gilmore-lie-groups: table evidence

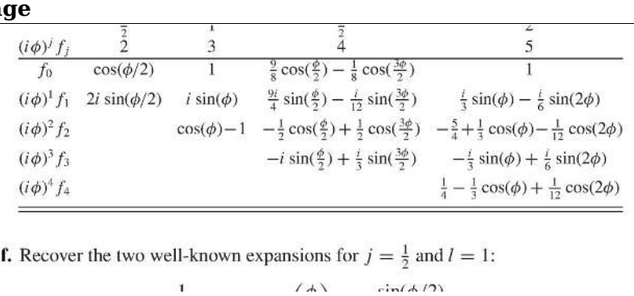
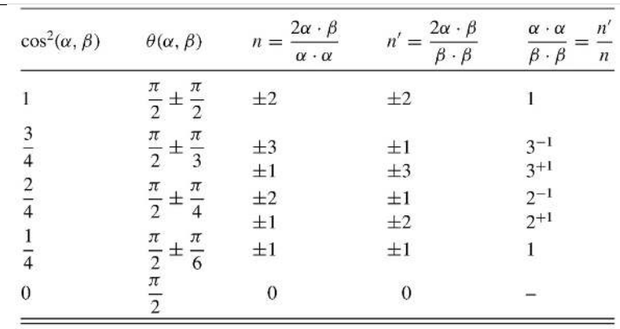
Tables

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_001	26	0.877	`\begin{tabular}[t]{|l|l|l|l|l|} \hline & I & (123) & (321) & Basis functions \\ \hline \(\Gamma^{1}\) & 1 & 1 & 1 & \(v_{1}=s_{1}+s_{2}+s_{3}\) \\ \hline \(\Gamma^{2}\) & 1 & \(\omega\) & \(\omega^{2}\) & \(v_{2}=s_{1}+\omega s_{2}+\omega^{2} s_{3}\) \\ \hline \(\Gamma^{3}\) & 1 & \(\omega^{2}\) & \(\omega\) & \(v_{3}=s_{1}+\omega^{2} s_{2}+\omega s_{3}\) \\ \hline \end{tabular}`	| | I | (123) | (321) | Basis functions | | Γ^1 | 1 | 1 | 1 | $v_1 = s_1 + s_2 + s_3$ | | Γ^2 | 1 | ω | ω^2 | $v_2 = s_1 + \omega s_2 + \omega^2 s_3$ | | Γ^3 | 1 | ω^2 | ω | $v_3 = s_1 + \omega^2 s_2 + \omega s_3$ |	Γ^3 1 ω^2 ω $v_3 = s_1 + \omega^2 s_2 + \omega s_3$ $\omega^3 = +1$ $\omega = e^{2\pi i/3} = \frac{-1+i\sqrt{3}}{}$
gilmore-lie-groups_TAB_002	30	0.669	`\begin{tabular}[t]{|l|l|l|l|l|l|} \hline & \(I\) & (12)(34) & (13)(24) & (14)(23) & Basis functions \\ \hline \(\Gamma^{1}\) & 1 & 1 & 1 & 1 & \(w_{1}=t_{1}+t_{2}+t_{3}+t_{4}\) \\ \hline \(\Gamma^{2}\) & 1 & 1 & -1 & -1 & \(w_{2}=t_{1}+t_{2}-t_{3}-t_{4}\) \\ \hline \(\Gamma^{3}\) & 1 & -1 & 1 & -1 & \(w_{3}=t_{1}-t_{2}+t_{3}-t_{4}\) \\ \hline \(\Gamma^{4}\) & 1 & -1 & -1 & 1 & \(w_{4}=t_{1}-t_{2}-t_{3}+t_{4}\) \\ \hline \end{tabular}`	| | I | (12)(34) | (13)(24) | (14)(23) | Basis functions | | Γ^1 | 1 | 1 | 1 | 1 | $w_1 = t_1 + t_2 + t_3 + t_4$ | | Γ^2 | 1 | 1 | -1 | -1 | $w_2 = t_1 + t_2 - t_3 - t_4$ | | Γ^3 | 1 | -1 | 1 | -1 | $w_3 = t_1 - t_2 + t_3 - t_4$ | | Γ^4 | 1 | -1 | -1 | 1 | $w_4 = t_1 - t_2 - t_3 + t_4$ |	Γ^1 1 1 1 1 $w_1 = t_1 + t_2 + t_3 + t_4$ Γ^2 1 1 -1 -1 $w_2 = t_1 + t_2 - t_3 - t_4$ Γ^3 1 -1 1 -1 $w_3 = t_1 - t_2 + t_3 - t_4$ Γ^4 1 -1 -1 1 $w_4 = t_1 - t_2 - t_3 + t_4$ The linear combinations of these roots that transform under each of the
gilmore-lie-groups_TAB_003	62	0.837	`\begin{tabular}[t]{|l|l|} \hline Group & Dimension \\ \hline \(U T(p, q)\) & \(p^{2}+q^{2}+p q\) \\ \hline \(H T(p, q)\) & \(p(p+q)\) \\ \hline \(U T(p, q, r)\) & \(p^{2}+q^{2}+r^{2}+p q+p r+q r\) \\ \hline \(\operatorname{Sol}(n)\) & \(n(n+1) / 2\) \\ \hline \(\operatorname{Nil}(n)\) & \(n(n-1) / 2\) \\ \hline \(A(p, q)\) & \(p q\) \\ \hline \end{tabular}`	| Group | Dimension | | $UT(p,q)$ | $p^2 + q^2 + pq$ | | $HT(p,q)$ | $p(p+q)$ | | $UT(p,q,r)$ | $p^2 + q^2 + r^2 + pq + pr + qr$ | | Sol(n) | $n(n+1)/2$ | | Nil(n) | $n(n-1)/2$ | | $A(p,q)$ | pq |	Group Dimension $UT(p,q)$ $p^2 + q^2 + pq$ $HT(p,q)$ $p(p+q)$ $UT(p,q,r)$ $p^2 + q^2 + r^2 + pq + pr + qr$ $Sol(n)$ $n(n+1)/2$ $Nil(n)$ $n(n-1)/2$ $A(p,q)$ pq ens to these dimensions if the matrix groups are over the
gilmore-lie-groups_TAB_004	62	1.000	`\begin{tabular}[t]{|l|l|l|} \hline Transformation & \(\mathbf{F}=d \mathbf{p} / d t\) & \(\left(\nabla^{2}-\frac{1}{c^{2}} \frac{\partial^{2}}{\partial t^{2}} \right) A_{\mu}=-\frac{4 \pi}{c} j_{\mu}\) \\ \hline Galilean & invariant & not invariant \\ \hline Poincaré & not invariant & invariant \\ \hline \end{tabular}`	| Transformation | $\mathbf{F} = d\mathbf{p}/dt$ | $\left(\nabla^2 - \frac{1}{c^2}\frac{\partial^2}{\partial t^2}\right) A_\mu = -\frac{4\pi}{c} j_\mu$ | | Galilean | invariant | not invariant | | Poincaré | not invariant | invariant |	Poincaré not invariant invariant ow do you reconcile these results? ow that the group of 2×2 matrices $SU(2)$ is parameterized by two compl

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_005	86	■1.000	`\begin{tabular}[t]{|l|l|l|l|l|} \hline \multicolumn{2}{|c|}{In group} & Relation & \multicolumn{2}{|c|}{In algebra} \\ \hline \(S^{t}=S^{+1}\) & \(\operatorname{det}(S)=+1\) & \(S=e^{\Sigma}\) & \(\operatorname{Tr} \Sigma=0\) & \(\Sigma^{t}=+\Sigma\) \\ \hline \(O^{t}=O^{-1}\) & \(\operatorname{det}(O)=+1\) & \(O=e^{A}\) & \(\operatorname{Tr} A=0\) & \(A^{t}=-A\) \\ \hline \end{tabular}`	<table><tr><th colspan="2">In group</th><th>Relation</th><th colspan="2">In algebra</th></tr><tr><td>$S^t = S^{+1}$</td><td>$\det(S) = +1$</td><td>$S = e^{\Sigma}$</td><td>$\operatorname{Tr}\Sigma = 0$</td><td>$\Sigma^t = +\Sigma$</td></tr><tr><td>$O^t = O^{-1}$</td><td>$\det(O) = +1$</td><td>$O = e^A$</td><td>$\operatorname{Tr} A = 0$</td><td>$A^t = -A$</td></tr></table>	<table><tr><td>$S^t = S^{+1}$</td><td>$\det(S) = +1$</td><td>$S = e^{\Sigma}$</td><td>$\operatorname{Tr}\Sigma = 0$</td><td>$\Sigma^t = +\Sigma$</td></tr><tr><td>$O^t = O^{-1}$</td><td>$\det(O) = +1$</td><td>$O = e^A$</td><td>$\operatorname{Tr} A = 0$</td><td>$A^t = -A$</td></tr></table>
gilmore-lie-groups_TAB_006	90	■1.000	`\begin{tabular}[t]{|l|l|l|l|l|l|l|} \hline & \(X_{\eta}\) & \(X_{R}\) & \(X_{L}\) & \(X_{r}\) & \(X_{l}\) & \(X_{\delta}\) \\ \hline \(X_{\eta}\) & 0 & \(2 X_{R}\) & \(-2 X_{L}\) & \(X_{r}\) & \(-X_{l}\) & 0 \\ \hline \(X_{R}\) & & 0 & \(-4 X_{\eta}\) & 0 & \(-2 X_{r}\) & 0 \\ \hline \(X_{L}\) & & & 0 & \(2 X_{l}\) & 0 & 0 \\ \hline \(X_{r}\) & & & & 0 & \(-X_{\delta}\) & 0 \\ \hline \(X_{l}\) & & & & & 0 & 0 \\ \hline \(X_{\delta}\) & & & & & & 0 \\ \hline \end{tabular}`	<table><tr><th></th><th>X_η</th><th>X_R</th><th>X_L</th><th>X_r</th><th>X_l</th><th>X_δ</th></tr><tr><td>X_η</td><td>0</td><td>$2X_R$</td><td>$-2X_L$</td><td>X_r</td><td>$-X_l$</td><td>0</td></tr><tr><td>X_R</td><td></td><td>0</td><td>$-4X_\eta$</td><td>0</td><td>$-2X_r$</td><td>0</td></tr><tr><td>X_L</td><td></td><td></td><td>0</td><td>$2X_l$</td><td>0</td><td>0</td></tr><tr><td>X_r</td><td></td><td></td><td></td><td>0</td><td>$-X_\delta$</td><td>0</td></tr><tr><td>X_l</td><td></td><td></td><td></td><td></td><td>0</td><td>0</td></tr><tr><td>X_δ</td><td></td><td></td><td></td><td></td><td></td><td>0</td></tr></table>	<table><tr><td>X_R</td><td>0</td><td>$-4X_\eta$</td><td>0</td><td>$-2X_r$</td><td>0</td></tr><tr><td>X_L</td><td></td><td>0</td><td>$2X_l$</td><td>0</td><td>0</td></tr><tr><td>X_r</td><td></td><td></td><td>0</td><td>$-X_\delta$</td><td>0</td></tr><tr><td>X_l</td><td></td><td></td><td></td><td>0</td><td>0</td></tr><tr><td>X_δ</td><td></td><td></td><td></td><td></td><td>0</td></tr></table><table><tr><td></td><td>$\hat{n}+\frac{1}{2}I$</td><td>$a^\dagger a^\dagger$</td><td>aa</td><td>$a^\dagger$</td><td>a</td></tr><tr><td>$\hat{n}+\frac{1}{2}I$</td><td>0</td><td>$2a^\dagger a^\dagger$</td><td>$-2aa$</td><td>$a^\dagger$</td><td>$-a$</td></tr></table>
gilmore-lie-groups_TAB_007	90	■0.600	`\begin{tabular}[t]{|l|l|l|l|l|l|l|l|} \hline & \(\hat{n}+\frac{1}{2} I\) & \(a^{\dagger} a^{\dagger}\) & \(a a\) & \(a^{\dagger}\) & \(a\) & I \\ \hline \(\hat{n}+\frac{1}{2} I\) & 0 & \(2 a^{\dagger} a^{\dagger}\) & \(-2 a a\) & \(a^{\dagger}\) & \(-a\) & 0 \\ \hline \(a^{\dagger} a^{\dagger}\) & & 0 & \(-4\left(\hat{n}+\frac{1}{2} I\right)\) & 0 & \(-2 a^{\dagger}\) & 0 \\ \hline \(a a\) & & & 0 & \(2 a\) & 0 & 0 \\ \hline \(a^{\dagger}\) & & & & 0 & \(-I\) & 0 \\ \hline \(a\) & & & & & 0 & 0 \\ \hline I & & & & & & 0 \\ \hline \end{tabular}`	<table><tr><td></td><td>$\hat{n}+\frac{1}{2}I$</td><td>$a^\dagger a^\dagger$</td><td>aa</td><td>$a^\dagger$</td><td>a</td><td>I</td></tr><tr><td>$\hat{n}+\frac{1}{2}I$</td><td>0</td><td>$2a^\dagger a^\dagger$</td><td>$-2aa$</td><td>$a^\dagger$</td><td>$-a$</td><td>0</td></tr><tr><td>$a^\dagger a^\dagger$</td><td></td><td>0</td><td>$-4\left(\hat{n}+\frac{1}{2}I\right)$</td><td>0</td><td>$-2a^\dagger$</td><td>0</td></tr><tr><td>aa</td><td></td><td></td><td>0</td><td>$2a$</td><td>0</td><td>0</td></tr><tr><td>$a^\dagger$</td><td></td><td></td><td></td><td>0</td><td>$-I$</td><td>0</td></tr><tr><td>a</td><td></td><td></td><td></td><td></td><td>0</td><td>0</td></tr><tr><td>I</td><td></td><td></td><td></td><td></td><td></td><td>0</td></tr></table>	<table><tr><td>aa</td><td>0</td><td>$2a$</td><td>0</td><td>0</td></tr><tr><td>$a^\dagger$</td><td></td><td>0</td><td>$-I$</td><td>0</td></tr><tr><td>a</td><td></td><td></td><td>0</td><td>0</td></tr><tr><td>I</td><td></td><td></td><td></td><td>0</td></tr></table>ble inherits the antisymmetry of the commutator, so only one h ited. It is clear that there is an isomorphism between this ma e algebra of the photon energy operator $\hat{n} = a^\dagger a + \frac{1}{2}$, two-phc
gilmore-lie-groups_TAB_008	100	■0.817	`\begin{tabular}[t]{|l|l|l|l|} \hline \(\mu\) & \(\sigma\) & Algebra & Singular subspace \\ \hline +1 & +1 & \(\mathfrak{s} \mathfrak{o}(3,2)\) & \\ \hline -1 & +1 & \(\mathfrak{s} \mathfrak{o}(4,1)\) & \\ \hline -1 & -1 & \(\mathfrak{s} \mathfrak{o}(5)\) & \\ \hline +1 & 0 & Poincare & translations \(t_{\mu}\) \\ \hline 0 & 0 & Galilei & translations \(t_{\mu}\), boosts \(\mathbf{b}\) \\ \hline \end{tabular}`	<table><tr><th>μ</th><th>σ</th><th>Algebra</th><th>Singular subspace</th></tr><tr><td>+1</td><td>+1</td><td>$\mathfrak{so}(3,2)$</td><td></td></tr><tr><td>-1</td><td>+1</td><td>$\mathfrak{so}(4,1)$</td><td></td></tr><tr><td>-1</td><td>-1</td><td>$\mathfrak{so}(5)$</td><td></td></tr><tr><td>+1</td><td>0</td><td>Poincare</td><td>translations t_μ</td></tr><tr><td>0</td><td>0</td><td>Galilei</td><td>translations t_μ, boosts $\mathbf{b}$</td></tr></table>	<table><tr><td>-1</td><td>-1</td><td>$\mathfrak{so}(5)$</td><td></td></tr><tr><td>$+1$</td><td>0</td><td>Poincare</td><td>translations t_μ</td></tr><tr><td>0</td><td>0</td><td>Galilei</td><td>translations t_μ, boosts $\mathbf{b}$</td></tr></table>t $\mathfrak{g} = A$, where A is a Lie algebra of $n \times n$ matrices is negative-definite: tr $A^2 \leq 0$, $= 0 \Rightarrow A = 0$. Then sl ıy neighborhood of the identity I_n if t becomes large en

