Homework 4

Problem 1

(1 point) Find the variance of $X \sim Bin(n, p)$ using indicator random variables I_j such that $X = I_1 + I_2 + \dots + I_n$.

Problem 2

(1 point) Derive the Poisson expectation and variance from its PMF.

Problem 3

(1 point) Find the mode, median, and expected value of a random variable X with PDF $\varphi(x) = 3x^2$, where $x \in [0, 1]$. Note: mode, $Mo(X)$, is the "most probable" (in some sense) value of X, i. e. the maximum of PMF/PDF. The median, $Me(X)$, is $F^{-1}(1/2)$ for X with CDF F.

Problem 4

(1 point) Assume that the device repair time is a random variable $X \sim Expo(\lambda)$. Find the probability that the device repair will take at least 20 days if the average device repair time is 15 days.

Problem 5

(2 points) Consider the Negative Hypergeometric distribution with parameters w, b, and r: an urn contains w white balls and b black balls, which are randomly drawn one by one without replacement, until r white balls have been obtained. Assuming $r \leq w$, we say that the number of black balls drawn before drawing the r-th white ball $X \sim NHGeom(w, b, r)$. Find the $E(X)$ using indicator random variables.

Problem 6

(2 points) Suppose that Bernoulli trials are being performed in continuous time; i. e. the trials take place at points on a timeline. Assume that the trials are at regularly spaced times $0, \Delta t, 2\Delta t, \dots$, where t is a small positive number. Let the probability of success of each trial be $\lambda\Delta t$, where λ is a positive constant. Let G be the number of failures before the first success in discrete time, and T be the time of the first success (in continuous time).

a) Relate G to T and find the CDF of T.

b) Show that as $\Delta t \to 0$, the CDF of T converges to the $Expo(\lambda)$ CDF, evaluating all the CDFs at a fixed $t \geq 0$.