Lezione 15 dicembre

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ESERCIZIO 1

$$X \setminus (A \cup B) = (X \setminus A) \cap (X \setminus B)$$

$x \in X \setminus (A \cup B) \iff \cancel{x \in X}$ e $x \notin A \cup B$

$\iff \cancel{x \in X}$ e $(x \notin A$ e $x \notin B)$

$\iff x \in X \setminus A$ e $x \in X \setminus B$

$\iff x \in (X \setminus A) \cap (X \setminus B)$

$$X \setminus (A \cap B) = (X \setminus A) \cup (X \setminus B)$$

$x \in X \setminus (A \cap B) \iff x \notin A \cap B \iff x \notin A$ oppure $x \notin B$

$\iff x \in X \setminus A$ oppure $x \in X \setminus B$

$\iff x \in (X \setminus A) \cup (X \setminus B)$

ESERCIZIO 2

$$\begin{cases} a \equiv 16 \pmod{19} \\ a \equiv 17 \pmod{20} \end{cases}$$

$a = 16 + 19k$

$\Downarrow$

$16 + 19k \equiv 17 \pmod{20}$

$\Downarrow$

$19k \equiv 1 \pmod{20}$ $\qquad 1 = 20 - 19 = 1 \cdot 20 + (-1) 19$

$k \equiv -1 \pmod{20}$

$a = 16 - 19 = -3$ $\qquad S = [-3]_{380} = [377]_{380}$

$s = 1137$

$\boxed{\text{ESERCIZIO 3}}$ $\qquad GL_2(\mathbb{R}) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ad - bc \neq 0 \right\}$

$$N = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \mid ad - cb = 3 \right\}$$

1) l'elemento neutro di $GL_2(\mathbb{R})$ è $\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$. $\det \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = 1$

$\Rightarrow \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \notin N$

2) Thm di Binet $\qquad \det(A \cdot B) = \det(A) \det(B)$

$\Rightarrow$ se $A, B \in N$, $\quad \det(AB) = 3 \cdot 3 = 9 \Rightarrow AB \notin N$

3) $\begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 3 & 0 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} \rightarrow \det \begin{pmatrix} 9 & 0 \\ 0 & 1 \end{pmatrix} = 9$

$\boxed{\text{ESERCIZIO 4}}$

$$\underset{a_{ik}}{\begin{pmatrix} a_{11} & a_{12} & \cdots & & a_{1n} \\ 0 & a_{22} & \cdots & & a_{2n} \\ 0 & 0 & a_{33} & \cdots & a_{3n} \\ \vdots & & & & \\ 0 & \cdots & & 0 & a_{nn} \end{pmatrix}} \quad \underset{b_{kj}}{\begin{pmatrix} b_{11} & b_{12} & \cdots & & b_{1n} \\ 0 & b_{22} & \cdots & & b_{2n} \\ 0 & 0 & b_{33} & \cdots & b_{3n} \\ \vdots & & & & \\ 0 & \cdots & & 0 & b_{nn} \end{pmatrix}} \leftarrow$$

$$c_{ij} = a_{i1} b_{1j} + a_{i2} b_{2j} + \dots + a_{in} b_{nj} \leftarrow$$

se $j \geq i$ si ha $a_{ij} \neq 0$

$j < i$, si ha $a_{ij} = 0$

$a_{i1}, a_{i2}, \dots, a_{i,i-1} = 0$

$b_{j+1,j}, b_{j+2,j} \dots, b_{n,j} = 0$