



Image Filtering

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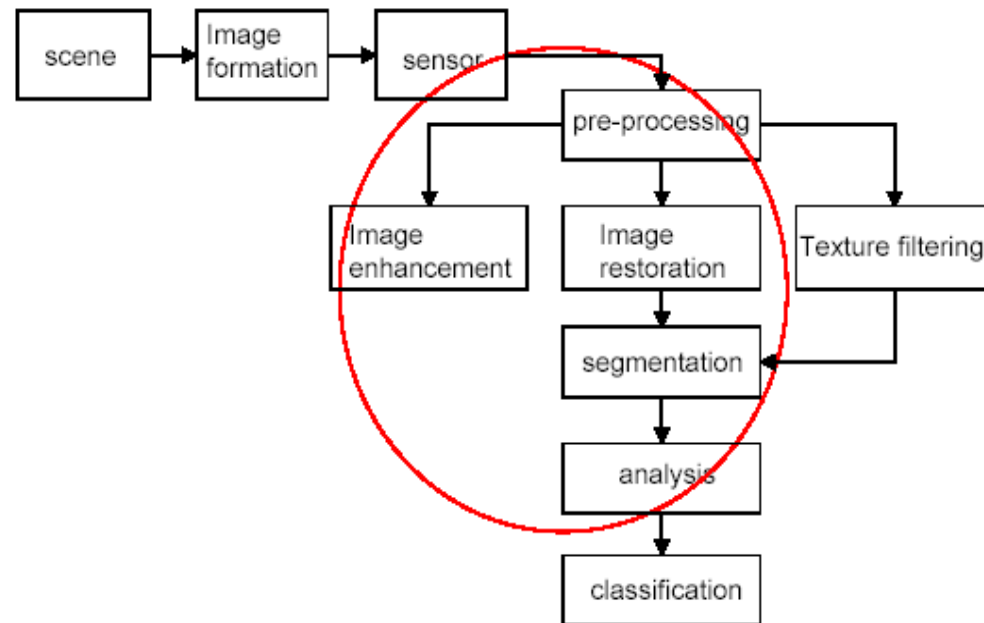
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Background



Background (cont.)

- Trasformazione dell'immagine:

$$g(x, y) = T[f(x, y)]$$

- f è l'immagine input
 - g è l'immagine elaborata
 - T è l'operatore di trasformazione
-
- T è lineare se gode delle seguenti proprietà:
 - *Omogeneo* se $T\{af[n]\} = aT\{f[n]\}$
 - *Associativo* se $T\{f_1[n] + f_2[n]\} = T\{f_1[n]\} + T\{f_2[n]\}$

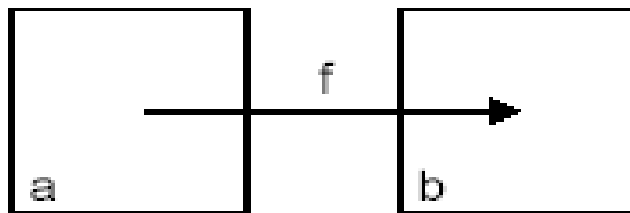


Background (cont.)

- Classificazione delle Trasformazioni *T*:
 - Tecniche operanti nel dominio *spaziale*
 - Manipolazione diretta dei pixel
 - Tecniche operanti nel dominio delle *frequenze*
 - Trasformazione dell'immagine nel dominio delle frequenze e manipolazione delle frequenze
 - Tecniche Ibride
 - Combinazione di tecniche

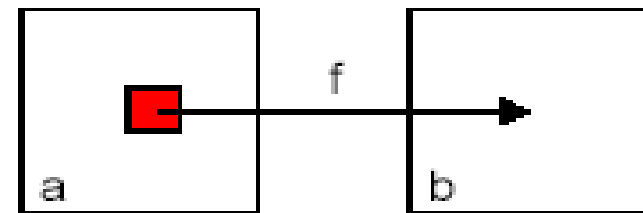
Classificazione delle Tecniche

point operations



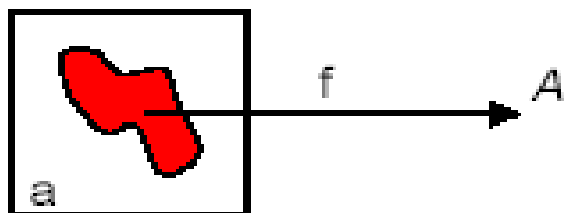
$$b(x,y) = f\{a(x,y)\}$$

local operations



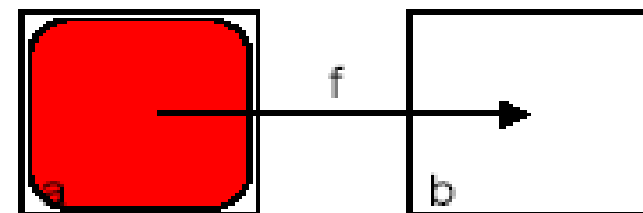
$$b(x,y) = f\{a(x_1,y_1), \dots, a(x_m,y_m)\}$$

object operations



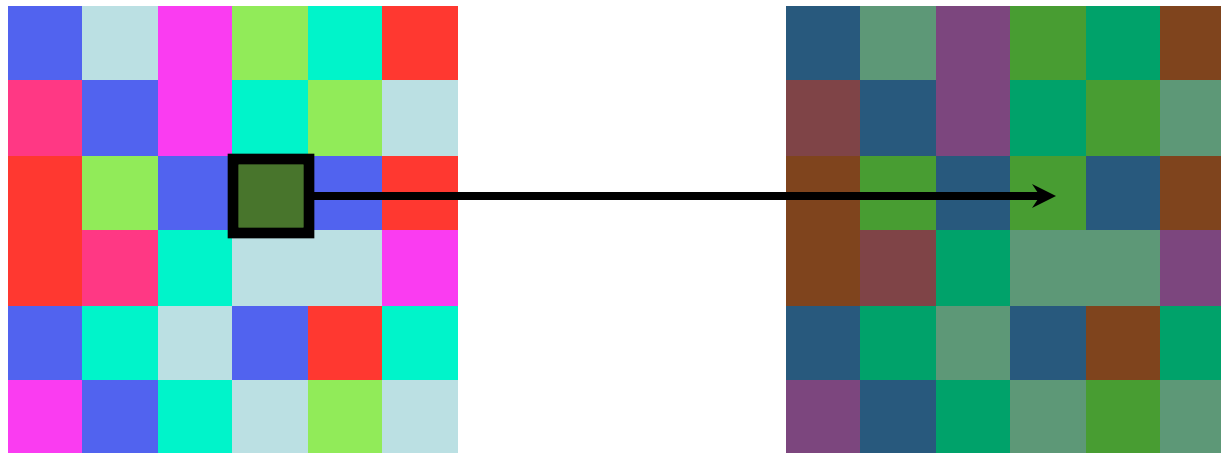
$$A = f\{a(x_i,y_i) \mid (x_i,y_i) \in \text{object}\}$$

