AER510 – Aerospace Propulsion
Project 1 – Engine Performance Optimization
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1. Problem Description

The purpose of this document is to detail the procedure and results of the engine optimization project. A client has tasked us with developing the initial layout of a new turbofan engine for a transonic passenger aircraft. The engine must have a specific thrust of 125 Ns/kg at a cruise condition of Mach 0.85 and ambient pressure and temperature 23kPa and 230K, respectively. The main parameters that are allowed to be altered are core compression ratio π_c, fan compression ratio π_f, and bypass ratio β. These three parameters have the largest effect of the weight and fuel consumption of the engine. As such, the optimization process will focus on minimizing a cost function based on these parameters. The cost function, C, is defined as follows:

$$C = W + F, \quad \text{where} \quad W = \frac{\beta}{\beta_0} + \frac{\pi_C}{\pi_{C0}} \text{ and } F = \frac{\text{TSFC}}{\text{TSFC}_0}$$

C is the total cost to be minimized, while W and F represent the weight and fuel penalties associated with the bypass ratio, and core and fan compression ratios. This cost function has empirical values $\beta_0 = 15$, $\pi_{c0} = 100$, and $\text{TFSC}_0 = 0.5 \times 10^{-5}$ kg/Ns.

This report will outline the major sections of the engine, along with associated equations and assumptions. The optimization process will be detailed, and results will be discussed.

2 Approach

2.1 Engine Outline

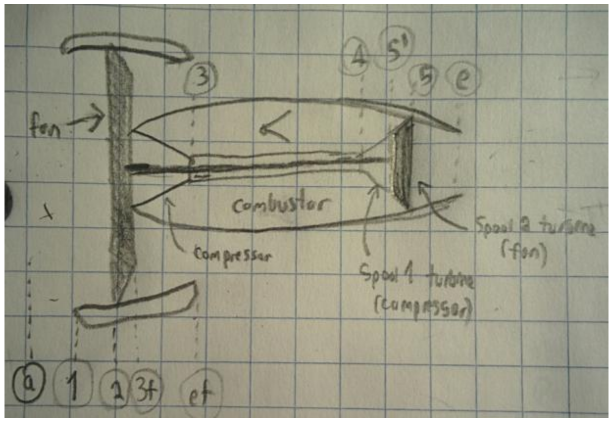

Figure 1: Outline of Bypass Engine (Components and Stages): Stages 1-2, Diffuser; Stages 2-3f, Fan; Stages 3f-ef, Fan Nozzle; Stages 3f-3, Core Compressor; Stages 3-4, Combustor; Stages 4-5', Spool 1 Turbine; Stages 5'-5, Spool 2 Turbine; Stages 5-e, Core Nozzle.

The main components of the bypass engine are defined by the stage numbers as seen in Figure 1. This section will detail the equations governing the performance of each component, based on certain assumptions and empirical data.

The engine diffuser is considered adiabatic, with an efficiency of $N_d = 0.98$. Therefore the stagnation temperatures at stages a, 1, and 2 are equal, and can define the pre-fan stagnation pressure as:

$$T_{0a} = T_{01} = T_{02} = T_a\left(1 + \frac{\gamma - 1}{2} M^2\right) \qquad P_{02} = P_a\left(1 + N_d\left(\frac{T_{02}}{T_a} - 1\right)\right)^{\frac{\gamma}{\gamma-1}}$$

The fan compression ratio π_f is used to determine the post-fan temperature and pressure at Stage 3f. This ratio will indirectly have an effect on the cost function, as it affects the core inlet parameters and fan nozzle exit velocity. The polytropic efficiency and fan nozzle efficiency are empirically determined as $e_f = 0.93$ and $N_{fn} = 0.95$, respectively. As a result, the post fan parameters are found via:

$$P_{03f} = \pi_f P_{02} \ , \ T_{03f} = T_{02}\pi_f^{\frac{\gamma-1}{\gamma}\frac{1}{e_f}} \ , \ u_{ef} = \sqrt{\left(2c_p N_{nf} T_{03f}\left[1 - \left(\frac{P_e}{P_{03f}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right)}$$

It is assumed (for this initial optimization) that the fan exit and core exit pressures are atmospheric, therefore $P_e = P_a$.

