Principal Component Analysis (PCA)

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1. Primer paso: cargar las librerias que necesitas.

```
library(ggplot2)
library(dplyr)
library(missMDA) # Imputate
library(ggfortify) # autoplot()
library(cluster) #pam
library(factoextra) #get_pca_var()
library(data.table) # data.table()
library(devtools)
install_github("vqv/ggbiplot") #ggbiplot
library(ggbiplot)
```

2. Segundo paso: cargar los datos.

```
channel <- read.csv("data/channel_form.csv", header=TRUE)
head(channel)
```

```
##      Forma NAN_Am NADBO NAtemp  nit NASatO2 Elevacion Ancho Velocidad Rocas
## 1 Trapecio   0.03  2.38  27.33 0.35   92.04        23    16         5    20
## 2 Trapecio   0.03  2.95  27.81   NA  100.03        31    11         0    20
## 3 Trapecio   0.03  3.13  24.27   NA   96.82        35    14        10    30
## 4 Trapecio   1.15  4.73  27.06 7.54   64.35         9     5         2     0
## 5 Trapecio   0.50  8.16  26.60   NA  110.39        43    11         9    10
## 6 Trapecio   0.53  8.57  23.82   NA  106.09        23    11         5    20
##   Canto grava arena Limo
## 1    25    30    20    0
## 2    45    20    15    0
## 3    30    20    10    0
## 4     0     0    50   50
## 5    40    10    20   20
## 6    60    20     0    0
```

2.1 Vamos a examinar los datos

```
summary(channel)
```

```
##     Forma               NAN_Am           NADBO            NAtemp
##  Length:138         Min.   :0.0200   Min.   : 1.310   Min.   :14.67
```

```
##  Class :character   1st Qu.:0.0400   1st Qu.: 1.930   1st Qu.:24.30
##  Mode  :character   Median :0.2150   Median : 3.000   Median :26.05
##                     Mean   :0.3201   Mean   : 6.164   Mean   :25.84
##                     3rd Qu.:0.5000   3rd Qu.: 8.585   3rd Qu.:27.70
##                     Max.   :1.5000   Max.   :34.900   Max.   :32.18
##                                      NA's   :35
##       nit            NASatO2         Elevacion          Ancho
##  Min.   :  0.00   Min.   : 23.43   Min.   :   3.00   Min.   : 1.000
##  1st Qu.:  0.40   1st Qu.: 86.24   1st Qu.:  25.25   1st Qu.: 2.000
##  Median :  0.92   Median : 94.59   Median :  53.00   Median : 3.000
##  Mean   : 12.00   Mean   : 91.05   Mean   : 230.89   Mean   : 3.822
##  3rd Qu.:  1.62   3rd Qu.:100.52   3rd Qu.: 269.25   3rd Qu.: 3.000
##  Max.   :324.11   Max.   :122.73   Max.   :2370.00   Max.   :16.000
##  NA's   :57                                          NA's   :3
##    Velocidad          Rocas           Canto           grava
##  Min.   : 0.000   Min.   : 0.00   Min.   : 0.00   Min.   : 0.0
##  1st Qu.: 3.000   1st Qu.: 0.00   1st Qu.: 0.00   1st Qu.: 2.5
##  Median :11.000   Median :10.00   Median :25.00   Median :20.0
##  Mean   : 9.133   Mean   :16.25   Mean   :25.65   Mean   :17.8
##  3rd Qu.:14.000   3rd Qu.:30.00   3rd Qu.:40.00   3rd Qu.:25.0
##  Max.   :16.000   Max.   :90.00   Max.   :80.00   Max.   :80.0
##  NA's   :3        NA's   :3       NA's   :4       NA's   :3
##      arena            Limo
##  Min.   :  0.00   Min.   :  0.00
##  1st Qu.: 10.00   1st Qu.:  0.00
##  Median : 15.00   Median : 10.00
##  Mean   : 19.79   Mean   : 20.62
##  3rd Qu.: 25.00   3rd Qu.: 25.00
##  Max.   :100.00   Max.   :100.00
##  NA's   :3        NA's   :3
```

2.1 Remover la(s) variable(s) que tiene(n) mucho(s) NAs y las Etiquetas (a la funcion lo le gusta), luego las agregamos.

```
channel_1 <- select(channel, -Forma, -nit, -NADBO)
summary(channel_1)
```

```
##      NAN_Am           NAtemp         NASatO2         Elevacion
##  Min.   :0.0200   Min.   :14.67   Min.   : 23.43   Min.   :   3.00
##  1st Qu.:0.0400   1st Qu.:24.30   1st Qu.: 86.24   1st Qu.:  25.25
##  Median :0.2150   Median :26.05   Median : 94.59   Median :  53.00
##  Mean   :0.3201   Mean   :25.84   Mean   : 91.05   Mean   : 230.89
##  3rd Qu.:0.5000   3rd Qu.:27.70   3rd Qu.:100.52   3rd Qu.: 269.25
##  Max.   :1.5000   Max.   :32.18   Max.   :122.73   Max.   :2370.00
##
##      Ancho          Velocidad          Rocas           Canto
##  Min.   : 1.000   Min.   : 0.000   Min.   : 0.00   Min.   : 0.00
##  1st Qu.: 2.000   1st Qu.: 3.000   1st Qu.: 0.00   1st Qu.: 0.00
##  Median : 3.000   Median :11.000   Median :10.00   Median :25.00
##  Mean   : 3.822   Mean   : 9.133   Mean   :16.25   Mean   :25.65
##  3rd Qu.: 3.000   3rd Qu.:14.000   3rd Qu.:30.00   3rd Qu.:40.00
##  Max.   :16.000   Max.   :16.000   Max.   :90.00   Max.   :80.00
##  NA's   :3        NA's   :3        NA's   :3       NA's   :4
```

```
##      grava           arena             Limo
##  Min.   : 0.0   Min.   :  0.00   Min.   :  0.00
##  1st Qu.: 2.5   1st Qu.: 10.00   1st Qu.:  0.00
##  Median :20.0   Median : 15.00   Median : 10.00
##  Mean   :17.8   Mean   : 19.79   Mean   : 20.62
##  3rd Qu.:25.0   3rd Qu.: 25.00   3rd Qu.: 25.00
##  Max.   :80.0   Max.   :100.00   Max.   :100.00
##  NA's   :3      NA's   :3        NA's   :3
```

2.2 Vamos a imputar datos. Esto es comun para set de datos de campo, los cuales tienden a tener ceros (por mal funcionamiento de los equipos, condiciones climticas adversas que no puedemos ir al campo). Se realiza como un paso preliminar para para realizar un PCA en un set de datos completos.