global operations



$$b(x,y) = f\{a(x_1,y_1), \dots, a(x_N,y_N)\}$$

Point Operations





Point Processing

Original



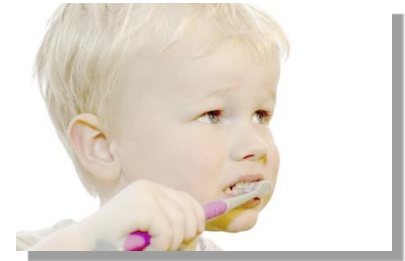
Darken



Lower Contrast



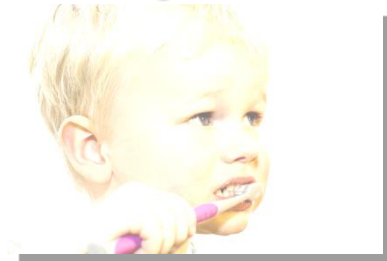
Nonlinear Lower Contrast



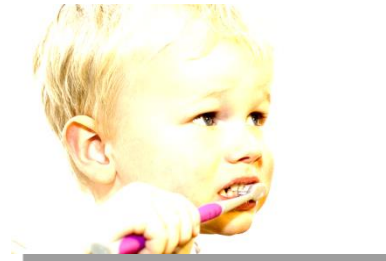
Invert



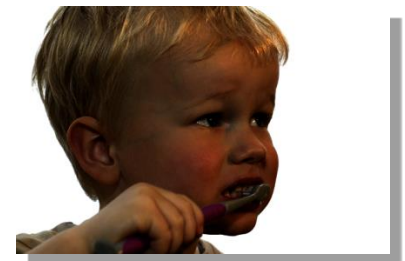
Lighten



Raise Contrast



Nonlinear Raise Contrast





Point Processing

Original



$$x$$

Darken



$$x - 128$$

Lower Contrast



$$x / 2$$

Nonlinear Lower Contrast



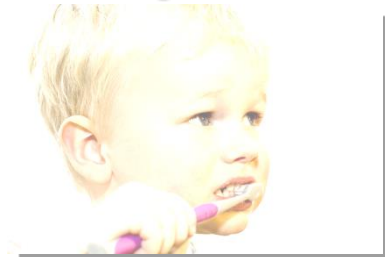
$$((x / 255.0) ^ 0.33) * 255.0$$

Invert



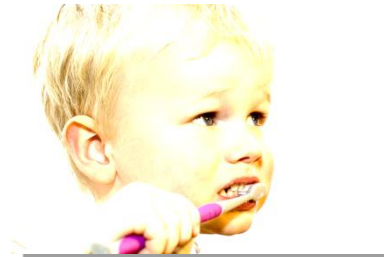
$$255 - x$$

Lighten



$$x + 128$$

Raise Contrast



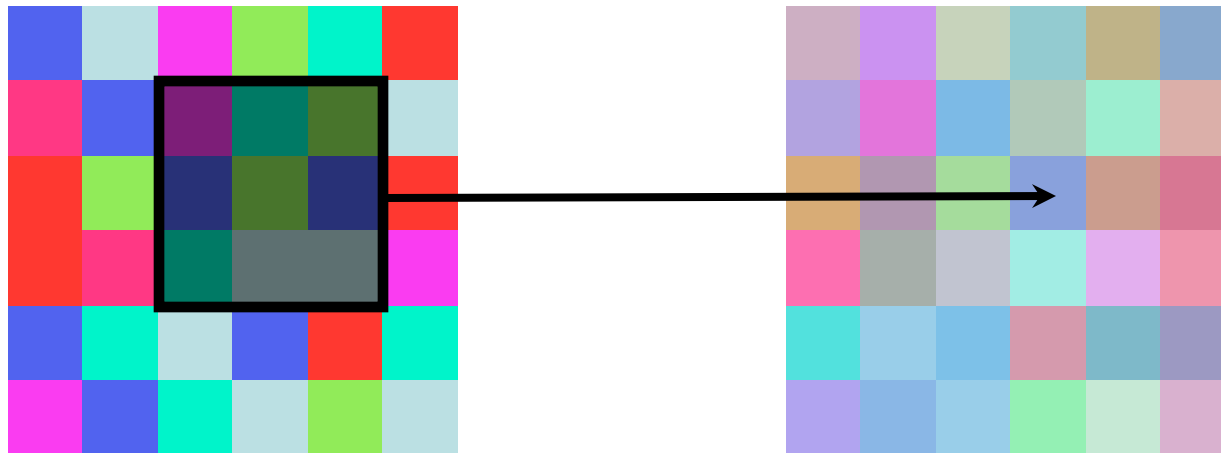
$$x * 2$$

Nonlinear Raise Contrast



$$((x / 255.0) ^ 2) * 255.0$$

Neighborhood Operations



Neighborhood operations



Image



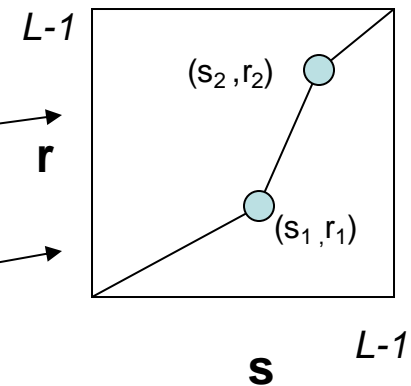
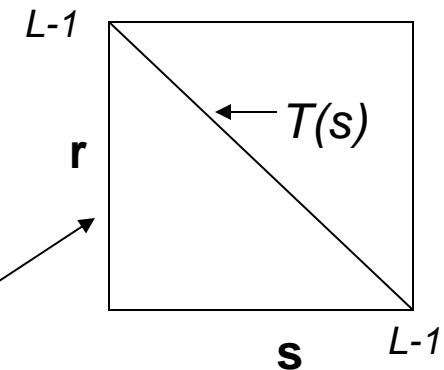
Edge detection



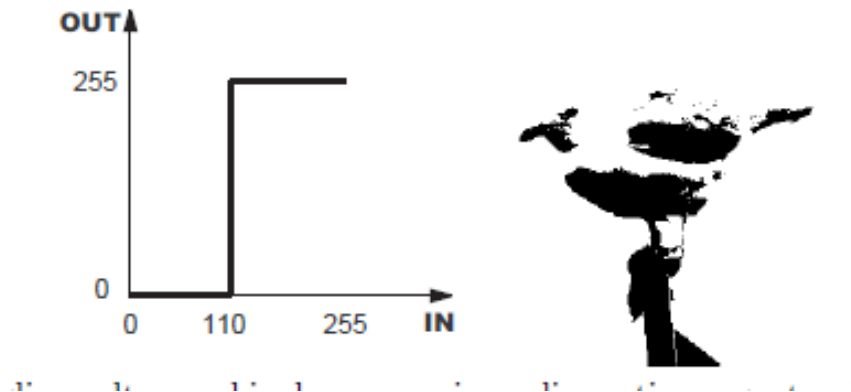
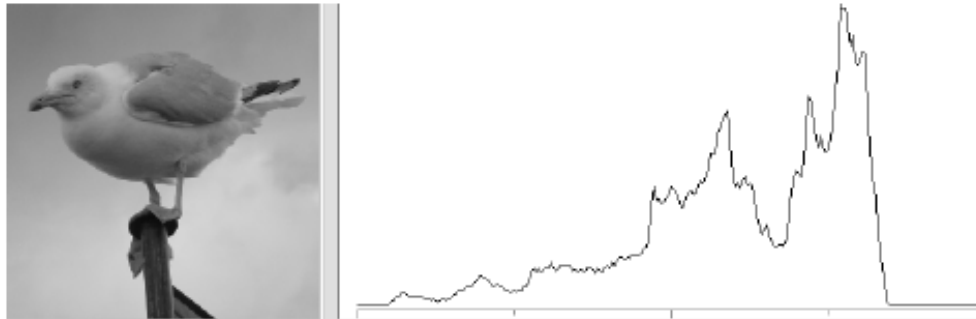
Blur

Tecniche Operanti nel Dominio Spaziale

- Point Processing: La trasformazione T è definita solo nel punto (x,y)
 - s = input gray level
 - r = output gray level
 - T dipende solo da un punto
- Negativo (T è monotona decrescente)
- Binarizzazione (T è monotona crescente)
- Contrast Stretching (T è monotona crescente)



