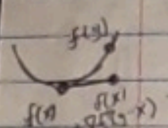
(b.) Now we assume f convex.

Suppose $x \in \operatorname{dom} f \cap \operatorname{dom} g$ and satisfies (6).
Then, if $y \in \operatorname{dom} g$, $\nabla f(x)^T(y-x) + g(y) - g(x) \geq 0$.



Thus, $f(x) + g(x) \leq f(x) + \nabla f(x)^T(y-x) + g(y)$.
But $f(x) + \nabla f(x)^T(y-x) \leq f(y)$ by the convexity of f and the fact that x is a minimum. It follows that:
$f(x) + g(x) \leq f(y) + g(y)$, $\forall\, x,y \in \operatorname{dom} f \cap \operatorname{dom} g$.
$\Rightarrow$ x is a mininimum.

(c1) Take $g(x) = \|x\|_1$. $\operatorname{dom}(g) = \mathbb{R}$.

(6) becomes $\nabla f(\hat{x})^T(y-\hat{x}) + \|y\|_1 - \|x\|_1 \geq 0, \forall y \in \operatorname{dom} g$

$$\Leftrightarrow \nabla f(\hat{x})^T(y-\hat{x}) + \sum_i |y_i| - \sum_i |x_i| \geq 0.$$

$$\Leftrightarrow \sum_i \left[\frac{\partial}{\partial x_i} f(\hat{x})(y_i - \hat{x}_i) + |y_i| - |x_i|\right] \geq 0$$

$$\Leftrightarrow \frac{\partial}{\partial x_i} f(\hat{x})(y_i - \hat{x}_i) + |y_i| - |x_i| \geq 0, \quad \forall i.$$

If $x_i = 0$, $\frac{\partial}{\partial x_i} f(\hat{x}) y_i + |y_i| \geq 0$

$$\Leftrightarrow \begin{cases} \frac{\partial}{\partial x_i} f(\hat{x}) \geq -|y_i|/y_i = -1 & \text{if } y_i > 0. \\ \frac{\partial}{\partial x_i} f(\hat{x}) \leq -|y_i|/y_i = 1 & \text{if } y_i < 0. \\ \frac{\partial}{\partial x_i} f(\hat{x}) = 0 & \text{if } y_i = x_i = 0. \end{cases}$$

$$\Leftrightarrow \left|\frac{\partial f(\hat{x})}{\partial x_i}\right| \leq 1 \quad \text{if } \hat{x}_i = 0.$$

If $x_i > 0$, $\frac{\partial}{\partial x_i} f(\hat{x})(y_i - x_i) \geq |x_i| - |y_i|$

If $\frac{\partial}{\partial x_i} f(\hat{x}) = -1 \Rightarrow x_i - y_i \geq |x_i| - |y_i|$

$\Rightarrow |x_i| - y_i \geq |x_i| - |y_i| \Rightarrow y_i \leq |y_i|$, so the condition is always satisfied.

If $x_i < 0$ and $\frac{\partial}{\partial x_i} f(\hat{x}) = 1$, we have $y_i - x_i \geq |x_i| - |y_i|$

$\Rightarrow y_i - x_i \geq -x_i - |y_i| \Rightarrow y_i \geq -|y_i|$, which is always satisfied.

$$\left|\frac{\partial}{\partial x_i} f(\hat{x})\right| \leq \frac{\big||x_i| - |y_i|\big|}{|y_i - x_i|} \leq 1, \quad \forall x_i, y_i:$$

$$= \begin{cases} \frac{|x_i - y_i|}{|y_i - x_i|} = 1 \text{ if } x_i, y_i > 0, & \frac{|-x_i + y_i|}{|y_i - x_i|} = 1 \text{ if } x_i, y_i < 0. \\ \frac{|x_i + y_i|}{|y_i - x_i|} < 1 \text{ if } x_i > 0, y_i < 0, & 1 \text{ if } x_i = 0 \text{ or } y_i = 0 \\ \frac{|(x_i + y_i)|}{|x_i - y_i|} < 1 \text{ if } x_i < 0, y_i > 0, & \text{undefined if both } x_i = 0 \ \& \ y_i = 0. \end{cases}$$

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Now, if $\hat{x}_i > 0$, if $y_i > \hat{x}_i$,

$\frac{\partial}{\partial x_i} f(\hat{x}) \geq \frac{|\hat{x}_i| - |y_i|}{|y_i| - |\hat{x}_i|} = -1$ and if $0 < y_i < \hat{x}_i$,

$\frac{\partial}{\partial x_i} f(\hat{x}) \leq \frac{|\hat{x}_i| - |y_i|}{|y_i| - |\hat{x}_i|} = -1$

Thus, $\frac{\partial}{\partial x_i} f(\hat{x}) = -1$.

