Descriptive Statistics

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Why describe data?

- Determine if our sample reflects the population of interest.
- Identify outliers.
- Obtain metrics necessary for inferential tests.
- Understand the distribution of our data values (i.e., test for normality).
- Identify the type of statistical test to run.

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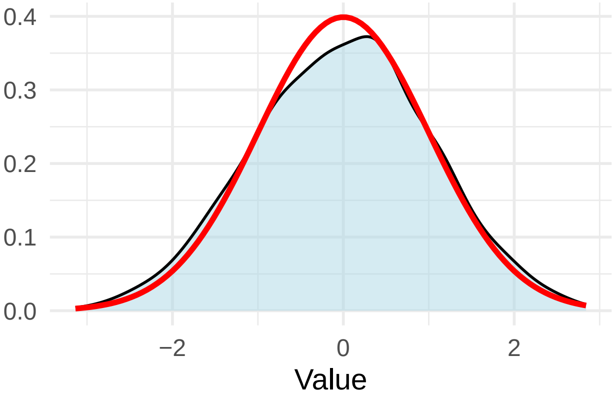
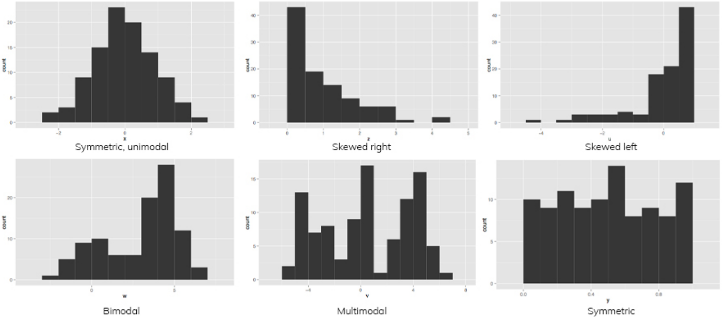
Data description and visualization

- We can examine our data and run statistical tests to see if the distribution approximates a normal curve.
- Typically, we start by visualizing our data.



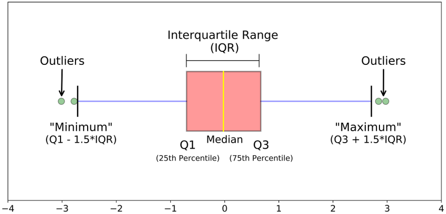
Histogram basic

- Continuous data are most commonly visualized using Histograms.
- Histograms display the distribution of data by grouping values into intervals or bins, allowing for an understanding of the frequency and spread of the data.



Box and Whisker Basics

- Box plots are used to visualize the distribution of continuous data, showing the **median**, **interquartile range (IQR)**, and **potential outliers**.
- The **box** represents the middle 50% of the data (from the first quartile $Q1$ to the third quartile $Q3$).
- The **line inside the box** shows the median (50th percentile).
- **Whiskers** extend from the box to the smallest and largest values within 1.5 times the IQR from $Q1$ and $Q3$.
- **Data points outside the whiskers** are considered potential outliers.



Metrics to Describe data distribution.

- Data and their associated distributions can be described in four primary way:
  - Central Tendency (mean, median, mode)
  - Variability (standard deviation, variance, quantiles)
  - Skew
  - Kurtosis (Peakedness)

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Central tendency

- **Mean** $\left(\frac{\sum X}{n}\right)$:
  - Most often used measure of central tendency.
  - Works well with normal and relatively normal curves.
- **Median (50th Percentile)**:
  - No formula. Rank order observations then find the middle.
  - The second most used measure of central tendency.
  - Works best with highly skewed populations.
- **Mode (Most Frequent Score)**:
  - Least used measure of central tendency.
  - Works best for highly irregular and multimodal distributions.

Central tendency: Mean

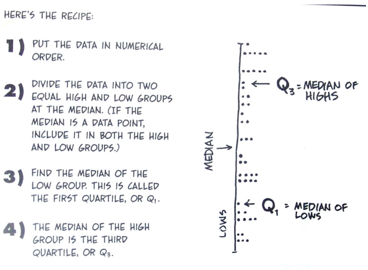
- $\left(\bar{X} = \frac{\sum X}{n}\right)$
  - where X represents individual data points and n is the number of observations.
- Sample mean is the measure of central tendency that best represents the population mean.
- Mean is **very** sensitive to extreme scores that can "skew" or distort findings.

