

Inferential Statistics

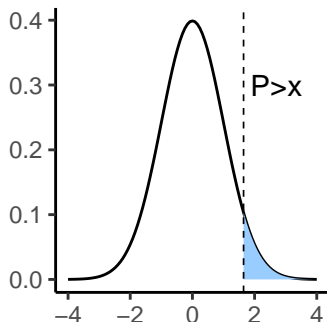
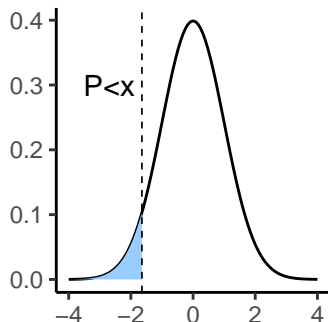
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Expanding on Hypothesis Testing

- 1-tailed
 - ▶ Hypothesis includes an **expected direction**.

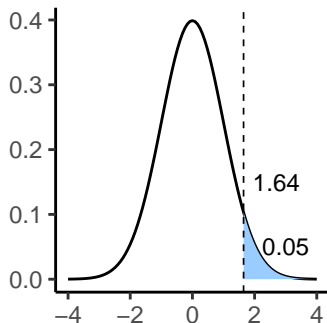
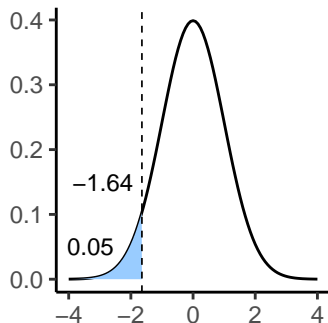


- Decrease
- Cooler
- Smaller
- Lower

- Increase
- Warmer
- Higher
- Expand

Expanding on Hypothesis Testing

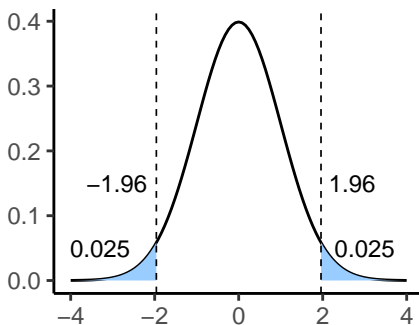
- 1-tailed - hypothesis includes an **expected direction**.



- If your obtained test statistic falls beyond the critical value (lightblue) for your given Alpha threshold = Significant result, **reject the null**.

Expanding on Hypothesis Testing

- 2-tailed tests:
 - ▶ Have **no expected directionality** hypothesized.
 - ▶ Splits the 5% of the area under the curve that would be considered significant between both tails of the normal distribution curve.
 - ▶ Are therefore less powerful tests (more likely to find a significant result).



Significant or Not?

• Not significant:

- Accept the null hypothesis.
- There is **no** difference between the sample and population mean.
- Obtained test statistic $<$ critical value threshold.
- p-value $>$ alpha threshold (usually 0.05).

Significant:

- Reject the null hypothesis,
- There **is** a difference between the sample and population mean.
- Obtained test statistic $>$ critical value threshold.
- p-value $<$ alpha threshold (usually 0.05).

Basic steps for an **Inferential Test**

contents. . .

- A statement of null hypothesis.
- Choose the appropriate test.
- Set the level of Type I error risk (α)
- Analyze data distribution
- Compute the test statistic (obtained) value
- Assess significance:
 - ▶ Determine the critical value needed to reject the null hypothesis and compare it to your -calculated test statistic
 - ▶ Determine the p-value associated with your calculated test statistic
- Summarize

Basic steps for an **Inferential Test**

- We can select an appropriate test simply by answering some questions.
- What type of data do we have?
 - ▶ If we have **frequency data**, we select the Chi-square family.
 - ▶ **Continuous and categorical variables** will have a variety of different tests depending on the research question:
- What type of research question are we considering?
 - ▶ If **comparing one sample mean** to a **population** = **z-test**.
 - ▶ If focus is on **differences** we go to the family of tests concerned with comparing groups or treatments (i.e. **t-tests**, **ANOVA**).
 - ▶ If the focus is on **relationships** between variables, we go to the **correlation tests**.
 - ▶ If we want to **predict outcomes** we go to the **regression family**.

$$y = \beta_0 + \beta_1 x + \epsilon$$

where:

- y is the dependent variable,
- x is the independent variable,
- β_0 is the intercept,
- β_1 is the slope,
- ϵ is the error term.