Mas informacion aca: https://www.rdocumentation.org/packages/missMDA/versions/1.18/topics/imputePCA

```
channel_2 <- imputePCA(channel_1, ncp=2, scale = TRUE, method = c("Regularized","EM"),
                       row.w = NULL, ind.sup=NULL,quanti.sup=NULL,quali.sup=NULL,
                       coeff.ridge = 1, threshold = 1e-06, seed = NULL, nb.init = 1,
                       maxiter = 1000)

# class(channel_2)
 as.data.frame(channel_2)
```

```
##     completeObs.NAN_Am completeObs.NAtemp completeObs.NASatO2
## 1               0.0300              27.33               92.04
## 2               0.0300              27.81              100.03
## 3               0.0300              24.27               96.82
## 4               1.1500              27.06               64.35
## 5               0.5000              26.60              110.39
## 6               0.5300              23.82              106.09
## 7               0.1200              22.92              104.97
## 8               0.0500              27.24               82.73
## 9               0.0300              26.53               77.81
## 10              0.1000              23.85              100.97
## 11              0.2800              23.84              101.41
## 12              0.5000              21.92               95.65
## 13              0.5000              22.78               93.89
## 14              0.5000              26.10              101.27
## 15              0.0300              25.82               77.78
## 16              0.0300              26.93               84.61
## 17              0.0300              26.08               78.53
## 18              0.1600              26.13              101.74
## 19              0.2200              27.33              101.61
## 20              0.0300              15.68               80.77
## 21              0.1500              25.96              110.90
## 22              0.0300              28.73              107.50
## 23              0.0300              27.91               94.30
## 24              0.0400              29.24              100.20
## 25              0.0300              29.11               98.75
## 26              0.0300              27.52               96.70
## 27              0.0400              26.60               70.86
## 28              0.2800              28.96              107.91
## 29              0.5300              29.57              102.19
```

```
## 30          0.4100          28.04          108.53
## 31          0.5700          30.62          122.73
## 32          0.6400          28.76           91.06
## 33          0.5000          28.63           96.41
## 34          0.5000          29.35           96.81
## 35          0.7700          29.87           98.19
## 36          0.5500          27.32          113.66
## 37          0.5900          27.48          104.84
## 38          0.6100          25.59          100.15
## 39          0.7000          30.50           86.63
## 40          0.1800          27.31           75.13
## 41          0.6900          23.72           95.63
## 42          0.7400          23.16           96.42
## 43          0.9100          26.67           88.13
## 44          0.5000          25.59           94.67
## 45          0.5000          25.66           98.25
## 46          0.0400          26.83           76.25
## 47          0.9500          26.30           90.15
## 48          1.4400          26.90           85.65
## 49          1.0500          27.15           89.74
## 50          0.7700          26.02           93.99
## 51          0.8800          27.18           93.44
## 52          0.3900          24.00           94.52
## 53          0.3000          23.97           94.67
## 54          0.5000          25.92           89.17
## 55          0.5000          25.70           85.60
## 56          0.7100          28.17           88.76
## 57          0.5000          26.92           87.13
## 58          0.7400          27.70           93.87
## 59          0.5000          30.13           84.95
## 60          0.0700          24.38          102.07
## 61          0.0700          25.93          108.25
## 62          0.0600          24.31          104.89
## 63          0.0200          25.40          107.65
## 64          0.0400          25.82          109.81
## 65          0.0600          26.55          107.03
## 66          1.1600          25.44           93.92
## 67          0.0300          30.89          103.13
## 68          0.0300          26.67           88.36
## 69          0.5000          19.28           96.10
## 70          0.5000          22.50           98.99
## 71          0.5000          23.34           98.24
## 72          0.5000          18.20           98.95
## 73          0.5000          22.62           97.35
## 74          0.5000          24.69           99.36
## 75          0.5000          26.25           97.75
## 76          0.5000          25.09           98.15
## 77          0.0300          28.09           63.61
## 78          0.0800          29.00           39.55
## 79          0.0300          24.29           87.73
## 80          0.2900          28.13           94.16
## 81          0.6500          24.02           95.92
## 82          0.3800          24.19           94.77
## 83          0.3100          24.41           97.41
```

```
## 84          0.0500          25.60           96.95
## 85          0.1800          24.92          103.08
## 86          0.1000          24.79          102.53
## 87          0.5000          27.18           87.88
## 88          0.1632          32.18          110.47
## 89          0.2000          30.38           44.79
## 90          0.0400          28.97           86.18
## 91          0.0400          27.14           75.82
## 92          0.6100          29.07           98.27
## 93          1.5000          30.90           93.30
## 94          0.6400          29.04           91.26
## 95          0.3900          29.31           56.80
## 96          0.2300          28.78           84.20
## 97          0.5800          25.26           51.68
## 98          0.3700          27.40           76.85
## 99          0.0500          27.69           60.43
## 100         0.0500          27.77           70.80
## 101         0.0500          28.57           60.58
## 102         0.0500          28.21           23.43
## 103         0.0500          26.80           38.76
## 104         0.7400          27.36           88.39
## 105         0.3100          24.96          102.86
## 106         0.6100          26.39           89.33
## 107         0.0400          31.99           61.38
## 108         0.0300          29.82           70.69
## 109         0.0300          29.90           77.13
## 110         0.2000          25.72          102.60
## 111         0.5000          22.67           99.85
## 112         0.5000          22.78           97.70
## 113         0.0300          25.45           94.93
## 114         0.0300          23.32           93.07
## 115         0.5000          17.27           91.61
## 116         0.0300          26.55           82.44
## 117         0.0300          25.19           77.37
## 118         0.2100          25.14          100.62
## 119         0.0300          19.00           86.46
## 120         0.0300          14.67           78.55
## 121         0.1546          25.34          105.20
## 122         0.0300          16.58           84.20
## 123         0.0300          22.06           88.88
## 124         0.0300          23.62           96.58
## 125         0.0300          26.00           94.24
## 126         0.0300          23.75           86.41
## 127         0.1000          25.07           82.38
## 128         0.5600          24.50          103.03
## 129         0.1600          23.79           97.39
## 130         0.2300          23.72          100.08
## 131         0.3500          17.35          104.25
## 132         0.5500          20.20          102.14
## 133         0.1100          21.83          102.92
## 134         0.0300          25.95          105.50
## 135         0.0400          26.88           92.18
## 136         0.0300          26.01           88.20
## 137         0.0300          24.81           91.41
```

```
## 138             0.0300              25.58               89.52
##     completeObs.Elevacion completeObs.Ancho completeObs.Velocidad
## 1                      23       16.00000000                5.00000
## 2                      31       11.00000000                0.00000
## 3                      35       14.00000000               10.00000
## 4                       9        5.00000000                2.00000
## 5                      43       11.00000000                9.00000
## 6                      23       11.00000000                5.00000
## 7                      86       11.00000000               13.00000
## 8                      26        3.00000000               11.00000
## 9                      24        3.00000000               14.00000
## 10                     53       11.00000000                4.00000
## 11                     24       11.00000000                3.00000
## 12                    619        2.00000000               14.00000
## 13                    598        3.00000000               14.00000
## 14                    583        3.00000000               14.00000
## 15                    114        2.00000000               11.00000
## 16                     46        3.00000000               14.00000
## 17                     46        3.00000000               16.00000
## 18                    158       11.00000000                4.00000
## 19                     34       11.00000000                4.00000
## 20                   1818       -1.86157257               17.60742
## 21                    205       11.00000000                4.00000
## 22                     38        3.00000000               13.00000
## 23                     98        3.00000000               14.00000
## 24                     49        3.00000000               15.00000
## 25                     29        3.00000000               14.00000
## 26                     99        3.00000000               14.00000
## 27                     20        3.00000000               14.00000
## 28                     82        1.00000000               11.00000
## 29                     43        2.00000000               11.00000
## 30                     17        3.00000000                2.00000
## 31                    149        3.00000000                3.00000
## 32                     10        1.00000000               14.00000
## 33                     28        1.00000000               14.00000
## 34                     18        1.00000000               12.00000
## 35                     85        1.00000000               14.00000
## 36                    130        2.00000000                2.00000
## 37                     51        2.00000000               10.00000
## 38                    198        1.00000000                3.00000
## 39                     13        1.00000000                3.00000
## 40                     53        2.00000000                3.00000
## 41                    492        2.00000000               14.00000
## 42                    428        2.00000000               14.00000
## 43                     49        3.00000000               11.00000
## 44                     67        3.00000000               12.00000
## 45                     67        1.00000000               11.00000
## 46                    100        2.00000000                9.00000
## 47                     83        1.00000000               14.00000
## 48                     63        1.00000000               12.00000
## 49                     60        3.00000000               12.00000
## 50                     25        3.00000000               11.00000
## 51                     30        3.00000000               11.00000
## 52                     50        2.00000000                3.00000
```

```
## 53          36        2.00000000          3.00000
## 54          22        3.00000000         11.00000
## 55          11        2.00000000         12.00000
## 56          71        3.00000000         14.00000
## 57          15        3.00000000         12.00000
## 58          85        3.00000000          9.00000
## 59          21        3.00000000         11.00000
## 60         659        2.00000000         13.00000
## 61         615        3.00000000         14.00000
## 62         517        3.00000000         14.00000
## 63         422        2.00000000         14.00000
## 64         363        3.00000000         14.00000
## 65         117        3.00000000         14.00000
## 66         244        2.00000000          9.00000
## 67          15        2.00000000         11.00000
## 68          22        3.00000000         14.00000
## 69        1114        1.00000000         10.00000
## 70         353        1.00000000         14.00000
## 71         314        1.00000000         14.00000
## 72        1630        1.00000000         14.00000
## 73         628        1.00000000         14.00000
## 74         137        1.00000000         14.00000
## 75          51        2.00000000         14.00000
## 76          27        2.00000000          8.00000
## 77          27       12.00000000          0.00000
## 78          15       12.00000000          0.00000
## 79          16       12.00000000          0.00000
## 80          15        5.00000000          2.00000
## 81           6       11.00000000          2.00000
## 82           3       11.00000000          2.00000
## 83          10       11.00000000          2.00000
## 84           8       11.00000000          2.00000
## 85          86       11.00000000          4.00000
## 86          26       11.00000000          2.00000
## 87           9        2.00000000         11.00000
## 88          28        5.00000000          2.00000
## 89          27        5.00000000          1.00000
## 90          21        3.00000000         11.00000
## 91          13        3.00000000          2.00000
## 92          23        3.00000000          2.00000
## 93          23        3.00000000          2.00000
## 94          11        3.00000000          2.00000
## 95          27        2.00000000          3.00000
## 96          19        2.00000000          2.00000
## 97          43        1.00000000          2.00000
## 98          46        2.00000000          3.00000
## 99          44        3.00000000         12.00000
## 100         53        2.00000000         14.00000
## 101         42        3.00000000          2.00000
## 102         50        3.00000000          2.00000
## 103         42        2.00000000          2.00000
## 104         58        3.00000000         12.00000
## 105         43        2.00000000          3.00000
## 106         51        2.00000000         11.00000
```

```
## 107                  15        3.00000000              2.00000
## 108                  22        2.00000000             11.00000
## 109                  13        2.00000000             11.00000
## 110                 115        5.00000000              9.00000
## 111                 491        2.00000000             14.00000
## 112                 524        3.00000000             14.00000
## 113                  98        2.00000000             14.00000
## 114                 275        3.27196421             10.43203
## 115                1488        2.00000000             14.00000
## 116                 196        2.00000000             15.00000
## 117                 291        2.00000000             11.00000
## 118                 223       11.00000000              5.00000
## 119                1346       -0.03968418             15.13673
## 120                2370        2.00000000             15.00000
## 121                  17       11.00000000              4.00000
## 122                1412        2.00000000             15.00000
## 123                 490        2.00000000             15.00000
## 124                 252        3.00000000             15.00000
## 125                 162        2.00000000             14.00000
## 126                 494        2.00000000             15.00000
## 127                 428        2.00000000             15.00000
## 128                 358        1.00000000              4.00000
## 129                 363        1.00000000              3.00000
## 130                 371        2.00000000              6.00000
## 131                1420        1.00000000              5.00000
## 132                 828        1.00000000              4.00000
## 133                 952        1.00000000             14.00000
## 134                 422        2.00000000             13.00000
## 135                 144        3.00000000             15.00000
## 136                 200        3.00000000             14.00000
## 137                 327        2.00000000             13.00000
## 138                  60        3.00000000             15.00000
##     completeObs.Rocas completeObs.Canto completeObs.grava completeObs.arena
## 1            20.00000          25.00000          30.00000         20.000000
## 2            20.00000          45.00000          20.00000         15.000000
## 3            30.00000          30.00000          20.00000         10.000000
## 4             0.00000           0.00000           0.00000         50.000000
## 5            10.00000          40.00000          10.00000         20.000000
## 6            20.00000          60.00000          20.00000          0.000000
## 7             0.00000          80.00000          20.00000         20.000000
## 8             0.00000          30.00000          20.00000         25.000000
## 9             5.00000          25.00000          35.00000         20.000000
## 10            0.00000          70.00000           5.00000         20.000000
## 11            0.00000          70.00000          20.00000         10.000000
## 12           30.00000          30.00000          20.00000         20.000000
## 13           20.00000          30.00000          20.00000         10.000000
## 14           20.00000          30.00000          20.00000         10.000000
## 15            0.00000          15.00000          30.00000         25.000000
## 16            0.00000           5.00000          20.00000         40.000000
## 17            0.00000           1.00000          40.00000         40.000000
## 18           40.00000          50.00000           5.00000          5.000000
## 19            0.00000          70.00000          15.00000         15.000000
## 20           51.77559          35.62091          16.63549         -6.150990
## 21            0.00000          80.00000          10.00000         10.000000
```

```
## 22          0.00000         40.00000         30.00000         20.000000
## 23         25.00000         25.00000         15.00000         25.000000
## 24         10.00000         60.00000         10.00000         10.000000
## 25          5.00000         25.00000         30.00000         25.000000
## 26         25.00000         40.00000          0.00000         10.000000
## 27         15.00000         15.00000          5.00000         30.000000
## 28         60.00000          0.00000         20.00000         20.000000
## 29          0.00000         50.00000         50.00000          0.000000
## 30          0.00000         33.30000         33.30000         33.300000
## 31         90.00000         10.00000          0.00000          0.000000
## 32         15.00000         20.00000         40.00000         10.000000
## 33         10.00000         70.00000         10.00000          0.000000
## 34         10.00000         20.00000         50.00000         10.000000
## 35         10.00000         20.00000         50.00000         10.000000
## 36          0.00000         30.00000          0.00000         70.000000
## 37         35.00000         50.00000          0.00000         15.000000
## 38         90.00000          0.00000          0.00000         10.000000
## 39         33.30000         33.30000          0.00000          0.000000
## 40          0.00000          0.00000         50.00000         50.000000
## 41         50.00000         20.00000         10.00000         10.000000
## 42         20.00000         40.00000         20.00000         10.000000
## 43          0.00000         10.00000         30.00000         50.000000
## 44         20.00000         40.00000         20.00000         10.000000
## 45         10.00000         60.00000         20.00000          5.000000
## 46         20.00000         35.00000         25.00000         15.000000
## 47         50.00000         30.00000         10.00000          5.000000
## 48         10.00000         60.00000         20.00000          5.000000
## 49         10.00000          5.00000          5.00000         30.000000
## 50          0.00000         70.00000         20.00000          5.000000
## 51          0.00000         10.00000         40.00000         40.000000
## 52          0.00000          0.00000          0.00000         50.000000
## 53          0.00000          0.00000         10.00000         20.000000
## 54          0.00000         20.00000         60.00000         10.000000
## 55          0.00000          0.00000         80.00000         20.000000
## 56          5.00000         50.00000         20.00000         15.000000
## 57          0.00000         10.00000         70.00000         10.000000
## 58          5.00000         60.00000         20.00000         10.000000
## 59          0.00000         10.00000         60.00000         20.000000
## 60         10.00000         70.00000         20.00000          0.000000
## 61         30.00000         30.00000         30.00000         10.000000
## 62         50.00000         30.00000         10.00000         10.000000
## 63         30.00000         40.00000         20.00000         10.000000
## 64         30.00000         40.00000         20.00000          5.000000
## 65         10.00000         70.00000         10.00000         10.000000
## 66         25.00000         30.00000         25.00000         15.000000
## 67          0.00000          0.00000         40.00000         40.000000
## 68         10.00000         40.00000         30.00000         15.000000
## 69         75.00000         15.00000          0.00000         10.000000
## 70         40.00000         20.00000         20.00000         20.000000
## 71         50.00000         10.00000         20.00000         20.000000
## 72         30.00000         25.00000         25.00000          0.000000
## 73         30.00000         20.00000         20.00000         20.000000
## 74         20.00000         60.00000         10.00000         10.000000
## 75         30.00000         25.00000         25.00000          0.000000
```

```
## 76          40.00000          30.00000          20.00000          10.000000
## 77           0.00000           0.00000          15.00000          85.000000
## 78           0.00000           0.00000          10.00000          90.000000
## 79           0.00000           0.00000          20.00000          80.000000
## 80           0.00000           0.00000           0.00000          50.000000
## 81           0.00000           0.00000           0.00000          50.000000
## 82           0.00000           0.00000           0.00000          50.000000
## 83           0.00000           0.00000           0.00000          50.000000
## 84           0.00000           0.00000           0.00000           0.000000
## 85           0.00000          65.00000          20.00000          10.000000
## 86           0.00000           0.00000           0.00000          50.000000
## 87           0.00000           0.00000           0.00000           0.000000
## 88           0.00000          37.75740          80.00000          10.000000
## 89           0.00000           0.00000          60.00000          20.000000
## 90           0.00000           0.00000           0.00000          40.000000
## 91           0.00000           0.00000           0.00000          20.000000
## 92           0.00000           0.00000           0.00000          50.000000
## 93           0.00000           0.00000           0.00000           0.000000
## 94           0.00000           0.00000           0.00000           0.000000
## 95           0.00000           0.00000           0.00000           0.000000
## 96           0.00000           0.00000           0.00000           0.000000
## 97           0.00000           0.00000           0.00000           0.000000
## 98           0.00000           0.00000          10.00000           0.000000
## 99           0.00000           0.00000           0.00000           0.000000
## 100          0.00000          15.00000           5.00000          15.000000
## 101          0.00000           0.00000           0.00000           0.000000
## 102          0.00000           0.00000           0.00000          15.000000
## 103          0.00000           0.00000           0.00000           5.000000
## 104          0.00000           0.00000           0.00000          50.000000
## 105          0.00000           0.00000           0.00000         100.000000
## 106          0.00000           0.00000          50.00000          50.000000
## 107          0.00000           0.00000           0.00000          50.000000
## 108          0.00000           0.00000           0.00000          50.000000
## 109          0.00000           5.00000          35.00000          30.000000
## 110          0.00000          80.00000          10.00000          10.000000
## 111         35.00000          20.00000           0.00000          35.000000
## 112         40.00000          30.00000          20.00000          10.000000
## 113         25.00000          30.00000          25.00000          15.000000
## 114         21.52682          29.80883          18.66657          16.030134
## 115         20.00000          40.00000          20.00000          10.000000
## 116         35.00000          40.00000          10.00000          10.000000
## 117         30.00000          30.00000          20.00000          15.000000
## 118         40.00000          50.00000           5.00000           5.000000
## 119         41.33221          34.05129          17.51130           1.523453
## 120         25.00000          25.00000          35.00000          10.000000
## 121         30.00000          40.00000          10.00000          10.000000
## 122         35.00000          40.00000          15.00000          10.000000
## 123         20.00000          30.00000          35.00000          15.000000
## 124         25.00000          30.00000          30.00000          15.000000
## 125         15.00000          20.00000          15.00000          25.000000
## 126         45.00000          25.00000          10.00000          15.000000
## 127         55.00000          30.00000           5.00000           5.000000
## 128         30.00000          60.00000           0.00000          10.000000
## 129         10.00000          40.00000          25.00000          25.000000
```

