

Principal Component Analysis (PCA)

Pablo E. Gutiérrez-Fonseca

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1. Primer paso: cargar las librerías que necesitas.

```
library(ggplot2)
library(dplyr)
library(missMDA) # Imputate
library(ggfortify) # autoplot()
library(cluster) #pam
library(factoextra) #get_pca_var()
library(data.table) # data.table()
library(devtools)

install_github("vqv/ggbiplot") #ggbiplot
library(ggbiplot)
```

2. Segundo paso: cargar los datos.

```
channel <- read.csv("data/channel_form.csv", header=TRUE)
head(channel)
```

```
##      Forma NAN_Am NADBO NAtemp  nit NASat02 Elevacion Ancho Velocidad Rocas
## 1 Trapecio  0.03  2.38  27.33 0.35   92.04        23   16         5    20
## 2 Trapecio  0.03  2.95  27.81  NA   100.03        31   11         0    20
## 3 Trapecio  0.03  3.13  24.27  NA    96.82        35   14        10    30
## 4 Trapecio  1.15  4.73  27.06 7.54   64.35         9    5         2     0
## 5 Trapecio  0.50  8.16  26.60  NA   110.39        43   11         9    10
## 6 Trapecio  0.53  8.57  23.82  NA   106.09        23   11         5    20
##      Canto grava arena Limo
## 1    25    30    20     0
## 2    45    20    15     0
## 3    30    20    10     0
## 4     0     0    50    50
## 5    40    10    20    20
## 6    60    20     0     0
```

- 2.1 Vamos a examinar los datos

```
summary(channel)
```

```
##      Forma      NAN_Am      NADBO      NAtemp
## Length:138      Min.    :0.0200      Min.    : 1.310      Min.    :14.67
## Class :character 1st Qu.:0.0400      1st Qu.: 1.930      1st Qu.:24.30
## Mode  :character Median :0.2150      Median : 3.000      Median :26.05
##                      Mean  :0.3201      Mean  : 6.164      Mean  :25.84
##                      3rd Qu.:0.5000      3rd Qu.: 8.585      3rd Qu.:27.70
##                      Max.   :1.5000      Max.   :34.900      Max.   :32.18
##                      NA's    :35
##      nit      NASat02      Elevacion      Ancho
## Min.    : 0.00      Min.    : 23.43      Min.    : 3.00      Min.    : 1.000
## 1st Qu.: 0.40      1st Qu.: 86.24      1st Qu.: 25.25      1st Qu.: 2.000
## Median : 0.92      Median : 94.59      Median : 53.00      Median : 3.000
## Mean   : 12.00      Mean   : 91.05      Mean   : 230.89      Mean   : 3.822
## 3rd Qu.: 1.62      3rd Qu.:100.52      3rd Qu.: 269.25      3rd Qu.: 3.000
## Max.   :324.11      Max.   :122.73      Max.   :2370.00      Max.   :16.000
## NA's    :57                      NA's    :3
##      Velocidad      Rocas      Canto      grava
## Min.    : 0.000      Min.    : 0.00      Min.    : 0.00      Min.    : 0.0
## 1st Qu.: 3.000      1st Qu.: 0.00      1st Qu.: 0.00      1st Qu.: 2.5
## Median :11.000      Median :10.00      Median :25.00      Median :20.0
## Mean   : 9.133      Mean   :16.25      Mean   :25.65      Mean   :17.8
## 3rd Qu.:14.000      3rd Qu.:30.00      3rd Qu.:40.00      3rd Qu.:25.0
## Max.   :16.000      Max.   :90.00      Max.   :80.00      Max.   :80.0
## NA's    :3          NA's    :3          NA's    :4          NA's    :3
##      arena      Limo
## Min.    : 0.00      Min.    : 0.00
## 1st Qu.: 10.00      1st Qu.: 0.00
## Median : 15.00      Median : 10.00
## Mean   : 19.79      Mean   : 20.62
## 3rd Qu.: 25.00      3rd Qu.: 25.00
## Max.   :100.00      Max.   :100.00
## NA's    :3          NA's    :3
```

2.1 Remover la(s) variable(s) que tiene(n) mucho(s) NAs y las Etiquetas (a la funcion lo le gusta), luego las agregamos.

```
channel_1 <- select(channel, -Forma)
summary(channel_1)
```

```
##      NAN_Am      NADBO      NAtemp      nit
## Min.    :0.0200      Min.    : 1.310      Min.    :14.67      Min.    : 0.00
## 1st Qu.:0.0400      1st Qu.: 1.930      1st Qu.:24.30      1st Qu.: 0.40
## Median :0.2150      Median : 3.000      Median :26.05      Median : 0.92
## Mean   :0.3201      Mean   : 6.164      Mean   :25.84      Mean   : 12.00
## 3rd Qu.:0.5000      3rd Qu.: 8.585      3rd Qu.:27.70      3rd Qu.: 1.62
## Max.   :1.5000      Max.   :34.900      Max.   :32.18      Max.   :324.11
##                      NA's    :35                      NA's    :57
##      NASat02      Elevacion      Ancho      Velocidad
## Min.    : 23.43      Min.    : 3.00      Min.    : 1.000      Min.    : 0.000
## 1st Qu.: 86.24      1st Qu.: 25.25      1st Qu.: 2.000      1st Qu.: 3.000
## Median : 94.59      Median : 53.00      Median : 3.000      Median :11.000
## Mean   : 91.05      Mean   : 230.89      Mean   : 3.822      Mean   : 9.133
## 3rd Qu.:100.52      3rd Qu.: 269.25      3rd Qu.: 3.000      3rd Qu.:14.000
```

```
## Max. :122.73 Max. :2370.00 Max. :16.000 Max. :16.000
## NA's :3 NA's :3
## Rocas Canto grava arena
## Min. : 0.00 Min. : 0.00 Min. : 0.0 Min. : 0.00
## 1st Qu.: 0.00 1st Qu.: 0.00 1st Qu.: 2.5 1st Qu.: 10.00
## Median :10.00 Median :25.00 Median :20.0 Median : 15.00
## Mean :16.25 Mean :25.65 Mean :17.8 Mean : 19.79
## 3rd Qu.:30.00 3rd Qu.:40.00 3rd Qu.:25.0 3rd Qu.: 25.00
## Max. :90.00 Max. :80.00 Max. :80.0 Max. :100.00
## NA's :3 NA's :4 NA's :3 NA's :3
## Limo
## Min. : 0.00
## 1st Qu.: 0.00
## Median : 10.00
## Mean : 20.62
## 3rd Qu.: 25.00
## Max. :100.00
## NA's :3
```

2.2 Vamos a imputar datos. Esto es comun para set de datos de campo, los cuales tienden a tener ceros (por mal funcionamiento de los equipos, condiciones climáticas adversas que no podemos ir al campo). Se realiza como un paso preliminar para para realizar un PCA en un set de datos completos.

Mas informacion aca: <https://www.rdocumentation.org/packages/missMDA/versions/1.18/topics/imputePCA>

```
df1 <- select(channel_1, Elevacion, Ancho, Velocidad, Rocas, Canto, grava, arena, Limo)
df1
```

```
## Elevacion Ancho Velocidad Rocas Canto grava arena Limo
## 1 23 16 5 20.0 25.0 30.0 20.0 0.0
## 2 31 11 0 20.0 45.0 20.0 15.0 0.0
## 3 35 14 10 30.0 30.0 20.0 10.0 0.0
## 4 9 5 2 0.0 0.0 0.0 50.0 50.0
## 5 43 11 9 10.0 40.0 10.0 20.0 20.0
## 6 23 11 5 20.0 60.0 20.0 0.0 0.0
## 7 86 11 13 0.0 80.0 20.0 20.0 0.0
## 8 26 3 11 0.0 30.0 20.0 25.0 25.0
## 9 24 3 14 5.0 25.0 35.0 20.0 15.0
## 10 53 11 4 0.0 70.0 5.0 20.0 5.0
## 11 24 11 3 0.0 70.0 20.0 10.0 0.0
## 12 619 2 14 30.0 30.0 20.0 20.0 0.0
## 13 598 3 14 20.0 30.0 20.0 10.0 20.0
## 14 583 3 14 20.0 30.0 20.0 10.0 20.0
## 15 114 2 11 0.0 15.0 30.0 25.0 15.0
## 16 46 3 14 0.0 5.0 20.0 40.0 35.0
## 17 46 3 16 0.0 1.0 40.0 40.0 19.0
## 18 158 11 4 40.0 50.0 5.0 5.0 0.0
## 19 34 11 4 0.0 70.0 15.0 15.0 0.0
## 20 1818 NA NA NA NA NA NA NA
## 21 205 11 4 0.0 80.0 10.0 10.0 0.0
## 22 38 3 13 0.0 40.0 30.0 20.0 10.0
## 23 98 3 14 25.0 25.0 15.0 25.0 10.0
## 24 49 3 15 10.0 60.0 10.0 10.0 10.0
```

## 25	29	3	14	5.0	25.0	30.0	25.0	15.0
## 26	99	3	14	25.0	40.0	0.0	10.0	25.0
## 27	20	3	14	15.0	15.0	5.0	30.0	35.0
## 28	82	1	11	60.0	0.0	20.0	20.0	0.0
## 29	43	2	11	0.0	50.0	50.0	0.0	0.0
## 30	17	3	2	0.0	33.3	33.3	33.3	0.0
## 31	149	3	3	90.0	10.0	0.0	0.0	0.0
## 32	10	1	14	15.0	20.0	40.0	10.0	15.0
## 33	28	1	14	10.0	70.0	10.0	0.0	10.0
## 34	18	1	12	10.0	20.0	50.0	10.0	10.0
## 35	85	1	14	10.0	20.0	50.0	10.0	10.0
## 36	130	2	2	0.0	30.0	0.0	70.0	0.0
## 37	51	2	10	35.0	50.0	0.0	15.0	0.0
## 38	198	1	3	90.0	0.0	0.0	10.0	0.0
## 39	13	1	3	33.3	33.3	0.0	0.0	33.3
## 40	53	2	3	0.0	0.0	50.0	50.0	0.0
## 41	492	2	14	50.0	20.0	10.0	10.0	10.0
## 42	428	2	14	20.0	40.0	20.0	10.0	10.0
## 43	49	3	11	0.0	10.0	30.0	50.0	10.0
## 44	67	3	12	20.0	40.0	20.0	10.0	10.0
## 45	67	1	11	10.0	60.0	20.0	5.0	5.0
## 46	100	2	9	20.0	35.0	25.0	15.0	5.0
## 47	83	1	14	50.0	30.0	10.0	5.0	5.0
## 48	63	1	12	10.0	60.0	20.0	5.0	5.0
## 49	60	3	12	10.0	5.0	5.0	30.0	50.0
## 50	25	3	11	0.0	70.0	20.0	5.0	5.0
## 51	30	3	11	0.0	10.0	40.0	40.0	10.0
## 52	50	2	3	0.0	0.0	0.0	50.0	50.0
## 53	36	2	3	0.0	0.0	10.0	20.0	70.0
## 54	22	3	11	0.0	20.0	60.0	10.0	10.0
## 55	11	2	12	0.0	0.0	80.0	20.0	0.0
## 56	71	3	14	5.0	50.0	20.0	15.0	10.0
## 57	15	3	12	0.0	10.0	70.0	10.0	10.0
## 58	85	3	9	5.0	60.0	20.0	10.0	5.0
## 59	21	3	11	0.0	10.0	60.0	20.0	10.0
## 60	659	2	13	10.0	70.0	20.0	0.0	0.0
## 61	615	3	14	30.0	30.0	30.0	10.0	0.0
## 62	517	3	14	50.0	30.0	10.0	10.0	0.0
## 63	422	2	14	30.0	40.0	20.0	10.0	0.0
## 64	363	3	14	30.0	40.0	20.0	5.0	5.0
## 65	117	3	14	10.0	70.0	10.0	10.0	0.0
## 66	244	2	9	25.0	30.0	25.0	15.0	5.0
## 67	15	2	11	0.0	0.0	40.0	40.0	20.0
## 68	22	3	14	10.0	40.0	30.0	15.0	5.0
## 69	1114	1	10	75.0	15.0	0.0	10.0	0.0
## 70	353	1	14	40.0	20.0	20.0	20.0	0.0
## 71	314	1	14	50.0	10.0	20.0	20.0	0.0
## 72	1630	1	14	30.0	25.0	25.0	0.0	20.0
## 73	628	1	14	30.0	20.0	20.0	20.0	10.0
## 74	137	1	14	20.0	60.0	10.0	10.0	0.0
## 75	51	2	14	30.0	25.0	25.0	0.0	20.0
## 76	27	2	8	40.0	30.0	20.0	10.0	0.0
## 77	27	12	0	0.0	0.0	15.0	85.0	0.0
## 78	15	12	0	0.0	0.0	10.0	90.0	0.0

