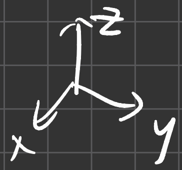
$^{A}P_{B}$ — A → frame; B → corrdinates of orgin frame B



Homogenous Transformation

$$^{A}P = {}^{A}_{B}R\,{}^{B}P + {}^{A}P_{B}$$

For rotation $R : [\hat{x}\ \hat{y}\ \hat{z}]$

$$\hat{z} = \hat{x} \times \hat{y}$$

$$\Rightarrow {}^{A}P = {}^{A}_{B}T\, P^{B} \qquad P^{B} \rightarrow \begin{bmatrix} {}^{B}P \\ 1 \end{bmatrix}, \qquad {}^{A}P \rightarrow \begin{bmatrix} {}^{A}P \\ 1 \end{bmatrix}$$

$$ {}^{A}_{B}T = \left[\begin{array}{c|c} {}^{A}_{B}R & {}^{A}P_{B} \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

这样做是为了让 ${}^{A}_{B}T$ 成为一个正方形矩阵

$$ {}^{A}_{B}T^{-1} = {}^{B}_{A}T$$

$$= \left[\begin{array}{c|c} {}^{A}_{B}R^{T} & {}^{A}_{B}R^{T}\cdot(-{}^{A}P_{B}) \\ \hline 0\ 0\ 0 & 1 \end{array}\right]$$

Forward Kinematics

$$ {}^{0}T_{E} = F(Q) \Rightarrow \underbrace{{}^{0}}_{\text{base}}\underbrace{T_{E}}_{\text{end}} = {}^{0}_{1}T(q_1)\cdot{}^{1}_{2}T(q_2)\cdot{}^{2}_{3}T(q_3)\cdots$$

joint state

D-H parameter

$$T = [Z_1][X_1][Z_2][X_2]\cdots$$

$$[Z_i] = \begin{bmatrix} \cos\theta_i & -\sin\theta_i & 0 & 0 \\ \sin\theta_i & \cos\theta_i & 0 & 0 \\ 0 & 0 & 1 & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \qquad [X_i] = \begin{bmatrix} 1 & 0 & 0 & r \\ 0 & \cos\alpha & -\sin\alpha & 0 \\ 0 & \sin\alpha & \cos\alpha & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$[Z_i]$: z-axis screw motion; $[X_i]$: X-axis screw motion

其实 D-H 就是把 frame 看成了 screw motion ☆

The reason why we need to use z_{i-1} axis and then X_i:

the $[z_i]$ already fit x_{i-1} to X_i so we need to fix z_{i-1} to z_i

therefore we need to look at X_i ☆

Inverse Kinematics

$$ {}^0T_E = F(Q) \Rightarrow \underset{\text{end}}{\overset{\text{base}}{{}^0T_E}} = {}^0_1T(q_1)\cdot{}^1_2T(q_2)\cdot{}^2_3T(q_3)\cdots $$

joint state

know ${}^0T_E$, find $q_1, q_2, q_3 \cdots$ to satisfy ${}^0T_E$

Kinetic

Vector form

$$ \vec{P}_1 = {}^A\vec{P}_B + {}^B\vec{P}_1 \qquad {}^B\vec{P}_1 = x\hat{i}_B + y\hat{j}_B + z\hat{k}_B $$

$$ \vec{V}_1 = {}^A\vec{V}_B + \underbrace{(\dot{x}\hat{i}_B + \dot{y}\hat{j}_B + \dot{z}\hat{k}_B)}_{{}^A_BR\,{}^BV_1} + (x\dot{\hat{i}}_B + y\dot{\hat{j}}_B + z\dot{\hat{k}}_B) $$

$$ \dot{\hat{i}}_B = \vec{w}\times\hat{i}_B \qquad \dot{\hat{j}}_B = \vec{w}\times\hat{j}_B \qquad \dot{\hat{k}}_B = \vec{w}\times\hat{k}_B $$

W是在 frame A 下的

$$ \Rightarrow \vec{V}_1 = {}^A\vec{V}_B + (\dot{x}\hat{i}_B + \dot{y}\hat{j}_B + \dot{z}\hat{k}_B) + W\times\underbrace{(x\hat{i}_B + y\hat{j}_B + z\hat{k}_B)}_{{}^BP_1} $$

$$ \Rightarrow \vec{V}_1 = {}^A\vec{V}_B + (\dot{x}\hat{i}_B + \dot{y}\hat{j}_B + \dot{z}\hat{k}_B) + \vec{W}\times{}^BP_1 $$

