Homework 3

Problem 1

(1 point) Let $X \sim Unif(-1, 3)$. Find the PDF of $Y = X^2$.

Problem 2

(1 point) Let $X \sim Bin(n, p)$. Find the PMF of $Y = e^X$.

Problem 3

(1 point) Find the PMF of distribution of a random variable X, which is equal to the number of failures in a series of Bernoulli trials with success probability p. Trials are carried out not a fixed number of times, but instead until there are r successes. Do you remember what this distribution of a random variable is called? Note: be careful with the binomial coefficient.

Problem 4

(1 point) An airline overbooks a flight, selling more tickets for the flight than there are seats on the plane (figuring that it's likely that some people won't show up). The plane has 100 seats, and 110 people have booked the flight. Each person will show up for the flight with probability 0.9, independently. Find the probability that there will be enough seats for everyone who shows up for the flight.

Problem 5

(2 points) Prove that if $X \sim HGeom(w, b, n)$ and $N = w + b \to \infty$ such that $p = \frac{w}{(w+b)}$ remains fixed, then the PMF of X converges to the $Bin \sim (n, p)$ PMF.

Problem 6

(2 points) Consider two independent random variables $X \sim F_X(X)$ and $Y \sim F_Y(Y)$. Find the CDF of random variables $Z_1 = max(X, Y)$, meaning that for every outcome w we have $Z_1(w) = max\{X(w), Y(w)\}$, and $Z_2 = min(X, Y)$.