Introduction:

The goal of this project is to analyze and plot the temperature distribution contour of a two-dimensional printed circuit board (PCB) in space, subject to a given heat flux at arbitrary locations. In the vacuum of space, the absence of a surrounding medium eliminates the possibility of convective heat transfer, necessitating a focus on conduction and radiation as the primary mechanisms of heat transfer.

Printed circuit boards are critical components in electronic systems, serving as the backbone for electrical connections and the mounting of various electronic components. In the harsh environment of space, managing the thermal performance of PCBs is crucial to ensure the reliability and functionality of onboard electronic systems. The absence of atmospheric convection in space poses unique challenges for thermal management, making it essential to thoroughly understand how heat is conducted through the PCB material and how it is radiated away into the surrounding vacuum.

This report delves into the theoretical and computational aspects of modeling heat transfer in a 2D PCB, taking into account the specific conditions encountered in space. The analysis includes the following key components:

Heat Conduction: Understanding the conduction process within the PCB material, where heat is transferred through the solid medium from regions of higher temperature to regions of lower temperature.

Radiative Heat Transfer: Examining the radiation mechanism by which the PCB dissipates heat into the vacuum of space. Radiation is a critical mode of heat transfer in space and depends on factors such as the emissivity of the PCB surface and the temperature distribution.

Heat Flux Input: Defining and applying heat flux at arbitrary locations on the PCB to simulate realistic operational conditions. This includes accounting for various sources of heat generation within the electronic components mounted on the PCB.

The objective of this analysis is to generate a detailed temperature distribution contour of the PCB, which will provide insights into the thermal behavior and identify potential hotspots. Such information is invaluable for optimizing the design and placement of components to ensure effective thermal management and avoid overheating, which can lead to component failure.

By leveraging computational techniques and advanced simulation tools, this report aims to contribute to the broader field of thermal analysis in space electronics, providing a robust methodology for evaluating the thermal performance of PCBs in space environments.

Approach:

Our approach involves the following steps:

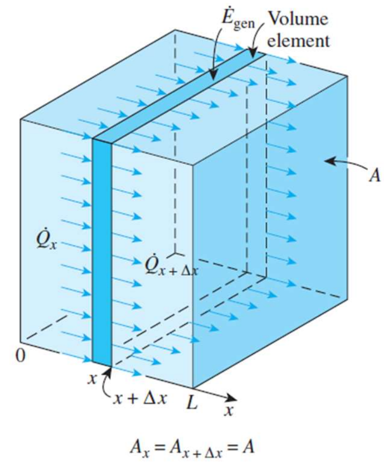
1. **Formulation of the Heat Conduction Problem**: Utilizing the given heat conduction equation, we set up the problem considering the specific properties of the PCB material and the initial and boundary conditions relevant to a space environment.
2. **Numerical Simulation**: Implementing advanced computational methods to solve the transient heat conduction equation. This includes discretizing the equation and applying numerical techniques to obtain the temperature distribution over time.
3. **Incorporation of Radiative Heat Transfer**: Accounting for radiative heat loss from the PCB surface, which is significant in the vacuum of space. This involves calculating the radiative heat flux based on the emissivity of the PCB material and the temperature distribution.
4. **Analysis of Results**: Generating temperature distribution contours and analyzing the data to identify critical thermal characteristics of the PCB. This includes examining the effects of different heat flux inputs and material properties on the overall thermal performance.

By comprehensively addressing these aspects, this report aims to provide a thorough understanding of the thermal behavior of PCBs in space environments. The insights gained from this study will be instrumental in optimizing the thermal management strategies for electronic systems used in space missions, ensuring their robustness and longevity in the harsh conditions of space

1D Heat Conduction Equation:

The transient heat conduction equation for a one-dimensional plane wall, noting that the area A is constant, is given by:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right) + \dot{e_{gen}} = \rho c\frac{\partial T}{\partial t}$$



This equation forms the foundation of our analysis, describing the distribution of temperature T over time t within a plane wall, where K represents the thermal conductivity, ρ is the density, c is the specific heat capacity, and $\dot{e_{gen}}$ denotes the volumetric heat generation rate.

For constant Conductivity:

$$K\frac{\partial^2 T}{\partial x^2} + \dot{e_{gen}} = \rho c\frac{\partial T}{\partial t}$$

We are considering Steady State Flow:

$$\frac{d}{dx}(K\frac{dT}{dx}) + \dot{e_{gen}} = 0$$

Control Volume Formulation for 1D Heat transfer:

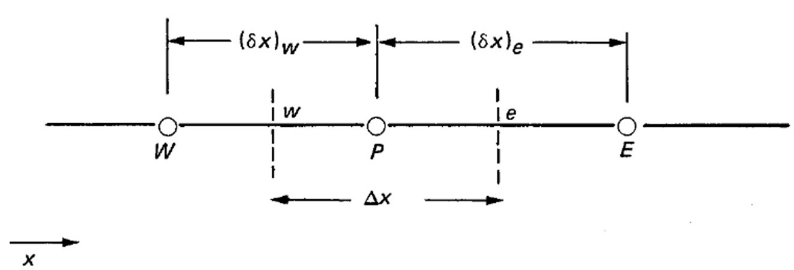
Lets create a control volume across point P and study the heat flow through the control volume.

If we consider the heat conduction in positive x-direction, heat goes in from west and out from east. We are further considering the source heat (S) from remaining y and z direction going in.

Considering T_P, T_E and T_W are the temperature at point P, east and west nodes respectively.

General Equation:

$$\frac{d}{dx}(K\frac{dT}{dx}) + S = 0$$



Integrating the general equation we get:

$$\left(K\frac{dT}{dx}\right)_e - \left(K\frac{dT}{dx}\right)_w + \int_w^e S dx = 0$$

$$K\frac{(T_E - T_P)}{\delta x} - K\frac{(T_P - T_W)}{\delta x} + S\Delta x = 0$$

Here the source is a linear function with coefficients S_P and S_C is as $S = S_C + S_P T_P$

$$K\frac{(T_E - T_P)}{\delta x} - \frac{(T_P - T_W)}{\delta x} + (S_C + S_P T_P)\Delta x = 0$$

Comparing with :

$$a_P T_P = a_E T_E + a_W T_W + b$$

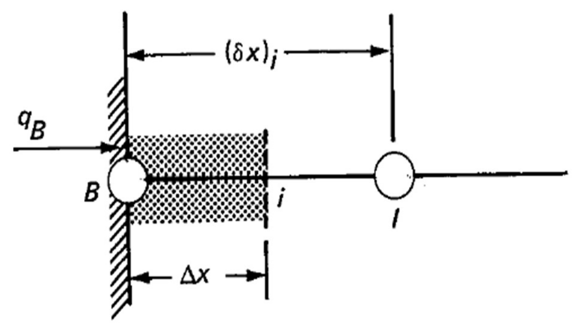
We get:

$a_E = \frac{K}{\delta x}$ $\qquad$ $a_W = \frac{K}{\delta x}$

$a_p = a_E + a_W - S_P \Delta x$ $\qquad$ $b = S_C \Delta x$

Boundary Equations:

- **Given Boundary Temperature**: No Additional equation required
- **Given Boundary Heat Flux** : Additional equation required is as follows:



General Equation: $\qquad \frac{d}{dx}(K\frac{dT}{dx}) + S = 0$

Integrating the general equation we get:

$$q_B - \left(K\frac{dT}{dx}\right)_I + \int_B^i S dx = 0$$

$S = S_C + S_P T_B$

$$q_B - \frac{K(T_B - T_I)}{\delta x} + (S_C + S_P T_B)\frac{\Delta x}{2}$$

Comparing with : $a_B T_B = a_I T_I + b$

$a_I = \frac{K}{\delta x}$ $\qquad$ $b = S_C \Delta x + q_B$ $\qquad$ $a_B = a_I - S_P \Delta$

- **Boundary heat flux specified via a heat transfer coefficient and the temperature of the surrounding fluid:**

  $q_B = h(T_f - T_B)$

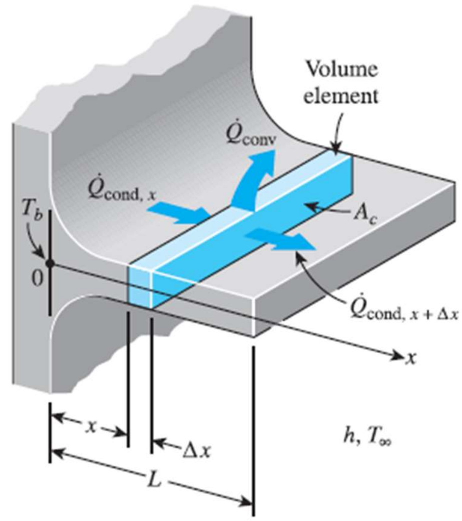
  $a_I = \frac{K}{\delta x}$ $\qquad$ $b = S_C \Delta x + hT_f$ $\qquad$ $a_B = a_I - S_P \Delta x + h$

1D Conduction and Convection:

$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{conv}$$

$$\dot{Q}_{conv} = \mathrm{hp}\Delta x(T - T_\infty)$$

$$\dot{Q}_{cond} = -KA_c\frac{dT}{dx}$$



Where,

p: perimeter

A_c : Cross Sectional Area

h: Convection Coefficient

Finally, the general equation come out to be:

$$\frac{d}{dx}\left(KA_c\frac{dT}{dx}\right) - \mathrm{hp}(T - T_\infty) = 0$$

Integrating with respect to x, we get:

$$\left(KA_c\frac{dT}{dx}\right)_e - \left(KA_c\frac{dT}{dx}\right)_w - \int_w^e \mathrm{hp}(T - T_\infty)dx = 0$$

We are considering a 1D heat conduction of unit breadth and width

A_c = 1 p = 4

$$\mathrm{K}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta\mathrm{x}} - \mathrm{K}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta\mathrm{x}} - 4h(T - T_\infty)\Delta\mathrm{x} = 0$$

Comparing with :

$$\mathrm{a_P T_P} = \mathrm{a_E T_E} + \mathrm{a_W T_W} + \mathrm{b}$$

We get:

$\mathrm{a_E} = \frac{\mathrm{K}}{\delta\mathrm{x}}$ $\mathrm{a_W} = \frac{\mathrm{K}}{\delta\mathrm{x}}$

$\mathrm{a_p} = \mathrm{a_E} + \mathrm{a_W} + 4\mathrm{h}\Delta\mathrm{x}$ $\mathrm{b} = 4\mathrm{h}T_\infty\Delta\mathrm{x}$

Boundary Equation:

General Equation: $\frac{d}{dx}\left(KA_c\frac{dT}{dx}\right) - \mathrm{hp}(T - T_\infty) = 0$

Integrating the general equation we get:

$$q_B - \left(KA_c\frac{dT}{dx}\right)_I - \int_B^i \mathrm{hp}(T - T_\infty)dx = 0$$

$$\mathrm{q_B} - \frac{\mathrm{K(T_B - T_I)}}{\delta\mathrm{x}} - 4\mathrm{h}(T - T_\infty)\frac{\Delta\mathrm{x}}{2}$$

Comparing with : $a_B T_B = a_I T_I + b$

$a_I = \frac{K}{\delta x}$

$b = 2hT_\infty \Delta x + q_B$

$a_B = a_I + 2h\Delta x$

Que. Plot the temperature distribution curve of a 1D copper rod of 1m length with only conduction and convection possible when one end is maintained at 400K and the other end is adiabatic with surrounding temperature 200K:

Given:

Length (L_x)= 1m

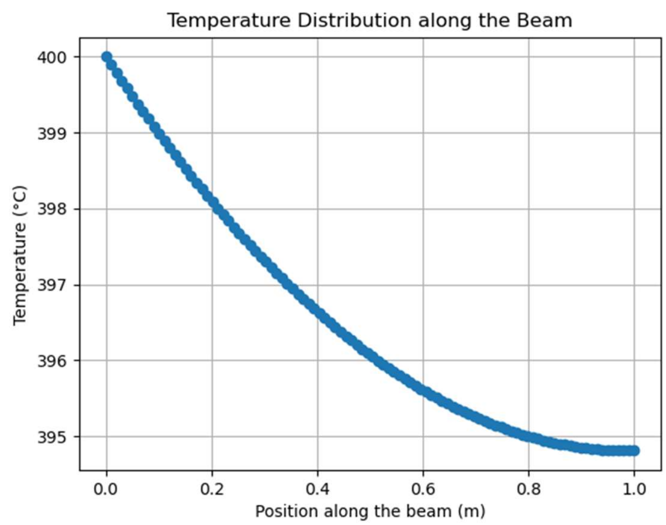
Number of nodes (n)= 100

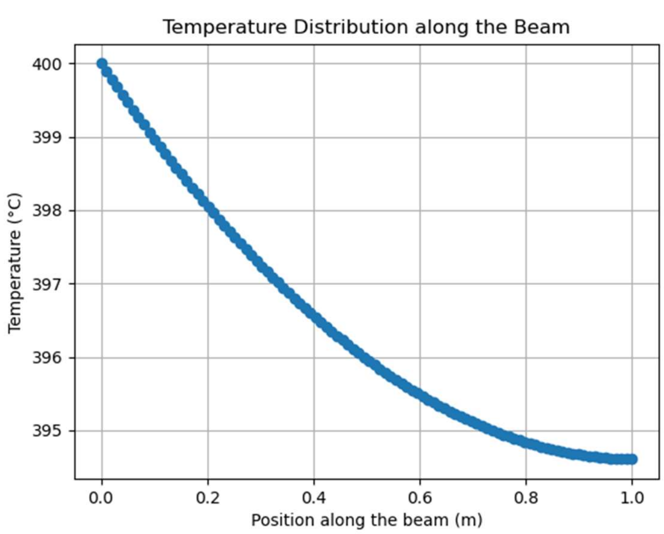
Boundary Temperature(T_B) = 400K

Ambient Temperature (T_∞)= 300K

Thermal Conductivity (K)= 400 W/mK

Solution:





Verification:

Validation Using Exact solution for adiabatic fin tip:

$$T(x) = T_\infty + (T_B - T_\infty)\frac{\cosh m(L-x)}{\cosh mL}$$

$$m = \sqrt{hp/KA}$$

1D Conduction and Radiation:

$$\dot{Q}_{cond,x} = \dot{Q}_{cond,x+\Delta x} + \dot{Q}_{Radiaiton}$$

$$\dot{Q}_{Radiation} = \varepsilon\sigma p \Delta x (T^4 - T_\infty^4)$$

$$\dot{Q}_{cond} = -KA_c \frac{dT}{dx}$$

Where,
p: perimeter
A_c : Cross-Sectional Area
h: Convection Coefficient

Finally, the general equation come out to be:

$$\frac{d}{dx}\left(KA_c \frac{dT}{dx}\right) - \varepsilon\sigma p (T^4 - T_\infty^4) = 0$$

Integrating with respect to x, we get:

$$\left(KA_c \frac{dT}{dx}\right)_e - \left(KA_c \frac{dT}{dx}\right)_w - \int_w^e \varepsilon\sigma p (T^4 - T_\infty^4) dx = 0$$

We are considering a 1D heat conduction of unit breadth and width
A_c = 1 p = 4

Linearizing the Radiation Equation:

If $\mathrm{S} = -\varepsilon\sigma(\mathrm{T}^4 - \mathrm{T}_\infty^4)$

$$S = S^* + \left(\frac{dS}{dT}\right)^* (T_P - T_P^*)$$

$\mathrm{S} = \varepsilon\sigma(3\mathrm{T}^4 + \mathrm{T}_\infty^4) - 4\,\varepsilon\sigma\mathrm{T}^3 T$

Discretized equation:

$$\mathrm{K}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta\mathrm{x}} - \mathrm{K}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta\mathrm{x}} - 4[\varepsilon\sigma(3T_P^4 + \mathrm{T}_\infty^4) - 4\,\varepsilon\sigma T_P^3 \mathrm{T_P}]\Delta\mathrm{x} = 0$$

Comparing with :

$$\mathrm{a_P T_P} = \mathrm{a_E T_E} + \mathrm{a_W T_W} + \mathrm{b}$$

We get:

$\mathrm{a_E} = \frac{\mathrm{K}}{\delta\mathrm{x}}$ $\mathrm{a_W} = \frac{\mathrm{K}}{\delta\mathrm{x}}$

$\mathrm{a_p} = \mathrm{a_E} + \mathrm{a_W} + 16\,\varepsilon\sigma T_P^3\,\Delta\mathrm{x}$ $\mathrm{b} = 4\varepsilon\sigma(3T_P^4 + \mathrm{T}_\infty^4)\,\Delta\mathrm{x}$

Boundary Condition:

$$q_B - \frac{K(T_B - T_I)}{\delta x} - 4[\varepsilon\sigma(3T_P^4 + T_\infty^4) - 4\,\varepsilon\sigma T_P^3 T_P\,]\frac{\Delta x}{2}$$

Comparing with : $a_B T_B = a_I T_I + b$

$a_I = \frac{K}{\delta x}$ $\qquad$ $b = 2\varepsilon\sigma(3T_P^4 + T_\infty^4)\Delta x + q_B$

$a_B = a_I + 8\,\varepsilon\sigma T_P^3 \Delta x$

Que. Plot the temperature distribution curve of a 1D copper rod of 1m length with only conduction and Radiation (with emissivity 0.9) possible when one end is maintained at 400K and the other end is adiabatic with surrounding temperature 200K:

Given:

Length (L_x)= 1m

Number of nodes (n)= 100

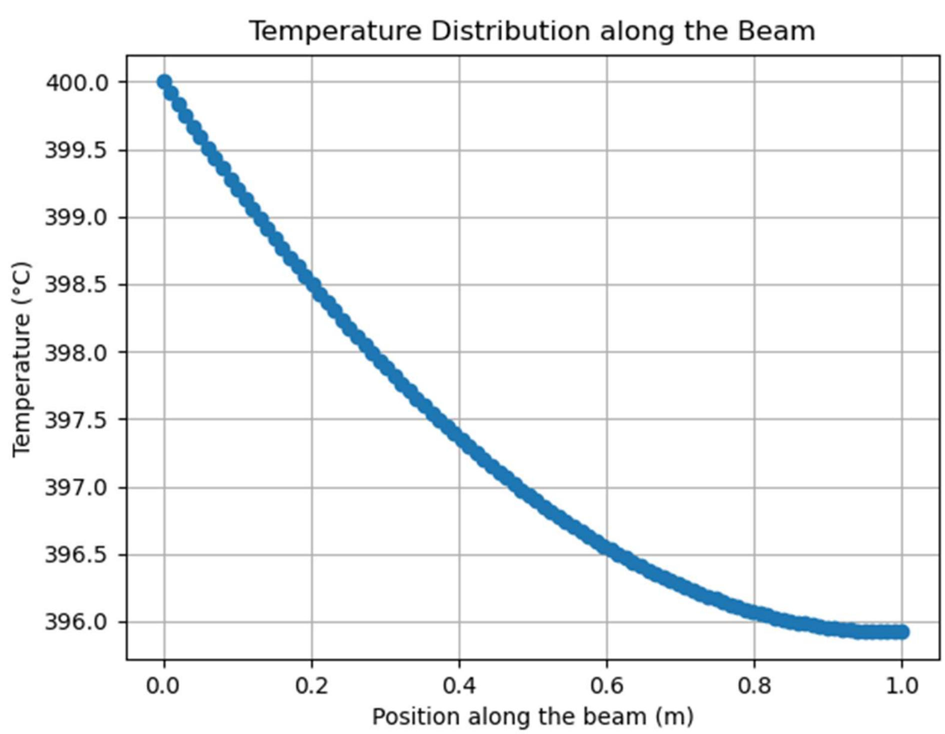
Boundary Temperature(T_B) = 400K

Ambient Temperature (T_∞)= 300K

Thermal Conductivity (K)= 400 W/mK

Emissivity (ε)= 0.9

Solution:



2D Heat Transfer Equation

We have a plate of unit thickness, so the conduction in 2 dimensions is as follows:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+e_{\dot{g}en}= \rho c\frac{\partial T}{\partial t}$$

