Lezione 22-09

DEF - Un intervallo chiuso è un sottoinsieme di $\mathbb{R}$ della forma

$$[a,b] = \{x\in\mathbb{R} \mid a\le x\le b\}\subseteq\mathbb{R}$$

è ben definita se $a\le b$

se $a=b \Rightarrow [a,b]=\{a\}$

$a>b \Rightarrow [a,b]=\emptyset$

Gli intervalli aperti sono $(a,b) =]a,b[= \{x\in\mathbb{R} \mid a<x<b\}\subseteq\mathbb{R}$, $a,b\notin(a,b)$

$(a,b] = \{x\in\mathbb{R} \mid a<x\le b\}\subseteq\mathbb{R}$, $a\notin(a,b]$ e $b\in(a,b]$

$[a,b) = \{x\in\mathbb{R} \mid a\le x< b\}\subseteq\mathbb{R}$, $a\in[a,b)$ e $b\notin[a,b)$

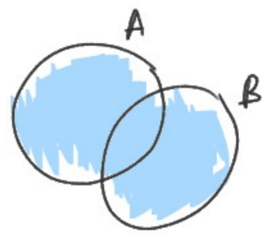
$$[5,3] = \{x\in\mathbb{R} \mid x\ge 5 \wedge x\le 3\} = \emptyset$$

$$[3,5] = \{x\in\mathbb{R} \mid x\ge 3 \wedge x\le 5\}$$

OPERAZIONI TRA INSIEMI

1) UNIONE

Dati A, B, $\quad A\cup B := \{x \mid x\in A \vee x\in B\}$



$A=\{a,b,c\}$

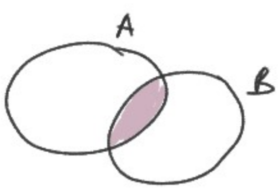
$B=\{d,a,f\}$

$A\cup B = \{a,b,c,d,f\}$

2) INTERSEZIONE

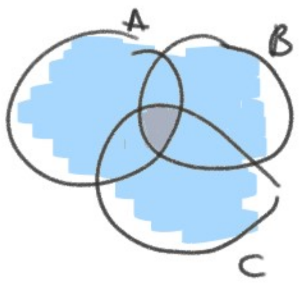
Dati A, B $\quad A\cap B := \{x \mid x\in A \wedge x\in B\}$

$A\cap B = \{a\}$



- $\cap$ e $\cup$ sono COMMUTATIVE: $A\cap B = B\cap A$, $\quad A\cup B = B\cup A$
- $\cap$ e $\cup$ sono ASSOCIATIVE: $(A\cup B)\cup C = A\cup(B\cup C) = A\cup B\cup C$

$(A\cap B)\cap C = A\cap(B\cap C) = A\cap B\cap C$



$$A_1 \cap A_2 \cap \ldots \cap A_n = \bigcap_{i=1}^{n} A_i$$

$$A_1 \cup \ldots \cup A_n = \bigcup_{i=1}^{n} A_i$$

$$\{A_i\}_{i \in I} \qquad \bigcup_{i \in I} A_i \qquad \bigcap_{i \in I} A_i$$

Es.- $\{\{n\}\}_{n \in \mathbb{N}}$ $\downarrow$

$$\{\{1\}, \{2\}, \{3\}, \ldots\}$$

$$\underbrace{\underbrace{\{1\} \cup \{2\}}_{\{1,2\}} \cup \{3\}}_{\{1,2,3\}} \cup \{4\} \cup \ldots = \mathbb{N}$$

$$\underbrace{\underbrace{\{1\} \cap \{2\}}_{\emptyset} \cap \{3\}}_{\emptyset} \cap \ldots = \emptyset$$