Binarizzazione



Negativo

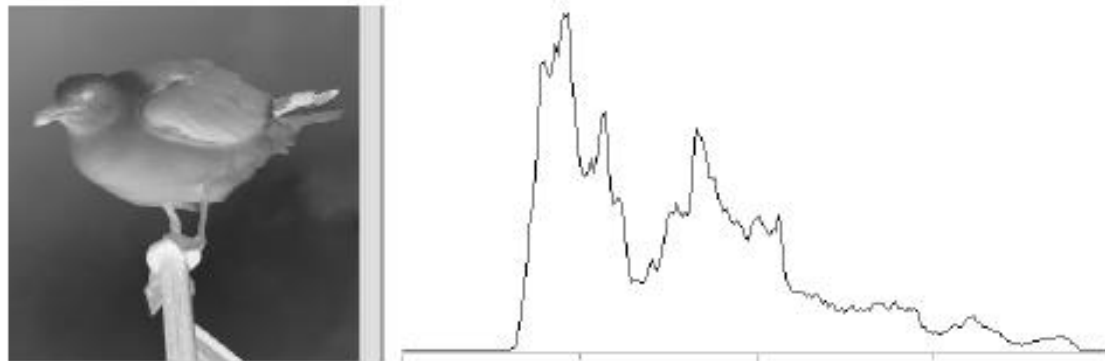
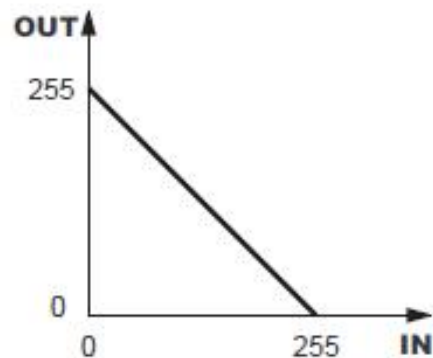
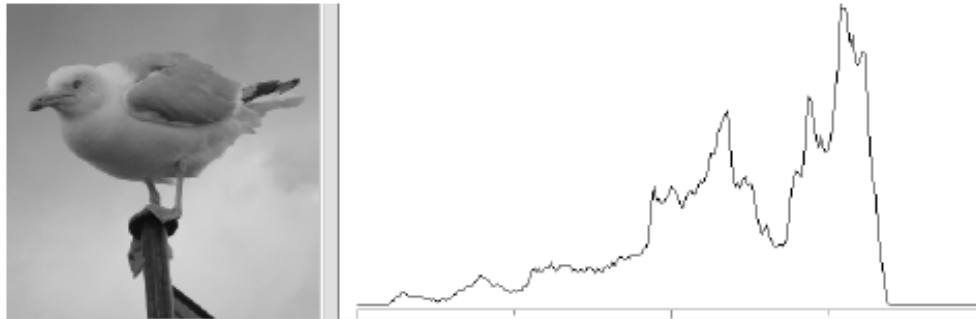
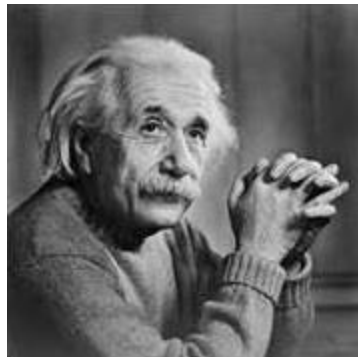
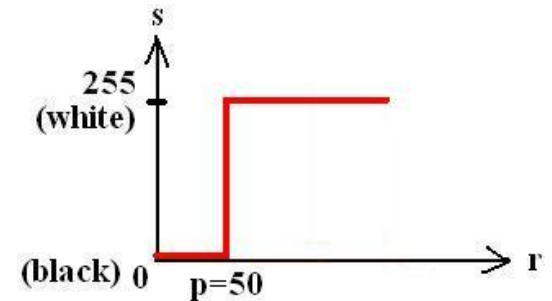
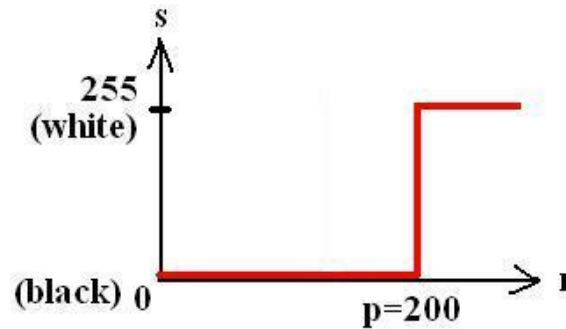
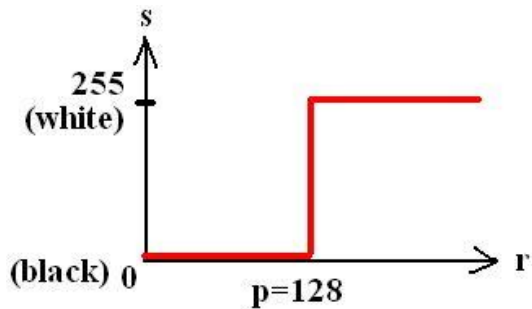
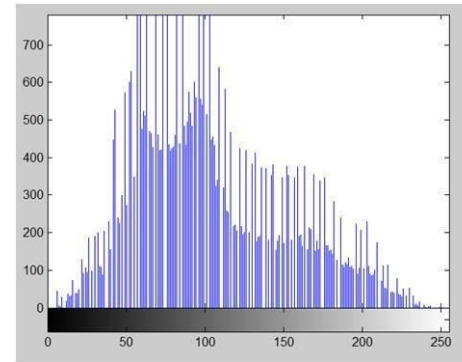


Image transformation



Original
image





Tecniche Operanti nel Dominio Spaziale (cont.)

- La trasformazione T è definita in un intorno predefinito di (x,y)
 - Filtri Spaziali (mask processing)
 - T dipende da un intorno del punto
 - Lowpass (passa basso): attenua o elimina le alte frequenze
 - » Contorni e dettagli
 - Highpass (passa alto): attenua o elimina le basse frequenze
 - » Contrasto e intensità
 - Bandpass (passa banda): attenua frequenze in una banda predefinita

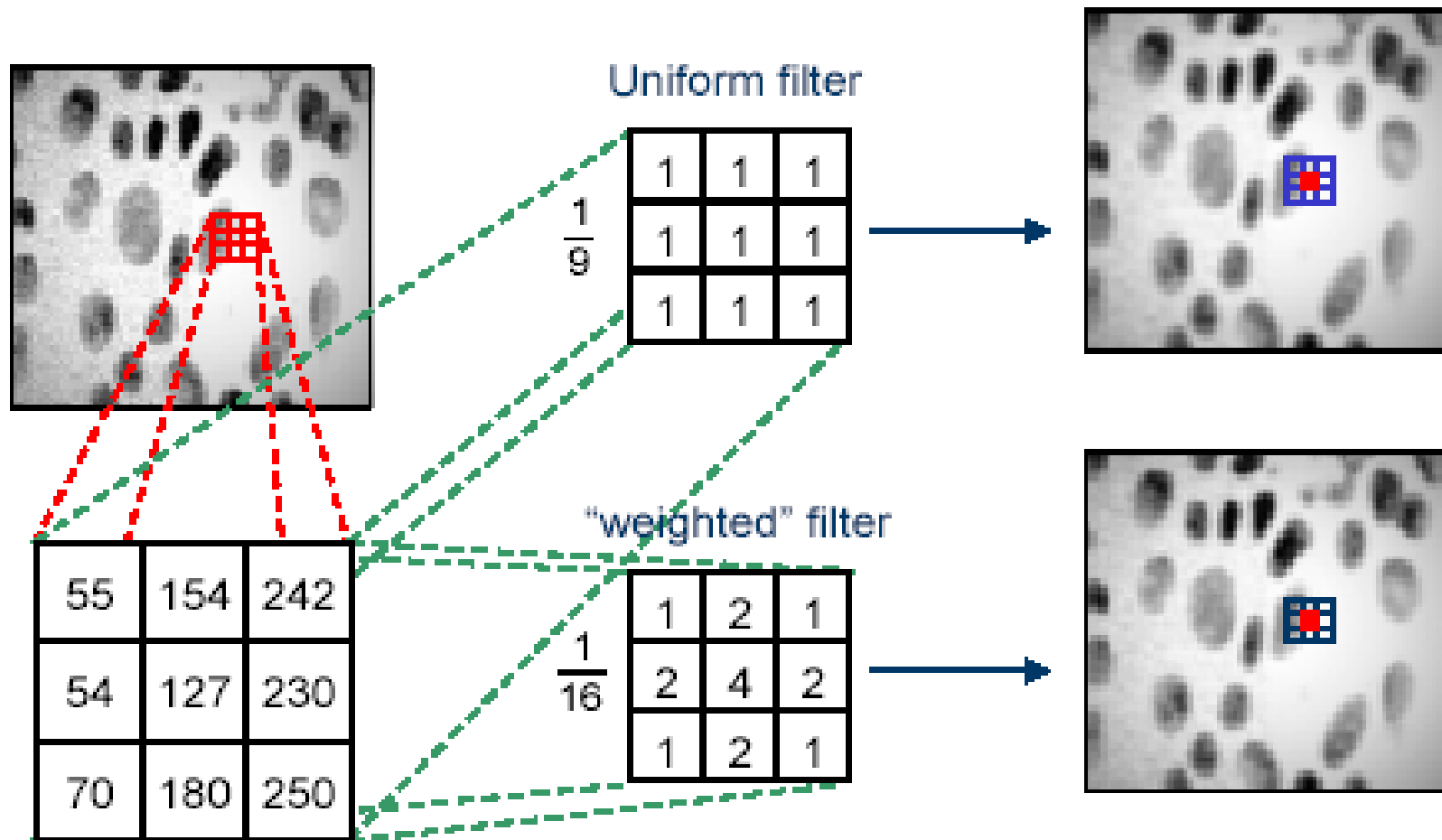
Filtri Spaziali

- Filtro lineare (mask) 3x3
- $R = w_1 z_1 + w_2 z_2 + w_3 z_3 + w_4 z_4 + w_5 z_5 + w_6 z_6 + w_7 z_7 + w_8 z_8 + w_9 z_9$

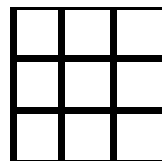
w_1	w_2	w_3
w_4	w_5	w_6
w_7	w_8	w_9

- Dove z_1, z_2, \dots, z_9 rappresentano i pixel dell'immagine ai quali è sovrapposto il filtro e w_1, w_2, \dots, w_9 i pesi caratterizzanti il filtro

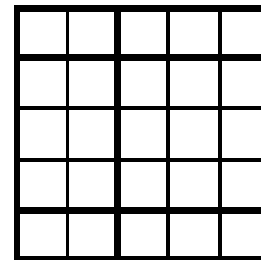
Filtri Spaziali (cont.)



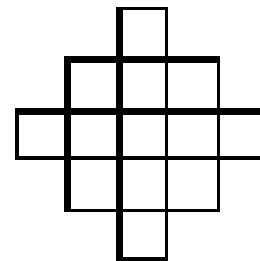
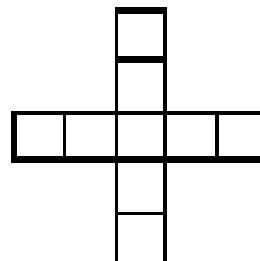
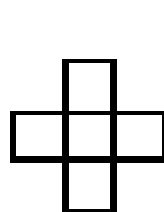
Filtri Spaziali (cont.)



3 x 3



5 x 5





Filtri Spaziali (cont.)