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## 130          25.00000          25.00000          20.00000          20.000000
## 131          40.00000          40.00000          10.00000          10.000000
## 132          40.00000          40.00000          10.00000          10.000000
## 133          50.00000          20.00000          20.00000          10.000000
## 134          30.00000          40.00000          20.00000          10.000000
## 135          50.00000          30.00000          10.00000           5.000000
## 136          15.00000          30.00000          30.00000          20.000000
## 137          40.00000          30.00000          20.00000           8.000000
## 138          30.00000          25.00000          10.00000          30.000000
##     completeObs.Limo fittedX.1 fittedX.2 fittedX.3   fittedX.4   fittedX.5
## 1           0.000000 0.3025114  27.61564  97.46164  -72.005885  6.64221363
## 2           0.000000 0.3022691  27.30437  98.02139  -27.062976  6.38371568
## 3           0.000000 0.3029569  26.21004  99.20761  134.690965  5.35975727
## 4          50.000000 0.3420183  28.22703  76.54125  -65.343011  4.23065606
## 5          20.000000 0.3052248  26.84294  97.16788   47.625471  5.75258262
## 6           0.000000 0.2976537  26.05510 102.11811  144.456411  5.61871295
## 7           0.000000 0.2926368  25.89967 104.88392  154.988946  5.85582773
## 8          25.000000 0.3217697  26.57467  89.14244  127.113127  4.27671377
## 9          15.000000 0.3200263  26.08279  90.71795  194.825340  3.96998487
## 10          5.000000 0.2999178  26.82671  99.88566   37.090579  6.13509131
## 11          0.000000 0.2969452  26.74572 101.50886   41.707444  6.28544953
## 12          0.000000 0.3165432  23.86777  95.59468  510.369027  2.26205094
## 13         20.000000 0.3196855  24.29149  93.40444  456.029721  2.40358253
## 14         20.000000 0.3171160  24.71381  94.11670  388.002945  2.97106542
## 15         15.000000 0.3245904  26.23241  88.19025  184.040755  3.76157911
## 16         35.000000 0.3281721  26.71536  85.69371  122.106393  3.92287480
## 17         19.000000 0.3249554  26.42140  87.73969  157.283158  3.90222606
## 18          0.000000 0.3029803  26.01762  99.46574  162.894534  5.18701585
## 19          0.000000 0.2971904  27.20904 100.73425  -25.470948  6.67885303
## 20          1.183094 0.3269562  20.10400  95.58778 1086.267354 -1.86157223
## 21          0.000000 0.2934247  26.61949 103.47381   51.609001  6.43659024
## 22         10.000000 0.3075309  26.24161  96.84054  141.198283  5.04570933
## 23         10.000000 0.3169708  25.70591  92.79839  242.523914  3.86358743
## 24         10.000000 0.3080806  25.71790  97.29621  219.144255  4.53918323
## 25         15.000000 0.3141634  26.37454  93.28591  137.886747  4.66776855
## 26         25.000000 0.3178916  25.40812  92.74864  288.324164  3.53008245
## 27         35.000000 0.3322468  26.26145  84.26140  198.415627  3.21473740
## 28          0.000000 0.3177652  25.08190  93.27057  335.736582  3.24963260
## 29          0.000000 0.3040325  25.99890  98.95765  168.193251  5.09167196
## 30          0.000000 0.3080835  27.14412  95.29360   10.521598  5.80642740
## 31          0.000000 0.3131910  24.90871  95.83646  349.944875  3.43784377
## 32         15.000000 0.3187275  25.78738  91.79201  234.878803  3.80460458
## 33         10.000000 0.3096604  25.34692  97.01449  277.255433  4.09133508
## 34         10.000000 0.3146174  26.04233  93.52149  187.586616  4.33858664
## 35         10.000000 0.3145457  25.87413  93.79389  212.016575  4.19447333
## 36          0.000000 0.3137006  27.36155  92.13601   -7.621018  5.57952508
## 37          0.000000 0.3106825  25.26210  96.61447  292.148359  3.93951723
## 38          0.000000 0.3247336  24.32313  90.79649  463.680250  2.05413259
## 39         33.300000 0.3272819  26.33452  86.68017  175.650190  3.65101887
## 40          0.000000 0.3237455  27.48718  86.85871   -1.564062  4.93986469
## 41         10.000000 0.3215490  23.81311  93.12930  530.540198  1.83907570
## 42         10.000000 0.3156825  24.32313  95.39286  441.664811  2.73109869
## 43         10.000000 0.3217012  26.84173  88.80247   87.880360  4.51916562
## 44         10.000000 0.3138763  25.35582  94.86106  286.207342  3.78392601
```

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## 45          5.000000 0.3095625  25.20764  97.25962  297.390052  3.97488962
## 46          5.000000 0.3195323  25.79402  91.37395  235.864729  3.75030815
## 47          5.000000 0.3200265  24.44137  93.02096  434.934468  2.51127259
## 48          5.000000 0.3145890  25.37761  94.46859  284.753656  3.74998552
## 49         50.000000 0.3331192  26.55864  83.40139  157.065003  3.41358986
## 50          5.000000 0.3071895  25.67801  97.80473  222.811770  4.57038750
## 51         10.000000 0.3184465  26.74792  90.58694   93.686508  4.67923201
## 52         50.000000 0.3346792  27.22040  81.68060   64.056013  3.88500267
## 53         70.000000 0.3365289  26.88952  81.20556  116.956877  3.45261012
## 54         10.000000 0.3141200  26.09598  93.69880  178.528957  4.42346566
## 55          0.000000 0.3156286  26.14305  92.86662  175.313419  4.35245441
## 56         10.000000 0.3135664  25.92219  94.22377  202.604788  4.31042551
## 57         10.000000 0.3152378  26.23798  92.93192  160.476117  4.46605277
## 58          5.000000 0.3095654  26.00612  96.13781  180.595467  4.68426183
## 59         10.000000 0.3172580  26.90404  90.97142   67.958572  4.90686161
## 60          0.000000 0.3054470  24.06452 100.95355  454.597149  3.26683497
## 61          0.000000 0.3093171  24.28401  98.68023  431.902939  3.17243319
## 62          0.000000 0.3129393  23.78999  97.53399  512.980527  2.46248363
## 63          0.000000 0.3091141  24.30206  98.75802  428.769942  3.20365264
## 64          5.000000 0.3082405  24.46878  98.96770  402.255710  3.41715935
## 65          0.000000 0.3025784  25.18272 100.84128  284.047445  4.47511161
## 66          5.000000 0.3173427  25.23457  93.27088  312.376599  3.41690163
## 67         20.000000 0.3187693  27.23547  89.73889   23.152089  5.08836296
## 68          5.000000 0.3124209  25.71204  95.10036  230.559394  4.20934745
## 69          0.000000 0.3252245  22.07487  93.70178  793.752586  0.01943017
## 70          0.000000 0.3176760  24.00924  94.82096  492.430447  2.30304745
## 71          0.000000 0.3198039  24.02490  93.71837  495.315736  2.15780710
## 72         20.000000 0.3236816  21.82608  94.83439  826.393329 -0.08626760
## 73         10.000000 0.3208709  23.96081  93.26643  507.285176  2.02105282
## 74          0.000000 0.3097007  24.60439  98.03588  385.970605  3.42845446
## 75         20.000000 0.3177526  24.98755  93.40931  349.507282  3.16672530
## 76          0.000000 0.3142629  25.08916  95.03893  326.156144  3.51803060
## 77          0.000000 0.3232347  29.42481  84.39935 -286.246130  6.70000666
## 78          0.000000 0.3313157  29.78632  79.78837 -319.471086  6.41685575
## 79          0.000000 0.3153684  28.70070  89.41005 -199.455878  6.64485124
## 80         50.000000 0.3315562  28.18888  81.90767  -85.211199  4.97926017
## 81         50.000000 0.3263293  28.21429  84.52636 -101.641293  5.39277654
## 82         50.000000 0.3262634  28.25011  84.50954 -107.041766  5.42953788
## 83         50.000000 0.3253554  28.25554  84.96302 -110.044912  5.50227771
## 84        100.000000 0.3338985  27.94913  81.05456  -44.442395  4.59100216
## 85          5.000000 0.2982642  26.79135 100.77501   38.241724  6.22734109
## 86         50.000000 0.3234966  28.25732  85.90448 -114.826607  5.64288804
## 87        100.000000 0.3445656  26.68879  77.40600  165.869215  2.67312412
## 88         10.000000 0.2987864  27.55169  99.44297  -71.712265  6.86398718
## 89         20.000000 0.3322656  28.21135  81.51588  -86.772686  4.94616985
## 90         60.000000 0.3362807  27.40646  80.60628   40.734228  3.93057180
## 91         80.000000 0.3440501  27.69236  76.25964   17.811280  3.60353666
## 92         50.000000 0.3325533  28.08400  81.54846  -67.443627  4.81147437
## 93        100.000000 0.3441195  27.88020  75.96084   -9.497282  3.76527627
## 94        100.000000 0.3435953  27.66293  76.53193   21.010794  3.61139797
## 95        100.000000 0.3546317  27.72231  70.84402   39.170215  2.83870148
## 96        100.000000 0.3461212  27.56689  75.38393   41.203420  3.33712395
## 97        100.000000 0.3578692  27.15985  69.98915  129.322233  2.09671099
## 98         90.000000 0.3454039  27.25896  76.18027   84.503574  3.11712049
```

```
## 99        100.000000 0.3513609  26.91005  73.64472  150.032615  2.36149928
## 100        65.000000 0.3386848  26.54160  80.59896  173.095528  2.98217343
## 101       100.000000 0.3522620  27.75255  72.00500   28.981523  3.04282465
## 102        85.000000 0.3610334  28.06300  67.11508    4.904673  2.66266418
## 103        95.000000 0.3591191  27.59679  68.74134   68.445870  2.39152966
## 104        50.000000 0.3349892  27.14655  81.62682   75.612804  3.79619053
## 105         0.000000 0.3230258  27.78312  86.80895  -46.605254  5.25669191
## 106         0.000000 0.3192732  26.69775  90.23751  103.035314  4.57282172
## 107        50.000000 0.3427667  28.71811  75.47211 -135.358492  4.61109063
## 108        50.000000 0.3400906  27.61241  78.38256   19.875524  3.82863356
## 109        30.000000 0.3282381  27.13900  85.06580   60.296090  4.29442365
## 110         0.000000 0.3007926  25.80544 100.87442  188.610732  5.16208448
## 111        10.000000 0.3216932  24.34725  92.30664  452.756576  2.30297130
## 112         0.000000 0.3145431  23.82393  96.67189  511.917429  2.37268322
## 113         5.000000 0.3144264  25.05958  94.99742  330.880728  3.47951750
## 114        14.124392 0.3175308  25.08732  93.38198  334.373940  3.27197682
## 115        10.000000 0.3204532  22.23650  95.89797  758.504098  0.51992745
## 116         5.000000 0.3183068  24.74607  93.46675  386.180344  2.91067363
## 117         5.000000 0.3214074  25.00304  91.53163  356.131143  2.90713986
## 118         0.000000 0.3035680  25.73703  99.56097  205.370198  4.89369401
## 119         5.043606 0.3231126  21.83061  95.11700  824.346247 -0.03967971
## 120         5.000000 0.3266223  20.74905  94.85228  991.095919 -1.26334808
## 121        10.000000 0.3048660  26.46894  97.87486  101.461639  5.44705460
## 122         0.000000 0.3205761  21.84192  96.38923  816.522696  0.16008236
## 123         0.000000 0.3152985  24.15214  95.82775  465.743215  2.60786141
## 124         0.000000 0.3118753  24.60547  96.93007  391.102377  3.26676770
## 125        25.000000 0.3221069  25.59808  90.34144  270.789644  3.38361459
## 126         5.000000 0.3213790  23.92684  93.05608  513.491016  1.95285691
## 127         5.000000 0.3219666  23.81942  92.90839  530.632992  1.81344923
## 128         0.000000 0.3117519  24.79331  96.72920  363.324939  3.44292923
## 129         0.000000 0.3139138  25.56591  94.54726  255.567458  3.96781861
## 130        10.000000 0.3176383  25.16571  93.21741  323.167363  3.33360817
## 131         0.000000 0.3174992  22.33151  97.26479  737.420770  0.82530304
## 132         0.000000 0.3165383  23.53844  96.05927  558.532192  1.96974781
## 133         0.000000 0.3176516  22.78753  96.54753  671.084096  1.21916054
## 134         0.000000 0.3098463  24.46162  98.16230  407.209154  3.29069358
## 135         5.000000 0.3162675  24.56051  94.76271  408.363621  2.89829845
## 136         5.000000 0.3151881  25.43238  94.08751  278.198622  3.75385375
## 137         2.000000 0.3164060  24.34198  94.99897  440.666803  2.69373633
## 138         5.000000 0.3182087  25.33838  92.68547  299.297302  3.44438818
##     fittedX.6  fittedX.7   fittedX.8 fittedX.9 fittedX.10 fittedX.11
## 1    7.064905  6.8383986  34.7270956 22.745199  27.091865  9.6484365
## 2    7.535290  8.7812608  35.7287604 22.865790  25.690783  7.9437439
## 3    9.147952 15.5129061  38.0674118 22.831399  20.793773  3.6107962
## 4    5.098568  0.5110474   2.7645809 10.886339  30.576542 55.0964430
## 5    8.144087 11.4481957  34.6663393 22.057559  23.675968  8.9919486
## 6    9.520514 16.8134680  42.5488333 24.434622  19.999054 -2.8278568
## 7    9.886152 18.0986079  46.8115304 25.952682  19.207538 -8.9599832
## 8    8.102070 12.0448449  22.6546868 17.172906  22.782872 25.5934092
## 9    8.881743 15.2026271  25.2832150 17.768617  20.543246 21.4319414
## 10   8.310007 11.8901146  38.7872112 23.640300  23.503540  3.1925063
## 11   8.510107 12.5828441  41.2838283 24.538041  23.084361 -0.3893778
## 12  12.276093 29.1416417  33.7633494 19.150651  10.539817  7.4032771
## 13  11.560646 26.3158831  30.2383931 18.149028  12.499898 12.7574999
```

```
## 14  10.999837 23.8663100  31.1065013 18.848350  14.346453 11.9444718
## 15   8.536853 13.9825764  21.3837317 16.386299  21.300208 27.0487031
## 16   7.721392 10.7618089  17.3658467 15.244588  23.534279 33.1516356
## 17   8.245429 12.7889100  20.6080306 16.248170  22.154991 28.3188641
## 18   9.434110 16.7028010  38.5536165 22.854413  19.930907  2.7436696
## 19   7.813016  9.6983026  39.8814905 24.392887  25.167716  2.0236478
## 20  17.607628 51.7761699  35.6223312 16.636142  -6.151505  1.1810038
## 21   8.792268 13.5909384  44.3200871 25.605992  22.451924 -4.7721056
## 22   8.978738 15.0231377  34.4696437 21.464529  21.021287  8.6907646
## 23   9.525060 17.7327763  28.6187448 18.737101  18.794951 16.3885538
## 24   9.744608 18.2304629  35.4193281 21.382393  18.681900  6.8550892
## 25   8.603491 13.7733794  29.0244019 19.468938  21.742176 16.4610556
## 26   9.944298 19.5173648  28.6913573 18.509302  17.476168 16.0007737
## 27   8.289086 13.3101462  15.4237779 14.101994  21.574240 35.4416984
## 28  10.433882 21.5453853  29.6432267 18.597744  16.010153 14.3516139
## 29   9.433917 16.7510449  37.7940260 22.544005  19.866644  3.7918196
## 30   7.618851  9.3994235  31.6803897 21.159464  25.080931 13.4720127
## 31  10.814174 22.9120408  33.6122172 19.986735  15.147239  8.6146660
## 32   9.356715 17.1152943  27.0553208 18.201353  19.193467 18.6611143
## 33  10.255346 20.4258952  35.1772781 20.969760  17.047061  6.8386845
## 34   9.086495 15.8011192  29.5459012 19.385483  20.260201 15.4101969
## 35   9.339098 16.8471818  30.0416794 19.433020  19.504204 14.5528807
## 36   7.144773  7.6917591  26.7940212 19.453043  26.161901 20.5390161
## 37  10.354460 20.8852939  34.6140507 20.678635  16.685710  7.5477848
## 38  11.378666 25.7951811  26.2760664 16.640964  12.736622 18.3492119
## 39   8.312778 13.1771450  19.0478127 15.568977  21.808882 30.4253163
## 40   6.689270  6.2676575  18.7454731 16.442490  26.914088 31.9561780
## 41  12.223862 29.1579915  30.0596404 17.668624  10.388564 12.5491638
## 42  11.620400 26.3775343  33.2317635 19.336042  12.566703  8.5865154
## 43   7.705859 10.3956509  22.0075570 17.151696  23.979810 26.7580617
## 44  10.129477 20.0995050  31.9140668 19.713039  17.166160 11.4273262
## 45  10.465534 21.2945297  35.6174060 21.020596  16.420361  6.0872460
## 46   9.325317 17.0223715  26.4193656 17.960654  19.238386 19.5600894
## 47  11.328152 25.3659135  29.5836307 18.024147  13.178764 13.8203743
## 48  10.077971 19.9187471  31.3093061 19.497442  17.277293 12.2970559
## 49   7.822853 11.4139097  13.9747070 13.795950  22.923987 37.7608346
## 50   9.827862 18.5347904  36.2086946 21.653960  18.486198  5.7088779
## 51   7.932605 11.1859217  24.7545902 18.135444  23.497808 22.8123968
## 52   6.794876  7.2160359  11.0419531 13.228371  25.922386 42.5123823
## 53   7.238631  9.1457749  10.4874165 12.729130  24.472556 42.9730698
## 54   9.019820 15.5009407  29.7875867 19.525236  20.491567 15.1224792
## 55   8.909375 15.1124366  28.5048604 19.068680  20.731068 16.9680385
## 56   9.293628 16.6126330  30.6683476 19.717134  19.701434 13.7194508
## 57   8.778328 14.5497976  28.5565280 19.170287  21.149650 16.9866499
## 58   9.275391 16.3503755  33.5231647 20.895405  20.002892  9.7931317
## 59   7.731663 10.2957394  25.2588865 18.465017  24.175946 22.2544575
## 60  12.279194 28.6372988  41.7752002 22.424053  11.214289 -3.6529078
## 61  11.848694 27.0292409  38.2259646 21.237499  12.271734  1.5393412
## 62  12.488269 29.8551109  36.7367397 20.235883  10.123192  3.1552985
## 63  11.827227 26.9305961  38.3347323 21.295148  12.348868  1.4039990
## 64  11.602060 25.9544485  38.5692224 21.529301  13.080528  1.2349339
## 65  10.689203 21.8981820  41.0499133 23.104077  16.177444 -1.5614924
## 66  10.217624 20.6272786  29.5678727 18.699759  16.687192 14.6039329
## 67   7.197321  8.1463343  23.2290783 17.963393  25.691350 25.4215674
```

```
## 68   9.637434 17.9875352  32.0992684 20.090949  18.737059 11.5093405
## 69  14.716424 39.6843321  31.7893385 16.844878   2.658655  8.4527887
## 70  12.034995 28.1928360  32.5222077 18.791337  11.195791  9.2810885
## 71  11.954825 27.9589682  30.8458840 18.155281  11.305995 11.6489312
## 72  15.128438 41.3240685  33.6268851 17.343035   1.513445  5.6348616
## 73  12.021839 28.2871038  30.1938017 17.847547  11.038508 12.5026466
## 74  11.360946 25.0208392  37.0917371 21.073378  13.716370  3.4388432
## 75  10.574834 22.1303661  29.9000455 18.616166  15.586616 13.9005853
## 76  10.516603 21.7257841  32.3156767 19.639459  15.976968 10.6076532
## 77   3.815007 -5.6968938  14.0613177 16.292860  35.598014 40.3907026
## 78   3.060390 -8.4551666   6.9039748 13.830346  37.371654 50.7834479
## 79   5.104330 -0.7072411  22.0036731 18.747892  32.201500 28.5480583
## 80   5.434832  1.4203425  10.9045379 14.007494  30.209010 43.6349602
## 81   5.536564  1.5993313  14.8547857 15.559906  30.224874 38.1149634
## 82   5.484930  1.3817585  14.8115391 15.573938  30.384365 38.2100514
## 83   5.501085  1.4065470  15.4950972 15.843460  30.391690 37.2558550
## 84   5.729608  2.7541315   9.7326136 13.347359  29.177293 45.0496651
## 85   8.406879 12.2154802  40.1506373 24.138183  23.313825  1.2449150
## 86   5.548077  1.5151231  16.9189239 16.396670  30.364780 35.2591461
## 87   7.323177  9.8716225   4.8372436 10.367370  23.722783 50.7106906
## 88   7.259693  7.4739923  37.7571936 23.864293  26.735039  5.3341459
## 89   5.382395  1.2355659  10.3004883 13.792759  30.323145 44.5043473
## 90   6.474796  5.9609548   9.3237647 12.722546  26.787239 45.1025633
## 91   5.841189  3.6908766   2.6039883 10.364591  28.215804 54.8085798
## 92   5.564519  2.0055891  10.4130394 13.726915  29.757162 44.2241225
## 93   5.559376  2.5233544   2.0585235 10.314683  29.059877 55.7544878
## 94   5.897209  3.9024008   3.0306613 10.504613  28.075197 54.1814641
## 95   5.513959  2.8246826  -5.6063430  7.209119  28.548775 66.3609621
## 96   5.972884  4.3345262   1.3410996  9.767429  27.691727 56.4606621
## 97   6.265799  6.0990111  -6.6206495  6.332704  26.085986 67.2446548
## 98   6.450995  6.2873382   2.6991561 10.028971  26.296666 54.2589512
## 99   6.811927  8.0644545  -0.9646046  8.309523  24.843043 59.0662931
## 100  7.699611 11.1613451   9.7422614 12.141384  22.952005 43.6849396
## 101  5.532168  2.7898690  -3.8644551  7.910033  28.639997 63.9451647
## 102  4.835213  0.3033211 -11.4185609  5.249905  30.197516 74.8457737
## 103  5.581186  3.3131380  -8.7259870  5.892495  28.069865 70.6190589
## 104  6.896665  7.6533554  10.9972406 13.147576  25.596865 42.5042456
## 105  6.267414  4.4815634  18.5231924 16.610712  28.228363 32.5522535
## 106  7.985291 11.4433360  24.2507233 17.897101  23.288260 23.4714458
## 107  4.346657 -2.5777776   0.9027443 10.587008  32.793909 58.1810515
## 108  6.066098  4.4406655   5.8563112 11.556034  27.782763 50.1670200
## 109  7.088225  8.1344735  16.2052084 15.158983  25.436251 35.1873339
## 110  9.808772 18.1573084  40.7907501 23.538864  18.937873 -0.5999498
## 111 11.423919 25.8414551  28.5493975 17.542527  12.787763 15.1816236
## 112 12.394854 29.5417908  35.4152693 19.753029  10.305563  5.0426313
## 113 10.556324 21.8984191  32.2675573 19.595392  15.847321 10.6467957
## 114 10.432066 21.5269163  29.8091397 18.666686  16.030063 14.1239473
## 115 14.602960 38.9905568  35.0325572 18.240418   3.294250  4.0559127
## 116 10.919955 23.5899564  30.1069092 18.488770  14.513542 13.3784207
## 117 10.454141 21.7993184  27.0508485 17.525518  15.724719 17.9144555
## 118  9.836622 18.4023770  38.8370996 22.723092  18.682999  2.0764232
## 119 15.136880 41.3326171  34.0523068 17.511765   1.523097  5.0421101
## 120 16.655143 47.8036305  34.1889121 16.635138  -3.263640  3.8121123
## 121  8.711087 13.7869977  35.9219751 22.222637  21.991228  6.8705712
```

```
## 122 15.187773 41.4258109  35.9719629 18.265279   1.526206  2.3586228
## 123 11.885499 27.4609547  33.9748100 19.476989  11.792332  7.3794678
## 124 11.301263 24.8742590  35.4177825 20.425708  13.762025  5.7893625
## 125  9.548592 18.0699485  24.9541829 17.224546  18.407595 21.4284556
## 126 12.058907 28.4647808  29.8923493 17.701545  10.895610 12.8931363
## 127 12.203305 29.0920569  29.7221914 17.543297  10.424712 13.0288484
## 128 11.024602 23.7191512  35.0205139 20.433219  14.602472  6.5272681
## 129  9.815364 18.7963121  31.3348417 19.669166  18.109431 12.4419708
## 130 10.312351 21.0345849  29.5211431 18.622477  16.383821 14.6034241
## 131 14.540250 38.5923418  37.0537695 19.105220   3.665069  1.3102378
## 132 12.767070 31.1811122  34.6299959 19.203398   9.062122  5.8707449
## 133 13.856512 35.7589481  35.7418396 18.988825   5.713946  3.5893665
## 134 11.569848 25.8954785  37.3539723 21.052281  13.078540  2.9337277
## 135 11.250980 24.8700948  32.1602510 19.124886  13.642726 10.3183165
## 136  9.980336 19.5410628  30.7054159 19.310531  17.534286 13.1972280
## 137 11.572976 26.2142425  32.6263214 19.117659  12.664877  9.4543850
## 138 10.039774 19.9287868  28.6304012 18.425744  17.169214 16.0193796
```