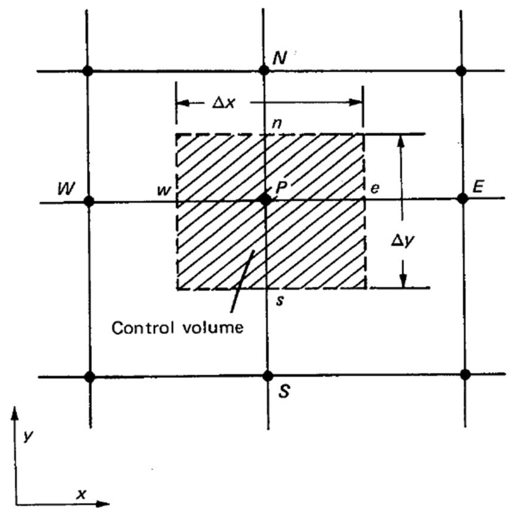
Here if we see the differential equation for 2D steady state Heat Transfer it goes like:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+e_{\dot{g}en}=0$$

Lets create a control volume across point P and study the heat flow through the control volume.

If we consider the heat conduction in positive x and y-direction, then heat is going in from west to east and south to north. Further considering the source heat (S) z direction going in.

Considering T_P, T_E, T_w, T_N and T_S are the temperature at point P, east, west, north and south node respectively**.**



General Equation:

$$\frac{\partial}{\partial x}\left(K\frac{\partial T}{\partial x}\right)+\frac{\partial}{\partial y}\left(K\frac{\partial T}{\partial y}\right)+S=0$$

Integrating For x and y:

$$\int_s^n\left[\left(K\frac{\partial T}{\partial x}\right)_e-\left(K\frac{\partial T}{\partial x}\right)_w\right]dy+\int_w^e\left[\left(K\frac{\partial T}{\partial y}\right)_n-\left(K\frac{\partial T}{\partial y}\right)_s\right]dx+\int_s^n\int_w^e S\,dx\,dy\ =0$$

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e-\left(K\frac{\partial T}{\partial x}\right)_w\right]\Delta y+\left[\left(K\frac{\partial T}{\partial y}\right)_n-\left(K\frac{\partial T}{\partial y}\right)_s\right]\Delta\mathrm{x}+\mathrm{S}\,\Delta x\,\Delta y=0$$

S = $S_C + S_P T_P$

Discretized Equation:

$$\mathrm{K\Delta y}\frac{(\mathrm{T_E}-\mathrm{T_P})}{\delta\mathrm{x}}-\mathrm{K\Delta y}\frac{(\mathrm{T_P}-\mathrm{T_W})}{\delta\mathrm{x}}+\mathrm{K\Delta x}\frac{(\mathrm{T_N}-\mathrm{T_P})}{\delta y}-\mathrm{K\Delta x}\frac{(\mathrm{T_P}-\mathrm{T_S})}{\delta y}+(S_C+S_PT_P)\Delta\mathrm{x}\,\Delta\mathrm{y}=0$$

Comparing with :

$$a_PT_P=a_ET_E+a_WT_W+a_NT_N+a_ST_S+b$$

We get:

$a_E = \frac{K\Delta y}{\delta x}$ $\qquad$ $a_W = \frac{K\Delta y}{\delta x}$

$a_N = \frac{K\Delta x}{\delta y}$ $\qquad$ $a_S = \frac{K\Delta x}{\delta y}$

$a_p = a_E + a_W + a_N + a_S - S_P \Delta x \Delta y$

$b = S_C \Delta x \, \Delta y$

Boundary Condition

- Edge Boundary Condition (Half control volume)
- Corner Boundary Condition (Quarter control volume)

Let 'B' be the boundary node and 'T_B' be its temperature.

The derivation and solutions for all the four edge boundary conditions are as follows:

1. **For the Top boundary,**

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\frac{\Delta y}{2} + \left[q_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\Delta x + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$q_n \Delta x - K\Delta x\frac{(T_B - T_S)}{\delta y} + K\frac{\Delta y}{2}\frac{(T_E - T_B)}{\delta x} - K\frac{\Delta y}{2}\frac{(T_B - T_W)}{\delta x} + (S_C + S_P T_P)\Delta x\frac{\Delta y}{2} = 0$$

Comparing with : $\qquad a_B T_B = a_E T_E + a_W T_W + a_S T_S + b$

We get:

$a_E = \frac{K\Delta y}{2\delta x}$ $\qquad$ $a_W = \frac{K\Delta y}{2\delta x}$ $\qquad$ $a_S = \frac{K\Delta x}{\delta y}$

$a_p = a_E + a_W + a_S - S_P \Delta x\frac{\Delta y}{2}$ $\qquad$ $b = S_C \Delta x\frac{\Delta y}{2} + q_n \Delta x$

2. **For the Bottom boundary,**

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\frac{\Delta y}{2} + \left[\left(K\frac{\partial T}{\partial y}\right)_n - q_s\right]\Delta x + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$K\Delta x\frac{(T_N - T_B)}{\delta y} - q_s\Delta x + K\frac{\Delta y}{2}\frac{(T_E - T_B)}{\delta x} - K\frac{\Delta y}{2}\frac{(T_B - T_W)}{\delta x} + (S_C + S_P T_P)\Delta x\frac{\Delta y}{2} = 0$$

Comparing with : $a_B T_B = a_E T_E + a_W T_W + a_N T_N + b$

We get:

$a_E = \frac{K\Delta y}{2\delta}$ $\quad a_W = \frac{K\Delta y}{2\delta x}$ $\quad a_N = \frac{K\Delta x}{\delta y}$

$a_p = a_E + a_W + a_N - S_P\Delta x\frac{\Delta y}{2}$ $\quad b = S_C\Delta x\frac{\Delta y}{2} + q_s\Delta x$

3. **For the right boundary,**

$$\left[q_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\Delta y + \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\frac{\Delta x}{2} + S\frac{\Delta x}{2}\Delta y = 0$$

$$S = S_C + S_P T_P$$

$$q_e\Delta y - K\Delta y\frac{(T_B - T_W)}{\delta x} + K\frac{\Delta x}{2}\frac{(T_N - T_B)}{\delta y} - K\frac{\Delta x}{2}\frac{(T_B - T_S)}{\delta y} + (S_C + S_P T_P)\frac{\Delta x}{2}\Delta y = 0$$

Comparing with : $a_B T_B = a_E T_E + a_W T_W + a_N T_N + b$

We get:

$a_E = \frac{K\Delta y}{2\delta x}$ $\quad a_W = \frac{K\Delta y}{2\delta x}$ $\quad a_N = \frac{K\Delta x}{\delta y}$

$a_p = a_E + a_W + a_N - S_P\Delta x\frac{\Delta y}{2}$ $\quad b = S_C\Delta x\frac{\Delta y}{2} + q_e\Delta y$