## 79	16	12	0	0.0	0.0	20.0	80.0	0.0
## 80	15	5	2	0.0	0.0	0.0	50.0	50.0
## 81	6	11	2	0.0	0.0	0.0	50.0	50.0
## 82	3	11	2	0.0	0.0	0.0	50.0	50.0
## 83	10	11	2	0.0	0.0	0.0	50.0	50.0
## 84	8	11	2	0.0	0.0	0.0	0.0	100.0
## 85	86	11	4	0.0	65.0	20.0	10.0	5.0
## 86	26	11	2	0.0	0.0	0.0	50.0	50.0
## 87	9	2	11	0.0	0.0	0.0	0.0	100.0
## 88	28	5	2	0.0	NA	80.0	10.0	10.0
## 89	27	5	1	0.0	0.0	60.0	20.0	20.0
## 90	21	3	11	0.0	0.0	0.0	40.0	60.0
## 91	13	3	2	0.0	0.0	0.0	20.0	80.0
## 92	23	3	2	0.0	0.0	0.0	50.0	50.0
## 93	23	3	2	0.0	0.0	0.0	0.0	100.0
## 94	11	3	2	0.0	0.0	0.0	0.0	100.0
## 95	27	2	3	0.0	0.0	0.0	0.0	100.0
## 96	19	2	2	0.0	0.0	0.0	0.0	100.0
## 97	43	1	2	0.0	0.0	0.0	0.0	100.0
## 98	46	2	3	0.0	0.0	10.0	0.0	90.0
## 99	44	3	12	0.0	0.0	0.0	0.0	100.0
## 100	53	2	14	0.0	15.0	5.0	15.0	65.0
## 101	42	3	2	0.0	0.0	0.0	0.0	100.0
## 102	50	3	2	0.0	0.0	0.0	15.0	85.0
## 103	42	2	2	0.0	0.0	0.0	5.0	95.0
## 104	58	3	12	0.0	0.0	0.0	50.0	50.0
## 105	43	2	3	0.0	0.0	0.0	100.0	0.0
## 106	51	2	11	0.0	0.0	50.0	50.0	0.0
## 107	15	3	2	0.0	0.0	0.0	50.0	50.0
## 108	22	2	11	0.0	0.0	0.0	50.0	50.0
## 109	13	2	11	0.0	5.0	35.0	30.0	30.0
## 110	115	5	9	0.0	80.0	10.0	10.0	0.0
## 111	491	2	14	35.0	20.0	0.0	35.0	10.0
## 112	524	3	14	40.0	30.0	20.0	10.0	0.0
## 113	98	2	14	25.0	30.0	25.0	15.0	5.0
## 114	275	NA	NA	NA	NA	NA	NA	NA
## 115	1488	2	14	20.0	40.0	20.0	10.0	10.0
## 116	196	2	15	35.0	40.0	10.0	10.0	5.0
## 117	291	2	11	30.0	30.0	20.0	15.0	5.0
## 118	223	11	5	40.0	50.0	5.0	5.0	0.0
## 119	1346	NA	NA	NA	NA	NA	NA	NA
## 120	2370	2	15	25.0	25.0	35.0	10.0	5.0
## 121	17	11	4	30.0	40.0	10.0	10.0	10.0
## 122	1412	2	15	35.0	40.0	15.0	10.0	0.0
## 123	490	2	15	20.0	30.0	35.0	15.0	0.0
## 124	252	3	15	25.0	30.0	30.0	15.0	0.0
## 125	162	2	14	15.0	20.0	15.0	25.0	25.0
## 126	494	2	15	45.0	25.0	10.0	15.0	5.0
## 127	428	2	15	55.0	30.0	5.0	5.0	5.0
## 128	358	1	4	30.0	60.0	0.0	10.0	0.0
## 129	363	1	3	10.0	40.0	25.0	25.0	0.0
## 130	371	2	6	25.0	25.0	20.0	20.0	10.0
## 131	1420	1	5	40.0	40.0	10.0	10.0	0.0
## 132	828	1	4	40.0	40.0	10.0	10.0	0.0

```
## 133      952      1      14 50.0 20.0 20.0 10.0 0.0
## 134      422      2      13 30.0 40.0 20.0 10.0 0.0
## 135      144      3      15 50.0 30.0 10.0  5.0 5.0
## 136      200      3      14 15.0 30.0 30.0 20.0 5.0
## 137      327      2      13 40.0 30.0 20.0  8.0 2.0
## 138       60      3      15 30.0 25.0 10.0 30.0 5.0
```

```
df1a <- imputePCA(df1, ncp=2)
df1a
```

```
## $completeObs
##      Elevacion      Ancho Velocidad      Rocas      Canto      grava      arena
## [1,]      23 16.0000000  5.000000 20.00000 25.00000 30.00000 20.00000
## [2,]      31 11.0000000  0.000000 20.00000 45.00000 20.00000 15.00000
## [3,]      35 14.0000000 10.000000 30.00000 30.00000 20.00000 10.00000
## [4,]       9  5.0000000  2.000000  0.00000  0.00000  0.00000 50.00000
## [5,]      43 11.0000000  9.000000 10.00000 40.00000 10.00000 20.00000
## [6,]      23 11.0000000  5.000000 20.00000 60.00000 20.00000  0.00000
## [7,]      86 11.0000000 13.000000  0.00000 80.00000 20.00000 20.00000
## [8,]      26  3.0000000 11.000000  0.00000 30.00000 20.00000 25.00000
## [9,]      24  3.0000000 14.000000  5.00000 25.00000 35.00000 20.00000
## [10,]     53 11.0000000  4.000000  0.00000 70.00000  5.00000 20.00000
## [11,]     24 11.0000000  3.000000  0.00000 70.00000 20.00000 10.00000
## [12,]    619  2.0000000 14.000000 30.00000 30.00000 20.00000 20.00000
## [13,]    598  3.0000000 14.000000 20.00000 30.00000 20.00000 10.00000
## [14,]    583  3.0000000 14.000000 20.00000 30.00000 20.00000 10.00000
## [15,]    114  2.0000000 11.000000  0.00000 15.00000 30.00000 25.00000
## [16,]     46  3.0000000 14.000000  0.00000  5.00000 20.00000 40.00000
## [17,]     46  3.0000000 16.000000  0.00000  1.00000 40.00000 40.00000
## [18,]    158 11.0000000  4.000000 40.00000 50.00000  5.00000  5.00000
## [19,]     34 11.0000000  4.000000  0.00000 70.00000 15.00000 15.00000
## [20,]   1818 -0.6363273 17.068792 45.49584 43.78348 23.15703 -5.265541
## [21,]    205 11.0000000  4.000000  0.00000 80.00000 10.00000 10.00000
## [22,]     38  3.0000000 13.000000  0.00000 40.00000 30.00000 20.00000
## [23,]     98  3.0000000 14.000000 25.00000 25.00000 15.00000 25.00000
## [24,]     49  3.0000000 15.000000 10.00000 60.00000 10.00000 10.00000
## [25,]     29  3.0000000 14.000000  5.00000 25.00000 30.00000 25.00000
## [26,]     99  3.0000000 14.000000 25.00000 40.00000  0.00000 10.00000
## [27,]     20  3.0000000 14.000000 15.00000 15.00000  5.00000 30.00000
## [28,]     82  1.0000000 11.000000 60.00000  0.00000 20.00000 20.00000
## [29,]     43  2.0000000 11.000000  0.00000 50.00000 50.00000  0.00000
## [30,]     17  3.0000000  2.000000  0.00000 33.30000 33.30000 33.30000
## [31,]    149  3.0000000  3.000000 90.00000 10.00000  0.00000  0.00000
## [32,]     10  1.0000000 14.000000 15.00000 20.00000 40.00000 10.00000
## [33,]     28  1.0000000 14.000000 10.00000 70.00000 10.00000  0.00000
## [34,]     18  1.0000000 12.000000 10.00000 20.00000 50.00000 10.00000
## [35,]     85  1.0000000 14.000000 10.00000 20.00000 50.00000 10.00000
## [36,]    130  2.0000000  2.000000  0.00000 30.00000  0.00000 70.00000
## [37,]     51  2.0000000 10.000000 35.00000 50.00000  0.00000 15.00000
## [38,]    198  1.0000000  3.000000 90.00000  0.00000  0.00000 10.00000
## [39,]     13  1.0000000  3.000000 33.30000 33.30000  0.00000  0.00000
## [40,]     53  2.0000000  3.000000  0.00000  0.00000 50.00000 50.00000
## [41,]    492  2.0000000 14.000000 50.00000 20.00000 10.00000 10.00000
## [42,]    428  2.0000000 14.000000 20.00000 40.00000 20.00000 10.00000
```

