CS130 - LAB - Bresenham's line algorithm / midpoint algorithm

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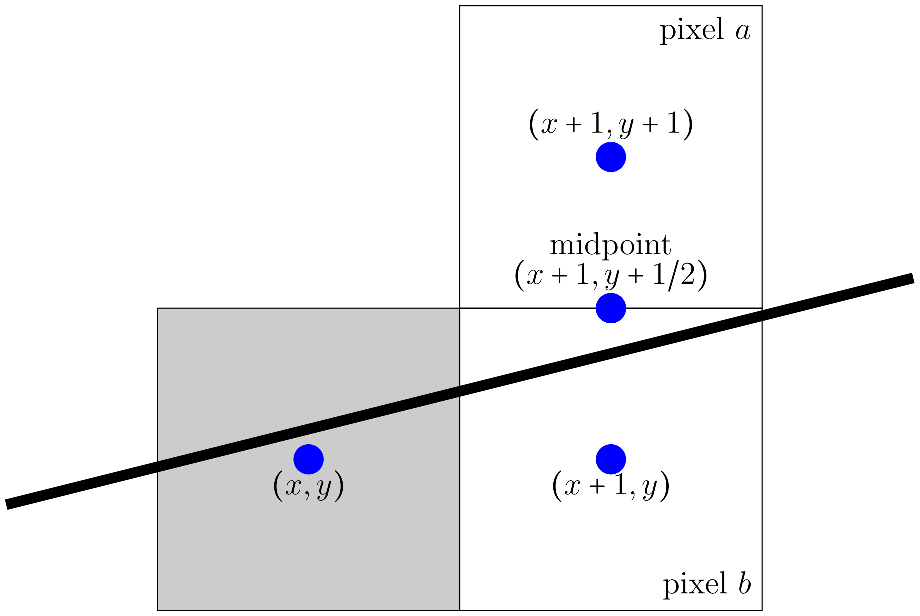
Part 1: Bresenham's line algorithm / midpoint algorithm

References: https://en.wikipedia.org/wiki/Bresenham%27s_line_algorithm

This Lab consists of implementing the midpoint algorithm to draw continuous lines using only integer operations. Recall the line equation is $y = mx + n$, where m is the slope and n is the y intercept. Given two points $p_0 = (x_0, y_0)$ and $p_1 = (x_1, y_1)$, the slope is calculated as $m = \frac{y_1 - y_0}{x_1 - x_0} = \frac{dy}{dx}$. Consider $0 \le m \le 1$ (line in angle between 0 and 45 degrees). The idea is to determine which of the two pixels (a or b) we should draw. Complete the missing coordinates of the points below.



In particular, we can evaluate the midpoint between a and b using a function $f(x,y)$ that returns a positive number if the line is above the midpoint and negative if the line is below the midpoint. The function $f(x,y)$ should be zero if the midpoint lies on the line

$$f(x,y) = 0 = mx + n - y \tag{1}$$

Rewrite the right-hand-side (RHS) of Equation (1) in the form $Ax + By + C$, where A, B, and C depends on dx, dy and n only. Remove any denominator by multiplying $f(x,y)$ by dx to ensure we have an integer solution.

$$f(x,y)dx = Ax + By + C \tag{2}$$

where $A = dy, B = -dx, C = ndx$

We want to make a decision on whether to draw pixel a or b using $f(x,y)$ on a sequence of points from p_0 to p_1. Define variables da and db to hold the difference of $f(x,y)$ from a and b to the previous midpoint $(x+1, y+1/2)$. Use Equation (2) to rewrite Equations (3) and (4) as function of A and B (you won't need C).

$$da = f(x+2, y+3/2) - f(x+1, y+1/2) = A + B \tag{3}$$

$$db = f(x+2, y+1/2) - f(x+1, y+1/2) = A \tag{4}$$

Final detail, we need to calculate the difference between the second and first point in order to use Equations (3) and (4) in a loop.

