5.14 The percentage of wins for the Chicago Bulls basketball team going into the playoffs for the 1996–97 season was 87.7. Round the 87.7 to 90 in order to use Table A.1.

(a) What is the probability that the Bulls sweep (4-0) the initial best-of-7 playoff series?

(b) What is the probability that the Bulls win the initial best-of-7 playoff series?

(c) What very important assumption is made in answering parts (a) and (b)?

(a)

$$(0.9)^4 = 0.6561$$

(b)

$$\binom{4}{4}(0.9)^4 + \binom{4}{3}(0.9)^4(0.1) + \binom{5}{3}(0.9)^4(0.1)^2 + \binom{6}{3}(0.9)^4(0.1)^3$$

$$= 0.6561 + 0.26244 + 0.06561 + 0.013122$$

$$= 0.99727$$

(c)

the probability that the Bull win is always 0.9

5.26 Assuming that 6 in 10 automobile accidents are due mainly to a speed violation, find the probability that among 8 automobile accidents, 6 will be due mainly to a speed violation

(a) by using the formula for the binomial distribution;

(b) by using Table A.1.

(a)

$$P(X=6) = \binom{8}{6}(0.6)^6(0.4)^2$$

$$= 28(0.6)^6(0.4)^2$$

$$= 0.2090$$

(b)

$$P(X=6) = P(X\leq 6) - P(X\leq 5)$$

$$= 0.8936 - 0.6846$$

$$= 0.2090$$

5.50 Find the probability that a person flipping a coin gets

(a) the third head on the seventh flip;

(b) the first head on the fourth flip.

(a) $b^*(7;3,0.5) = \binom{6}{2}(0.5)^3(0.5)^4$

$= 15\cdot\left(\frac{1}{2}\right)^7 = 0.1172$

(b) $g(4;1,0.5) = \left(\frac{1}{2}\right)\left(\frac{1}{2}\right)^3 = \left(\frac{1}{2}\right)^4$

$= \frac{1}{16}$

5.56 On average, 3 traffic accidents per month occur at a certain intersection. What is the probability that in any given month at this intersection

(a) exactly 5 accidents will occur?

(b) fewer than 3 accidents will occur?

(c) at least 2 accidents will occur?

(a)

$$P(X=5) = \frac{e^{-3}\cdot(3)^5}{5!}$$

$$= \sum_{x=0}^{5} p(x;3) - \sum_{x=0}^{4} P(x;3)$$

$$= 0.9161 - 0.8153$$

$$= 0.1008$$

(b)

$$P(X<3) = P(2;3)$$

$$= \sum_{x=0}^{2} P(x;3)$$

$$= 0.4232$$

(c)

$$P(X \geq 2) = 1 - P(X \leq 1)$$

$$= 1 - \sum_{x=0}^{1} P(x;3)$$

$$= 1 - 0.1991$$

$$= 0.8009$$

5.80 Service calls come to a maintenance center according to a Poisson process, and on average, 2.7 calls are received per minute. Find the probability that

(a) no more than 4 calls come in any minute;

(b) fewer than 2 calls come in any minute;

(c) more than 10 calls come in a 5-minute period.

(a)

$$P(X \leq 4) = \sum_{x=0}^{4} P(x; 2.7)$$

$$= 0.8629$$

(b)

$$P(X < 2) = \sum_{x=0}^{1} P(x; 2.7)$$

$$= 0.2487$$

(c) $\lambda t = 5 \cdot 2.7 = 13.5$

$$P(X > 10) = 1 - P(X \leq 10)$$

$$= 1 - \sum_{x=0}^{10} P(x; 13.5)$$

$$= 0.7888$$

Matlab

1 (a) b(3; 5, 0.2) = 0.0512

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HW5_2_a.m | HW5_2_b.m | HW5_1_a.m | HW5_1_b.m | HW5_1_c.m | HW5_1_d.m | HW5_1_e.m

```
x = input("number of successes(x) = ");
n = input("number of trials(n) = ") ;
p = input("probability of success(p) = ");
myBinomial(x, n, p) %call the function
function b = myBinomial(k, n, p)
    if k <= 0
        b = p^k * (1-p)^(n-k); % do the function
    else
        b = nchoosek(n, k) * p^k * (1-p)^(n-k); % do the function
    end
end
```