Ex $A_n = [0, n] \subseteq \mathbb{R}$

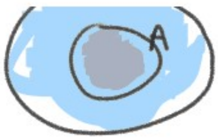
$$\underbrace{[0,1] \cap [0,2]} \cap [0,3] \cap \ldots = A_1 = [0,1]$$

$$A_1 \cap A_2 = [0,1] = A_1$$

$$\underbrace{A_1 \cap A_2}_{A_1} \cap A_3 = A_1$$

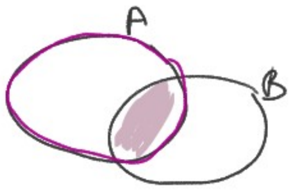
- $A \subseteq B \Rightarrow$ $A \cap B = A$
  $A \cup B = B$

- $A \subseteq B \Rightarrow$ $A \cap B = A$, $A \cup B = B$



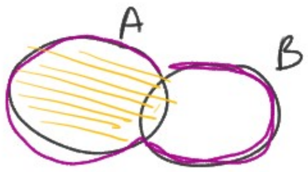
- $A \cap B \subseteq A$

  $A \cap B \subseteq B$



- $A \subseteq A \cup B$

  $B \subseteq A \cup B$



- A insieme, $\mathcal{P}(A)$

  $\forall X \in \mathcal{P}(A)$ $(\Leftrightarrow X \subseteq A)$

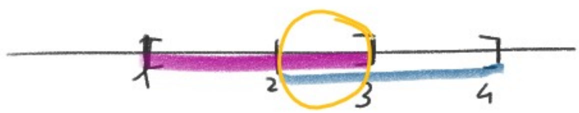
$$\left.\begin{array}{l} X \cap \emptyset = \emptyset \\ X \cup A = A \\ X \cup \emptyset = X \\ X \cap A = X \end{array}\right\}$$

in $\mathcal{P}(A)$, $\emptyset$ è elemento neutro per $\cup$

A è elemento neutro per $\cap$

ES - $[1,2] \cap [3,5] = \emptyset$

$$\underbrace{[1,3]}_{1 \le x \le 3} \cap \underbrace{(2,4]}_{2 < x \le 4} = (2,3]$$



ES - $3\mathbb{Z} = \{3z \mid z \in \mathbb{Z}\}$ = multipli di 3 = $\{ \ldots -6, -3, 0, 3, 6, \ldots \}$

$6\mathbb{Z} = \{6z \mid z \in \mathbb{Z}\}$ = multipli di 6

$$\left.\begin{array}{l} 3\mathbb{Z} \cap 6\mathbb{Z} = 6\mathbb{Z} \\ 3\mathbb{Z} \cup 6\mathbb{Z} = 3\mathbb{Z} \end{array}\right\} \text{---} \; 6\mathbb{Z} \subseteq 3\mathbb{Z}$$

$3 \in 3\mathbb{Z}$ ma $3 \notin 6\mathbb{Z}$

✓ $6\mathbb{Z} \subseteq 3\mathbb{Z}$ ma $3\mathbb{Z} \not\subseteq 6\mathbb{Z}$

$\forall\, n \in 6\mathbb{Z}, \quad n = 6 \cdot z = 3 \cdot (2z) \in 3\mathbb{Z}$

3) DIFFERENZA DI INSIEMI

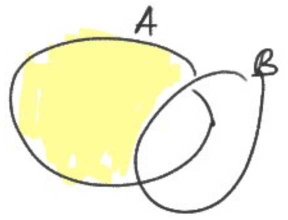
A, B insiemi qualsiasi

$A \setminus B = \{ x \in A \mid x \notin B \}$

$B \setminus A = \{ x \in B \mid x \notin A \}$

$A = \{ a, b, c \}$ $\quad A \setminus B = \{ b, c \}$

$B = \{ a, f, g \}$



$B \setminus A = \{ f, g \}$

$A \setminus \phi = A$

$\emptyset \setminus A = \phi$

- In $\mathcal{P}(A)$, $X \in \mathcal{P}(A)$

  $A \setminus X$ è detto **COMPLEMENTO** di X

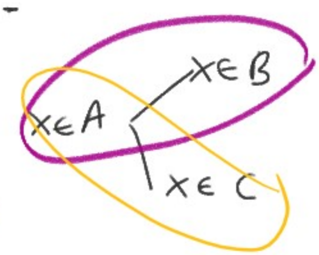
  Si indica anche con $\bar{X}$ oppure X^c

$(A \setminus X) \cup X = A$

$(A \setminus X) \cap X = \phi$

Due insiemi S, T si dicono **DISGIUNTI** se $S \cap T = \phi$.

