Inferential Statistics

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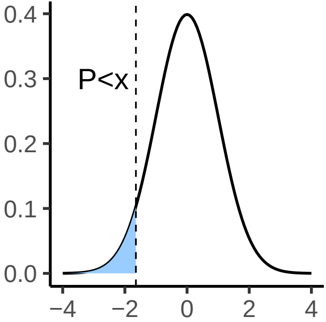
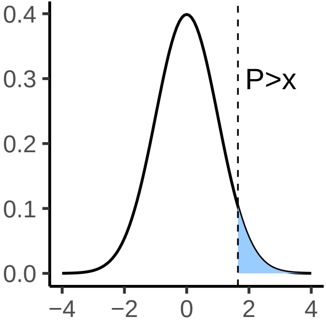
Fall 2024

Expanding on Hypothesis Testing

- 1-tailed
  - Hypothesis includes an **expected direction**.



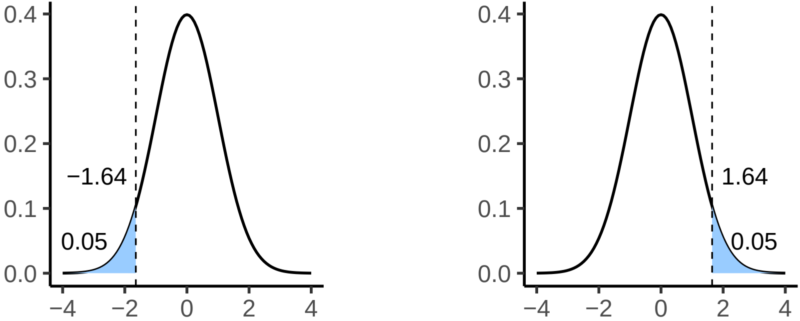
- Decrease
- Cooler
- Smaller
- Lower



- Increase
- Warmer
- Higher
- Expand

Expanding on Hypothesis Testing

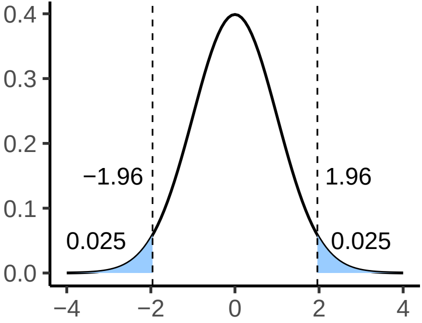
- 1-tailed - hypothesis includes an **expected direction**.



- If your obtained test statistic falls beyond the critical value (lightblue) for your given Alpha threshold = Significant result, **reject the null**.

Expanding on Hypothesis Testing

- 2-tailed tests:
  - Have **no expected directionality** hypothesized.
  - Splits the 5% of the area under the curve that would be considered significant between both tails of the normal distribution curve.
  - Are therefore less powerful tests (more likely to find a significant result).



Significant or Not?

- **Not significant**:
  - Accept the null hypothesis.
  - There is **no** difference between the sample and population mean.
  - Obtained test statistic $<$ critical value threshold.
  - p-value $>$ alpha threshold (usually 0.05).

Significant:

- Reject the null hypothesis,
- There **is** a difference between the sample and population mean.
- Obtained test statistic $>$ critical value threshold.
- p-value $<$ alpha threshold (usually 0.05).

Basic steps for an **Inferential Test**

contents...

- A statement of null hypothesis.
- Choose the appropriate test.
- Set the level of Type I error risk (alpha)
- Analyze data distribution
- Compute the test statistic (obtained) value
- Assess significance:
  - Determine the critical value needed to reject the null hypothesis and compare it to your - calculated test statistic
  - Determine the p-value associated with your calculated test statistic
- Summarize

Basic steps for an **Inferential Test**

- We can select an appropriate test simply by answering some questions.
- What type of data do we have?
  - If we have **frequency data**, we select the Chi-square family.
  - **Continuous and categorical variables** will have a variety of different tests depending on the research question:
- What type of research question are we considering?
  - If **comparing one sample mean** to a **population** = z-test.
  - If focus is on **differences** we go to the family of tests concerned with comparing groups or treatments (i.e. t-tests, ANOVA).
  - If the focus is on **relationships** between variables, we go to the correlation tests.
  - If we want to **predict outcomes** we go to the regression family.

$y = \beta_0 + \beta_1 x + \epsilon$

where:
- y is the dependent variable,
- x is the independent variable,
- β_0 is the intercept,
- β_1 is the slope,
- ϵ is the error term.

