Wireless Channels

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Large-scale vs. Small-scale Fading

- Large-scale propagation models
  - the mean signal strength for an arbitrary transmitter-receiver (T-R) separation distance
    → useful in estimating the radio coverage area of a transmitter
  - They characterize signal strength over large T-R separation distances (several hundreds or thousands of meters).
  - Ex: As a mobile moves away from the transmitter over much larger distance
    → The local average received signal will gradually decrease.

- Small-scale fading models
  - the variability of the signal strength in close spatial proximity to a particular location
  - They characterize the rapid fluctuations of the received signal strength over very short travel distances (a few wavelengths) or short time durations (on the order of seconds).
  - Ex: As the mobile moves over small distance
    → The instantaneous received signal strength may fluctuate rapidly.

Large-scale vs. Small-scale Fading

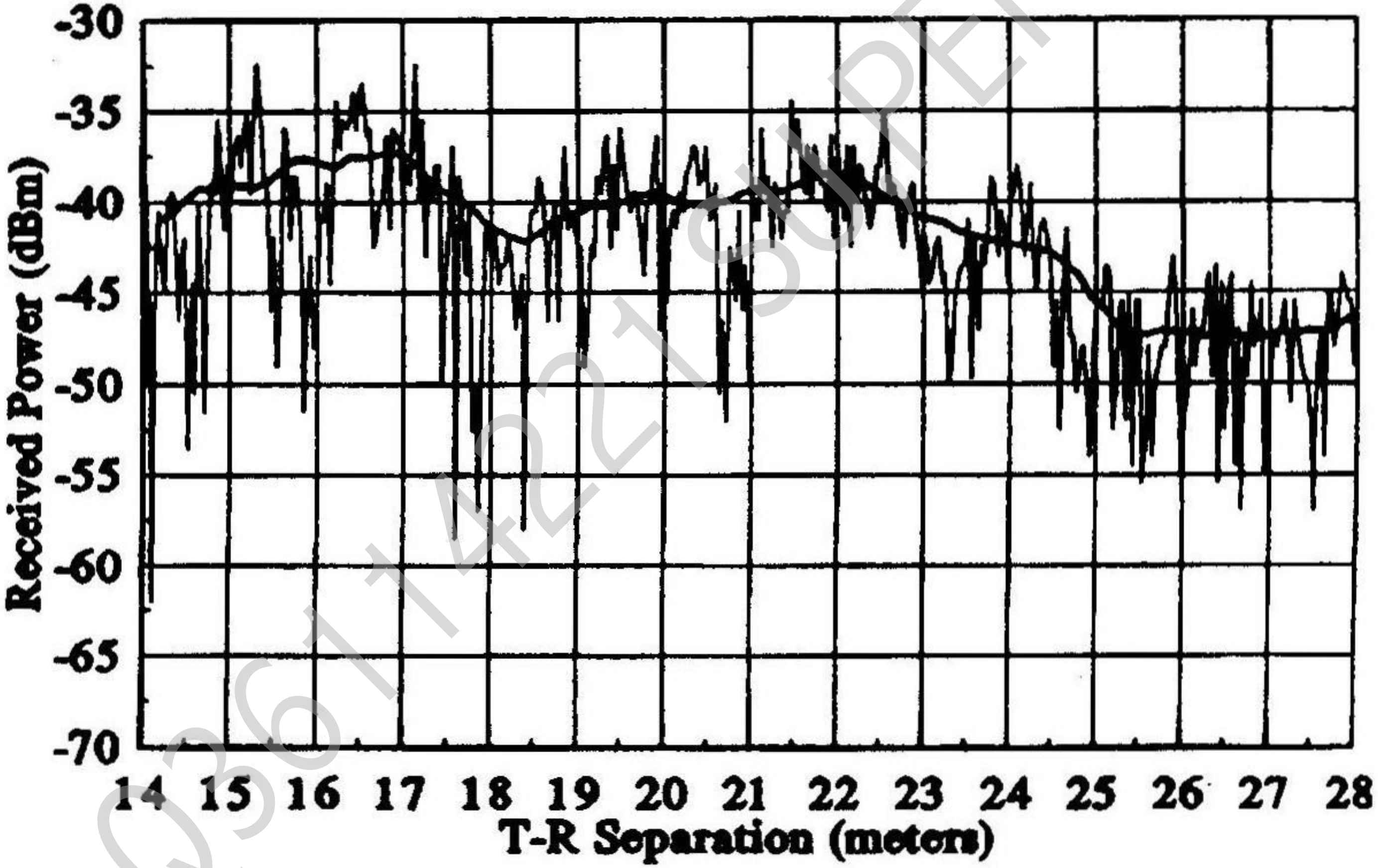

Figure 4.1 Small-scale and large-scale fading.

Free Space Propagation Model

- The received power (Friis free space equation)

$$P_r(d) = \frac{P_t G_t G_r \lambda^2}{(4\pi)^2 d^2 L}$$

  - P_t : the transmitted power
  - G_t : the transmitter antenna gain
  - G_r : the receiver antenna gain
  - d : the T-R separation distance in meters
  - L : the system loss factor ($L \geq 1$)
    - Not related to propagation
    - Transmission line attenuation / Filter losses / Antenna losses
    - $L = 1 \rightarrow$ No loss in the system hardware
  - λ : the wavelength in meters
    - $\lambda = \frac{c}{f}$
    - c : the speed of light given in meters/s
    - f : the carrier frequency in Hertz
- $P_r(d) \propto \lambda^2 \Rightarrow$ The received power fall offs quadratically with the carrier frequency.
- $P_r(d) \propto \frac{1}{d^2} \Rightarrow$ The received power decays with distance at a rate of 20 dB/decade.

Free Space Propagation Model

- Path loss for the free space model
  - Represent signal attenuation as positive quantity measured in dB

$$PL(dB) = 10\log\frac{P_t}{P_r} = -10\log\left[\frac{G_t G_r \lambda^2}{(4\pi)^2 d^2}\right]$$

- P_r is valid for values of d which are in the far-field of the transmitting antenna.

- Far-field / Fraunhofer region
  - The region beyond the far-field / Fraunhofer distance $\quad d_f = \frac{2D^2}{\lambda}$
    - D: the largest physical linear dimension of the antenna
  - To be in the far-field region, d_f must satisfy

$$d_f \gg D \qquad \text{and} \qquad d_f \gg \lambda$$

Free Space Propagation Model

- The Friis free space model does not hold for $d = 0$

  $\rightarrow$ Use a close-in distance, d_0, as a known received power reference point
- The received power in free space at a distance greater than d_0 is given by

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^2 \qquad d \geq d_0 \geq d_f$$

- The value of $P_r(d_0)$ may be predicted from the Friis free space model, or be measured in the radio environment.
- Because of the large dynamic range of received power levels, we often use dBm or dBW units to express received power level

$$P_r(d)\mathrm{dBm} = 10\log\left[\frac{P_r(d_0)}{0.001\mathrm{W}}\right] + 20\log\left(\frac{d_0}{d}\right) \qquad d \geq d_0 \geq d_f$$

where $P_r(d_0)$ is in units of watts.

Log-distance Path Loss Model

- The average large-scale path loss for an arbitrary T-R separation

$$\overline{PL}(d) \propto \left(\frac{d}{d_0}\right)^n \qquad \overline{PL}(\text{dB}) = \overline{PL}(d_0) + 10n\log\left(\frac{d}{d_0}\right)$$

- n : the path loss exponent (the rate at which the path loss increases with distance)

Table 4.2 Path Loss Exponents for Different Environments

Environment	Path Loss Exponent, *n*
Free space	2
Urban area cellular radio	2.7 to 3.5
Shadowed urban cellular radio	3 to 5
In building line-of-sight	1.6 to 1.8
Obstructed in building	4 to 6
Obstructed in factories	2 to 3