- Smoothing Filter (preprocessing step)
 - Blurring e Noise reduction
 - Blurring è usato per rimuovere piccoli dettagli prima di estrarre (grandi) oggetti
 - Lowpass
 - Lineari
 - » Filtro Media (aritmetica)
 - Non Lineari
 - » Filtro Mediano
 - » Filtro Max
 - » Filtro Min

Filtri Spaziali: La Media

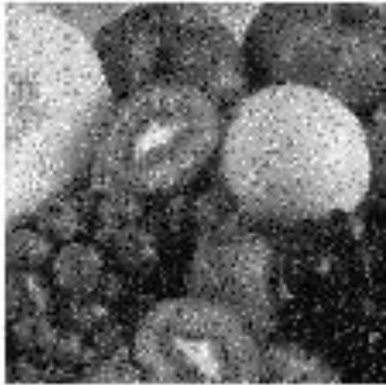
- Filtro Media

- S= insieme di pixel connessi con (x,y)
- M= numero di pixel coinvolti ($|S|+1$)

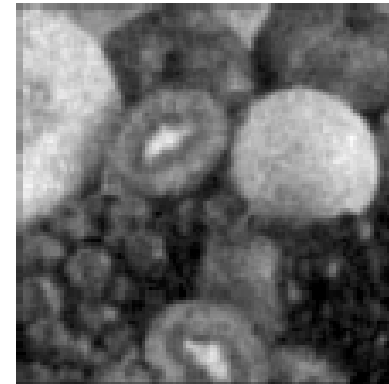
$$g(x, y) = \frac{1}{M} \sum_{f(n,m) \in S} f(n, m)$$

$$\frac{1}{9} \begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$$

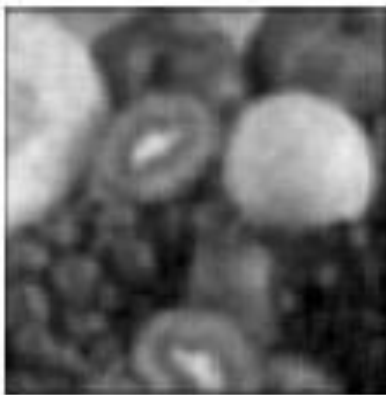
Filtri Spaziali: La Media (cont.)



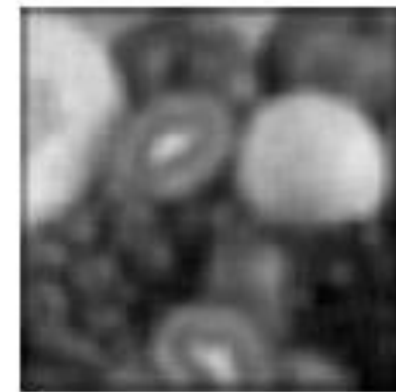
Salt & Pepper
Noise



3 X 3 Average



5 X 5 Average



7 X 7 Average

Filtri Spaziali: La Media (cont.)



(a)



(b)



(c) PSNR = 9.2000 dB

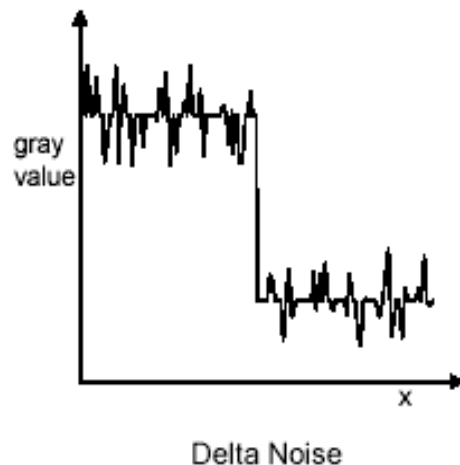


(d) PSNR = 12.4450 dB

Filtri Spaziali: La Media Adattiva

S = neighborhood of pixel (x,y)
M = number of pixels in neighborhood S
T = predefined threshold value

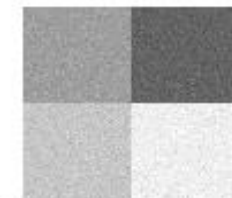
$$g(x,y) = \begin{cases} \frac{1}{M} \sum_{(n,m) \in S} f(n,m) & \text{if } \frac{1}{M} \sum_{(n,m) \in S} |f(n,m) - f(x,y)| < T \\ f(x,y) & \text{otherwise} \end{cases}$$



Adaptive Averaging



Original



Original+Delta Noise



3x3 Average



5x5 Average



9x9 Average



15x15 Average

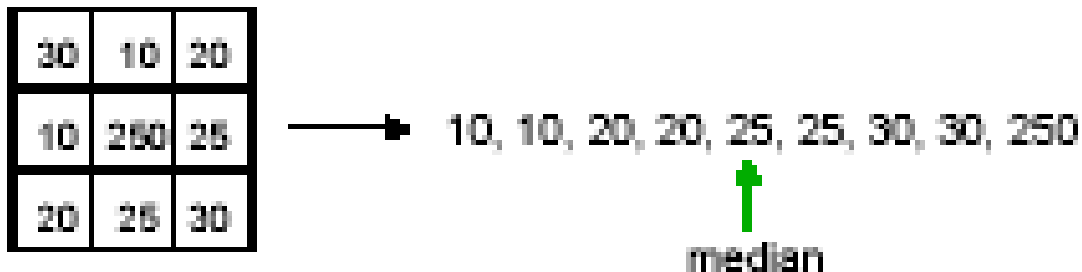


Adaptive 9x9 Average
T=25

Filtri Spaziali: La Mediana

- Filtro Mediana

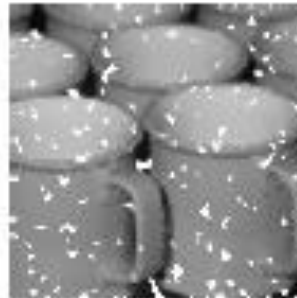
$$g(x, y) = \text{mediana}\{f(x, y)\}_{f(x, y) \in S}$$



Filtri Spaziali: La Mediana (cont.)



Large Noise



Median



Median x 2



Median x 4



Median x 8



Median x 6

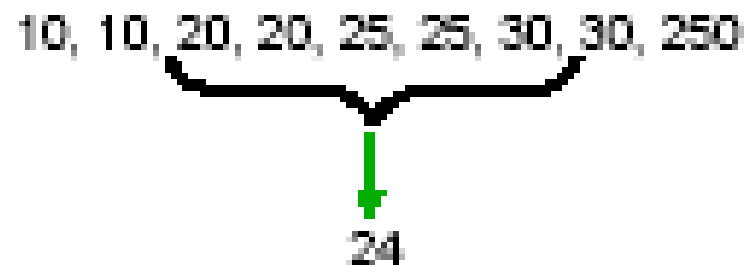


Median x 7

Filtri Spaziali

- Filtro Ibrido:
 - Media + Mediana

Median + Average: average the k central values.



Media vs Mediana

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100



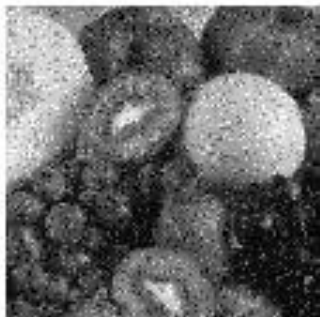
100	100	100	100	100
100	100	200	100	100
100	200	200	200	100
100	100	195	100	100
100	100	100	100	100

100	100	100	100	100
100	200	205	203	100
100	195	200	200	100
100	200	205	195	100
100	100	100	100	100

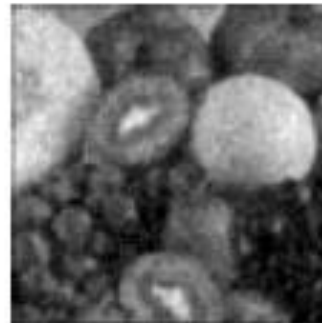


100	100	100	100	100
100	144	167	145	100
100	167	200	168	100
100	144	166	144	100
100	100	100	100	100

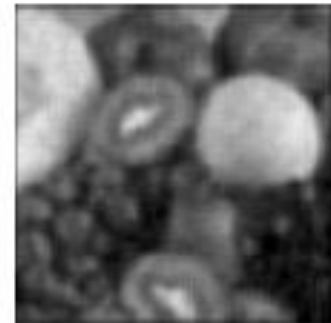
Media vs Mediana



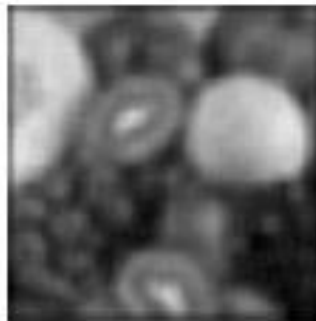
Salt & Pepper
Noise



3 X 3 Average



5 X 5 Average

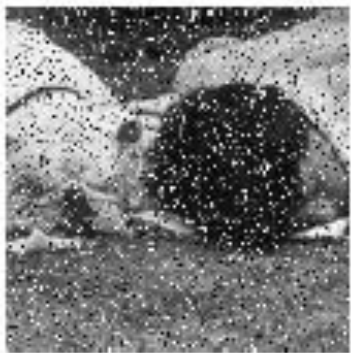


7 X 7 Average

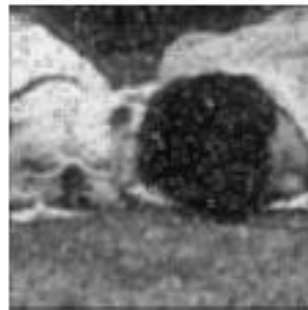


Median

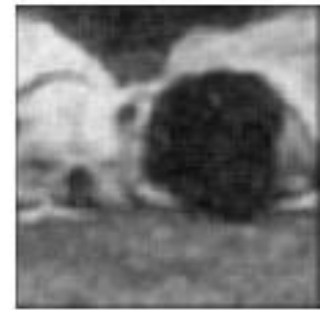
Media vs Mediana (cont.)



