

2.11.2025

Q3. The Markov algorithm functions as follows:

1. initialize a starting point  $x_0$ .
2. Draw a horizontal slice by sampling  $y \sim U(0, p(x))$ .
3. determine an interval  $(L, R)$  where  $p(x) \geq y$ .
4. Sample a new  $x$  with the interval  $(L, R)$ , using rejection sampling.
5. If the new  $x$  satisfies  $p(x) \geq y$ , accept it; otherwise, reduce the interval and resample.
6. repeat the above steps until convergence.

We can use slice sampling as an alternative to Metropolis-Hastings when a full conditional distribution is only known up to normalization constants. There are some advantages for using slice sampling:

1. Doesn't need distribution proposal
2. Acceptance Guaranteed
3. Handles Multimodal Distributions

The blue curve in the plot generated by the given code is a Beta density with distribution  $\text{Beta}(1.5, 3)$ . The slice sampling samples from this distribution by randomly selecting horizontal slices and sampling from the region beneath the curve. The black dashed lines in the histogram represent the steps of slice sampling where the interval is iteratively narrowed.

The Markov chain was initialized at  $x_0 = 0.25$

A uniform draw was used to determine the slice level  $y$ .

10 iterations (black dashed lines) were visualized to show the adaption process before the total 100 iterations generated the final histogram.