If $\hat{x}_i < 0$, if $y_i < \hat{x}_i$,

$\frac{\partial}{\partial x_i} f(\hat{x}) \leq \frac{|\hat{x}_i| - |y_i|}{-|y_i| + \hat{x}_i} = 1$ and if $0 < y_i < \hat{x}_i$,

$\frac{\partial}{\partial x_i} f(\hat{x}) \geq \frac{|\hat{x}_i| - |y_i|}{-|y_i| + \hat{x}_i} = 1$

Thus, $\frac{\partial}{\partial x_i} f(\hat{x}) = 1$.

Thus, we've established both directions of the equivalence.

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$\boxed{\text{T } 4.13}$

$$\min \; c^T x$$
$$\text{s.t. } Ax \preceq b, \; \forall A \in \mathcal{A},$$

$x \in \mathbb{R}^n$, $\mathcal{A} \subseteq \mathbb{R}^{m \times n} = \{ A \in \mathbb{R}^{m \times n} \mid \bar{A}_{ij} - V_{ij} \leq A_{ij} \leq \bar{A}_{ij} + V_{ij}, \; i = 1, \dots, m \}$

We could write:

$$\min \; c^T x$$
$$\text{s.t. } (\bar{A} + tV) x \leq b$$
$$-1 \leq t \leq 1.$$

But this is not an LP.

$$\boxed{\begin{array}{l} \text{Let } Y = \begin{bmatrix} X \\ T \end{bmatrix}, \; Q = \begin{bmatrix} \bar{A} & V \end{bmatrix}, \; C^T = \begin{bmatrix} c & 0 \end{bmatrix}^T \\ \\ \min \; C^T Y \\ \text{s.t. } QY \leq b \\ \begin{bmatrix} -1^T & 1^T \end{bmatrix} Y \leq 0 \\ \begin{bmatrix} -1^T & -1^T \end{bmatrix} Y \leq 0 \end{array}}$$

Because we then have:

$$\min \; C^T Y = \min \; c^T X + 0^T T$$
$$\text{s.t. } \bar{A} X + TV \leq b$$
$$T \leq X \Rightarrow tX \leq X \Rightarrow t \leq 1$$
$$-T \leq X \Rightarrow -tX \leq X \Rightarrow -t \leq 1$$
$$\Rightarrow t \geq -1.$$

$\boxed{\text{A 3.5}}$

$$\min \frac{\max_{i=1,\dots,m} (a_i^T x + b_i)}{\min_{i=1,\dots,p} (c_i^T x + d_i)} \qquad ①$$

$$\text{s.t.} \quad Fx \leq g,$$

$x \in \mathbb{R}^n$, $c_i^T x + d_i > 0$, $\max_{i=1,\dots,m}(a_i^T x + b_i) \geq 0$.
for all x s.t. $Fx \leq g$.

Note

$$\min_x \max_{a,b} (a^T x, b^T x) \Leftrightarrow \min t \quad \text{s.t.} \quad a^T x \leq t, \quad b^T x \leq t$$

Let's try: $$\min_{y,z} \max_i (a_i^T y + b_i z) \qquad ②$$

$$\text{s.t.} \quad Fy - gz \leq 0$$
$$\min_i (c_i^T y + d_i z) = 1$$
$$z \geq 0$$

Let $y = \dfrac{x}{\min_i (c_i^T x + d_i)}$, $z = \dfrac{1}{\min_i (c_i^T x + d_i)}$.