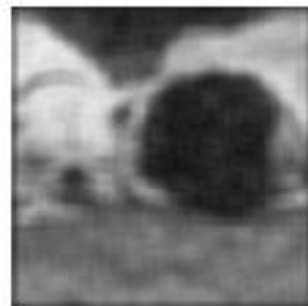
Salt & Pepper
Noise



3 X 3 Average



5 X 5 Average



7 X 7 Average



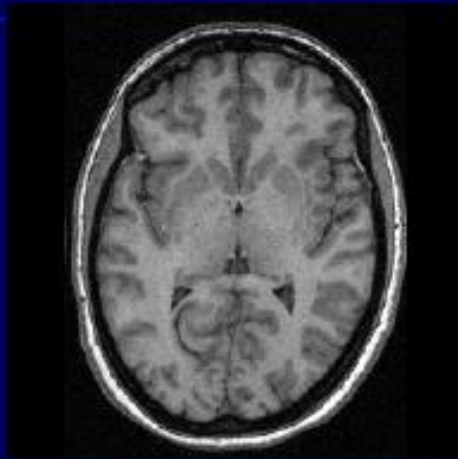
Median

Media vs Mediana (cont.)

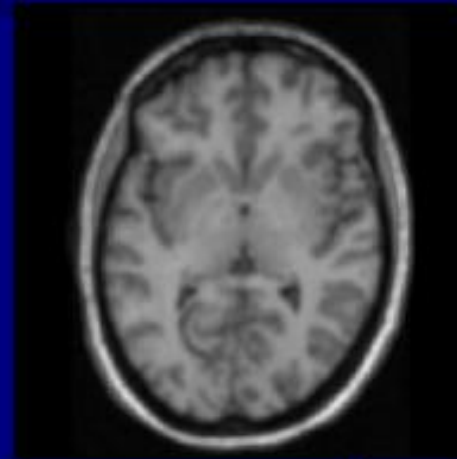
- Media
 - Eccessiva velatura (*blurring*) al crescere della maschera e delle iterazioni con conseguente perdita dei dettagli
 - Preserva con la versione adattiva le forme (scalini)
- Mediana
 - Riduzione del rumore (*noise reduction*) senza (eccessiva) perdita di dettagli (*blurring limitato*)
 - Non preserva le forme (scalini)

Media vs Mediana (cont.)

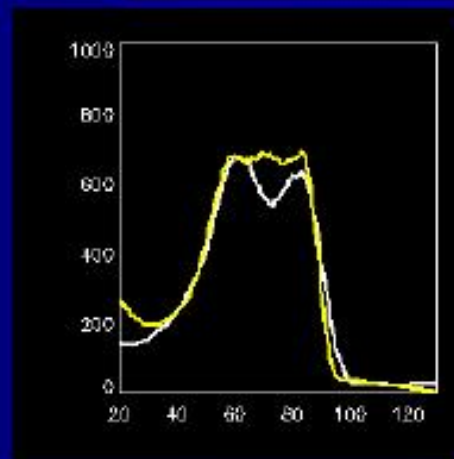
T1-weighted MRI slice



Boxcar mean, 5x5

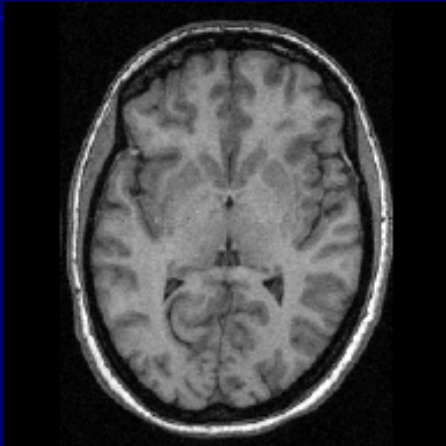


Distinct histogram peaks in original are poorly differentiated after smoothing.

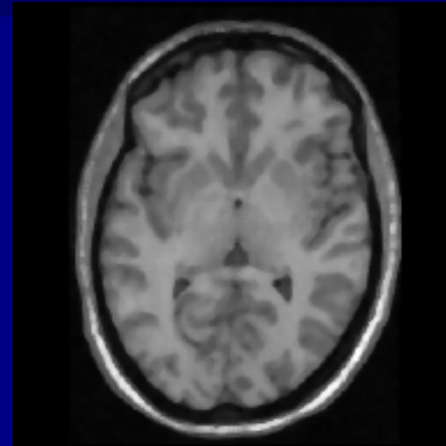


Media vs Mediana (cont.)

T1-weighted MRI slice

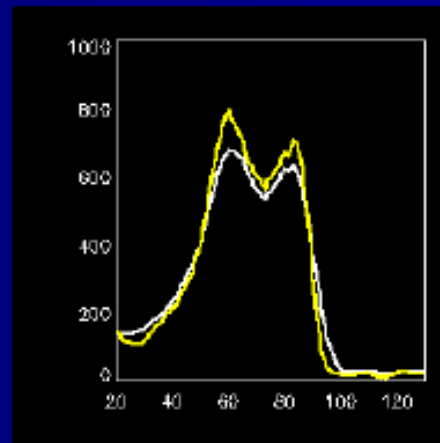


Square median, 5x5

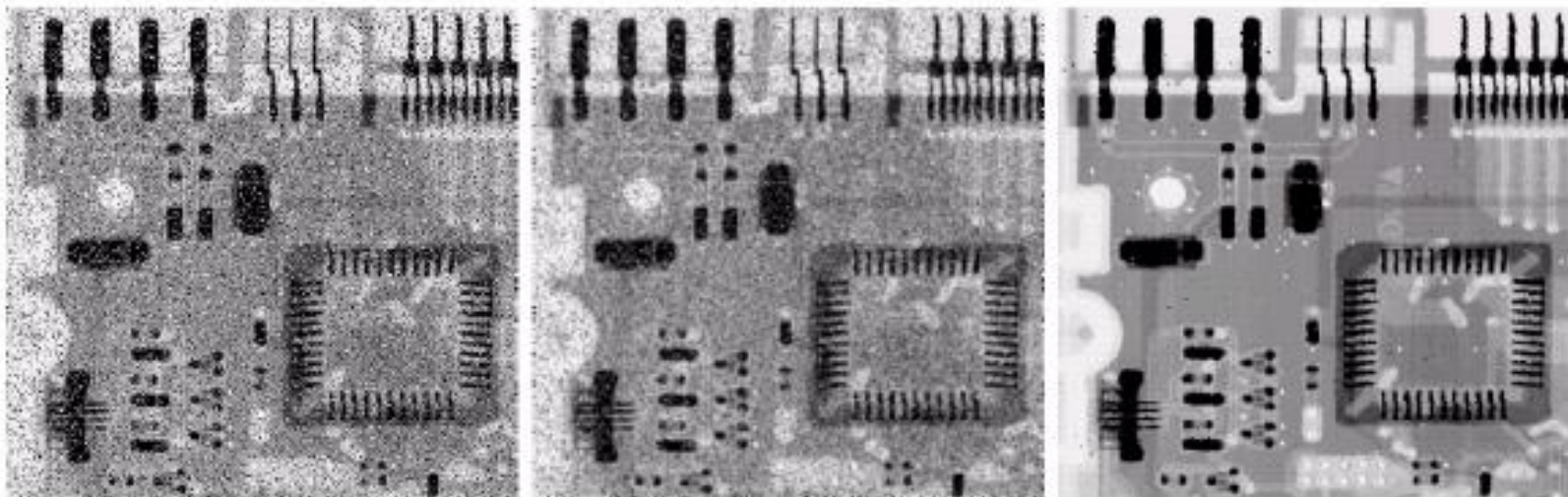


Note that bright rim, mid-hemisphere boundary have been suppressed.

Histogram peaks are accentuated, rather than smeared together.



Media vs Mediana (cont.)