Log-normal Shadowing

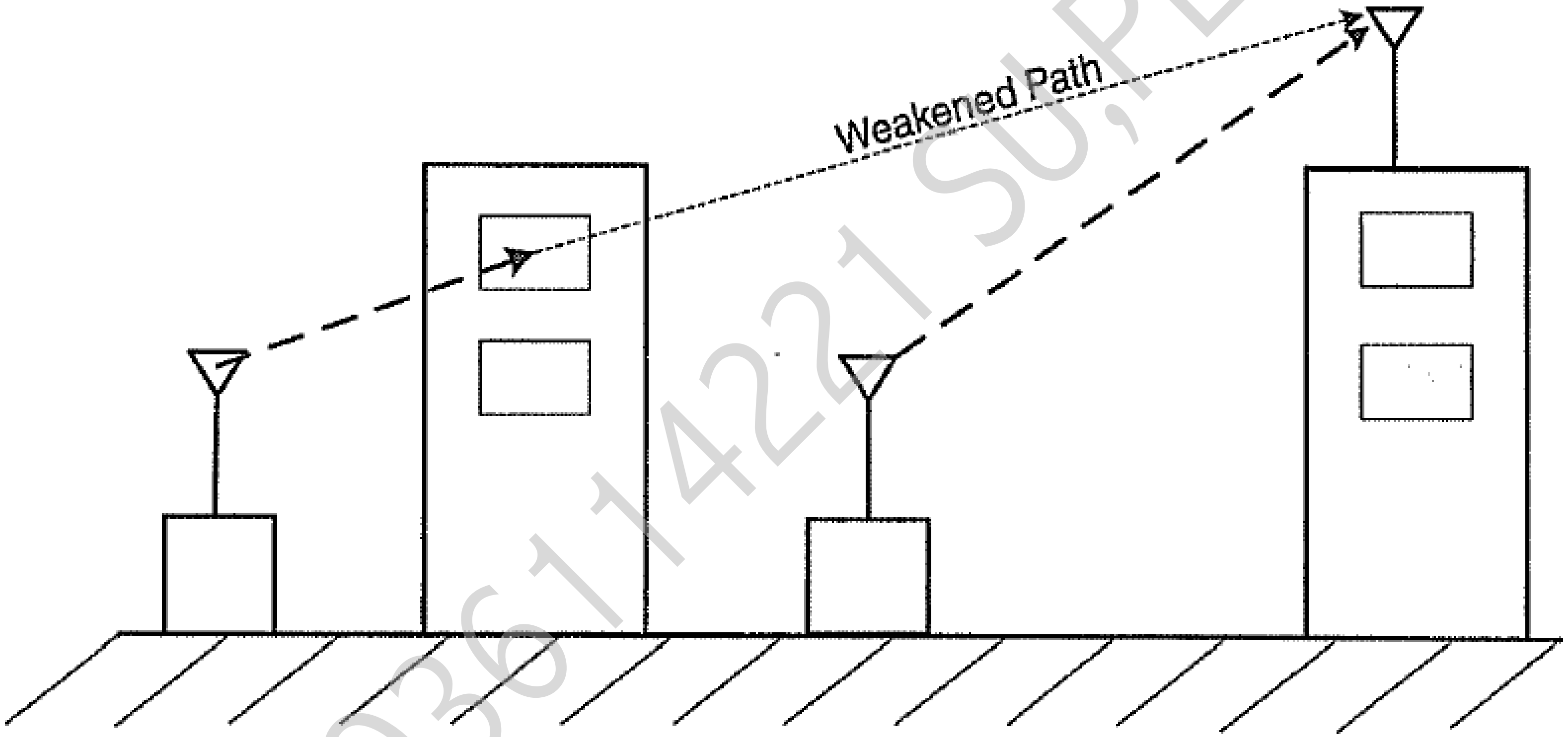

Figure 3.3 Shadowing causes large random fluctuations about the pathloss model: Figure from [28], courtesy of IEEE.

Log-normal Shadowing

- The surrounding environmental clutter may be vastly different at two different locations having the same T-R separation.

$$PL(d)[\mathrm{dB}] = \overline{PL}(d) + X_\sigma = \overline{PL}(d_0) + 10n\log(\frac{d}{d_0}) + X_\sigma$$

$$P_r(d)[\mathrm{dBm}] = P_t[\mathrm{dBm}] - PL(d)[\mathrm{dB}] \qquad \text{(antenna gains included in } PL(d)\text{)}$$

  - X_σ : a zero-mean Gaussian random variable (in dB) with standard deviation σ (also in dB)
  - In practice, the values of n and σ are computed from measured data by linear regression.

- The log normal distribution describes the random *shadowing* effects which occur over a large number of measurement locations which have the same T-R separation, but have different levels of clutter on the propagation path.

Log-normal Shadowing

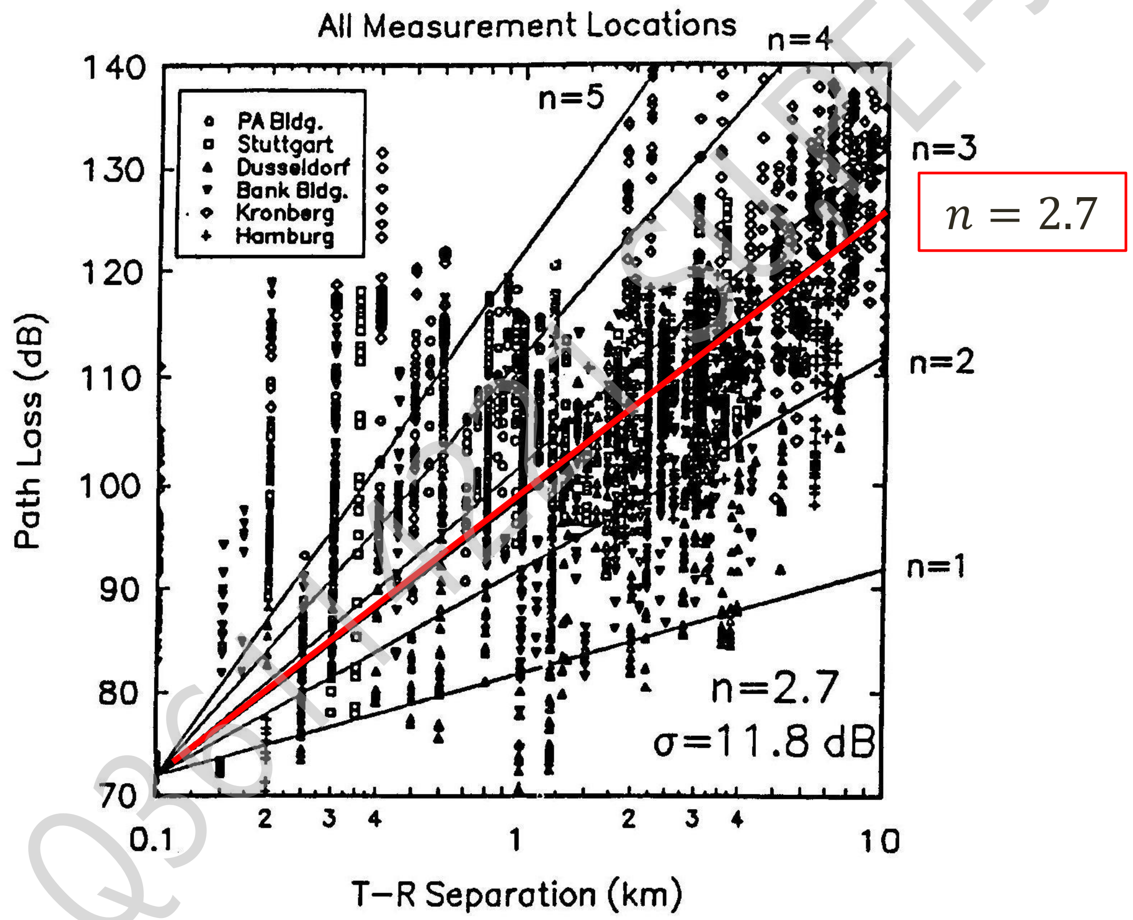

Figure 4.17 Scatter plot of measured data and corresponding MMSE path loss model for many cities in Germany. For this data, $n = 2.7$ and $\sigma = 11.8$ dB [from [Sei91] © IEEE].

Percentage of Coverage Area

- The percentage of area with a received signal that is equal or greater than γ

$$U(\gamma) = \frac{1}{\pi R^2} \int Pr[P_r(r) > \gamma] dA = \frac{1}{\pi R^2} \int_0^{2\pi} \int_0^R Pr[P_r(r) > \gamma]\, r dr d\theta$$

$$Pr[P_r(r) > \gamma] = \mathrm{Q}\left(\frac{\gamma - \overline{P_r(r)}}{\sigma}\right) = \frac{1}{2} - \mathrm{erf}\left(\frac{\gamma - [P_t - (\overline{PL}(d_0) + 10n \log(r/d_0))]}{\sigma\sqrt{2}}\right)$$

  - By choosing the signal level such that $\overline{P_r}(R) = \gamma$, $U(\gamma)$ can be shown to be

$$U(\gamma) = \frac{1}{2}\left[1 + \exp\left(\frac{1}{b^2}\right)\left(1 - \mathrm{erf}(\frac{1}{b})\right)\right]$$