Command Window

```
>> HW5_2_a
>> HW5_2_b
>> HW5_1_a
number of successe(x) =
Operation terminated by user during HW5_1_a


>> HW5_1_a
number of successes(x) = 3
number of trials(n) = 5
probability of success(p) = 0.2

ans =

    0.0512

fx >>
```

1 (b) P(2; 6.5) = 0.0318

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HW5_2_a.m | HW5_2_b.m | HW5_1_a.m | HW5_1_b.m | HW5_1_c.m | HW5_1_d.m | HW5_1_e.m

```
x = input("number of event(x) = ");
p = input("average number of events(lambda) = ");
myPoisson(x, p) %call the function
function p = myPoisson(k, lambda)
% lambda: average number of events
% k: number of events
    p = exp(-lambda) * (lambda^k / factorial(k)); % The formula
end
```

Command Window

```
>> HW5_1_a
number of successes(x) = 3
number of trials(n) = 5
probability of success(p) = 0.2

ans =

    0.0512

>> HW5_1_b
number of event =
Operation terminated by user during HW5_1_b


>> HW5_1_b
number of event(x) = 2
average number of events(lambda) = 6.5

ans =

    0.0318
```

1(c)

n	r	p = 0.10	p = 0.20	p = 0.25	p = 0.30	p = 0.40	p = 0.50	p = 0.60	p = 0.70	p = 0.80	p = 0.90
1	0	0.9	0.75	0.8	0.7	0.6	0.5	0.4	0.3	0.2	0.1
	1	1	1	1	1	1	1	1	1	1	1
2	0	0.81	0.5625	0.64	0.49	0.36	0.25	0.16	0.09	0.04	0.01
	1	0.99	0.9375	0.96	0.91	0.84	0.75	0.64	0.51	0.36	0.19
	2	1	1	1	1	1	1	1	1	1	1
3	0	0.729	0.42188	0.512	0.343	0.216	0.125	0.064	0.027	0.008	0.001
	1	0.972	0.84375	0.896	0.784	0.648	0.5	0.352	0.216	0.104	0.028
	2	0.999	0.98438	0.992	0.973	0.936	0.875	0.784	0.657	0.488	0.271
	3	1	1	1	1	1	1	1	1	1	1
4	0	0.6561	0.31641	0.4096	0.2401	0.1296	0.0625	0.0256	0.0081	0.0016	0.0001
	1	0.9477	0.73828	0.8192	0.6517	0.4752	0.3125	0.1792	0.0837	0.0272	0.0037
	2	0.9963	0.94922	0.9728	0.9163	0.8208	0.6875	0.5248	0.3483	0.1808	0.0523
	3	0.9999	0.99609	0.9984	0.9919	0.9744	0.9375	0.8704	0.7599	0.5904	0.3439
	4	1	1	1	1	1	1	1	1	1	1
5	0	0.59049	0.2373	0.32768	0.16807	0.07776	0.03125	0.01024	0.00243	0.00032	1e-05
	1	0.91854	0.63281	0.73728	0.52822	0.33696	0.1875	0.08704	0.03078	0.00672	0.00046
	2	0.99144	0.89648	0.94208	0.83692	0.68256	0.5	0.31744	0.16308	0.05792	0.00856
	3	0.99954	0.98438	0.99328	0.96922	0.91296	0.8125	0.66304	0.47178	0.26272	0.08146
	4	0.99999	0.99902	0.99968	0.99757	0.98976	0.96875	0.92224	0.83193	0.67232	0.40951
	5	1	1	1	1	1	1	1	1	1	1
6	0	0.53144	0.17798	0.26214	0.11765	0.046656	0.015625	0.004096	0.000729	6.4e-05	1e-06
	1	0.88574	0.53394	0.65536	0.42017	0.23328	0.10938	0.04096	0.010935	0.0016	5.5e-05
	2	0.98415	0.83057	0.90112	0.74431	0.54432	0.34375	0.1792	0.07047	0.01696	0.00127
	3	0.99873	0.9624	0.98304	0.92953	0.8208	0.65625	0.45568	0.25569	0.09888	0.01585
	4	0.99995	0.99536	0.9984	0.98906	0.95904	0.89062	0.76672	0.57983	0.34464	0.11426
	5	1	0.99976	0.99994	0.99927	0.9959	0.98438	0.95334	0.88235	0.73786	0.46856
	6	1	1	1	1	1	1	1	1	1	1
7	0	0.4783	0.13348	0.20972	0.082354	0.027994	0.0078125	0.0016384	0.0002187	1.28e-05	1e-07
	1	0.85031	0.44495	0.57672	0.32942	0.15863	0.0625	0.018842	0.0037908	0.0003712	6.4e-06
	2	0.97431	0.75641	0.85197	0.64707	0.4199	0.22656	0.096256	0.028796	0.004672	0.0001765
	3	0.99727	0.92944	0.96666	0.87396	0.71021	0.5	0.28979	0.12604	0.033344	0.002728
	4	0.99982	0.98712	0.99533	0.9712	0.90374	0.77344	0.5801	0.35293	0.14803	0.025691
	5	0.99999	0.99866	0.99963	0.99621	0.98116	0.9375	0.84137	0.67058	0.42328	0.14969