- $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$



se $x \in A \cap (B \cup C) \iff x \in A$ e $x \in B \cup C \iff$

$\iff x \in A$ e $x \in B \iff x \in A \cap B$ oppure $x \in A \cap C$

oppure

$x \in A$ e $x \in C$

$\iff x \in (A \cap B) \cup (A \cap C)$

$x \in A$ e $x \in C$

$\Longleftarrow x \in (A \cap B) \cup (A \cap C)$

- $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$ DA FARE A CASA

4) PRODOTTO CARTESIANO DI INSIEMI

A, B insiemi – il prodotto cartesiano di A e B è l'insieme

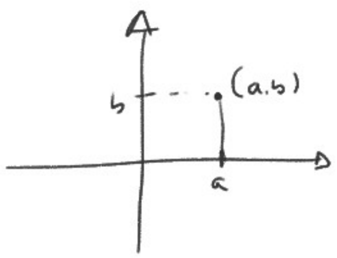
$$A \times B := \{ \underbrace{(a,b)}_{\text{coppie ordinate}} \mid a \in A \wedge b \in B \}$$

$A = \{a, b, c\}$
$B = \{1, 2\}$

$A \times B = \{ (a,1), (a,2), (b,1), (b,2), (c,1), (c,2) \}$
$B \times A = \{ (1,a), (1,b), (1,c), (2,a), (2,c), (2,b) \}$

Se $|A| = n$ e $|B| = m$ $\quad |A \times B| = n \cdot m$

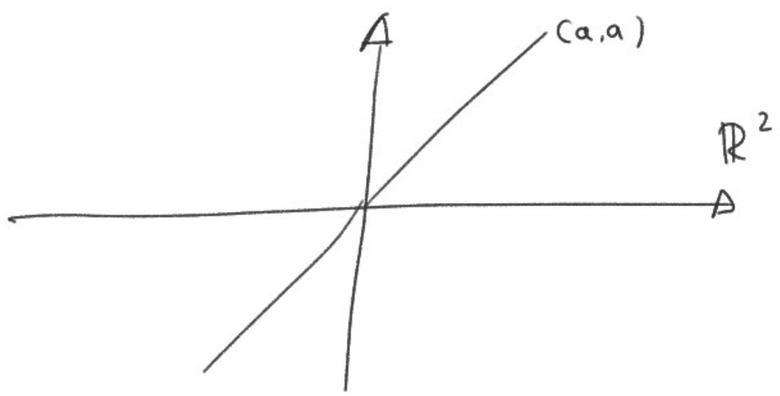
$\mathbb{R} \times \mathbb{R} = \mathbb{R}^2 \longrightarrow$ il piano cartesiano



$(1,2) \neq (2,1)$

$A = \{a, b, c, d\}$
$B = \{d, e, f\}$

$A \times B = \{ (a,d), (a,e), (a,f), (b,d), (b,e), (b,f), (c,d), (c,f), (c,e), (d,d), (d,e), (d,f) \}$



DEF - Una RELAZIONE è un sottoinsieme del prodotto cartesiano di due insiemi S, T

$R \subseteq S \times T$ -

Ex - $A \times B = \{(a,1), (a,2), (b,1), (b,2), (c,1), (c,2)\}$

$R = A \times B \subseteq A \times B$

$R = \emptyset \subseteq A \times B$

$R = \{(a,1), (b,2)\} \subseteq A \times B$

Es $A = \{$ insieme degli abitanti di Salerno $\}$

$R = \{(x,y) \in A \times A \mid x \text{ è sposato con } y\}$

Es - $\leq \subseteq \mathbb{R}^2$

$(x,y) \in \leq \iff x \leq y$