Basic steps for an **Inferential Test**

- We can select an appropriate test simply by answering some questions.
 - ▶ How many variables, how many groups are we including? (multivariate)
 - ▶ Are observations independent from each other or purposely paired?
 - ▶ Is my data not normally distributed? (non-parametric tests)

Statistical shorthand **z-test** example

$$z_{100} = 1.4, p = 0.16$$

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$$z_{100} = 1.4, p = 0.16$$



$$\underbrace{z}_{\text{Test statistic}} \underbrace{100}_{\text{Sample size}} = \underbrace{1.4}_{\text{z-value}}, \underbrace{p = 0.16}_{\text{p-value}}$$

- Every statistical test has a letter designating the type of test.
- Because power is so closely tied to sample size, we report sample size.
- For clarity (and to confirm the correct-tailed test is used to assess the p-value), include the calculated test statistic value.
- Always report the p-value.
- Some tests will also include a secondary metric to assess how meaningful results are (if significant).

How to summarize your analysis: Key Components of a Statistical Summary

Before installing a new air quality monitoring instrument, we tested to see if a sample of measurements taken at the testing lab differed significantly different from the long-term mean for the larger network population.

Using a 2-tailed, one-sample z-test for on our normally distributed samples ($W = 0.78$).

While the mean of the new instrument sample was slightly higher (sample mean = 117 vs. population mean = 108), we found no significant difference ($z(100) = 1.4$, $p = 0.16$).

Based on these results we approve the installation of this instrument at the new site. However, we would suggest statistical comparisons of the current and new unit side-by-side, prior to decommissioning the current instrument.

How to summarize your analysis: Key Components of a Statistical Summary

Introduction/Background:

Before installing a new air quality monitoring instrument, we tested to see if a sample of measurements taken at the testing lab differed significantly different from the long-term mean for the larger network population.

Methods:

Using a 2-tailed, one-sample z-test for on our normally distributed samples ($W = 0.78$).

Results:

While the mean of the new instrument sample was slightly higher (sample mean = 117 vs. population mean = 108), we found no significant difference $z_{100} = 1.4$, $p = 0.16$.

Implications:

Based on these results we approve the installation of this instrument at the new site. However, we would suggest statistical comparisons of the current and new unit side-by-side, prior to decommissioning the current

Our First Inferential Tests!

One sample z-test

When would you run a one-sample z-test?

- Use the One Sample z-test when:
 - ▶ You want to test for a difference between one sample mean and a larger population mean.
 - ▶ There is only one group (sample) being tested against the larger population.
 - ▶ You know (or can estimate) the mean and standard deviation of the population.
 - ▶ Data is normally distributed.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where:

- \bar{X} : Sample mean.
- μ : Population mean.
- σ : Population standard deviation. - n : Sample size.

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 - ▶ There is only one group (sample) being tested against the larger population.
 - ▶ You know (or can estimate) the mean and standard deviation of the population.
 - ★ Expert Information.
 - ★ Scientific Literature.
 - ★ Data archives.
 - ★ Meaningful (hypothesized value).
 - ▶ Data is normally distributed.

Flowcharts: Keeping it simple with questions

One sample z-test

Assume you have a sample of **20 observations**. The mean of this **sample is 150**, and you want to determine whether there is a **difference between the sample mean and the larger population mean**. Since you are not specifying a direction for this difference, you will conduct a two-tailed test.

The **population mean is 164**, and the **population standard deviation is 33**.

Next, calculate the obtained value for this **one-sample z-test**.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

$$z = \frac{150 - 164}{\frac{33}{\sqrt{20}}}$$