##	[43,]	49	3.0000000	11.000000	0.00000	10.00000	30.00000	50.000000
##	[44,]	67	3.0000000	12.000000	20.00000	40.00000	20.00000	10.000000
##	[45,]	67	1.0000000	11.000000	10.00000	60.00000	20.00000	5.000000
##	[46,]	100	2.0000000	9.000000	20.00000	35.00000	25.00000	15.000000
##	[47,]	83	1.0000000	14.000000	50.00000	30.00000	10.00000	5.000000
##	[48,]	63	1.0000000	12.000000	10.00000	60.00000	20.00000	5.000000
##	[49,]	60	3.0000000	12.000000	10.00000	5.00000	5.00000	30.000000
##	[50,]	25	3.0000000	11.000000	0.00000	70.00000	20.00000	5.000000
##	[51,]	30	3.0000000	11.000000	0.00000	10.00000	40.00000	40.000000
##	[52,]	50	2.0000000	3.000000	0.00000	0.00000	0.00000	50.000000
##	[53,]	36	2.0000000	3.000000	0.00000	0.00000	10.00000	20.000000
##	[54,]	22	3.0000000	11.000000	0.00000	20.00000	60.00000	10.000000
##	[55,]	11	2.0000000	12.000000	0.00000	0.00000	80.00000	20.000000
##	[56,]	71	3.0000000	14.000000	5.00000	50.00000	20.00000	15.000000
##	[57,]	15	3.0000000	12.000000	0.00000	10.00000	70.00000	10.000000
##	[58,]	85	3.0000000	9.000000	5.00000	60.00000	20.00000	10.000000
##	[59,]	21	3.0000000	11.000000	0.00000	10.00000	60.00000	20.000000
##	[60,]	659	2.0000000	13.000000	10.00000	70.00000	20.00000	0.000000
##	[61,]	615	3.0000000	14.000000	30.00000	30.00000	30.00000	10.000000
##	[62,]	517	3.0000000	14.000000	50.00000	30.00000	10.00000	10.000000
##	[63,]	422	2.0000000	14.000000	30.00000	40.00000	20.00000	10.000000
##	[64,]	363	3.0000000	14.000000	30.00000	40.00000	20.00000	5.000000
##	[65,]	117	3.0000000	14.000000	10.00000	70.00000	10.00000	10.000000
##	[66,]	244	2.0000000	9.000000	25.00000	30.00000	25.00000	15.000000
##	[67,]	15	2.0000000	11.000000	0.00000	0.00000	40.00000	40.000000
##	[68,]	22	3.0000000	14.000000	10.00000	40.00000	30.00000	15.000000
##	[69,]	1114	1.0000000	10.000000	75.00000	15.00000	0.00000	10.000000
##	[70,]	353	1.0000000	14.000000	40.00000	20.00000	20.00000	20.000000
##	[71,]	314	1.0000000	14.000000	50.00000	10.00000	20.00000	20.000000
##	[72,]	1630	1.0000000	14.000000	30.00000	25.00000	25.00000	0.000000
##	[73,]	628	1.0000000	14.000000	30.00000	20.00000	20.00000	20.000000
##	[74,]	137	1.0000000	14.000000	20.00000	60.00000	10.00000	10.000000
##	[75,]	51	2.0000000	14.000000	30.00000	25.00000	25.00000	0.000000
##	[76,]	27	2.0000000	8.000000	40.00000	30.00000	20.00000	10.000000
##	[77,]	27	12.0000000	0.000000	0.00000	0.00000	15.00000	85.000000
##	[78,]	15	12.0000000	0.000000	0.00000	0.00000	10.00000	90.000000
##	[79,]	16	12.0000000	0.000000	0.00000	0.00000	20.00000	80.000000
##	[80,]	15	5.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[81,]	6	11.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[82,]	3	11.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[83,]	10	11.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[84,]	8	11.0000000	2.000000	0.00000	0.00000	0.00000	0.000000
##	[85,]	86	11.0000000	4.000000	0.00000	65.00000	20.00000	10.000000
##	[86,]	26	11.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[87,]	9	2.0000000	11.000000	0.00000	0.00000	0.00000	0.000000
##	[88,]	28	5.0000000	2.000000	0.00000	37.37683	80.00000	10.000000
##	[89,]	27	5.0000000	1.000000	0.00000	0.00000	60.00000	20.000000
##	[90,]	21	3.0000000	11.000000	0.00000	0.00000	0.00000	40.000000
##	[91,]	13	3.0000000	2.000000	0.00000	0.00000	0.00000	20.000000
##	[92,]	23	3.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[93,]	23	3.0000000	2.000000	0.00000	0.00000	0.00000	0.000000
##	[94,]	11	3.0000000	2.000000	0.00000	0.00000	0.00000	0.000000
##	[95,]	27	2.0000000	3.000000	0.00000	0.00000	0.00000	0.000000
##	[96,]	19	2.0000000	2.000000	0.00000	0.00000	0.00000	0.000000