Then, if x feasible in ①, y, z are feasible in ②, with the same objective value:

$$\max_i \left(\frac{a_i^T x + b_i}{\min_i (c_i^T x + d_i)} \right) = \frac{\max_i (a_i^T x + b_i)}{\min_i (c_i^T x + d_i)}$$

And constraints:

$$F \frac{x}{\min_i (c_i^T x + d_i)} - g \frac{1}{\min_i (c_i^T x + d_i)} \leq 0 \Leftrightarrow Fx \leq g$$

$$\min_i \left(c_i^T \frac{x}{\min_i (c_i^T x + d_i)} + d_i \frac{1}{\min_i (c_i^T x + d_i)} \right) = \frac{\min_i (c_i^T x + d_i)}{\min_i (c_i^T x + d_i)} = 1.$$

Conversely, if (y, z) feasible in ②, with $z \neq 0$, then $x = y/z$ is feasible, with the same objective value. Following the logic in the text, we can get arbitrarily close in ① to the objective value in ②.

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This is an LP, except for the term
$\min_i (c_i^T y + d_i z) = 1$.

$\Rightarrow \quad c_i^T y + d_i z \geq 1$.

If we add $\sum_i (c_i^T y + d_i z)$ to the objective,
this will force $\min(c_i^T y + d_i z) = 1$.

We have:

$$\boxed{\begin{array}{ll} \min & t + \sum_i (c_i^T y + d_i z) \\ \text{s.t.} & a_i^T y + b_i z \leq t \\ & Fy - gz \preceq 0 \\ & c_i^T y + d_i z \geq 1 \\ & z \geq 0 \end{array}}$$

$\boxed{A\ 3.39}$ $\min\ c^T x$
s.t. $Gx \preceq h$.
c random.

$\bar{c} = \sum_{i=1}^{m} p_i c_i \quad \Sigma = \sum_{i=1}^{m} p_i (c_i - \bar{c})(c_i - \bar{c})^T$

Let $f_\gamma(y) = \frac{1}{\gamma} \log \sum_{i=1}^{m} p_i e^{\gamma y_i}$, $p_i > 0$ $\sum_i p_i = 1$.

$\min\ f_\gamma(Cx)$
s.t. $Gx \preceq h$
$C = \begin{bmatrix} - c_1 - \\ \vdots \\ - c_m - \end{bmatrix}$

(a.) $\lim_{\gamma \to 0} f_\gamma(Cx) = \lim_{\gamma \to 0} \frac{1}{\gamma} \log \sum_i p_i e^{\gamma c_i^T x}$

$\frac{\partial}{\partial \gamma} \gamma = 1$, $\frac{\partial}{\partial \gamma} \log \sum (f_i(\gamma)) = \frac{1}{\sum f(\gamma)} \frac{\partial}{\partial \gamma} \sum f(\gamma)$

$= \frac{1}{\sum f(\gamma)} \sum \frac{\partial}{\partial \gamma} f(\gamma) = \frac{1}{\sum f(\gamma)} \sum_i p_i c_i^T x\, e^{\gamma c_i^T x}$

$= \frac{\sum_i p_i c_i^T x\, e^{\gamma c_i^T x}}{\sum_i p_i e^{\gamma c_i^T x}}$

$\lim_{\gamma \to 0} f_\gamma(Cx) = \lim_{\gamma \to 0} \frac{\sum_i p_i c_i^T x\, e^{\gamma c_i^T x}}{\sum_i p_i e^{\gamma c_i^T x}}$

(by L'Hôpital's rule)

$= \sum_i p_i c_i^T x \,/\, \sum_i p_i = \bar{c}^T x$

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Let $q = \max_i c_i^T x$.

Then, $f_\gamma(cx) = \frac{1}{\gamma} \log \sum_i p_i e^{\gamma q} e^{-\gamma a_i}$, $a_i \geq 0$

$$\Rightarrow \lim_{\gamma \to \infty} f_\gamma(cx) = \lim_{\gamma \to \infty} \frac{1}{\gamma} \log \left(p_q e^{\gamma q} \right)$$

$$= \lim_{\gamma \to \infty} \left(\frac{1}{\gamma} \log (p_q) + \frac{1}{\gamma} \gamma q \right) = q = \max_i c_i^T x$$

Let $l = \min_i c_i^T x$

Then, $f_\gamma(cx) = \frac{1}{\gamma} \log \sum_i p_i e^{\gamma l} e^{\gamma b_i}$, $b_i \geq 0$.