  - Let $b = (10n \log e)/\sigma\sqrt{2}$

Percentage of Coverage Area

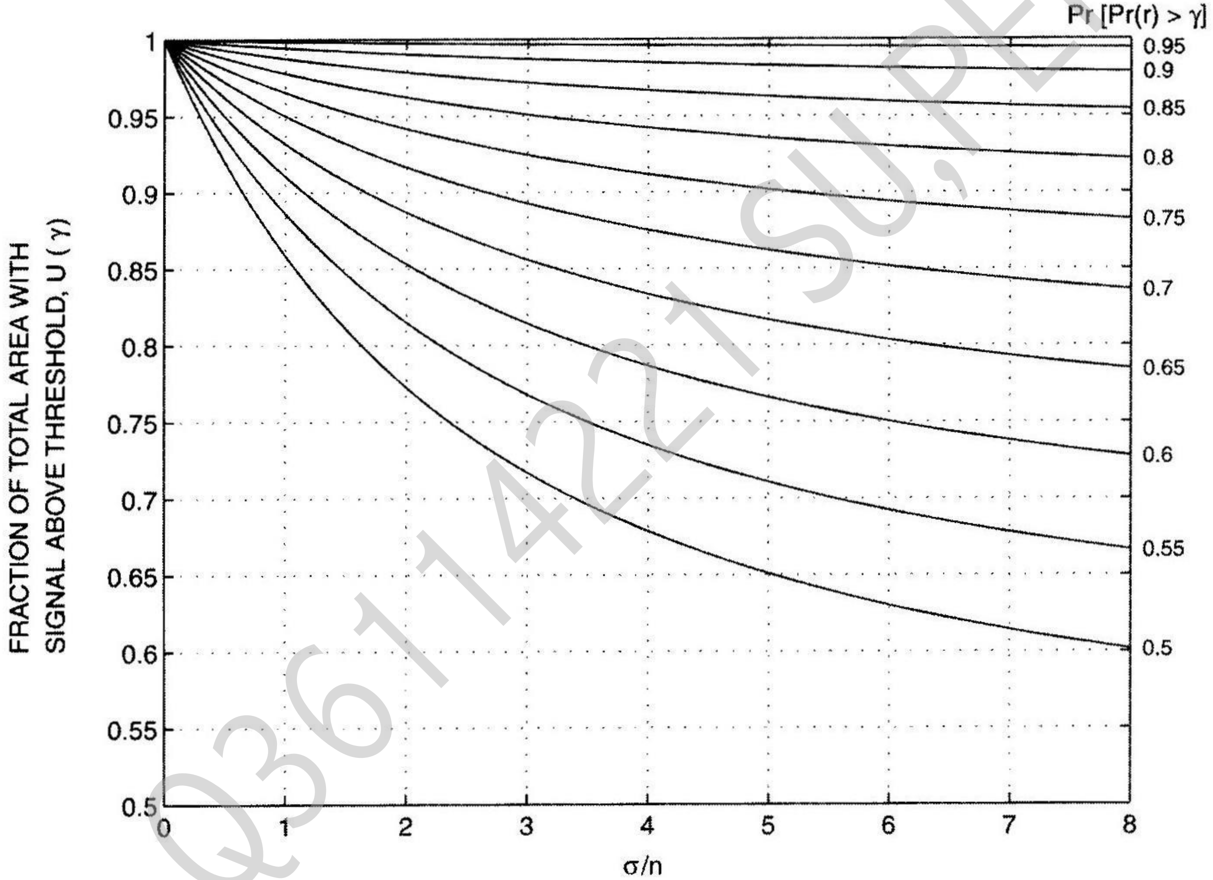

Figure 4.18 Family of curves relating fraction of total area with signal above threshold, $U(\gamma)$ as a function of probability of signal above threshold on the cell boundary.

Example 4.9

Four received power measurements were taken at distances of 100 m, 200 m, 1 km, and 3 km from a transmitter. These measured values are given in the following table. It is assumed that the path loss for these measurements follows the model in Equation (4.69.a), where $d_0 = 100$ m: (a) find the minimum mean square error (MMSE) estimate for the path loss exponent, n; (b) calculate the standard deviation about the mean value; (c) estimate the received power at $d = 2$ km using the resulting model; (d) predict the likelihood that the received signal level at 2 km will be greater than -60 dBm; and (e) predict the percentage of area within a 2 km radius cell that receives signals greater than –60 dBm, given the result in (d).

Distance from Transmitter	Received Power
100 m	0 dBm
200 m	–20 dBm
1000 m	–35 dBm
3000 m	–70 dBm

Solution

The MMSE estimate may be found using the following method. Let p_i be the received power at a distance d_i and let $\hat{p}_i$ be the estimate for p_i using the $(d/d_0)^n$ path loss model of Equation (4.67). The sum of squared errors between the measured and estimated values is given by

$$P_r(d) = P_r(d_0)\left(\frac{d_0}{d}\right)^n \qquad d \geq d_0 \geq d_f$$

$$J(n) = \sum_{i=1}^{k} (p_i - \hat{p}_i)^2$$

The value of n which minimizes the mean square error can be obtained by equating the derivative of $J(n)$ to zero, and then solving for n.

(a)Using Equation (4.68), we find $\hat{p}_i = p_i(d_0) - 10n\log(d_i § 100\text{ m})$. Recognizing that $P(d_0) = 0$ dBm, we find the following estimates for $\hat{p}_i$ in dBm:

$$\hat{p}_1 = 0,\ \ \hat{p}_2 = -3n,\ \ \hat{p}_3 = -10n,\ \ \hat{p}_4 = -14.77n.$$

The sum of squared errors is then given by

$$J(n) = (0-0)^2 + (-20-(-3n))^2 + (-35-(-10n))^2 + (-70-(-14.77n))^2$$

$$= 6525 - 2887.8n + 327.153n^2$$

$$\frac{dJ(n)}{dn} = 654.306n - 2887.8.$$

Setting this equal to zero, the value of n is obtained as $n = 4.4$.

(b) The sample variance $\sigma^2 = J(n)/4$ at $n = 4.4$ can be obtained as follows.

$$J(n) = (0+0) + (-20+13.2)^2 + (-35+44)^2 + (-70+64.988)^2$$
$$= 152.36.$$
$$\sigma^2 = 152.36/4 = 38.09\,\mathrm{dB}^2$$

therefore

$\sigma = 6.17$ dB, which is a biased estimate. In general, a greater number of measurements are needed to reduce σ^2.

(c) The estimate of the received power at $d = 2$ km is given by

$$\hat{p}(d = 2\text{ km}) = 0 - 10(4.4)\log(2000/100) = -57.24\text{ dBm}.$$

A Gaussian random variable having zero mean and $\sigma = 6.17$ dB could be added to this value to simulate random shadowing effects at $d = 2$ km.

(d) The probability that the received signal level will be greater than -60 dBm is given by

$$Pr[P_r(d) > -60\text{ dBm}] = Q\left(\frac{\gamma - \overline{P_r(d)}}{\sigma}\right) = Q\left(\frac{-60+57.24}{6.17}\right) = 67.4\,\%.$$

(e) If 67.4% of the users on the boundary receive signals greater than -60 dBm, then Equation (4.78) or Figure 4.18 may be used to determine that 88% of the cell area receives coverage above -60 dBm.

Multipath Propagation

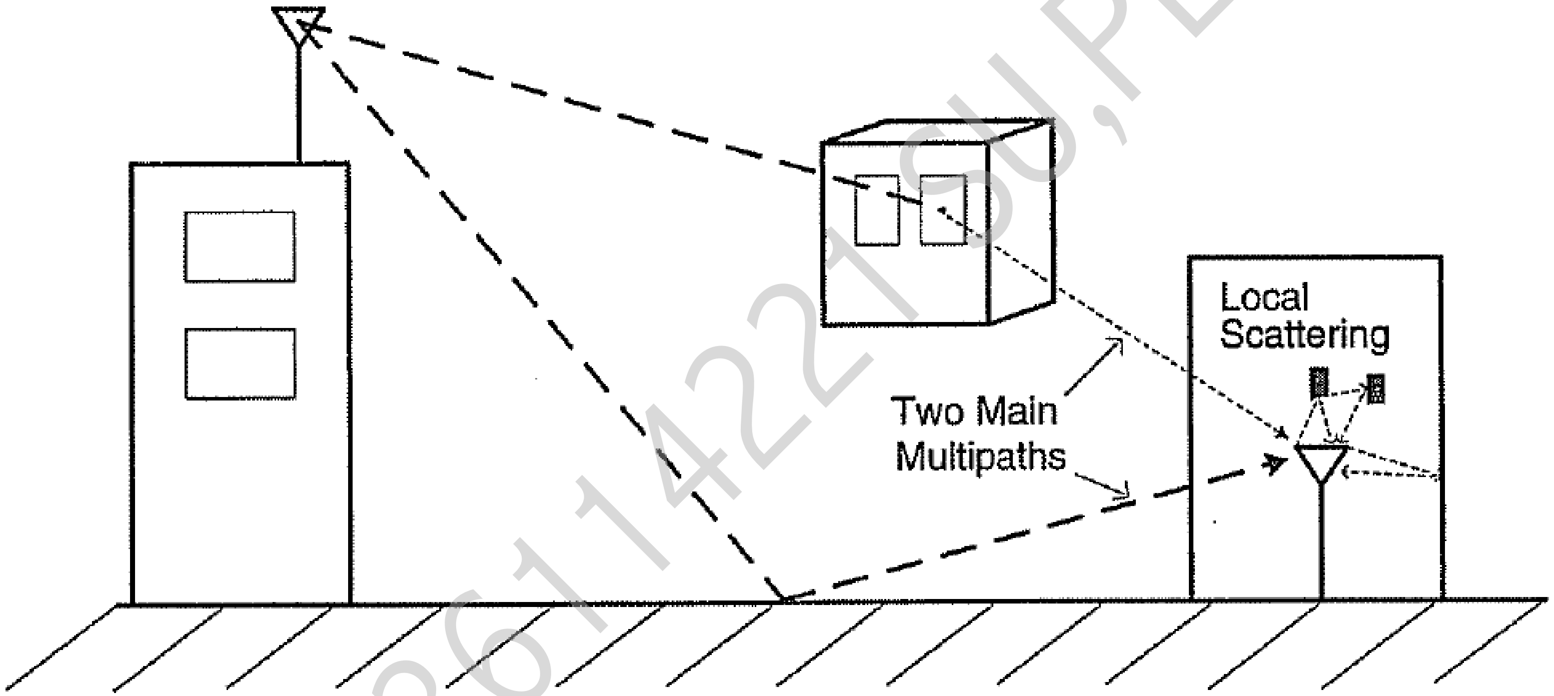

Figure 3.10 A channel with a few major paths of different lengths, with the receiver seeing a number of locally scattered versions of those paths

Impulse Response Model of a Multipath Channel

- Let $x(t)$ represent the transmitted signal, then the received signal $y(d,t)$ at position d can be expressed as

$$y(d,t) = x(t) \otimes h(d,t) = \int_{-\infty}^{\infty} x(\tau)h(d,t-\tau)d\tau$$

- For a causal system, $h(d,t) = 0$ for $t < 0$, thus this equation reduces to

$$y(d,t) = \int_{-\infty}^{t} x(\tau)h(d,t-\tau)d\tau$$

Figure 5.2 The mobile radio channel as a function of time and space.