##	[97,]	43	1.0000000	2.000000	0.00000	0.00000	0.00000	0.000000
##	[98,]	46	2.0000000	3.000000	0.00000	0.00000	10.00000	0.000000
##	[99,]	44	3.0000000	12.000000	0.00000	0.00000	0.00000	0.000000
##	[100,]	53	2.0000000	14.000000	0.00000	15.00000	5.00000	15.000000
##	[101,]	42	3.0000000	2.000000	0.00000	0.00000	0.00000	0.000000
##	[102,]	50	3.0000000	2.000000	0.00000	0.00000	0.00000	15.000000
##	[103,]	42	2.0000000	2.000000	0.00000	0.00000	0.00000	5.000000
##	[104,]	58	3.0000000	12.000000	0.00000	0.00000	0.00000	50.000000
##	[105,]	43	2.0000000	3.000000	0.00000	0.00000	0.00000	100.000000
##	[106,]	51	2.0000000	11.000000	0.00000	0.00000	50.00000	50.000000
##	[107,]	15	3.0000000	2.000000	0.00000	0.00000	0.00000	50.000000
##	[108,]	22	2.0000000	11.000000	0.00000	0.00000	0.00000	50.000000
##	[109,]	13	2.0000000	11.000000	0.00000	5.00000	35.00000	30.000000
##	[110,]	115	5.0000000	9.000000	0.00000	80.00000	10.00000	10.000000
##	[111,]	491	2.0000000	14.000000	35.00000	20.00000	0.00000	35.000000
##	[112,]	524	3.0000000	14.000000	40.00000	30.00000	20.00000	10.000000
##	[113,]	98	2.0000000	14.000000	25.00000	30.00000	25.00000	15.000000
##	[114,]	275	3.6434578	9.451479	17.41934	26.46385	18.01685	18.782967
##	[115,]	1488	2.0000000	14.000000	20.00000	40.00000	20.00000	10.000000
##	[116,]	196	2.0000000	15.000000	35.00000	40.00000	10.00000	10.000000
##	[117,]	291	2.0000000	11.000000	30.00000	30.00000	20.00000	15.000000
##	[118,]	223	11.0000000	5.000000	40.00000	50.00000	5.00000	5.000000
##	[119,]	1346	0.6728487	14.738674	36.90731	38.48545	21.58466	2.090840
##	[120,]	2370	2.0000000	15.000000	25.00000	25.00000	35.00000	10.000000
##	[121,]	17	11.0000000	4.000000	30.00000	40.00000	10.00000	10.000000
##	[122,]	1412	2.0000000	15.000000	35.00000	40.00000	15.00000	10.000000
##	[123,]	490	2.0000000	15.000000	20.00000	30.00000	35.00000	15.000000
##	[124,]	252	3.0000000	15.000000	25.00000	30.00000	30.00000	15.000000
##	[125,]	162	2.0000000	14.000000	15.00000	20.00000	15.00000	25.000000
##	[126,]	494	2.0000000	15.000000	45.00000	25.00000	10.00000	15.000000
##	[127,]	428	2.0000000	15.000000	55.00000	30.00000	5.00000	5.000000
##	[128,]	358	1.0000000	4.000000	30.00000	60.00000	0.00000	10.000000
##	[129,]	363	1.0000000	3.000000	10.00000	40.00000	25.00000	25.000000
##	[130,]	371	2.0000000	6.000000	25.00000	25.00000	20.00000	20.000000
##	[131,]	1420	1.0000000	5.000000	40.00000	40.00000	10.00000	10.000000
##	[132,]	828	1.0000000	4.000000	40.00000	40.00000	10.00000	10.000000
##	[133,]	952	1.0000000	14.000000	50.00000	20.00000	20.00000	10.000000
##	[134,]	422	2.0000000	13.000000	30.00000	40.00000	20.00000	10.000000
##	[135,]	144	3.0000000	15.000000	50.00000	30.00000	10.00000	5.000000
##	[136,]	200	3.0000000	14.000000	15.00000	30.00000	30.00000	20.000000
##	[137,]	327	2.0000000	13.000000	40.00000	30.00000	20.00000	8.000000
##	[138,]	60	3.0000000	15.000000	30.00000	25.00000	10.00000	30.000000
##			Limo					
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## [125,] 25.000000
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##
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##      [,1]      [,2]      [,3]      [,4]      [,5]      [,6]
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## [2,]  40.316182  6.33027358  7.366733  7.0270957  31.7723778  23.106454
## [3,] 130.752942  5.64824076  8.648592 11.6836478  34.9401167  24.135119
## [4,] -70.087579  5.29957377  4.422942  0.9739984  7.2781077  9.611777
## [5,]  77.037619  5.65034666  7.588999  8.8381801  28.5974636  20.877156
## [6,] 144.004515  5.71938369  8.963026 12.3997954  37.2981626  25.409562
## [7,] 145.285463  6.18030349  9.329428 12.5588035  42.5524160  28.515067
## [8,] 170.415404  4.32859832  8.455546 13.5240528  25.0320153  17.883084
## [9,] 234.931763  4.11178735  9.569625 16.8993128  30.2779177  20.388679
## [10,]  30.004264  6.57342613  7.342962  6.5288424  33.2420127  24.075467
## [11,]  46.472936  6.71307940  7.771650  7.4289819  36.7397454  25.995864
## [12,] 420.819135  2.43975764 12.004487 26.4171584  33.7983986  20.728586
## [13,] 384.288609  2.53084636 11.350232 24.4997516  30.4774198  19.101818
## [14,] 382.170398  2.54992635 11.322507 24.3912998  30.4376021  19.098122
## [15,] 194.784578  4.12257268  8.784495 14.7744131  25.6393482  18.014160
## [16,] 139.187589  4.14484972  7.702657 11.8332418  19.2969905  14.774076
## [17,] 171.429775  4.38203689  8.515131 13.5883620  25.7438037  18.295872
## [18,] 187.789794  4.91770696  9.234740 14.5610535  33.6126190  22.810827
## [19,]  45.089678  6.66730998  7.710453  7.3466453  36.0691377  25.610980
## [20,] 792.412731 -0.63633887 17.068936 45.4962393  43.7842775  23.157440
## [21,]  89.690150  6.33793085  8.347787  9.6444852  37.7086032  26.164113
## [22,] 226.721213  4.38882617  9.612443 16.5190925  32.3717176  21.708263
## [23,] 291.972656  3.35681208 10.137765 19.7720995  28.6803286  18.904093
## [24,] 318.669580  3.58678781 10.835356 21.2320010  34.3901242  22.040761
## [25,] 222.081565  4.14973778  9.343853 16.2260026  29.1751181  19.855246
## [26,] 330.490924  2.77919461 10.471240 21.6986123  26.8509418  17.456244
## [27,] 211.476480  3.40735591  8.584960 15.5173373  19.7000098  14.333069
## [28,] 391.778598  2.12928451 11.201028 24.8171727  26.9197371  16.920434
## [29,] 291.312798  4.32092917 10.838210 19.9277844  39.2750330  25.196558
## [30,]  73.116852  5.65618485  7.515864  8.6316090  28.1974313  20.676690
## [31,] 428.454975  1.58838617 11.525290 26.6533614  25.2785480  15.601580
## [32,] 316.308888  3.26613950 10.551429 21.0435244  30.5607314  19.790841

```

##	[33,]	370.674669	3.13566048	11.528871	23.8980694	35.5593644	22.245224
##	[34,]	286.145296	3.72950552	10.298451	19.5370575	32.1154771	20.997130
##	[35,]	317.128383	3.48956188	10.732959	21.1311216	33.1311681	21.308273
##	[36,]	3.169643	5.66196365	6.138327	4.9268883	19.9717477	16.454170
##	[37,]	338.899267	3.08747220	10.865484	22.2050524	31.2601144	19.993270
##	[38,]	433.087665	1.28321455	11.390971	26.8384606	22.4492911	13.879283
##	[39,]	270.398557	2.76642764	9.274662	18.5123424	19.5878813	13.712166
##	[40,]	36.592332	5.70797672	6.832647	6.7067497	24.4421427	18.792173
##	[41,]	470.611570	1.53300093	12.317110	28.8759022	29.6615317	17.805544
##	[42,]	391.464391	2.70807114	11.623143	24.9149752	33.2897456	20.702968
##	[43,]	104.725441	4.99808712	7.653280	10.1761268	24.6582302	18.279351
##	[44,]	303.103194	3.52149316	10.479520	20.3943691	31.8224841	20.663736
##	[45,]	327.658761	3.54402569	10.981292	21.6997998	34.9820769	22.307414
##	[46,]	278.549460	3.70420121	10.129669	19.1296198	30.9351494	20.368260
##	[47,]	453.381237	1.85589673	12.215677	28.0268738	31.1940128	18.876938
##	[48,]	337.854775	3.47175384	11.129232	22.2257023	35.3903782	22.453746
##	[49,]	162.086297	3.44146971	7.634497	12.9073485	14.2242742	11.548789
##	[50,]	268.934479	4.33850619	10.409132	18.7456375	36.8174908	23.948949
##	[51,]	129.154911	4.93363425	8.088189	11.4576763	26.8399387	19.343999
##	[52,]	-5.953439	4.57152984	5.151140	4.2279168	6.8192943	8.736135
##	[53,]	56.558326	3.86737015	5.864963	7.4006033	6.4326100	7.918554
##	[54,]	216.259403	4.61039082	9.569735	16.0086291	33.5846074	22.526349
##	[55,]	208.846788	4.79929946	9.563097	15.6532572	34.7973602	23.315665
##	[56,]	276.578784	3.95282702	10.274730	19.0743751	33.4539200	21.881288
##	[57,]	219.135184	4.60949186	9.625881	16.1608127	33.9154733	22.695607
##	[58,]	249.416501	4.32397727	10.012801	17.7086852	34.3435190	22.664680
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##	[60,]	441.583162	2.83795232	12.709361	27.5959936	40.6672751	24.608802
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##	[65,]	332.832285	3.73814741	11.227153	22.0122832	37.7441605	23.897592
##	[66,]	309.419772	3.35001580	10.477405	20.6951189	30.6728034	19.922155
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##	[69,]	606.159437	0.08959728	13.926702	35.7719260	29.7470467	16.580972
##	[70,]	420.188308	2.23656351	11.841655	26.3435570	31.4742509	19.355512
##	[71,]	435.751920	1.94413634	11.932709	27.1103060	30.0815335	18.382727
##	[72,]	613.375689	0.35468767	14.265436	36.2066683	33.5368530	18.761720
##	[73,]	422.630209	2.06830327	11.765376	26.4396592	29.9009844	18.399056
##	[74,]	388.449857	2.97165089	11.758649	24.8073829	35.8503474	22.250636
##	[75,]	374.537452	2.56268463	11.181159	23.9894226	29.6742345	18.717001
##	[76,]	327.735251	3.15872724	10.697669	21.6276633	30.7258361	19.781299
##	[77,]	-252.867366	8.27840828	3.016552	-8.1207967	18.5924217	18.044702
##	[78,]	-268.118204	8.33798269	2.759357	-8.9170190	17.4444954	17.507081
##	[79,]	-240.864452	8.24808985	3.231234	-7.4908671	19.6792942	18.576657
##	[80,]	-69.240294	5.29194178	4.434033	1.0173792	7.2940348	9.613256
##	[81,]	-165.678203	6.50045402	3.423234	-3.8530128	9.2432798	11.677162
##	[82,]	-166.101845	6.50427002	3.417689	-3.8747032	9.2353162	11.676423
##	[83,]	-165.113346	6.49536602	3.430627	-3.8240923	9.2538978	11.678148
##	[84,]	-71.955213	4.91454293	4.101111	0.7989135	2.7944253	6.969003
##	[85,]	58.328573	6.47660778	7.830862	8.0103438	35.5270200	25.164761
##	[86,]	-162.853921	6.47501404	3.460201	-3.7084104	9.2963700	11.682090

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## [87,] 167.784309 2.42143587 6.992201 13.0075323 3.6076505 5.195917
## [88,] 105.500133 6.13870481 8.512681 10.4427167 37.3768331 25.818512
## [89,] 28.304295 5.77233285 6.716541 6.2803682 24.1723335 18.710063
## [90,] 78.865265 3.87237626 6.309344 8.5834341 9.1316913 9.310149
## [91,] 18.263942 3.94544411 5.166232 5.3871722 2.7585158 6.098864
## [92,] -36.388254 4.88274437 4.780207 2.6769938 6.6575590 8.926519
## [93,] 57.052307 3.29937728 5.454387 7.3144598 0.2033956 4.217867
## [94,] 55.357738 3.31464127 5.432207 7.2276984 0.1715414 4.214911
## [95,] 84.239198 3.01741872 5.782808 8.6990901 0.3040770 4.021816
## [96,] 72.348614 3.10495457 5.612688 8.0864262 -0.1360784 3.872528
## [97,] 91.598917 2.87491589 5.822743 9.0608358 -0.4012260 3.534087
## [98,] 95.346692 3.22130332 6.153119 9.3278909 3.8774897 6.037557
## [99,] 167.626509 2.49906676 7.046534 13.0145227 4.4483337 5.696212
## [100,] 205.441910 2.96316829 8.137032 15.1097868 14.0676178 11.047925
## [101,] 59.735375 3.27520930 5.489507 7.4518321 0.2538313 4.222549
## [102,] 32.832920 3.74004344 5.302040 6.1184332 2.2113164 5.637116
## [103,] 66.252483 3.23403530 5.587782 7.7889723 0.5703917 4.349060
## [104,] 76.162946 4.06462587 6.398231 8.4782783 10.9396603 10.408314
## [105,] -100.382499 6.16380092 4.464022 -0.4601600 13.2548761 13.443062
## [106,] 122.396868 5.09164188 8.071614 11.1308738 27.7881884 19.970216
## [107,] -37.517967 4.89292036 4.765420 2.6191528 6.6363229 8.924548
## [108,] 76.179531 3.98826697 6.342050 8.4640578 10.0963226 9.907772
## [109,] 149.430826 4.24479998 7.978980 12.3957015 21.6174105 16.054499
## [110,] 225.952215 4.86854723 9.952262 16.5732105 37.5911582 24.812398
## [111,] 360.737683 2.37021307 10.766107 23.2202395 25.9080959 16.612238
## [112,] 446.622722 2.26189538 12.382616 27.7490842 34.8875005 21.132018
## [113,] 342.606866 3.14892837 10.984208 22.4136366 32.3798413 20.622765
## [114,] 246.495848 3.64345629 9.451493 17.4193801 26.4639244 18.016885
## [115,] 541.151343 1.35975232 13.582420 32.5789020 36.1035283 20.964157
## [116,] 415.627131 2.42545328 11.891333 26.1392530 33.0247272 20.318385
## [117,] 351.562353 2.93071613 10.999640 22.8449567 31.0255443 19.734949
## [118,] 207.729582 4.75766718 9.510217 15.5858343 34.2040816 22.974161
## [119,] 625.418066 0.67284021 14.738778 36.9075954 38.4860230 21.584956
## [120,] 684.314159 0.17105507 15.530938 39.9287380 39.9111481 21.876411
## [121,] 111.033768 5.39046558 8.068275 10.5879364 29.7495315 21.240869
## [122,] 591.555696 0.99272892 14.306553 35.1767927 38.0141022 21.623543
## [123,] 398.952102 2.89176725 11.907005 25.3480047 36.2106777 22.365624
## [124,] 363.465758 3.18921768 11.426093 23.5267255 35.2978999 22.157912
## [125,] 272.819036 3.17718506 9.626453 18.7218047 24.4218797 16.557595
## [126,] 461.775332 1.78245135 12.327151 28.4570771 31.3757721 18.905807
## [127,] 510.983587 1.33441196 12.967700 30.9755774 32.2477349 18.960198
## [128,] 340.017021 3.05454565 10.863199 22.2577611 31.0280815 19.845079
## [129,] 219.565128 4.21277463 9.340790 16.1051445 29.5747139 20.116018
## [130,] 273.854575 3.38461409 9.800415 18.8176855 26.8408719 17.983136
## [131,] 514.921643 1.35184474 13.058398 31.1876662 32.9074292 19.314569
## [132,] 420.562022 2.18222794 11.808827 26.3526124 30.9170388 19.021380
## [133,] 559.855648 0.97642331 13.668248 33.4940726 34.0767282 19.585596
## [134,] 416.248214 2.61152354 12.041301 26.2089521 35.1581565 21.578390
## [135,] 441.033841 2.09996636 12.152373 27.4209605 32.4325692 19.727994
## [136,] 299.625719 3.67259764 10.522645 20.2400089 33.0831050 21.444430
## [137,] 427.641458 2.32699362 12.055814 26.7563130 33.3586198 20.403459
## [138,] 302.007111 3.28132888 10.280137 20.2888075 29.0339329 19.019491
##      [,7]      [,8]
## [1,] 30.6914169 8.0686240

```

```

## [2,] 28.1931415 10.6551478
## [3,] 24.2174595 5.7455437
## [4,] 32.4764177 49.6462663
## [5,] 26.4555189 15.7982101
## [6,] 23.6872588 1.9964757
## [7,] 23.7749444 -6.4065957
## [8,] 22.1615813 21.6649203
## [9,] 19.4079246 13.3855979
## [10,] 28.6970704 8.2861494
## [11,] 28.0538314 2.7195236
## [12,] 11.1534809 8.0783350
## [13,] 12.7029672 13.3271832
## [14,] 12.7970360 13.3871824
## [15,] 21.0834835 20.7359020
## [16,] 23.4060927 30.7848109
## [17,] 22.1356817 20.5280327
## [18,] 21.6181493 7.9687307
## [19,] 28.0974430 3.7898900
## [20,] -5.2658646 -7.2475348
## [21,] 26.1389009 1.2450713
## [22,] 19.8346940 10.0217817
## [23,] 16.8009670 16.0408612
## [24,] 15.7593271 6.9534205
## [25,] 19.9547892 15.1273303
## [26,] 15.0198073 19.0345998
## [27,] 20.1693565 30.2662894
## [28,] 12.2681075 19.0315366
## [29,] 17.1234715 -0.9087978
## [30,] 26.6206179 16.4313033
## [31,] 10.5749046 21.7209972
## [32,] 15.7595393 13.0752365
## [33,] 13.4551041 5.1737546
## [34,] 17.1577391 10.5354299
## [35,] 15.7937719 8.9647010
## [36,] 29.5359112 29.4679456
## [37,] 14.7639057 11.9958589
## [38,] 10.2885559 26.2550891
## [39,] 17.5189399 30.5485232
## [40,] 28.1578293 22.3749338
## [41,] 8.8019997 14.7830654
## [42,] 12.4583036 8.8407903
## [43,] 25.1026415 22.1481952
## [44,] 16.3877368 11.0337386
## [45,] 15.3718138 6.0221574
## [46,] 17.4663869 12.4103577
## [47,] 9.6185041 12.3014534
## [48,] 14.9250021 5.3867909
## [49,] 22.2370544 38.9396853
## [50,] 18.0609785 2.9835035
## [51,] 24.0653535 18.7007299
## [52,] 29.5822446 50.4921959
## [53,] 26.7629572 51.2199071
## [54,] 20.3382605 8.0632407
## [55,] 20.7048255 6.1102418

