OFDM Part Ⅲ Assignment

Q36114221 蘇沛錦

· 非週期訊號

$$x(t) * y(t) \xrightarrow[\mathcal{F}]{(CTFT)} X(f)\cdot Y(f)$$

linear convolution
aperiodic convolution

A samples, B samples

$$x(n) * y(n) \xrightarrow[\mathcal{F}]{(DTFT)} X(e^{jw})\cdot Y(e^{jw})$$

A+B-1 samples

$$x(n) \xrightarrow{DFT} X_k$$

$$y(n) \xrightarrow{DFT} Y_k$$

$$\Longrightarrow x(n) \circledast y(n) \xrightarrow{DFT} X_k \cdot Y_k$$

· 週期訊號 If $x(n)$, $y(n)$ periodic, f.p. $= N$

$$x(n) \xrightarrow{DTFS} a_k \qquad y(n) \xrightarrow{DTFS} b_k$$

then

N samples, N samples

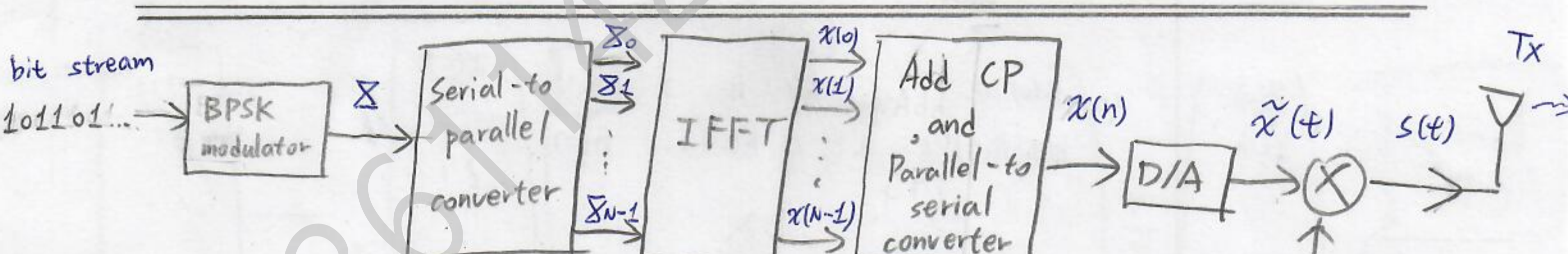
$$z(n) = x(n) \circledast y(n) \xrightarrow{DTFS} c_k = a_k \cdot b_k$$

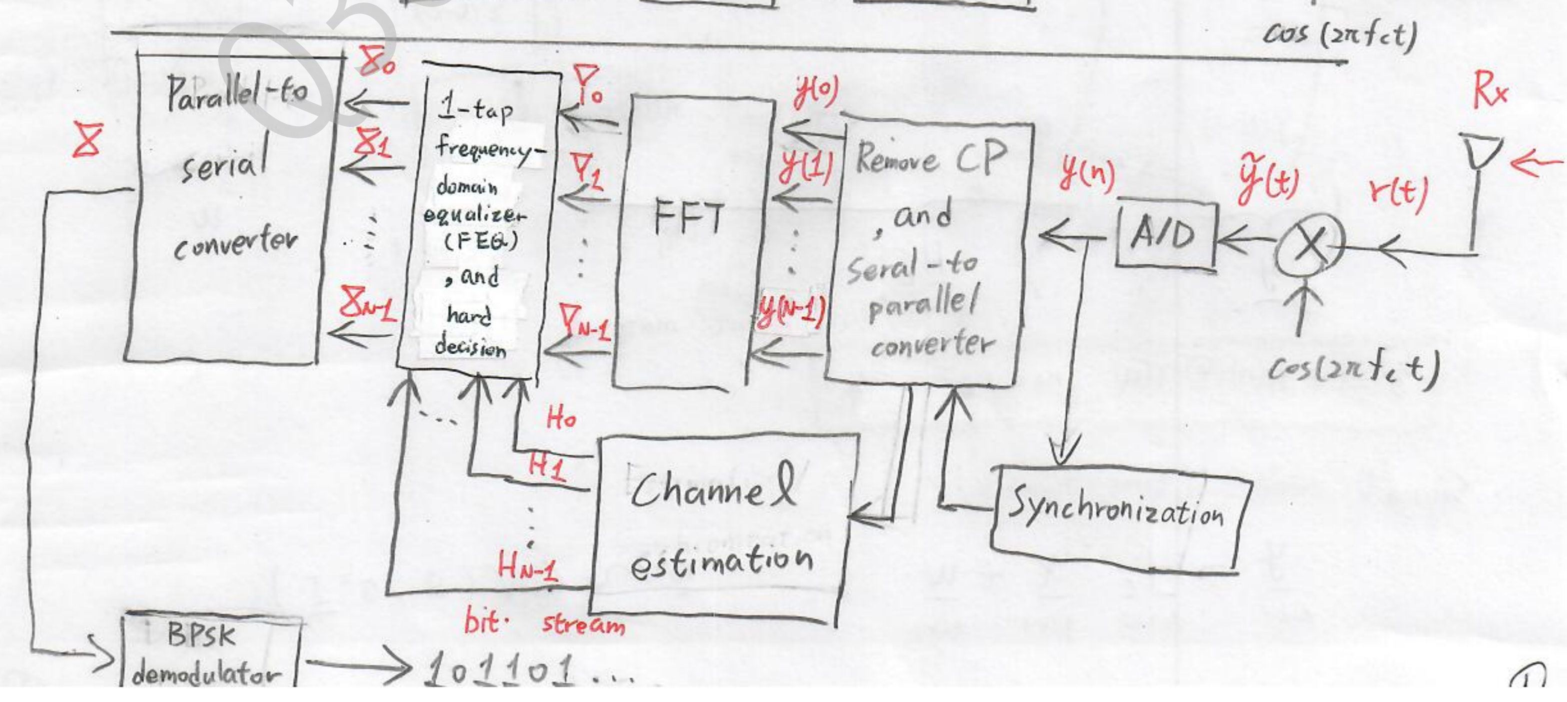
circular convolution
periodic convolution