4. **For the left boundary,**

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - q_w\right]\Delta y + \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\frac{\Delta x}{2} + S\frac{\Delta x}{2}\Delta y = 0$$

$$S = S_C + S_P T_P$$

$$K\Delta y\frac{(T_E - T_B)}{\delta x} - q_e\Delta y + K\frac{\Delta x}{2}\frac{(T_N - T_B)}{\delta y} - K\frac{\Delta x}{2}\frac{(T_B - T_S)}{\delta y} + (S_C + S_P T_P)\frac{\Delta x}{2}\Delta y = 0$$

Comparing with : $a_B T_B = a_E T_E + a_W T_W + a_N T_N + b$

We get:

$a_E = \frac{K\Delta y}{2\delta x}$ $\quad a_W = \frac{K\Delta y}{2\delta x}$ $\quad a_N = \frac{K\Delta x}{\delta y}$

$a_p = a_E + a_W + a_N - S_P\Delta x\frac{\Delta y}{2}$ $\quad b = S_C\Delta x\frac{\Delta y}{2} + q_e\Delta y$

The derivation and solutions for all the four corner boundary conditions are as follows:

1. **For the Top-right corner,**

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - q_w\right]\frac{\Delta y}{2} + \left[q_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\frac{\Delta x}{2} + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$K\frac{\Delta y}{2}\frac{(T_E - T_B)}{\delta x} - q_w\frac{\Delta y}{2} + q_n\frac{\Delta x}{2} - K\frac{\Delta x}{2}\frac{(T_B - T_S)}{\delta y} + (S_C + S_P T_P)\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Comparing with : $a_B T_B = a_E T_E + a_S T_S + b$

We get:

$a_E = \frac{K\Delta y}{2\delta}$ $a_S = \frac{K\Delta x}{2\delta y}$

$a_p = a_E + a_S - S_P\frac{\Delta x}{2}\frac{\Delta y}{2}$ $b = S_C\frac{\Delta x}{2}\frac{\Delta y}{2} - q_w\frac{\Delta y}{2} + q_n\frac{\Delta x}{2}$

2. **For the Top-left corner,**

$$\left[q_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\frac{\Delta y}{2} + \left[q_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\frac{\Delta x}{2} + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$q_e\frac{\Delta y}{2} - K\frac{\Delta y}{2}\frac{(T_B - T_W)}{\delta x} + q_n\frac{\Delta x}{2} - K\frac{\Delta x}{2}\frac{(T_B - T_S)}{\delta y} + (S_C + S_P T_P)\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Comparing with : $a_B T_B = a_W T_W + a_S T_S + b$

We get:

$a_W = \frac{K\Delta y}{2\delta x}$ $a_S = \frac{K\Delta x}{2\delta y}$

$a_p = a_W + a_S - S_P\frac{\Delta x}{2}\frac{\Delta y}{2}$ $b = S_C\frac{\Delta x}{2}\frac{\Delta y}{2} + q_e\frac{\Delta y}{2} + q_n\frac{\Delta x}{2}$

3. For Bottom-right corner,

$$\left[q_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\frac{\Delta y}{2} + \left[\left(K\frac{\partial T}{\partial y}\right)_n - q_s\right]\frac{\Delta x}{2} + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$q_e\frac{\Delta y}{2} - K\frac{\Delta y}{2}\frac{(T_B - T_W)}{\delta x} + K\frac{\Delta x}{2}\frac{(T_N - T_B)}{\delta y} - q_s\frac{\Delta x}{2} + (S_C + S_P T_P)\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Comparing with : $a_B T_B = a_W T_W + a_N T_N + b$

We get:

$a_W = \frac{K\Delta y}{2\delta x}$ $a_N = \frac{K\Delta x}{2\delta y}$

$a_p = a_W + a_N - S_P\frac{\Delta x}{2}\frac{\Delta y}{2}$ $b = S_C\frac{\Delta x}{2}\frac{\Delta y}{2} + q_e\frac{\Delta y}{2} - q_s\frac{\Delta x}{2}$

4. For Bottom-left corner,

$$\left[\left(K\frac{\partial T}{\partial y}\right)_e - q_w\right]\frac{\Delta y}{2} + \left[\left(K\frac{\partial T}{\partial y}\right)_n - q_s\right]\frac{\Delta x}{2} + S\,\Delta x\frac{\Delta y}{2} = 0$$

$$S = S_C + S_P T_P$$

$$K\frac{\Delta y}{2}\frac{(T_E - T_B)}{\delta x} - q_w\frac{\Delta y}{2} + K\frac{\Delta x}{2}\frac{(T_N - T_B)}{\delta y} - q_s\frac{\Delta x}{2} + (S_C + S_P T_P)\frac{\Delta x}{2}\frac{\Delta y}{2} = 0$$

Comparing with : $a_B T_B = a_E T_E + a_N T_N + b$

We get:

$a_E = \frac{K\Delta y}{2\delta x}$ $a_N = \frac{K\Delta x}{2\delta}$

$a_p = a_E + a_N - S_P\frac{\Delta x}{2}\frac{\Delta y}{2}$ $b = S_C\frac{\Delta x}{2}\frac{\Delta y}{2} - q_w\frac{\Delta y}{2} - q_s\frac{\Delta x}{2}$

All the boundary conditions collectively:

Edge Boundary Condition Equations:

1. $q_n \Delta \mathrm{x} - K \Delta \mathrm{x} \frac{(T_{\mathrm{B}} - T_{\mathrm{S}})}{\delta y} + K \frac{\Delta \mathrm{y}}{2} \frac{(T_E - T_{\mathrm{B}})}{\delta x} - K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{B}} - T_W)}{\delta x} + (S_C + S_P T_P) \Delta \mathrm{x} \frac{\Delta \mathrm{y}}{2} = 0$
2. $K \Delta \mathrm{x} \frac{(T_{\mathrm{N}} - T_{\mathrm{B}})}{\delta y} - q_s \Delta \mathrm{x} + K \frac{\Delta \mathrm{y}}{2} \frac{(T_E - T_{\mathrm{B}})}{\delta x} - K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{B}} - T_W)}{\delta x} + (S_C + S_P T_P) \Delta \mathrm{x} \frac{\Delta \mathrm{y}}{2} = 0$
3. $q_e \Delta \mathrm{y} - K \Delta \mathrm{y} \frac{(T_{\mathrm{B}} - T_{\mathrm{W}})}{\delta x} + K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{N}} - T_{\mathrm{B}})}{\delta y} - K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{S}})}{\delta y} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \Delta \mathrm{y} = 0$
4. $K \Delta \mathrm{y} \frac{(T_{\mathrm{E}} - T_{\mathrm{B}})}{\delta x} - q_e \Delta \mathrm{y} + K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{N}} - T_{\mathrm{B}})}{\delta y} - K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{S}})}{\delta y} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \Delta \mathrm{y} = 0$