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_009	151	1.000	\begin{tabular}[t]{|l|l|l|} \hline \(\left[\begin{array}{cc|c}0 & \theta & v_{1} \\ -\theta & 0 & v_{2} \\ \hline v_{1} & v_{2} & 0\end{array}\right]\) & \(\left[\begin{array}{cc|cc}0 & \theta & v_{1} & t_{1} \\ -\theta & 0 & v_{2} & t_{2} \\ \hline v_{1} & v_{2} & 0 & t_{3} \\ 0 & 0 & 0 & 0\end{array}\right]\) & \(\left[\begin{array}{cc|cc}0 & \theta & v_{1} & t_{1} \\ -\theta & 0 & v_{2} & t_{2} \\ \hline 0 & 0 & 0 & t_{3} \\ 0 & 0 & 0 & 0\end{array}\right]\) \\ \hline Lorentz & Poincare & Galilei \\ \hline \end{tabular}	$\left[\begin{array}{cc\|c}0 & \theta & v_{1} \\ -\theta & 0 & v_{2} \\ \hline v_{1} & v_{2} & 0\end{array}\right]$ $\left[\begin{array}{cc\|cc}0 & \theta & v_{1} & t_{1} \\ -\theta & 0 & v_{2} & t_{2} \\ \hline v_{1} & v_{2} & 0 & t_{3} \\ 0 & 0 & 0 & 0\end{array}\right]$ $\left[\begin{array}{cc\|cc}0 & \theta & v_{1} & t_{1} \\ -\theta & 0 & v_{2} & t_{2} \\ \hline 0 & 0 & 0 & t_{3} \\ 0 & 0 & 0 & 0\end{array}\right]$ Lorentz Poincare Galilei	

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_010	167	■ 0.608	\begin{tabular}[t]{|l|l|l|} \hline \(\left[H_{1}, H_{2}\right]\) & = & 0 \\ \hline \(\left[\mathbf{H}, E_{ \pm 2 \mathbf{e}_{1}}\right]\) & = & \((\pm 2 / \sqrt{12}, 0) E_{ \pm 2 \mathrm{e}_{1}}\) \\ \hline \(\left[\mathbf{H}, E_{ \pm 2 \mathbf{e}_{2}}\right]\) & = & \((0, \pm 2 / \sqrt{12}) E_{ \pm 2 \mathrm{e}_{2}}\) \\ \hline \(\left[\mathbf{H}, E_{ \pm \mathbf{e}_{1} \pm \mathbf{e}_{2}}\right]\) & = & \((\pm / \sqrt{12}, \pm / \sqrt{12}) E_{ \pm \mathbf{e}_{1} \pm \mathbf{e}_{2}}\) \\ \hline \(\left[E_{+2 \mathrm{e}_{1}}, E_{-2 \mathrm{e}_{1}}\right]\) & = & \((2 / \sqrt{12}) H_{1}\) \\ \hline \(\left[E_{+2 \mathrm{e}_{2}}, E_{-2 \mathrm{e}_{2}}\right]\) & = & \((2 / \sqrt{12}) H_{2}\) \\ \hline \(\left[E_{ \pm \mathbf{e}_{1} \pm \mathbf{e}_{2}}, E_{-\left(\pm \mathbf{e}_{1} \pm \mathbf{e}_{2}\right)}\right]\) & = & \((1 / \sqrt{12})\left(\pm H_{1} \pm H_{2}\right)\) \\ \hline \(\left[E_{+2 \mathbf{e}_{1}}, E_{-\left(\mathbf{e}_{1}+\mathbf{e}_{2}\right)}\right]\) & = & \(*(1 / \sqrt{6}) E_{\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \(\left[E_{-\mathbf{e}_{1}+\mathbf{e}_{2}}, E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}\right]\) & = & \((1 / \sqrt{6}) E_{-2 \mathrm{e}_{1}}\) \\ \hline \(\left[E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}, E_{+2 \mathrm{e}_{1}}\right]\) & = & \((1 / \sqrt{6}) E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \(\left[E_{-2 \mathrm{e}_{1}}, E_{\mathbf{e}_{1}-\mathbf{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \(\left[E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}, E_{+\mathbf{e}_{1}+\mathbf{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{+2 \mathrm{e}_{1}}\) \\ \hline \(\left[E_{+\mathbf{e}_{1}+\mathbf{e}_{2}}, E_{-2 \mathrm{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \(\left[E_{+2 \mathrm{e}_{2}}, E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}\right]\) & = & \(*(1 / \sqrt{6}) E_{-\mathbf{e}_{1}+\mathbf{e}_{2}}\) \\ \hline \(\left[E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}, E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}\right]\) & = & \((1 / \sqrt{6}) E_{-2 \mathrm{e}_{2}}\) \\ \hline \(\left[E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}, E_{+2 \mathrm{e}_{2}}\right]\) & = & \((1 / \sqrt{6}) E_{+\mathbf{e}_{1}+\mathbf{e}_{2}}\) \\ \hline \(\left[E_{-2 \mathrm{e}_{2}}, E_{+\mathbf{e}_{1}+\mathbf{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{+\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \(\left[E_{+\mathbf{e}_{1}+\mathbf{e}_{2}}, E_{-\mathbf{e}_{1}+\mathbf{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{+2 \mathrm{e}_{2}}\) \\ \hline \(\left[E_{-\mathbf{e}_{1}+\mathbf{e}_{2}}, E_{-2 \mathrm{e}_{2}}\right]\) & = & \((-1 / \sqrt{6}) E_{-\mathbf{e}_{1}-\mathbf{e}_{2}}\) \\ \hline \end{tabular}	$[H_1, H_2] = 0$ $[\mathbf{H}, E_{\pm 2\mathbf{e}_1}] = (\pm 2/\sqrt{12}, 0)E_{\pm 2\mathbf{e}_1}$ $[\mathbf{H}, E_{\pm 2\mathbf{e}_2}] = (0, \pm 2/\sqrt{12})E_{\pm 2\mathbf{e}_2}$ $[\mathbf{H}, E_{\pm\mathbf{e}_1\pm\mathbf{e}_2}] = (\pm/\sqrt{12}, \pm/\sqrt{12})E_{\pm\mathbf{e}_1\pm\mathbf{e}_2}$ $[E_{+2\mathbf{e}_1}, E_{-2\mathbf{e}_1}] = (2/\sqrt{12})H_1$ $[E_{+2\mathbf{e}_2}, E_{-2\mathbf{e}_2}] = (2/\sqrt{12})H_2$ $[E_{\pm\mathbf{e}_1\pm\mathbf{e}_2}, E_{-(\pm\mathbf{e}_1\pm\mathbf{e}_2)}] = (1/\sqrt{12})(\pm H_1 \pm H_2)$ $[E_{+2\mathbf{e}_1}, E_{-(\mathbf{e}_1+\mathbf{e}_2)}] = *(1/\sqrt{6})E_{\mathbf{e}_1-\mathbf{e}_2}$ $[E_{-\mathbf{e}_1+\mathbf{e}_2}, E_{-\mathbf{e}_1-\mathbf{e}_2)}] = (1/\sqrt{6})E_{-2\mathbf{e}_1}$ $[E_{-\mathbf{e}_1-\mathbf{e}_2}, E_{+2\mathbf{e}_1}] = (1/\sqrt{6})E_{+\mathbf{e}_1-\mathbf{e}_2}$ $[E_{-2\mathbf{e}_1}, E_{\mathbf{e}_1-\mathbf{e}_2}] = (-1/\sqrt{6})E_{-\mathbf{e}_1-\mathbf{e}_2}$ $[E_{+\mathbf{e}_1-\mathbf{e}_2}, E_{+\mathbf{e}_1+\mathbf{e}_2}] = (-1/\sqrt{6})E_{+2\mathbf{e}_1}$ $[E_{+\mathbf{e}_1+\mathbf{e}_2}, E_{-2\mathbf{e}_2}] = (-1/\sqrt{6})E_{+\mathbf{e}_1-\mathbf{e}_2}$ $[E_{+2\mathbf{e}_2}, E_{-\mathbf{e}_1-\mathbf{e}_2}] = *(1/\sqrt{6})E_{-\mathbf{e}_1+\mathbf{e}_2}$ $[E_{-\mathbf{e}_1-\mathbf{e}_2}, E_{+\mathbf{e}_1-\mathbf{e}_2}] = (1/\sqrt{6})E_{-2\mathbf{e}_2}$ $[E_{+\mathbf{e}_1-\mathbf{e}_2}, E_{+2\mathbf{e}_2}] = (1/\sqrt{6})E_{+\mathbf{e}_1+\mathbf{e}_2}$ $[E_{-2\mathbf{e}_2}, E_{+\mathbf{e}_1+\mathbf{e}_2}] = (-1/\sqrt{6})E_{+\mathbf{e}_1-\mathbf{e}_2}$ $[E_{+\mathbf{e}_1+\mathbf{e}_2}, E_{-\mathbf{e}_1+\mathbf{e}_2}] = (-1/\sqrt{6})E_{+2\mathbf{e}_2}$ $[E_{-\mathbf{e}_1+\mathbf{e}_2}, E_{-2\mathbf{e}_2}] = (-1/\sqrt{6})E_{-\mathbf{e}_1-\mathbf{e}_2}$ **9.8 Conclusion**