```

```

## [56,] 17.6245512 8.3776026
## [57,] 20.2181999 7.5389698
## [58,] 18.8695189 6.9070807
## [59,] 21.8089110 11.6898007
## [60,] 10.4104401 -2.8736617
## [61,] 10.7438424 4.0171059
## [62,] 8.7778790 8.4091540
## [63,] 10.9245379 5.2437642
## [64,] 11.5785992 5.9747905
## [65,] 15.2157211 1.6126423
## [66,] 16.0721591 12.8839268
## [67,] 24.0267616 25.0346418
## [68,] 17.4540832 6.8621828
## [69,] 2.7143168 14.8828912
## [70,] 11.1175796 11.7952075
## [71,] 10.3798228 14.0503278
## [72,] 2.4948544 8.8328788
## [73,] 10.9643790 14.3162483
## [74,] 12.6645239 4.7392949
## [75,] 13.1188766 14.5950282
## [76,] 15.2507266 12.8310629
## [77,] 41.0013020 31.2275110
## [78,] 41.6547779 33.0372425
## [79,] 40.4920648 29.5097783
## [80,] 32.4387902 49.6222667
## [81,] 36.8255415 46.3356790
## [82,] 36.8443553 46.3476788
## [83,] 36.8004565 46.3196792
## [84,] 32.4363914 56.8156197
## [85,] 27.4876460 4.6802348
## [86,] 36.7001165 46.2556800
## [87,] 21.6875842 55.9332039
## [88,] 25.4194015 1.8034084
## [89,] 28.5227454 22.7920833
## [90,] 25.8353337 46.9410875
## [91,] 28.3819314 57.0305600
## [92,] 30.9451834 50.6977962
## [93,] 26.5685758 61.1857368
## [94,] 26.6438308 61.2337362
## [95,] 25.3498757 61.0721353
## [96,] 25.8719425 61.7555012
## [97,] 24.9997141 62.2132670
## [98,] 24.9496016 55.3750839
## [99,] 21.7179121 54.5880749
## [100,] 20.2847945 39.2659766
## [101,] 26.4494221 61.1097379
## [102,] 27.7122344 57.9313561
## [103,] 26.1653645 60.6147083
## [104,] 26.0067222 44.0441350
## [105,] 34.0027510 40.0322548
## [106,] 24.3951978 17.1720032
## [107,] 30.9953535 50.7297957
## [108,] 25.9826656 45.3932639
## [109,] 23.0100138 27.0906820

```

```
## [110,] 20.0135196 1.6708221
## [111,] 13.6347849 20.5956861
## [112,] 10.0242532 6.3811277
## [113,] 14.6282779 10.2110873
## [114,] 18.7829344 19.5070874
## [115,] 5.8107786 4.6008481
## [116,] 11.3653674 9.3069251
## [117,] 14.1884804 12.3932101
## [118,] 20.7386213 7.0573680
## [119,] 2.0906054 0.9366399
## [120,] -0.5161512 -1.2403393
## [121,] 24.9599471 14.0145767
## [122,] 3.5989671 1.6324652
## [123,] 12.2026274 4.1812028
## [124,] 13.7717668 5.5794387
## [125,] 17.5438107 22.8197291
## [126,] 9.2463890 12.0253439
## [127,] 7.0596080 10.7163547
## [128,] 14.7072722 12.3689967
## [129,] 20.0788961 14.4836979
## [130,] 17.5641504 18.9518389
## [131,] 6.9009097 9.6679065
## [132,] 11.0853865 12.6872406
## [133,] 4.9143860 7.8758025
## [134,] 11.3964346 5.8951305
## [135,] 10.2074978 10.2985614
## [136,] 16.5788226 9.0110338
## [137,] 10.8348036 8.7937654
## [138,] 16.3599020 15.4927121
```

```
df2 <- select(channel_1, Elevacion, NAN_Am, NATemp, NASatO2, nit, NADBO)
df2a <- imputePCA(df2, ncp=2, scale = TRUE, method = c("Regularized","EM"),
  row.w = NULL, ind.sup=NULL, quanti.sup=NULL, quali.sup=NULL,
  coeff.ridge = 1, threshold = 1e-06, seed = NULL, nb.init = 1,
  maxiter = 1000)

df1b <- as.data.frame(df1a) # Sustrata
df2b <- as.data.frame(df2a) # Physicochemical

new_channel <- do.call("merge", c(lapply(list(df1b, df2b), data.frame, row.names=NULL),
  by = 0, all = TRUE, sort = FALSE))[-1]

new_channel2 <- select(new_channel,
  completeObs.Elevacion.x, completeObs.Ancho, completeObs.Velocidad,
  completeObs.Rocas, completeObs.Canto, completeObs.grava, completeObs.arena,
  completeObs.Limo, completeObs.NAN_Am, completeObs.NATemp, completeObs.NASatO2,
  )
```

3. Vamos a correr el PCA

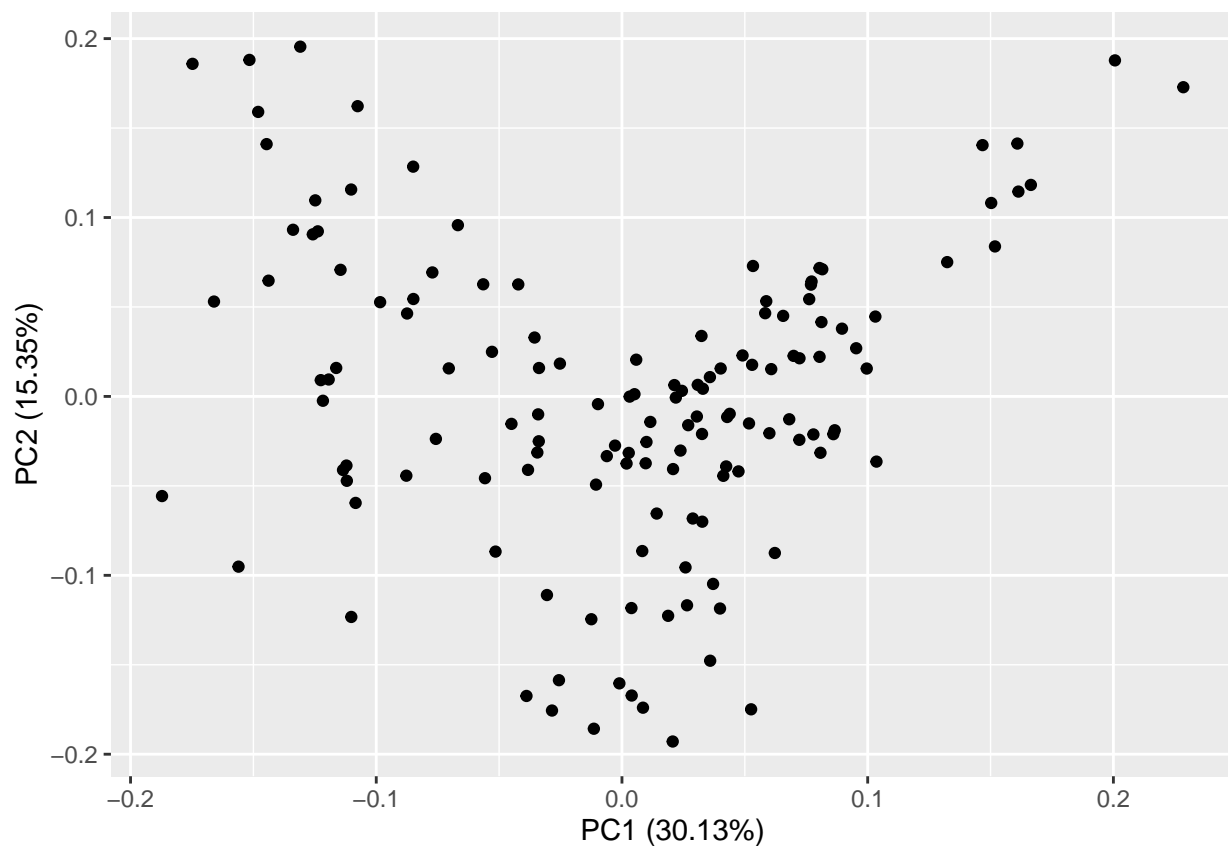
```
channel.pca <- prcomp(new_channel2, center = TRUE, scale = TRUE)
summary(channel.pca)
```



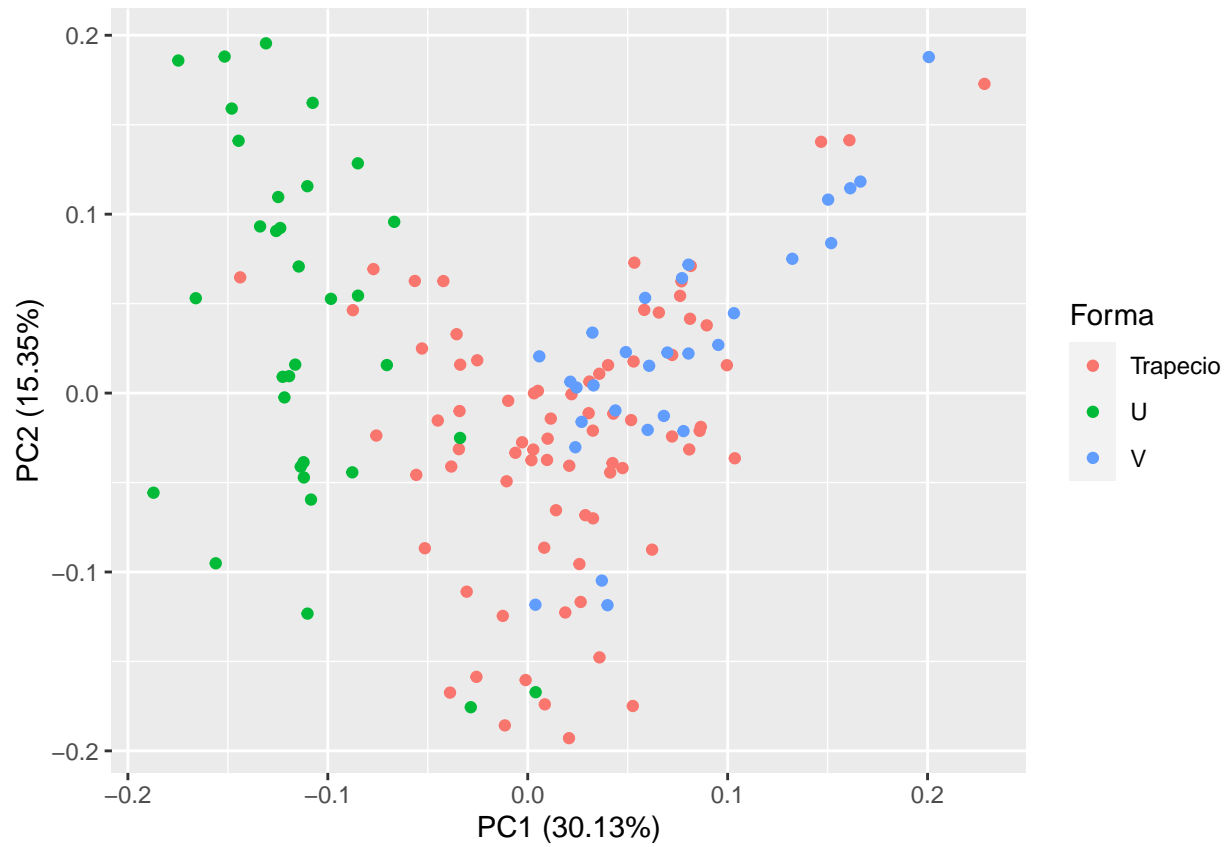
```
## Importance of components:
##               PC1    PC2    PC3    PC4    PC5    PC6    PC7
## Standard deviation  1.8204 1.2996 1.1989 1.1224 1.00580 0.88356 0.75458
## Proportion of Variance 0.3013 0.1535 0.1307 0.1145 0.09197 0.07097 0.05176
## Cumulative Proportion 0.3013 0.4548 0.5855 0.7000 0.79195 0.86292 0.91468
##               PC8    PC9    PC10    PC11
## Standard deviation  0.66732 0.59198 0.37028 0.07501
## Proportion of Variance 0.04048 0.03186 0.01246 0.00051
## Cumulative Proportion 0.95517 0.98702 0.99949 1.00000
```

