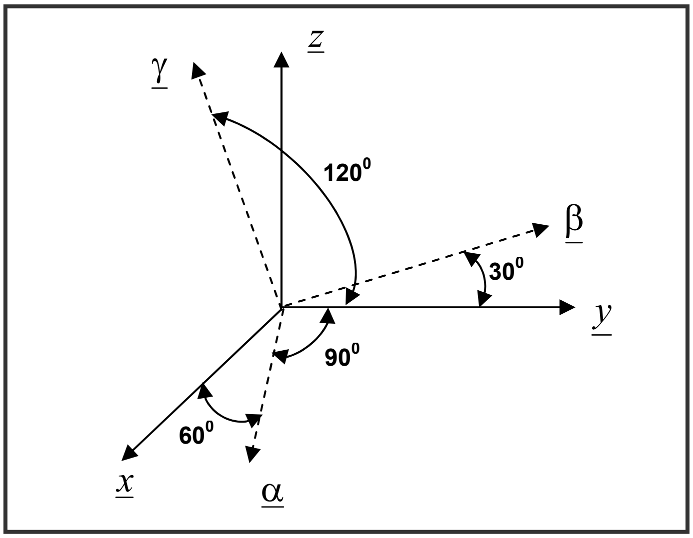
ME4524 – Robotics and Automation

Exercise # 1 – Solutions

1. Determine the overall rotation matrix that carries axes $\underline{x}\underline{y}\underline{z}$ into the axes $\underline{\alpha}\underline{\beta}\underline{\gamma}$ as shown in the Figure below.



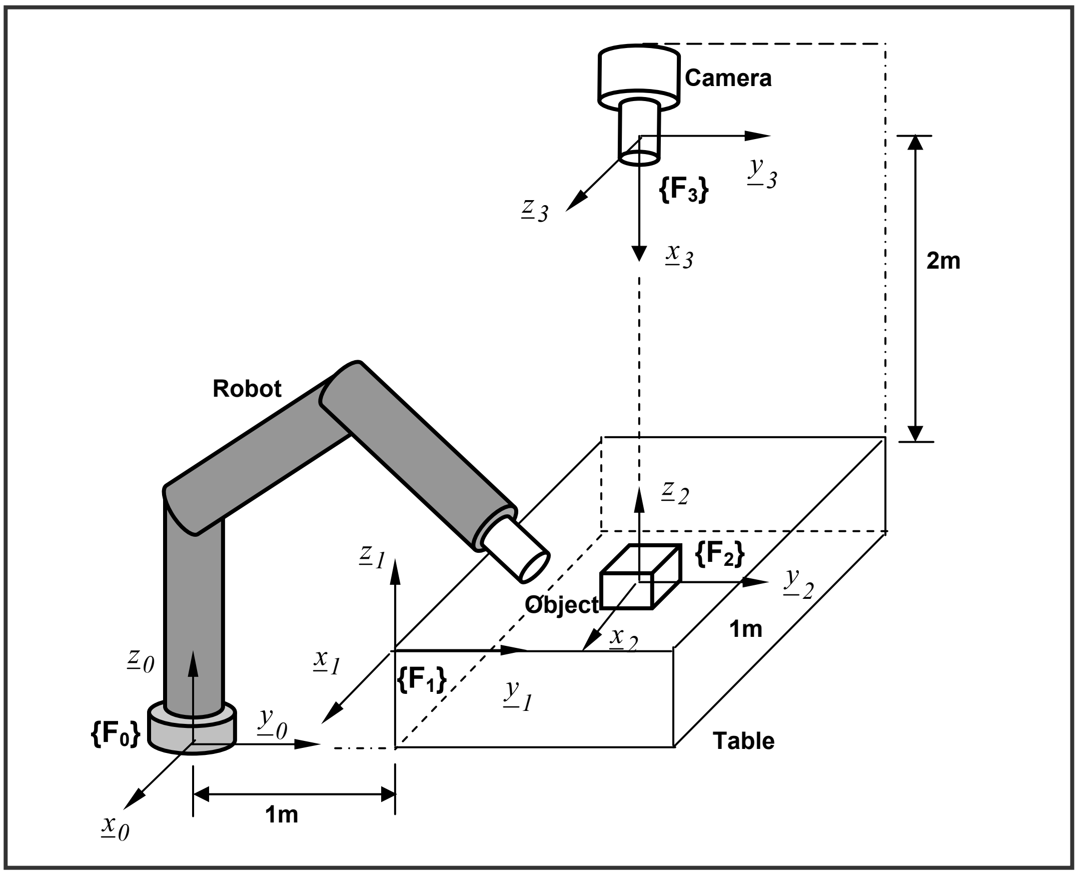
Answer: The $\underline{\alpha}\underline{\beta}\underline{\gamma}$ reference frame is obtained by rotating 60 degrees about the $\underline{y}$ axis followed by a rotation of 30 degrees about the $\underline{\alpha}$ axis.