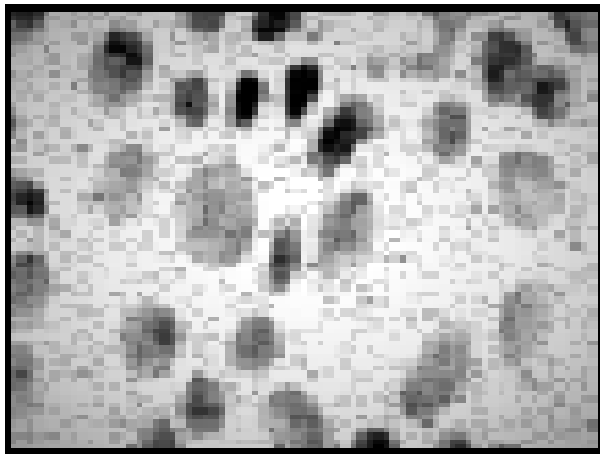


a b c

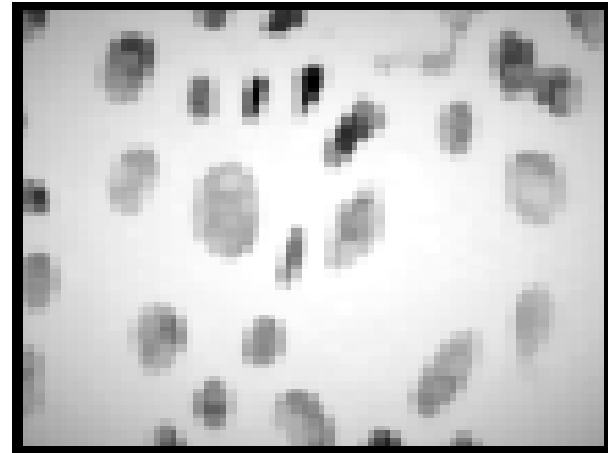
FIGURE 3.37 (a) X-ray image of circuit board corrupted by salt-and-pepper noise. (b) Noise reduction with a 3×3 averaging mask. (c) Noise reduction with a 3×3 median filter. (Original image courtesy of Mr. Joseph E. Pascente, Lixi, Inc.)

Max/Min Filtering

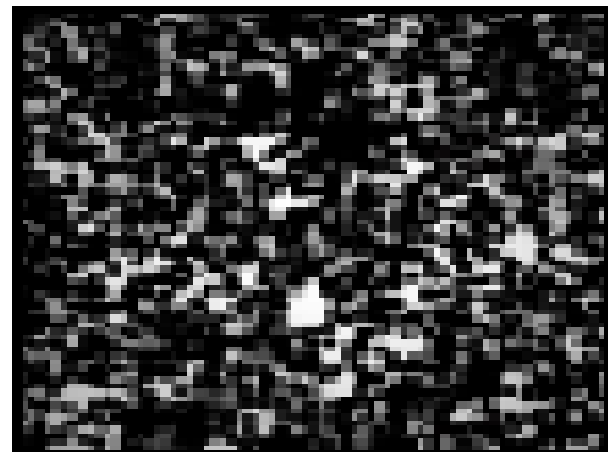
original image A



$\text{maxf}(A)$



$\text{minf}(A)$



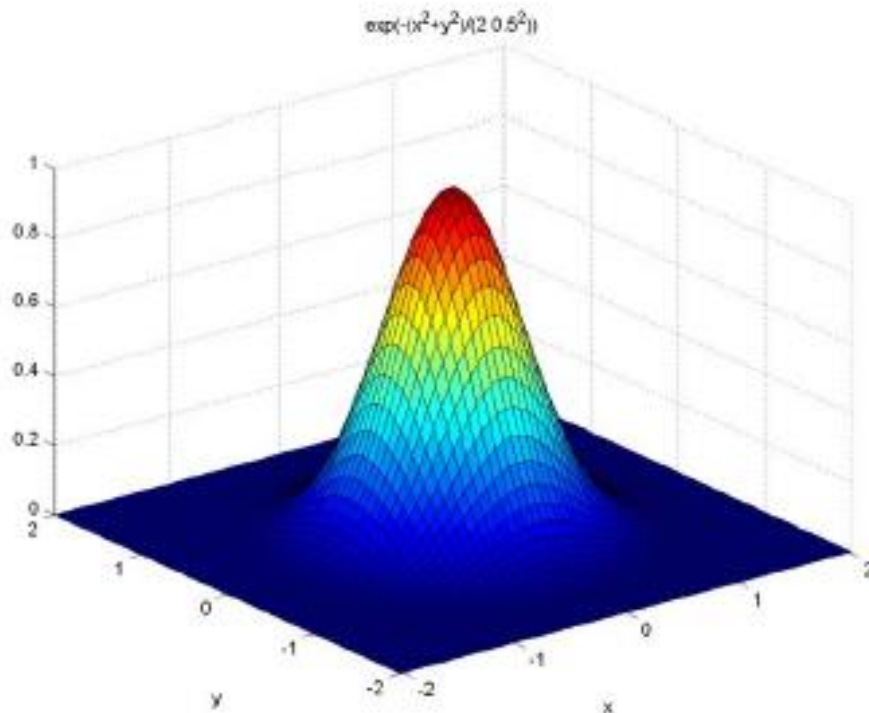
Smoothing Gaussiano

Filtro Gaussiano

Il funzionamento é simile a quello del filtro mediano con la differenza che il contributo di ciascun pixel limitrofo ha un peso diverso, definito dalla distribuzione spaziale della gaussiana.

Nel campo dell'elaborazione delle immagini la gaussiana viene troncata limitandone l'estensione ad una zona di dimensioni $N * N$ (i valori piú frequenti per N sono 3, 4 e 5).

Smoothing Gaussiano (cont.)



$$\begin{bmatrix} 0.0113 & 0.0838 & 0.0113 \\ 0.0838 & 0.6193 & 0.0838 \\ 0.0113 & 0.0838 & 0.0113 \end{bmatrix}$$

$$\begin{bmatrix} 0.0000 & 0.0000 & 0.0002 & 0.0000 & 0.0000 \\ 0.0000 & 0.0113 & 0.0837 & 0.0113 & 0.0000 \\ 0.0002 & 0.0837 & 0.6187 & 0.0837 & 0.0002 \\ 0.0000 & 0.0113 & 0.0837 & 0.0113 & 0.0000 \\ 0.0000 & 0.0000 & 0.0002 & 0.0000 & 0.0000 \end{bmatrix}$$

Gaussian Smoothing

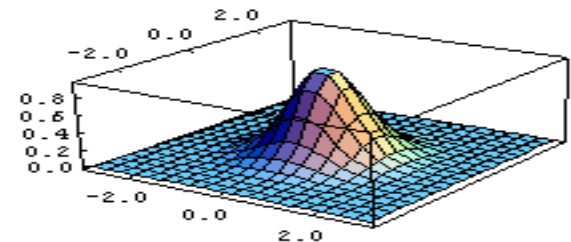


- A Gaussian kernel gives less weight to pixels further from the center of the window (5x5, sigma=0.5)

0.0000	0.0000	0.0002	0.0000	0.0000
0.0000	0.0113	0.0837	0.0113	0.0000
0.0002	0.0837	0.6187	0.0837	0.0002
0.0000	0.0113	0.0837	0.0113	0.0000
0.0000	0.0000	0.0002	0.0000	0.0000

- This kernel is an approximation of a Gaussian function:

$$h(u, v) = \frac{1}{2\pi\sigma^2} e^{-\frac{u^2+v^2}{\sigma^2}}$$



Gaussian Smoothing



original



$\sigma = 2$



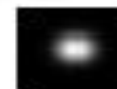
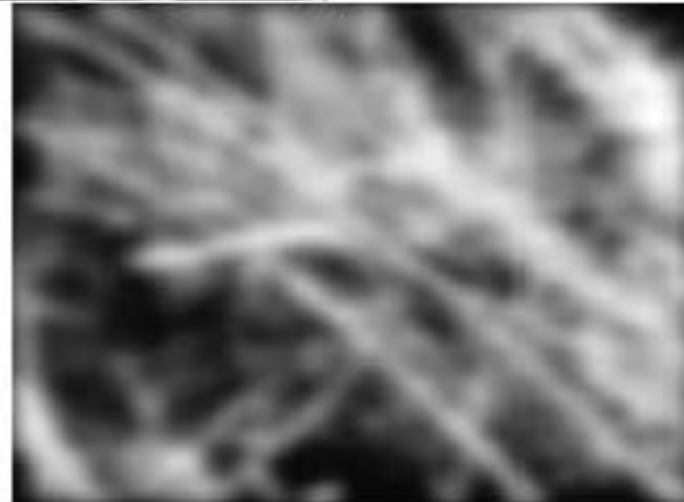
$\sigma = 2.8$