Impulse Response Model of a Multipath Channel

- Substituting $d = vt$ in the equation, we obtain

  (Because the receiver moves along the ground at a constant velocity v)

$$y(vt, t) = \int_{-\infty}^{t} x(\tau) h(vt, t - \tau) d\tau$$

- Since v is a constant, $y(vt, t)$ is just a function of t.

$$y(t) = \int_{-\infty}^{t} x(\tau) h(vt, t - \tau) d\tau = x(t) \otimes h(vt, t) = x(t) \otimes h(d, t)$$

Impulse Response Model of a Multipath Channel

- $x(t)$: the transmitted bandpass waveform
- $y(t)$: the received waveform
- $h(t,\tau)$: the impulse response of the time varying multipath channel
  - t : the time variations due to motion
  - τ : the channel multipath delay for a fixed value of τ

$$y(t) = \int_{-\infty}^{\infty} x(\tau)h(t,\tau)d\tau = x(t) \otimes h(t,\tau)$$

- $h_b(t)$: a complex baseband impulse response
- $c(t)$: the complex envelope of $x(t)$, i.e. $x(t) = Re\{c(t)\exp(\mathrm{j}2\pi f_c t)\}$
- $r(t)$: the complex envelope of $y(t)$, i.e. $y(t) = Re\{r(t)\exp(\mathrm{j}2\pi f_c t)\}$

$$r(t) = c(t) \otimes \frac{1}{2} h_b(t,\tau)$$

Impulse Response Model of a Multipath Channel

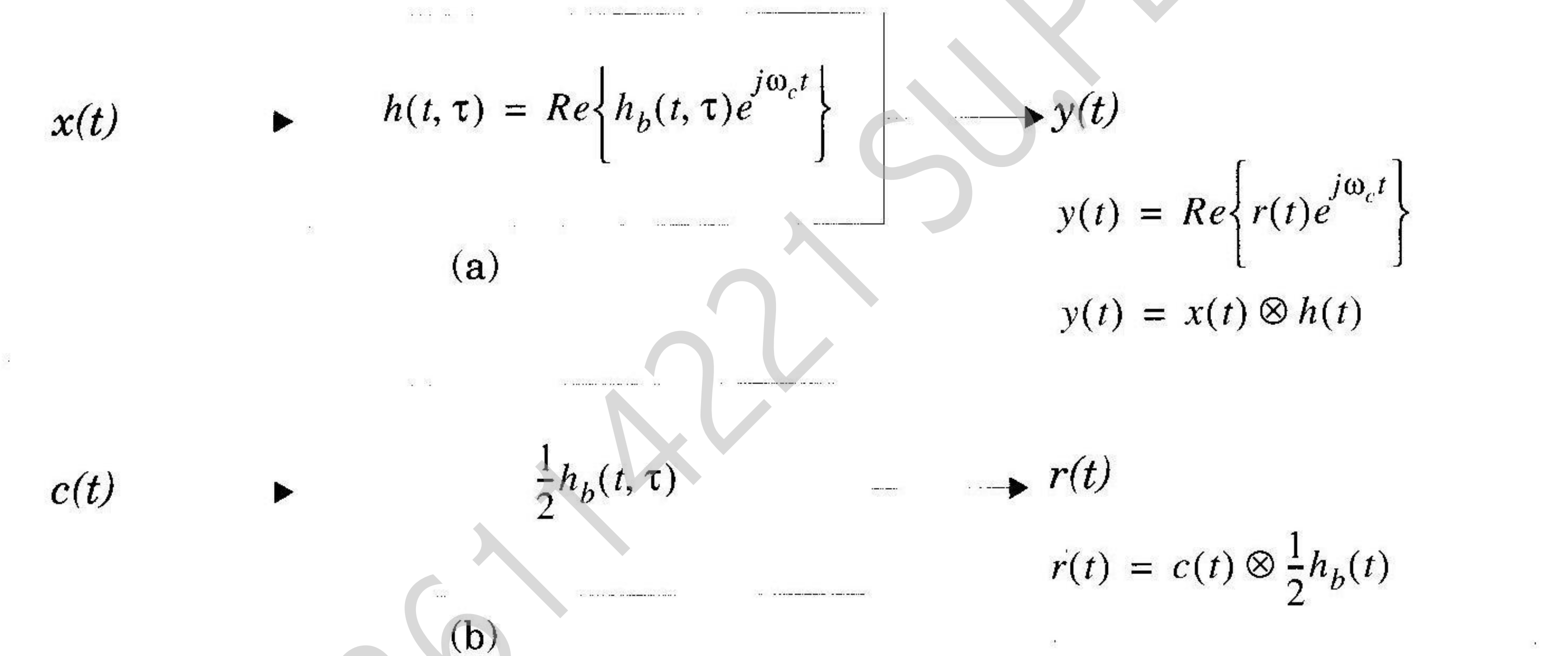

Figure 5.3 (a) Bandpass channel impulse response model; (b) baseband equivalent channel impulse response model.

Impulse Response Model of a Multipath Channel

- The baseband impulse response of a multipath

$$h_b(t,\tau) = \sum_{i=0}^{N-1} a_i(t,\tau)\exp\left[\mathrm{j}\left(2\pi f_c \tau_i(t) + \phi_i(t,\tau)\right)\right]\delta(\tau - \tau_i)$$

  - $a_i(t,\tau)$: the real amplitudes of ith multipath component at time t
  - $\tau_i(t)$: excess delays of ith multipath component at time t
  - $2\pi f_c \tau_i(t)$: the phase shift due to free space propagation of the ith multipath component
  - $\phi_i(t,\tau)$: any additional phase shifts which are encountered in the channel
  - τ_i : *excess delay*
    - The relative delay of the ith multipath component as compared to the first arriving component.
    - $\tau_0 = 0$ (the first arriving signal at the receiver)
    - $\Delta\tau = \tau_{i+1} - \tau_i$, and $\tau_i = i\Delta\tau$ for $i = 0$ to $N-1$
    - N : the total number of possible equally-spaced multipath components (including τ_0)
    - The *maximum excess delay* of the channel is given by $N\Delta\tau$.

Impulse Response Model of a Multipath Channel

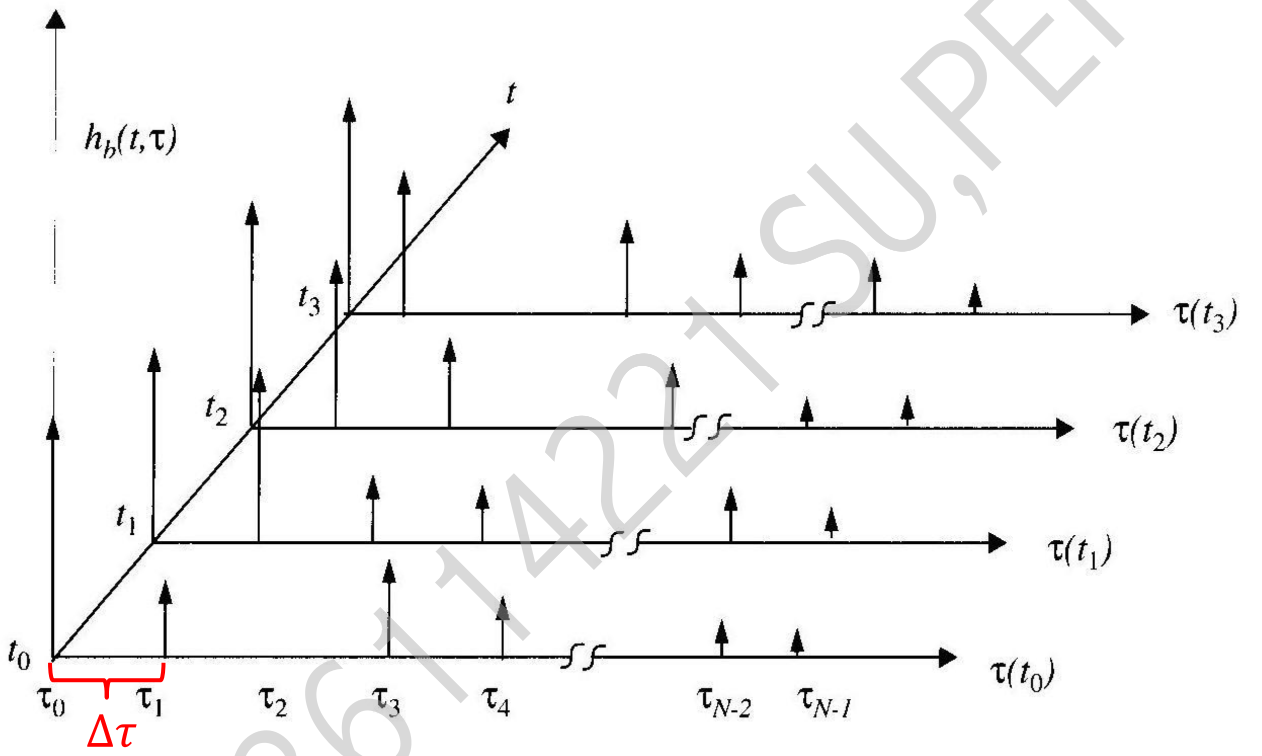

Figure 5.4 An example of the time varying discrete-time impulse response model for a multipath radio channel. Discrete models are useful in simulation where modulation data must be convolved with the channel impulse response [Tra02].

Impulse Response Model of a Multipath Channel

- If the channel impulse response is assumed to be time invariant, or is at least wide sense stationary (WSS) over a small-scale time or distance interval, then the channel response may be simplified as

$$h_b(\tau) = \sum_{i=0}^{N-1} a_i \exp[\mathrm{j}\theta_i]\,\delta(\tau - \tau_i)$$

  - θ_i lumps together all the mechanism for phase shifts of the ith multipath component

- **Q**: Why delta(t) is involved in the equation?
- **A**: The unit impulse function $\delta(\cdot)$ determines the specific multipath bins that have components at time t and excess delay τ_i.