1(d)

r	m = 5.5	m = 6.0	m = 6.5	m = 7.0	m = 7.5	m = 8.0	m = 8.5	m = 9.0	m = 9.5
0	0.0040868	0.0024788	0.0015034	0.00091188	0.00055308	0.00033546	0.00020347	0.00012341	7.4852e-05
1	0.026564	0.017351	0.011276	0.0072951	0.0047012	0.0030192	0.0019329	0.0012341	0.00078594
2	0.088376	0.061969	0.043036	0.029636	0.020257	0.013754	0.0092832	0.0062322	0.0041636
3	0.2017	0.1512	0.11185	0.081765	0.059145	0.04238	0.030109	0.021226	0.01486
4	0.35752	0.28506	0.22367	0.17299	0.13206	0.099632	0.074364	0.054964	0.040263
5	0.52892	0.44568	0.36904	0.30071	0.24144	0.19124	0.1496	0.11569	0.088528
6	0.68604	0.6063	0.52652	0.44971	0.37815	0.31337	0.25618	0.20678	0.16495
7	0.80949	0.74398	0.67276	0.59871	0.52464	0.45296	0.3856	0.3239	0.26866
8	0.89436	0.84724	0.79157	0.72909	0.66197	0.59255	0.52311	0.45565	0.39182
9	0.94622	0.91608	0.87738	0.8305	0.77641	0.71662	0.65297	0.58741	0.52183
10	0.97475	0.95738	0.93316	0.90148	0.86224	0.81589	0.76336	0.70599	0.64533
11	0.98901	0.97991	0.96612	0.94665	0.92076	0.88808	0.84866	0.80301	0.75199
12	0.99555	0.99117	0.98397	0.973	0.95733	0.9362	0.90908	0.87577	0.83643
13	0.99831	0.99637	0.9929	0.98719	0.97844	0.96582	0.94859	0.92615	0.89814
14	0.9994	0.9986	0.99704	0.99428	0.98974	0.98274	0.97257	0.95853	0.94001
15	0.9998	0.99949	0.99884	0.99759	0.99539	0.99177	0.98617	0.97796	0.96653
16	0.99994	0.99983	0.99957	0.99904	0.99804	0.99628	0.99339	0.98889	0.98227
17	0.99998	0.99994	0.99985	0.99964	0.99921	0.99841	0.997	0.99468	0.99107
18	0.99999	0.99998	0.99995	0.99987	0.9997	0.99935	0.9987	0.99757	0.99572
19	1	0.99999	0.99998	0.99996	0.99989	0.99975	0.99947	0.99894	0.99804
20	1	1	1	0.99999	0.99996	0.99991	0.99979	0.99956	0.99914
21	1	1	1	1	0.99999	0.99997	0.99992	0.99983	0.99964
22	1	1	1	1	1	0.99999	0.99997	0.99993	0.99985
23	1	1	1	1	1	1	0.99999	0.99998	0.99994
24	1	1	1	1	1	1	1	0.99999	0.99998