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_011	172	1.000	\begin{tabular}[t]{|l|l|l|l|l|} \hline \multirow[b]{3}{*}{\((i \phi)^{j} f_{j}\)} & \multicolumn{4}{|c|}{Representation} \\ \hline & \(\frac{1}{2}\) & 1 & \(\frac{3}{2}\) & 2 \\ \hline & 2 & 3 & 4 & 5 \\ \hline \(f_{0}\) & \(\cos (\phi / 2)\) & 1 & \(\frac{9}{8} \cos \left(\frac{\phi}{2}\right)-\frac{1}{8} \cos \left(\frac{3 \phi}{2}\right)\) & 1 \\ \hline \((i \phi)^{1} f_{1}\) & \(2 i \sin (\phi / 2)\) & \(i \sin (\phi)\) & \(\frac{9 i}{4} \sin \left(\frac{\phi}{2}\right)-\frac{i}{12} \sin \left(\frac{3 \phi}{2}\right)\) & \(\frac{i}{3} \sin (\phi)-\frac{i}{6} \sin (2 \phi)\) \\ \hline \((i \phi)^{2} f_{2}\) & & \(\cos (\phi)-1\) & \(-\frac{1}{2} \cos \left(\frac{\phi}{2}\right)+\frac{1}{2} \cos \left(\frac{3 \phi}{2}\right)\) & \(-\frac{5}{4}+\frac{1}{3} \cos (\phi)-\frac{1}{12} \cos (2 \phi)\) \\ \hline \((i \phi)^{3} f_{3}\) & & & \(-i \sin \left(\frac{\phi}{2}\right)+\frac{i}{3} \sin \left(\frac{3 \phi}{2}\right)\) & \(-\frac{i}{3} \sin (\phi)+\frac{i}{6} \sin (2 \phi)\) \\ \hline \((i \phi)^{4} f_{4}\) & & & & \(\frac{1}{4}-\frac{1}{3} \cos (\phi)+\frac{1}{12} \cos (2 \phi)\) \\ \hline \end{tabular}	*(not rendered)*	
gilmore-lie-groups_TAB_012	175	0.995	\begin{tabular}[t]{|l|l|l|l|l|} \hline \(\cos ^{2}(\alpha, \beta)\) & \(\theta(\alpha, \beta)\) & \(n= \frac{2 \alpha \cdot \beta}{\alpha \cdot \alpha}\) & \(n^{\prime}=\frac{2 \alpha \cdot \beta}{\beta \cdot \beta}\) & \(\frac{\alpha \cdot \alpha}{\beta \cdot \beta}=\frac{n^{\prime}}{n}\) \\ \hline 1 & \(\frac{ \pi}{2} \pm \frac{\pi}{2}\) & \(\pm 2\) & \(\pm 2\) & 1 \\ \hline 3 & \(\frac{\pi}{2} \pm \frac{\pi}{3}\) & \(\pm 3\) & \(\pm 1\) & \(3^{-1}\) \\ \hline & & \(\pm 1\) & \(\pm 3\) & \(3^{+1}\) \\ \hline \(\frac{2}{4}\) & \(\frac{\pi}{2} \pm \frac{\pi}{4}\) & \(\pm 2\) & \(\pm 1\) & \(2^{-1}\) \\ \hline & & \(\pm 1\) & \(\pm 2\) & \(2^{+1}\) \\ \hline \(\frac{1}{4}\) & \(\frac{\pi}{2} \pm \frac{ \pi}{6}\) & \(\pm 1\) & \(\pm 1\) & 1 \\ \hline 0 & \(\frac{\pi}{2}\) & 0 & 0 & - \\ \hline \end{tabular}	$\cos^2(\alpha,\beta)$ | $\theta(\alpha,\beta)$ | $n=\frac{2\alpha\cdot\beta}{\alpha\cdot\alpha}$ | $n'=\frac{2\alpha\cdot\beta}{\beta\cdot\beta}$ | $\frac{\alpha\cdot\alpha}{\beta\cdot\beta}=\frac{n'}{n}$ 1 | $\frac{\pi}{2}\pm\frac{\pi}{2}$ | ±2 | ±2 | 1 $\frac{3}{4}$ | $\frac{\pi}{2}\pm\frac{\pi}{3}$ | ±3 | ±1 | 3^{-1} | | ±1 | ±3 | 3^{+1} $\frac{2}{4}$ | $\frac{\pi}{2}\pm\frac{\pi}{4}$ | ±2 | ±1 | 2^{-1} | | ±1 | ±2 | 2^{+1} $\frac{1}{4}$ | $\frac{\pi}{2}\pm\frac{\pi}{6}$ | ±1 | ±1 | 1 0 | $\frac{\pi}{2}$ | 0 | 0 | -	

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_013	180	0.508	`\begin{tabular}[t]{|l|l|l|l|l|l|l|} \hline & \(\alpha_{1}\) & \(\alpha_{2}\) & \(\alpha_{3}\) & & \(\alpha_{l-1}\) & \(\alpha_{l}\) \\ \hline \(A_{l-1}\) & \(\mathbf{e}_{1}-\mathbf{e}_{2}\) & \(\mathbf{e}_{2}-\mathbf{e}_{3}\) & \(\mathbf{e}_{3}-\mathbf{e}_{4}\) & ... & \(\mathbf{e}_{l-1}-\mathbf{e}_{l}\) & \\ \hline \(D_{l}\) & " & " & " & ... & " & \(\mathbf{e}_{l-1}+\mathbf{e}_{l}\) \\ \hline \(B_{l}\) & " & " & " & ... & " & \(\mathbf{e}_{l}\) \\ \hline \(C_{l}\) & " & " & " & ... & " & \(2\mathbf{e}_{l}\) \\ \hline \end{tabular}`	α_1 α_2 α_3 α_{l-1} α_l A_{l-1} $\mathbf{e}_1-\mathbf{e}_2$ $\mathbf{e}_2-\mathbf{e}_3$ $\mathbf{e}_3-\mathbf{e}_4$ ... $\mathbf{e}_{l-1}-\mathbf{e}_l$ D_l " " " ... " $\mathbf{e}_{l-1}+\mathbf{e}_l$ B_l " " " ... " $\mathbf{e}_l$ C_l " " " ... " $2\mathbf{e}_l$	A_{l-1} $\mathbf{e}_1-\mathbf{e}_2$ $\mathbf{e}_2-\mathbf{e}_3$ $\mathbf{e}_3-\mathbf{e}_4$ $\cdots$ $\mathbf{e}_{l-1}-\mathbf{e}_l$ D_l " " " $\cdots$ " $\mathbf{e}_{l-1}+\mathbf{e}_l$ B_l " " " $\cdots$ " $\mathbf{e}_l$ C_l " " " $\cdots$ " $2\mathbf{e}_l$ Inner products among the fundamental roots are summarized conv
gilmore-lie-groups_TAB_014	182	0.999	`\begin{tabular}[t]{|l|l|l|l|} \hline \(p\) & \(q\) & \(r\) & Root space \\ \hline \(p\) & 2 & 2 & \(D_{p+2}\) \\ \hline 3 & 3 & 2 & \(E_{6}\) \\ \hline 4 & 3 & 2 & \(E_{7}\) \\ \hline 5 & 3 & 2 & \(E_{8}\) \\ \hline \end{tabular}`	p q r Root space p 2 2 D_{p+2} 3 3 2 E_6 4 3 2 E_7 5 3 2 E_8	3 3 2 E_6 4 3 2 E_7 5 3 2 E_8 grams are summarized in T : root spaces. Each root spa
gilmore-lie-groups_TAB_015	184	1.000	`\begin{tabular}[t]{|l|l|l|l|} \hline Canonical form & Boson operators & Coordinates and derivatives & Fermion operators \\ \hline \(H_{i}\) & \(b_{i}^{\dagger} b_{i}\) & \(x^{i} \partial_{i}\) & \(f_{i}^{\dagger} f_{i}\) \\ \hline \(E_{+\mathbf{e}_{i}-\mathbf{e}_{j}}\) & \(b_{i}^{\dagger} b_{j}\) & \(x^{i} \partial_{j}\) & \(f_{i}^{\dagger} f_{j}\) \\ \hline \end{tabular}`	Canonical form Boson operators Coordinates and derivatives Fermion operators H_i $b_i^\dagger b_i$ $x^i\partial_i$ $f_i^\dagger f_i$ $E_{+\mathbf{e}_i-\mathbf{e}_j}$ $b_i^\dagger b_j$ $x^i\partial_j$ $f_i^\dagger f_j$	H_i $b_i^\dagger b_i$ $x^i\partial_i$ $f_i^\dagger f_i$ $E_{+\mathbf{e}_i-\mathbf{e}_j}$ $b_i^\dagger b_j$ $x^i\partial_j$ $f_i^\dagger f_j$ that the following identifications are appropriate for the eigenopera paces C_l and D_l:

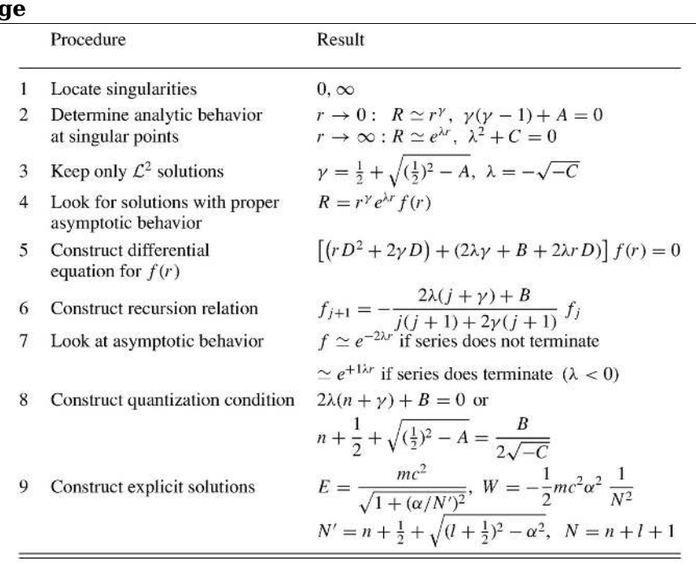
Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_016	184	0.960	\begin{tabular}[t]{|l|l|l|l|l|} \hline \multirow[b]{2}{*}{Canonical form} & \multicolumn{2}{|c|}{\(C_{l}\)} & \multicolumn{2}{|c|}{\(D_{l}\)} \\ \hline & Boson operators & Coordinates and derivatives & Fermion operators & Coordinates and derivatives \\ \hline \(H_{i}\) & \(b_{i}^{\dagger} b_{i}+\frac{1}{2}\) & \(x^{i} \partial_{i}\) & \(f_{i}^{\dagger} f_{i}+\frac{1}{2}\) & \(x^{i} \partial_{i}+\frac{1}{2}\) \\ \hline \(E_{+\mathbf{e}_{i}-\mathbf{e}_{j}}\) & \(b_{i}^{\dagger} b_{j}\) & \(x^{i} \partial_{j}\) & \(f_{i}^{\dagger} f_{j}\) & \(x^{i} \partial_{j}\) \\ \hline \(E_{+\mathbf{e}_{i}+\mathbf{e}_{j}}\) & \(b_{i}^{\dagger} b_{j}^{\dagger}\) & \(x^{i} x^{j}\) & \(f_{i}^{\dagger} f_{j}^{\dagger}\) & \(x^{i} x^{j}\) \\ \hline \(E_{-\mathbf{e}_{i}-\mathbf{e}_{j}}\) & \(b_{i} b_{j}\) & \(\partial_{i} \partial_{j}\) & \(f_{i} f_{j}\) & \(\partial_{i} \partial_{j}\) \\ \hline \(E_{+2 \mathrm{e}_{i}}\) & \(b_{i}^{\dagger} b_{i}^{\dagger}\) & \(x^{i} x^{i}\) & & \\ \hline \(E_{-2 \mathbf{e}_{i}}\) & \(b_{i} b_{i}\) & \(\partial_{i} \partial_{i}\) & & \\ \hline \end{tabular}	*(not rendered)*	H_i $b_i^\dagger b_i + \frac{1}{2}$ $x^i \partial_i$ $f_i^\dagger f_i + \frac{1}{2}$ $x^i \partial_i + \frac{1}{2}$ $E_{+\mathbf{e}_i - \mathbf{e}_j}$ $b_i^\dagger b_j$ $x^i \partial_j$ $f_i^\dagger f_j$ $x^i \partial_j$ $E_{+\mathbf{e}_i + \mathbf{e}_j}$ $b_i^\dagger b_j^\dagger$ $x^i x^j$ $f_i^\dagger f_j^\dagger$ $x^i x^j$ $E_{-\mathbf{e}_i - \mathbf{e}_j}$ $b_i b_j$ $\partial_i \partial_j$ $f_i f_j$ $\partial_i \partial_j$ $E_{+2\mathbf{e}_i}$ $b_i^\dagger b_i^\dagger$ $x^i x^i$ $E_{-2\mathbf{e}_i}$ $b_i b_i$ $\partial_i \partial_i$
gilmore-lie-groups_TAB_017	195	0.899	\begin{tabular}[t]{|l|l|l|l|} \hline Mapping & Real form & Root space & Condition \\ \hline \multirow[t]{4}{*}{Block submatrix} & \(\mathfrak{s} \mathfrak{o}(p, q)\) & \(D_{n}\) & \(p+q=2 n\) \\ \hline & \(\mathfrak{s} \mathfrak{o}(p, q)\) & \(B_{n}\) & \(p+q=2 n+1\) \\ \hline & \(\mathfrak{s} \mathfrak{u}(p, q)\) & \(A_{n-1}\) & \(p+q=n\) \\ \hline & \(\mathfrak{s} \mathfrak{p}(p, q)\) & \(C_{n}\) & \(p+q=n\) \\ \hline \multirow[t]{2}{*}{Subfield restriction} & \(\mathfrak{s l}(n ; \mathbb{R})\) & \(A_{n-1}\) & \\ \hline & \(\mathfrak{s} \mathfrak{p}(2 n ; \mathbb{R})\) & \(C_{n}\) & \\ \hline \multirow[t]{2}{*}{Field embedding} & \(\mathfrak{s} \mathfrak{o}^{*}(2 n)\) & \(D_{n}\) & \\ \hline & \(\mathfrak{s} \mathfrak{u}^{*}(2 n)\) & \(A_{2 n-1}\) & \\ \hline \end{tabular}	*(not rendered)*	Block submatrix $\mathfrak{so}(p,q)$ D_n $p+q=2n$ $\mathfrak{so}(p,q)$ B_n $p+q=2n+1$ $\mathfrak{su}(p,q)$ A_{n-1} $p+q=n$ $\mathfrak{sp}(p,q)$ C_n $p+q=n$ Subfield restriction $\mathfrak{sl}(n;\mathbb{R})$ A_{n-1} $\mathfrak{sp}(2n;\mathbb{R})$ C_n Field embedding $\mathfrak{so}^*(2n)$ D_n $\mathfrak{su}^*(2n)$ A_{2n-1}