The core compressor ratio π_c is the second major design parameter to optimize. This will have an effect on the weight of the engine (as larger compressor ratios require more compressor blades or more complex compressor systems). The compressor polytropic efficiency was defined to be e_c = 0.92, which is used to find the post-compressor stagnation temperature and pressure:

$$P_{03} = \pi_c P_{03f} \ , \ T_{03} = T_{03f}\pi_c^{\frac{\gamma-1}{\gamma}\frac{1}{e_c}}$$

The combustor was estimated to have an efficiency of π_b = 0.95, which results in a stagnation pressure loss (non-ideal combustion). The fuel-air ratio through the core was also determined using a fuel heating value of Qr = 43MJ/kg and combustion efficiency N_b = 0.99. Both of these values are limited by the maximum turbine blade temperature T_{04} = 1380K:

$$P_{04} = \pi_b P_{03} \ , \ f = \frac{\left(\frac{T_{04}}{T_{03}} - 1\right)}{\frac{(N_b Q_r)}{c_p T_{03}} - \frac{T_{04}}{T_{03}}}$$

The power extracted by the 1st spool high-power turbine (HPT) must equal the power required to operate the compressor. For an assumed mechanical efficiency N_m = 1 and polytropic turbine efficiency e_t = 0.93, the post-LPT stagnation parameters are found via:

$$P_{HPT} = -P_c \dashrightarrow (m_{aH} + m_f)c_p\left(T_{05}' - T_{04}\right) = m_{aH}\, c_p\left(T_{03f} - T_{03}\right)$$

$$\dashrightarrow \ T_{05}' = T_{04} + \left(\frac{1}{1+f}\right)(T_{03f} - T_{03}) \qquad P_{05}' = P_{04}\left(\frac{T_{05}'}{T_{04}}\right)^{\frac{\gamma}{\gamma-1}\frac{1}{e_t}}$$

The power extracted by the low-power turbine (LPT) is found in the same manner, though now accounting for the extra bypass flow around the fan:

$$P_{LPT} = -P_f \dashrightarrow (m_{aH} + m_f)c_p\left(T_{05} - T_{05}'\right) = (m_{aH} + m_{aC})c_p\left(T_{02} - T_{03f}\right)$$

$$\dashrightarrow \ T_{05} = T_{05}' + \left(\frac{1+\beta}{1+f}\right)(T_{02} - T_{03f}) \qquad P_{05} = P_{05}'\left(\frac{T_{05}}{T_{05}'}\right)^{\frac{\gamma}{\gamma-1}\frac{1}{e_t}}$$

The flow through the core nozzle exits out to atmospheric pressure, at a velocity of:

$$u_e = \sqrt{\left(2 c_p N_n T_{05}\left[1 - \left(\frac{P_e}{P_{05}}\right)^{\frac{Y-1}{Y}}\right]\right)}$$

The resulting specific thrust equation for both bypass fan flow and core flow is as follows:

$$\frac{\tau}{m_{aH} + m_{aC}} = \frac{\beta}{\beta+1}(u_{ef} - u) + \left(\frac{1+f}{1+\beta}\right)u_e - \frac{u}{1+\beta} + \ (\text{Pressure Terms})$$

The final parameter of importance in this investigation is the Thrust Specific Fuel Consumption. This can be simply calculated by using the fuel-air ratio, Specific Thrust, and a bypass ratio parameter:

$$\mathrm{TSFC} = \frac{1}{\beta + 1}\left(\frac{f}{\text{Specific Thrust}}\right)$$

2.2 Optimization Approach

The specific thrust for the engine must be equal to 125 Ns/kg. This equation is dependent on the bypass ratio β, fan exit velocity u_{ef} and core exit velocity u_e. These exit velocities are indirectly dependant on the fan and compression ratios defined earlier, through their pressure and temperature terms. Therefore the optimization methodology is as follows:

1. **Assume trial values for π_c and π_f**. Based on π_f assumption, one can directly calculate fan exit velocity u_{ef}. The π_c assumption will allow for calculation of all engine parameters up to the 2nd Spool Turbine exit stagnation temperature and pressure (T_{05} and P_{05}). These are dependent on the bypass ratio β, therefore making the core exit velocity u_e dependent on the bypass ratio.
2. **Set up a system of equations for β**. The final equation for the specific thrust (based on trial values of π_c and π_f) will depend on bypass ratio, and the 2nd Spool Turbine exit temperature and pressure (through core exit velocity u_e). As a result, the set of 3 equations with 3 unknowns to solve is as follows:

$$0 = \frac{\beta}{\beta+1}(u_{ef} - u) + \left(\frac{1+f}{1+\beta}\right)\sqrt{\left(2c_p N_n T_{05}\left[1 - \left(\frac{P_e}{P_{05}}\right)^{\frac{\gamma-1}{\gamma}}\right]\right)} - \frac{u}{1+\beta} - 125$$

$$0 = T_{05}' + \left(\frac{1+\beta}{1+f}\right)(T_{02} - T_{03f}) - T_{05}$$

$$0 = P_{05}'\left(\frac{T_{05}}{T_{05}'}\right)^{\frac{\gamma}{\gamma-1}\frac{1}{e_t}} - P_{05}$$

The three unknowns are β, P_{05}, and T_{05}. The first equation is the specific thrust equation, with the expanded form of u_e and 125 Ns/kg replacing the specific thrust value. The second and third equations are the 2nd-Spool Turbine exit temperature and pressures, respectively.

3. **Solve for β and TSFC corresponding to trial π_c and π_f values**. This system of equations was solved using the fsolve function in MATLAB, using an initial guess for values of B, P_{05}, and T_{05}. This guess was set to values that are reasonable for these parameters, such as B = 5, P05 = 50000Pa, and T05 = 500K. As a result, the value of beta

corresponding to the desired specific thrust has been determined. Using this information the TSFC can be calculated, and therefore the Cost Values, Weight and Fuel penalties associated with the values of B, π_c and π_f.

4. **Iterate in the solve-space and determine optimized values.** The process is then repeated for different trial values of π_c and π_f. The fan compression ratio is iterated from values of 1.1 to 2.0. Larger fan pressure ratios will require larger increasingly complex fan designs, as well as reducing the effectiveness of low-pressure ratio compressors. The core compression ratio is iterated from 10 to 60. Increasing the compressor ratio to higher levels will directly increase the weight of the engine and require more power from the turbine, resulting in diminishing thrust results. Pressure ratios less than 10 require much larger bypass ratios to obtain desired thrust, therefore increasing weight.

3 Results and Recommendation

The analysis was performed using trial values of π_c and π_f between the ranges as described above. The first analysis iterated between each π_c in increments of 1, while incrementing π_f by 0.025, for a total of 1887 data points. The data for every major parameter is saved in matrix form (51x37 matrices), with the columns representing the fan ratio, and rows representing the compressor ratio. The results can be seen in Figure 2:

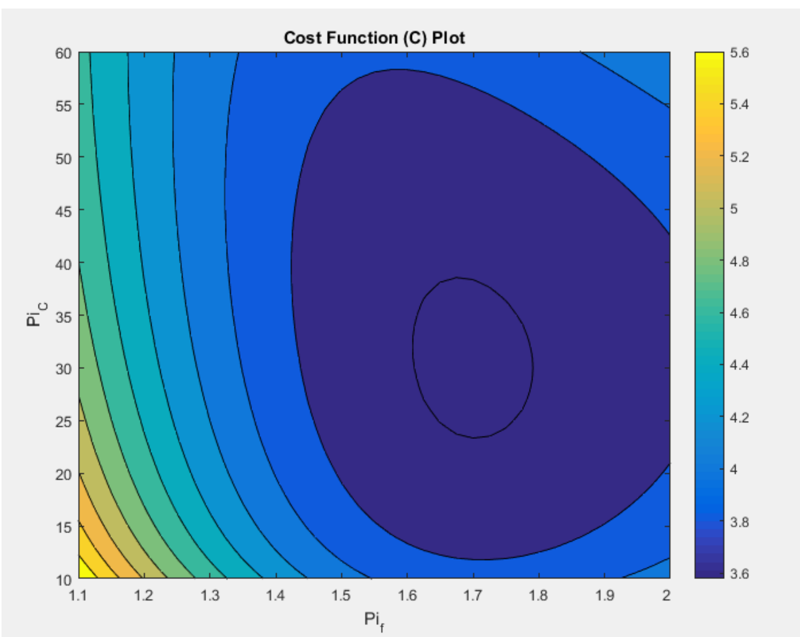

Figure 2: Cost Matrix as a function of π_c and π_f

It can be seen that the cost function is minimized for π_c and π_f values of approximately 30 and 1.7, respectively. The precise values are found by finding the minimum value of the Cost Function Matrix, and using it's matrix element coordinates to find the values of all major parameters (in their matrices). The bypass ratio at this location can be determined from a contour plot of the bypass ratios:

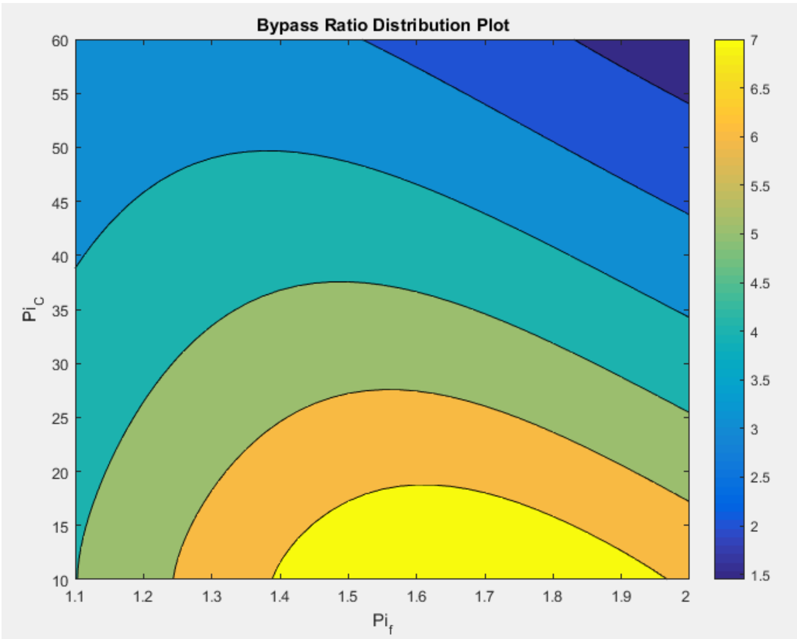

Figure 3: Bypass Ratio Matrix as a function of π_c and π_f

For a compressor ratio of approximately 30 and fan ratio of approximately 1.7, the bypass ratio is approximately 5. To find the actual results for these parameters, and all other important analysis parameters, the location of the minimized cost function value is found in the Cost Function Matrix – this was found to be at Row 21, Column 25, corresponding to a π_c of 31 and π_f of 1.72. These index values were used to find the optimized value of each other parameter, which is summarized in Table 1:

PARAMETER	Optimized Engine Value
Cost Function	**3.58**
Weight Penalty	0.668
Fuel Penalty	2.91
Compressor Ratio	31
Fan Ratio	1.72
Bypass Ratio	**5.5276**
TSFC	1.4550×10^{-5}
Fuel-Air Ratio	0.0119
T02	263.23 K
T03f	309.84 K
T03	890.99 K
T04	1380 K
T05’	805.66 K
T05	504.99 K
P02	36563 Pa
P03f	62157 Pa
P03	1864700 Pa
P04	1771500 Pa
P05’	238830 Pa
P05	41950 Pa
Core Exit Velocity	381.53 m/s
Fan Exit Velocity	389.07 m/s

Table 1: Optimized Engine Station Parameter Values

Therefore, in order to meet the requirement of a specific thrust of 125 Ns/kg, the engine must be designed with a bypass ratio of 5.53, compressor ratio $\pi_f = 31$, and fan ratio $\pi_c = 1.72$. These numbers are based on the given Cost Function and flight condition data – more thorough analysis can be performed using various flight conditions and a detailed cost function.

