Homework Problems for Lectures # 03 & 04

Assigned Date: December 09, 2022

1. Consider a 3G mobile in the 2GHz band moving at a velocity of 60 mph, and the underlying wireless channel is Rayleigh fading in nature. Let $h(t) = x(t) + \mathrm{j}y(t)$ be the channel estimate at time t, normalized to unit power. Employ the Jakes' model as you deem appropriate to answer the question below.

   (a) (2%) What is the maximum Doppler frequency f_d and coherence time T_c?

   (b) (1%) What is the joint distribution $f(x(0), y(0))$?

   (c) (1%) What is the correlation between $h(0)$ and $h(0.5T_c)$?

2. Compute the SVDs below by inspection/intuition and describe clearly how you arrived at each decomposition (3%)

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & \sqrt{5} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}, \begin{bmatrix} -1 & 0 & 0 \\ 0 & -\sqrt{5} & 0 \\ 0 & 0 & \sqrt{3} \end{bmatrix}, \begin{bmatrix} 1 & 2 \\ 1 & 2 \end{bmatrix}$$

3. In class we considered the Alamouti code for a "$r \times t = 1 \times 2$" MISO system. On similar lines, extend the Alamouti scheme to a "$r \times t = 2 \times 2$" MIMO system and describe the receiver processing. (3%)

4. Consider a 3×2 MIMO system with total transmit power $P_t = 13$ dB and per receiver noise variance $\sigma_n^2 = 0$ dB. Let the MIMO channel matrix be as given below

$$\begin{bmatrix} 1 & 3 \\ 3 & 1 \\ 1 & 1 \end{bmatrix}$$

   (a) (1%) Compute the MIMO-ZF receiver matrix?

   (b) (1%) Compute the MIMO-MMSE receiver matrix?

5. Consider the channel $\mathbf{y} = \mathbf{h}x + \mathbf{z}$, where $\mathbf{z} \sim \mathcal{CN}(0, \mathbf{K_z})$, $\mathbf{h}$ is a complex deterministic vector, and $\mathbf{x}$ is the zero mean unknown complex random variable to be estimated. The noise $\mathbf{z}$ and the data symbol x are assumed to be uncorrelated.

   (a) (2%) Consider the following estimate of x from $\mathbf{y}$ unsing the vector $\mathbf{c}$ (normalized so that $\|\mathbf{c}\| = 1$)

$$\widehat{x} := \alpha \mathbf{c}^* \mathbf{y} = \alpha \mathbf{c}^* \mathbf{h} x + \alpha \mathbf{c}^* \mathbf{z}.$$

   Show thta the constant α minimizes the mean square error $\mathbb{E}\{|x - \widehat{x}|^2\}$ is equal to

$$\frac{\mathbb{E}\{|x|^2\}|\mathbf{c}^*\mathbf{y}|^2}{\mathbb{E}\{|x|^2\}|\mathbf{c}^*\mathbf{y}|^2 + \mathbf{c}^*\mathbf{K_z}\mathbf{c}} \frac{\mathbf{h}^*\mathbf{c}}{|\mathbf{h}^*\mathbf{c}|}$$

(b) (2%) Calculate the minimal mean square error (denoted by MMSE) of the above linear estimate (by suing the value of α obtained in (a)). Show that

$$\frac{\mathbb{E}\{|x|^2\}}{\text{MMSE}} = 1 + \text{SNR} := 1 + \frac{\mathbb{E}\{|x|^2\}|\mathbf{c}^*\mathbf{y}|^2}{\mathbf{c}^*\mathbf{K_z}\mathbf{c}}$$

6. Let the MIMO channel matrix be as given below

$$\mathbf{H} = \begin{bmatrix} 1 & -0.8 \\ 0.2 & -1 \end{bmatrix}$$

Consider a transmit power of $P = -3$ dB on each transmit antenna, i.e., $\mathbb{E}\{|x|^2\} = -3$ dB on each transmit antenna. Let the transmit constellation be BPSK on each transmit antenna (i.e., $\pm\sqrt{P}$).

(a) (1%) Derive the ZF receiver decoding matrix for the above MIMO system.

(b) (1%) Let a received noisy output vector of the above MIMO channel be given as

$$\mathbf{y} = \begin{bmatrix} -1.50 \\ 0.27 \end{bmatrix}.$$

Employing ZF decoding, compute the linear estimate and the decoded transmitted synbols on each transmit antenna.

(c) (1%) What is the vector transmit constellation corresponding to BPSK?

(d) (1%) Employ the optimal ML decoder and compute the decoded vector belonging ro the transmit constellation above corresponding to the received vecor.