N samples

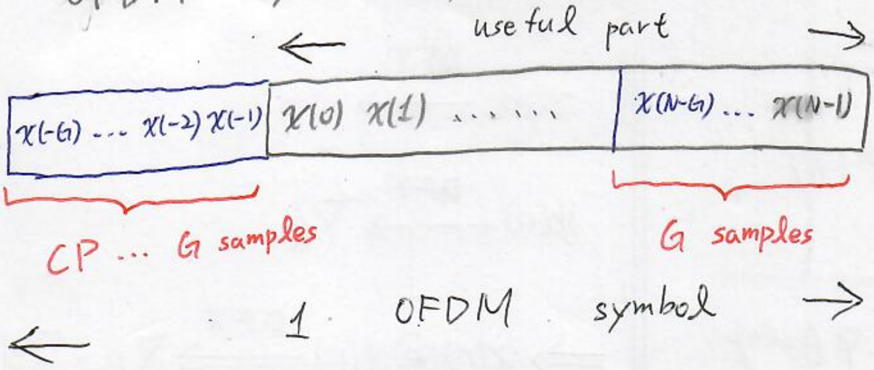
$$z(n) = \sum_{k=\langle N\rangle} c_k \cdot \phi_k(n) = \sum_{k=0}^{N-1} c_k \cdot \phi_k(n), \quad n = 0, 1, \ldots, N-1$$

$$\phi_k(n) = e^{j\left(\frac{2\pi}{N}\right)\cdot kn}$$

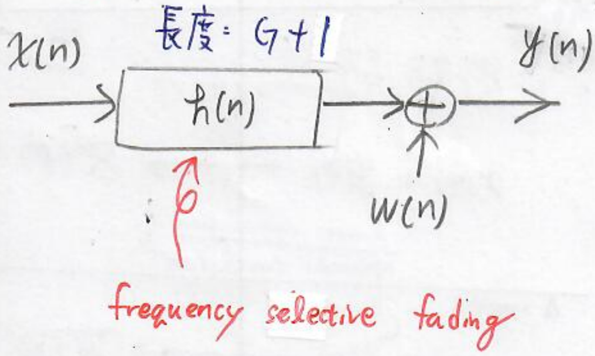




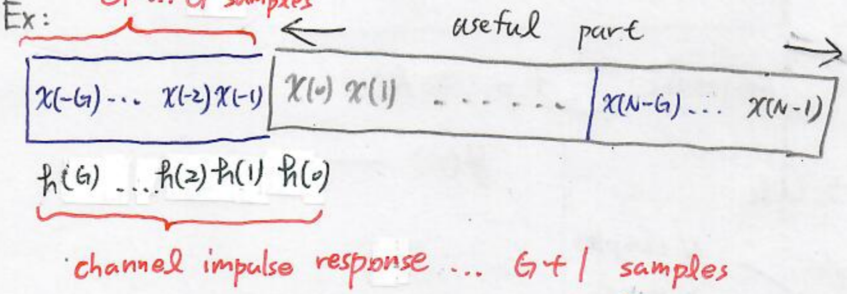
· OFDM Symbol



· Signal model (Time-domain)