1(e)

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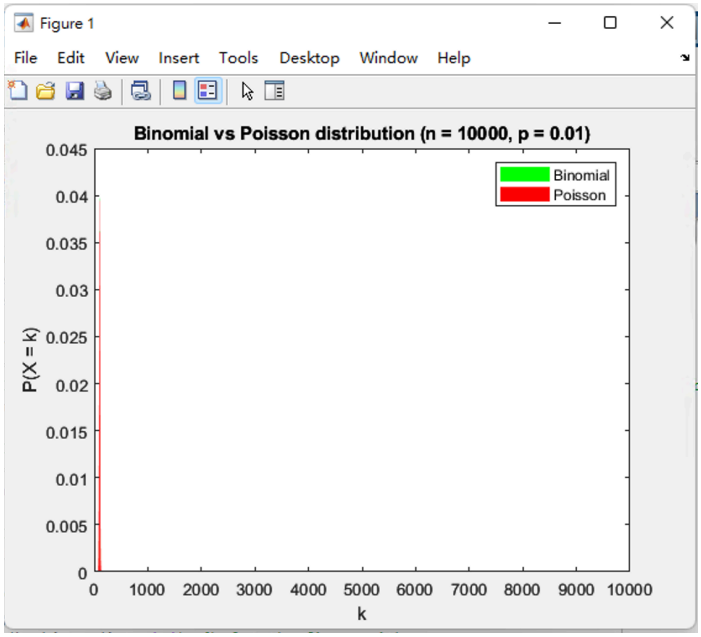
HW5_2_a.m | HW5_2_b.m | HW5_1_a.m | HW5_1_b.m | HW5_1_c.m | HW5_1_d.m | HW5_1_e.m

```
% Define parameters of the Binomial distributions
n_values = 10000;
p_values = [0.01, 0.1, 0.2, 0.5];

% Generate plots for each (n, p) condition
for i = 1:length(p_values)
    % Calculate the Poisson parameter lambda using the approximation from Binomial distribution
    lambda = n_values * p_values(i);

    % Calculate the Binomial distribution
    k = 0:n_values;
    binom_dist = binopdf(k, n_values, p_values(i));

    % Calculate the Poisson distribution
    poisson_dist = poisspdf(k, lambda);

    % Plot both distributions on the same figure
    figure();
    bar(k, binom_dist, 'g'); % plot the figure with green
```

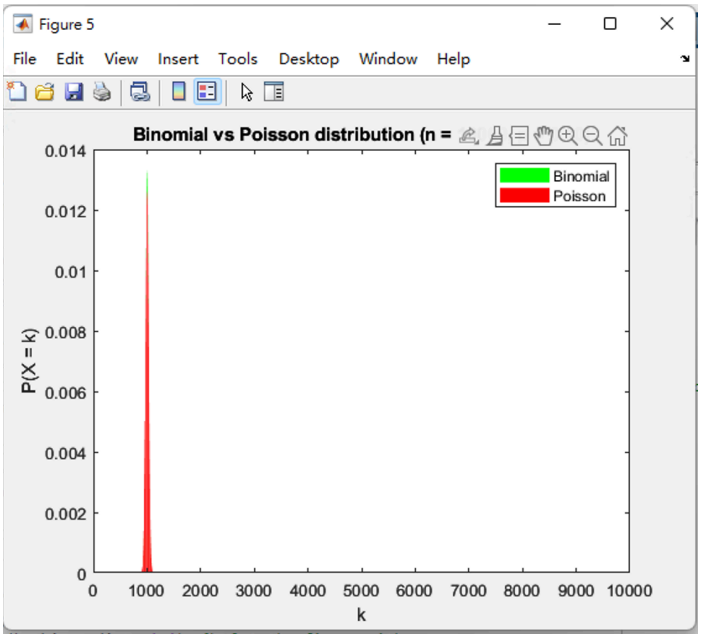
Command Window

```
    22          1          1          1          1          1    0.99999    0.99997
    23          1          1          1          1          1          1    0.99999
    24          1          1          1          1          1          1          1

>> HW5_1_e
For (n, p) = (10000, 0.01), the maximum absolute error is 0.0002
For (n, p) = (10000, 0.10), the maximum absolute error is 0.0007
For (n, p) = (10000, 0.20), the maximum absolute error is 0.0011
For (n, p) = (10000, 0.50), the maximum absolute error is 0.0023
```

