1.12.2025

Q1.

$P(\text{pass}) = 0.9$, $P(\text{quickly} \mid \text{pass}) = 0.6$, $P(\text{quickly} \mid \text{not pass}) = 0.3$

$P(\text{not pass}) = 1 - P(\text{pass}) = 1 - 0.9 = 0.1$

$P(\text{quickly}) = P(\text{quickly} \mid \text{pass}) \cdot P(\text{pass}) + P(\text{quickly} \mid \text{not pass}) \cdot P(\text{not pass})$

$= 0.6 \times 0.9 + 0.3 \times 0.1 = 0.54 + 0.03 = 0.57$

$P(\text{pass} \mid \text{quickly}) = \frac{P(\text{quickly} \mid \text{pass}) \cdot P(\text{pass})}{P(\text{quickly})} = \frac{0.6 \times 0.9}{0.57} = \frac{0.54}{0.57} \approx 94.7\%$

The proportion of students who answer this question quickly and will pass is about 94.7%.

Q2. likelihood: $p(x \mid \theta, n) = \frac{n! \cdot \prod_{i=1}^{k} \theta_i^{x_i}}{x_1! + x_2! + \cdots + x_k!}$

Prior: $R(\theta \mid \alpha) = \frac{\prod_{i=1}^{k} \theta_i^{\alpha_i - 1}}{B(\alpha)}$, where $\alpha = (\alpha_1, \alpha_2, \cdots \alpha_k)$

and $B(\alpha) = \frac{\prod_{i=1}^{k} \Gamma(\alpha_i)}{\Gamma(\sum_{i=1}^{k} \alpha_i)}$

Posterior: $P(\theta \mid x, n, \alpha) \propto P(x \mid \theta, n) \cdot P(\theta \mid \alpha) = \prod_{i=1}^{k} \theta_i^{x_i} \cdot \prod_{i=1}^{k} \theta_i^{\alpha_i - 1}$

$= \prod_{i=1}^{k} \theta_i^{x_i + \alpha_i - 1}$

$P(\theta \mid x, n, \alpha) = \text{Dirichlet}(\alpha' + x)$, where, $x = (x_1, x_2, \ldots, x_k)$

and $\alpha' = (\alpha_1, \alpha_2, \cdots \alpha_k)$