Therefore $R = R_{\underline{y}}(60^\circ) \cdot R_{\underline{\alpha}}(30^\circ)$

$$R = \begin{bmatrix} \cos 60 & 0 & \sin 60 \\ 0 & 1 & 0 \\ -\sin 60 & 0 & \cos 60 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos 30 & -\sin 30 \\ 0 & \sin 30 & \cos 30 \end{bmatrix}$$

$$R = \begin{bmatrix} 0.5 & 0.433 & 0.75 \\ 0 & 0.866 & -0.5 \\ -0.866 & 0.25 & 0.433 \end{bmatrix}$$

2. Consider the diagram shown below. A robot base is set up 1m from a table. A frame $\{\mathbf{F_0}\}$ is attached to the base of the robot such that the $\underline{y}_0$ passes through the two legs of the table. The tabletop is 1m high and 1m square. A frame $\{\mathbf{F_1}\}$ is fixed to the edge of the table. A cube measuring 20 cm on each side is placed at the center of the table and a frame $\{\mathbf{F_2}\}$ is defined at the center of the cube. A camera is situated directly above the center of the cube (2m above the tabletop) and a frame $\{\mathbf{F_3}\}$ is attached to the camera. Find the homogenous transformation relating each of these frames to the base frame $\{\mathbf{F_0}\}$. Also find the homogenous transformation matrix that relates the frame $\{\mathbf{F_2}\}$ to $\{\mathbf{F_3}\}$.



Answer: Treat Frame 0 as the fixed frame and all other frames as moving frames.

Frame 0 to Frame 1: Translate 1 m along Y_0.

Translate 1 m along Z_0.

Since all translations are in the fixed frame we can trivially set up the R_{01} transformation matrix.

$$T_{01} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Frame 0 to Frame 2: Translate 1.5 m along Y_0.
Translate 1.1 m along Z_0.
Translate -0.5 m along X_0.

Again, we can trivially set up T_{02}. $T_{02} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 1.1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Frame 0 to Frame 3: Translate 1.5 m along Y_0.
Translate 3 m along Z_0.
Translate -0.5 m along X_0.
Rotate 90° about Y_0.

$$T_{03} = \begin{bmatrix} 1 & 0 & 0 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & -0.5 \\ 0 & 1 & 0 & 1.5 \\ -1 & 0 & 0 & 3 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Frame 2 to Frame 3: Translate 1.9 m along Z_2.
Rotate 90° about Y_2.

$$T_{23} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 1.9 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Now find $(T_{23})^{-1} = T_{32}$ using the formula from Lecture 2.

$$(T_{AB})^{-1} = \left[\begin{array}{c|c} (R_{AB})^T & -(R_{AB})^T \, {}^A p_{AB} \\ \hline [0] & 1 \end{array} \right] = T_{32} = \begin{bmatrix} 0 & 0 & -1 & 1.9 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Or, this can be done directly through:
Frame 2 to Frame 3: Translate 1.9 m along X_3.
Rotate -90° about Y_3.

$$T_{32} = \begin{bmatrix} 0 & 0 & -1 & 1.9 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Similarly, $T_{30} = (T_{03})^{-1} = \begin{bmatrix} 0 & 0 & -1 & 3 \\ 0 & 1 & 0 & -1.5 \\ 1 & 0 & 0 & 0.5 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

3. Compute the homogenous transformation representing a translation of 3 units along x-axis followed by a rotation of 90^0 about the current z-axis followed by a translation of 1 unit along the fixed y-axis. Sketch the frames at the end of each transformation. Find the coordinates of the relocated origin with respect to the original frame in each case.