3.1 Vamos a ver el grafico.

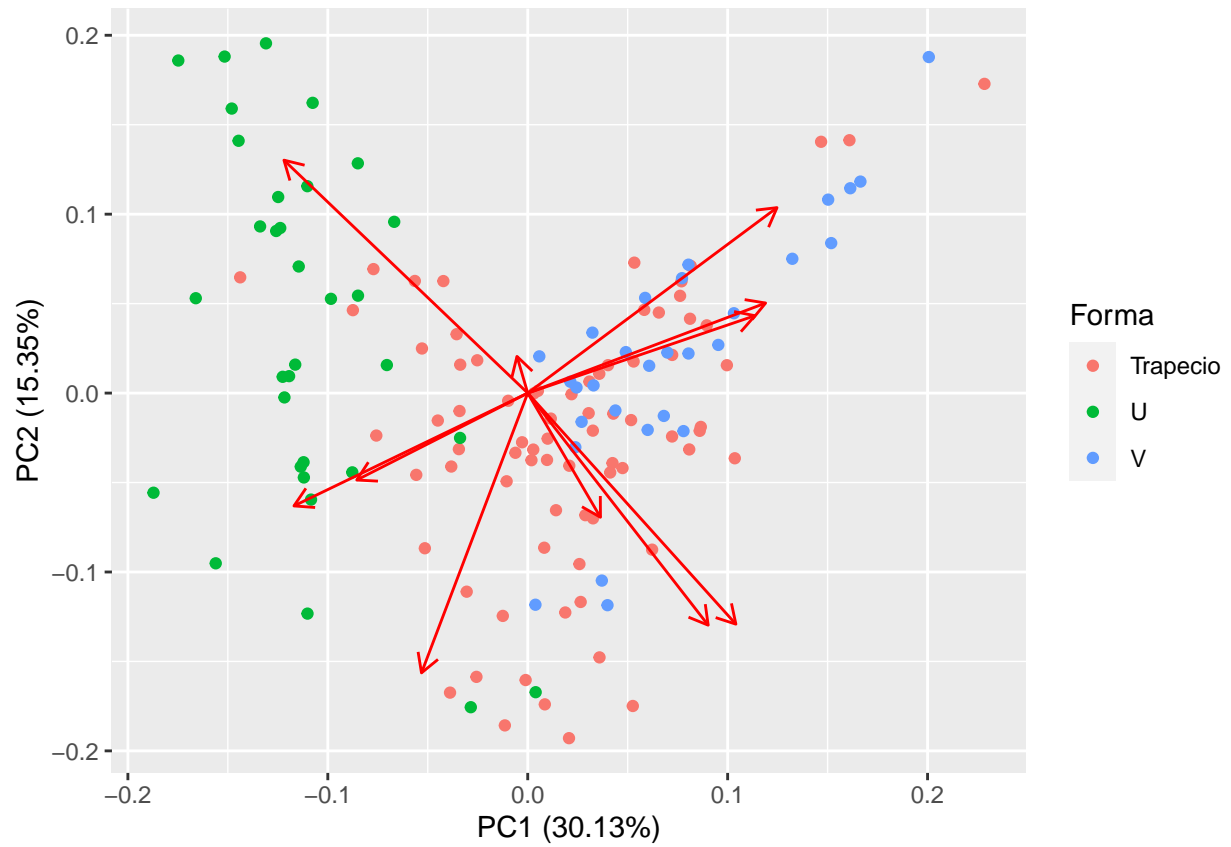
```
autoplot(channel.pca)
```



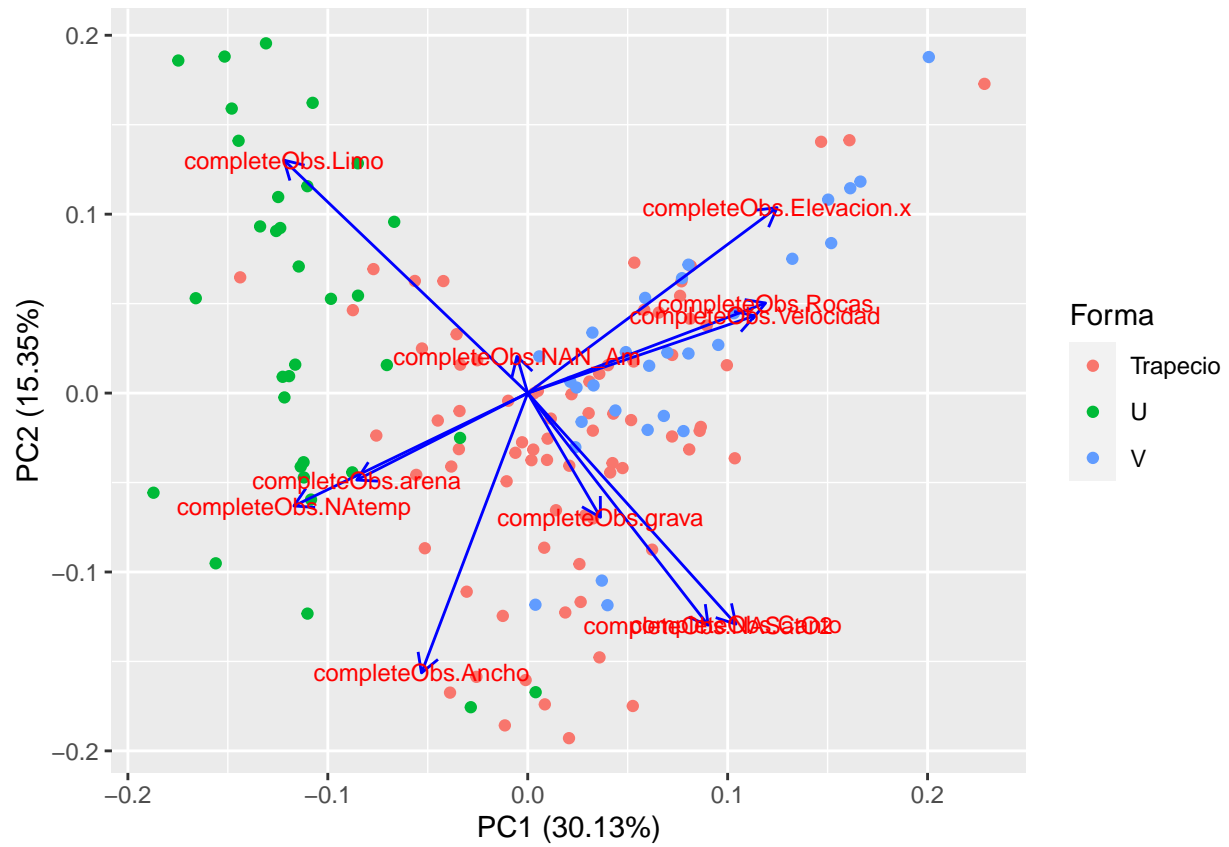
```
autoplot(channel.pca, data = channel, colour = 'Forma')
```



```
autoplot(channel.pca, data = channel, colour = 'Forma', loadings = TRUE)
```

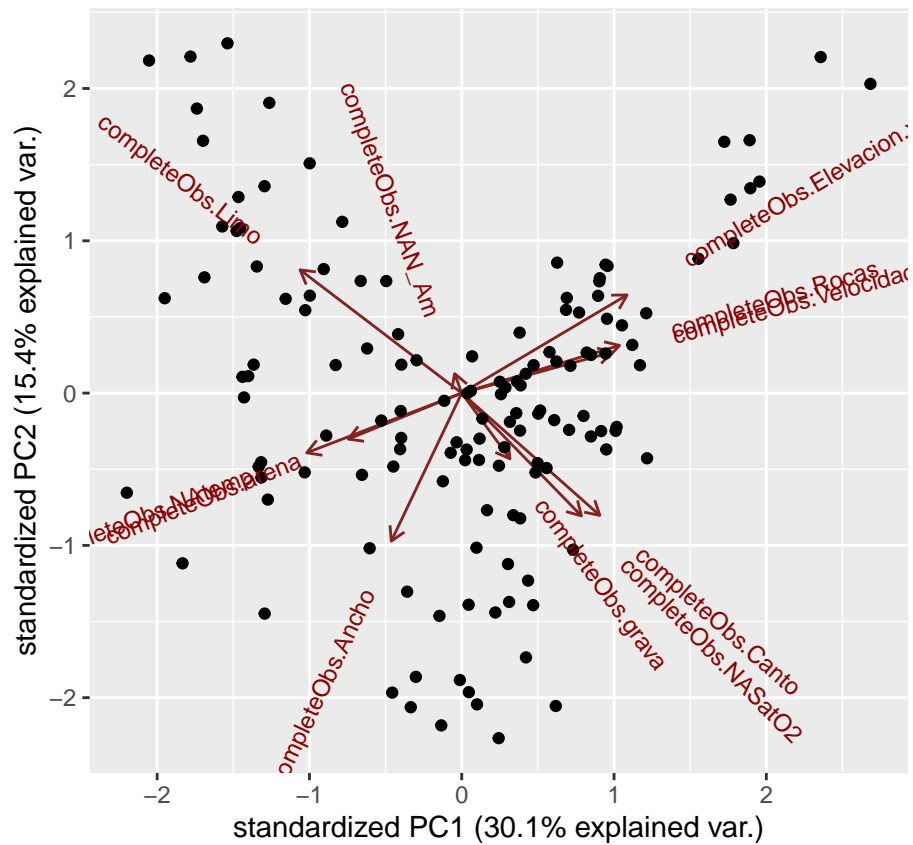


```
autoplot(channel.pca, data = channel, colour = 'Forma', loadings = TRUE,
         loadings.colour = 'blue',
         loadings.label = TRUE, loadings.label.size = 3)
```



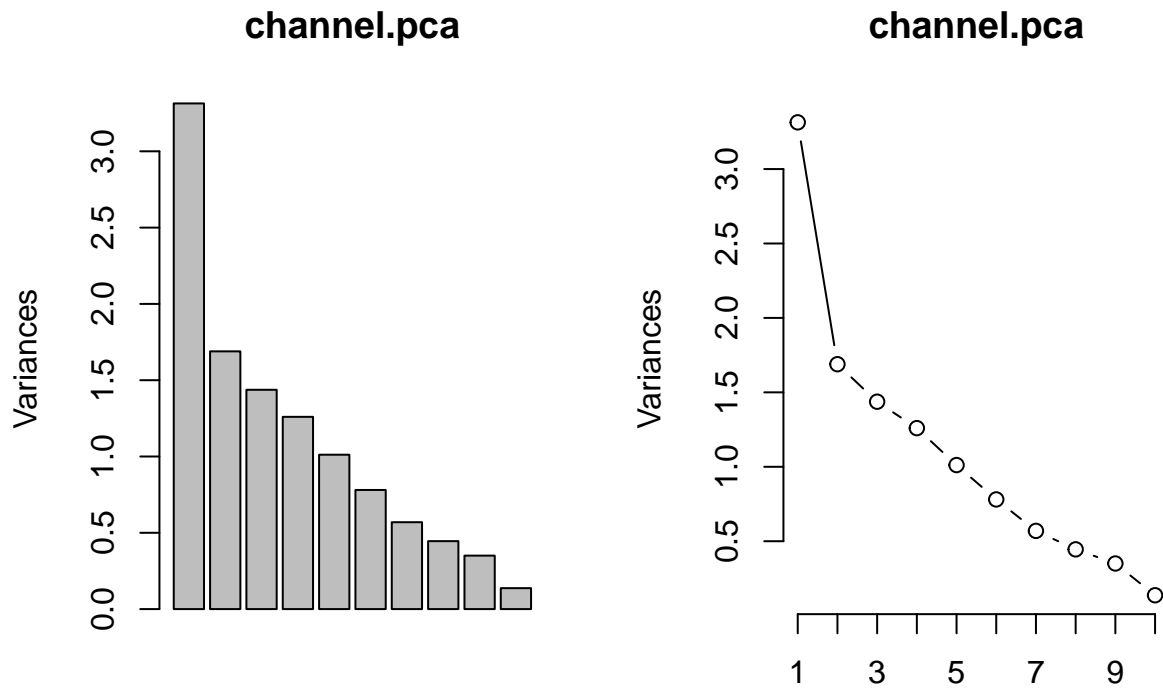
Otra manera de ver el grafico

```
ggbiplot(channel.pca, labels=rownames(channel$Forma))
```



