

Homework 5

Problem 1

(1 point) Find all moments of the normal distribution using MGF.

Problem 2

(1 point) For independent and identically distributed $X, Y \sim N(0, 1)$ find $E(|X - Y|)$. Use MGF of independent normal distributions.

Problem 3

(1 point) Let $X, Y \sim \text{Expo}(1)$ be independent and identically distributed random variables. Find the correlation between $\max(X, Y)$ and $\min(X, Y)$.

Problem 4

(1 point) Consider the Log-Normal distribution $Y \sim LN(\mu, \sigma^2)$, where $Y = e^X$ and $X \sim N(\mu, \sigma^2)$. Check that the MGF of the Log-Normal distribution doesn't exist. Despite this, obtain all moments of the Log-Normal, using the MGF of the Normal.

Problem 5

(2 points) The distribution function of a continuous random variable X , distributed according to the Cauchy law, is $F(x) = A + B \arctg \frac{x}{a}$ for $a > 0$.

Find the constants A and B , the PDF, the probability $P(-a \leq X \leq a)$. What are the mathematical expectation and variance of this random variable?

Problem 6

(2 points) Find the skewness $A = \frac{\mu_3}{\sigma^3}$ and kurtosis $E = \frac{\mu_4}{\sigma^4} - 3$ of a random variable distributed according to the Laplace law with a probability density function $\phi(x) = \frac{1}{2}e^{-|x|}$.