4 Trends and Discussion

The optimized engine results are based on the balancing of cost function and specific thrust requirement. The individual performance parameters that led to the final recommendation are compared in Figure 4:

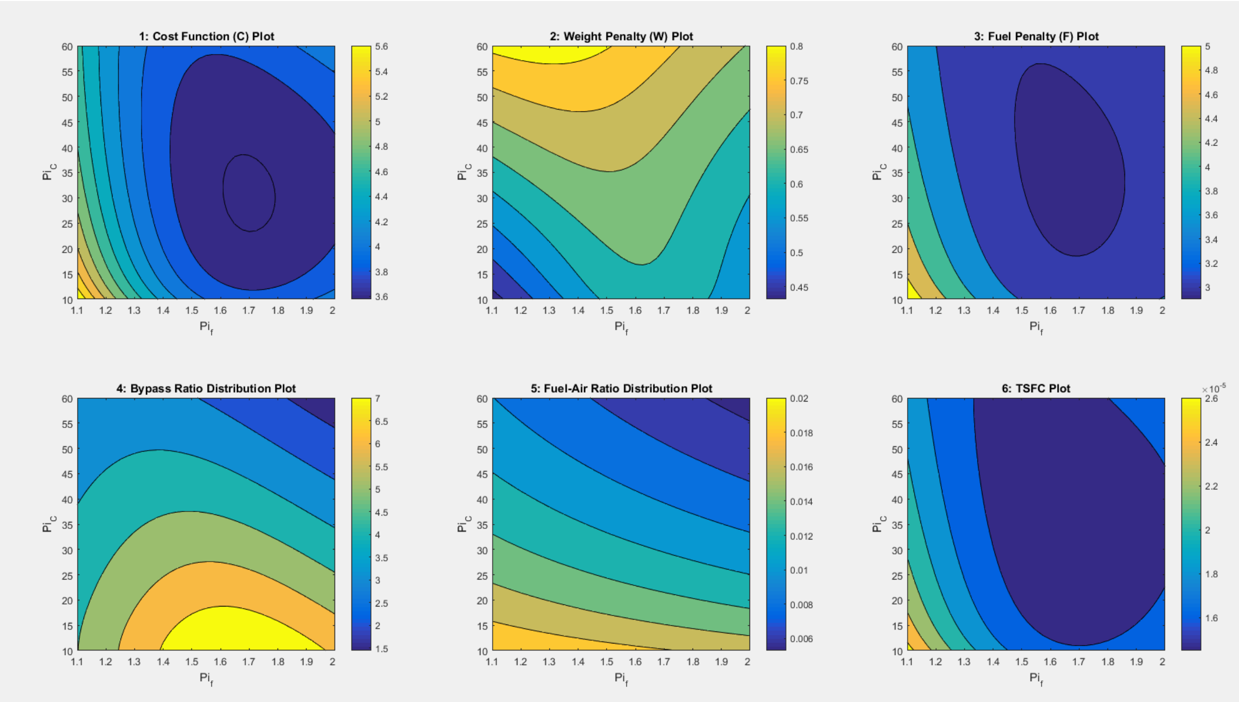

Figure 4: Comparison of 1. Cost Function, 2. Weight Penalty, 3. Fuel Penalty, 4. Bypass Ratio, 5. Fuel-Air Requirements, 6. TSFC, as functions of π_c and π_f

The cost function C is the summation of weight (W) and fuel (F) penalties. It can be seen that the minimized cost range is greatly dependant on the fuel penalty. The TSFC (primary contributor to the Fuel Penalty) is minimized for smaller values of fuel-air ratio and larger bypass ratios. As π_c and π_f were increased, the fuel-air ratio would decrease but the bypass ratio would also decrease. The bypass ratio decreased due to thermodynamic limitations imposed by the increasing compressor ratio, in addition to the higher-efficiency fan compression resulting in less need for bypass flow. Therefore, an optimized balance was reached near the center of the distribution.