$\sigma = 4$



Mean vs. Gaussian filtering



Gaussian Smoothing

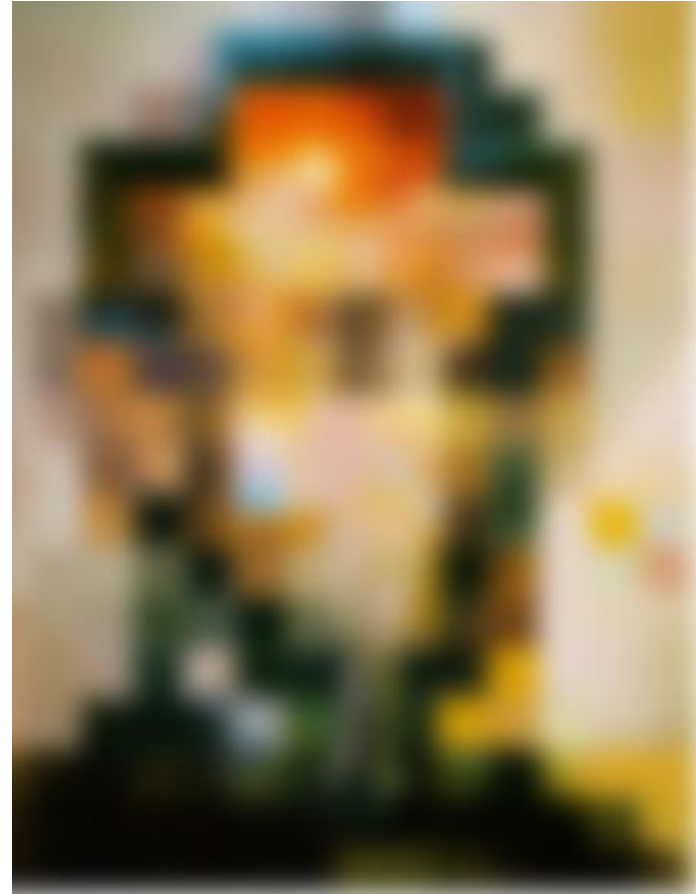


by Charles Allen Gillbert



by Harmon & Julesz

Gaussian Smoothing



Smoothing

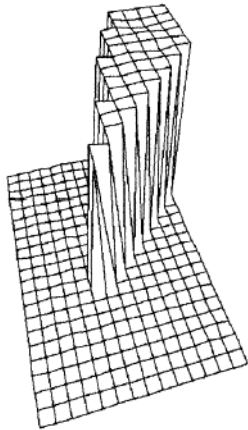


Immagine originale

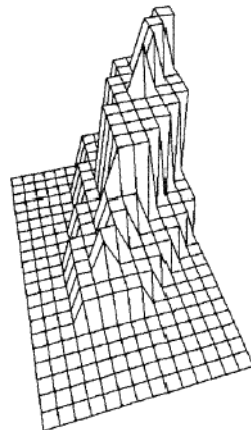


Immagine filtrata

Media

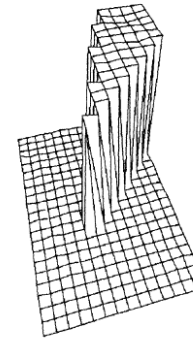


Immagine originale

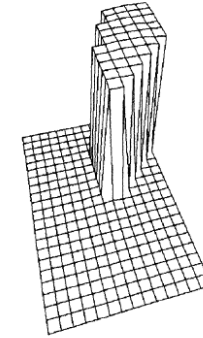


Immagine filtrata

Mediana

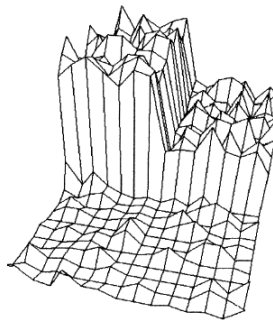


Immagine originale

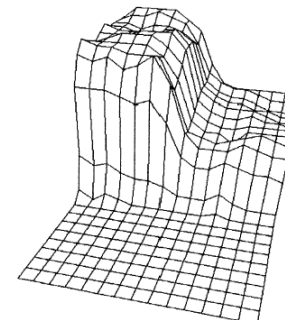


Immagine filtrata

Gaussiana

noisy



Gaussian filter



Median filter



Weighted Averaging Filter: Example





Original



Noise



Mean



Median



Mode



Circular



Pyramid



Cone

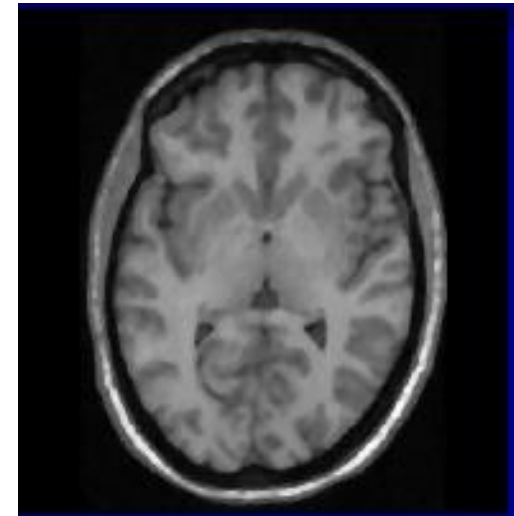
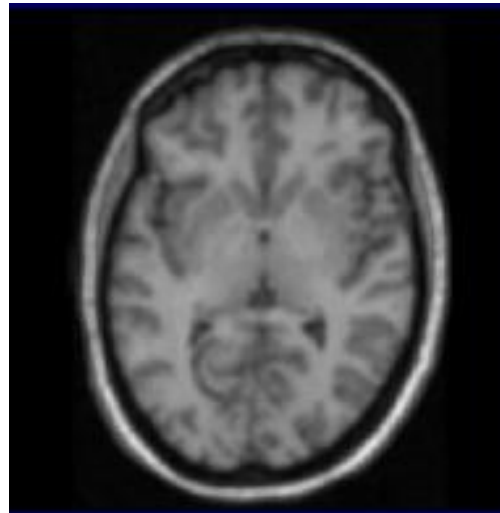
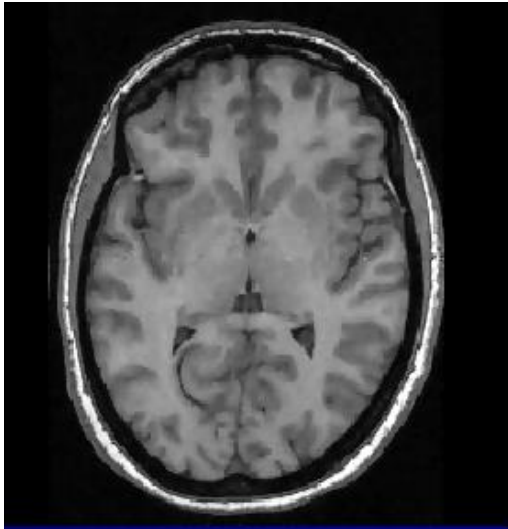
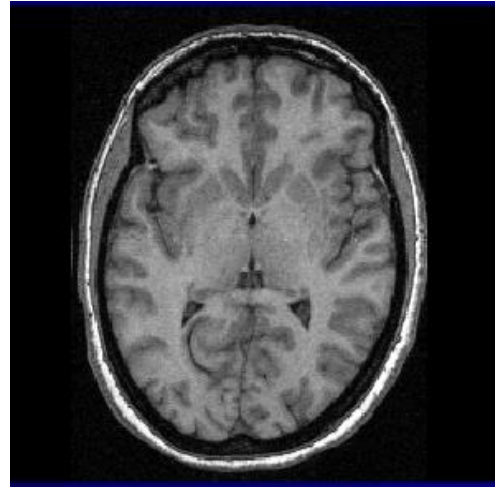