Central tendency: Median

- Percentiles are used to define the percent of cases equal to and below a certain point on a distribution.
  - The median **is the 50th percentile**, meaning *half of all observations fall at or below this value*.
- But lots of other percentiles are also important.

A little about Percentiles

- Quartiles (i.e., divide data into four equal parts: 25%, 50%, 75%) are a common percentile used to represent the value below which.
  - 25% (Q1 or first quartile)
  - 75% (Q3 or third quartile)



When to use What

- Use the **Mode** when the data are categorical:
  - **Mode**: is the value that occurs most frequently in your data.
  - This is because having the same value occur for measurements with many significant digits is highly unlikely.

- Use the **Median** when you have extreme scores:
  - **Median**: is simply the value that falls in the middle of all your data.

- Use the **Mean** the rest of the time.

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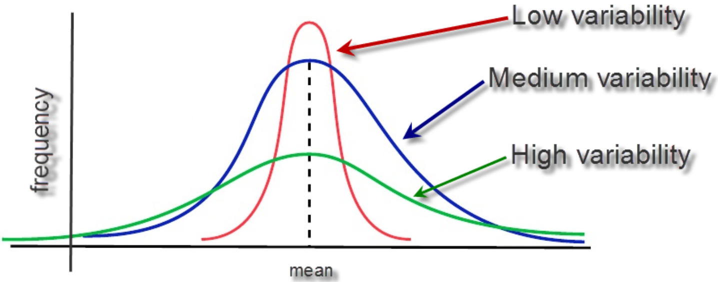
Variability



Variability: Standard Deviation

- **Standard Deviation** measures how spread out the numbers in a dataset are around the mean.
- The sample standard deviation s is calculated as:

$$s = \sqrt{\frac{\sum(x_i - \bar{x})^2}{n-1}}$$

- s: The **sample standard deviation**, which measures the spread or variability of the data points around the sample mean $\bar{x}$.
- n: The **sample size**, or the number of observations in the dataset.
- x_i: Each individual data point in the sample.
- $\bar{x}$: The **sample mean**, or the average of the sample data.

Variability

- **Variance** measures the average of the squared differences from the mean, indicating how spread out the data points are.

- The variance σ^2 is calculated as:

$$s^2 = \frac{\sum(x_i - \bar{x})^2}{n-1}$$

- **Variance is always larger than the Standard Deviation (SD)** because variance is the square of SD. For example, if the SD is 3, the variance will be $3^2 = 9$.

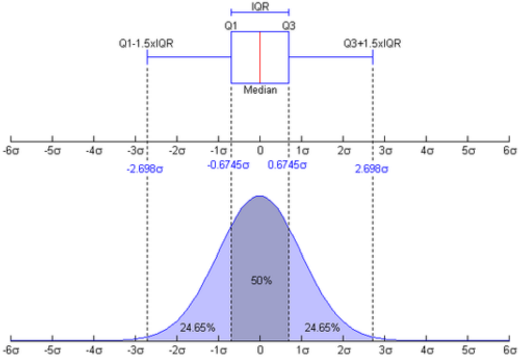
Variability: Range

- **Range** is the difference between the largest and smallest values in a dataset, providing a measure of the spread or dispersion of the data.
- The range is calculated as:

$$\text{Range} = \max(x) - \min(x)$$

Percentiles are useful for spread too

- You can use percentiles to get a feel for how spread out the data is and where most of your observations are contained:
  - Inter-quartile range (IQR) = Q3 - Q1



Identifying outliers

- An outlier is an observation that lies outside the overall pattern of a distribution (Moore and McCabe 1999).
- Usually, the presence of an outlier indicates some sort of problem. (e.g. an error in measurement or sample selection).
- But they may also be an indicator of novel data or identification of unique and exciting observations.