The distributions of the major engine stage parameters were also compared. The plots for T_{02}, P_{02}, and T_{04} have been omitted – T_{02} and P_{02} are the entry parameters that are entirely based on flight condition (and therefore will not change in this analysis). T04 is the turbine inlet temperature, which also does not change in this analysis. See Figure 5 for the temperatures at major stages, and Figure for Pressure at major stages.

Observing Figure 5, it can be seen that as π_f increases, the stagnation temperature of the fluid after fan (T_{03f}) also increases. This is because the fluid is being compressed, and will therefore have more energy. After the core compressor, the stagnation temperature (T_{03}) will typically increase. Higher post-compressor temperatures are achieved with higher π_c values. Increasing π_f will also increase the resulting temperature (as this will increase the T_{03f} entering the compressor. Noting that T04 is constant (turbine inlet temperature) at 1480K, we can see the post-1st-spool turbine temperature (T05') has a larger decrease at higher π_c values. This is because the first spool powers the core compressor; large π_c values require more power to maintain, therefore requiring the turbine to extract more energy from the flow. A similar trend can be seen for the post-2nd-spool turbine temperature (T05), though now higher π_f values will increase the decrease the temperature more (energy extracted from flow to power the fan).

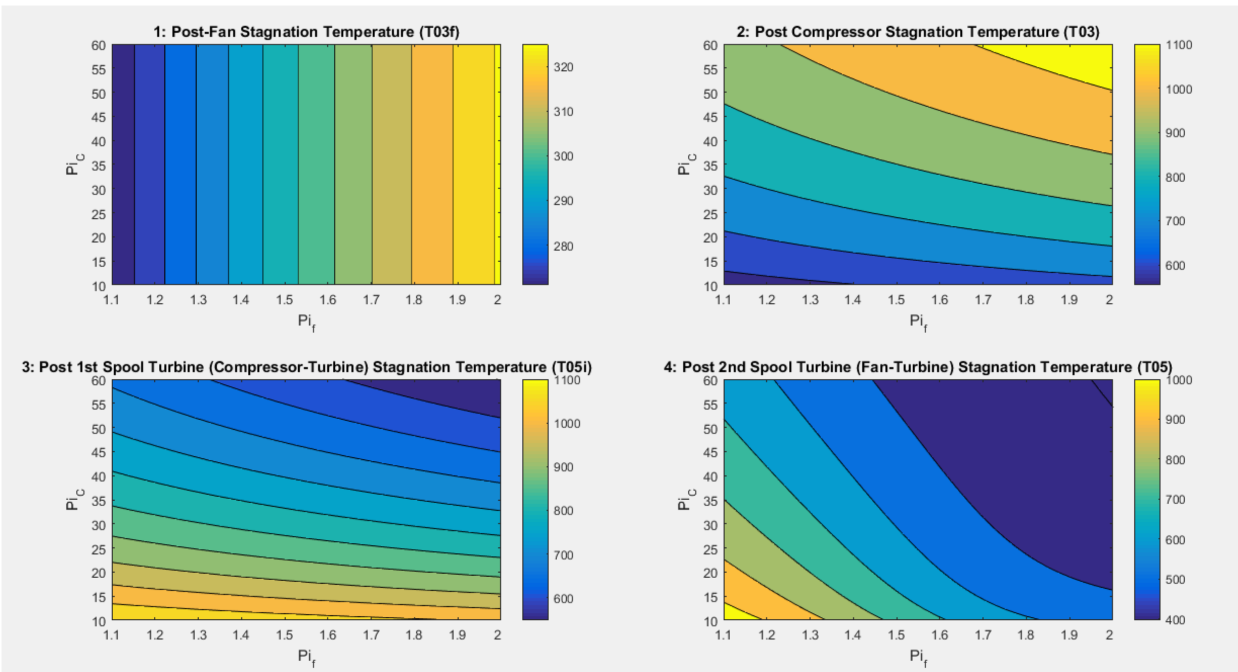

Figure 5: Comparison of 1. Post-Fan Stagnation Temperature, 2. Post Compressor Stagnation Temperature, 3. Post-1st Spool Turbine Stagnation Temperature, 4. Post 2nd Spool Turbine Stagnation Temperature