$$\Rightarrow \lim_{\gamma \to -\infty} f_\gamma(cx) = \lim_{\gamma \to -\infty} \frac{1}{\gamma} \log \left(p_l e^{\gamma l} \right)$$

$$= \lim_{\gamma \to -\infty} \left(\frac{1}{\gamma} \log (p_l) + \frac{1}{\gamma} \gamma l \right) = l = \min_i c_i^T x.$$

(b.) Is $f_\gamma(y)$ monotonically increasing?

$$\frac{\partial}{\partial \gamma} f_\gamma(y) = -\frac{1}{\gamma^2} \log \sum_i p_i e^{\gamma y_i} + \frac{\sum p_i y_i e^{\gamma y_i}}{\gamma \sum_i p_i e^{\gamma y_i}}$$

$$= \left(\gamma \sum_i p_i e^{\gamma y_i} \right)^{-1} \left(\sum p_i y_i e^{\gamma y_i} - \frac{1}{\gamma} \log \sum_i p_i e^{\gamma y_i} \right)$$

$$= \left(\gamma \sum_i p_i e^{\gamma y_i} \right)^{-1} \left(\sum_i p_i y_i e^{\gamma y_i} - f_\gamma(y) \right)$$

$$\frac{\partial}{\partial \gamma} f_\gamma(y) \Big|_{\gamma = 0} = \frac{1}{\gamma} \left(\sum_i p_i y_i - \frac{1}{\gamma} \log \sum_i p_i \right) \Big|_{\gamma = 0}$$

Instead. Is $\frac{1}{\gamma} \log \sum p_i e^{\gamma y_i} \geq \sum p_i y_i$, if $\gamma > 0$?

$$\Leftrightarrow \sum p_i e^{\gamma y_i} \geq e^{\gamma \bar{y}} \Leftrightarrow \sum_i p_i e^{\gamma y_i} \geq \sum_i p_i e^{\gamma \bar{y}}$$

$$\Leftrightarrow \sum_i p_i \left(e^{\gamma y_i} - e^{\gamma \bar{y}} \right) \geq 0 \Leftrightarrow e^{\gamma} \sum_i p_i \left(e^{y_i} - e^{\bar{y}} \right) \geq 0$$

$$\Leftrightarrow \sum_i p_i (e^{y_i} - e^{\bar{y}}) \geq 0$$

$$= \sum_i p_i e^{\bar{y}} (e^{a_i} - e^0) \geq 0, \ \sum a_i = 0$$

Clearly, the positive a_i terms will dominate the negative a_i's.

If $\gamma < 0$, we have that
$\sum_i p_i e^{\gamma y_i} \leq e^{\gamma \bar{y}} \Leftrightarrow \sum_i p_i e^{\bar{y}}(e^{a_i} - 1) \leq 0.$

(c.) Let $\hat{y} = (\bar{c}^T x) 1$

$$\nabla f_\gamma(y)_j = \frac{\partial}{\partial y_j} \frac{1}{\gamma} \log \sum p_i e^{\gamma y_i}$$

$$= \frac{1}{\gamma} \frac{1}{\sum p_i e^{\gamma y_i}} \frac{\partial}{\partial y_j} \sum p_i e^{\gamma y_i} = \frac{\gamma}{\gamma} \frac{p_j e^{\gamma y_j}}{\sum p_i e^{\gamma y_i}} = ?$$

$$\nabla f_\gamma(x)_j = \frac{\partial}{\partial x_j} \frac{1}{\gamma} \log \sum p_i e^{\gamma \bar{c}^T x_i}$$

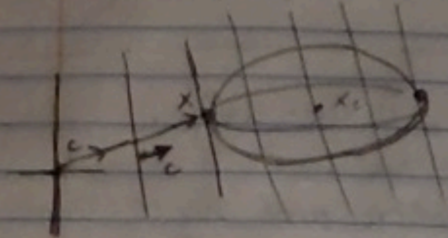
$$= \frac{1}{\gamma} \frac{1}{\sum p_i e^{\gamma \bar{c}^T x_i}} \frac{\partial}{\partial x_j} \sum p_i e^{\gamma \bar{c}^T x_i}$$

$$= \frac{1}{\gamma} \frac{1}{\sum p_i e^{\gamma \bar{c}^T x_i}} p_j \gamma \bar{c}^T e^{\gamma \bar{c}^T x_i}$$

$$= p_j \sum p_i c_i$$

$\boxed{\text{T 4.21 (b)}}$

min $c^T x$
s.t. $(x-x_c)^T A (x-x_c) \le 1$,
$A \in S^n_{++}$ & $c \neq 0$.