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_018	195	■0.500	`\begin{tabular}[t]{|l|l|l|l|l|} \hline \(A_{1}\) & ~ & \(B_{1}\) & ~ & \(C_{1}\) \\ \hline \(\mathfrak{s} \mathfrak{u}(2)\) & ~ & \(\mathfrak{s} \mathfrak{o}(3) \) & ~ & \(\mathfrak{s} \mathfrak{p}(1)=\mathfrak{u} \mathfrak{s} \mathfrak{p}(2)\) \\ \hline \(\mathfrak{s u}(1,1)=\mathfrak{s l}(2 ; \mathbb{R})\) & ~ & \(\mathfrak{s} \mathfrak{o}(2,1)\) & ~ & \(\mathfrak{s} \mathfrak{p}(2 ; \mathbb{R})\) \\ \hline \(D_{2}\) & = & \(A_{1}\) & + & \(A_{1}\) \\ \hline \(\mathfrak{s} \mathfrak{o}(4)\) & = & \(\mathfrak{s} \mathfrak{o}(3) \) & + & \(\mathfrak{s} \mathfrak{o}(3)\) \\ \hline \(\mathfrak{s} \mathfrak{o}^{*}(4)\) & ~ & \(\mathfrak{s} \mathfrak{o}(3)\) & + & \(\mathfrak{s} \mathfrak{o}(2,1) \) \\ \hline \(\mathfrak{s} \mathfrak{o}(3,1)\) & & \(\mathfrak{s l}(2 ; \mathbb{C})\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(2,1)\) & ~ & \(\mathfrak{s} \mathfrak{o}(2,1)\) & + & \(\mathfrak{s} \mathfrak{o}(2,1)\) \\ \hline \(B_{2}\) & = & \(C_{2}\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(5)\) & ~ & \(\mathfrak{s} \mathfrak{p}(2)=\mathfrak{u} \mathfrak{s} \mathfrak{p}(4)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(4,1)\) & ~ & \(\mathfrak{s} \mathfrak{p}(1,1)=\mathfrak{u} \mathfrak{s} \mathfrak{p}(2,2)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(3,2)\) & ~ & \(\mathfrak{s} \mathfrak{p}(4 ; R)\) & & \\ \hline \(D_{3}\) & = & \(A_{3}\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(6) \) & ~ & \(\mathfrak{s} \mathfrak{u}(4)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(5,1)\) & ~ & \(\mathfrak{s} \mathfrak{u}^{*}(4)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}^{*}(6)\) & ~ & \(\mathfrak{s} \mathfrak{u}(3,1)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(4,2)\) & ~ & \(\mathfrak{s} \mathfrak{u}(2,2)\) & & \\ \hline \(\mathfrak{s} \mathfrak{o}(3,3)\) & ~ & \(\mathfrak{s l}(4 ; \mathbb{R})\) & & \\ \hline \end{tabular}`	A_1 ‖ B_1 ‖ C_1 $\mathfrak{su}(2)$ ‖ $\mathfrak{so}(3)$ ‖ $\mathfrak{sp}(1) = \mathfrak{usp}(2)$ $\mathfrak{su}(1,1) = \mathfrak{sl}(2;\mathbb{R})$ ‖ $\mathfrak{so}(2,1)$ ‖ $\mathfrak{sp}(2;\mathbb{R})$ D_2 ‖ = ‖ A_1 ‖ + ‖ A_1 $\mathfrak{so}(4)$ ‖ = ‖ $\mathfrak{so}(3)$ ‖ + ‖ $\mathfrak{so}(3)$ $\mathfrak{so}^*(4)$ ‖ $\mathfrak{so}(3)$ ‖ + ‖ $\mathfrak{so}(2,1)$ $\mathfrak{so}(3,1)$ ‖ $\mathfrak{sl}(2;\mathbb{C})$ $\mathfrak{so}(2,2)$ ‖ $\mathfrak{so}(2,1)$ ‖ + ‖ $\mathfrak{so}(2,1)$ B_2 ‖ = ‖ C_2 $\mathfrak{so}(5)$ ‖ $\mathfrak{sp}(2) = \mathfrak{usp}(4)$ $\mathfrak{so}(4,1)$ ‖ $\mathfrak{sp}(1,1) = \mathfrak{usp}(2,2)$ $\mathfrak{so}(3,2)$ ‖ $\mathfrak{sp}(4;R)$ D_3 ‖ = ‖ A_3 $\mathfrak{so}(6)$ ‖ $\mathfrak{su}(4)$ $\mathfrak{so}(5,1)$ ‖ $\mathfrak{su}^*(4)$ $\mathfrak{so}^*(6)$ ‖ $\mathfrak{su}(3,1)$ $\mathfrak{so}(4,2)$ ‖ $\mathfrak{su}(2,2)$ $\mathfrak{so}(3,3)$ ‖ $\mathfrak{sl}(4;\mathbb{R})$	D_2 = A_1 + A_1 $\mathfrak{so}(4)$ = $\mathfrak{so}(3)$ + $\mathfrak{so}(3)$ $\mathfrak{so}^*(4)$ ~ $\mathfrak{so}(3)$ + $\mathfrak{so}(2, 1)$ $\mathfrak{so}(3, 1)$ ~ $\mathfrak{sl}(2; \mathbb{C})$ $\mathfrak{so}(2, 2)$ ~ $\mathfrak{so}(2, 1)$ + $\mathfrak{so}(2, 1)$ B_2 = C_2 $\mathfrak{so}(5)$ ~ $\mathfrak{sp}(2) = \mathfrak{usp}(4)$ $\mathfrak{so}(4, 1)$ ~ $\mathfrak{sp}(1, 1) = \mathfrak{usp}(2, 2)$ $\mathfrak{so}(3, 2)$ ~ $\mathfrak{sp}(4; R)$ D_3 = A_3 $\mathfrak{so}(6)$ ~ $\mathfrak{su}(4)$ $\mathfrak{so}(5, 1)$ ~ $\mathfrak{su}^*(4)$ $\mathfrak{so}^*(6)$ ~ $\mathfrak{su}(3, 1)$ $\mathfrak{so}(4, 2)$ ~ $\mathfrak{su}(2, 2)$ $\mathfrak{so}(3, 3)$ ~ $\mathfrak{sl}(4; \mathbb{R})$

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_019	196	■ 1.000	`\begin{tabular}[t]{|l|l|l|l|} \hline \multirow[b]{2}{*}{Root space} & \multirow[b]{2}{*}{Class \({ }_{\text {rank(character) }}\)} & \multicolumn{2}{|l|}{Maximal compact subgroup} \\ \hline & & Root space & Dimension \\ \hline \multirow[t]{2}{*}{ \(G_{2}\)} & \(G_{2(-14)}\) & \(G_{2}\) & 14 \\ \hline & \(G_{2(+2)}\) & \(A_{1}+A_{1}\) & 6 \\ \hline \multirow[t]{3}{*}{\(F_{4}\)} & \(F_{4(-52)}\) & \(F_{4} \) & 52 \\ \hline & \(F_{4(-20)}\) & \(B_{4}\) & 36 \\ \hline & \(F_{4(+4)}\) & \(C_{3}+A_{1}\) & 24 \\ \hline \multirow[t]{5}{*}{\(E_{6}\)} & \(E_{6(-78)}\) & \(E_{6}\) & 78 \\ \hline & \(E_{6(-26)}\) & \(F_{4}\) & 52 \\ \hline & \(E_{6(-14)}\) & \(D_{5}+D_{1}\) & 46 \\ \hline & \(E_{6(+2)}\) & \(A_{5}+A_{1}\) & 38 \\ \hline & \(E_{6(+6)}\) & \(C_{4}\) & 36 \\ \hline \multirow[t]{4}{*}{\(E_{7}\)} & \(E_{7(-133)}\) & \(E_{7}\) & 133 \\ \hline & \(E_{7(-25)}\) & \(E_{6}+D_{1}\) & 79 \\ \hline & \(E_{7(-5)}\) & \(D_{6}+A_{1}\) & 69 \\ \hline & \(E_{7(+7)}\) & \(A_{7}\) & 63 \\ \hline \multirow[t]{3}{*}{\(E_{8}\)} & \(E_{8(-248)}\) & \(E_{8}\) & 248 \\ \hline & \(E_{8(-24)}\) & \(E_{7}+A_{1}\) & 136 \\ \hline & \(E_{8(+8)}\) & \(D_{8}\) & 120 \\ \hline \end{tabular}`	*(not rendered)*	Root space / Class$_{\text{rank(character)}}$ / Maximal compact subgroup: Root space, Dimension G_2: $G_{2(-14)}$, G_2, 14; $G_{2(+2)}$, A_1+A_1, 6 F_4: $F_{4(-52)}$, F_4, 52; $F_{4(-20)}$, B_4, 36; $F_{4(+4)}$, C_3+A_1, 24 E_6: $E_{6(-78)}$, E_6, 78; $E_{6(-26)}$, F_4, 52; $E_{6(-14)}$, D_5+D_1, 46; $E_{6(+2)}$, A_5+A_1, 38; $E_{6(+6)}$, C_4, 36 E_7: $E_{7(-133)}$, E_7, 133; $E_{7(-25)}$, E_6+D_1, 79; $E_{7(-5)}$, D_6+A_1, 69; $E_{7(+7)}$, A_7, 63 E_8: $E_{8(-248)}$, E_8, 248; $E_{8(-24)}$, E_7+A_1, 136; $E_{8(+8)}$, D_8, 120 completeness, we list the real forms for the exceptional Li
gilmore-lie-groups_TAB_020	199	■ 1.000	`\begin{tabular}[t]{|l|l|l|} \hline Quadrant & Operator & Edge \\ \hline I & \(J_{+}\) & \(n_{2}=0\) \\ \hline I & \(J_{-}\) & \(n_{1}=0\) \\ \hline III & \(J_{+}\) & \(n_{1}=-1\) \\ \hline III & \(J_{-}\) & \(n_{2}=-1\) \\ \hline \end{tabular}`	Quadrant / Operator / Edge I / J_+ / $n_2=0$ I / J_- / $n_1=0$ III / J_+ / $n_1=-1$ III / J_- / $n_2=-1$	III / J_+ / $n_1=-1$ III / J_- / $n_2=-1$ elements for the shift operators $K_\pm$ in su(1 $i\sqrt{(j\mp m)(j\pm m+1)}=\sqrt{\left(m\pm\frac{1}{2}\right)^2-}$