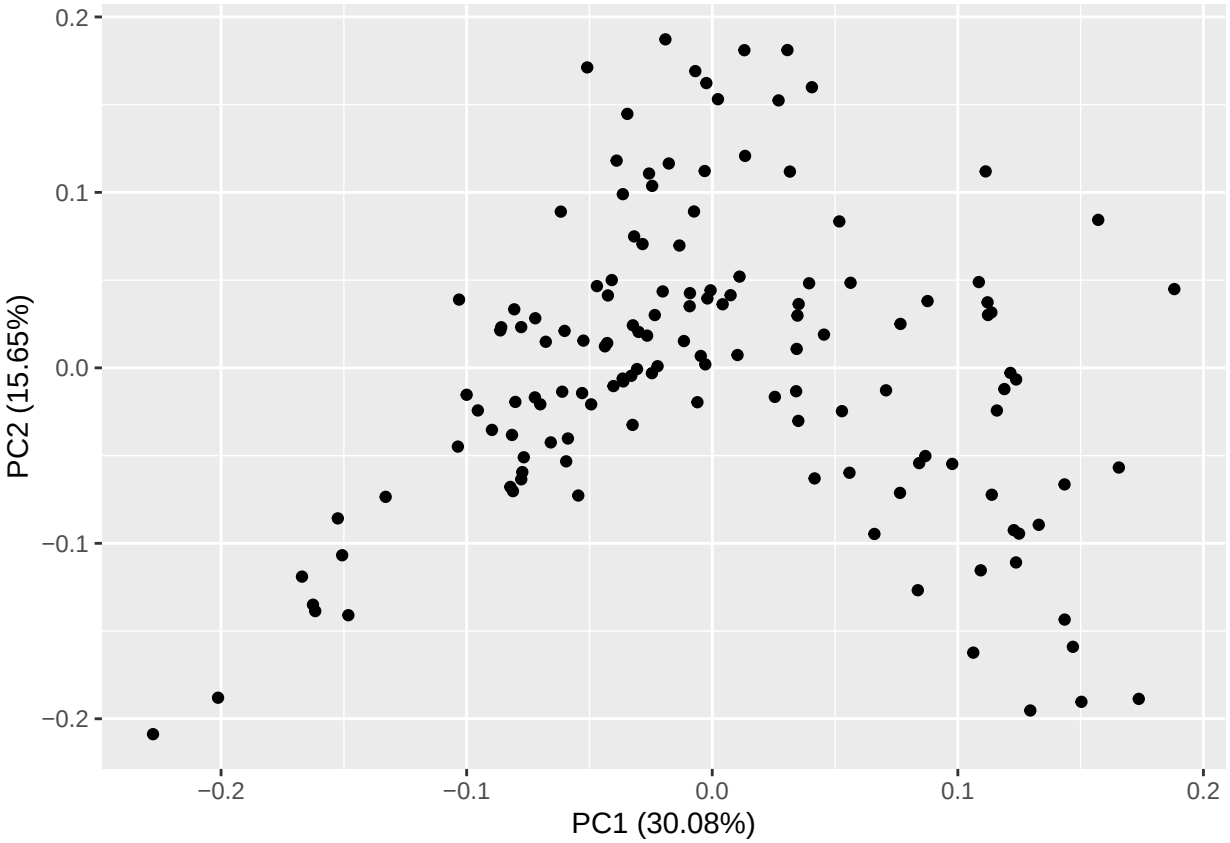
3. Vamos a correr el PCA

```
channel.pca <- prcomp(channel_2$completeObs, center = TRUE, scale = TRUE)
summary(channel.pca)
```