Clearly, the optimal x is the point on the boundary of the ellipsoid at which a hyperplane with normal vector c is tangent to the ellipsoid
(Actually, there are 2 such points: one a maximum and one a minimum.)

That is, we want x such that x is parallel to c and $(x-x_c)^T A (x-x_c) = 1$.

Actually, we can set $x = -\alpha\hat{c}$, where $\hat{c} = \frac{c}{\|c\|}$.
Then, $c^T x = -\alpha\|c\|$.

Then, $(-\alpha\hat{c} - x_c)^T A (-\alpha\hat{c} - x_c) = 1$

$\Rightarrow \alpha^2 \hat{c}^T A \hat{c} + \alpha \hat{c}^T A x_c + \alpha x_c^T A \hat{c} + x_c^T A x_c = 1$

$\Rightarrow \alpha^2 \hat{c}^T A \hat{c} + 2\alpha \hat{c}^T A x_c + (x_c^T A x_c - 1) = 0$

$$\Rightarrow \alpha = \frac{-2\hat{c}^T A x_c \pm \sqrt{(2\hat{c}^T A x_c)^2 - 4(\hat{c}^T A \hat{c})(x_c^T A x_c - 1)}}{2\hat{c}^T A \hat{c}}$$

We want the positive sign, since this will decrease $-\alpha\|c\|$.

$$\boxed{x = -\alpha\hat{c}, \quad \alpha = \frac{-\hat{c}^T A x_c + \sqrt{(\hat{c}^T A x_c)^2 - (\hat{c}^T A \hat{c})(x_c^T A x_c - 1)}}{\hat{c}^T A \hat{c}} \quad (\hat{c} = c/\|c\|)}$$

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$\boxed{\text{A7.9}}$ $\min g(x) = \max_k \| f_k(x) - y^{(k)} \|_2$.

$f_k(x) = \frac{1}{c_k^T x + d_k}(A_k x + b_k)$, $P_k = \begin{bmatrix} A_k & b_k \\ c_k^T & d_k \end{bmatrix}$, $k = 1, \ldots, N$.

(a.) Let $g(a_k) = \max_k \| a_k - y_k \|_2$

Let $r(a_k) = \| a_k - y_k \|_2$

$r(a_k)$ is a convex function and hence quasiconvex. $r(f_k(x))$ is then quasi-convex since $f_k(x)$ is a linear fractional function. Then, we see that $g(r(f_k(x)))$ is a maximum of quasiconvex functions and hence also quasiconvex.

$$\begin{aligned} \min \quad & t \\ \text{s.t.} \quad & \| f_k - y_k \|_2 \leq t \end{aligned}$$

$$\Updownarrow$$

$$\begin{aligned} \min \quad & t \\ \text{s.t.} \quad & \left\| \frac{A_k x + b_k - (c_k^T x + d_k) y_k}{c_k^T x + d_k} \right\| \leq t \end{aligned}$$

$$\| A_k x + b_k - (c_k^T x + d_k) y_k \| \leq t$$

In [1]:
```
import numpy as np
import cvxpy as cp
import matplotlib.pyplot as plt
```

In [29]:
```
P1 = np.array([[1,0,0,0],[0,1,0,0],[0,0,1,0]])
P2 = np.array([[1,0,0,0],[0,0,1,0],[0,-1,0,10]])
P3 = np.array([[1,1,1,-10],[-1,1,1,0],[-1,-1,1,10]])
P4 = np.array([[0,1,1,0],[0,-1,1,0],[-1,0,0,10]])

y1 = np.array([0.98, 0.93])
y2 = np.array([1.01, 1.01])
y3 = np.array([0.95, 1.05])
y4 = np.array([2.04, 0.00])

P = np.array([P1,P2,P3,P4])
```

In [17]:
```
def f_Abcd(P):
    A = P[0:2,0:3]
    b = P[0:2,3]
    c = P[2,0:3]
    d = P[2,3]
    return A, b, c, d
```