Corner Boundary Condition Equations:

1. $K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{E}} - T_{\mathrm{B}})}{\delta x} - q_w \frac{\Delta \mathrm{y}}{2} + q_n \frac{\Delta \mathrm{x}}{2} - K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{S}})}{\delta y} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \frac{\Delta \mathrm{y}}{2} = 0$
2. $q_e \frac{\Delta \mathrm{y}}{2} - K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{W}})}{\delta x} + q_n \frac{\Delta \mathrm{x}}{2} - K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{S}})}{\delta y} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \frac{\Delta \mathrm{y}}{2} = 0$
3. $q_e \frac{\Delta \mathrm{y}}{2} - K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{B}} - T_{\mathrm{W}})}{\delta x} + K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{N}} - T_{\mathrm{B}})}{\delta y} - q_s \frac{\Delta \mathrm{x}}{2} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \frac{\Delta \mathrm{y}}{2} = 0$
4. $K \frac{\Delta \mathrm{y}}{2} \frac{(T_{\mathrm{E}} - T_{\mathrm{B}})}{\delta x} - q_w \frac{\Delta \mathrm{y}}{2} + K \frac{\Delta \mathrm{x}}{2} \frac{(T_{\mathrm{N}} - T_{\mathrm{B}})}{\delta y} - q_s \frac{\Delta \mathrm{x}}{2} + (S_C + S_P T_P) \frac{\Delta \mathrm{x}}{2} \frac{\Delta \mathrm{y}}{2} = 0$

2D Conduction and Convection

$\dot{Q}_{conv} = 2\ \Delta x \Delta y\ h\ (T - T_\infty)$

$\dot{Q}_{x,cond} = -KA_x \frac{dT}{dx}$ $\qquad\qquad$ $\dot{Q}_{y,cond} = -KA_y \frac{dT}{dy}$

Finally, the general equation comes out to be:

$$\frac{\partial}{\partial x}\left(KA_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(KA_y \frac{\partial T}{\partial y}\right) - 2\mathrm{h}(T - T_\infty) = 0$$

Integrating For x and y:

$$\int_s^n \left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right] dy + \int_w^e \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right] dx - \int_s^n \int_w^e 2\mathrm{h}(T - T_\infty)dx\,dy \ = 0$$

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right]\Delta y + \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right]\Delta \mathrm{x} - 2\mathrm{h}(T - T_\infty)\Delta x\ \Delta y = 0$$

Discretized Equation:

$$\mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta \mathrm{x}} - \mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta \mathrm{x}} + \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_N} - \mathrm{T_P})}{\delta y} - \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_P} - \mathrm{T_S})}{\delta y} - 2\mathrm{h}(T - T_\infty)\Delta\mathrm{x}\ \Delta\mathrm{y} = 0$$

Comparing with:

$$\mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta \mathrm{x}} - \mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta \mathrm{x}} + \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_N} - \mathrm{T_P})}{\delta y} - \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_P} - \mathrm{T_S})}{\delta y} + (S_C + S_P T_P)\Delta\mathrm{x}\ \Delta\mathrm{y} = 0$$

We get:

$S_C = 2hT_\infty$ and $S_P = -2h$

Rest all boundary conditions are the same by replacing the S_C and S_P.

2D Conduction and Radiation

$\dot{Q}_{radiation} = 2\,\Delta x \Delta y \varepsilon \sigma A(T^4 - T_\infty^4)$

$\dot{Q}_{x,cond} = -KA_x \frac{dT}{dx} \qquad \dot{Q}_{y,cond} = -KA_y \frac{dT}{dy}$

Finally, the general equation come out to be:

$$\frac{\partial}{\partial x}\left(KA_x \frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(KA_y \frac{\partial T}{\partial y}\right) - \varepsilon \sigma A(T^4 - T_\infty^4) = 0$$

Integrating For x and y:

$$\int_s^n \left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right] dy + \int_w^e \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right] dx - \int_s^n \int_w^e 2\varepsilon \sigma A(T^4 - T_\infty^4)\, dx\, dy \; = 0$$

$$\left[\left(K\frac{\partial T}{\partial x}\right)_e - \left(K\frac{\partial T}{\partial x}\right)_w\right] \Delta y + \left[\left(K\frac{\partial T}{\partial y}\right)_n - \left(K\frac{\partial T}{\partial y}\right)_s\right] \Delta \mathrm{x} - 2\varepsilon \sigma A(T^4 - T_\infty^4)\Delta x\, \Delta y = 0$$

Discretized Equation:

$$\mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta\mathrm{x}} - \mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta\mathrm{x}} + \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_N} - \mathrm{T_P})}{\delta y} - \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_P} - \mathrm{T_S})}{\delta y} - 2[\varepsilon\sigma\big(3\mathrm{T_P}^4 + \mathrm{T}_\infty^4\big) - 4\,\varepsilon\sigma\mathrm{T_P}^3\mathrm{T_P}]\Delta\mathrm{x}\,\Delta\mathrm{y} = 0$$

Comparing with:

$$\mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_E} - \mathrm{T_P})}{\delta\mathrm{x}} - \mathrm{K}\Delta\mathrm{y}\frac{(\mathrm{T_P} - \mathrm{T_W})}{\delta\mathrm{x}} + \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_N} - \mathrm{T_P})}{\delta y} - \mathrm{K}\Delta\mathrm{x}\frac{(\mathrm{T_P} - \mathrm{T_S})}{\delta y} + (S_C + S_P T_P)\Delta\mathrm{x}\,\Delta\mathrm{y} = 0$$

We get:

$S_C = 2\varepsilon\sigma\big(3\mathrm{T_P}^4 + \mathrm{T}_\infty^4\big)$ and $S_P = -8\,\varepsilon\sigma\mathrm{T_P}^3\mathrm{T_P}$

Rest all boundary conditions are same by replacing the S_C and S_P.

Coding Approach

The coding approach is structured as follows:

1. Problem Setup

- Define Physical Properties: Initialize the thermal conductivity.
- Define Geometry and Discretization: Set the dimensions of the PCB and discretize the spatial domain into a finite number of nodes.
- Initialize Temperature Distribution: Establish the initial temperature distribution across the PCB.
- Set Time Parameters: Determine the time step and total simulation time.

2. Discretization of the Heat Conduction Equation

- Finite Difference Method (FDM): Discretize the one-dimensional transient heat conduction equation using the finite difference method. This involves converting the partial differential equation into a system of algebraic equations.
- Spatial and Temporal Discretization: Use central difference for spatial derivatives and forward difference for the time derivative.

3. Implementation of Boundary Conditions

- Radiative Heat Loss: Incorporate radiative heat transfer at the boundaries by adding a term to account for radiative heat loss based on the surface emissivity and temperature.
- Heat Flux Inputs: Define and apply heat flux inputs at specified locations to simulate the heat generated by electronic components on the PCB.

