CS 267A - Homework 1

Peter Racioppo (103953689)

April 9, 2020

Problem 1

1. Knowledge Base:
   $\neg b \Rightarrow c$
   $\neg a \Rightarrow \neg c$
   c
   $\neg a \vee \neg b$

2. Truth Table:
   There are $2^3 = 8$ models.

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	T	T	F	F	F	T	T	F
T	T	F	F	F	T	T	T	F
T	F	T	F	T	F	T	T	T
T	F	F	F	T	T	F	T	T
F	T	T	T	F	F	T	F	T
F	T	F	T	F	T	T	T	T
F	F	T	T	T	F	T	F	T
F	F	F	T	T	T	F	T	T

3. Logical Consistency:
   Prop. 3 is True in 4 models.

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	T	T
F	T	T	T	F	F	T	F	T
F	F	T	T	T	F	T	F	T

   Prop. 1 is true in all 4 of these models.
   Prop. 2 is satisfied in 2 of these models.

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	T	T	F	F	F	T	T	F
T	F	T	F	T	F	T	T	T

   Prop. 4 is satisfied in 1 of these models.

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	F	T	F	T	F	T	T	T

Thus, the three testimonies are consistent, and they are satisfied by exactly one model.

4. Who Lied?
   If everyone is innocent, we take the first row of the truth table.

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	T	T	F	F	F	T	T	F

   Prop. 1-3 are all true, but Prop. 4 ($\neg a \vee \neg b$) is false. Thus, C lied.

5. Guilt/Innocence:
   If all of the testimonies are true, only one model is satisfied:

a	b	c	$\neg a$	$\neg b$	$\neg c$	$\neg b \Rightarrow c$	$\neg a \Rightarrow \neg c$	$\neg a \vee \neg b$
T	F	T	F	T	F	T	T	T

   Thus, B is guilty.

Problem 2

1. $(a \vee b \vee \neg c) \wedge (a \vee \neg d)$
   If a = 1, we have $1 \wedge 1 = 1$. Thus, this statement is satisfiable.
   DPLL:
   $S = (a \vee b \vee \neg c) \wedge (a \vee \neg d)$
   if $(a = 1)$
       $S = 1$
   else if $(a = 0)$
       $S = (b \vee \neg c) \wedge \neg d$
       if $(d = 1)$
           $S = 0$
       else if $(d = 0)$
           $S = b \wedge \neg c$
           if $(b = 0)$
               $S = 0$
           else if $(b = 1)$
               $S = \neg c$
               if $(c = 0)$
                   $S = 1$
               else if $(c = 1)$
                   $S = 0$

   SAT: (For example: $a = 1, b = 0, c = 0$)

2. $\neg(a \vee b) \wedge (\neg c \vee (c \wedge d)) \Rightarrow \neg c \wedge d$
   If $d = 0$, we have $\neg a \wedge \neg b \wedge \neg c \Rightarrow 0$.
   Then, if $a \vee b \vee c$, we have $0 \Rightarrow 0$, which is True. Thus, the statement is satisfiable.
   DPLL:
   $S = \neg(a \vee b) \wedge (\neg c \vee (c \wedge d)) \Rightarrow \neg c \wedge d$
   if ($d = 0$)
      $S = \neg a \wedge \neg b \wedge \neg c \Rightarrow 0$
      if ($a = 1$)
         $S = 1$
      else if ($a = 0$)
         $\neg b \wedge \neg c \Rightarrow 0$
            $S =$ True if $b \vee c$ and $S =$ False otherwise.
   else if ($d = 1$)
      $\neg a \wedge \neg b \Rightarrow \neg c$
      if ($c = 1$)
         $S = \neg a \wedge \neg b \Rightarrow 0$
            $S =$ True if $a \vee b$ and $S =$ False otherwise.
      else if ($c = 0$)
         $S = \neg a \wedge \neg b \Rightarrow 1$
            $S =$ False if $a \vee b$ and $S =$ True otherwise.
   SAT: (For example: $a = 1, b = 1, c = 1, d = 0$)

3. DPLL:
   $S = (x \vee y \vee z) \wedge (x \vee y \vee \neg z) \wedge (x \vee \neg y \vee z) \wedge (x \vee \neg y \vee \neg z) \wedge (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$
   if ($x = 0$)
      $S = (y \vee z) \wedge (y \vee \neg z) \wedge (\neg y \vee z) \wedge (\neg y \vee \neg z)$
      if ($y = 0$)
         $S = z \wedge \neg z = 0$
      else if ($y = 1$)
         $S = z \wedge \neg z = 0$
   else if ($x = 1$)
      $S = (y \vee z) \wedge (y \vee \neg z) \wedge (\neg y \vee z) \wedge (\neg y \vee \neg z)$
      if ($y = 0$)
         $S = z \wedge \neg z = 0$
      else if ($y = 1$)
         $S = z \wedge \neg z = 0$

   Thus, this statement is not satisfiable. UNSAT.