$$dinit = f(x0+1, y0+1/2) - f(x0, y0) = A + (1/2)B \tag{5}$$

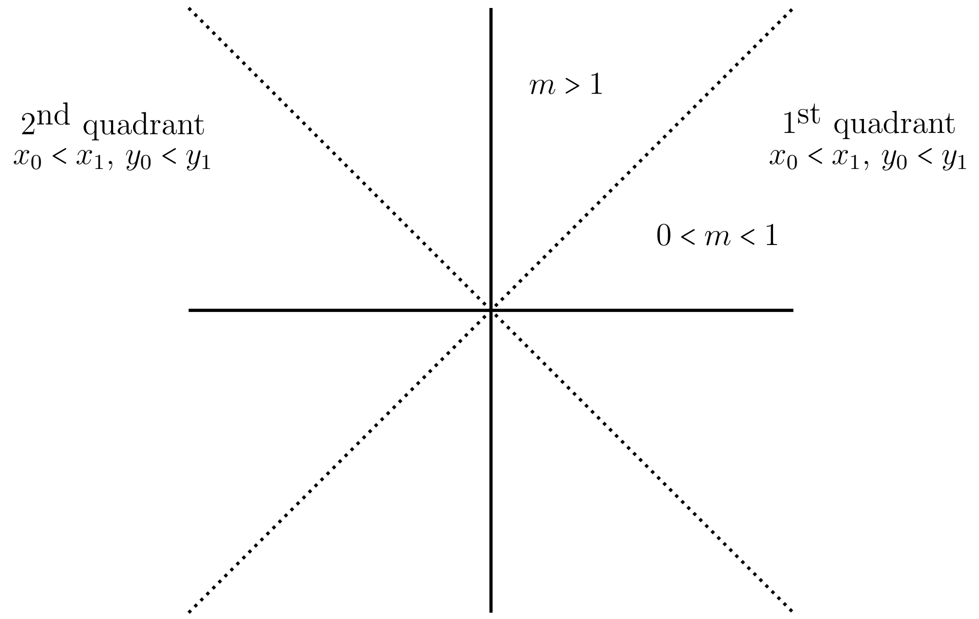
We only care whether d is positive or negative, hence, we can multiply $dinit$, da and db by 2 to get equations containing only integers.

Using Equations (2) to (5), complete the midpoint algorithm below:

```
MPA(x0, y0, x1, y1):
  dx = x1 - x0
  dy = y1 - y0
  D = 2 * dy - dx // initialize D using dinit
  y = y0
  for x = x0 to x1:
    set_pixel(x, y)
    if D > 0:
       y = y + 1
       D = D - 2 * dx // D is positive, use da
    D = D + 2 * dy // D is negative, use db
```

Part 2: Implementing the midpoint algorithm

We considered $0 \leq m \leq 1$ (line in angle between 0 and 45 degrees) to write the code in the previous part. How can we manage other angles? Feel free to think a little bit about it and to look at the Wikipedia page (link on Part 1). Here is a free blank space and a diagram.



Download the skeleton code on the website and write the midpoint algorithm in the function `draw_line_MPA` in `application.cpp`. To draw a pixel, you can use `set_pixel(x, y, linecolor)`, where linecolor is given to you as argument of `draw_line_MPA`. The following commands are available:

- Click and hold to draw lines.
- Type "c"to clear old lines.
- Type "a"to generate 1K random lines.
- Type "A"to generate 1M random lines.
- Type "m"to toggle between the MPA and the DDA algorithms.
- Type "["or "]"to change point-size.

Make sure your code runs faster than the DDA algorithm. Why is the DDA algorithm slower? Feel free to take a look at `draw_line_DDA` code.

Run your MPA algorithm 3 times with 1K and 1M lines without increasing the point-size. Run the DDA algorithm 3 times with 1K and 1M lines without increasing the point-size. Fill the table below with the running time (in seconds).

DDA (1K / 1M): Run 1 = 0.016/4.719; Run 2 = 0.016/4.781; Run 3 = 0.015/4.766
MPA (1K / 1M): Run 1 = 0.016/4.422; Run 2 = 0/4.437; Run 3 = 0/4.391

The DDA algorithm is slower because it has to deal with floating points. This is seen mainly with the inclusion of 'm' causing us to always have to round 'cx' or 'cy' depending on if the absolute value of our slope is less than or greater than 1.