Identifying outliers

- The first and third quantiles (Q1 and Q3) are often calculated to identify outliers.
- One method for systematically identifying outliers uses:
  - Q1 - (1.5 * the inter-quartile range)
  - Q3 + (1.5 * the inter-quartile range)
- Others identify outliers as any values below the 0.5th or above the 99.5th percentile.

When to use What

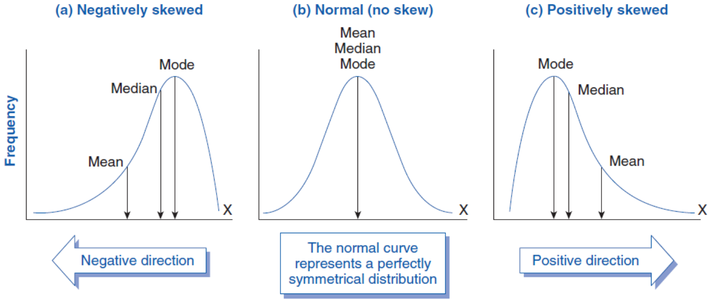
- Use the **Standard deviation (SD)** in most cases.
  - SD quantifies how far, on average, each observation is from the mean.
  - The larger the SD, the more highly variable your data.
- Use **range (R)** when describing predictive models.
  - R is simply the maximum minus the minimum value in your data set
  - R is important when modeling or making predictions, since your algorithms are valid only over the range of values used to calibrate your predictive model
- Use the **IQR** to identify and test potential outliers in your data.

Metrics to Describe data distribution.

- Data and their associated distributions can be described in four primary way:
  - Central Tendency (mean, median, mode)
  - Variability (standard deviation, variance, quantiles)
  - **Skew**
  - Kurtosis (Peakedness)

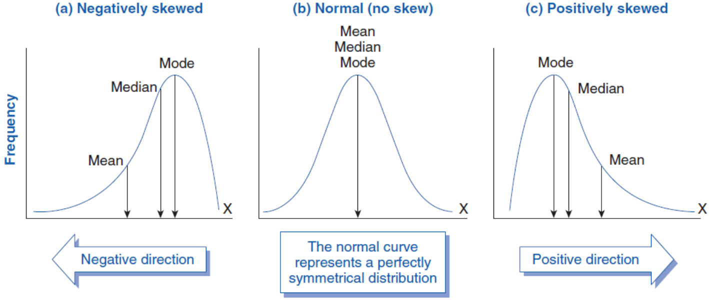
Skewness

- **Skewness**: This metric quantifies how balanced (symmetrical) your distribution curve is.



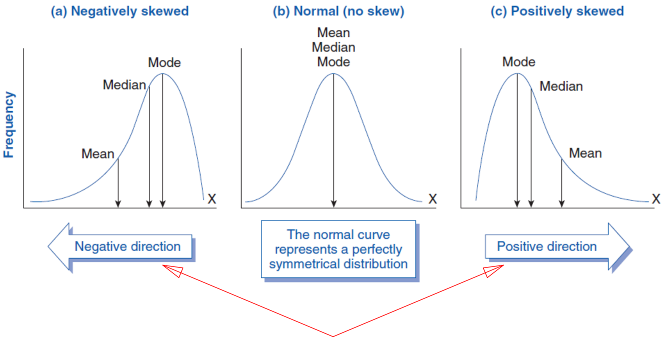
Skewness

- A **normal distribution** will have its mean and median values located somewhere near the center of its range.



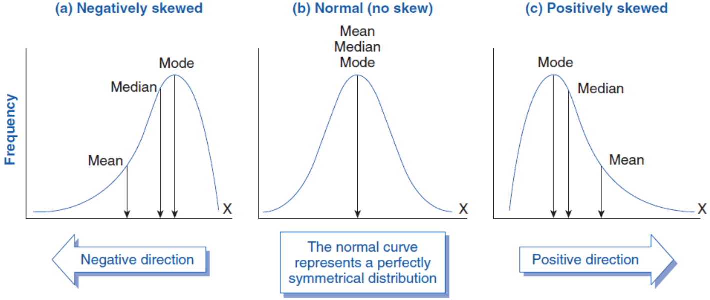
Skewness

- Skew of this peak away from center is common when extreme values pull the median away from the mean.