4. Numerical Solution

- Iterative Solution: Implement an iterative scheme to solve the discretized equations at each time step. Depending on stability requirements, choose between an explicit or implicit method.
- Update Temperature Distribution: Update the temperature at each node based on the computed values from the previous time step.

5. Visualization and Analysis

- Temperature Contour Plots: Generate contour plots of the temperature distribution at different time intervals to visualize the thermal behavior of the PCB.
- Validation and Verification: Validate the numerical model against analytical solutions or experimental data, if available, to ensure accuracy.
- Parametric Studies: Conduct additional simulations to analyze the effects of different parameters, such as material properties, heat flux values, and boundary conditions.

6. Interpretation of Results

- Analyze Temperature Gradients and Hotspots: Examine the temperature contours to identify critical thermal characteristics, including hotspots and temperature gradients.

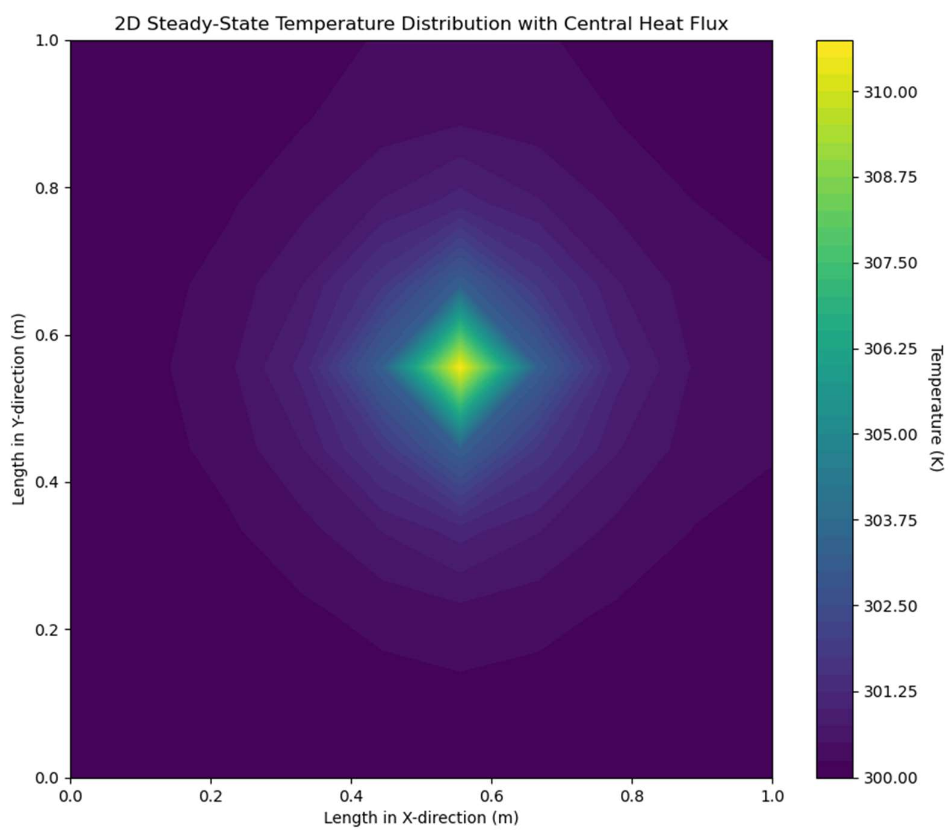
Que.

Plot the temperature distribution contour of a 2D PCB plate of arbitrary dimension in space (only conduction and radiation possible) with given heat flux at arbitrary locations.

Given:

Length (L_x)= 1m
Breadth(L_y) = 1m
Thickness(z) = 1mm
Number of nodes in x-direction (n_x)= 10
Number of nodes in y-direction (n_y)=10
Ambient Temperature (T_∞)= 300K
Thermal Conductivity (K)= 200 W/mK
Emissivity (ε)= 0.9
Heat flux at the centre (q)= 600 W/m^2

Solution:



(Not Exactly at the centre because of even number of nodes)

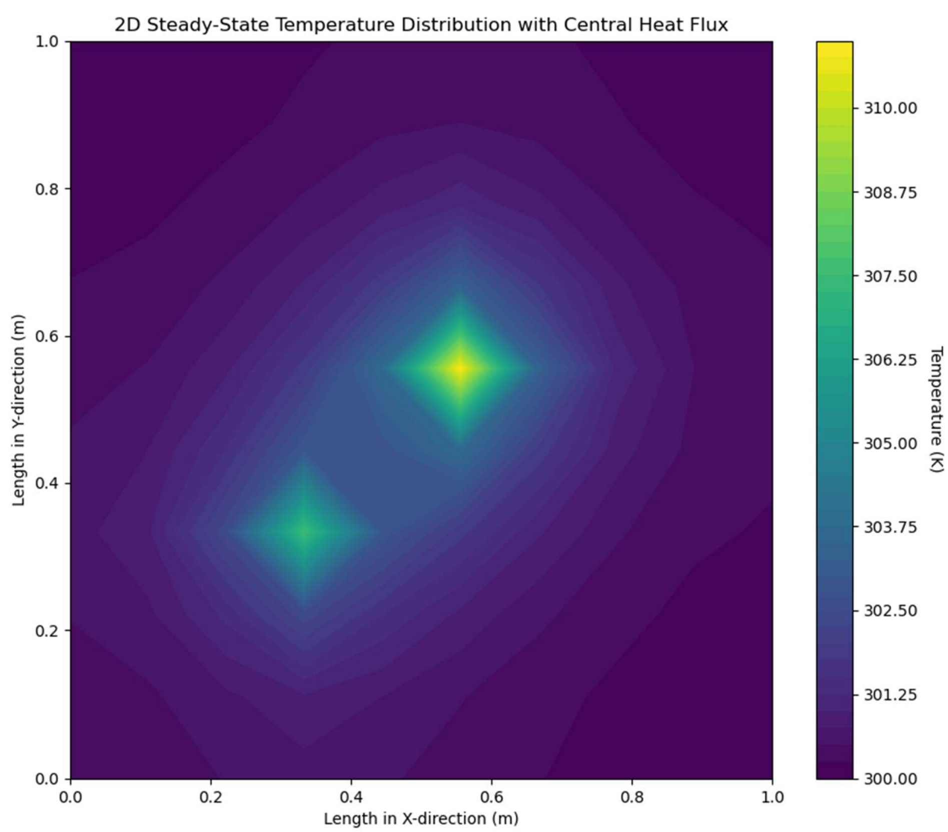
Que.

Plot the temperature distribution contour of a 2D PCB plate of arbitrary dimension in space (only conduction and radiation possible) with given heat flux at arbitrary locations.

