Lezione 25-10

martedì 25 ottobre 2022 11:09

CORREZIONE ESERCIZI

(c) $33x \equiv 24 \pmod{12}$

$MCD(33, 12) = 3$ poiché $3 \mid 24$ si ha che

$33x \equiv 24 \pmod{12}$ ha soluzione $\Leftrightarrow$ $11x \equiv 8 \pmod 4$ ha soluzione

$$\begin{cases} 11 = 4 \cdot 2 + 3 \\ 4 = 3 \cdot 1 + 1 \\ 3 = 1 \cdot 3 + 0 \end{cases} \rightarrow 1 = 4 - 3 = 4 - (11 - 4 \cdot 2) =$$

$$= 4 + (-1)11 + (2) \cdot 4 =$$

$$= 3 \cdot 4 + (-1)11$$

$$S = 8(-1) = -8 \equiv 0 \pmod 4$$

$$S = [0]_4 = [0]_{12} \cup \left[0 + \frac{12}{3}\right]_{12} \cup \left[0 + \frac{12 \cdot 2}{3}\right]_{12} =$$

$$= [0]_{12} \cup [4]_{12} \cup [8]_{12}$$

(d) $3x \equiv 5 \pmod 8$ $\quad MCD(8,3) = 1$

$8 = 3 \cdot 2 + 2$

$3 = 2 \cdot 1 + 1 \rightarrow 1 = 3 - 2 = 3 + (-1)(8 - 3 \cdot 2) =$

$2 = 1 \cdot 2 + 0$

$$= 3 + (-1)8 + (+2) \cdot 3 =$$

$$= (-1)8 + (3) \cdot 3$$

$$S = 3 \cdot 5 = 15 \equiv 7 \pmod 8$$

$$S = [7]_8 = \{7 + 8k \mid k \in \mathbb{Z}\}$$

(e) $87x \equiv 27 \pmod{12}$ $\quad MCD(87, 12) = 3$ $\quad 29x \equiv 9 \pmod 4$

$97 = 12 \cdot 7 + 3$

$12 = 3 \cdot 4 + 0$

$3 = 97 + (-7)12$

$1 = 29 + (-7)4$, $u = 1$

$s = 9 \equiv 1 \pmod 4$

$S = [1]_4$

(2)

$$\begin{cases} a \equiv 2 \pmod 3 \\ a \equiv 3 \pmod 4 \\ a \equiv 6 \pmod 7 \end{cases}$$

(i) $a = 2 + 3k$, $k \in \mathbb{Z}$

(ii) $2 + 3k \equiv 3 \pmod 4 \rightarrow 3k \equiv 1 \pmod 4$

$1 = 4 - 3 = 4 + (-1)3$

$k = -1 \equiv 3 \pmod 4$

$a = 2 + 3 \cdot 3 = 11 \rightsquigarrow [11]_{12} = 11 + 12k$, $k \in \mathbb{Z}$