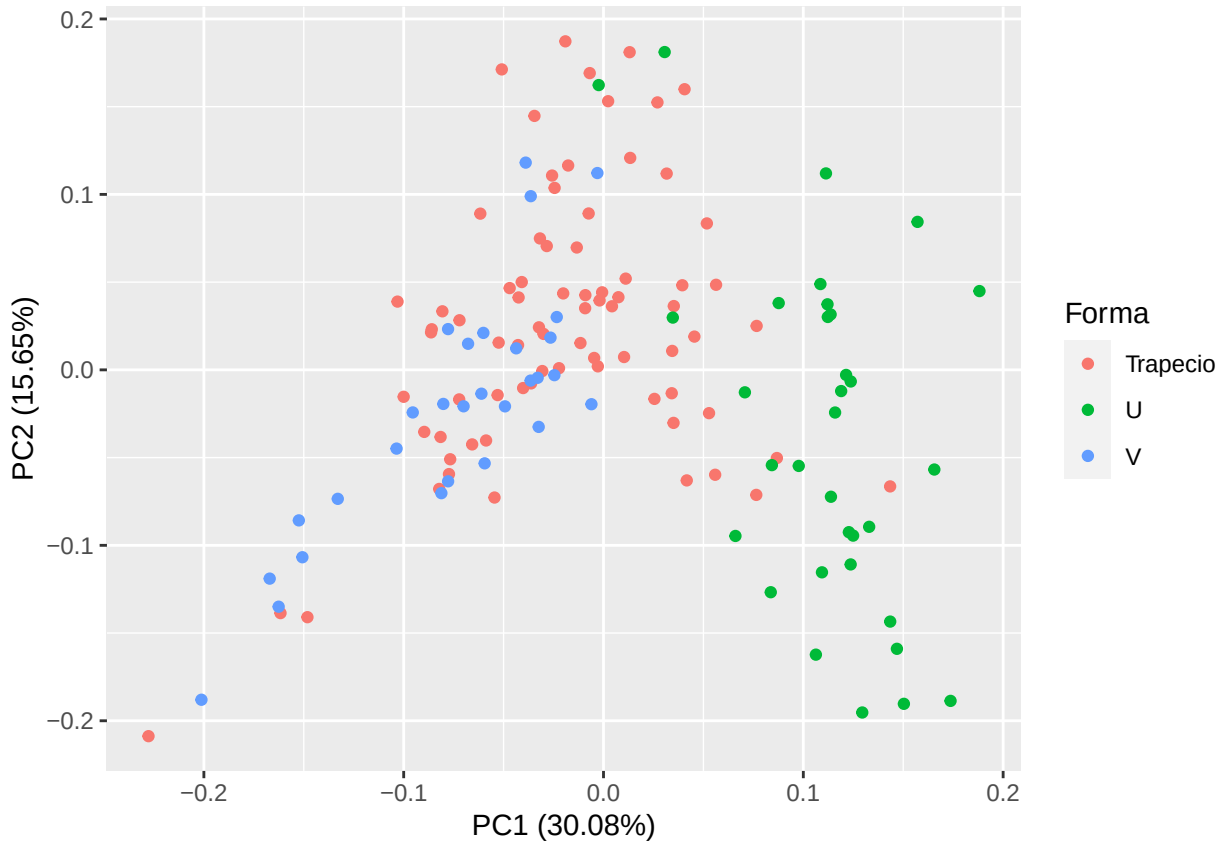
```
## Importance of components:
##                           PC1    PC2    PC3    PC4     PC5     PC6     PC7
## Standard deviation     1.8192 1.3122 1.1975 1.1185 1.00559 0.87738 0.75302
## Proportion of Variance 0.3009 0.1565 0.1304 0.1137 0.09193 0.06998 0.05155
## Cumulative Proportion  0.3009 0.4574 0.5878 0.7015 0.79343 0.86341 0.91496
##                           PC8     PC9    PC10    PC11
## Standard deviation     0.6667 0.59031 0.36992 0.07547
## Proportion of Variance 0.0404 0.03168 0.01244 0.00052
## Cumulative Proportion  0.9554 0.98704 0.99948 1.00000
```