Impulse Response Model of a Multipath Channel

- When measuring or predicting $h_b(\tau)$, a probing pulse $p(t)$ which approximates a delta function is used at the transmitter.

$$p(t) \approx \delta(t-\tau)$$

- *Power delay profile* of the channel
  - The spatial average of $|h_b(t;\tau)|^2$ over a local area

$$P(\tau) \approx k\overline{|h_b(t;\tau)|^2}$$

  - k : The gain relates the transmitted power in the probing pulse $p(t)$ to the total power received in a multipath delay profile

Time Dispersion Parameters

- mean excess delay ($\bar{\tau}$)
  - The first moment of the power delay profile

$$\bar{\tau} = \frac{\sum_k a_k^2 \tau_k}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k}{\sum_k P(\tau_k)}$$

- rms delay spread (σ_τ)
  - The square root of the second central moment of the power delay profile

$$\sigma_\tau = \sqrt{\overline{\tau^2} - (\bar{\tau})^2}$$

$$\overline{\tau^2} = \frac{\sum_k a_k^2 \tau_k^2}{\sum_k a_k^2} = \frac{\sum_k P(\tau_k)\tau_k^2}{\sum_k P(\tau_k)}$$

Time Dispersion Parameters

- maximum excess delay (X dB)
  - The time delay during which multipath energy falls to X dB below the maximum

$$\tau_X - \tau_0$$

    - τ_X : the maximum delay at which a multipath component is within X dB of the strongest arriving multipath signal (which does not necessarily arrive at τ_0)
    - τ_0 : the first arriving signal

  - The temporal extent of the multipath that is above a particular threshold.

  - The value of τ_X is sometimes called *excess delay spread* of a power delay profile, but in all cases must be specified with a threshold that relates the multipath noise floor to the maximum received multipath component.

  - The noise threshold is used to differentiate between received multipath components and thermal noise. If the threshold is set too low, then noise will be processed as multipath.

Time Dispersion Parameters

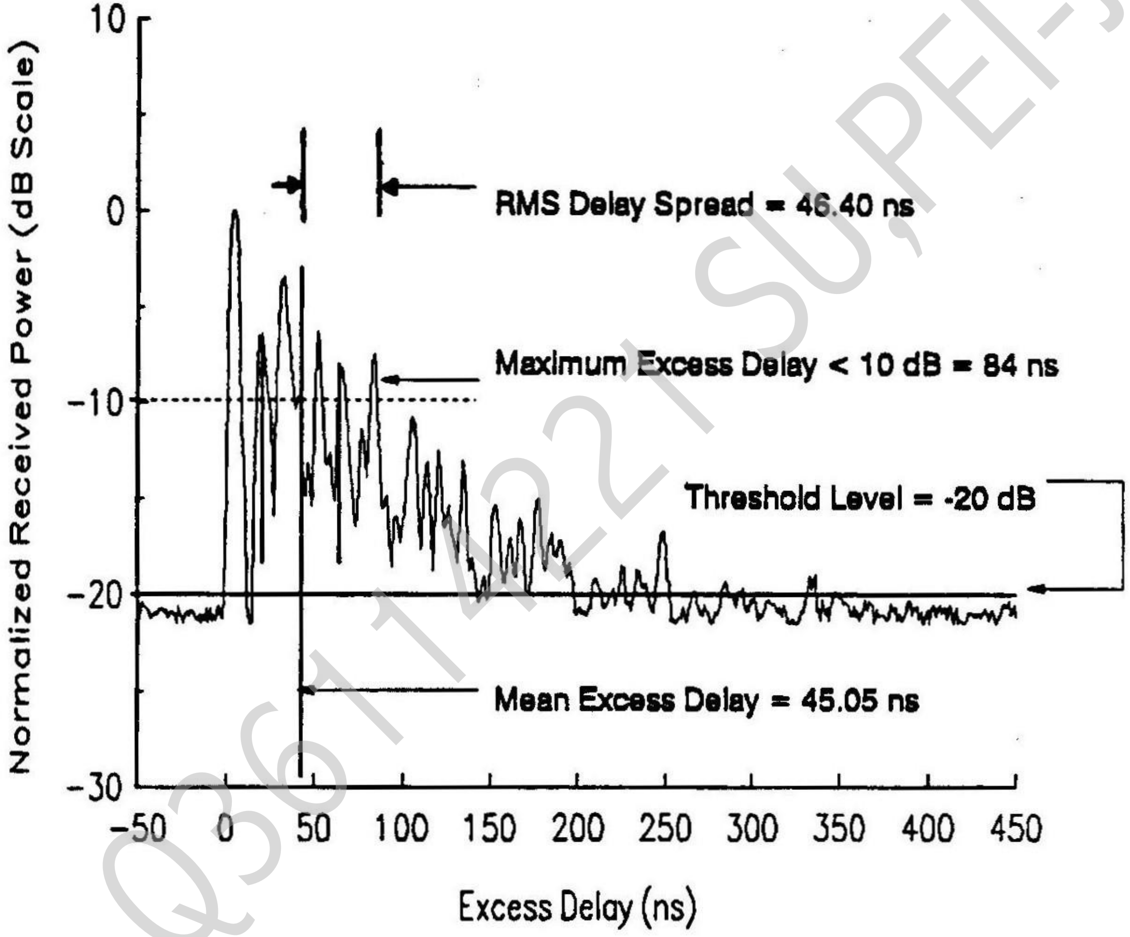

Figure 5.10 Example of an indoor power delay profile; rms delay spread, mean excess delay, maximum excess delay (10 dB), and threshold level are shown.

Coherence Bandwidth

- Coherence bandwidth, B_C, is a statistical measure of the range of frequencies over which the channel can be considered “flat” (i.e., a channel which passes all spectral components with approximately equal gain and linear phase).

- B_C is a rough measure for the maximum separation between a frequency f_1 and a frequency f_2 where the channel response is correlated.

$$|f_1 - f_2| \le B_c \quad \Rightarrow \quad H(f_1) \approx H(f_2)$$

$$|f_1 - f_2| > B_c \quad \Rightarrow \quad H(f_1) \text{ and } H(f_2) \text{ are uncorrelated}$$

- The bandwidth over which the frequency correlation is above 0.9

$$B_C \approx \frac{1}{50\sigma_\tau}$$

- The bandwidth over which the frequency correlation is above 0.5

$$B_C \approx \frac{1}{5\sigma_\tau}$$

- The rms delay spread and coherence bandwidth are inversely proportional to one another.

Doppler Shift

- The relative motion between the base station (BS) and the mobile:
  - The difference in path length

$$\Delta l = d\cos\theta = v\Delta t\cos\theta$$

  - The phase change in the received signal due to Δl

$$\Delta\phi = \frac{2\pi\Delta l}{\lambda} = \frac{2\pi v\Delta t}{\lambda}\cos\theta$$

- The apparent change in frequency (Doppler shift)

$$f_d = \frac{1}{2\pi}\cdot\frac{\Delta\phi}{\Delta t} = \frac{v}{\lambda}\cos\theta$$

  - Mobile is moving toward the BS → Doppler shift is positive
  - Mobile is moving away from the BS → Doppler shift is negative

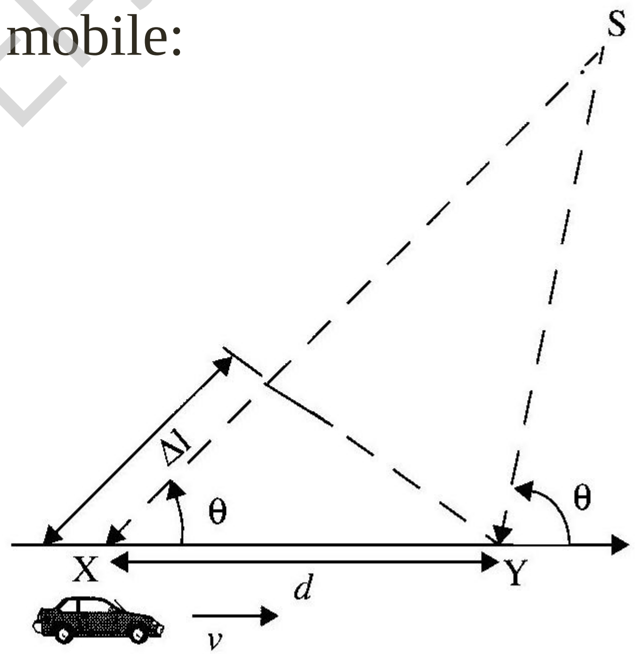

Figure 5.1 Illustration of Doppler effect.