Skewness

- **Positive Skew**: the "slide" takes you in a positive direction.
  - The mean is bigger than the median (which is why the slide is being pulled to higher values).
- **Negative Skew**: the "slide" takes you in a negative direction.
  - The mean is smaller than the median (which is why the slide is being pulled to lower values).



Calculating Skew

- Negative value = Negative Skew.
- Positive value = Positive Skew
- Zero = Normal distribution

$$\text{Skewness} = \frac{3(\bar{x} - \text{Median})}{\text{SD}}$$

- where:
  - $\bar{x}$ is the **sample mean**, representing the average of all data points.
  - **Median** is the middle value in a dataset when sorted in ascending or descending order.
  - **SD** (Standard Deviation) measures the spread of the data points around the mean.

Skewness: Significant?

- To determine if this deviation from zero in the skew statistic is likely a significant departure from normality, compare it to the **standard error of skew (ses)**.
- If the skew you have calculated is more than **2 times the ses**, then you likely have significant skew, which means you have **non normal data** and should consider a nonparametric test for your statistical analyses

$$ses = \sqrt{\frac{6}{n}} \qquad \text{Skewness} = \frac{3(\bar{x} - \text{Median})}{\text{SD}}$$

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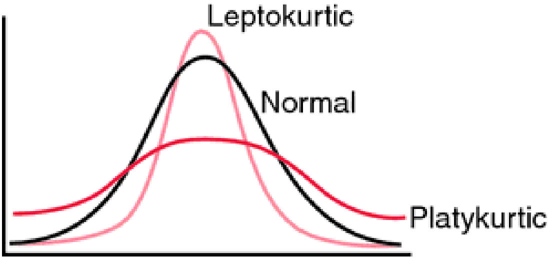
  - **Kurtosis (Peakedness)**

Kurtosis

- **Kurtosis** is simply a measure of how pointy or flat the peak of your distribution curve is.
- Any deviation from a bell shape, with the peak either too flat (platykurtic) or too peaked (leptokurtic), suggests that your data are not normally distributed.



Kurtosis

- Positive values = Leptokurtic.
- Zero = Mesokurtic = normal (bell-shaped).
- Negative values = Platykurtic.

$$\text{Kurtosis} = \frac{\sum\left(\left(\frac{x_i - \bar{x}}{\text{SD}}\right)^4 - 3\right)}{n}$$

- where:
  - x_i represents each individual data point.
  - $\bar{x}$ is the **sample mean**, the average of all data points.
  - **SD** (Standard Deviation) is the measure of how spread out the data points are from the mean.
  - n is the number of data points in the sample.
  - The subtraction of 3 is to adjust for the kurtosis of a normal distribution, which has a kurtosis of 3.

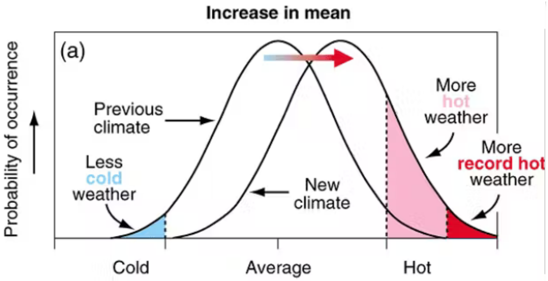
Kurtosis: Significant?

- To determine if this deviation from zero in the kurtosis statistic is likely a significant departure from normality, compare it to the **standard error of kurtosis (sek)**.
- If the kurtosis you have calculated is more than **twice the sek**, you likely have **non normal data** and should consider a nonparametric test for your statistical analyses.

$$sek = \sqrt{\frac{24}{n}} \qquad \text{Kurtosis} = \frac{\sum \left(\frac{x_i - \bar{x}}{\text{SD}}\right)^4 - 3}{n}$$

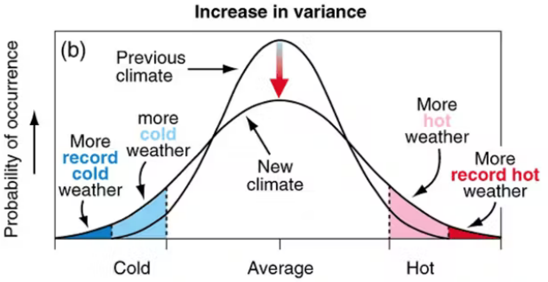
Some Visual Examples

- How can climate change?
- Change in **central tendency**.



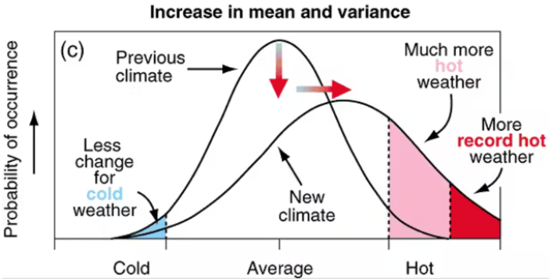
Some Visual Examples

- How can climate change?
- Change in **spread and shape**.



Some Visual Examples

- How can climate change?
- Change in **both**.



Data Distributions

Data Distributions

- There are various types of data distributions, each with its own unique properties and implications.
- **In nature, most data are normally distributed**.
- The central limit theorem (CLT) states that the distribution of sample means approximates a normal distribution as the sample size gets larger, regardless of the population's distribution.

$$\bar{X}_n \sim N\left(\mu, \frac{\sigma}{\sqrt{n}}\right)$$

This means that $\bar{X}_n$ (the average of your sample) will approximately follow a normal distribution with a mean of μ and a standard deviation of $\frac{\sigma}{\sqrt{n}}$, especially if your sample size n is large.

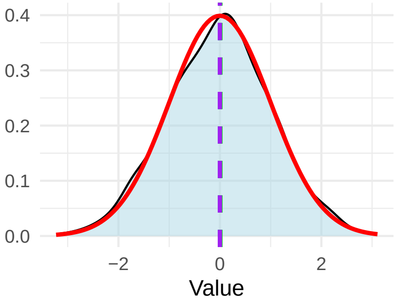
Why do we care if our data is normal?

- The math "under the hood" of many analyses **expects that data is normally distributed** - if it isn't, you'll still get an answer, but it won't actually be saying what you think it is saying.

Why do we care about **skew** and **kurtosis**?

- Because many statistical analyses assume a normal distribution of the data, testing for normality must always be a precursor to any analysis.

- Normally Distributed Data is:
  - Unimodal (one mode)
  - Symmetrical (no SKEW)
  - Bell Shaped (no KURTOSIS)
  - Mean, Mode and Median are all centered
  - Asymptotic (tails never reach 0)



Why do we care about **skew** and **kurtosis**?

- We can examine all of these different descriptors individually, but the easiest and most complete way to test for normality is to test the **goodness of fit** for a normal distribution.

```r
# Generate data from
# a normal distribution
set.seed(123)
data <- rnorm(100,mean=0,sd=1)
# Shapiro-Wilk normality test
shapiro.test(data)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  data
## W = 0.99388, p-value = 0.9349
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- Interpretate shapiro.tets:
  - Null hypothesis (H_0): The data is normally distributed
  - Alternative hypothesis (H_1): The data is not normally distributed.
- **Larger p-value ($p > 0.05$)**: Fail to reject the null hypothesis; the data is likely normally distributed.
- **Smaller p-value ($p < 0.05$)**: Reject the null hypothesis; the data is not normally distributed.

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What to do about non-normal data?

- Once you discover that your data is non-normal you have several options:
  - Analyze and potentially remove outliers
  - Transform the data mathematically
  - Conduct non-parametric analyses

Outliers?

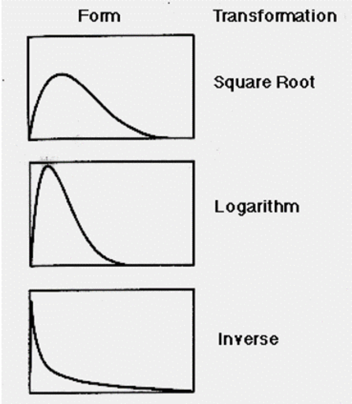
- How to find outliers:
  - Outlier box plots (visual) use the IQR * 1.5 threshold.
  - **IQR**: $Q_3 - Q_1$
  - **Lower Threshold**: $Q_1 - (1.5 * \text{IQR})$
  - **Upper Threshold**: $Q_3 + (1.5 * \text{IQR})$
- These can help identify potential outliers but **do not justify their removal**.
- Sometimes outliers are **real, correct (although extreme) observations** that we are truly interested in.
- We can only remove outliers if we know the data is **incorrect**

Working with non-normal Data.

- Transformations:
  - To transform your data, apply a mathematical function to each observation, then use these numbers in your statistical test.
  - There are an infinite number of transformations you could use, but it is better to use one common to our field.

Working with non-normal Data.

- Non-normal data happens. Especially with counts, percents, rare events.
- Common transformations in our field:
  - square-root:
    - for moderate skew.
  - log:
    - for positively skewed data.
  - Inverse:
    - for severe skew.
  - Rank



Square root Transformation

- **Square-root transformation**: This consists of taking the square root of each observation.
- In R use: **sqrt(X)**

```
data <- c(1, 4, 9, 16, 25, 36, 49, 64, 81, 100)
# Apply square-root transformation
sqrt_data <- sqrt(data)
# Goodness of fit test
shapiro.test(sqrt_data)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  sqrt_data
## W = 0.97016, p-value = 0.8924
```

Square root Transformation

- If you apply a square root to a continuous variable that contains **values negative values, decimals and values above 1**, you are treating some numbers differently than others.
- So a constant must be added to move the minimum value of the distribution to 1.

Log Transformation

- Many variables in biology have log-normal distributions.
- In R use: **log(X)**

```r
# Sample data (e.g., positive values)
data <- c(1, 10, 100, 1000, 10000, 100000)
# Apply log transformation (log base 10)
log_data <- log10(data)
# Goodness of fit test
shapiro.test(log_data)
```

```
##
##  Shapiro-Wilk normality test
##
## data:  log_data
## W = 0.98189, p-value = 0.9606
```

Log Transformation

- The logarithm of any **negative number is undefined and log functions treat decimals differently than numbers $>$1**.
- So a constant must be added to move the minimum value of the distribution to 1.

Inverse transformation

- **Inverse transformation**: This consists of taking the inverse X^{-1} of a number.
- In R use: `1/X` or `X^-1`

```
# Sample data (e.g., positive values)
data <- c(1, 2, 4, 8, 16, 32, 64)
# Apply inverse transformation (reciprocal)
inverse_data <- 1 / data
```

- Tends to make big numbers small and small numbers big a constant must be added to move the minimum value of the distribution to 1

Reflecting Transformations

- Each of these transformations can be adjusted for negative skew by taking the reflection.
- To reflect a value, multiply data by -1, and then add a constant to bring the minimum value back above 1.0.
- For example:
  - sqrt(x) for positively skewed data,
  - sqrt((x*-1) + c) for negatively skewed data
  - log10(x) for positively skewed data,
  - log10(x*-1 + c) for negatively skewed data

Rank Transform

- Rank transformation requires sorting your data and then creating a new column where each observation is assigned a rank.

- For tied values, assign the average rank.
- Perform all subsequent analyses on the ranked data instead of the original values.

Transformation Rules to Live By

- Transformations work by altering the relative distances between data points.
- If done correctly, all data points remain in the same relative order as prior to transformation.
- However, this might be undesirable if the original variables were meant to be substantively interpretable.
- Therefore. . .

Transformation Rules to Live By

- Don't mess with your data unless you have to.
- Are there true outliers? Remove and retest.
- If you have to mess with it, make sure you know what you are doing. Try different transformations to see which is best.
- Include these details in your methods.
- Back transform to original units for reports of central tendency and variability.
- Sometimes transformations don't work, don't panic, you will just get to run **nonparametric tests**.