3.2 Ver graficamente lo que explica cada axis.

```
layout(matrix(1:2, ncol=2))
screeplot(channel.pca)
screeplot(channel.pca, type="lines")
```



3.3 Vamos a ver la contribucion de cada una de las variables. Usamos otra libreria. factoextra

```
get_eigenvalue(channel.pca)
```

	eigenvalue	variance.percent	cumulative.variance.percent
## Dim.1	3.313755251	30.12504773	30.12505
## Dim.2	1.689049493	15.35499539	45.48004
## Dim.3	1.437244748	13.06586135	58.54590
## Dim.4	1.259763350	11.45239409	69.99830
## Dim.5	1.011629652	9.19663320	79.19493
## Dim.6	0.780673684	7.09703349	86.29197
## Dim.7	0.569391016	5.17628197	91.46825
## Dim.8	0.445319958	4.04836326	95.51661
## Dim.9	0.350437019	3.18579108	98.70240
## Dim.10	0.137109816	1.24645287	99.94885
## Dim.11	0.005626012	0.05114556	100.00000

```
res.var <- get_pca_var(channel.pca)
res.var$contrib          # Contributions to the PCs
```

	Dim.1	Dim.2	Dim.3	Dim.4	Dim.5
## completeObs.Elevacion.x	15.29621110	10.5676990	6.6474128	4.3496592	0.8767749
## completeObs.Ancho	2.77771564	24.0608974	20.3460480	0.3823721	1.2441681
## completeObs.Velocidad	12.70587345	1.8406123	13.6779648	2.8497532	1.4993696
## completeObs.Rocas	13.93190883	2.4914189	2.6855542	3.9529342	1.9580543

```
## completeObs.Canto      10.66628290 16.4122996  0.1153770  4.4867880 12.5906780
## completeObs.grava      1.30143186  4.7421878 30.9010043 17.2113363  0.3104132
## completeObs.arena      7.19595359  2.3235895  2.3293132 17.3249555 35.1004115
## completeObs.Limo       14.62778901 16.6759938  0.4949031  4.8118433  6.2978302
## completeObs.NAN_Am     0.02842016  0.4138249  5.9566920 31.2616088 32.6711964
## completeObs.NAtemp     13.45278080  3.9142178 16.6886573  3.7314123  2.0509508
## completeObs.NASat02    8.01563266 16.5572589  0.1570733  9.6373371  5.4001531
##                          Dim.6      Dim.7      Dim.8      Dim.9
## completeObs.Elevacion.x 9.185024  0.238732001 1.00279477 3.220621e+00
## completeObs.Ancho       2.520523  5.676961709 0.01584748 3.784466e+01
## completeObs.Velocidad   4.450552 21.845733874 0.14825648 3.950071e+01
## completeObs.Rocas       29.924538 23.017661922 4.01893997 1.260225e+00
## completeObs.Canto       5.170944 10.866802835 12.98206646 6.416521e+00
## completeObs.grava       9.535450 21.805367681  2.87461644 2.075289e-05
## completeObs.arena       3.612484 13.791520805  1.08894421 6.335258e-01
## completeObs.Limo        2.372910  0.562086854 16.23212704 1.715186e+00
## completeObs.NAN_Am      17.207962  0.004598295  5.72331166 6.114050e+00
## completeObs.NAtemp      13.707101  0.765766615  0.65948897 2.106257e+00
## completeObs.NASat02     2.312512  1.424767409 55.25360652 1.188222e+00
##                          Dim.10      Dim.11
## completeObs.Elevacion.x 48.57727559 3.779553e-02
## completeObs.Ancho       5.12981047 9.952647e-04
## completeObs.Velocidad   1.45642015 2.475280e-02
## completeObs.Rocas       0.08799712 1.667077e+01
## completeObs.Canto       0.07559349 2.021665e+01
## completeObs.grava       0.49725283 1.082092e+01
## completeObs.arena       0.56807769 1.603122e+01
## completeObs.Limo        0.08040327 3.612893e+01
## completeObs.NAN_Am      0.61644440 1.890604e-03
## completeObs.NAtemp      42.88140533 4.196242e-02
## completeObs.NASat02     0.02931968 2.411734e-02
```

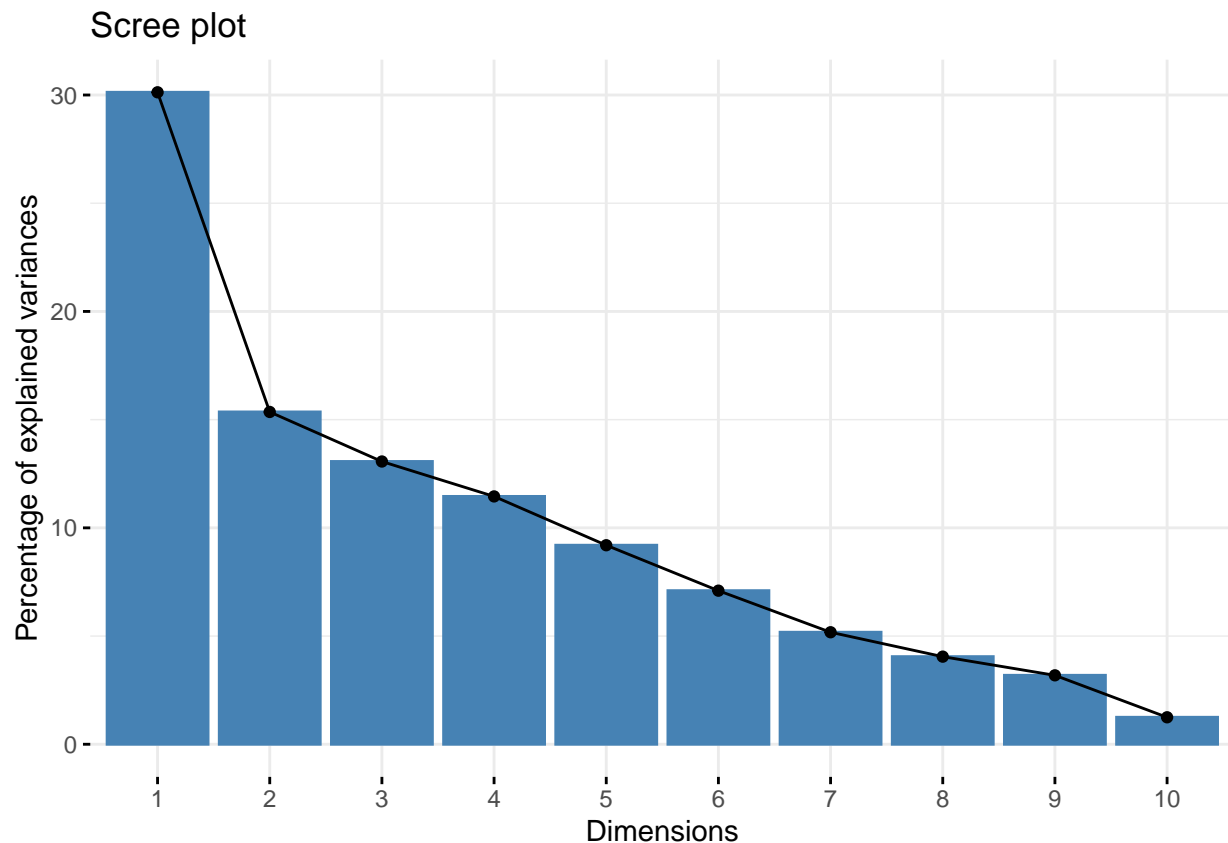
```
res.var$coord      # Coordinates
```

```
##                          Dim.1      Dim.2      Dim.3      Dim.4
## completeObs.Elevacion.x 0.71195435  0.42248511 -0.30909479  0.2340842
## completeObs.Ancho      -0.30339199 -0.63749546 -0.54076104  0.0694045
## completeObs.Velocidad   0.64887714  0.17632031  0.44338001  0.1894733
## completeObs.Rocas       0.67946255  0.20513727 -0.19646371 -0.2231538
## completeObs.Canto       0.59452040 -0.52650913 -0.04072162 -0.2377455
## completeObs.grava       0.20766865 -0.28301572  0.66642559  0.4656416
## completeObs.arena      -0.48831986 -0.19810749 -0.18296976  0.4671760
## completeObs.Limo       -0.69622491  0.53072195 -0.08433842 -0.2462069
## completeObs.NAN_Am     -0.03068835  0.08360447  0.29259570 -0.6275526
## completeObs.NAtemp     -0.66767674 -0.25712463  0.48975183 -0.2168109
## completeObs.NASat02    0.51538185 -0.52882918 -0.04751345 -0.3484360
##                          Dim.5      Dim.6      Dim.7      Dim.8
## completeObs.Elevacion.x -0.09417916  0.2677780 -0.036868938  0.066825484
## completeObs.Ancho       0.11218901  0.1402749 -0.179789071 -0.008400714
## completeObs.Velocidad   0.12315871 -0.1863982  0.352686328  0.025694663
## completeObs.Rocas      -0.14074181 -0.4833353 -0.362022788 -0.133780199
## completeObs.Canto       0.35689078  0.2009184  0.248746053 -0.240440706
## completeObs.grava      -0.05603777  0.2728383 -0.352360334  0.113142568
## completeObs.arena      -0.59589107 -0.1679336  0.280227908 -0.069636814
```

```
## completeObs.Limo      0.25240982  0.1361054  0.056572715  0.268858515
## completeObs.NAN_Am    -0.57490131  0.3665215  0.005116862 -0.159646638
## completeObs.NAtemp     0.14404175 -0.3271204 -0.066031858 -0.054192583
## completeObs.NASat02   -0.23372965 -0.1343621  0.090069405  0.496039653
##                      Dim.9      Dim.10      Dim.11
## completeObs.Elevacion.x -0.1062367582  0.25807792  0.0014582116
## completeObs.Ancho       0.3641726255  0.08386581 -0.0002366299
## completeObs.Velocidad   0.3720552567  0.04468663 -0.0011800828
## completeObs.Rocas       0.0664552221 -0.01098420 -0.0306251429
## completeObs.Canto       -0.1499528738  0.01018067 -0.0337252275
## completeObs.grava       0.0002696773 -0.02611096 -0.0246735933
## completeObs.arena       -0.0471180304  0.02790861 -0.0300319602
## completeObs.Limo        0.0775283498  0.01049956 -0.0450845632
## completeObs.NAN_Am      0.1463758711  0.02907242 -0.0003261374
## completeObs.NAtemp      -0.0859133478  0.24247601  0.0015364930
## completeObs.NASat02     -0.0645288183  0.00634036  0.0011648368
```

4 Otras formas de visualizar los datos.

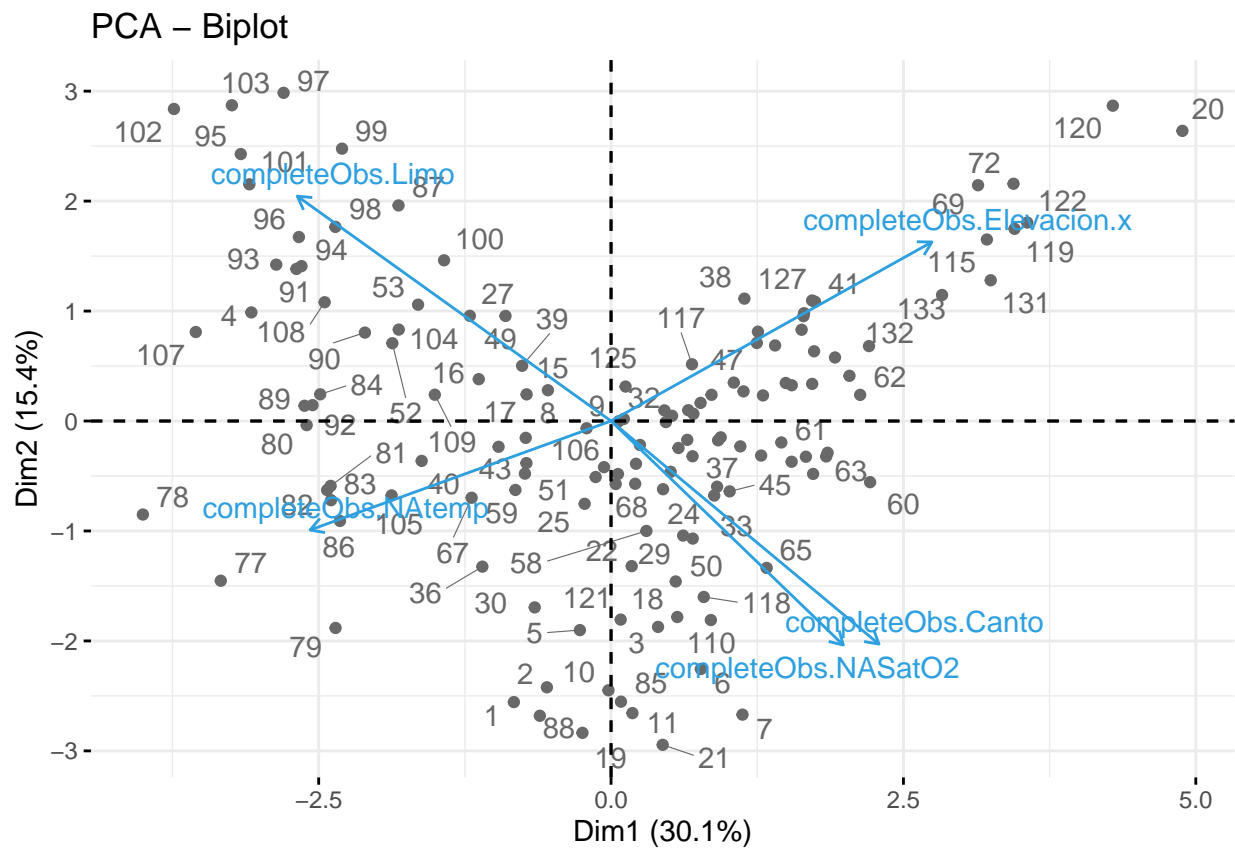
```
fviz_eig(channel.pca)
```



```
fviz_pca_biplot(channel.pca, repel = TRUE,
  col.var = "#2E9FDF", # Variables color
  col.ind = "#696969", # Individuals color
  select.var = list(contrib = 5))
```

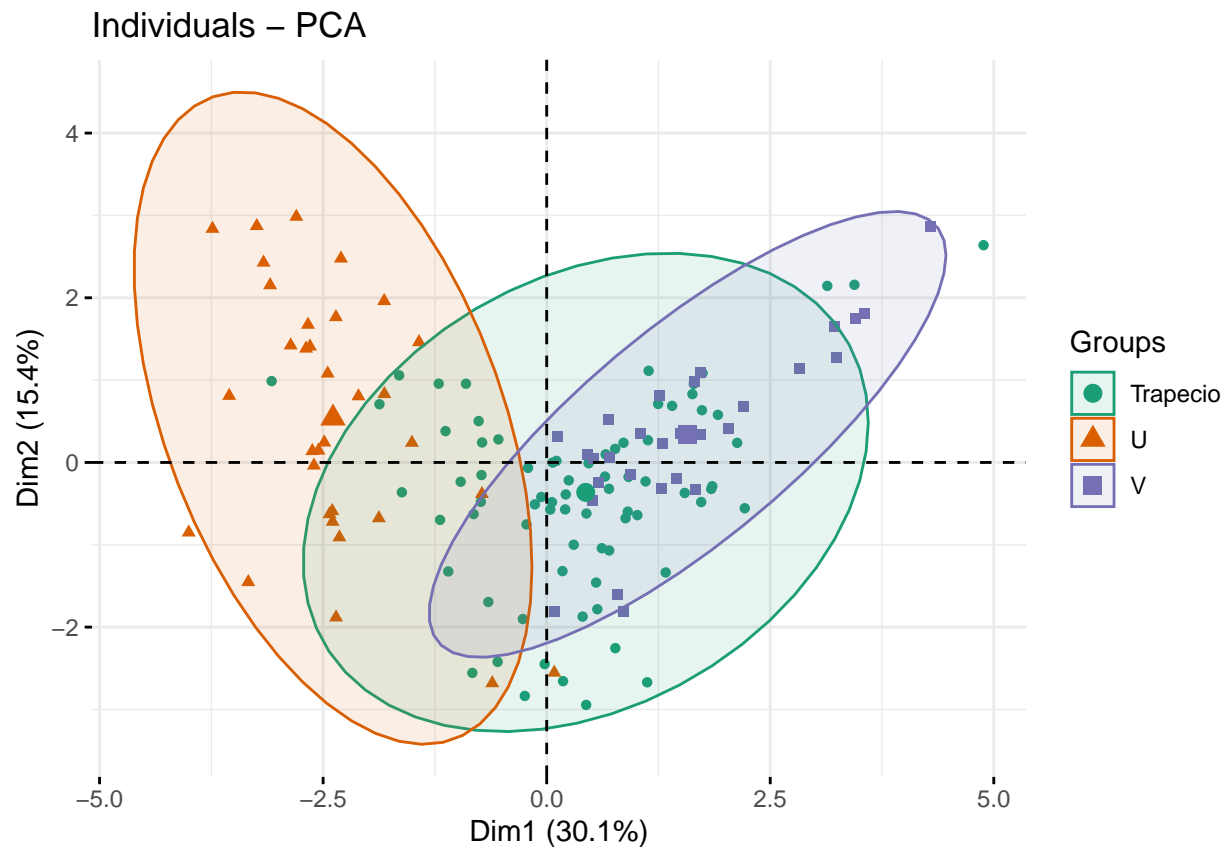


```
## Warning: ggrepel: 41 unlabeled data points (too many overlaps). Consider
## increasing max.overlaps
```



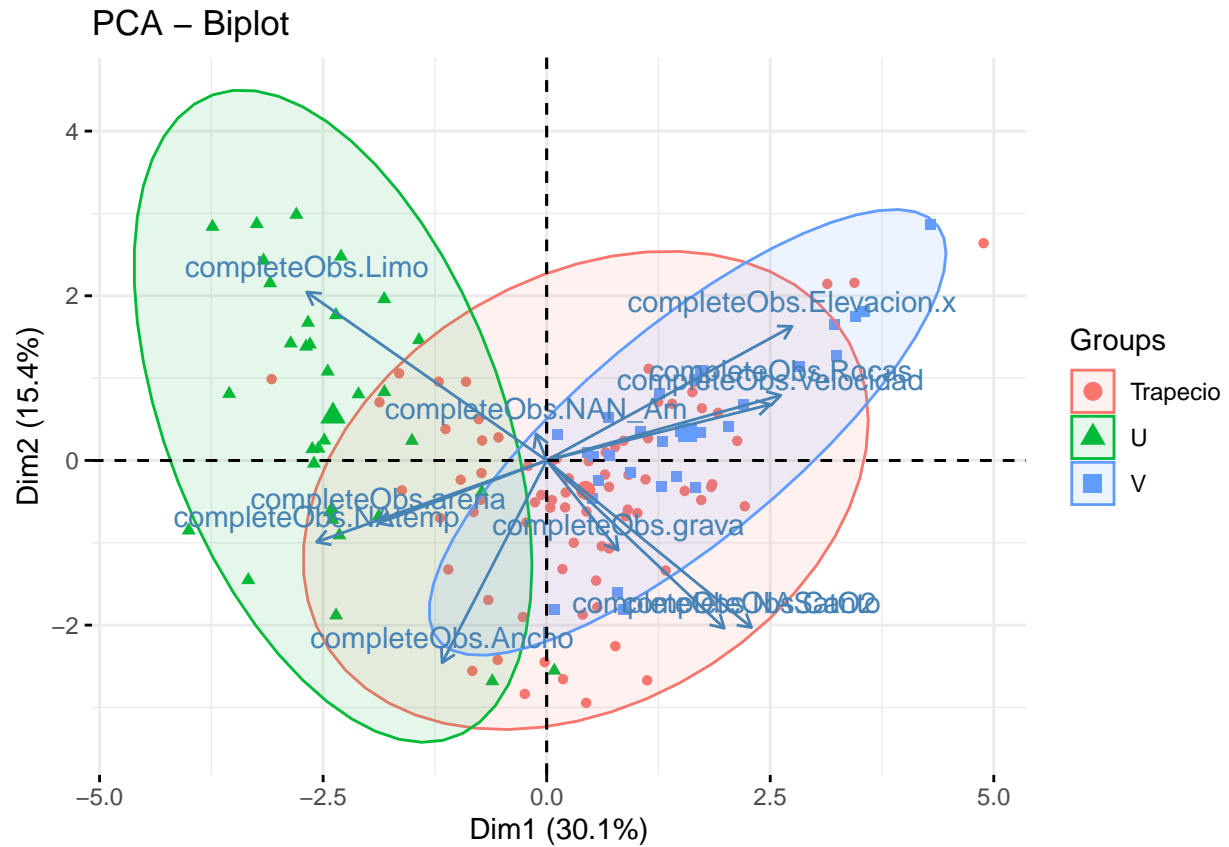
4.1 Con las elipses.

```
fviz_pca_ind(channel.pca, label="none", habillage=channel$Forma,
  addEllipses=TRUE, ellipse.level=0.95, palette = "Dark2")
```



4.1

```
fviz_pca_biplot(channel.pca, label = "var", habillage=channel$Forma,  
  addEllipses=TRUE, ellipse.level=0.95,  
  ggtheme = theme_minimal())
```



5. Convertirlo en una data.frame para trabajarlo en ggplot2

```
data <- data.table(PC1=channel.pca$x[,1], PC2=channel.pca$x[,2], Forma= channel[,1])
data <- data[order(channel$Forma),]

ggplot(data, aes(x=PC1,y=PC2)) +
  geom_point(size = 2, aes(color=Forma))
```