- Ex:



$$\Longrightarrow \begin{pmatrix} y(0) \\ y(1) \\ \vdots \\ \vdots \\ y(N-1) \end{pmatrix} = \underbrace{\begin{pmatrix} h(G) & \cdots & h(2) & \boxed{h(1)} & h(0) & 0 & \cdots & 0 \\ \vdots & \ddots & & & & \ddots & & \vdots \\ \vdots & & \ddots & & & & \ddots & \vdots \\ 0 & \cdots & 0 & h(G) & \cdots & h(2) & h(1) & h(0) \end{pmatrix}}_{\underline{H}} \begin{pmatrix} x(-G) \\ \vdots \\ x(-2) \\ \boxed{x(-1)} \\ x(0) \\ x(1) \\ \vdots \\ x(N-G) \\ \vdots \\ \boxed{x(N-1)} \end{pmatrix} + \begin{pmatrix} w(0) \\ w(1) \\ \vdots \\ \vdots \\ w(N-1) \end{pmatrix}$$

CP (G samples)

Remove "CP" $\Longrightarrow$

$$\underbrace{\begin{pmatrix} y(0) \\ y(1) \\ \vdots \\ \vdots \\ y(N-1) \end{pmatrix}}_{\underline{y}} = \underbrace{\begin{pmatrix} h(0) & 0 & \cdots & 0 & h(G) & \cdots & h(2) & \boxed{h(1)} \\ h(1) & h(0) & 0 & \cdots & 0 & h(G) & \cdots & h(2) \\ \vdots & & \ddots & & & \ddots & & \vdots \\ \vdots & & & \ddots & & & \ddots & \vdots \\ 0 & \cdots & \cdots & & & \cdots & h(1) & h(0) \end{pmatrix}}_{\underline{H}_c} \underbrace{\begin{pmatrix} x(0) \\ x(1) \\ \vdots \\ x(N-G) \\ \vdots \\ \boxed{x(N-1)} \end{pmatrix}}_{\underline{x}} + \underbrace{\begin{pmatrix} w(0) \\ w(1) \\ \vdots \\ \vdots \\ w(N-1) \end{pmatrix}}_{\underline{w}}$$

circulant matrix

$$\boxed{y(n) = h(n) \circledast x(n), \quad n = 0, 1, 2, \ldots N-1}$$

· Signal model (time-domain)

$$\underset{N\times 1}{\underline{y}} = \underset{N\times N}{\underline{H}_c} \cdot \underset{N\times 1}{\underline{x}} + \underset{N\times 1}{\underline{w}} \qquad \underline{w} \sim \mathcal{CN}(\underline{0}, \sigma^2 \underline{I})$$

$$\boxed{y(n) = h(n) \circledast x(n)\ ,\ n = 0, 1, \ldots, N-1} \xrightarrow{\text{DFT}} \boxed{\bar{Y}_k = H_k \cdot \bar{X}_k\ ,\ k = 0, 1, \ldots, N-1}$$

$$\Rightarrow \underbrace{\begin{pmatrix} \bar{Y}_0 \\ \bar{Y}_1 \\ \vdots \\ \bar{Y}_{N-1} \end{pmatrix}}_{\underline{\bar{Y}}} = \begin{pmatrix} H_0 \bar{X}_0 \\ H_1 \bar{X}_1 \\ \vdots \\ H_{N-1} \bar{X}_{N-1} \end{pmatrix} + \begin{pmatrix} \bar{W}_0 \\ \bar{W}_1 \\ \vdots \\ \bar{W}_{N-1} \end{pmatrix}$$

$$\Rightarrow \underline{\bar{Y}} = \underbrace{\begin{pmatrix} H_0 & 0 & \cdots & 0 \\ 0 & H_1 & 0 \cdots & 0 \\ \vdots & & \ddots & 0 \\ 0 & \cdots & 0 & H_{N-1} \end{pmatrix}}_{\underline{D}_H} \underbrace{\begin{pmatrix} \bar{X}_0 \\ \bar{X}_1 \\ \vdots \\ \bar{X}_{N-1} \end{pmatrix}}_{\underline{\bar{X}}} + \underbrace{\begin{pmatrix} \bar{W}_0 \\ \bar{W}_1 \\ \vdots \\ \bar{W}_{N-1} \end{pmatrix}}_{\underline{\bar{W}}}$$