(iii) $11 + 12k \equiv 6 \pmod 7$

$12k \equiv -5 \pmod 7 \Rightarrow 5k \equiv 2 \pmod 7$

$12 \equiv_7 5$, $-5 \equiv_7 2$

$7 = 5 \cdot 1 + 2$

$5 = 2 \cdot 2 + 1 \rightarrow 1 = 5 + (-2) \cdot 2 = 5 + (-2)(7 + (-1)5) =$

$1 = 1 \cdot 2 + 0$

$= 5 + (-2)7 + (2) \cdot 5 =$

$= 3 \cdot 5 + (-2) \cdot 7$

$= 3 \cdot 5 + (-2) \cdot 7$

$k = 6 \rightarrow$

$S = 11 + 12 \cdot 6 = 83 \quad , \quad S = [83]_{84}$

NB $\quad 12k \equiv -5 \pmod 7$

$12 \equiv 5 \pmod 7 \xrightarrow{(*)} 12k \equiv 5k \pmod 7$

$-5 \equiv 2 \pmod 7$

$12k \equiv -5 \ \wedge \ -5 \equiv 2 \Rightarrow 12k \equiv 2$

$5k \equiv 12k \ \wedge \ 12k \equiv 2 \Rightarrow 5k \equiv 2$

$a \equiv b \mod m$

$c \equiv c \mod m$ ✓

$a + c \equiv b + c \pmod m$

$ac \equiv bc \pmod m \ (*)$

(b) $\begin{cases} b \equiv 15 \pmod{18} \\ b \equiv 16 \pmod{19} \end{cases}$

(i) dalla I , $b = 15 + 18k$, $k \in \mathbb{Z}$

(ii) $15 + 18k \equiv 16 \pmod{19}$

$18k \equiv 1 \pmod{19}$

$19 = 18 + 1 \rightarrow 1 = 19 + (-1) \cdot 18$

$k = (-1) \cdot 1 = -1 \equiv 18 \pmod{19}$

$b = 15 + 18(-1) = -3 \rightarrow S = [-3]_{342} = [339]_{342}$

(3) $\begin{cases} x \equiv 1 \pmod 2 \\ x \equiv 9 \pmod{11} \\ x \equiv 51 \pmod{75} \end{cases}$

$MCD(2,11) = 1$

$MCD(2,75) = 1$

$MCD(11,75) = 1$

i) $x = 1 + 2k \quad , k \in \mathbb{Z}$

ii) $1 + 2k \equiv 9 \pmod{11}$

$2k \equiv 8 \pmod{11}$

$11 = 2 \cdot 5 + 1 \rightarrow 1 = 11 - 2 \cdot 5 = 11 + (-5) \cdot 2$

$1 = 2 \cdot 6 - 11 \qquad (-5 \equiv 6 \pmod{11})$

$k = (-5)\,8 = -40 \equiv 4 \pmod{11}$

$\Rightarrow x = 1 + 2 \cdot 4 = 9 \quad , \quad [9]_{22} \Rightarrow 9 + 22k$

iii) $9 + 22k \equiv 51 \pmod{75}$

$22k \equiv 42 \pmod{75}$

$75 = 22 \cdot 3 + 9$

$22 = 9 \cdot 2 + 4$

$9 = 4 \cdot 2 + 1 \rightarrow 1 = 9 + (-2)\,4 = 9 + (-2)(22 + (-2) \cdot 9) =$

$4 = 1 \cdot 4 + 0 \qquad = (-2) \cdot 22 + 5 \cdot 9 =$

$= (-2)\cdot 22 + 5(75 + (-3)\cdot 22) =$

$= (-2)\cdot 22 + 5\cdot 75 + (-15)\cdot 22 =$

$= (-17)\cdot 22 + 5\cdot 75$

$k = (-17)\,42 = -714$

$s = 9 + 22(-714) = -15699$

$[-15699]_{1650} = [801]_{1650}$

(b)
$$\begin{cases} x \equiv 2 \pmod 5 \\ x \equiv 1 \pmod 3 \\ x \equiv 6 \pmod{14} \\ x \equiv 5 \pmod{11} \end{cases}$$

i) $x = 2 + 5k, \quad k \in \mathbb{Z}$

ii) $2 + 5k \equiv 1 \pmod 3$

$5k \equiv -1 \pmod 3$

$2k \equiv 2 \pmod 3 \Rightarrow k \equiv 1 \pmod 3$

iii) $x = 2 + 5\cdot 1 = 7 \rightsquigarrow [7]_{15} = \{7 + 15h \mid h \in \mathbb{Z}\}$

iv) $7 + 15h \equiv 6 \pmod{14}$

$15h \equiv -1 \pmod{14}$

$h \equiv -1 \pmod{14}$

$s = 7 + 15(-1) = -8 \rightsquigarrow [-8]_{210} = \{-8 + 210t \mid t \in \mathbb{Z}\}$