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_021	206	0.877	`\begin{tabular}[t]{|l|l|l|l|} \hline Root space & Quotient \(G^{\prime} / H\) & Dimension \(P\) & Rank \(P\) \\ \hline \(A_{p+q-1}\) & \(S U(p, q) / S[U(p) \otimes U(q)]\) & \(2 p q\) & \(\min (p, q)\) \\ \hline \(A_{n-1}\) & \(S L(n ; \mathbb{R}) / S O(n)\) & \(\frac{1}{2}(n+2)(n-1)\) & \(n-1\) \\ \hline \(A_{2 n-1}\) & \(S U^{*}(2 n) / U \operatorname{Sp}(2 n)\) & \((2 n+1)(n-1)\) & \(n-1\) \\ \hline \(B_{p+q}\) & \(S O(p, q) / S O(p) \otimes S O(q)\) & \(p q\) & \(\min (p, q)\) \\ \hline \(D_{p+q}\) & \(S O(p, q) / S O(p) \otimes S O(q)\) & \(p q\) & \(\min (p, q)\) \\ \hline \(D_{n}\) & \(S O^{*}(2 n) / U(n)\) & \(n(n-1)\) & [\(n / 2\)] \\ \hline \(C_{p+q}\) & \(U S p(2 p, 2 q) / U S p(2 p) \otimes U S p(2 q)\) & \(4 p q\) & \(\min (p, q)\) \\ \hline \(C_{n}\) & \(S p(2 n ; \mathbb{R}) / U(n)\) & \(n(n+1)\) & \(n\) \\ \hline \end{tabular}`	Root space | Quotient G'/H | Dimension P | Rank P A_{p+q-1} | $SU(p,q)/S[U(p)\otimes U(q)]$ | $2pq$ | $\min(p,q)$ A_{n-1} | $SL(n;\mathbb{R})/SO(n)$ | $\frac{1}{2}(n+2)(n-1)$ | $n-1$ A_{2n-1} | $SU^*(2n)/U\,\mathrm{Sp}(2n)$ | $(2n+1)(n-1)$ | $n-1$ B_{p+q} | $SO(p,q)/SO(p)\otimes SO(q)$ | pq | $\min(p,q)$ D_{p+q} | $SO(p,q)/SO(p)\otimes SO(q)$ | pq | $\min(p,q)$ D_n | $SO^*(2n)/U(n)$ | $n(n-1)$ | $[\,n/2\,]$ C_{p+q} | $USp(2p,2q)/USp(2p)\otimes USp(2q)$ | $4pq$ | $\min(p,q)$ C_n | $Sp(2n;\mathbb{R})/U(n)$ | $n(n+1)$ | n	A_{2n-1} | $SU^*(2n)/USp(2n)$ | $(2n+1)(n-1)$ | $n-1$ B_{p+q} | $SO(p,q)/SO(p)\otimes SO(q)$ | pq | $\min(p,q)$ D_{p+q} | $SO(p,q)/SO(p)\otimes SO(q)$ | pq | $\min(p,q)$ D_n | $SO^*(2n)/U(n)$ | $n(n-1)$ | $[n/2]$ C_{p+q} | $USp(2p,2q)/USp(2p)\otimes USp(2q)$ | $4pq$ | $\min(p,q)$ C_n | $Sp(2n;\mathbb{R})/U(n)$ | $n(n+1)$ | n
gilmore-lie-groups_TAB_022	207	0.979	`\begin{tabular}[t]{|l|l|l|l|l|l|} \hline Root space & \(G^{\prime} / H\) & \(\operatorname{Dim} G^{\prime}\) & \(\operatorname{Dim} H\) & \(\operatorname{Dim} P\) & Rank \(P\) \\ \hline \(G_{2}\) & \(G_{2(+2)} /\left(A_{1} \oplus A_{1}\right)\) & 14 & 6 & 8 & 2 \\ \hline \(F_{4}\) & \(F_{4(-20)} / B_{4}\) & 52 & 36 & 16 & 1 \\ \hline & \(F_{4(+4)} /\left(C_{3} \oplus A_{1}\right)\) & 52 & 24 & 28 & 4 \\ \hline \multirow[t]{4}{*}{\(E_{6}\)} & \(E_{6(-26)} / F_{4}\) & 78 & 52 & 26 & 2 \\ \hline & \(E_{6(-14)} /\left(D_{5} \oplus D_{1}\right)\) & 78 & 46 & 32 & 2 \\ \hline & \(E_{6(+2)} /\left(A_{5} \oplus A_{1}\right)\) & 78 & 38 & 40 & 4 \\ \hline & \(E_{6(+6)} / C_{4}\) & 78 & 36 & 42 & 6 \\ \hline \multirow[t]{3}{*}{\(E_{7}\)} & \(E_{7(-25)} /\left(E_{6} \oplus D_{1}\right)\) & 133 & 79 & 54 & 3 \\ \hline & \(E_{7(-5)} /\left(D_{6} \oplus A_{1}\right)\) & 133 & 69 & 64 & 4 \\ \hline & \(E_{7(+7)} / A_{7}\) & 133 & 63 & 70 & 7 \\ \hline \multirow[t]{2}{*}{\(E_{8}\)} & \(E_{8(-24)} /\left(E_{7} \oplus A_{1}\right)\) & 248 & 136 & 112 & 4 \\ \hline & \(E_{8(+8)} / D_{8}\) & 248 & 120 & 128 & 8 \\ \hline \end{tabular}`	*(not rendered)*	Root space | G'/H | Dim G' | Dim H | Dim P | Rank P G_2 | $G_{2(+2)}/(A_1\oplus A_1)$ | 14 | 6 | 8 | 2 F_4 | $F_{4(-20)}/B_4$ | 52 | 36 | 16 | 1 | $F_{4(+4)}/(C_3\oplus A_1)$ | 52 | 24 | 28 | 4 E_6 | $E_{6(-26)}/F_4$ | 78 | 52 | 26 | 2 | $E_{6(-14)}/(D_5\oplus D_1)$ | 78 | 46 | 32 | 2 | $E_{6(+2)}/(A_5\oplus A_1)$ | 78 | 38 | 40 | 4 | $E_{6(+6)}/C_4$ | 78 | 36 | 42 | 6 E_7 | $E_{7(-25)}/(E_6\oplus D_1)$ | 133 | 79 | 54 | 3 | $E_{7(-5)}/(D_6\oplus A_1)$ | 133 | 69 | 64 | 4 | $E_{7(+7)}/A_7$ | 133 | 63 | 70 | 7 E_8 | $E_{8(-24)}/(E_7\oplus A_1)$ | 248 | 136 | 112 | 4 | $E_{8(+8)}/D_8$ | 248 | 120 | 128 | 8 subgroup in G'. To each there is a compact real form under $G'/H \to G/H$. For

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_023	212	1.000	`\begin{tabular}[t]{|l|l|l|l|} \hline Operation & \(i \sigma_{1}\) & \(i \sigma_{2}\) & \(i \sigma_{3}\) \\ \hline \(T_{1}=\) block matrix decomposition & -1 & -1 & +1 \\ \hline \(T_{2}=\) complex conjugation & -1 & +1 & -1 \\ \hline \(T_{3}=T_{1} T_{2}\) & +1 & -1 & -1 \\ \hline \end{tabular}`	Operation | $i\sigma_1$ | $i\sigma_2$ | $i\sigma_3$ $T_1 =$ block matrix decomposition | -1 | -1 | +1 $T_2 =$ complex conjugation | -1 | +1 | -1 $T_3 = T_1 T_2$ | +1 | -1 | -1	$T_1 =$ block matrix decomposition -1 -1 $+1$ $T_2 =$ complex conjugation -1 $+1$ -1 $T_3 = T_1\, T_2$ $+1$ -1 -1 es $\mathfrak{g}_{+,+} = 0, \mathfrak{g}_{+,-} = i\sigma_3, \mathfrak{g}_{-,+} = i\sigma_2, \mathfrak{g}_{-,-} = i\sigma_1$. Note tha
gilmore-lie-groups_TAB_024	232	1.000	`\begin{tabular}[t]{|l|l|l|} \hline Bessel function & $\rightarrow$ & harmonic oscillator eigenfunction \\ \hline Bessel equation & $\rightarrow$ & Schrödinger equation for harmonic oscilator \\ \hline \end{tabular}`	Bessel function | $\rightarrow$ | harmonic oscillator eigenfunction Bessel equation | $\rightarrow$ | Schrödinger equation for harmonic oscilator	with respect to the subalgebra J_-. Use a simple Inönü–Wigner contraction t show
gilmore-lie-groups_TAB_025	239	0.424	`\begin{tabular}[t]{|l|l|l|l|} \hline Equation & \(A \) & \(B\) & \(C\) \\ \hline Klein-Gordon & \(-l(l+1)+ \left(e^{2} / \hbar c\right)^{2}\) & \(2 E e^{2} /(\hbar c)^{2}\) & \(\left[E^{2}-\left(m c^{2}\right)^{2}\right] /(\hbar c)^{2}\) \\ \hline Schrödinger & \(-l(l+1)\) & \(2 m e^{2} / \hbar^{2}\) & \(2 \mathrm{~m} \mathrm{~W} / \hbar^{2}\) \\ \hline \end{tabular}`	Equation | A | B | C Klein-Gordon | $-l(l+1)+\left(e^2/\hbar c\right)^2$ | $2Ee^2/(\hbar c)^2$ | $\left[E^2-\left(mc^2\right)^2\right]/(\hbar c)^2$ Schrödinger | $-l(l+1)$ | $2me^2/\hbar^2$ | $2\,\mathrm{m}\,\mathrm{W}/\hbar^2$	Schrödinger $-l(l+1)$ $2me^2/\hbar^2$ $2mW/\hbar^2$ There is a standard procedure for solving simple ordinary differential equatic of the type presented in Eq. (14.9). This is the Frobenius method. The steps involv in this method, and the result of each step, are summarized in Table 14.1.

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_026	240	■ 0.655	\begin{tabular}[t]{|l|l|l|} \hline & Procedure & Result \\ \hline 1 & Locate singularities & \(0, \infty\) \\ \hline 2 & Determine analytic behavior at singular points & \[\begin{aligned} & r \rightarrow 0: R \simeq r^{\gamma}, \gamma(\gamma-1)+A=0 \\ & r \rightarrow \infty: R \simeq e^{\lambda r}, \quad \lambda^{2}+C=0 \end{aligned} \] \\ \hline 3 & Keep only \(\mathcal{L}^{2}\) solutions & \(\gamma=\frac{1}{2}+ \sqrt{\left(\frac{1}{2}\right)^{2}-A}, \lambda=- \sqrt{-C}\) \\ \hline 4 & Look for solutions with proper asymptotic behavior & \(R=r^{\gamma} e^{\lambda r} f(r) \) \\ \hline 5 & Construct differential equation for \(f(r)\) & \(\left[\left(r D^{2}+2 \gamma D\right)+(2 \lambda \gamma+B+2 \lambda r D)\right] f(r)=0\) \\ \hline 6 & Construct recursion relation & \(f_{j+1}=- \frac{2 \lambda(j+\gamma)+B}{j(j+1)+2 \gamma(j+1)} f_{j}\) \\ \hline 7 & Look at asymptotic behavior & \[\begin{aligned} & f \simeq e^{-2 \lambda r} \text { if series does not terminate } \\ & \simeq e^{+1 \lambda r} \text { if series does terminate }(\lambda<0) \end{aligned} \] \\ \hline 8 & Construct quantization condition & \[\begin{aligned} & 2 \lambda(n+\gamma)+B=0 \text { or } \\ & n+\frac{1}{2}+\sqrt{\left(\frac{1}{2} \right)^{2}-A}=\frac{B}{2 \sqrt{-C}} \end{aligned} \] \\ \hline 9 & Construct explicit solutions & \[\begin{aligned} & E=\frac{m c^{2}}{\sqrt{1+\left(\alpha / N^{\prime}\right)^{2}}}, W=-\frac{1}{2} m c^{2} \alpha^{2} \frac{1}{N^{2}} \\ & N^{\prime}=n+\frac{1}{2}+\sqrt{\left(l+\frac{1}{2}\right)^{2}-\alpha^{2}}, \quad N=n+l+1 \end{aligned} \] \\ \hline \end{tabular}	*(not rendered)*	 This is a dimensionless ratio of three physical constants that are fundamental in three "different" areas of physics: e (electromagnetism), $\hbar$ (quantum mechanics),