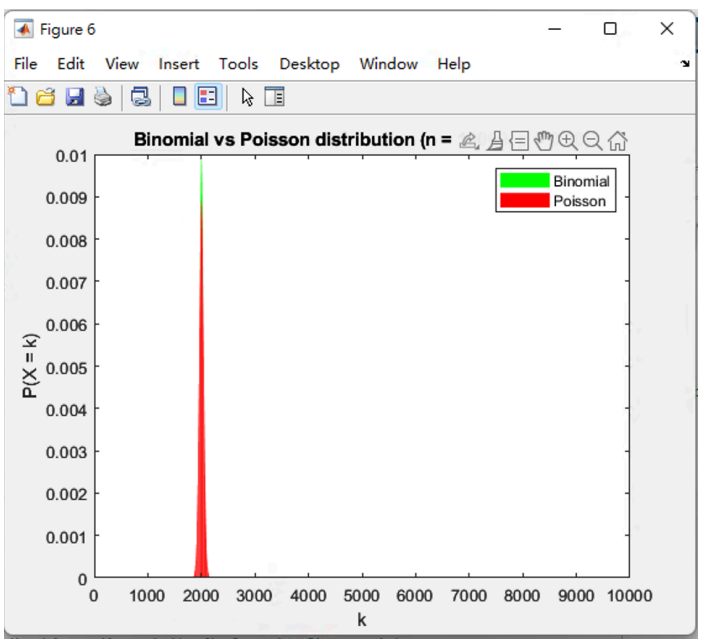
$(n,p) = (10^4, 0.01)$



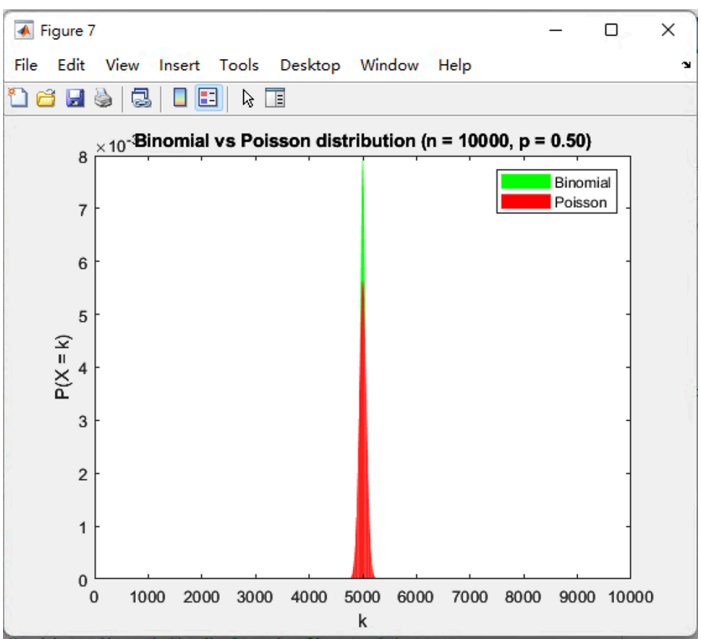
$(n,p) = (10^4, 0.1)$



$(n,p) = (10^4, 0.2)$

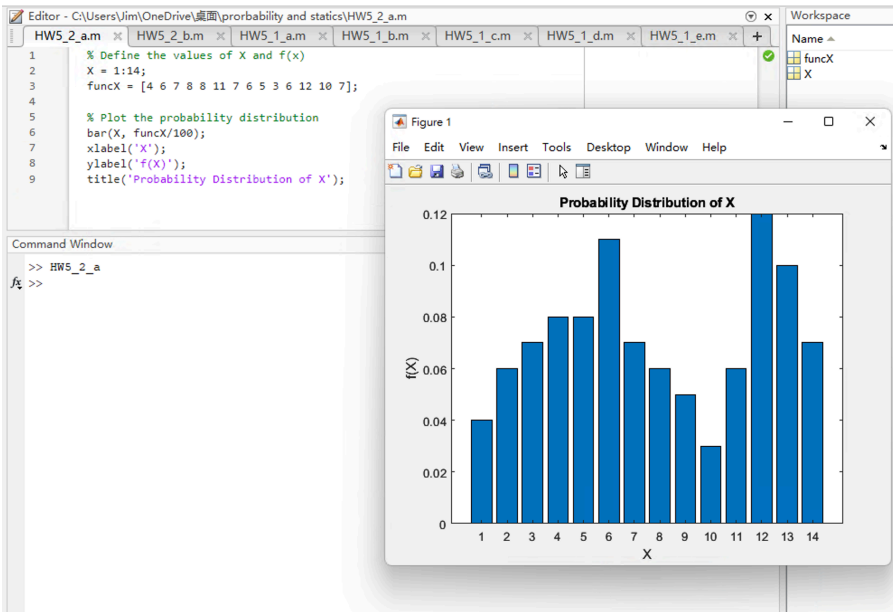


$(n,p) = (10^4, 0.5)$

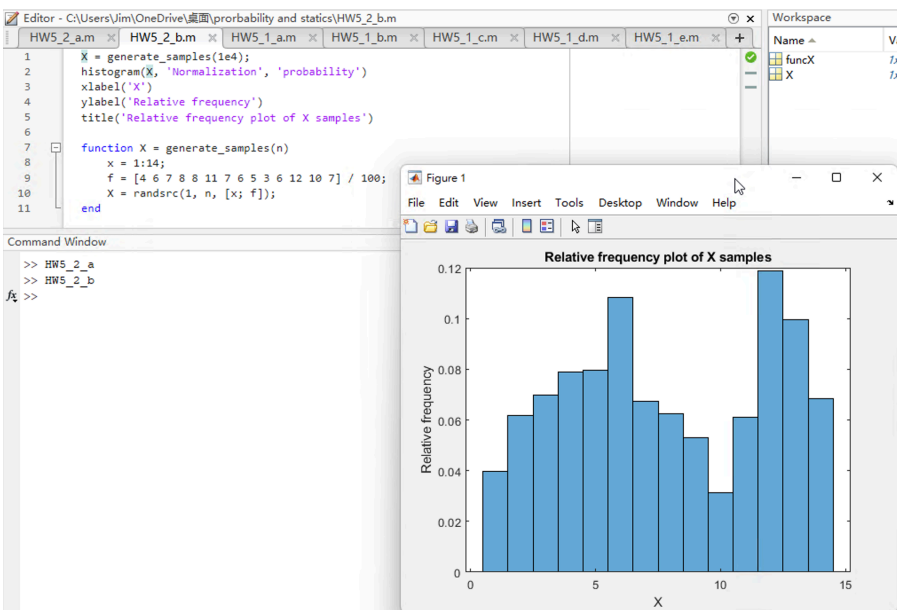


由上述四圖可發現 Binomial Distribution 的 $P(X=k)$ 之最大值皆大於 Poisson Distribution 的 $P(X=k)$ 最大值。另外，隨著 (n,p) 中 p 的值越小，兩者間的差距也會越來越小，產生最大值時的 k 值也越來越小

2(a)



2(b)



由結果可見，2(a) 和 2(b) 兩張圖十分的相似。
因為當樣本基數 ($X = 10^4$) 夠大的時候，會十分
趨進理論值，也就是 2(a) 的圖