Doppler Spread

- Doppler spread B_D is a measure of the spectral broadening caused by the time rate of the change of the mobile radio channel.
- B_D is defined as the range of frequencies over which the received Doppler spectrum is essentially non-zero.
- When a pure sinusoidal tone of frequency f_c is transmitted, and the received signal spectrum (Doppler spectrum) will have components in the range $f_c - f_d$ to $f_c + f_d$.
- <u>If the baseband signal bandwidth is much greater than B_D, the effects of Doppler spread are negligible at the receiver (i.e., a *slow fading* channel).</u>

Doppler Spectrum

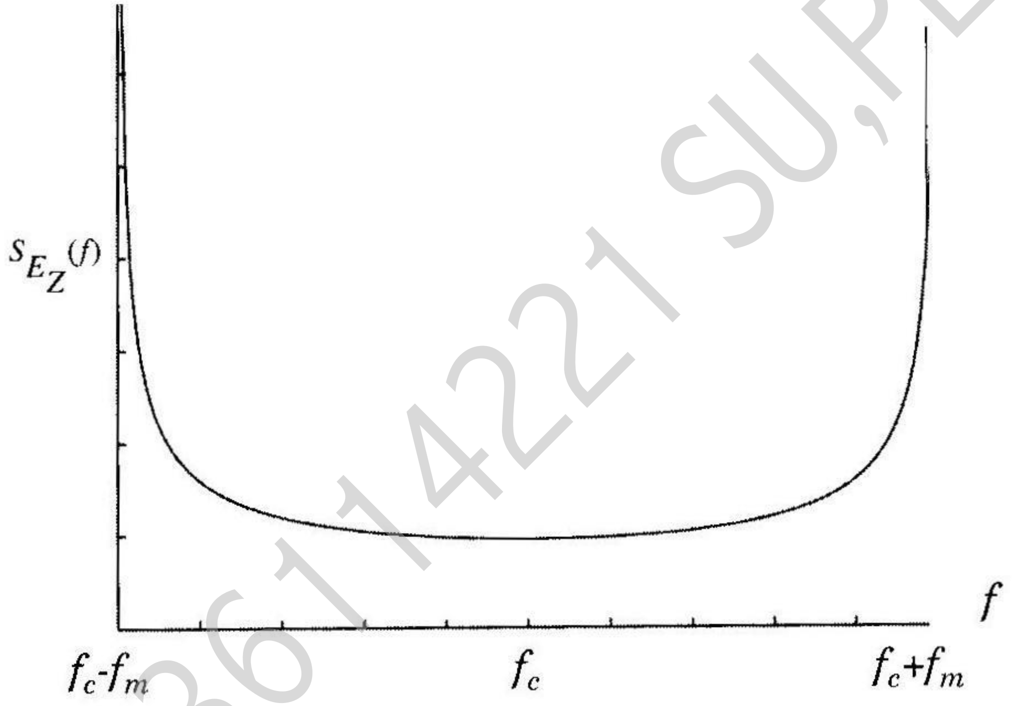

Figure 5.20 Doppler power spectrum for an unmodulated CW carrier [from [Gan72] © IEEE].

Coherence Time

- Coherence time T_C is the time domain dual of Doppler spread and is used to characterize the time varying nature of the frequency dispersiveness of the channel in the time domain.

- Doppler spread and coherence time are inversely proportional to one another:

$$T_C \approx \frac{1}{f_m}$$

  - f_m : the maximum Doppler shift given by $f_m = v/\lambda$

- T_C gives the time duration over which the channel is significantly correlated.

$$\boxed{\begin{array}{lll} |t_1 - t_2| \leq T_c & \Rightarrow & \mathbf{h}(t_1) \approx \mathbf{h}(t_2) \\ |t_1 - t_2| > t_c & \Rightarrow & \mathbf{h}(t_1) \text{ and } \mathbf{h}(t_2) \text{ are uncorrelated} \end{array}}$$

Parameters of Mobile Multipath Channels

- Delay Spread
- Coherence Bandwidth (B_C)

describe the time dispersive nature of the channel in a local area
(∵ presence of reflecting objects and scatters in the channel)

- Doppler Spread (B_D)
- Coherence Time (T_C)

describe the time varying nature of the channel in a small-scale region
(∵ relative motion between the mobile and BS or movement of object)

Small-Scale Fading
(Based on multipath time delay spread)

time dispersion

Flat Fading
1. BW of signal < BW of channel
2. Delay spread < Symbol period

Frequency Selective Fading
1. BW of signal > BW of channel
2. Delay spread > Symbol period

Small-Scale Fading
(Based on Doppler spread)

frequency dispersion
(time selective fading)

Fast Fading
1. High Doppler spread
2. Coherence time < Symbol period
3. Channel variations faster than baseband signal variations

Slow Fading
1. Low Doppler spread
2. Coherence time > Symbol period
3. Channel variations slower than baseband signal variations

Figure 5.11 Types of small-scale fading.

Flat Fading

- In flat fading, the multipath structure of the channel is such that the spectral characteristics of the transmitted signal are preserved at the receiver.

- However the strength of the received signal changes with time, due to fluctuations in the gain of the channel caused by multipath.

- Flat fading channels are also known as *amplitude varying channels* and are sometimes referred to as *narrowband channels*.

- To summarize, a signal undergoes flat fading if

$$B_S \ll B_C \quad \text{and} \quad T_S \gg \sigma_\tau$$

  - B_S : the bandwidth of the transmitted modulation
  - T_S : the reciprocal bandwidth (e.g., symbol period) of the transmitted modulation

Flat Fading

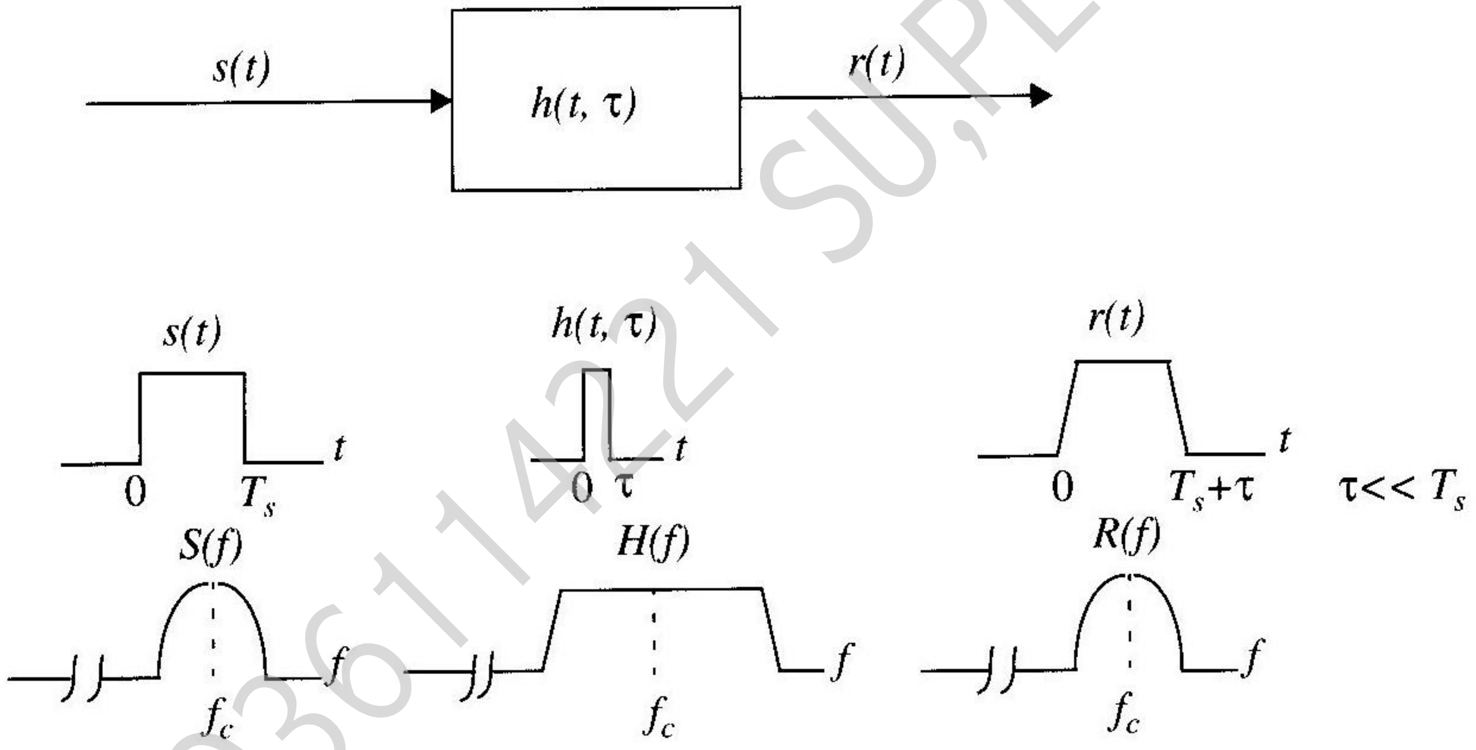

Figure 5.12 Flat fading channel characteristics.

Frequency Selective Fading

- For frequency selective fading, the received signal includes multiple versions of the transmitted waveform which are attenuated (faded) and delayed in time.

- Frequency selective fading is due to time dispersion of the transmitted symbols within the channel, and therefore the channel induce *intersymbol interference* (ISI).

- Frequency selective fading channels are also known as *wideband channels*.