	Procedure	Result
1	Locate singularities	$0, \infty$
2	Determine analytic behavior at singular points	$r \rightarrow 0: \; R \simeq r^{\gamma}, \; \gamma(\gamma-1)+A=0$ $r \rightarrow \infty: R \simeq e^{\lambda r}, \; \lambda^{2}+C=0$
3	Keep only $\mathcal{L}^{2}$ solutions	$\gamma=\frac{1}{2}+\sqrt{\left(\frac{1}{2}\right)^{2}-A}, \; \lambda=-\sqrt{-C}$
4	Look for solutions with proper asymptotic behavior	$R=r^{\gamma} e^{\lambda r} f(r)$
5	Construct differential equation for $f(r)$	$\left[\left(r D^{2}+2 \gamma D\right)+(2 \lambda \gamma+B+2 \lambda r D)\right] f(r)=0$
6	Construct recursion relation	$f_{j+1}=-\frac{2 \lambda(j+\gamma)+B}{j(j+1)+2 \gamma(j+1)} f_{j}$
7	Look at asymptotic behavior	$f \simeq e^{-2 \lambda r}$ if series does not terminate $\simeq e^{+1 \lambda r}$ if series does terminate $(\lambda<0)$
8	Construct quantization condition	$2 \lambda(n+\gamma)+B=0$ or $n+\frac{1}{2}+\sqrt{\left(\frac{1}{2}\right)^{2}-A}=\frac{B}{2 \sqrt{-C}}$
9	Construct explicit solutions	$E=\frac{m c^{2}}{\sqrt{1+(\alpha / N^{\prime})^{2}}}, \; W=-\frac{1}{2} m c^{2} \alpha^{2} \frac{1}{N^{2}}$ $N^{\prime}=n+\frac{1}{2}+\sqrt{\left(l+\frac{1}{2}\right)^{2}-\alpha^{2}}, \; N=n+l+1$

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_027	244	0.998	`\begin{tabular}[t]{|l|l|l|l|l|} \hline \(m\) & \(l=0 \) & \(l=1\) & \(l=2\) & \(l=3\) \\ \hline 0 & \(\sqrt{ \frac{1}{4 \pi}}\) & \(\sqrt{\frac{3}{4 \pi}} \cos \theta \) & \(\sqrt{\frac{5}{16 \pi}}\left(3 \cos ^{2} \theta-1 \right)\) & \(\sqrt{\frac{7}{16 \pi}}\left(5 \cos ^{3} \theta-3 \cos \theta\right)\) \\ \hline \(\pm 1\) & & \(\mp \sqrt{\frac{3}{8 \pi}} \sin \theta e^{ \pm i \phi}\) & \(\mp \sqrt{\frac{15}{8 \pi}} \cos \theta \sin \theta e^{ \pm i \phi}\) & \(\mp \sqrt{\frac{21}{64 \pi}} \sin \theta\left(5 \cos ^{2} \theta-1\right) e^{ \pm i \phi}\) \\ \hline \(\pm 2\) & & & \(\sqrt{\frac{15}{32 \pi}} \sin ^{2} \theta e^{ \pm 2 i \phi}\) & \(\sqrt{\frac{105}{32 \pi}} \sin ^{2} \theta \cos \theta e^{ \pm 2 i \phi}\) \\ \hline \(\pm 3\) & & & & \(\mp \sqrt{\frac{35}{64 \pi}} \sin ^{3} \theta e^{ \pm 3 i \phi}\) \\ \hline \end{tabular}`	m | $l=0$ | $l=1$ | $l=2$ | $l=3$ 0 | $\sqrt{\frac{1}{4\pi}}$ | $\sqrt{\frac{3}{4\pi}}\cos\theta$ | $\sqrt{\frac{5}{16\pi}}\left(3\cos^2\theta-1\right)$ | $\sqrt{\frac{7}{16\pi}}\left(5\cos^3\theta-3\cos\theta\right)$ ±1 | | $\mp\sqrt{\frac{3}{8\pi}}\sin\theta e^{\pm i\phi}$ | $\mp\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta e^{\pm i\phi}$ | $\mp\sqrt{\frac{21}{64\pi}}\sin\theta\left(5\cos^2\theta-1\right)e^{\pm i\phi}$ ±2 | | | $\sqrt{\frac{15}{32\pi}}\sin^2\theta e^{\pm 2i\phi}$ | $\sqrt{\frac{105}{32\pi}}\sin^2\theta\cos\theta e^{\pm 2i\phi}$ ±3 | | | | $\mp\sqrt{\frac{35}{64\pi}}\sin^3\theta e^{\pm 3i\phi}$	m | $l=0$ | $l=1$ | $l=2$ | $l=3$ 0 | $\sqrt{\frac{1}{4\pi}}$ | $\sqrt{\frac{3}{4\pi}}\cos\theta$ | $\sqrt{\frac{5}{16\pi}}(3\cos^2\theta-1)$ | $\sqrt{\frac{7}{16\pi}}(5\cos^3\theta-3\cos\theta)$ ±1 | | $\mp\sqrt{\frac{3}{8\pi}}\sin\theta\, e^{\pm i\phi}$ | $\mp\sqrt{\frac{15}{8\pi}}\cos\theta\sin\theta\, e^{\pm i\phi}$ | $\mp\sqrt{\frac{21}{64\pi}}\sin\theta(5\cos^2\theta-1)\, e^{\pm i\phi}$ ±2 | | | $\sqrt{\frac{15}{32\pi}}\sin^2\theta\, e^{\pm 2i\phi}$ | $\sqrt{\frac{105}{32\pi}}\sin^2\theta\cos\theta\, e^{\pm 2i\phi}$ ±3 | | | | $\mp\sqrt{\frac{35}{64\pi}}\sin^3\theta\, e^{\pm 3i\phi}$
gilmore-lie-groups_TAB_028	258	0.721	`\begin{tabular}[t]{|l|l|l|} \hline H & \(\frac{p^{2}}{2 m}-\frac{\alpha}{r}\) & \(\sqrt{p^{2}+m^{2}}-\frac{ \alpha}{r}\) \\ \hline \(A_{4}\) & \(\mathbf{r} \cdot \mathbf{p}-i\) & \(\mathbf{r} \cdot \mathbf{p}-i\) \\ \hline \(\Gamma_{4}\) & \(\frac{1}{2}(r \mathbf{p} \cdot \mathbf{p}-r)\) & \(\frac{1}{2}\left(r \mathbf{p} \cdot \mathbf{p}-r-\frac{\alpha^{2}}{r}\right)\) \\ \hline \(\Gamma_{5}\) & \(\frac{1}{2}(r \mathbf{p} \cdot \mathbf{p}+r)\) & \(\frac{1}{2}\left(r \mathbf{p} \cdot \mathbf{p}+r-\frac{\alpha^{2}}{r}\right)\) \\ \hline \(\Theta\) & \(r\left(H_{S}-W\right)\) & \(r\left\{ \left(H_{K G}+\frac{\alpha}{r}\right)^{2}-\left(E+\frac{ \alpha}{r}\right)^{2}\right\}\) \\ \hline & \(A\left(\Gamma_{5}+\Gamma_{4}\right)+B\left(\Gamma_{5}-\Gamma_{4} \right)+C\) & \(A\left(\Gamma_{5}+\Gamma_{4}\right)+B \left(\Gamma_{5}-\Gamma_{4}\right)+C\) \\ \hline \(A\) & \(1 / 2 m\) & 1 \\ \hline B & \(-W\) & \(m^{2}-E^{2}\) \\ \hline C & \(-\alpha\) & \(-2 \alpha E\) \\ \hline \end{tabular}`	H | $\frac{p^2}{2m}-\frac{\alpha}{r}$ | $\sqrt{p^2+m^2}-\frac{\alpha}{r}$ A_4 | $\mathbf{r}\cdot\mathbf{p}-i$ | $\mathbf{r}\cdot\mathbf{p}-i$ Γ_4 | $\frac{1}{2}(r\mathbf{p}\cdot\mathbf{p}-r)$ | $\frac{1}{2}\left(r\mathbf{p}\cdot\mathbf{p}-r-\frac{\alpha^2}{r}\right)$ Γ_5 | $\frac{1}{2}(r\mathbf{p}\cdot\mathbf{p}+r)$ | $\frac{1}{2}\left(r\mathbf{p}\cdot\mathbf{p}+r-\frac{\alpha^2}{r}\right)$ Θ | $r\left(H_S-W\right)$ | $r\left\{\left(H_{KG}+\frac{\alpha}{r}\right)^2-\left(E+\frac{\alpha}{r}\right)^2\right\}$ | $A\left(\Gamma_5+\Gamma_4\right)+B\left(\Gamma_5-\Gamma_4\right)+C$ | $A\left(\Gamma_5+\Gamma_4\right)+B\left(\Gamma_5-\Gamma_4\right)+C$ A | $1/2m$ | 1 B | $-W$ | m^2-E^2 C | $-\alpha$ | $-2\alpha E$	H | $\frac{p^2}{2m}-\frac{\alpha}{r}$ | $\sqrt{p^2+m^2}-\frac{\alpha}{r}$ A_4 | $\mathbf{r}\cdot\mathbf{p}-i$ | $\mathbf{r}\cdot\mathbf{p}-i$ Γ_4 | $\frac{1}{2}(r\mathbf{p}\cdot\mathbf{p}-r)$ | $\frac{1}{2}\left(r\mathbf{p}\cdot\mathbf{p}-r-\frac{\alpha^2}{r}\right)$ Γ_5 | $\frac{1}{2}(r\mathbf{p}\cdot\mathbf{p}+r)$ | $\frac{1}{2}\left(r\mathbf{p}\cdot\mathbf{p}+r-\frac{\alpha^2}{r}\right)$ Θ | $r(H_S-W)$ | $r\left\{\left(H_{KG}+\frac{\alpha}{r}\right)^2-\left(E+\frac{\alpha}{r}\right)^2\right\}$ | $A(\Gamma_5+\Gamma_4)+B(\Gamma_5-\Gamma_4)+C$ | $A(\Gamma_5+\Gamma_4)+B(\Gamma_5-\Gamma_4)+C$ A | $1/2m$ | 1 B | $-W$ | m^2-E^2 C | $-\alpha$ | $-2\alpha E$ In the event a magnetic field **B** is present, the momentum operators **p** should be replaced by $\pi=\mathbf{p}-\frac{q}{c}\mathbf{A}$. Under this condition the operators still close under