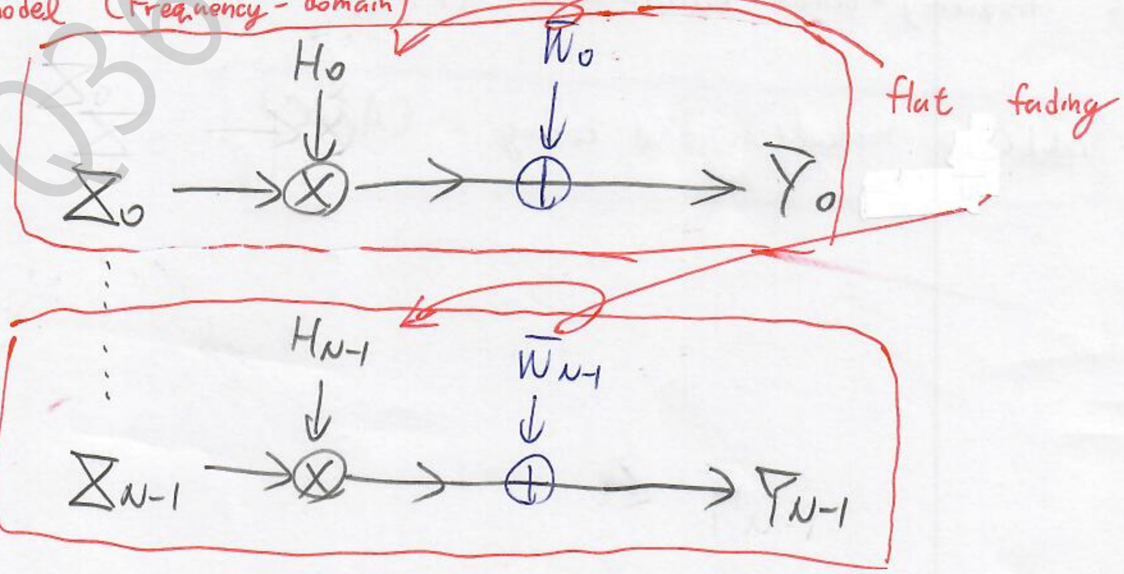
diagonal matrix

unitary matrix

$\because \underline{\bar{W}} = \underline{\tilde{F}}\, \underline{W} \sim \mathcal{CN}(\underline{0}, \sigma^2 \underline{I})$ **i.i.d.**

$\mathcal{CN}(\underline{0}, \sigma^2 \underline{I})$ **i.i.d.**

• Signal model (Frequency-domain)



OFDM is another example of <u>parallel AWGN channels</u>.

• Signal model (Frequency-domain)

$$\underline{y} = \underline{H}_c \cdot \underline{x} + \underline{w} \xrightarrow{\text{DFT}} \underline{Y} = \underline{\tilde{F}}\,\underline{H}_c \cdot \underline{x} + \underbrace{\underline{\tilde{F}} \cdot \underline{w}}_{\underline{\overline{W}}}$$

$$= \boxed{\underline{\tilde{F}}\,\underline{H}_c \cdot \underline{\tilde{F}}^H}\,\underline{X} + \underline{\overline{W}}$$

$$\boxed{\underline{Y} = \boxed{\underline{D}_H} \cdot \underline{X} + \underline{\overline{W}}}$$

$$\Longrightarrow \underline{\tilde{F}}\,\underline{H}_c\,\tilde{F}^H = \underline{D}_H$$

$$\Longrightarrow \boxed{\underline{H}_c = \underline{\tilde{F}}^H \underline{D}_H \underline{\tilde{F}}}$$

eigen decomposition

✓ ✓ (已知)

$$Y_k = H_k \cdot X_k + \overline{W}_k \quad , \quad k = 0, 1, \ldots, N-1$$

$$\Longrightarrow Y_k' = \frac{Y_k}{H_k} = X_k + \frac{\overline{W}_k}{H_k}$$

Implement hard decision   modulation symbol   Gaussian noise

- How to detect the modulation symbols on each sub-carrier?

1-tap frequency-domain equalization (FDE)

Adaptive modulation and coding (AMC)