Given:

Length (L_x)= 1m
Breadth(L_y) = 1m
Thickness(z) = 1mm
Number of nodes in x-direction (n_x)= 10
Number of nodes in y-direction (n_y)=10
Ambient Temperature (T_∞)= 300K
Thermal Conductivity (K)= 200 W/mK
Emissivity (ε)= 0.9
Heat flux at the centre (q)= 600 W/m^2
Heat flux at the $\frac{10}{3}, \frac{10}{3}$ (q)= 400 W/m^2

Solution:



Final Python Code:

```
import numpy as np

import matplotlib.pyplot as plt

from matplotlib.ticker import FormatStrFormatter

from tqdm import tqdm

import time


# Define parameters

Lx = 1  # Length in X-direction (m)

nx = 10  # Number of nodes in X-direction

dx = Lx / (nx - 1)  # Step size in X-direction

Ly = 1  # Length in Y-direction (m)

ny = 10  # Number of nodes in Y-direction

dy = Ly / (ny - 1)  # Step size in Y-direction

z = 0.001 #thickness(m)

Ta = 300  # Ambient temperature (K)

K = 200 # Thermal conductivity of aluminum (W/mK)

e = 0.9  # Emissivity

s = 5.67 * 10**(-8)  # Stefan-Boltzmann constant (W/m²K⁴)

q1 = 600 # Heat flux at the center (W/m²) - increased for more noticeable effect

q2 = 400 # Heat flux at the center (W/m²) - increased for more noticeable effect

relaxation_factor = 0.9  # Relaxation factor for convergence

convergence_criterion = 1e-5  # Convergence criterion


# Initialize temperature field

T = np.zeros([nx, ny]) + Ta


# Maximum number of iterations

max_iterations = 2000


# Initialize the progress bar

pbar = tqdm(total=max_iterations, desc="Iterating", unit="iteration")


# Start the timer
```

```
start_time = time.time()

# Iterate to solve the temperature field
converged = False
for l in range(max_iterations):
    T_old = T.copy()
    # Define coefficients
    aw = ae = K * z*dy / dx
    an = asn = K * z*dx / dy

    for j in range(1, ny - 1):
        for i in range(1, nx - 1):
            b = 2 * e * s * (3 * T_old[i, j]**4 + Ta**4) * dx * dy
            # Edge boundary conditions
            T[0, j] = (ae * T[1, j] + an * T[0, j + 1] * 0.5 + asn * T[0, j - 1] * 0.5 + b / 2) / (asn * 0.5 + an * 0.5 + ae + 4 * e * s * T_old[i, j]**3 * dx * dy)
            T[i, 0] = (ae * T[i + 1, 0] * 0.5 + aw * T[i - 1, 0] * 0.5 + an * T[i, 1] + b / 2) / (aw * 0.5 + an + ae * 0.5 + 4 * e * s * T_old[i, j]**3 * dx * dy)
            T[nx - 1, j] = (aw * T[nx - 2, j] + an * T[nx - 1, j + 1] * 0.5 + asn * T[nx - 1, j - 1] * 0.5 + b / 2) / (aw + an * 0.5 + asn * 0.5 + 4 * e * s * T_old[i, j]**3 * dx * dy)
            T[i, ny - 1] = (ae * T[i + 1, ny - 1] * 0.5 + aw * T[i - 1, ny - 1] * 0.5 + asn * T[i, ny - 2] + b / 2) / (aw * 0.5 + ae * 0.5 + asn + 4 * e * s * T_old[i, j]**3 * dx * dy)

            # Corner boundary conditions
            T[0, 0] = (ae * T[1, 0] * 0.5 + asn * T[0, 1] * 0.5 + b / 4) / (ae * 0.5 + asn * 0.5 + 2 * e * s * T_old[i, j]**3 * dx * dy)
            T[0, ny - 1] = (ae * T[1, ny - 1] * 0.5 + asn * T[0, ny - 2] * 0.5 + b / 4) / (ae * 0.5 + asn * 0.5 + 2 * e * s * T_old[i, j]**3 * dx * dy)
            T[nx - 1, 0] = (aw * T[nx - 2, 0] * 0.5 + an * T[nx - 1, 1] * 0.5 + b / 4) / (aw * 0.5 + an * 0.5 + 2 * e * s * T_old[i, j]**3 * dx * dy)
            T[nx - 1, ny - 1] = (aw * T[nx - 2, ny - 1] * 0.5 + asn * T[nx - 1, ny - 2] * 0.5 + b / 4) / (aw * 0.5 + asn * 0.5 + 2 * e * s * T_old[i, j]**3 * dx * dy)

            # Bulk equation
            ap = aw + ae + an + asn + 8 * e * s * T_old[i, j]**3 * dx * dy

            # Apply heat flux at the center
            if i == nx // 2 and j == ny // 2:
```

```
            T[i, j] = (ae * T[i + 1, j] + aw * T[i - 1, j] + an * T[i, j + 1] + asn * T[i, j - 1] + b + q1* dx * dy) / ap
          elif i == nx // 3 and j == ny // 3:
            T[i, j] = (ae * T[i + 1, j] + aw * T[i - 1, j] + an * T[i, j + 1] + asn * T[i, j - 1] + b + q2 * dx * dy) / ap
          else:
            T[i, j] = (ae * T[i + 1, j] + aw * T[i - 1, j] + an * T[i, j + 1] + asn * T[i, j - 1] + b) / ap

          # Apply relaxation factor
          T[i, j] = relaxation_factor * T[i, j] + (1 - relaxation_factor) * T_old[i, j]

    # Update the progress bar
    pbar.update(1)

    # Check for convergence
    if np.max(np.abs(T - T_old)) < convergence_criterion:
      converged = True
      print(f"Converged after {l} iterations.")
      break

  # Close the progress bar
  pbar.close()

  # Check if the solution converged
  if not converged:
    print("Error: Solution did not converge within the maximum number of iterations.")

  # Print the final temperature field
  print(T)

  # Create a meshgrid for plotting
  x = np.linspace(0, Lx, nx)
  y = np.linspace(0, Ly, ny)
  X, Y = np.meshgrid(x, y)

  # Plot the temperature field
```

```python
plt.figure(figsize=(10, 10))
contour = plt.contourf(X, Y, T, levels=50, cmap='viridis')
cbar = plt.colorbar(contour)
cbar.set_label('Temperature (K)', rotation=270, labelpad=15)
plt.xlabel("Length in X-direction (m)")
plt.ylabel("Length in Y-direction (m)")
plt.title("2D Steady-State Temperature Distribution with Central Heat Flux")
plt.grid(False)
plt.show()
```

References:

1. ***Numerical Heat Transfer and fluid flow*** *by Suhas V. Patankar*
2. ***Heat and mass Transfer*** *by Yunus A. Çengel*
3. ***ChatGPT***