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(* Peter Racioppo *)
(* Homework 5 *)

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(* Question 1 *)
(* A. *)

ClearAll["Global`*"]
T01 = {{Cos[θ1], 0, -Sin[θ1], 0},
       {Sin[θ1], 0, Cos[θ1], 0},
       {0, -1, 0, 0.4},
       {0, 0, 0, 1}};
T12 = {{Cos[θ2], 0, Sin[θ2], 0},
       {Sin[θ2], 0, -Cos[θ2], 0},
       {0, 1, 0, 0},
       {0, 0, 0, 1}};
T23 = {{Cos[θ3], -Sin[θ3], 0, 0},
       {Sin[θ3], Cos[θ3], 0, 0},
       {0, 0, 1, 0.1},
       {0, 0, 0, 1}};
T3e = {{1, 0, 0, 0.1},
       {0, 1, 0, 0},
       {0, 0, 1, 0.05},
       {0, 0, 0, 1}};

T02 = Simplify[T01.T12];
T03 = Simplify[T02.T23];
T0e = Simplify[T03.T3e];
T0e // MatrixForm;
R01 = T01[[1 ;; 3, 1 ;; 3]];
R02 = T02[[1 ;; 3, 1 ;; 3]];
R03 = T03[[1 ;; 3, 1 ;; 3]];
R0e = T0e[[1 ;; 3, 1 ;; 3]];

px = Cos[θ1] (0.1 * Cos[θ2] Cos[θ3] + 0.15 * Sin[θ2]) - 0.1 * Sin[θ1] Sin[θ3];
py = 0.1 * Cos[θ2] Cos[θ3] Sin[θ1] + 0.15 * Sin[θ1] Sin[θ2] + 0.1 * Cos[θ1] Sin[θ3];
pz = 0.4 + 0.15 * Cos[θ2] - 0.1 * Cos[θ3] Sin[θ2];

z00 = {0, 0, 1};
z01 = R01.{0, 0, 1};
z02 = R02.{0, 0, 1};
z03 = R03.{0, 0, 1};

J0 = Simplify[{{D[px, θ1], D[px, θ2], D[px, θ3]}, {D[py, θ1], D[py, θ2], D[py, θ3]},
               {D[pz, θ1], D[pz, θ2], D[pz, θ3]}, {z00[[1]], z01[[1]], z02[[1]]},
               {z00[[2]], z01[[2]], z02[[2]]}, {z00[[3]], z01[[3]], z02[[3]]}}];
J0 // MatrixForm;

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(* ----- *)
(* B. *)
R30 = Transpose[R03];
Re0 = Transpose[R0e];
Re0 // MatrixForm;

fa = {-8, 0, -10};
ga = R03.{0.04, 0, 0};

F = {fa[[1]], fa[[2]], fa[[3]], ga[[1]], ga[[2]], ga[[3]]};
F // MatrixForm;

τ = Simplify[-Transpose[J0].F];
τ // MatrixForm;

(* ----- *)
(* C. *)

f3 = {fmx, fmy, fmz};
g3 = {Nmx, Nmy, Nmz};
p3e = {0.1, 0, 0.05};
p3eskew = {{0, -0.05, 0},
           {0.05, 0, -0.1},
           {0, 0.1, 0}};
Re3 = {{1, 0, 0},
       {0, 1, 0},
       {0, 0, 1}};
fe = Simplify[Transpose[Re3].f3];
ge = Simplify[Transpose[Re3].g3 - Transpose[Re3].(p3eskew.f3)];
fe // MatrixForm;
ge // MatrixForm;

f0 = R0e.fe;
g0 = R0e.ge;
F1 = {f0[[1]], f0[[2]], f0[[3]], g0[[1]], g0[[2]], g0[[3]]};
F1 // MatrixForm;

(* Angles for B and C *)
θ1 = 90 Degree;
θ2 = 45 Degree;
θ3 = 0 Degree;
F // MatrixForm;
τ // MatrixForm;
F1 // MatrixForm;

(* ----- *)

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(* Question 2 *)
ClearAll["Global`*"]
T01 = {{Cos[θ1], 0, Sin[θ1], 0},
       {Sin[θ1], 0, -Cos[θ1], 0},
       {0, 1, 0, L0},
       {0, 0, 0, 1}};
T12 = {{Cos[θ2], -Sin[θ2], 0, L1 * Cos[θ2]},
       {Sin[θ2], Cos[θ2], 0, L1 * Sin[θ2]},
       {0, 0, 1, 0},
       {0, 0, 0, 1}};
T23 = {{Cos[θ3], -Sin[θ3], 0, L2 * Cos[θ3]},
       {Sin[θ3], Cos[θ3], 0, L2 * Sin[θ3]},
       {0, 0, 1, 0},
       {0, 0, 0, 1}};

T02 = Simplify[T01.T12];
T03 = Simplify[T02.T23];
T03 // MatrixForm;

px = Cos[θ1] (L1 Cos[θ2] + L2 Cos[θ2 + θ3]);
py = (L1 Cos[θ2] + L2 Cos[θ2 + θ3]) Sin[θ1];
pz = L0 + L1 Sin[θ2] + L2 Sin[θ2 + θ3];

R01 = T01[[1 ;; 3, 1 ;; 3]];
R02 = T02[[1 ;; 3, 1 ;; 3]];

z00 = {0, 0, 1};
z01 = R01.{0, 0, 1};
z02 = R02.{0, 0, 1};

J0 = Simplify[{{D[px, θ1], D[px, θ2], D[px, θ3]}, {D[py, θ1], D[py, θ2], D[py, θ3]},
               {D[pz, θ1], D[pz, θ2], D[pz, θ3]}, {z00[[1]], z01[[1]], z02[[1]]},
               {z00[[2]], z01[[2]], z02[[2]]}, {z00[[3]], z01[[3]], z02[[3]]}}];
J0 // MatrixForm;

θ1 = 90 Degree;
θ2 = 135 Degree; θ3 = -90 Degree;
L0 = 1;
L1 = 1;
L2 = 1.5;

K = {{4 * 10^5, 0, 0}, {0, 2 * 10^5, 0}, {0, 0, 1 * 10^5}};
Jt = Transpose[J0];
Ki = Inverse[K];
C1 = Simplify[J0.Ki.Jt];
C1 // MatrixForm;

C2 = Transpose[C1].C1;
C2 // MatrixForm;

eigs = Eigenvalues[C2];
eigs // MatrixForm;

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