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_029	267	0.953	`\begin{tabular}[t]{|l|l|} \hline Particle & Rest energy (MeV) \\ \hline electron \(e^{ \pm}\) & 0.511 \\ \hline mu meson \(\mu^{ \pm}\) & 105.7 \\ \hline tau meson \(\tau^{ \pm}\) & 1784.0 \\ \hline proton, antiproton \(p^{ \pm}\) & 938.26 \\ \hline deuteron \(d^{+}\) & 1875.6 \\ \hline tritium \(t^{+}\) & 2809.4 \\ \hline pi meson \(\pi^{ \pm}\) & 139.6 \\ \hline sigma meson \(\Sigma^{ \pm}\) & 1385.0 \\ \hline cascade meson \(\Xi^{-}\) & 1533.0 \\ \hline omega \(\Omega^{-}\) & 1672.0 \\ \hline \end{tabular}`	<table><tr><th>Particle</th><th>Rest energy (MeV)</th></tr><tr><td>electron $e^{\pm}$</td><td>0.511</td></tr><tr><td>mu meson $\mu^{\pm}$</td><td>105.7</td></tr><tr><td>tau meson $\tau^{\pm}$</td><td>1784.0</td></tr><tr><td>proton, antiproton $p^{\pm}$</td><td>938.26</td></tr><tr><td>deuteron d^{+}</td><td>1875.6</td></tr><tr><td>tritium t^{+}</td><td>2809.4</td></tr><tr><td>pi meson $\pi^{\pm}$</td><td>139.6</td></tr><tr><td>sigma meson $\Sigma^{\pm}$</td><td>1385.0</td></tr><tr><td>cascade meson Ξ^{-}</td><td>1533.0</td></tr><tr><td>omega Ω^{-}</td><td>1672.0</td></tr></table>	<table><tr><th>Particle</th><th>Rest energy (MeV)</th></tr><tr><td>electron $e^{\pm}$</td><td>0.511</td></tr><tr><td>mu meson $\mu^{\pm}$</td><td>105.7</td></tr><tr><td>tau meson $\tau^{\pm}$</td><td>1784.0</td></tr><tr><td>proton, antiproton $p^{\pm}$</td><td>938.26</td></tr><tr><td>deuteron d^{+}</td><td>1875.6</td></tr><tr><td>tritium t^{+}</td><td>2809.4</td></tr><tr><td>pi meson $\pi^{\pm}$</td><td>139.6</td></tr><tr><td>sigma meson $\Sigma^{\pm}$</td><td>1385.0</td></tr><tr><td>cascade meson Ξ^{-}</td><td>1533.0</td></tr><tr><td>omega Ω^{-}</td><td>1672.0</td></tr></table>
gilmore-lie-groups_TAB_030	269	1.000	`\begin{tabular}[t]{|l|l|l|l|l|l|l|l|} \hline Mercury & Venus & Earth & Mars & Jupiter & Saturn & Uranus & Neptune \\ \hline 0.39 & 0.72 & 1.00 & 1.52 & 5.20 & 9.54 & 19.18 & 30.06 \\ \hline \end{tabular}`	<table><tr><th>Mercury</th><th>Venus</th><th>Earth</th><th>Mars</th><th>Jupiter</th><th>Saturn</th><th>Uranus</th><th>Neptune</th></tr><tr><td>0.39</td><td>0.72</td><td>1.00</td><td>1.52</td><td>5.20</td><td>9.54</td><td>19.18</td><td>30.06</td></tr></table>	<table><tr><th>Mercury</th><th>Venus</th><th>Earth</th><th>Mars</th><th>Jupiter</th><th>Saturn</th><th>Uranus</th><th>Neptune</th></tr><tr><td>0.39</td><td>0.72</td><td>1.00</td><td>1.52</td><td>5.20</td><td>9.54</td><td>19.18</td><td>30.06</td></tr></table>
gilmore-lie-groups_TAB_031	274	1.000	`\begin{tabular}[t]{|l|l|l|l|} \hline Time period & Approach & Strengths & Weaknesses \\ \hline Nineteenth century & Manifestly covariant & Fields have elegant transformation properties & Many fields represent nonphysical states \\ \hline Twentieth century & Hilbert space & All linear superpositions represent physical states & Transformation properties are complicated \\ \hline \end{tabular}`	<table><tr><th>Time period</th><th>Approach</th><th>Strengths</th><th>Weaknesses</th></tr><tr><td>Nineteenth century</td><td>Manifestly covariant</td><td>Fields have elegant transformation properties</td><td>Many fields represent nonphysical states</td></tr><tr><td>Twentieth century</td><td>Hilbert space</td><td>All linear superpositions represent physical states</td><td>Transformation properties are complicated</td></tr></table>	<table><tr><th>Time period</th><th>Approach</th><th>Strengths</th><th>Weaknesses</th></tr><tr><td>Nineteenth century</td><td>Manifestly covariant</td><td>Fields have elegant transformation properties</td><td>Many fields represent nonphysical states</td></tr><tr><td>Twentieth century</td><td>Hilbert space</td><td>All linear superpositions represent physical states</td><td>Transformation properties are complicated</td></tr></table>
gilmore-lie-groups_TAB_032	280	0.815	`\begin{tabular}[t]{|l|l|l|} \hline & \(|k\rangle\) & \(\left.\begin{array}{cc}j & j^{\prime} \\ \mu & \mu^{\prime}\end{array}\right\}\) \\ \hline \(\{I, a\}\) & \(|k\rangle e^{i k \cdot a}\) & \(\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle_{\nu \nu^{\prime} ; \mu \mu^{\prime}}\) \\ \hline \{ \(\Lambda\), 0\} & \(|\Lambda k\rangle\) & \(\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle D_{\nu \nu^{\prime} ; \mu \mu^{\prime}}^{j j^{\prime}}(\Lambda)\) \\ \hline \end{tabular}`	<table><tr><td></td><td>$\|k\rangle$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \mu & \mu^{\prime}\end{array}\right\}$</td></tr><tr><td>$\{I, a\}$</td><td>$\|k\rangle e^{i k \cdot a}$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle_{\nu \nu^{\prime} ; \mu \mu^{\prime}}$</td></tr><tr><td>$\{\Lambda, 0\}$</td><td>$\|\Lambda k\rangle$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle D_{\nu \nu^{\prime} ; \mu \mu^{\prime}}^{j j^{\prime}}(\Lambda)$</td></tr></table>	<table><tr><td></td><td>$\|k\rangle$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \mu & \mu^{\prime}\end{array}\right\rangle$</td></tr><tr><td>$\{I, a\}$</td><td>$\|k\rangle e^{i k \cdot a}$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle \delta_{\nu \nu^{\prime} ; \mu \mu^{\prime}}$</td></tr><tr><td>$\{\Lambda, 0\}$</td><td>$\|\Lambda k\rangle$</td><td>$\left.\begin{array}{cc}j & j^{\prime} \\ \nu & \nu^{\prime}\end{array}\right\rangle D_{\nu \nu^{\prime} ; \mu \mu^{\prime}}^{j j^{\prime}}(\Lambda)$</td></tr></table>

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_033	290	1.000	`\begin{tabular}[t]{|l|l|l|} \hline & Electric & Magnetic \\ \hline Charge density & \(\rho_{e}(\mathbf{x}, t)=\sum_{j} e_{j} \mathbf{x}_{j}(t)\) & \(\rho_{m}(\mathbf{x}, t)=\sum_{j} m_{j} \mathbf{x}_{j}(t)\) \\ \hline Current density & \(\mathbf{J}_{e}(\mathbf{x}, t)=\sum_{j} e_{j} \frac{d \mathbf{x}_{j}(t)}{d t}\) & \(\mathbf{J}_{m}(\mathbf{x}, t)=\sum_{j} m_{j} \frac{d \mathbf{x}_{j}(t)}{d t}\) \\ \hline Conservation law & \(\nabla \cdot \mathbf{J}_{e}(\mathbf{x}, t)+\frac{\partial \rho_{e}(\mathbf{x}, t)}{\partial t}=0\) & \(\nabla \cdot \mathbf{J}_{m}(\mathbf{x}, t)+\frac{\partial \rho_{m}(\mathbf{x}, t)}{\partial t}=0\) \\ \hline \end{tabular}`	<table><tr><th></th><th>Electric</th><th>Magnetic</th></tr><tr><td>Charge density</td><td>$\rho_e(\mathbf{x},t)=\sum_j e_j\mathbf{x}_j(t)$</td><td>$\rho_m(\mathbf{x},t)=\sum_j m_j\mathbf{x}_j(t)$</td></tr><tr><td>Current density</td><td>$\mathbf{J}_e(\mathbf{x},t)=\sum_j e_j\frac{d\mathbf{x}_j(t)}{dt}$</td><td>$\mathbf{J}_m(\mathbf{x},t)=\sum_j m_j\frac{d\mathbf{x}_j(t)}{dt}$</td></tr><tr><td>Conservation law</td><td>$\nabla\cdot\mathbf{J}_e(\mathbf{x},t)+\frac{\partial\rho_e(\mathbf{x},t)}{\partial t}=0$</td><td>$\nabla\cdot\mathbf{J}_m(\mathbf{x},t)+\frac{\partial\rho_m(\mathbf{x},t)}{\partial t}=0$</td></tr></table>	<table><tr><td>Charge density</td><td>$\rho_e(\mathbf{x},t)=\sum_j e_j\mathbf{x}_j(t)$</td><td>$\rho_m(\mathbf{x},t)=\sum_j m_j\mathbf{x}_j(t)$</td></tr><tr><td>Current density</td><td>$\mathbf{J}_e(\mathbf{x},t)=\sum_j e_j\frac{d\mathbf{x}_j(t)}{dt}$</td><td>$\mathbf{J}_m(\mathbf{x},t)=\sum_j m_j\frac{d\mathbf{x}_j(t)}{dt}$</td></tr><tr><td>Conservation law</td><td>$\nabla\cdot\mathbf{J}_e(\mathbf{x},t)+\frac{\partial\rho_e(\mathbf{x},t)}{\partial t}=0$</td><td>$\nabla\cdot\mathbf{J}_m(\mathbf{x},t)+\frac{\partial\rho_m(\mathbf{x},t)}{\partial t}=0$</td></tr></table> The conservation equations enforce the conditions of charge conservation (both elec-