Observing Figure 6, it can be seen that as π_f increases, the post-fan pressure (P_{03f}) will increase accordingly. At the compressor, larger π_c values will result in an overall larger pressure (P_{03}). Higher π_f will also increase this pressure, as the post-fan flow goes into the compressor. Post-combustion pressure (P_{04}) is simply affected by the combustion pressure ratio, and will therefore follow the same trends for varying π_c and π_f as they did for P_{03}. After the first turbine (which powers the compressor), it can be seen that as the value of π_c increases, the pressure will decrease (as more energy is extracted by the turbine from the flow). For increasing π_f, however, the overall pressure increases, especially at lower π_c values. Since the compressor does not require as much power from the turbine (at lower π_c), the pressure at the turbine exit will be higher than normal (especially at large π_f). After the second turbine, it can be seen that higher π_f values demand more power from the turbine, with π_c having very little effect.

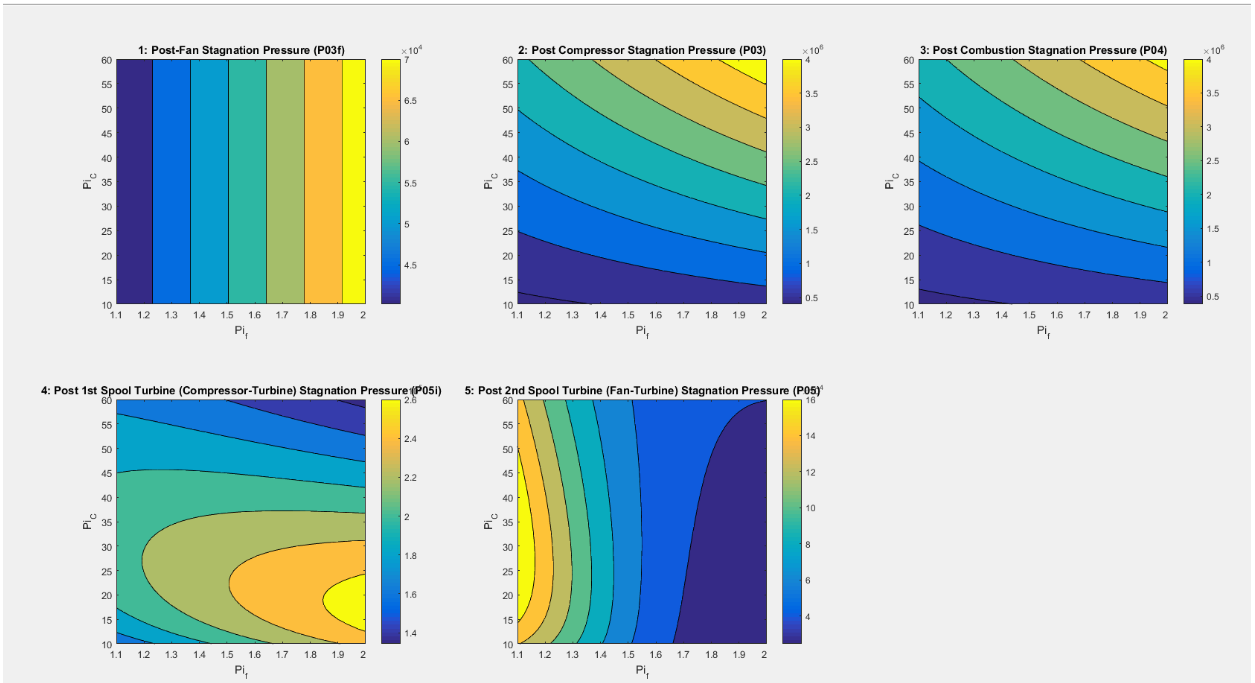

Figure 6 Comparison of 1. Post Fan Stagnation Pressure, 2. Post Compressor Stagnation Pressure, 3. Post Combustion Stagnation Pressure, 4. Post 1st Spool Turbine Stagnation Pressure, 5. Post 2nd Spool Turbine Stagnation Pressure