- To summarize, a signal undergoes frequency selective fading if

$$B_S > B_C \qquad \text{and} \qquad T_S < \sigma_\tau$$

Frequency Selective Fading

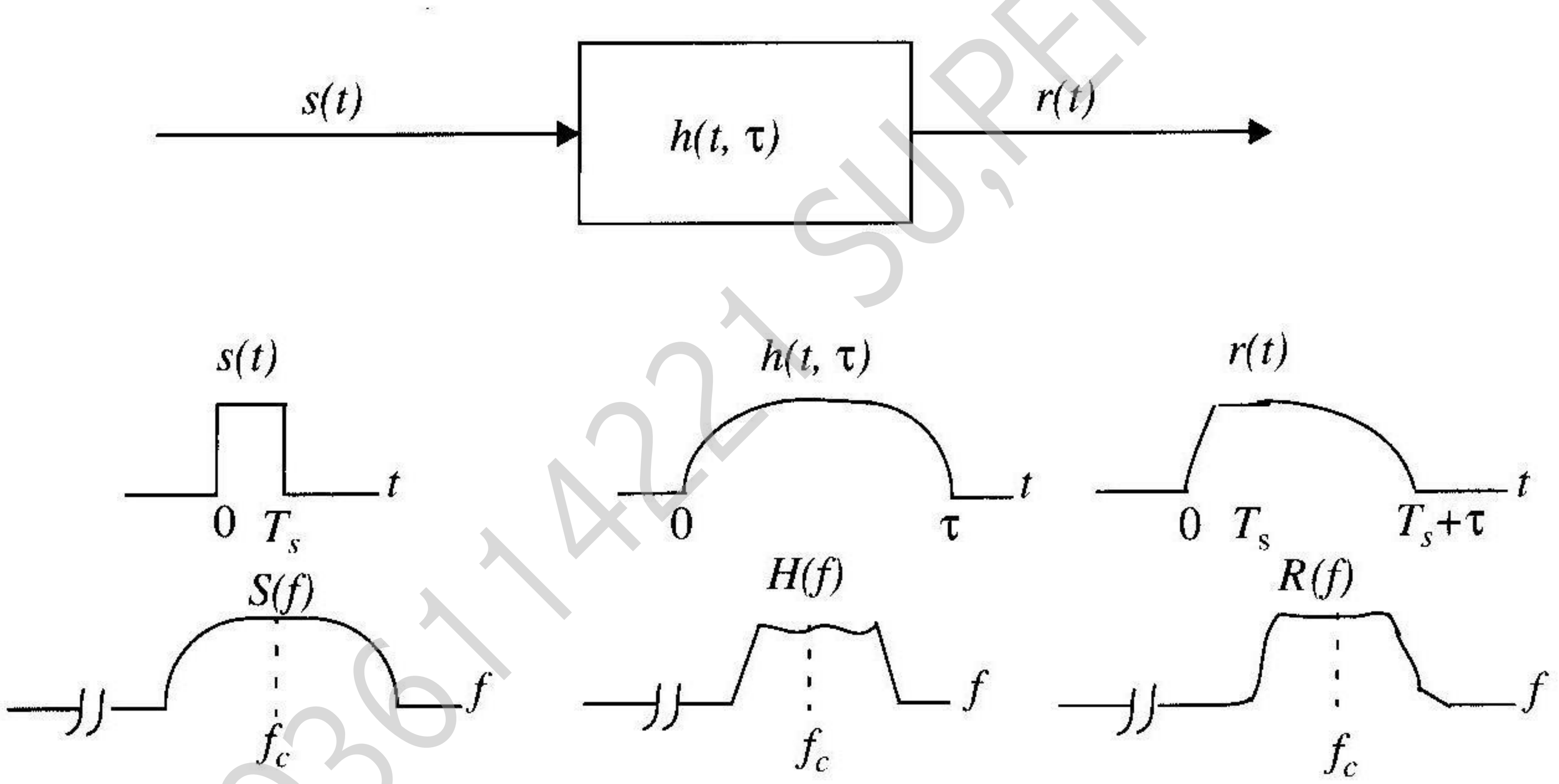

Figure 5.13 Frequency selective fading channel characteristics.

Fast Fading

- The velocity of the mobile (or velocity of objects in the channel) and the baseband signaling determines whether a signal undergoes fast fading or slow fading.

- In a fast fading channel, the channel impulse response changes rapidly within the symbol duration (i.e., $T_S > T_C$). This caused *frequency dispersion* (also called *time selective fading*) due to Doppler shift, which leads to signal distortion.

- Viewed in frequency domain, signal distortion due to fast fading increases with increasing Doppler spread relative to the bandwidth of the bandwidth of the transmitted signal.

- Therefore, a signal undergoes fast fading if

$$T_S > T_C \qquad \text{and} \qquad B_S < B_D$$

Slow Fading

- In a slow fading channel, the channel impulse response changes at a rate much slower than the transmitted baseband signal $s(t)$.

- In this case, the channel may be assumed to be static over one or several reciprocal bandwidth intervals.

- Therefore, a signal undergoes slow fading if

$$T_S \ll T_C \qquad \text{and} \qquad B_S \gg B_D$$

- Fast fading and slow fading deal with the relationship between the time rate of change in the channel and the transmitted signal, and not with propagation path loss models.

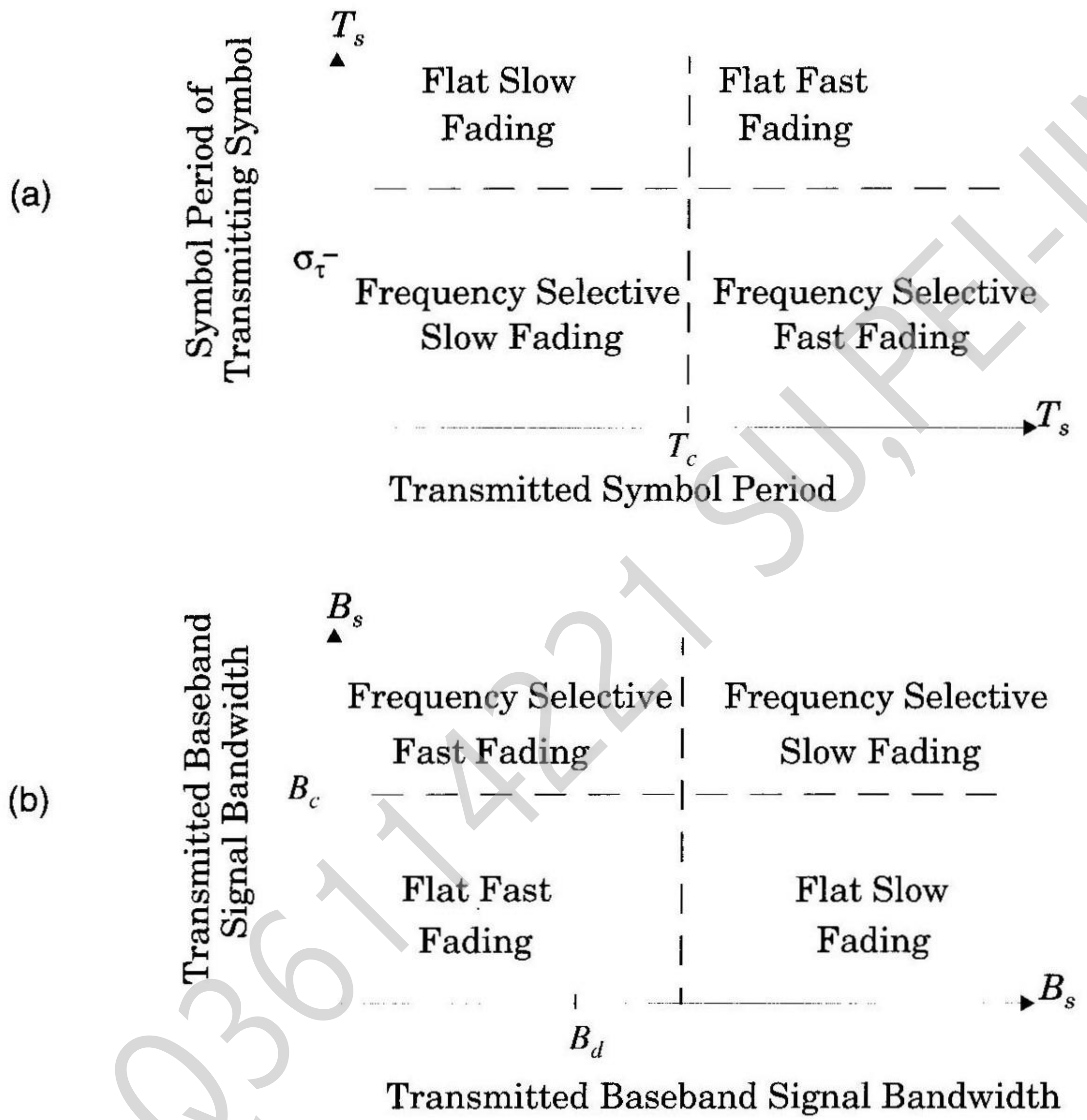

Figure 5.14 Matrix illustrating type of fading experienced by a signal as a function of: (a) symbol period; and (b) baseband signal bandwidth.

Rayleigh Fading Distribution

- The Rayleigh distribution is commonly used to describe the statistical time varying nature of the received envelope of a flat fading signal, or the envelope of an individual multipath component.

- The sum of two quadrature Gaussian noise signals obeys a Rayleigh distribution.

- The Rayleigh distribution has a probability density function (pdf) given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2}\exp\left(-\frac{r^2}{2\sigma^2}\right) & (0 \le r \le \infty) \\ 0 & (r < 0) \end{cases}$$

  - σ is the rms value of the received voltage signal before *envelope detection*.
  - σ^2 is the time average power of the received signal before envelope detection.