Gaussian

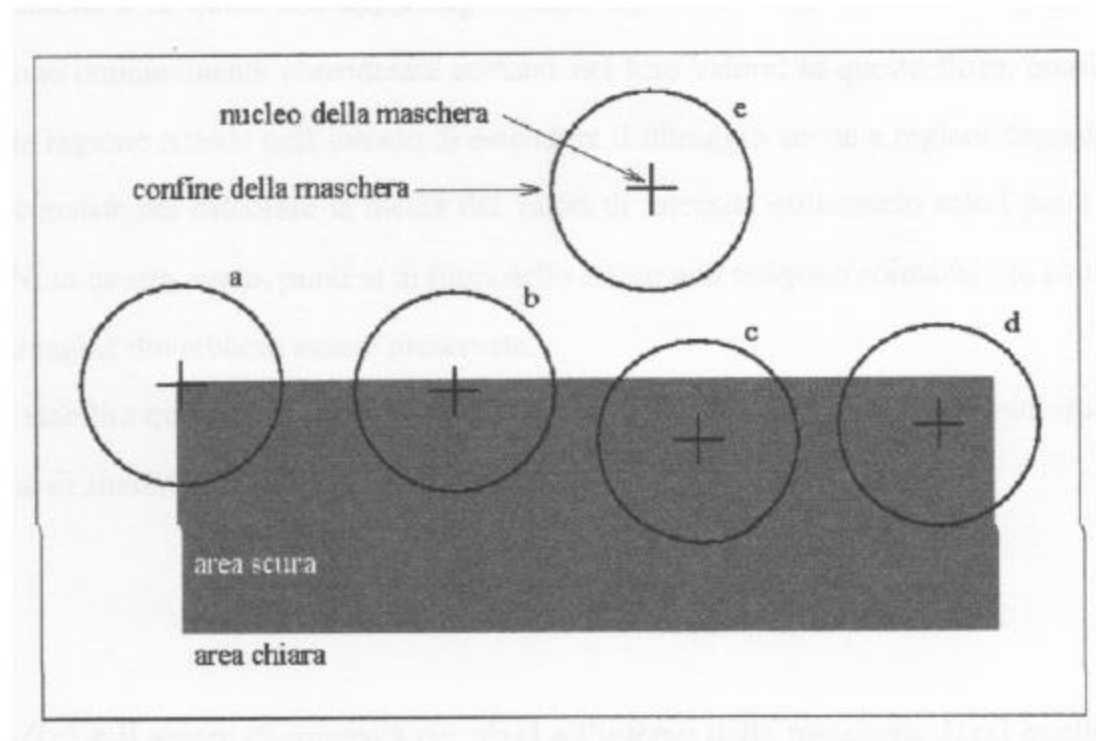


Hybrid

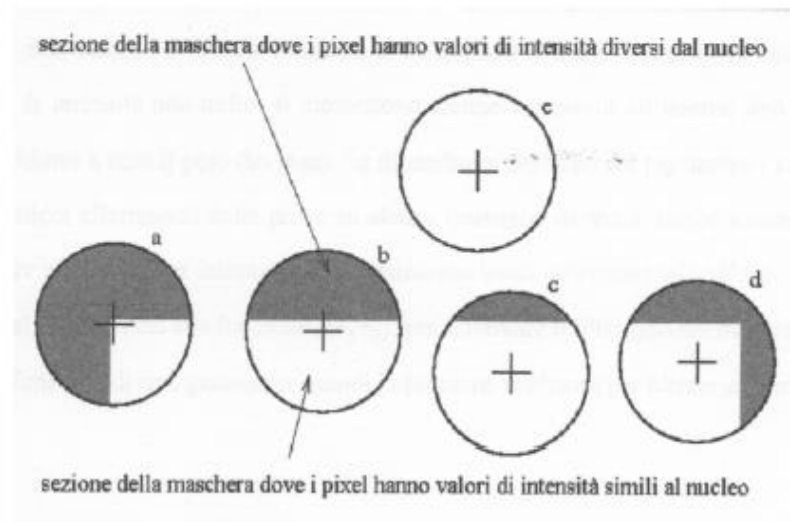
Originale, Gaussiano, Media, Mediana



SUSAN: Smoothing Segment Univalue Assimilating Values



SUSAN (cont.)



L'idea consiste nel calcolare la media dei valori di intensità utilizzando solo i pixel dell'USAN; in questo modo, punti al di fuori dello stesso non vengono coinvolti e le strutture dell'immagine dovrebbero essere preservate.

SUSAN (cont.)

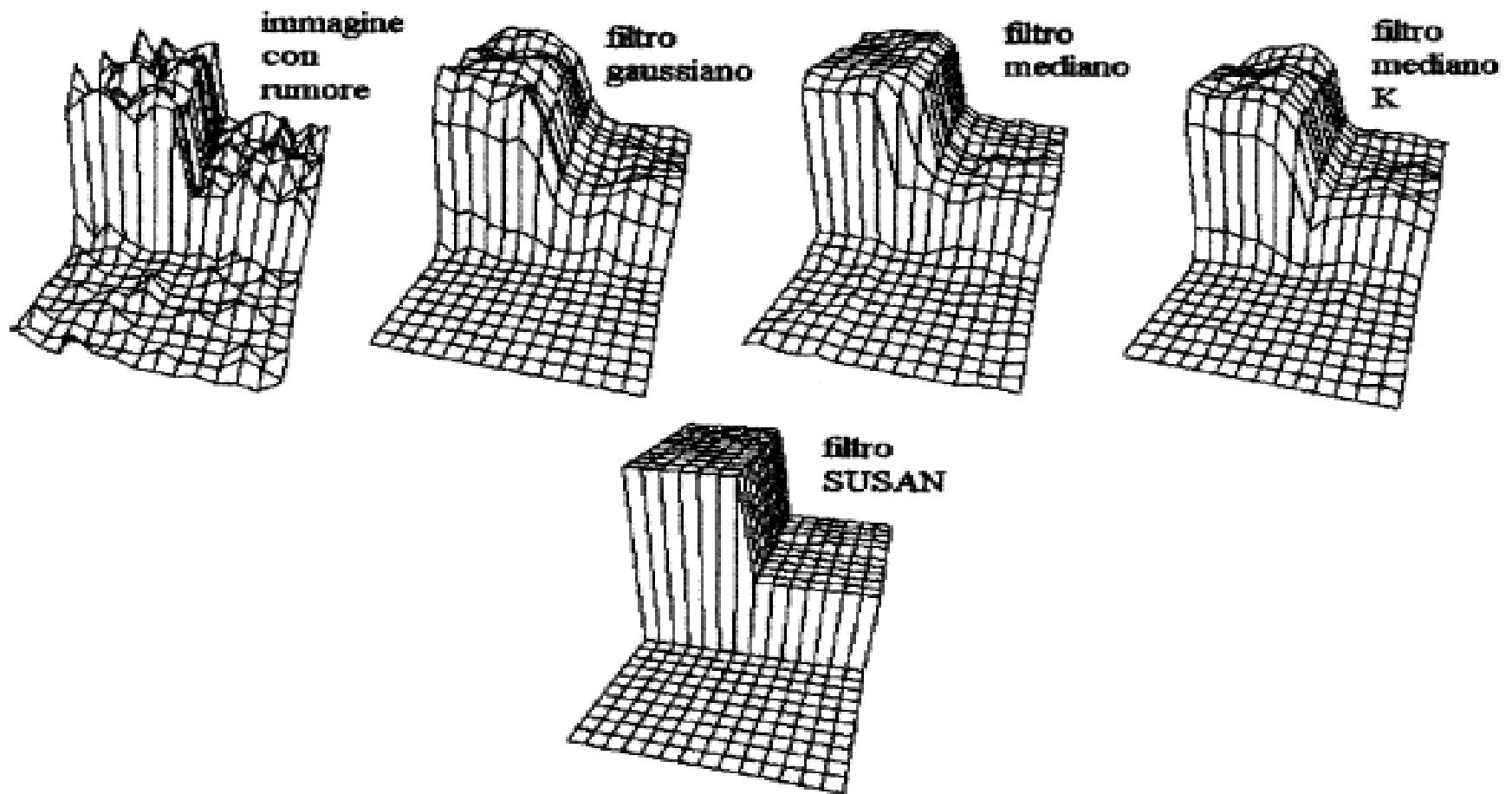
Per stabilire quali pixel fanno parte dell'USAN associato ad un nucleo si introduce la funzione di similitudine di intensità:

$$c(r, r_0) = e^{-\left(\frac{I(r) - I(r_0)}{t}\right)^2}$$

in cui $I(r)$ è il valore di intensità dei pixel all'interno della maschera, $I(r_0)$ quello del nucleo, t è un valore di soglia sull'intensità e $c(r, r_0)$ rappresenta il peso che il pixel in esame ha nel calcolo della media.

Oltre al peso dovuto alla funzione $c(r, r_0)$, per effettuare il filtraggio del rumore, si utilizza una funzione di tipo gaussiano. Se l'area dell'USAN è nulla il valore dell'intensità del pixel in esame è calcolato applicando il filtro mediano descritto precedentemente.

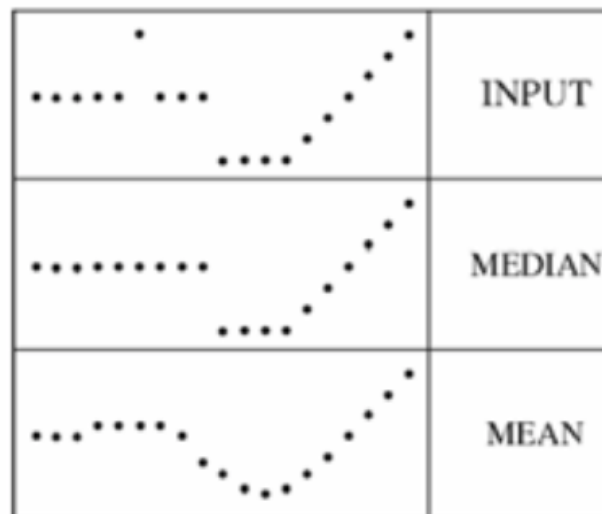
Smoothing: Confronto



Quale Filtro Scegliere

- What advantage does median filtering have over Gaussian filtering?
 - Robustness to outliers

filters have width 5 :



Source: K. Grauman

Median vs. Gaussian filtering

