

# Homework 5

## Problem 1

(1 point) Find all moments of the normal distribution using MGF.

## Problem 2

(1 point) For independent and identically distributed  $X, Y \sim N(0, 1)$  find  $E(|X - Y|)$ . Use MGF of independent normal distributions.

## Problem 3

(1 point) Let  $X, Y \sim Expo(1)$  be independent and identically distributed random variables. Find the correlation between  $\max(X, Y)$  and  $\min(X, Y)$ .

## Problem 4

(1 point) Consider the Log-Normal distribution  $Y \sim LN(\mu, \sigma^2)$ , where  $Y = e^X$  and  $X \sim N(\mu, \sigma^2)$ . Check that the MGF of the Log-Normal distribution doesn't exist. Despite this, obtain all moments of the Log-Normal, using the MGF of the Normal.

## Problem 5

(2 points) The distribution function of a continuous random variable  $X$ , distributed according to the Cauchy law, is  $F(x) = A + B \operatorname{arctg} \frac{x}{a}$  for  $a > 0$ .

Find the constants  $A$  and  $B$ , the PDF, the probability  $P(-a \leq X \leq a)$ . What are the mathematical expectation and variance of this random variable?

## Problem 6

(2 points) Find the skewness  $A = \frac{\mu_3}{\sigma^3}$  and kurtosis  $E = \frac{\mu_4}{\sigma^4} - 3$  of a random variable distributed according to the Laplace law with a probability density function  $\phi(x) = \frac{1}{2}e^{-|x|}$ .