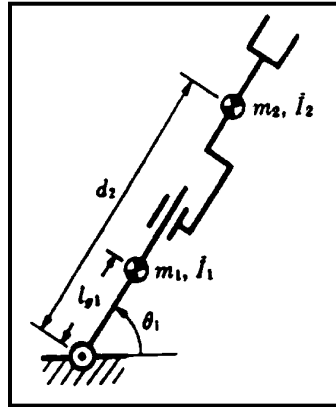


## ME4524 – Robotics and Automation

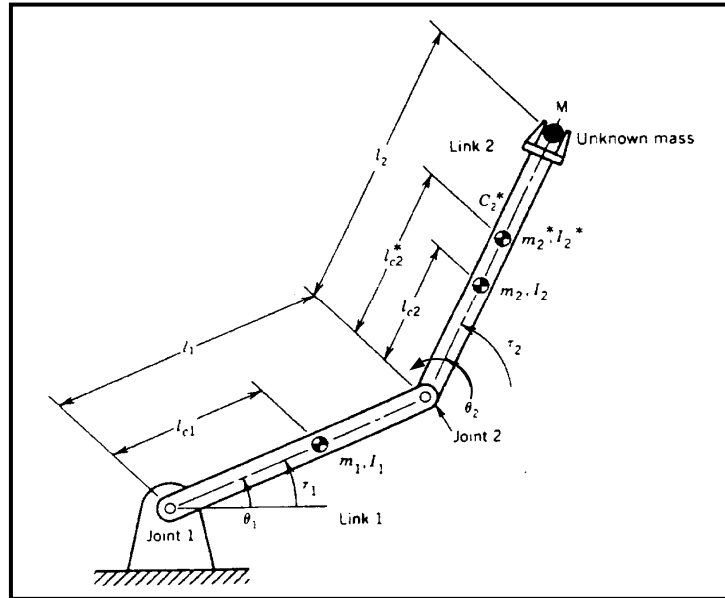
### Exercise # 6

1. Derive the equations of motion for the two-link manipulator shown in the Figure below, using the Lagrangian formulation. Assume that the gravity acts vertically downwards. (Note that  $\tilde{I}_i$  is the moment of inertia about the center of mass of link  $i$ ).

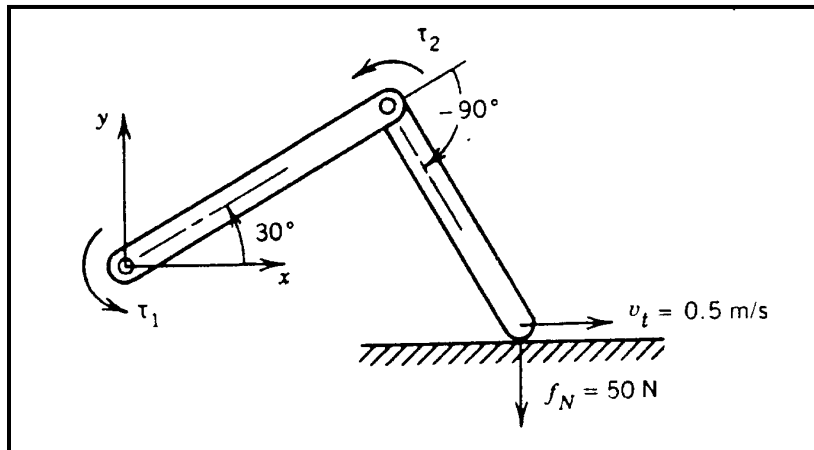


2. Consider the two-degree of freedom manipulator shown in the Figure below. When the manipulator grasps an unknown mass particle  $M$  at the tip of link 2, the mass properties of link 2 change from known values  $m_2, l_{c2}, I_2$  to  $m_2^*, l_{c2}^*, I_2^*$  where  $m_2^* = m_2 + M$ . It is required to identify the unknown mass properties by experiments.
  - (a) The unknown mass  $M$  is modeled as a point mass, and the centroids are assumed to be located on the centerline of each link. Derive the distance to  $l_{c2}^*$  and the centroidal moment of inertia to  $I_2^*$  as functions of the unknown mass.

- (b) Assume that the generalized coordinates are  $\theta_1$  and  $\theta_2$ , the actuators exert torques  $\tau_1$  and  $\tau_2$ , and the manipulator with unknown mass moves at angular velocities  $\dot{\theta}_1, \dot{\theta}_2$  and accelerations  $\ddot{\theta}_1, \ddot{\theta}_2$ . Determine the unknown mass  $M$  at the tip of link 2 from this set of data.



3. The figure below shows the same manipulator from the previous problem. The end-point of the manipulator is in contact with a smooth surface and while applying a normal force  $f_N$  to the surface, the manipulator is moving in a constant speed  $v_t$  along the tangential direction. Compute the required joint torques  $\tau_1$  and  $\tau_2$  in the case shown in the figure. The length of each link is 1 m and the mass of each link is 1 kg. (Hint: you need to consider both the static and dynamic effects).



4. The table below shows the link dimensions and mass properties of the two-degree of freedom manipulator shown in the Figure. Using Lagrangian formulation, compute the joint torques required to move the arm with the specified joint velocities and accelerations at the configuration given below.