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_ TAB_034	311	0.852	\begin{tabular}[t]{|l|l|l|l|l|l|l|l|} \hline \multicolumn{3}{|c|}{Infinitesimal generators} & \multicolumn{3}{|c|}{Canonical coordinates} & Constraint \\ \hline \(\xi(x, y)\) & \(\eta(x, y)\) & \(\zeta(x, y, p)\) & \(R(x, y)\) & \(S(x, y)\) & \(T(x, y, p)\) & \(d S / d R\) \\ \hline 0 & 1 & 0 & \(x\) & \(y\) & \(p\) & \(T\) \\ \hline 1 & 0 & 0 & \(y\) & \(x\) & \(p\) & \(1 / T\) \\ \hline 1 / a & -1 / b & 0 & \(a x+b y\) & \(b x-a y\) & \(p\) & \((b-a T) /(a+b T)\) \\ \hline \(x\) & 0 & \(-p\) & \(y\) & \(\ln x\) & \(x p\) & \(1 / T\) \\ \hline 0 & \(y\) & \(p\) & \(x\) & \(\ln y\) & \(p / y\) & \(T\) \\ \hline \(x / a\) & \(y / b\) & \((1 / b-1 / a) p\) & \(y^{b} / x^{a}\) & \(b \ln y\) & \(p / x^{(a / b-1)}\) & \((b T / R) /(b T-a R^{(1 / b)}\) \\ \hline \(x\) & \(y\) & 0 & \(y / x\) & \(\ln y\) & \(p\) & \((T / R) /(T-R)\) \\ \hline \(x\) & \(-y\) & \(-2 p\) & \(x y\) & \(\ln y\) & \(x^{2} p\) & \((T / R) /(T+R)\) \\ \hline \(2 x\) & \(y\) & \(-p\) & \(y^{2} / x\) & \(\ln y\) & \(y p\) & \((T / R) /(2 T-R)\) \\ \hline \(x\) & \(2 y\) & \(p\) & \(y / x^{2}\) & \(\ln y\) & \(p / x\) & \((T / R) /(T-2 R)\) \\ \hline \(y\) & 0 & \(-p^{2}\) & \(y\) & \(x / y\) & \(x-y / p\) & \(-T / R^{2}\) \\ \hline 0 & \(x\) & 1 & \(x\) & \(y / x\) & \(x p-y\) & \(T / R^{2}\) \\ \hline \(-y\) & \(x\) & \(1+p^{2}\) & \(\sqrt{x^{2}+y^{2}}\) & \(\tan ^{-1}(y / x)\) & \((y-x p) /(x+y p)\) & \(-T / R\) \\ \hline 1 & \(y / x\) & \((p x-y) / x^{2}\) & \(y / x\) & \(x\) & \((x p-y) / x^{2}\) & \(1 / T\) \\ \hline \(a\) & \(x\) & 1 & \(x^{2}-2 a y\) & \(x / a\) & \(x-a p\) & \(1 /(2 a T)\) \\ \hline \(a\) & \(y\) & \(p\) & \(x-a \ln y\) & \(x / a\) & \(p / y\) & \((1 / a) /(1-a T)\) \\ \hline \(x\) & \(b\) & \(-p\) & \(e^{y} / x^{b}\) & \(y / b\) & \(e^{y} * p^{b}\) & \(\left[1-b(R / T)^{(1 / b)}\right]\) \\ \hline \(y\) & \(b\) & \(-p^{2}\) & \(y^{2}-2 b x\) & \(y / b\) & \(y-b / p\) & \(1 /(2 b T)\) \\ \hline 0 & \(e^{f(x)}\) & \(f^{\prime} e^{f}\) & \(x\) & \(y / e^{f}\) & \(p-y f^{\prime}\) & \(T / e^{f(R)}\) \\ \hline \(x^{2}\) & \(x y\) & \(y-x p\) & \(y / x\) & \(1 / x\) & \(x p-y\) & \(1 / T\) \\ \hline \(x y\) & \(y^{2}\) & \(y p-x p^{2}\) & \(y / x\) & \(1 / y\) & \(y / p-x\) & \(1 /(T R^{2})\) \\ \hline \(x y\) & 0 & \(-y p-x p^{2}\) & \(y\) & \((\ln x) / y\) & \(y /(x p)-\ln x\) & \(T / R^{2}\) \\ \hline 0 & \(x y\) & \(y+x p\) & \(x\) & \((\ln y) / x\) & \(x p / y-\ln y\) & \(T / R^{2}\) \\ \hline \(g(y)\) & 0 & \(-g^{\prime} p^{2}\) & \(y\) & \(x / g\) & \(1 / p-x g^{\prime} / g\) & \(T / g(R)\) \\ \hline 0 & \(f(x)\) & \(f^{\prime}\) & \(x\) & \(y / f\) & \(f p-f^{\prime} y\) & \(T / f^{2}(R)\) \\ \hline \(f(x)\) & 0 & \(-f^{\prime} p\) & \(y\) & \(F\left(F^{\prime} f=1\right)\) & \(p f\) & \(1 / T\) \\ \hline 0 & \(g(y)\) & \(g^{\prime} p\) & \(x\) & \(G\left(G^{\prime} g=1\right)\) & \(p / g\) & \(T\) \\ \hline \(x^{k+1}\) & \(k x^{k} y\) & \(x^{k}\left(k^{2} y / x-p\right)\) & \(y / x^{k}\) & \(1 / x^{k}\) & \(x p-k y\) & \(-k / T\) \\ \hline \(k x y^{k}\) & \(y^{k+1}\) & \(y^{k}\left(p-k^{2} x p^{2} / y\right)\) & \(x / y^{k}\) & \(1 / y^{k}\) & \(y / p-k x\) & \(-k / T\) \\ \hline \end{tabular}	<table><tr><th colspan="3">Infinitesimal generators</th><th colspan="3">Canonical coordinates</th><th>Constraint</th></tr><tr><td>$\xi(x,y)$</td><td>$\eta(x,y)$</td><td>$\zeta(x,y,p)$</td><td>$R(x,y)$</td><td>$S(x,y)$</td><td>$T(x,y,p)$</td><td>dS/dR</td></tr><tr><td>0</td><td>1</td><td>0</td><td>x</td><td>y</td><td>p</td><td>T</td></tr><tr><td>1</td><td>0</td><td>0</td><td>y</td><td>x</td><td>p</td><td>$1/T$</td></tr><tr><td>$1/a$</td><td>$-1/b$</td><td>0</td><td>$ax+by$</td><td>$bx-ay$</td><td>p</td><td>$(b-aT)/(a+bT)$</td></tr><tr><td>x</td><td>0</td><td>$-p$</td><td>y</td><td>$\ln x$</td><td>xp</td><td>$1/T$</td></tr><tr><td>0</td><td>y</td><td>p</td><td>x</td><td>$\ln y$</td><td>p/y</td><td>T</td></tr><tr><td>x/a</td><td>y/b</td><td>$(1/b-1/a)p$</td><td>y^b/x^a</td><td>$b\ln y$</td><td>$p/x^{(a/b-1)}$</td><td>$(bT/R)/(bT-aR^{(1/b)})$</td></tr><tr><td>x</td><td>y</td><td>0</td><td>y/x</td><td>$\ln y$</td><td>p</td><td>$(T/R)/(T-R)$</td></tr><tr><td>x</td><td>$-y$</td><td>$-2p$</td><td>xy</td><td>$\ln y$</td><td>x^2p</td><td>$(T/R)/(T+R)$</td></tr><tr><td>$2x$</td><td>y</td><td>$-p$</td><td>y^2/x</td><td>$\ln y$</td><td>yp</td><td>$(T/R)/(2T-R)$</td></tr><tr><td>x</td><td>$2y$</td><td>p</td><td>y/x^2</td><td>$\ln y$</td><td>p/x</td><td>$(T/R)/(T-2R)$</td></tr><tr><td>y</td><td>0</td><td>$-p^2$</td><td>y</td><td>x/y</td><td>$x-y/p$</td><td>$-T/R^2$</td></tr><tr><td>0</td><td>x</td><td>1</td><td>x</td><td>y/x</td><td>$xp-y$</td><td>T/R^2</td></tr><tr><td>$-y$</td><td>x</td><td>$1+p^2$</td><td>$\sqrt{x^2+y^2}$</td><td>$\tan^{-1}(y/x)$</td><td>$(y-xp)/(x+yp)$</td><td>$-T/R$</td></tr><tr><td>1</td><td>y/x</td><td>$(px-y)/x^2$</td><td>y/x</td><td>x</td><td>$(xp-y)/x^2$</td><td>$1/T$</td></tr><tr><td>a</td><td>x</td><td>1</td><td>x^2-2ay</td><td>x/a</td><td>$x-ap$</td><td>$1/(2aT)$</td></tr><tr><td>a</td><td>y</td><td>p</td><td>$x-a\ln y$</td><td>x/a</td><td>p/y</td><td>$(1/a)/(1-aT)$</td></tr><tr><td>x</td><td>b</td><td>$-p$</td><td>e^y/x^b</td><td>y/b</td><td>e^y*p^b</td><td>$(bR)^{-1}/$</td></tr><tr><td></td><td></td><td></td><td></td><td></td><td></td><td>$[1-b(R/T)^{(1/b)}]$</td></tr><tr><td>y</td><td>b</td><td>$-p^2$</td><td>y^2-2bx</td><td>y/b</td><td>$y-b/p$</td><td>$1/(2bT)$</td></tr><tr><td>0</td><td>$e^{f(x)}$</td><td>$f'e^f$</td><td>x</td><td>y/e^f</td><td>$p-yf'$</td><td>$T/e^{f(R)}$</td></tr><tr><td>x^2</td><td>xy</td><td>$y-xp$</td><td>y/x</td><td>$1/x$</td><td>$xp-y$</td><td>$1/T$</td></tr><tr><td>xy</td><td>y^2</td><td>$yp-xp^2$</td><td>y/x</td><td>$1/y$</td><td>$y/p-x$</td><td>$1/(TR^2)$</td></tr><tr><td>xy</td><td>0</td><td>$-yp-xp^2$</td><td>y</td><td>$(\ln x)/y$</td><td>$y/(xp)-\ln x$</td><td>T/R^2</td></tr><tr><td>0</td><td>xy</td><td>$y+xp$</td><td>x</td><td>$(\ln y)/x$</td><td>$xp/y-\ln y$</td><td>T/R^2</td></tr><tr><td>$g(y)$</td><td>0</td><td>$-g'p^2$</td><td>y</td><td>x/g</td><td>$1/p-xg'/g$</td><td>$T/g(R)$</td></tr><tr><td>0</td><td>$f(x)$</td><td>f'</td><td>x</td><td>y/f</td><td>$fp-f'y$</td><td>$T/f^2(R)$</td></tr><tr><td>$f(x)$</td><td>0</td><td>$-f'p$</td><td>y</td><td>$F\,(F'f=1)$</td><td>pf</td><td>$1/T$</td></tr><tr><td>0</td><td>$g(y)$</td><td>$g'p$</td><td>x</td><td>$G\,(G'g=1)$</td><td>p/g</td><td>T</td></tr><tr><td>x^{k+1}</td><td>kx^ky</td><td>$x^k(k^2y/x-p)$</td><td>y/x^k</td><td>$1/x^k$</td><td>$xp-ky$</td><td>$-k/T$</td></tr><tr><td>kxy^k</td><td>y^{k+1}</td><td>$y^k(p-k^2xp^2/y)$</td><td>x/y^k</td><td>$1/y^k$</td><td>$y/p-kx$</td><td>$-k/T$</td></tr></table> where $D^{(n)} = \frac{\partial}{} + \frac{dy}{}\frac{\partial}{} + \frac{dy^{(1)}}{}\frac{\partial}{} + \dots + y^{(n+1)}\frac{\partial}{}$ (16.40)

Identifier	Page	Conf.	LaTeX source	Rendered	Image
gilmore-lie-groups_TAB_035	316	1.000	`\begin{tabular}[t]{|l|l|l|} \hline 1830 & Galois & solve algebraic equations \\ \hline 1874 & Lie & solve differential equations \\ \hline 1969 & Risch & integrate in closed form \\ \hline ? & - & solve determining equations for \(\xi, \eta\) \\ \hline ? & - & solve determining equations for \(R, S, T\) \\ \hline ? & - & solve \(F(R,-, T)=0\) for \(R\). \\ \hline \end{tabular}`	1830 | Galois | solve algebraic equations 1874 | Lie | solve differential equations 1969 | Risch | integrate in closed form ? | - | solve determining equations for ξ, η ? | - | solve determining equations for R, S, T ? | - | solve $F(R,-,T)=0$ for R.	1830 Galois solve algebraic equations 1874 Lie solve differential equations 1969 Risch integrate in closed form ? – solve determining equations for ξ, η ? – solve determining equations for R, S, T ? – solve $F(R,-,T)=0$ for R. It is clear that additional algorithms are possible and desirable.
gilmore-lie-groups_TAB_036	318	1.000	`\begin{tabular}[t]{|l|l|l|} \hline \(k\) & Potential type & Transformation \\ \hline -1 & Coulomb & \(\gamma^{2}=\beta^{3}\) \\ \hline 0 & no force & \(\gamma^{2}=\beta^{2}\) \\ \hline +1 & local gravitational potential & \(\gamma^{2}=\beta^{1}\) \\ \hline +2 & harmonic oscillator & \(\gamma^{2}=\beta^{0}\) \\ \hline \end{tabular}`	k | Potential type | Transformation -1 | Coulomb | $\gamma^2=\beta^3$ 0 | no force | $\gamma^2=\beta^2$ +1 | local gravitational potential | $\gamma^2=\beta^1$ +2 | harmonic oscillator | $\gamma^2=\beta^0$	k Potential type Transformation -1 Coulomb $\gamma^2=\beta^3$ 0 no force $\gamma^2=\beta^2$ $+1$ local gravitational potential $\gamma^2=\beta^1$ $+2$ harmonic oscillator $\gamma^2=\beta^0$
gilmore-lie-groups_TAB_037	319	1.000	`\begin{tabular}[t]{|l|l|l|} \hline \(v_{i}\) & Infinitesial & \(e^{\epsilon v_{i}} f(x, t)=\) \\ \hline \(v_{1}\) & \(\partial_{x}\) & \(f(x-\epsilon, t)\) \\ \hline \(v_{2}\) & \(\partial_{t}\) & \(f(x, t-\epsilon)\) \\ \hline \(v_{3}\) & \(u \partial_{u}\) & \(e^{\epsilon} f(x, t)\) \\ \hline \(v_{4}\) & \(x \partial_{x}+2 t \partial_{t}\) & \(f\left(e^{-\epsilon} x, e^{-2 \epsilon} t\right)\) \\ \hline \(v_{5}\) & \(2 t \partial_{x}-x u \partial_{u}\) & \(e^{-\epsilon x+\epsilon^{2} t} f(x-2 \epsilon t, t)\) \\ \hline \(v_{6}\) & \(4 x t \partial_{x}+4 t^{2} \partial_{t}-\left(x^{2}+2 t\right) u \partial_{u}\) & \(\lambda e^{-\epsilon \lambda^{2} x^{2}} f\left(\lambda^{2} x, \lambda^{2} t\right)\) where \(\lambda^{2}=1 /(1+4 \epsilon t)\) \\ \hline \end{tabular}`	v_i | Infinitesial | $e^{\epsilon v_i} f(x,t)=$ v_1 | ∂_x | $f(x-\epsilon,t)$ v_2 | ∂_t | $f(x,t-\epsilon)$ v_3 | $u\partial_u$ | $e^{\epsilon} f(x,t)$ v_4 | $x\partial_x+2t\partial_t$ | $f\left(e^{-\epsilon}x, e^{-2\epsilon}t\right)$ v_5 | $2t\partial_x-xu\partial_u$ | $e^{-\epsilon x+\epsilon^2 t} f(x-2\epsilon t,t)$ v_6 | $4xt\partial_x+4t^2\partial_t-\left(x^2+2t\right)u\partial_u$ | $\lambda e^{-\epsilon\lambda^2x^2} f\left(\lambda^2 x, \lambda^2 t\right)$ where $\lambda^2=1/(1+4\epsilon t)$	v_i Infinitesial $e^{\epsilon v_i} f(x,t)=$ v_1 ∂_x $f(x-\epsilon,t)$ v_2 ∂_t $f(x,t-\epsilon)$ v_3 $u\partial_u$ $e^{\epsilon} f(x,t)$ v_4 $x\partial_x+2t\partial_t$ $f(e^{-\epsilon}x, e^{-2\epsilon}t)$ v_5 $2t\partial_x-xu\partial_u$ $e^{-\epsilon x+\epsilon^2 t} f(x-2\epsilon t,t)$ v_6 $4xt\partial_x+4t^2\partial_t-(x^2+2t)u\partial_u$ $\lambda e^{-\epsilon\lambda^2x^2} f(\lambda^2 x, \lambda^2 t)$ where $\lambda^2=1/(1+4\epsilon t)$ xo-dimensional wave equation is
gilmore-lie-groups_TAB_038	320	1.000	`\begin{tabular}[t]{|l|l|} \hline Symmetry & Conserved quantity \\ \hline Space displacements & momentum \\ \hline Time displacements & energy \\ \hline Space-time displacements & four-momentum \\ \hline Rotations & angular momentum \\ \hline \end{tabular}`	Symmetry | Conserved quantity Space displacements | momentum Time displacements | energy Space-time displacements | four-momentum Rotations | angular momentum	Rotations angular momentum