Answer: Step 1: Translate along X_a axis (fixed) for 3 units.
Step 2: Rotate about Z_b axis (moving) 90^0.
Step 3: Translate along Y_a axis (fixed) for 1 unit.

$$D_{X_a}(3) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} R_{Z_b}(90\circ) = \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} D_{Y_a}(1) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

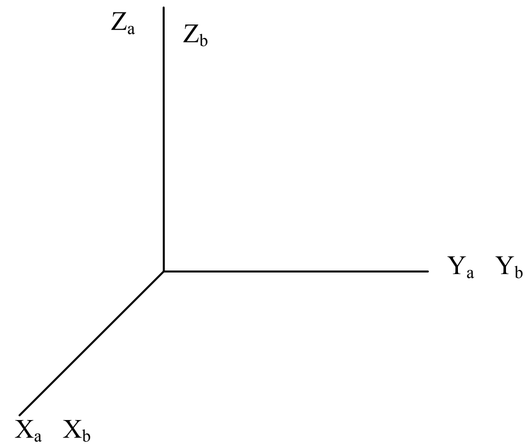
Step 1: $T = D_{X_A}(3) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

Step 2: $T = D_{X_A}(3) R_{z_b}(90\circ) = \begin{bmatrix} 1 & 0 & 0 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

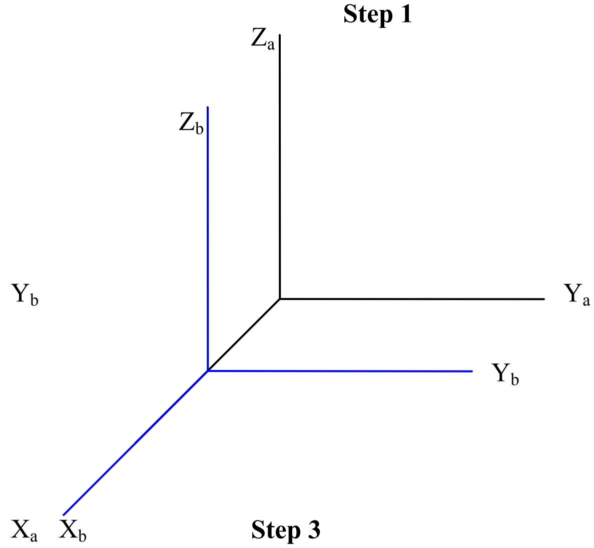
Step 3:

$$T = D_{Y_A}(1) D_{X_A}(3) R_{z_b}(90\circ) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 0 & -1 & 0 & 3 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

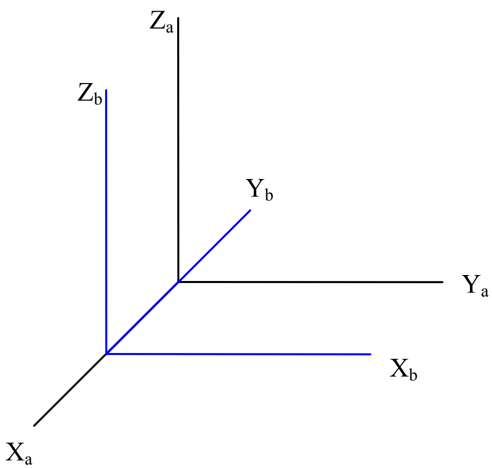
At each step, the first 3 numbers in the 4^{th} column of the rotation matrix give the location of the moving origin with respect to the fixed reference frame.



Step 1



Step 2



Step 3