CS267A: Homework #3

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Problem 1

In Homework 2, Problem 6, I used a Monte Carlo sampler to compute the probability of the truth of two CNF statements, over three trials. The estimates were, for the first statement, $Pr(\Delta = true) = 0.569, 0.548, 0.592$ and for the second statement, $Pr(\Delta = true) = 0.643, 0.652, 0.657$. The exact answers, using Dice, are:

1. **Solution:**
   Part 1: $Pr(\Delta = true) = 0.561600$.
   Part 2: $Pr(\Delta = true) = 0.665000$.
   The 3-trial-average Monte Carlo estimate thus differs from the true value by about 1.4% for statement 1 and 2.16% for statement 2.

2. **Dice Code (Part 1):**

```
let a = flip 0.3 in
let b = flip 0.6 in
let c = flip 0.1 in
let d = flip 0.8 in
let e = flip 0.4 in

let s1 = (a || b || !c) && (b || c || d || !e) && (!b || !d || e) &&
    (!a || !b) in
s1
```

3. **Dice Code (Part 2):**

```
let a = flip 0.2 in
let b = flip 0.1 in
let c = flip 0.8 in
let d = flip 0.3 in
let e = flip 0.5 in

let s2 = (!a || c || d) && (b || c || !d || e) && (!c || d || !e) in
s2
```

Problem 2

Solution:
Part 1: $P(b \vee e | j \wedge m) = 0.123889$
Part 2: $P(e | \neg b \wedge m) = 0.024308$

Dice Code (Part 1):

```
// Probability of, resp, burglary and earthquake
let b = flip 0.001 in
let e = flip 0.002 in

// Probability of burglary or earthquake
let b_or_e = b || e in

// Probability of alarm, given burglary and/or earthquake
let a =
    if (b) then
            (if (e) then
                    (flip 0.95)
             else
                    (flip 0.94)
            )
    else
            (if (e) then
                    (flip 0.29)
             else
                    (flip 0.01)
            )
    in

// Probability John calls given alarm
let j =
    if (a) then
            (flip 0.90)
    else
            (flip 0.05)
    in

// Probability Mary calls given alarm
let m =
    if (a) then
            (flip 0.70)
    else
            (flip 0.01)
    in

let tmp = observe j && m in
b_or_e
```

Dice Code (Part 2):

```
// Probability of, resp, burglary and earthquake
let b = flip 0.001 in
let e = flip 0.002 in

// Probability of alarm, given burglary and/or earthquake
let a =
    if (b) then
            (if (e) then
                    (flip 0.95)
             else
                    (flip 0.94)
            )
    else
            (if (e) then
                    (flip 0.29)
             else
                    (flip 0.01)
            )
    in

// Probability John calls given alarm
let j =
    if (a) then
            (flip 0.90)
    else
            (flip 0.05)
    in

// Probability Mary calls given alarm
let m =
    if (a) then
            (flip 0.70)
    else
            (flip 0.01)
    in

let tmp = observe !b && m in
e
```

Problem 3

Note: We can assume, without loss of generality, that the car is placed behind a random door and we always initially choose the first door, since we could switch door labels otherwise. (Equivalently, we could suppose that the car is always placed behind door 1, and we choose a random door).

Problem 3 continued on next page...

Solution:
Part 1: The probability that the car is behind the door we initially chose is 1/3 and the probability that it is behind the remaining door is 2/3.
Part 2: If we observe that the host shows a goat behind door 3, the probability that the car is behind door 1 is 0.5 and the prob. that it is behind door 2 is 0.5. If we observe that the host shows a goat behind door 2, the prob. that the car is behind door 1 is 0.75 and the prob. that it is behind door 3 is 0.25.

Dice Code (Part 1):

```
// Randomly choose a door.
let door = discrete(0.33333,0.33333,0.33333) in

let host =
    // If car is behind door 1,
    // host reveals doors 2 and 3
    // with equal probability.
    if ((door == int(3, 0))) then
            (flip 0.5)
    else
            // If car is behind door 2,
            // host shows door 3.
            (if (door == int(3, 1)) then
                    (true)
            // If car is behind door 3,
            // host shows door 2.
            else
                    (false)
            )
    in

// We observe host chose door 3.
let tmp = observe host in

door
```