3.1 Vamos a ver el grafico.

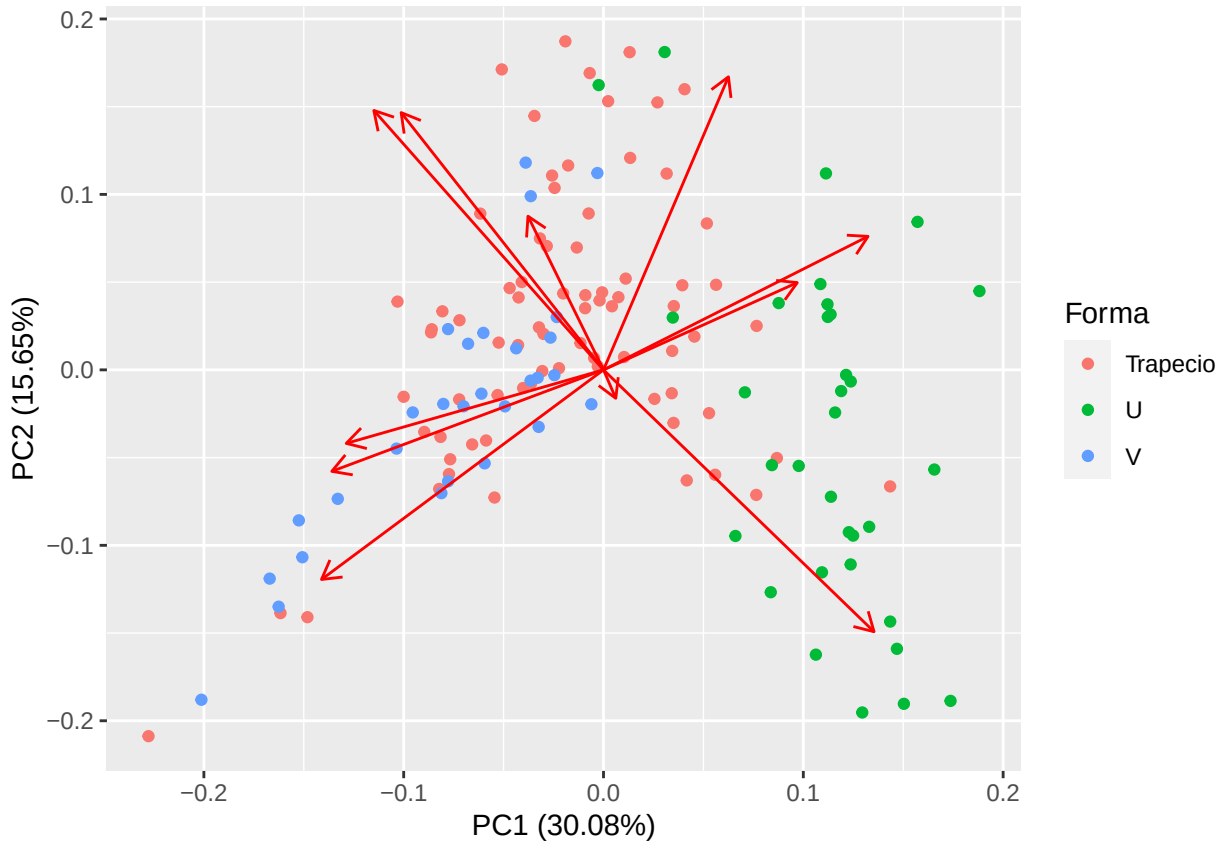
```
autoplot(channel.pca)
```



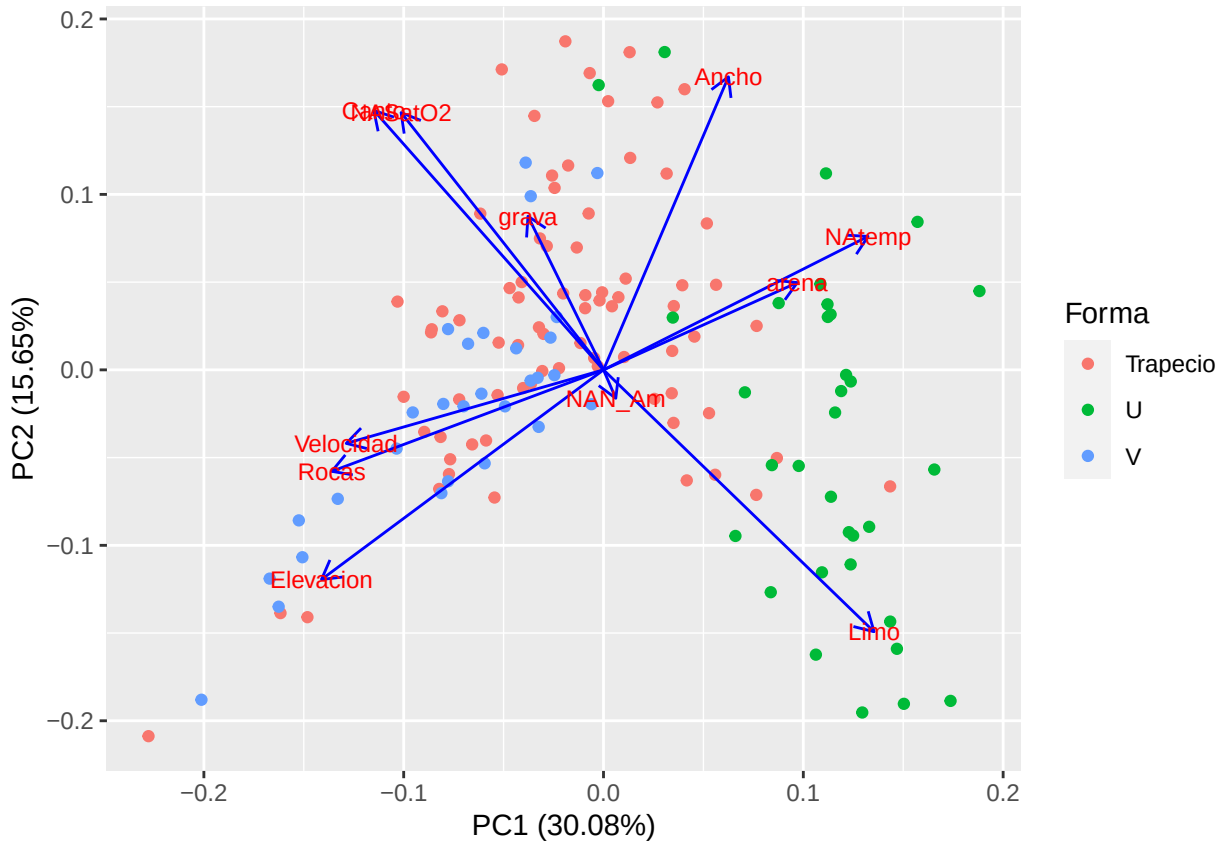

```
autoplot(channel.pca, data = channel, colour = 'Forma')
```




```
autoplot(channel.pca, data = channel, colour = 'Forma', loadings = TRUE)
```

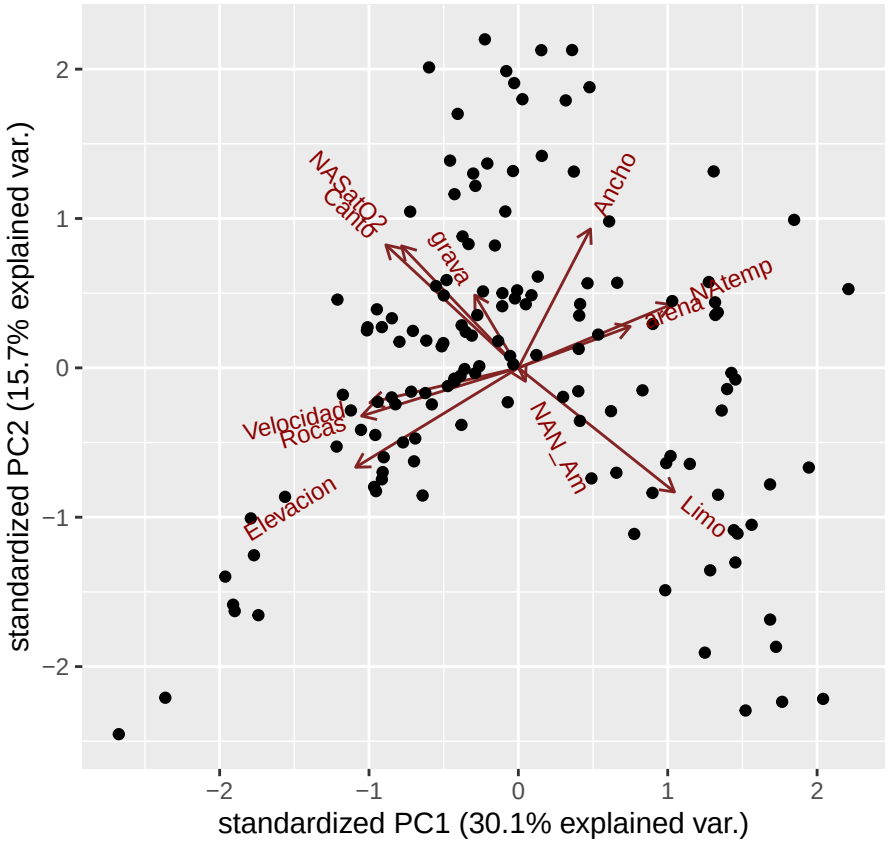



```
autoplot(channel.pca, data = channel, colour = 'Forma', loadings = TRUE,
         loadings.colour = 'blue',
         loadings.label = TRUE, loadings.label.size = 3)
```



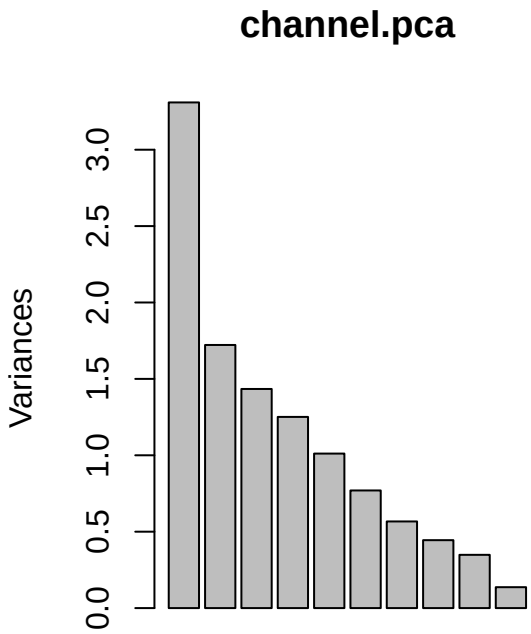
Otra manera de ver el grafico

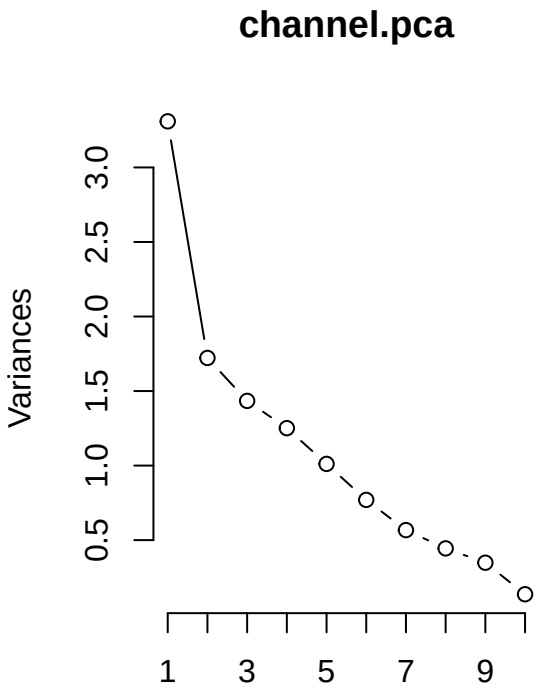
```
ggbiplot(channel.pca, labels=rownames(channel$Forma))
```



3.2 Ver graficamente lo que explica cada axis.

```
layout(matrix(1:2, ncol=2))
screeplot(channel.pca)
screeplot(channel.pca, type="lines")
```