Basic steps for an **Inferential Test**

- We can select an appropriate test simply by answering some questions.
  - How many variables, how many groups are we including? (multivariate)
  - Are observations independent from each other or purposely paired?
  - Is my data not normally distributed? (non-parametric tests)

Statistical shorthand **z-test** example

$$z_{100} = 1.4, p = 0.16$$

Statistical shorthand **z-test** example

$$z_{100} = 1.4,\ p = 0.16$$

$$\downarrow$$

$$\underbrace{z\underbrace{100}_{\text{Sample size}}}_{\text{Test statistic}} = \underbrace{1.4}_{\text{z-value}},\quad \underbrace{p = 0.16}_{\text{p-value}}$$

- Every statistical test has a letter designating the type of test.
- Because power is so closely tied to sample size, we report sample size.
- For clarity (and to confirm the correct-tailed test is used to assess the p-value), include the calculated test statistic value.
- Always report the p-value.
- Some tests will also include a secondary metric to assess how meaningful results are (if significant).

How to summarize your analysis: Key Components of a Statistical Summary

Before installing a new air quality monitoring instrument, we tested to see if a sample of measurements taken at the testing lab differed significantly different from the long-term mean for the larger network population.

Using a 2-tailed, one-sample z-test for on our normally distributed samples ($W = 0.78$).

While the mean of the new instrument sample was slightly higher (sample mean = 117 vs. population mean = 108), we found no significant difference ($z(100) = 1.4$, $p = 0.16$).

Based on these results we approve the installation of this instrument at the new site. However, we would suggest statistical comparisons of the current and new unit side-by-side, prior to decommissioning the current instrument.

How to summarize your analysis: Key Components of a Statistical Summary

Introduction/Background:

Before installing a new air quality monitoring instrument, we tested to see if a sample of measurements taken at the testing lab differed significantly different from the long-term mean for the larger network population.

Methods:

Using a 2-tailed, one-sample z-test for on our normally distributed samples (W = 0.78).

Results:

While the mean of the new instrument sample was slightly higher (sample mean = 117 vs. population mean = 108), we found no significant difference $z_{100} = 1.4$, $p = 0.16$.

Implications:

Based on these results we approve the installation of this instrument at the new site. However, we would suggest statistical comparisons of the current and new unit side-by-side, prior to decommissioning the current

Our First Inferential Tests!
One sample z-test

When would you run a one-sample z-test?

- Use the One Sample z-test when:
  - You want to test for a difference between one sample mean and a larger population mean.
  - There is only one group (sample) being tested against the larger population.
  - You know (or can estimate) the mean and standard deviation of the population.
  - Data is normally distributed.

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}}$$

where:
- $\bar{X}$: Sample mean.
- μ: Population mean.
- σ: Population standard deviation. - n: Sample size.

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  - There is only one group (sample) being tested against the larger population.
  - You know (or can estimate) the mean and standard deviation of the population.
    - Expert Information.
    - Scientific Literature.
    - Data archives.
    - Meaningful (hypothesized value).
  - Data is normally distributed.

Flowcharts: Keeping it simple with questions

One sample z-test

Assume you have a sample of **20 observations**. The mean of this **sample is 150**, and you want to determine whether there is a **difference between the sample mean and the larger population mean**. Since you are not specifying a direction for this difference, you will conduct a two-tailed test.

The **population mean is 164**, and the **population standard deviation is 33**.

Next, calculate the obtained value for this **one-sample z-test.**

$$z = \frac{\bar{X} - \mu}{\frac{\sigma}{\sqrt{n}}} \qquad z = \frac{150 - 164}{\frac{33}{\sqrt{20}}} = -1.897$$

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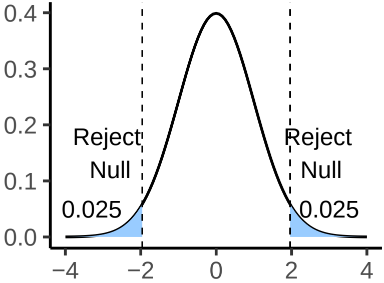

$$z = \frac{150 - 164}{\frac{33}{\sqrt{20}}} = -1.897$$

Critical Value Approach

- In the critical value approach, we compare the calculated test statistic to the critical value from the standard normal distribution. For a two-tailed test at the $\alpha = 0.05$ significance level, the critical values are approximately ± 1.96, which corresponds to the **0.025** in each tail.

- Decision Rule
- If $|\text{calculated test statistic}| >$ critical value = **Significant**
- If $|\text{calculated test statistic}| \leq$ critical value = **Not Significant**

Critical Value Approach

In this case:

$$|-1.897| < 1.96$$

- Thus, we fail to reject the null hypothesis, indicating that there is **no significant** difference between the sample mean and the population mean.

Finding the p-value

Now we need to determine the probability (p-value) associated with the calculated test statistic. You have several options to achieve this:

```
z_value <- -1.897
p_value <- 2 * pnorm(z_value)  # Two-tailed p-value
p_value  # This will give the result
```

```
## [1] 0.05782794
```

- Since this p-value (0.0578) is greater than the alpha threshold of 0.05, we conclude that the result is **not significant**.