In [37]:
```
A1, b1, c1, d1 = f_Abcd(P1)
A2, b2, c2, d2 = f_Abcd(P2)
A3, b3, c3, d3 = f_Abcd(P3)
A4, b4, c4, d4 = f_Abcd(P4)
```

In [53]:
```
# Define and solve the CVXPY problem.

x = cp.Variable(3)
f1 = cp.norm((A1@x+b1)/(c1@x+d1)-y1)
f2 = cp.norm((A2@x+b2)/(c2@x+d2)-y2)
f3 = cp.norm((A3@x+b3)/(c3@x+d3)-y3)
f4 = cp.norm((A4@x+b4)/(c4@x+d4)-y4)
cost = cp.maximum(f1,f2,f3,f4)
prob = cp.Problem(cp.Minimize(cost))
prob.solve(qcp=True)
```

```
---------------------------------------------------------------------------
TypeError                                 Traceback (most recent call last)
<ipython-input-53-e3fa284df401> in <module>
      6 f3 = cp.norm((A3@x+b3)/(c3@x+d3)-y3)
      7 f4 = cp.norm((A4@x+b4)/(c4@x+d4)-y4)
----> 8 cost = cp.max(f1,f2,f3,f4)
      9 prob = cp.Problem(cp.Minimize(cost))
     10 prob.solve(qcp=True)

TypeError: __init__() takes from 2 to 4 positional arguments but 5 were given
```

In [52]:
```
# Define and solve the CVXPY problem.

X = cp.Variable(4)
x = X[0:3]
t = X[3]
f1 = cp.norm((A1@x+b1)/(c1@x+d1)-y1)
f2 = cp.norm((A2@x+b2)/(c2@x+d2)-y2)
f3 = cp.norm((A3@x+b3)/(c3@x+d3)-y3)
f4 = cp.norm((A4@x+b4)/(c4@x+d4)-y4)
cost = t
constr = [f1 <= t,f2 <= t,f3 <= t,f4 <= t]

prob = cp.Problem(cp.Minimize(cost), constr)
prob.solve(qcp=True)
```

```
---------------------------------------------------------------------------
DQCPError                                 Traceback (most recent call last)
<ipython-input-52-14cbef0f19c8> in <module>
     12
     13 prob = cp.Problem(cp.Minimize(cost), constr)
---> 14 prob.solve(qcp=True)

/opt/anaconda3/lib/python3.7/site-packages/cvxpy/problems/problem.py in solve(self, *args, **kwargs)
    287         else:
    288             solve_func = Problem._solve
--> 289         return solve_func(self, *args, **kwargs)
    290
    291     @classmethod

/opt/anaconda3/lib/python3.7/site-packages/cvxpy/problems/problem.py in _solve(self, solver, warm_st
art, verbose, parallel, gp, qcp, **kwargs)
    549         if qcp and not self.is_dcp():
    550             if not self.is_dqcp():
--> 551                 raise error.DQCPError("The problem is not DQCP.")
    552             reductions = [dqcp2dcp.Dqcp2Dcp()]
    553             if type(self.objective) == Maximize:

DQCPError: The problem is not DQCP.
```

In [126]:

```
# Define and solve the CVXPY problem.

x = cp.Variable(3)
f1 = cp.norm((A1@x+b1)-(c1@x+d1)*y1)
f2 = cp.norm((A2@x+b2)-(c2@x+d2)*y2)
f3 = cp.norm((A3@x+b3)-(c3@x+d3)*y3)
f4 = cp.norm((A4@x+b4)-(c4@x+d4)*y4)
t_low = 0
t_high = 2
for i in np.arange(20):
    t = (t_low+t_high)/2
#     print(t_low)
#     print(t_high)
    cost = cp.sum(x)
    constr = [f1 <= t,f2 <= t,f3 <= t,f4 <= t]
    prob = cp.Problem(cp.Minimize(cost), constr)
    s = prob.solve(qcp=True)
#     print(x.value)
    if s > 10000:
        t_low = t
    else:
        t_high = t
    if t_high-t_low < 10e-4:
#         print("done")
        break
print(t)
```

```
0.2587890625
```

In [ ]: