Mae 171A: Final Project
Control Design for an Active Car Suspension System

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For the purpose of this analysis, we model a car suspension system using a one-dimensional "quarter car model." The model consists of a serial chain of two spring-mass-dampers, with the upper (larger) mass corresponding to the vehicle, the upper spring-damper comprising the suspension, which includes an active forcing input, and the lower spring-mass-damper representing the tire. Displacement of the tire due to uneven terrain is treated as a noise input.

The equations of motion for the system are:

$$m_1\ddot{x}_1(t) = -k_1\big(x_1(t) - x_2(t)\big) - b_1\big(\dot{x}_1(t) - \dot{x}_2(t)\big) + f(t)$$

$$m_2\ddot{x}_2(t) = k_1\big(x_1(t) - x_2(t)\big) + b_1\big(\dot{x}_1(t) - \dot{x}_2(t)\big) - k_2(x_2(t) - w(t)) - b_2(\dot{x}_2(t) - \dot{w}(t)) - f(t)$$

where m_1 is the mass of the car, k_1 is the upper spring constant, b_1 is the upper damping constant, m_2 is the lower mass (that of the tire), k_2 is the lower spring constant, and b_2 is the lower damping constant. The forcing term is denoted $f(t)$ and the displacement noise is denoted $w(t)$. The upper and lower mass's displacements from equilibrium are denoted $x_1(t)$ and $x_2(t)$, respectively. We here use system parameters: $m_1 = 2{,}500$ kg, $m_2 = 320$ kg, $k_1 = 80{,}000$ N/m, $k_2 = 500{,}000$ N/m, $b_1 = 350$ N·s/m, and $b_2 = 15{,}000$ N·s/m.

The equilibrium positions of masses one and two are, respectively, $r_1 = \left[\frac{(k_1+k_2)}{k_1k_2}m_1 + \frac{1}{k_1}m_2\right]g$ and $r_2 = \frac{(m_1+m_2)g}{k_2}$, where $g \approx 9.8\ \mathrm{m/s^2}$ is the gravitational acceleration at Earth's surface. With our system parameters, $r_1 = 36.2$ cm and $r_2 = 5.53$ cm.

We take the initial conditions for the displacement from equilibrium and noise as:
$x_1(0) = x_2(0) = 0, \dot{x}_1(0) = \dot{x}_2(0) = 0, w(0) = \dot{w}(0) = 0.$

Taking the Laplace transform of each side,

$$s^2X_1(s) - sx_1(0) - \dot{x}_1(0) = -\frac{k_1}{m_1}\big(X_1(s) - X_2(s)\big) - \frac{b_1}{m_1}\big[s\big(X_1(s) - X_2(s)\big) - (x_1(0) - x_2(0))\big] + \frac{F(s)}{m_1}$$

$$\begin{aligned} s^2X_2(s) - sx_2(0) - \dot{x}_2(0) &= \frac{k_1}{m_2}\big(X_1(s) - X_2(s)\big) + \frac{b_1}{m_2}\big[s\big(X_1(s) - X_2(s)\big) - (x_1(0) - x_2(0))\big] \\ &\quad - \frac{k_2}{m_2}\big(X_2(s) - W(s)\big) - \frac{b_2}{m_2}\big[s\big(X_2(s) - W(s)\big) - \big(x_2(0) - w(0)\big)\big] - \frac{F(s)}{m_2} \end{aligned}$$

Plugging in the initial conditions,

$$s^2X_1(s) = -\frac{k_1}{m_1}\big(X_1(s) - X_2(s)\big) - \frac{b_1}{m_1}\big[s\big(X_1(s) - X_2(s)\big)\big] + \frac{F(s)}{m_1}$$

$$\begin{aligned} s^2X_2(s) &= \frac{k_1}{m_2}\big(X_1(s) - X_2(s)\big) + \frac{b_1}{m_2}\big[s\big(X_1(s) - X_2(s)\big)\big] - \frac{k_2}{m_2}\big(X_2(s) - W(s)\big) \\ &\quad - \frac{b_2}{m_2}\big[s\big(X_2(s) - W(s)\big)\big] - \frac{F(s)}{m_2} \end{aligned}$$

Letting $Y(s) = X_1(s) - X_2(s)$,

$$s^2X_1(s) = -\frac{k_1}{m_1}Y(s) - \frac{b_1}{m_1}sY(s) + \frac{F(s)}{m_1} \tag{1}$$

$$s^2X_2(s) = \frac{k_1}{m_2}Y(s) + \frac{b_1}{m_2}sY(s) - \frac{k_2}{m_2}\big(X_2(s) - W(s)\big) - \frac{b_2}{m_2}s\big(X_2(s) - W(s)\big) - \frac{F(s)}{m_2} \tag{2}$$

Collecting terms in (2),

$$\left(s^2 + \frac{k_2+b_2s}{m_2}\right)X_2(s) = \left(\frac{k_1+b_1s}{m_2}\right)Y(s) + \left(\frac{k_2+b_2s}{m_2}\right)W(s) - \frac{F(s)}{m_2} \tag{3}$$

Subtracting (2) from (1) and collecting terms,

$$\left[s^2 + (k_1 + b_1s)\left(\frac{m_1+m_2}{m_1m_2}\right)\right]Y(s) = \left(\frac{k_2+b_2s}{m_2}\right)X_2(s) - \left(\frac{k_2+b_2s}{m_2}\right)W(s) + \left(\frac{m_1+m_2}{m_1m_2}\right)F(s) \tag{4}$$

Letting $W(s) = 0$ in (3) and (4), we have:

$$A(s)Y(s) = B(s)X_2(s) + EF(s) \tag{5}$$

$$C(s)X_2(s) = D(s)Y(s) - F(s)/m_2, \tag{6}$$

where $A(s) = s^2 + (k_1 + b_1s)E$, $B(s) = \frac{k_2+b_2s}{m_2}$, $C(s) = s^2 + \frac{k_2+b_2s}{m_2}$, $D(s) = \frac{k_1+b_1s}{m_2}$, $E = \left(\frac{m_1+m_2}{m_1m_2}\right)$

Substituting (6) into (5),

$A(s)Y(s) = B(s)C^{-1}(s)[D(s)Y(s) - F(s)/m_2] + EF(s)$

Collecting terms and rearranging, $Y(s) = \frac{\mathrm{E} - B(s)C^{-1}(s)/m_2}{A(s) - B(s)C^{-1}(s)D(s)}F(s)$.

Multiplying by $C(s)$ and substituting in E, $Y(s) = \frac{\left(\frac{m_1+m_2}{m_1m_2}\right)C(s) - \frac{B(s)}{m_2}}{A(s)C(s) - B(s)D(s)}F(s)$.

Now, letting $F(s) = 0$ in (3) and (4), we have:

$$A(s)Y(s) = B(s)\big(X_2(s) - W(s)\big) \tag{7}$$

$$C(s)X_2(s) = D(s)Y(s) + B(s)W(s) \tag{8}$$

Substituting (8) into (7),

$A(s)Y(s) = B(s)\left(C^{-1}(s)\big(D(s)Y(s) + B(s)W(s)\big) - W(s)\right)$

Collecting terms and rearranging, $Y(s) = \frac{B(s)\left(B(s)C^{-1}(s) - 1\right)}{A(s) - B(s)C^{-1}(s)D(s)}W(s)$.

Multiplying by $C(s)$, $Y(s) = \frac{B(s)(B(s) - C(s))}{A(s)C(s) - B(s)D(s)}W(s)$.

In summary,

$Y(s) = G_{F\to Y}(s)\,F(s)$ and $Y(s) = G_{W\to Y}(s)\,W(s)$, where

$$G_{F\to Y}(s) = \frac{\left(\frac{m_1+m_2}{m_1m_2}\right)C(s) - \frac{B(s)}{m_2}}{A(s)C(s) - B(s)D(s)},\ G_{W\to Y}(s) = \frac{B(s)(B(s) - C(s))}{A(s)C(s) - B(s)D(s)},$$

and where $A(s) = s^2 + (k_1 + b_1s)\left(\frac{m_1+m_2}{m_1m_2}\right)$, $B(s) = \frac{k_2+b_2s}{m_2}$, $C(s) = s^2 + \frac{k_2+b_2s}{m_2}$, $D(s) = \frac{k_1+b_1s}{m_2}$.

Substituting A, B, C, and D and simplifying, the input-to-output and noise-to-output transfer functions are, respectively:

$$G_{F\to Y}(s) = \frac{\frac{(m_1+m_2)s^2 + b_2 s + k_2}{m_1 m_2}}{s^4 + \frac{m_1 b_1 + m_1 b_2 + m_2 b_1}{m_1 m_2}s^3 + \frac{(m_1+m_2)k_1 + m_1 k_2 + b_1 b_2}{m_1 m_2}s^2 + \frac{b_1 k_2 + b_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}}$$

$$G_{W\to Y}(s) = -\frac{\frac{m_1}{m_1 m_2}\left(s^2 b_2 s + k_2\right)}{s^4 + \frac{m_1 b_1 + m_1 b_2 + m_2 b_1}{m_1 m_2}s^3 + \frac{(m_1+m_2)k_1 + m_1 k_2 + b_1 b_2}{m_1 m_2}s^2 + \frac{b_1 k_2 + b_2 k_1}{m_1 m_2}s + \frac{k_1 k_2}{m_1 m_2}}$$

With our system parameters, the denominator is then:
$800{,}000s^4 + 38{,}487{,}000s^3 + 1{,}480{,}850{,}000s^2 + 1{,}375{,}000{,}000s + 40{,}000{,}000{,}000$
and the roots are: $-23.9446 \pm 35.2083i, -0.1098 \pm 5.2504i$.

All poles have negative real part, so the system is stable.

The impulse and step responses of the two transfer functions are plotted below in Fig. 1 – 4.

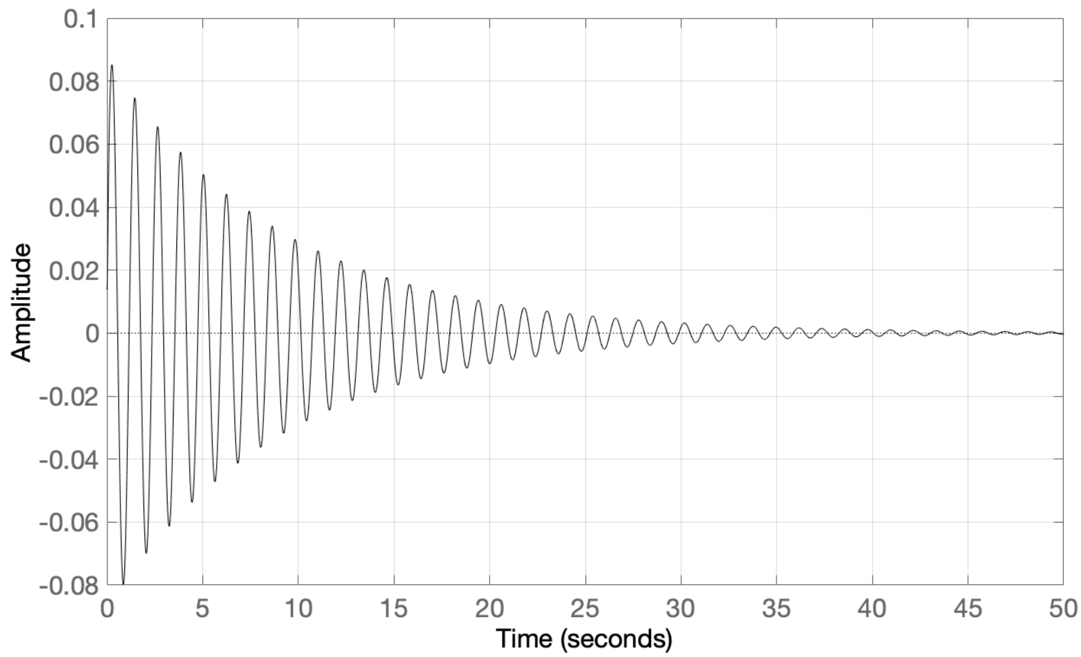

Fig. 1. Impulse Response of $G_{F\to Y}$

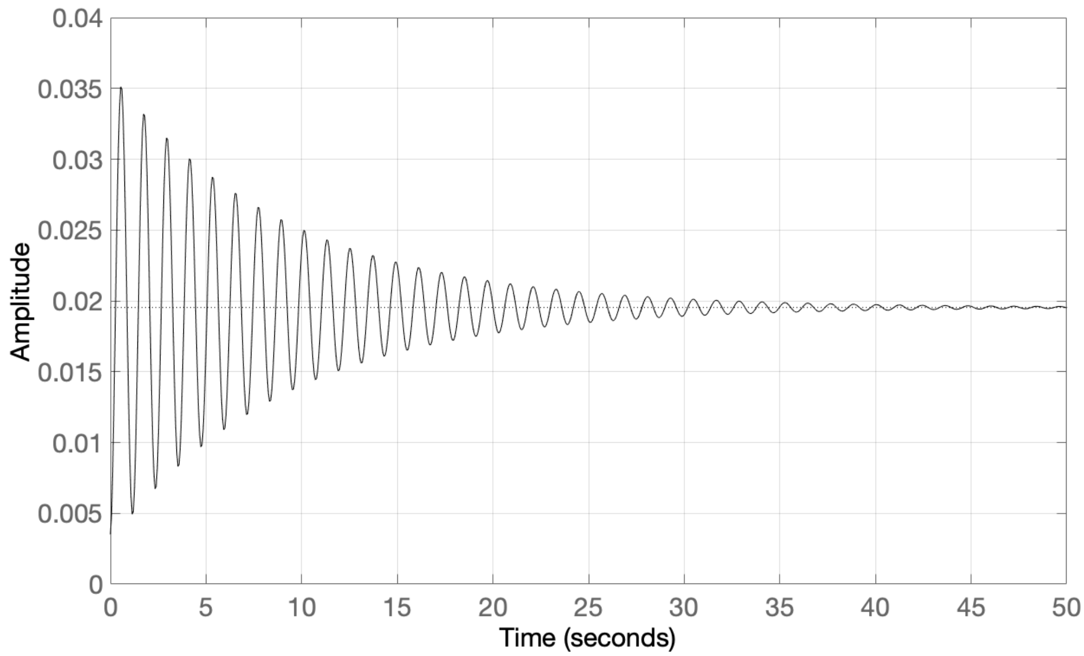

Fig. 2. Step Response of $G_{F \to Y}$

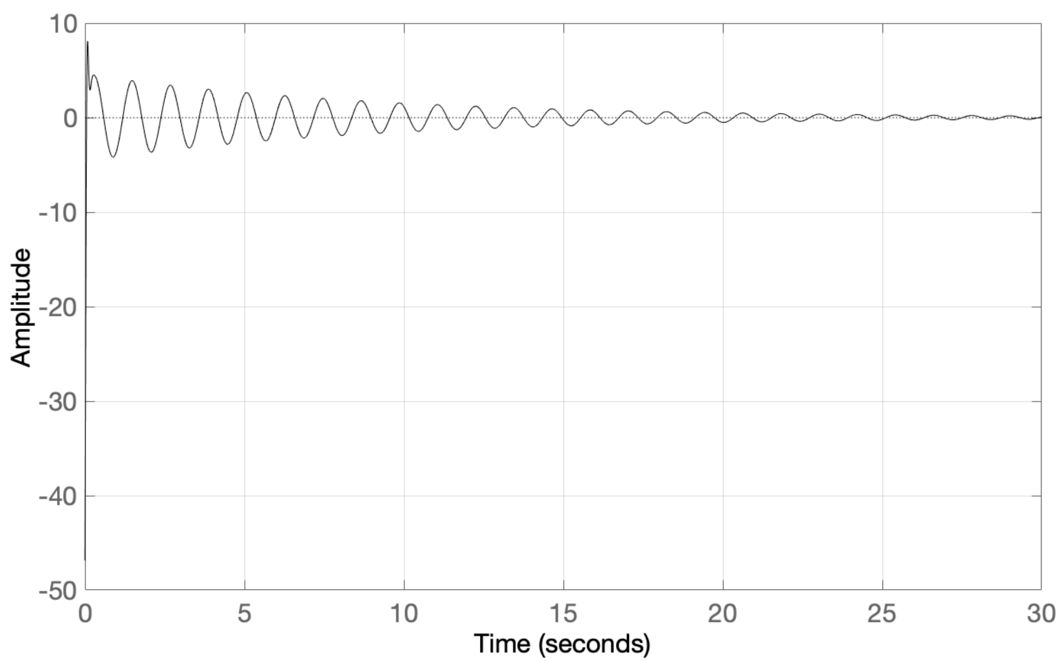

Fig. 3. Impulse Response of $G_{W \to Y}$

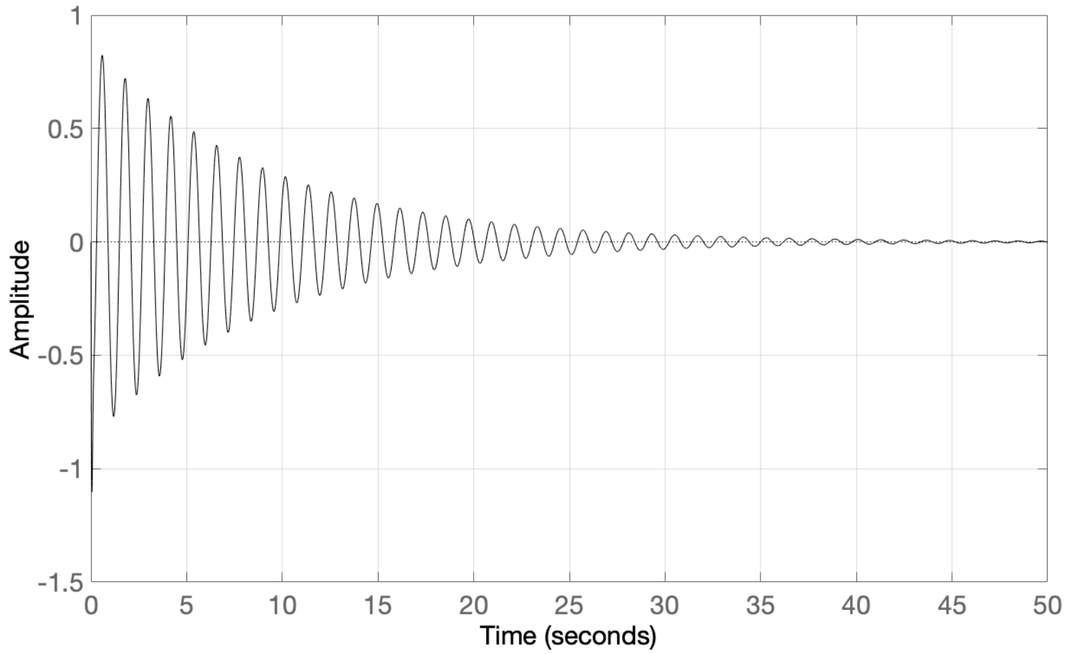

Fig. 4. Step Response of $G_{W\to Y}$

As shown in Fig. 1 – 4, the impulse and step responses of both transfer functions are underdamped and heavily oscillatory, and have long settling times of about 30 seconds.

Using the superposition principle, the system can be represented in block diagram form as:

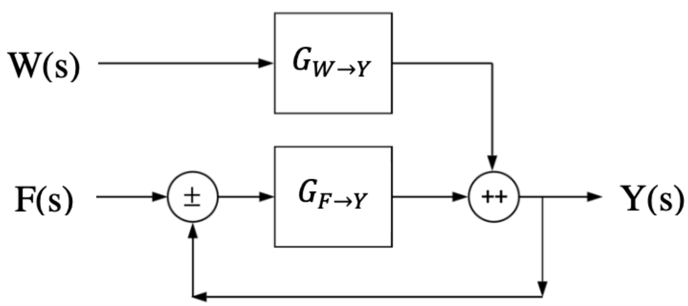

Fig. 5. Block Diagram, using the superposition principle

We can put the diagram in a more standard form by including the inverse of $G_{W\to Y}(s)$:

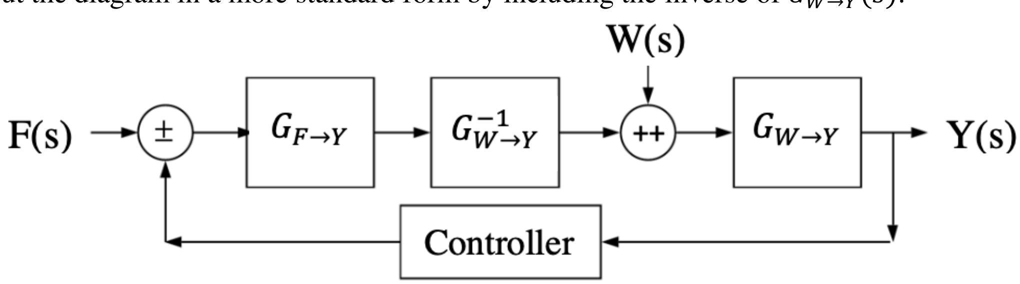

Fig. 6. A more standard block diagram

Recognizing that $G_{W\to Y}^{-1}(s) = G_{Y\to W}(s)$ and defining $G_{F\to W}(s) = G_{F\to Y}(s)G_{W\to Y}^{-1}(s) = -\frac{(m_1+m_2)s^2+b_2s+k_2}{m_1s^2(b_2s+k_2)}$, we can simplify the diagram as:

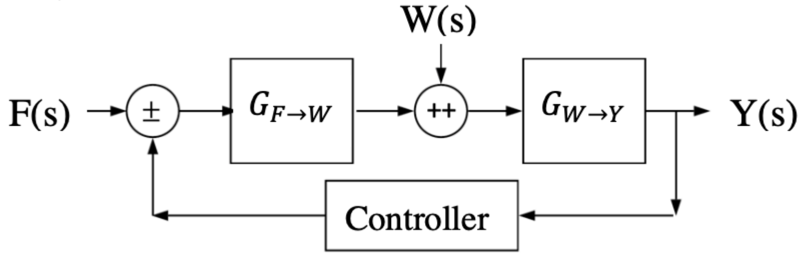

Fig. 7. Fully simplified block diagram of the system

(Equivalently, the ‘Controller’ block could be placed in the forward part of the diagram, before $G_{F\to W}$, rather than in the feedback section. A reference signal *r* can be given to the controller.)

Bode plots of the two transfer functions are displayed in Fig. 8 and 9.

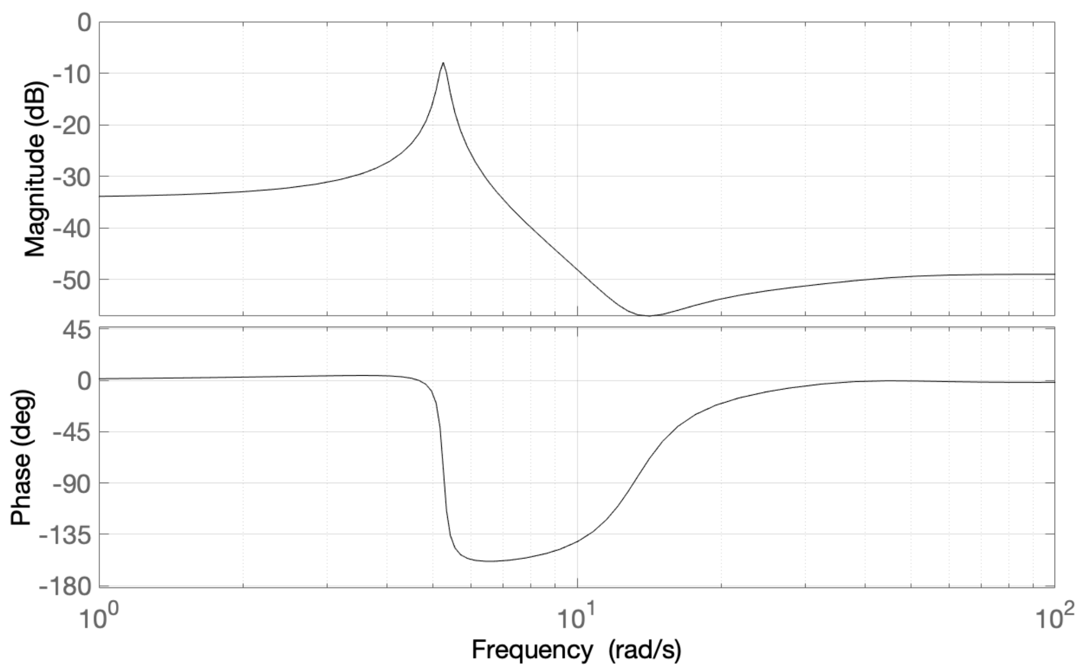

Fig. 8. Bode Plot of $G_{F\to Y}$

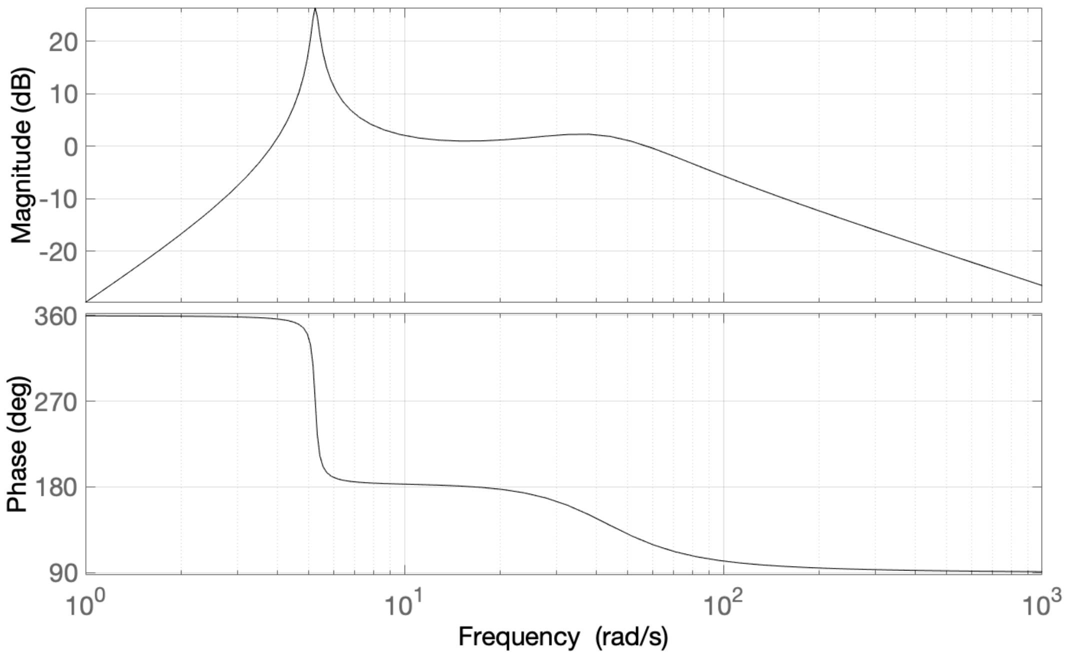

Fig. 9. Bode Plot of $G_{W\to Y}$

As shown in Fig. 8 and 9, both Bode plots peak between 1 and 10 Hz, indicating that the system is sensitive to both low frequency forcing and disturbances, owing to the high mass of the car. A second, much less pronounced peak exists in the noise-to-output Bode plot between 10 and 100 Hz, because of the existence of the second mass. The force-to-output Bode plot has a minimum just after 10 Hz, indicating that there is a band of frequencies at which forcing is most ineffective. The noise-to-output response decreases linearly at high frequencies, indicating that the system is able to damp out high-frequency oscillations effectively, but the response of the system to high-frequency forcing levels out after 10 Hz. Nyquist diagrams of the two transfer functions are shown in Fig. 10 – 12.

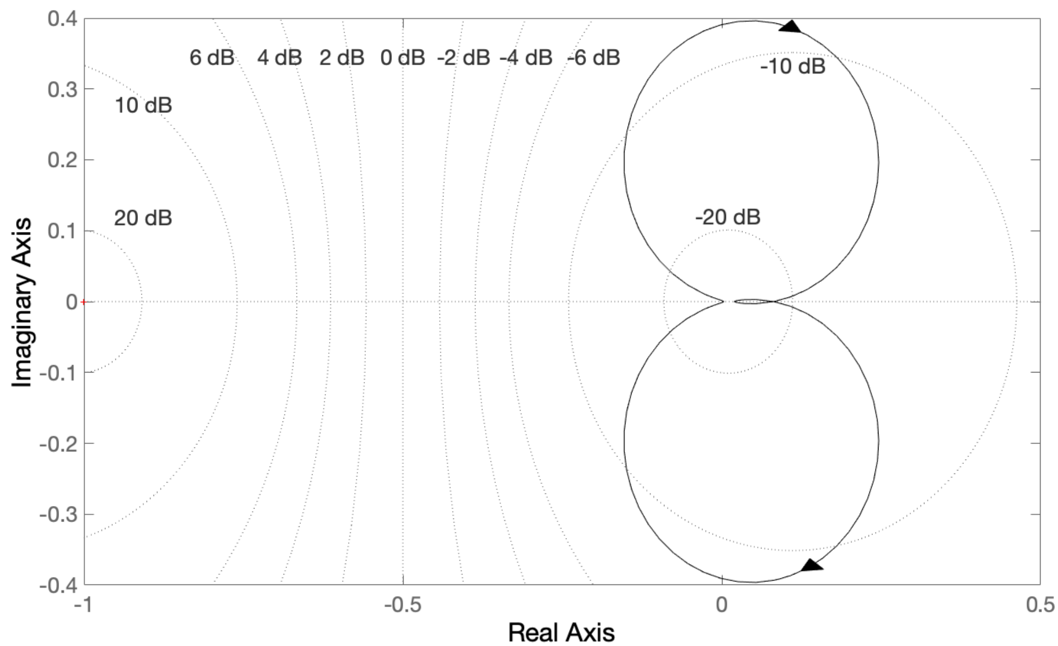

Fig. 10. Nyquist Diagram of $G_{F\to Y}$

Figure 10 shows that there are no net encirclements of the point –1 by the Nyquist diagram.

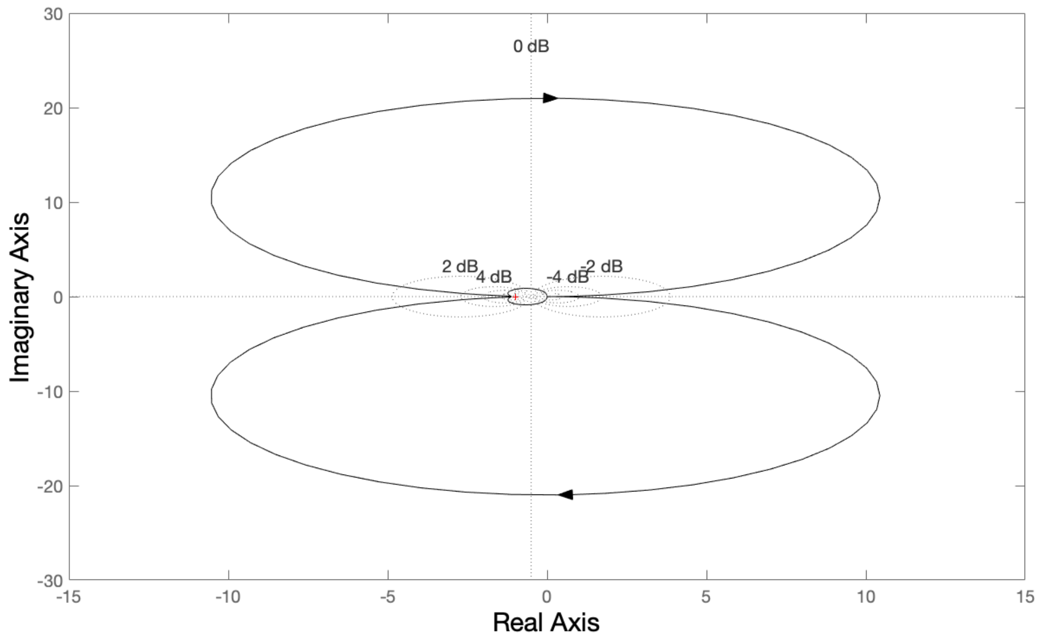

Fig. 11. Nyquist Diagram of $G_{W\rightarrow Y}$

In Fig. 11, the Nyquist diagram is seen to encircle the point –1. However, plotting only the part of the contour corresponding to positive ω, as in Fig. 12, we see that each section of the Nyquist diagram encircles –1 once, in opposite directions, so that there are zero net encirclements.

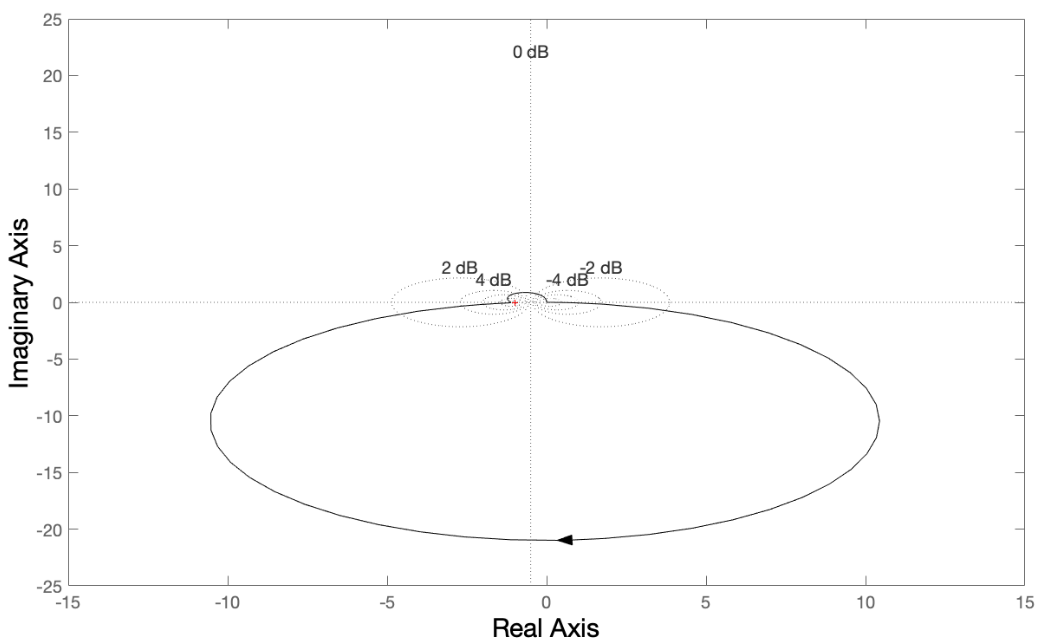

Fig. 12. Section of the Nyquist contour corresponding to nonnegative ω

A root locus plot of the input-to-output transfer function is displayed in Fig. 13. As shown in the figure, two poles are dominant, as they are much closer than the other pair to the imaginary axis. All poles stay in the left hand plane for all ω.

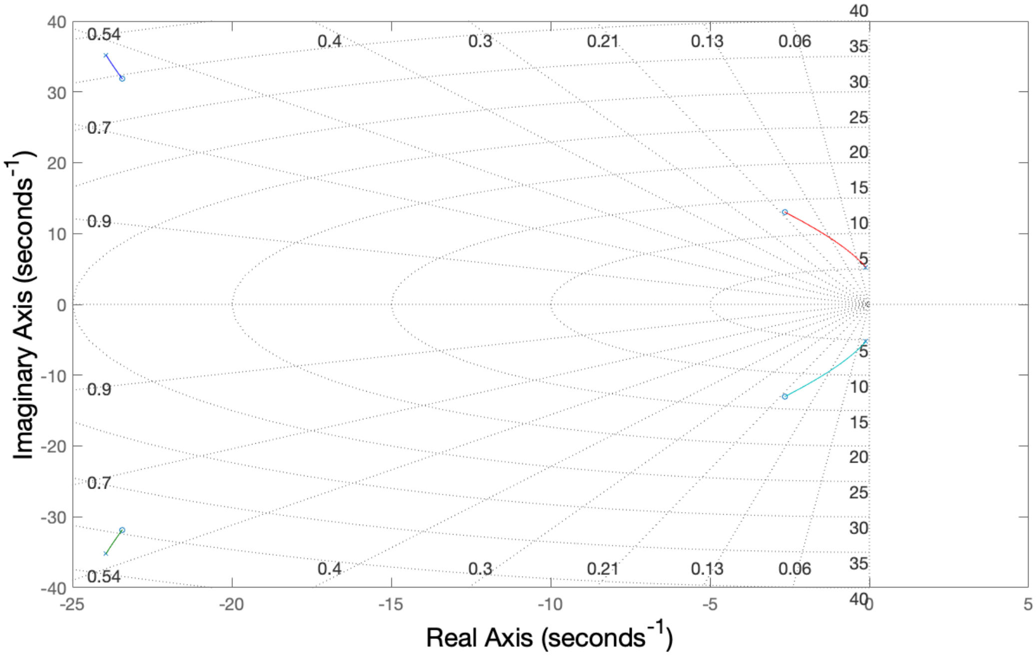

Fig. 13. Root Locus of the input-to-output transfer function

The step response of the input-to-output transfer function is replotted in Fig. 14. The step response takes about 33 seconds to settle to within 5% of its steady-state value.

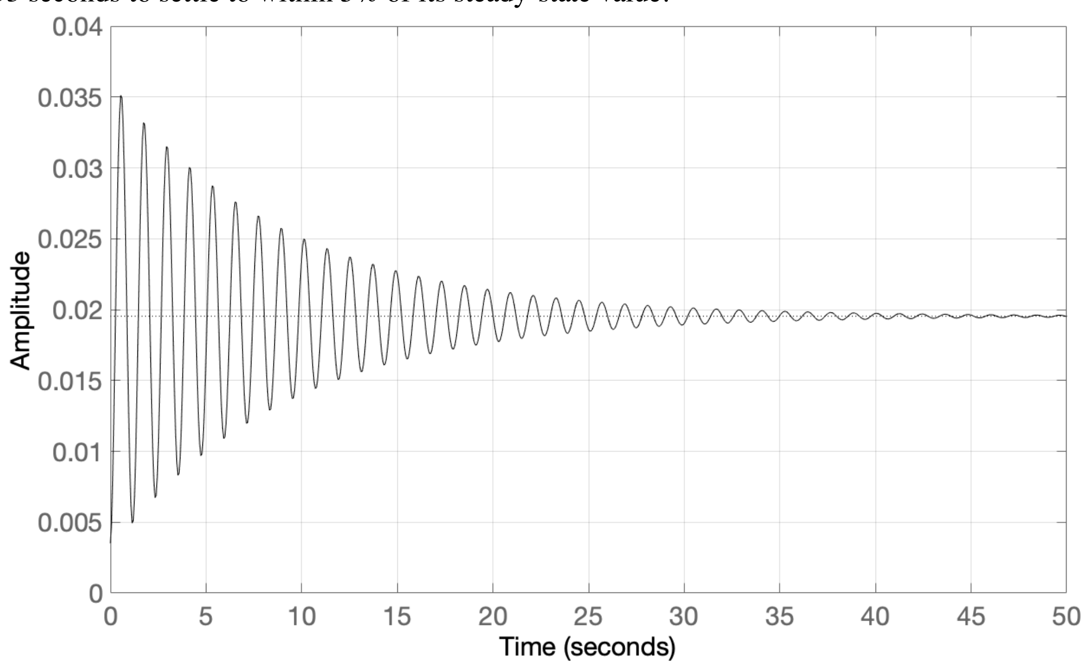

Fig. 14. Replotting the step response of $G_{F\to Y}$

The overshoot for a second-order system is $M_p = e^{-\pi\xi/\sqrt{1-\xi^2}}$ and the settling time is $T_s \approx 4.6/\xi\omega_n$, where $\omega_n = \sqrt{\sigma^2 + \omega^2}$ and $\xi = \sigma/\omega_n$. Our step-response specifications are a maximum overshoot of 5% and a settling time of one second. Approximating the system as second order, on account of the two dominant poles, we require $M_p \leq \tilde{M}_p = 0.05$ and $T_s \leq \tilde{T}_s = 1$. Thus, we need: $e^{-\pi\xi/\sqrt{1-\xi^2}} \leq 0.05$ and $4.6/\xi\omega_n \leq 1$, which implies that: $\sigma \geq 4.6$ and $\xi \geq \frac{\ln(0.05)}{\sqrt{(\ln(0.05))^2 - \pi^2}} \approx -0.69$. Thus, $\tilde{\theta} = \arccos(\tilde{\xi}) \approx \arccos(-0.69) \approx 2.33$ rad $\approx 133.6°$. In summary: the dominant poles must be placed in the part of the complex plane delineated by:

$\boxed{\tilde{\sigma} = 4.6 \text{ and } \tilde{\theta} \approx 133.6°}$

Choosing as an initial guess the complex conjugate pole pair $-5 \pm 2i$, which satisfies the above requirements, and leaving the non-dominant poles unchanged results in the input-to-output transfer function: $G_t(s) \approx \frac{[(m_1+m_2)s^2+b_2s+k_2]/(m_1m_2)}{(s^2+10s+29)(s^2+47.89s+1{,}812.97)}$.

The step response for this transfer function is plotted in Fig. 15.

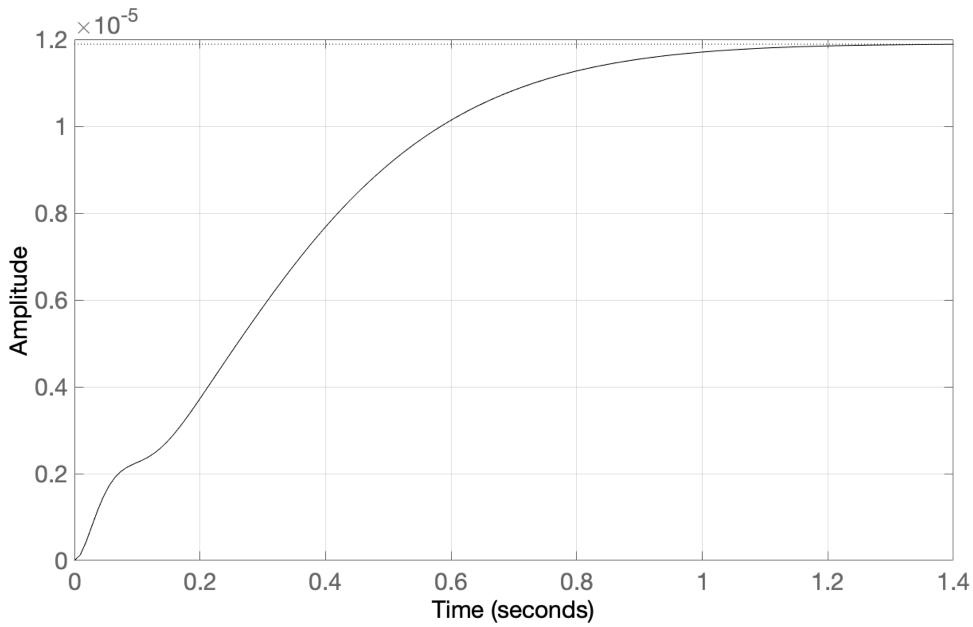

Fig. 15. Step response for the system, with dominant poles placed using second-order criteria.

From Fig. 15, the system is overdamped and the settling time is now on the order of a second, but the steady-state response is far too small, so the amplitude of the controller clearly needs to be increased.

I now turned to the Control System Designer in MATLAB (the SISO tool) to design a controller using full-state feedback (i.e. pole placement). To begin, I placed a pole on the real axis, with real part equal to about –5, in order to stay in the allowed region, but the system was unstable until the pole was moved back to about –18, and continued to exhibit heavy oscillation. I then added a pair of complex conjugate zeros, but the system became unstable, so added another pole so that one was placed on either side of the zeros. My intention here was to make a cross-shaped root locus between the controller pole-zeros and force the plant pole loci into the open left half plane. (This is essentially like cascading a PID controller with the single-pole controller.) I then played with shifting the two poles up and down the real axis. I hoped that moving the dominant pole closer to zero would decrease steady-state error, but found it most effective when kept in the region to the left of about –5 (as determined using the 2nd-order formulae above). In order to reach 95% of a steady-state amplitude of 1, the controller gain had to be increased to about 1,000. The overshoot was too large, so I tried placing the poles further to the left, to improve the transient response. I found that I was able to nearly meet the overshoot specification by placing the leftmost pole at about –14.5 and the rightmost pole at about –5.5. In order to sufficiently damp out oscillations, I found that the absolute value of the imaginary component of the zeros needed to be kept below about 2. Lastly, I decreased the settling time by moving the zeros toward the imaginary axis. A step-response nearly meeting the specifications used poles placed at –5.5 and –14.5 and zeros at $-1.8 \pm 6i$. The Bode plot, Root locus, and step response of this controller, $C = 1000\frac{0.03s^2+0.09s+1}{(1+0.07s)(1+0.18s)}$, are displayed in Fig. 16. As shown in the root locus, all plant and controller poles are stable for all values of ω. The step response has a steady-state error of about 5% and about twice the desired overshoot, but falls well within the desired 1 second settling time.

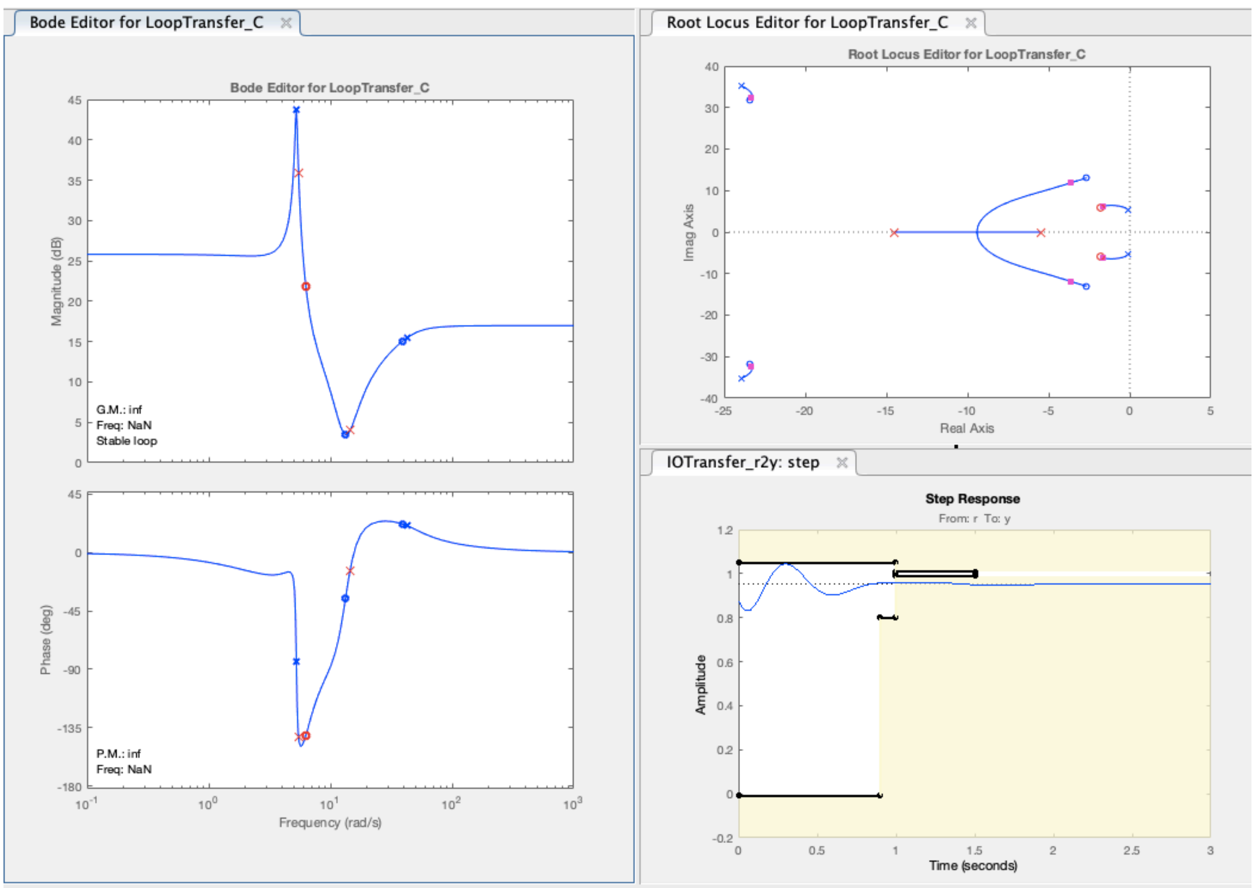

Fig. 16. SISOTOOL design of a controller with two real poles and a complex-conjugate zero pair.

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% MAE 171A - Controller Design for an Active Suspension System
% Peter Racioppo

clear all;
close all;
clc;

%% System Parameters
% Masses [kg]: m1=2500, m2=320
% Spring constants [N/m]: k1=80,000, k2=500,000
% Damping coefficients [M/m/s]: b1=350, b2=15,000

m1 = 2500;
m2 = 320;
k1 = 80000;
k2 = 500000;
b1 = 350;
b2 = 15000;

g = 9.8; % Gravitational acceleration at Earth's surface

% Equilibrium positions
r1 = (((k1+k2)/(k1*k2))*m1 + m2/k2)*g;
r2 = (m1+m2)*g/k2;

%% Symbolic Calculations 1 (W = 0)

syms k1 k2 m1 m2 b1 b2 s F

A = s^2 + (k1+b1*s)*(1/m1+1/m2);
B = (k2+b2*s)/m2;
C = (s^2+(k2+b2*s)/m2);
D = (k1+b1*s)/m2;
E = F*(1/m1+1/m2);
Num = -B/m2 + C*(1/m1+1/m2);
Denom = A-B*D/C;
G = simplify(Num/Denom);

Num2 = (k2 + b2*s + m1*s^2 + m2*s^2);
Denom2 = (m1*m2*(s^2 - ((k1 + b1*s)*(k2 + b2*s))/(m2*(m2*s^2 +↙
b2*s + k2)) + ((m1 + m2)*(k1 + b1*s))/(m1*m2)));
```

```
collect(Num2,s);
collect(Denom2,s);

Num3 = ((m1 + m2)*s^2 + b2*s + k2)*(m2*s^2 + b2*s + k2);
collect(Num3,s);

clear all;