$\hat{i}_B, \hat{j}_B, \hat{k}_B$ 需要在 frame A 下进行表示

skew-symmetric matrix

必须在 W 所表示的 frame 下

才能这样写

$$ \vec{W} = \begin{bmatrix} W_x \\ W_y \\ W_z \end{bmatrix} \qquad \text{So } W\times(x\hat{i}_B + y\hat{j}_B + z\hat{k}_B) = \begin{bmatrix} 0 & -W_z & W_y \\ W_z & 0 & -W_x \\ -W_y & W_x & 0 \end{bmatrix}\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} $$

在 frame A 下的坐标

matric form

$$ {}^AP_1 = {}^AP_B + {}^A_BR\,{}^BP_1 $$

$$ {}^AV_1 = {}^AV_B + {}^A_B\dot{R}\,{}^BP_1 + {}^A_BR\,{}^BV_1 $$

$$ = {}^AV_B + {}^A_BR\,{}^BV_1 + \underbrace{{}^A_B\dot{R}\,{}^A_BR^T}\,{}^AP_1 $$

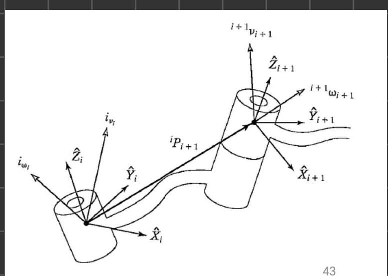
skew-symmetric matrix

Velocity "propagation"

把 joint i+1 放到 joint i 上然后测量其相对于 joint i 的Z轴的角速度

$$ {}^iW_{i+1} = {}^iW_i + {}^i_{i+1}R\,\dot{\theta}_{i+1}\,{}^{i+1}\hat{Z}_{i+1} $$

$$ {}^iV_{i+1} = {}^iV_i + {}^iW\times{}^iP_{i+1} $$



Jacobians

$$y_1 = f_1(x_1, x_2 \cdots x_6)$$
$$\vdots$$
$$y_6 = f_6(x_1, x_2 \cdots x_6)$$

$$\Rightarrow \quad Y = F(X)$$

$$\partial y_1 = \frac{\partial f_1}{\partial x_1}\partial x_1 + \frac{\partial f_1}{\partial x_2}\partial x_2 + \cdots + \frac{\partial f_1}{\partial x_6}\partial x_6$$
$$\partial y_2 = \frac{\partial f_2}{\partial x_1}\partial x_1 + \frac{\partial f_2}{\partial x_2}\partial x_2 + \cdots + \frac{\partial f_2}{\partial x_6}\partial x_6$$
$$\vdots$$
$$\partial y_6 = \frac{\partial f_6}{\partial x_1}\partial x_1 + \frac{\partial f_6}{\partial x_2}\partial x_2 + \cdots + \frac{\partial f_6}{\partial x_6}\partial x_6$$

So $\partial Y = J\,\partial X$

$\Downarrow$ dev. time

$\dot{Y} = J\dot{X}$

$$J = \begin{bmatrix} \frac{\partial f_1}{\partial x_1} & \frac{\partial f_2}{\partial x_2} & \cdots \\ \vdots & \ddots & \\ \frac{\partial f_6}{\partial x_1} & \cdots & \frac{\partial f_6}{\partial x_6} \end{bmatrix}$$

J is a map that maps $\dot{X}$ to $\dot{Y}$

In robotics we usually do that from $\dot{\theta}$ to v

$v = J\dot{\theta}$

changing frame

$$\begin{bmatrix} {}^B v \\ {}^B w \end{bmatrix} = {}^B J(\theta)\,\dot{\theta}$$

$$\begin{bmatrix} {}^A v \\ {}^A w \end{bmatrix} = \begin{bmatrix} {}^A_B R & 0 \\ 0 & {}^A_B R \end{bmatrix} \begin{bmatrix} {}^B v \\ {}^B w \end{bmatrix} {}^B J(\theta)\,\dot{\theta}$$

So ${}^A J = \begin{bmatrix} {}^A_B R & 0 \\ 0 & {}^A_B R \end{bmatrix} {}^B J$