3.3 Vamos a ver la contribucion de cada una de las variables. Usamos otra libreria. factoextra

```
get_eigenvalue(channel.pca)
```

```
##          eigenvalue variance.percent cumulative.variance.percent
## Dim.1  3.309339880      30.08490800                     30.08491
## Dim.2  1.721987545      15.65443223                     45.73934
## Dim.3  1.434048849      13.03680772                     58.77615
## Dim.4  1.251144650      11.37404227                     70.15019
## Dim.5  1.011205837       9.19278033                     79.34297
## Dim.6  0.769787383       6.99806712                     86.34104
## Dim.7  0.567041778       5.15492525                     91.49596
## Dim.8  0.444432998       4.04029998                     95.53626
## Dim.9  0.348471304       3.16792094                     98.70418
## Dim.10 0.136844213       1.24403830                     99.94822
## Dim.11 0.005695562       0.05177784                    100.00000
```

```
res.var <- get_pca_var(channel.pca)
res.var$contrib        # Contributions to the PCs
```

```
##                 Dim.1      Dim.2      Dim.3      Dim.4      Dim.5     Dim.6
## NAN_Am      0.02920886  0.2034616  5.8056413 31.9945477 34.3470814 15.246039
## NAtemp     13.57660503  4.4828567 15.4148551  3.7395130  2.1197838 14.516177
## NASatO2     7.94579553 16.6572006  0.4379407  8.8574581  5.4079970  2.598790
## Elevacion 15.42979758 11.0563517  5.5445224  4.1639816  0.9325291  9.624475
```

```
## Ancho      3.02902190 21.6083873 22.1140351  0.4570409  1.2134531  2.713345
## Velocidad 12.83990263  1.3599088 14.2241968  2.7802734  1.6200790  4.161196
## Rocas     14.32470621  2.5906170  2.5662488  3.2825439  1.7671606 29.965671
## Canto     10.23186117 16.9344566  0.2986754  4.7329896 12.2604278  5.124245
## grava      1.10633855  5.9387801 30.8293418 16.2613832  0.3136215  9.937735
## arena      7.28689335  1.9159508  2.4783815 18.4552620 33.9916827  3.692205
## Limo      14.19986918 17.2520289  0.2861611  5.2750067  6.0261840  2.420122
##                Dim.7       Dim.8        Dim.9      Dim.10       Dim.11
## NAN_Am     0.01132441  5.69938420 6.040932e+00  0.62044106  0.001939282
## NAtemp     0.73662346  0.59088977 2.080669e+00 42.69918930  0.042838114
## NASatO2    1.24823008 55.62178693 1.180103e+00  0.02000417  0.024693645
## Elevacion  0.24607958  0.96539511 3.314342e+00 48.68500085  0.037525452
## Ancho      5.68811760  0.01494364 3.793491e+01  5.22569343  0.001047357
## Velocidad 21.91572939  0.18791971 3.942194e+01  1.46336747  0.025485968
## Rocas     23.07674466  4.13345827 1.324401e+00  0.08968473 16.878763447
## Canto     11.02914223 12.77009548 6.399584e+00  0.08647903 20.132043196
## grava     21.59705383  2.73504867 2.422399e-04  0.47831503 10.802140459
## arena     13.91693263  1.00150996 6.374613e-01  0.55983412 16.063886205
## Limo       0.53402214 16.27956826 1.665410e+00  0.07199080 35.989636876
```

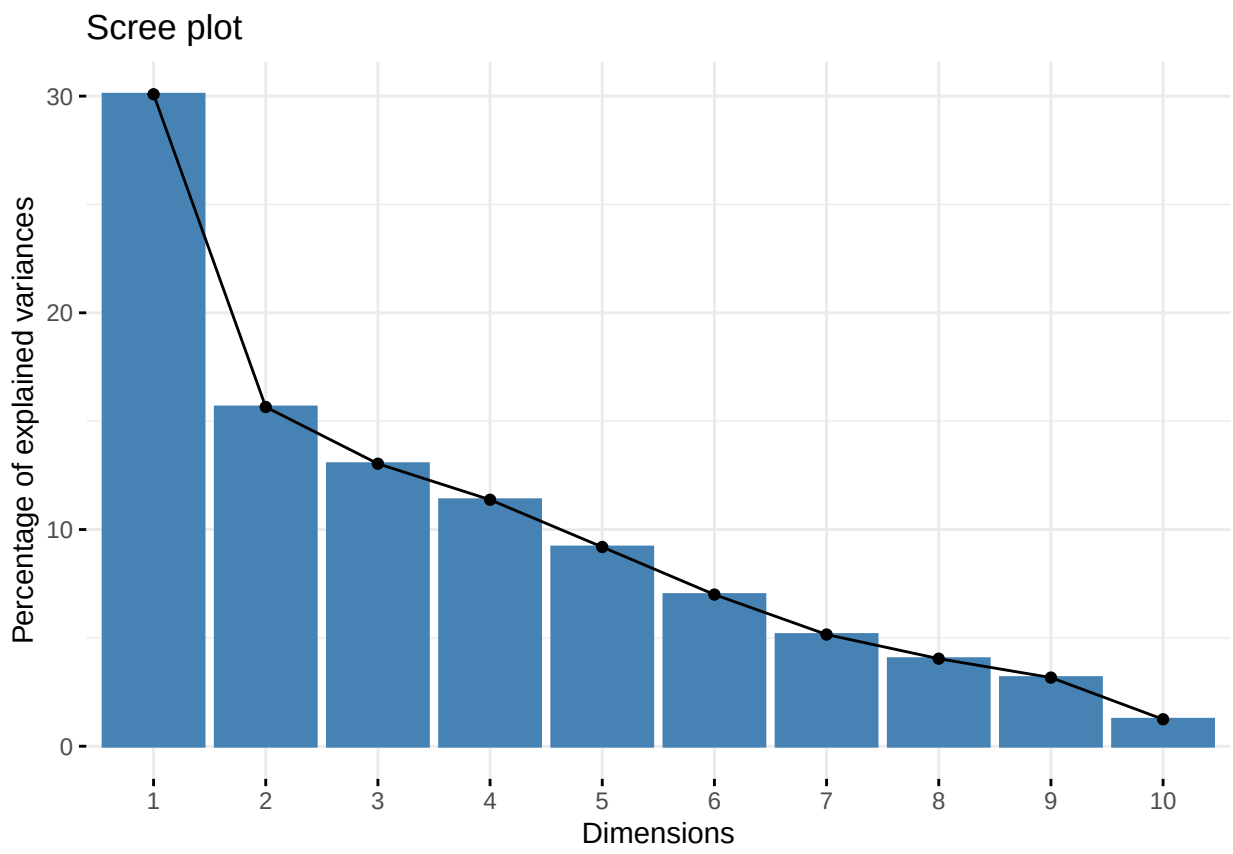
```
res.var$coord          # Coordinates
```

```
##                Dim.1       Dim.2       Dim.3       Dim.4       Dim.5
## NAN_Am     0.03109052 -0.05919107 -0.28854069 -0.63269113  0.58933835
## NAtemp     0.67029546  0.27783850 -0.47016652 -0.21630237 -0.14640826
## NASatO2   -0.51278980  0.53556971  0.07924824 -0.33289580  0.23385034
## Elevacion -0.71457991 -0.43633588  0.28197723  0.22824862  0.09710710
## Ancho      0.31660801  0.60999487  0.56313947  0.07561906 -0.11077233
## Velocidad -0.65185583 -0.15302764 -0.45164359  0.18650802 -0.12799349
## Rocas     -0.68851523 -0.21121104  0.19183655 -0.20265580  0.13367734
## Canto     -0.58189953  0.54000855  0.06544578 -0.24334450 -0.35210533
## grava     -0.19134394  0.31978908 -0.66491189  0.45105812  0.05631482
## arena      0.49106829  0.18163820  0.18852374  0.48052266  0.58628140
## Limo       0.68550852 -0.54504843  0.06406005 -0.25690069 -0.24685446
##                Dim.6        Dim.7        Dim.8         Dim.9       Dim.10
## NAN_Am     0.3425815 -0.008013373  0.159153838 -0.1450893281 -0.029138251
## NAtemp    -0.3342809  0.064629426  0.051245576  0.0851500600 -0.241725815
## NASatO2   -0.1414396 -0.084130767 -0.497193700  0.0641273732 -0.005232070
## Elevacion  0.2721911  0.037354705 -0.065502171  0.1074687420 -0.258113553
## Ancho      0.1445233  0.179593995  0.008149508 -0.3635825795 -0.084563935
## Velocidad -0.1789759 -0.352521406 -0.028899433 -0.3706401926 -0.044749678
## Rocas     -0.4802832  0.361738556  0.135537643 -0.0679349618  0.011078283
## Canto      0.1986097 -0.250079676  0.238232068  0.1493342367 -0.010878490
## grava      0.2765853  0.349949022 -0.110251797  0.0009187691  0.025584105
## arena     -0.1685886 -0.280917821  0.066716120  0.0471314081 -0.027678522
## Limo       0.1364910 -0.055028435 -0.268982849 -0.0761805491 -0.009925485
##                Dim.11
## NAN_Am     0.0003323447
## NAtemp    -0.0015620088
## NASatO2   -0.0011859350
## Elevacion -0.0014619458
## Ancho      0.0002442393
## Velocidad  0.0012048108
## Rocas      0.0310054912
```

```
## Canto       0.0338619707
## grava       0.0248040851
## arena       0.0302477874
## Limo        0.0452748515
```

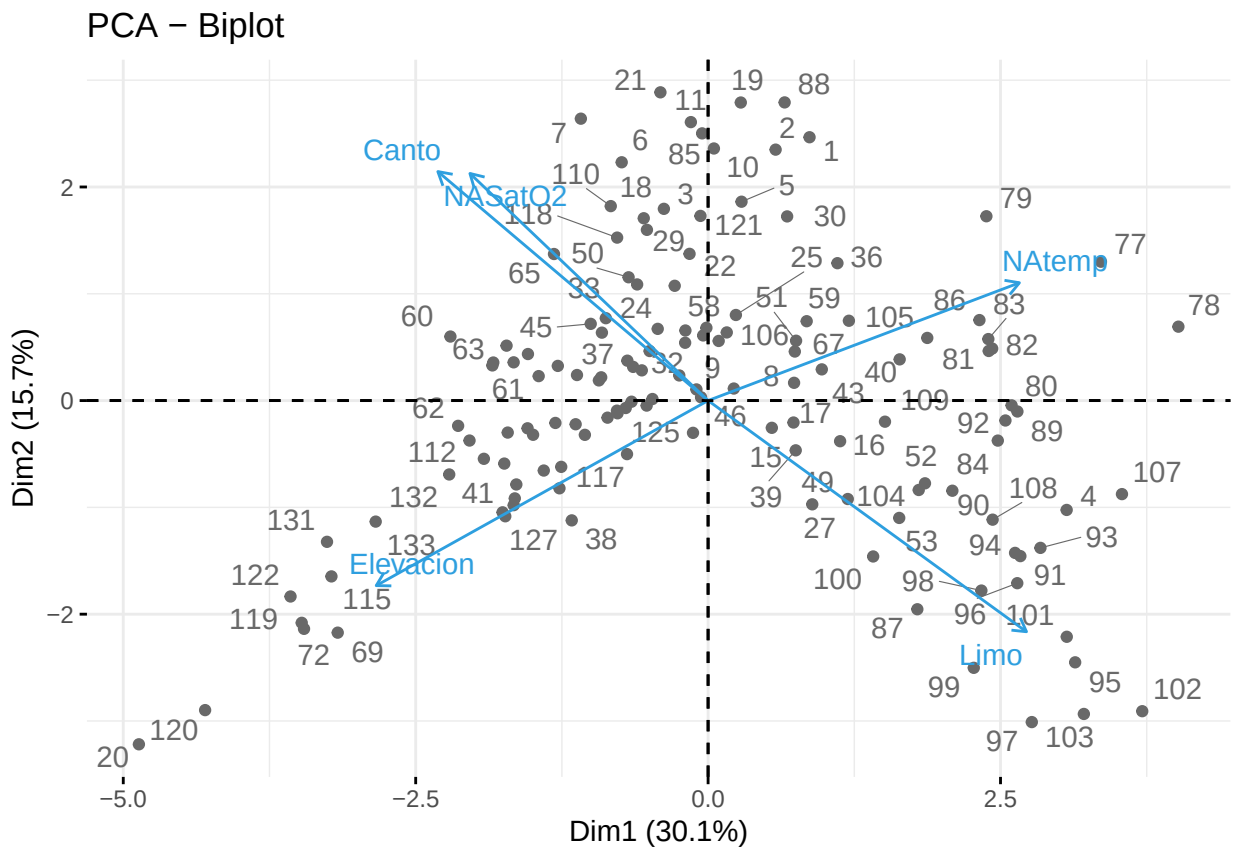
4 Otras formas de visualizar los datos.

```
fviz_eig(channel.pca)
```