%% Symbolic Calculations 2 (F = 0)

syms k1 k2 m1 m2 b1 b2 s F

A = s^2 + (k1+b1*s)*(1/m1+1/m2);
B = (k2+b2*s)/m2;
C = (s^2+(k2+b2*s)/m2);
D = (k1+b1*s)/m2;

Num = B*(B-C);
Denom = A*C-B*D;
G = simplify(Num/Denom);

Num2 = -(s^2*(k2 + b2*s));
Denom2 = (m2*(((k2 + b2*s)/m2 + s^2)*((1/m1 + 1/m2)*(k1 + b1*s)↙
+ s^2) - ((k1 + b1*s)*(k2 + b2*s))/m2^2));

collect(Num2,s);
collect(Denom2,s);

Num_f = -b2*s^3 - k2*s^2;
Denom_f = m2*s^4 + m2*(b2/m2 + b1*(1/m1 + 1/m2))*s^3 + m2*(k2/m2↙
+ k1*(1/m1 + 1/m2) - (b1*b2)/m2^2 + (b1*b2*(1/m1 + 1/m2))/m2)↙
*s^2 - m2*((b1*k2)/m2^2 + (b2*k1)/m2^2 - (b1*k2*(1/m1 + 1/m2))↙
/m2 - (b2*k1*(1/m1 + 1/m2))/m2)*s - m2*((k1*k2)/m2^2 - (k1*k2*↙
(1/m1 + 1/m2))/m2);

clear all;

%% G1(s) (Input to Output Transfer Function)

m1 = 2500;
m2 = 320;
```

```
k1 = 80000;
k2 = 500000;
b1 = 350;
b2 = 15000;

nvec = [m2*(m1 + m2),(b2*m2 + b2*(m1 + m2)),(b2^2 + k2*(m1 + m2)↙
+ k2*m2),2*b2*k2,k2^2];
dvec = [m1*m2^2,(b1*m2^2 + b1*m1*m2 + b2*m1*m2),(k1*m2^2 +↙
b1*b2*m2 + k1*m1*m2 + k2*m1*m2),(b1*k2*m2 + b2*k1*m2),k1*k2*m2];

Roots1 = roots(dvec);

sys1 = tf(nvec,dvec);

% Bode Plot
figure;
opts = bodeoptions('cstprefs');
opts.XLabel.FontSize = 30;
opts.YLabel.FontSize = 30;
opts.Title.String = '';
opts.TickLabel.FontSize = 30;
bode(sys1, opts, 'k');
grid on;

% Nyquist Plot
figure;
opts = nyquistoptions('cstprefs');
opts.XLabel.FontSize = 30;
opts.YLabel.FontSize = 30;
opts.Title.String = '';
opts.TickLabel.FontSize = 25;
nyquist(sys1, opts, 'k');
grid on;

% Impulse Response
figure;
impulse(sys1,'k');
title('', 'FontSize', 30);
xlabel('Time', 'FontSize', 30);
ylabel('Amplitude', 'FontSize', 30);
set(gca,'FontSize',30);
```

```
grid on;

% Step Response
figure;
step(sys1,'k');
title('', 'FontSize', 30);
xlabel('Time', 'FontSize', 30);
ylabel('Amplitude', 'FontSize', 30);
set(gca,'FontSize',30);
grid on;

% Root Locus
figure;
rlocus(sys1);
title('', 'FontSize', 30);
xlabel('Real Axis', 'FontSize', 30);
ylabel('Imaginary Axis', 'FontSize', 30);
set(gca,'FontSize',20);
grid on;

%% G2(s) (Noise to Output Transfer Function)

nvec = [-b2,-k2,0,0];
dvec = [m2,m2*(b2/m2 + b1*(1/m1 + 1/m2)),m2*(k2/m2 + k1*(1/m1 +↙
1/m2) - (b1*b2)/m2^2 + (b1*b2*(1/m1 + 1/m2))/m2),-m2*((b1*k2)↙
/m2^2 + (b2*k1)/m2^2 - (b1*k2*(1/m1 + 1/m2))/m2 - (b2*k1*(1/m1 +↙
1/m2))/m2),-m2*((k1*k2)/m2^2 - (k1*k2*(1/m1 + 1/m2))/m2)];

Roots2 = roots(dvec);

sys2 = tf(nvec,dvec);

% Bode Plot
figure;
opts = bodeoptions('cstprefs');
opts.XLabel.FontSize = 30;
opts.YLabel.FontSize = 30;
opts.Title.String = '';
opts.TickLabel.FontSize = 30;
bode(sys2, opts, 'k');
grid on;
```

```
% Nyquist Plot
figure;
opts = nyquistoptions('cstprefs');
opts.XLabel.FontSize = 30;
opts.YLabel.FontSize = 30;
opts.Title.String = '';
opts.TickLabel.FontSize = 20;
nyquist(sys2, opts, 'k');
grid on;

% Nyquist Plot (one sided)
figure;
opts = nyquistoptions('cstprefs');
opts.XLabel.FontSize = 30;
opts.YLabel.FontSize = 30;
% opts.Xlim = [-2 0];
% opts.Ylim = [-5 5];
opts.Title.String = '';
opts.TickLabel.FontSize = 20;
opts.ShowFullContour = 'off';
nyquist(sys2, opts, 'k');
grid on;

% Impulse Response
figure;
impulse(sys2, 'k');
title('', 'FontSize', 30);
xlabel('Time', 'FontSize', 30);
ylabel('Amplitude', 'FontSize', 30);
set(gca,'FontSize',30);
grid on;

% Step Response
figure;
step(sys2, 'k');
title('', 'FontSize', 30);
xlabel('Time', 'FontSize', 30);
ylabel('Amplitude', 'FontSize', 30);
set(gca,'FontSize',30);
grid on;
```

```
%% Controller Design
% Specifications: we want a maximum overshoot of 5% and a 1↙
seconds
% settling time within 1% of the final value. Find the↙
corresponding
% sector in the complex domain where the poles should be placed.↙
Apply
% the formulas used for a second order system. They are not↙
exact for
% this case, but they will work reasonably well since thereare↙
dominant
% poles.

% Choose -5 +/- 2i
% Denominator: s^4 + 57.8892*s^3 + 2320.86*s^2 + 19518.5*s +↙
52576.1

% syms s
% num_t_sym = (m1+m2)*s^2/(m1*m2) + b2*s/(m1*m2)+ k2/(m1*m2);
% denom_t_sym = s^4 + 57.8892*s^3 + 2320.86*s^2 + 19518.5*s +↙
52576.1;
% vpa(simplify(num_t_sym/denom_t_sym),2);
% % (3.5e-3*s^2+0.019*s+0.62)/(s^4+59.0*s^3+2.3e3*s^2+2.0e4*s+5.↙
3e4)

num_t = [(m1+m2)/(m1*m2), b2/(m1*m2), k2/(m1*m2)];
denom_t = [1,57.8892,2320.86,19518.5,52576.1];
sys_t = tf(num_t,denom_t);

% Step Response
figure;
step(sys_t, 'k');
title('', 'FontSize', 30);
xlabel('Time', 'FontSize', 30);
ylabel('Amplitude', 'FontSize', 30);
set(gca,'FontSize',30);
grid on;

% Sisotool / controlSystemDesigner(tf())
% controlSystemDesigner(sys1)
```

```
sisotool(sys1)

% ----------------------------
```