v) $-8 + 210t \equiv 5 \pmod{11}$

$210t \equiv 13 \pmod{11} \rightarrow t \equiv 2 \pmod{11}$

$210t \equiv 13 \pmod{11} \rightarrow t \equiv 2 \pmod{11}$

$210 \equiv 1$ $\quad 13 \equiv 2$

$S = -9 + 210 \cdot 2 = 420 - 8 = 412$

$S = [412]_{2310}$

(A.2) $S = \{a, b, c, d\}$

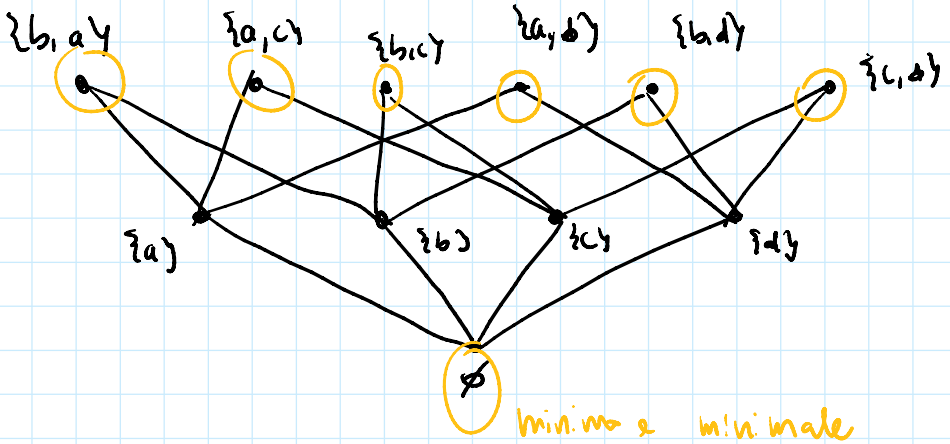
1) $|\mathcal{P}(S)| = 2^{|S|} = 2^4$

2) $A, B \subseteq S \quad \inf(A, B) = A \cap B \;,\; \sup(A, B) = A \cup B$

$\inf(\{a, c\}, \{a, d\}) = \{a\} \qquad \sup(\{a, c\}, \{a, d\}) = \{a, c, d\}$

3) $A = \{X \in \mathcal{P}(S) \mid |X| \leq 2\} =$

$= \{\emptyset, \{a\}, \{b\}, \{c\}, \{d\}, \{a, b\}, \{a, c\}, \{a, d\}, \{b, c\}, \{b, d\}, \{c, d\}\}$



$\{b, a\} \not\subseteq \{b, c\}$ e $\{b, c\} \not\subseteq \{b, a\}$

(7.3) $A = \{2,3,4\}$ $\quad B = A \times A$

$(a,b) \sqsubseteq (c,d) \iff a \leq c \ \text{ e } \ b \mid d$

(i) $\sqsubseteq$ è d'ordine:

(A) $(a,b) \sqsubseteq (a,b)$ vero perché $a \leq a$ e $b \mid b$

(B) $(a,b) \sqsubseteq (c,d)$ e $(c,d) \sqsubseteq (a,b) \Rightarrow (a,b) = (c,d)$

$a \leq c$ e $b \mid d$ $\quad$ $c \leq a$ e $d \mid b$ $\quad \Rightarrow \ a = c$ e $b = d$

(C) $(a,b) \sqsubseteq (c,d)$ e $(c,d) \sqsubseteq (e,f) \Rightarrow (a,b) \sqsubseteq (e,f)$

$a \leq c$ e $b \mid d$ $\quad$ $c \leq e$ e $d \mid f$ $\quad \Rightarrow \ a \leq e$ e $b \mid f$

(ii)

$(2,2) \not\sqsubseteq (2,3)$ $\quad 2 \leq 2$ ma $2 \nmid 3$

$(2,2) \sqsubseteq (2,4)$

$\sqsubseteq (3,2) \sqsubseteq (3,4)$

$\sqsubseteq (4,2) \sqsubseteq (4,4)$

$(3,2) \sqsubseteq (3,4) \sqsubseteq (4,4)$

$\sqsubseteq (4,2)$

$(2,3) \sqsubseteq (3,3) \sqsubseteq (4,3)$