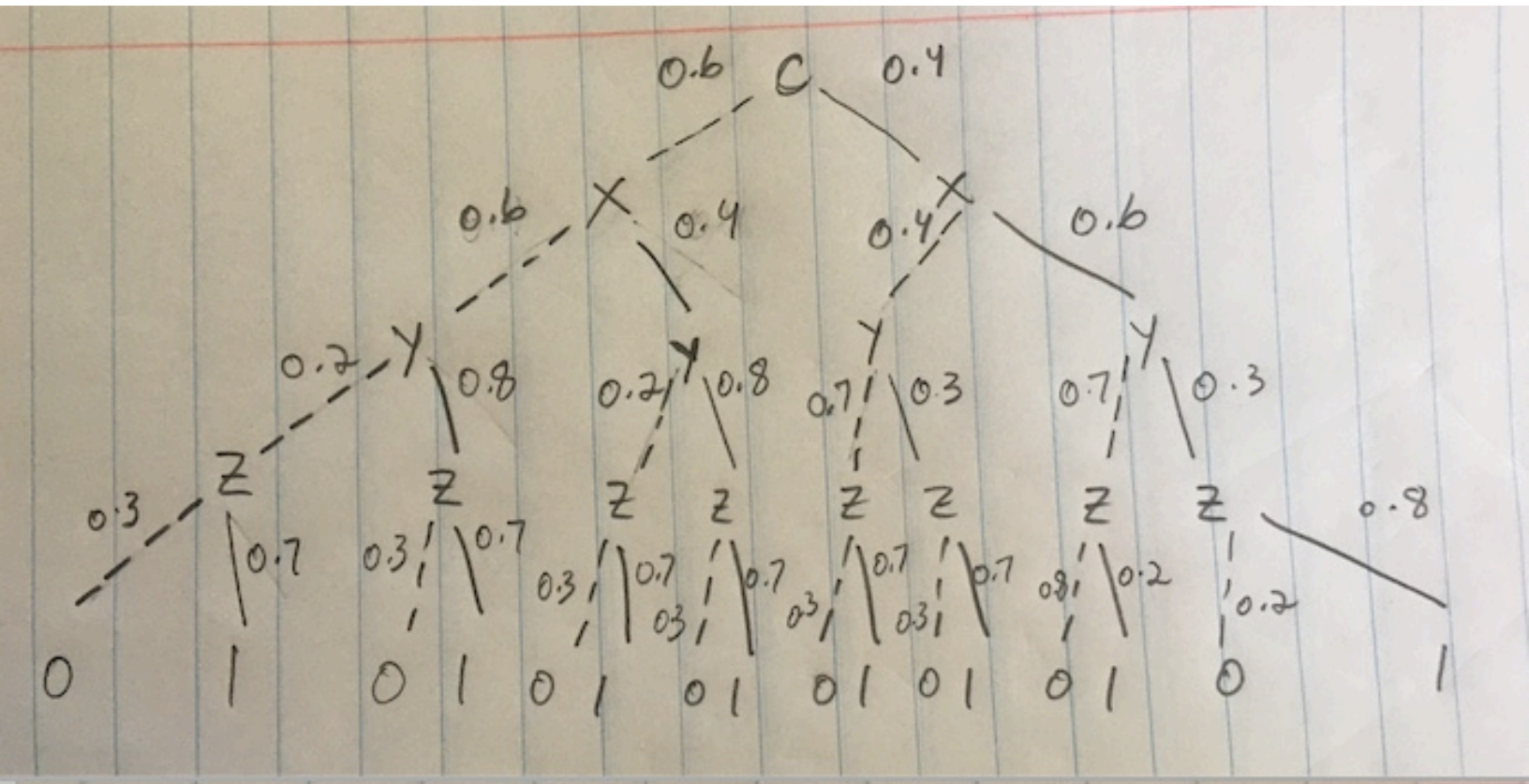
Dice Code (Part 2):

```
// Randomly choose a door.
let door = discrete(0.6,0.3,0.1) in

let host =
    // If car is behind door 1,
    // host reveals doors 2 and 3
    // with equal probability.
    if ((door == int(3, 0))) then
            (flip 0.5)
    else
            // If car is behind door 2,
            // host shows door 3.
            (if (door == int(3, 1)) then
```

```
                (true)
            // If car is behind door 3,
            // host shows door 2.
            else
                (false)
            )
        in

    // We observe host chose door 3.
    let tmp = observe host in

    // We observe host chose door 2.
    // let tmp = observe !host in

door
```

Problem 4

(See attached photos.)



The top diagram can be reduced to the bottom one. The y variable can be eliminated completely.