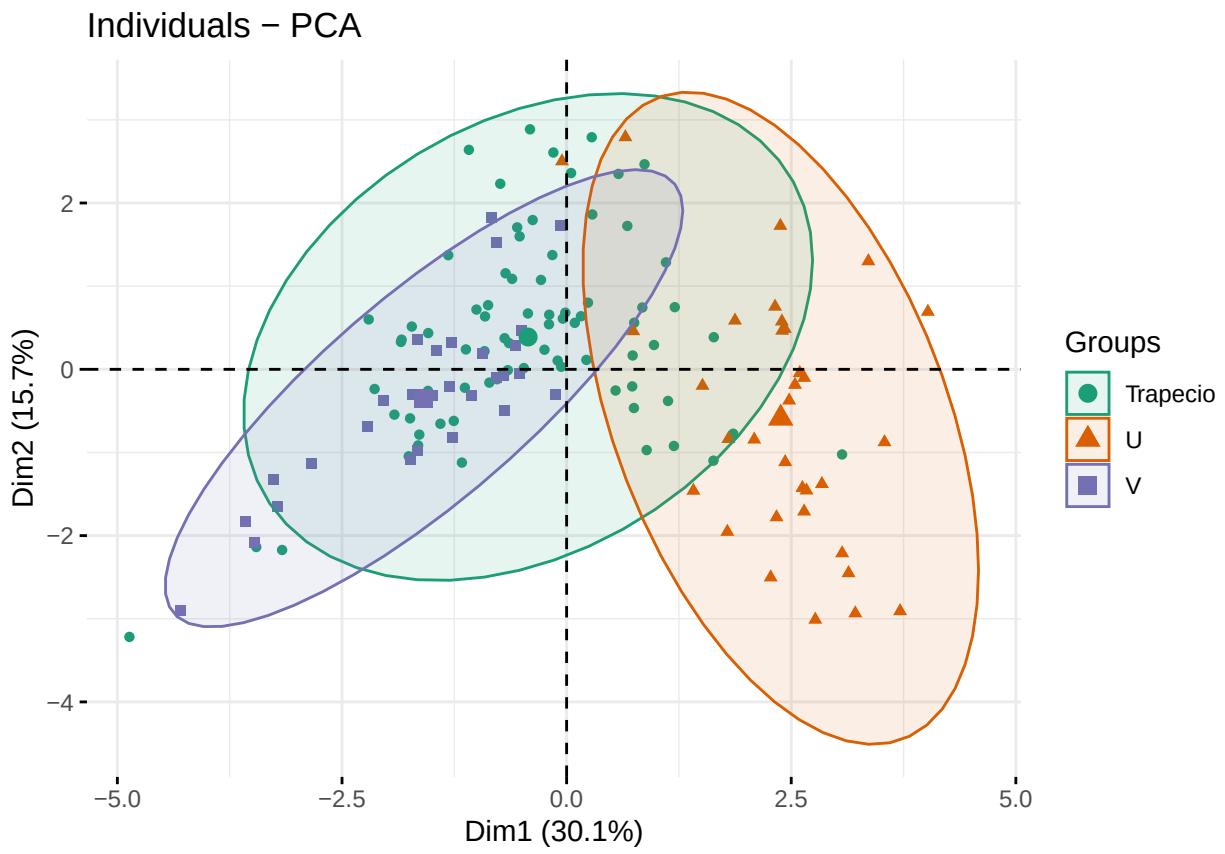

```
fviz_pca_biplot(channel.pca, repel = TRUE,
                col.var = "#2E9FDF", # Variables color
                col.ind = "#696969",  # Individuals color
                select.var = list(contrib = 5))
```

```
## Warning: ggrepel: 41 unlabeled data points (too many overlaps). Consider
## increasing max.overlaps
```



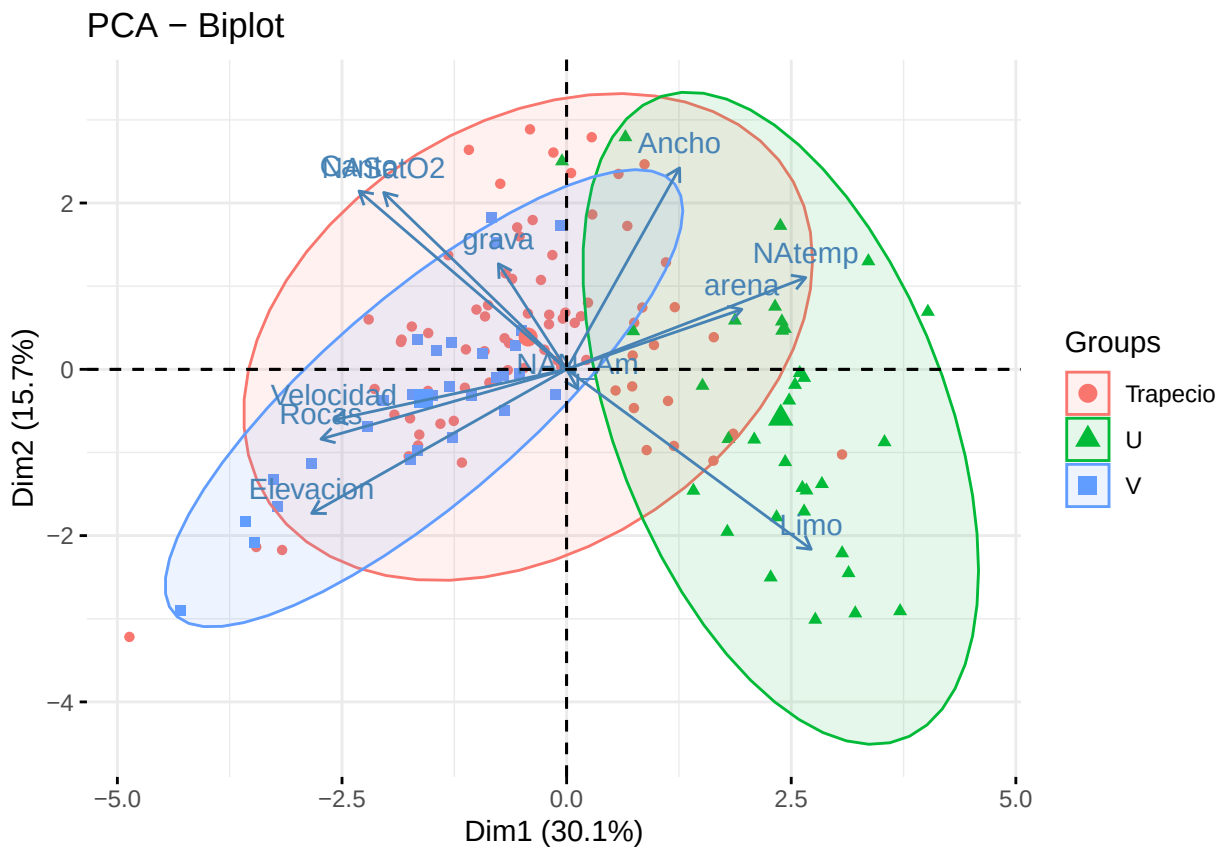
4.1 Con las elipses.

```
fviz_pca_ind(channel.pca, label="none", habillage=channel$Forma,
     addEllipses=TRUE, ellipse.level=0.95, palette = "Dark2")
```



4.1

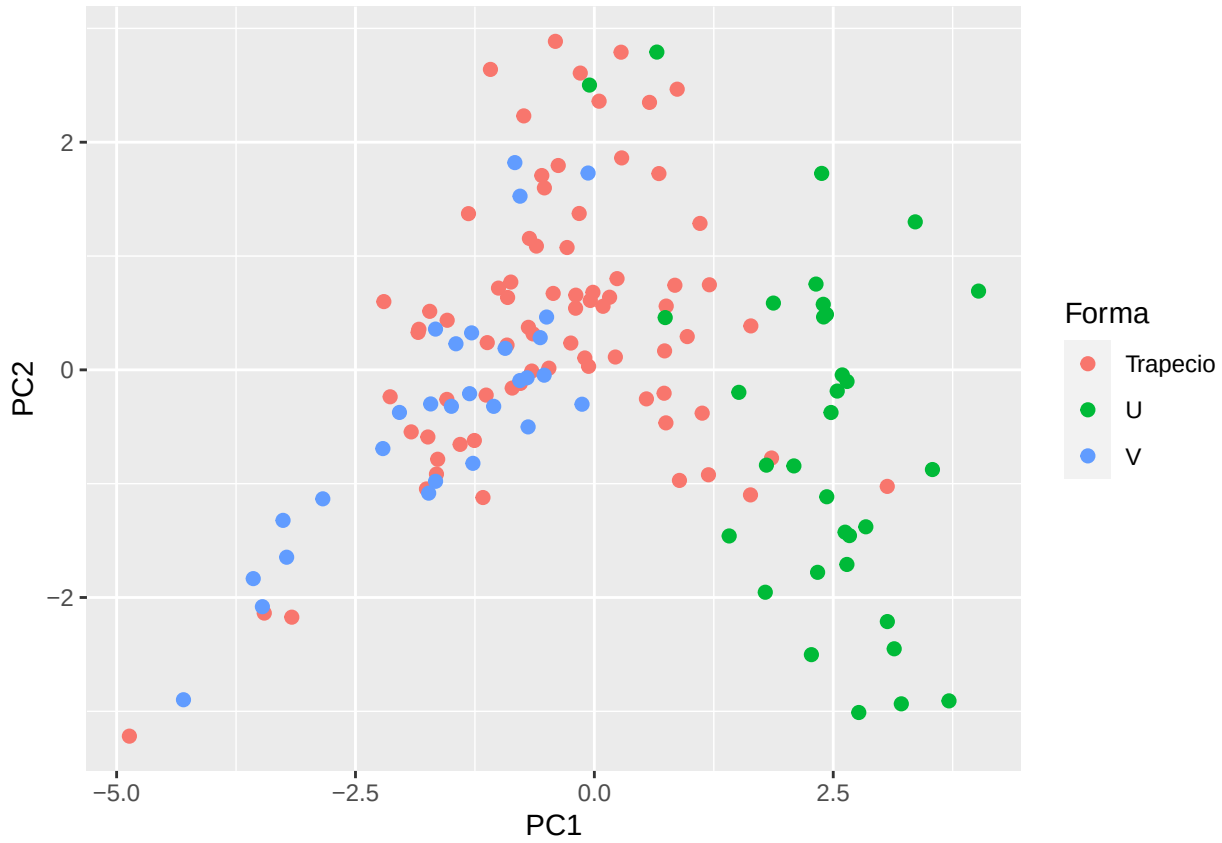
```
fviz_pca_biplot(channel.pca, label = "var", habillage=channel$Forma,
                addEllipses=TRUE, ellipse.level=0.95,
                ggtheme = theme_minimal())
```



5. Convertirlo en una data.frame para trabajarlo en ggplot2

```
data <- data.table(PC1=channel.pca$x[,1], PC2=channel.pca$x[,2], Forma=  channel[,1])
data <- data[order(channel$Forma),]

ggplot(data, aes(x=PC1,y=PC2)) +
  geom_point(size = 2, aes(color=Forma))
```


PC2
2
0
−2
−5.0
−2.5
0.0
2.5
PC1
Forma
Trapecio
U
V