Rayleigh Fading Distribution

- The mean value r_{mean} of the Rayleigh distribution is given by

$$r_{mean} = E[r] = \int_0^\infty rp(r)dr = \sigma\sqrt{\frac{\pi}{2}} = 1.2533\sigma$$

- The variance σ_r^2 of the Rayleigh distribution
  - σ_r^2 represents the ac power in the signal envelope

$$\sigma_r^2 = E[r^2] - E^2[r]$$

$$= \int_0^\infty r^2p(r)dr - \frac{\sigma^2\pi}{2}$$

$$= \sigma^2\left(2 - \frac{\pi}{2}\right)$$

$$= 0.4292\sigma^2$$

Let $r = \sigma$

$$\Rightarrow p(\sigma) = \frac{1}{\sigma}\exp\left(-\frac{1}{2}\right) \approx \frac{0.60653}{\sigma}$$

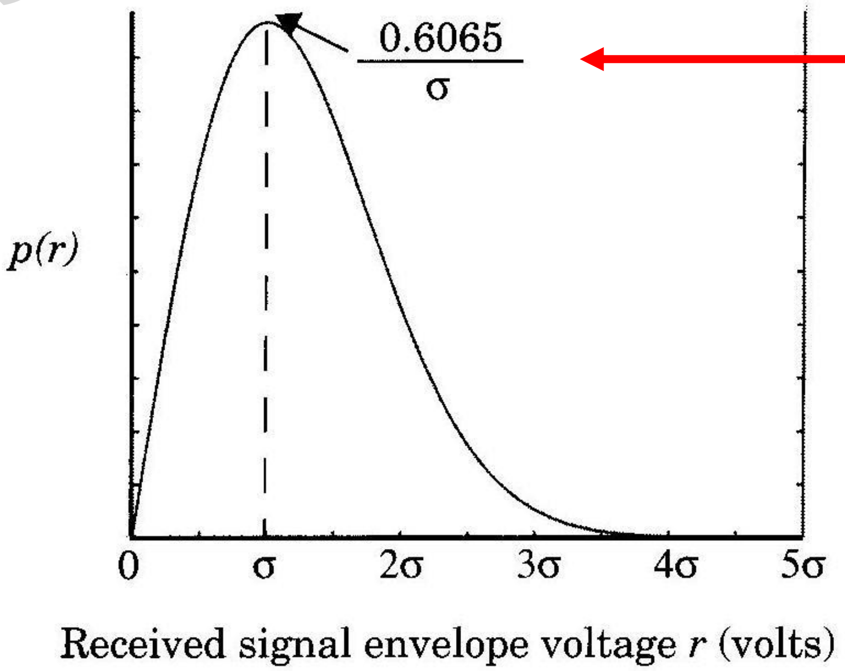

Figure 5.16 Rayleigh probability density function (pdf).

Rayleigh Fading Distribution

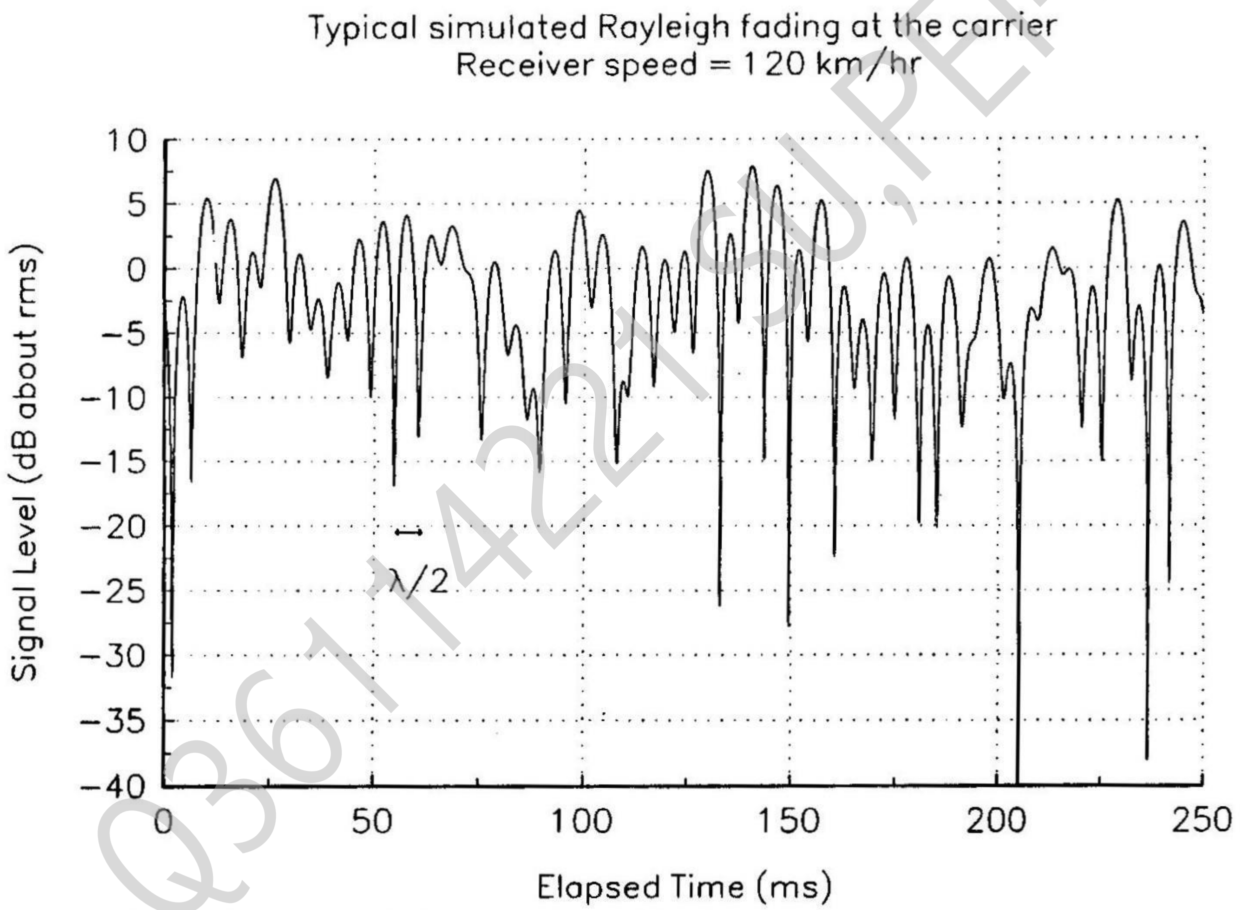

Figure 5.15 A typical Rayleigh fading envelope at 900 MHz [from [Fun93] © IEEE].

Ricean Fading Distribution

- When there is a nonfading signal component present, such as a line-of-sight (LOS) propagation path, the small-scale fading envelope distribution is Ricean.
- In such a situation, random multipath components arriving at different angles are superimposed on a stationary dominant (nonfading) signal.

- The Ricean distribution is given by

$$p(r) = \begin{cases} \frac{r}{\sigma^2} e^{-\frac{(r^2+A^2)}{2\sigma^2}} I_0\left(\frac{Ar}{\sigma^2}\right) & \text{for } (A \geq 0, r \geq 0) \\ 0 & \text{for } (r < 0) \end{cases}$$

  - The parameter A denotes the peak amplitude of the dominant signal.
  - $I_0(\cdot)$ is the modified Bessel function of the first kind and zero-order.

Ricean Fading Distribution

- The Ricean factor K is defined as the ratio between the deterministic signal power and variance of the multipath.

- It is given by

$$K = A^2/2\sigma^2$$

or, in terms of dB,

$$K(\mathrm{dB}) = 10\log\frac{A^2}{2\sigma^2}\ \mathrm{dB}$$

Ricean Fading Distribution

- As $A \rightarrow 0$, $K \rightarrow -\infty$ dB, and as the dominant path decreases in amplitude, the Ricean distribution degenerates to a Rayleigh distribution.

$$A = 0 \Rightarrow I_0\left(\frac{Ar}{\sigma^2}\right) = 1$$

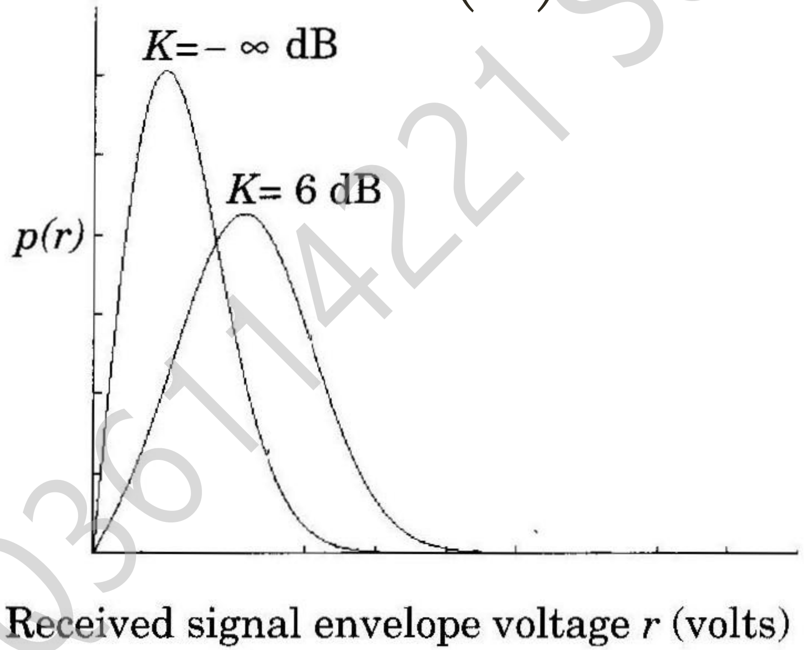

Figure 5.18 Probability density function of Ricean distributions: $K = -\infty$ dB (Rayleigh) and $K = 6$ dB. For K >> 1, the Ricean pdf is approximately Gaussian about the mean.

Two-ray Rayleigh Fading Model

- In modern communication systems with high data rates, it has become necessary to model the effects of multipath delay spread as well as fading.

- The impulse response of the model is represented as

$$h_b(t) = \alpha_1 \exp(j\phi_1)\,\delta(t) + \alpha_2 \exp(j\phi_2)\,\delta(t)$$

  - α_1 and α_2 are independent and Rayleigh distributed.
  - ϕ_1 and ϕ_2 are independent and uniformly distributed over $[0, 2\pi]$.
  - τ is the time delay between two rays.

- By setting α_2 equal to zero, the special case of Rayleigh fading channel is obtained as

$$h_b(t) = \alpha_1 \exp(j\phi_1)\,\delta(t)$$

Two-ray Rayleigh Fading Model

- The block diagram of the two-ray independent Rayleigh fading channel model:

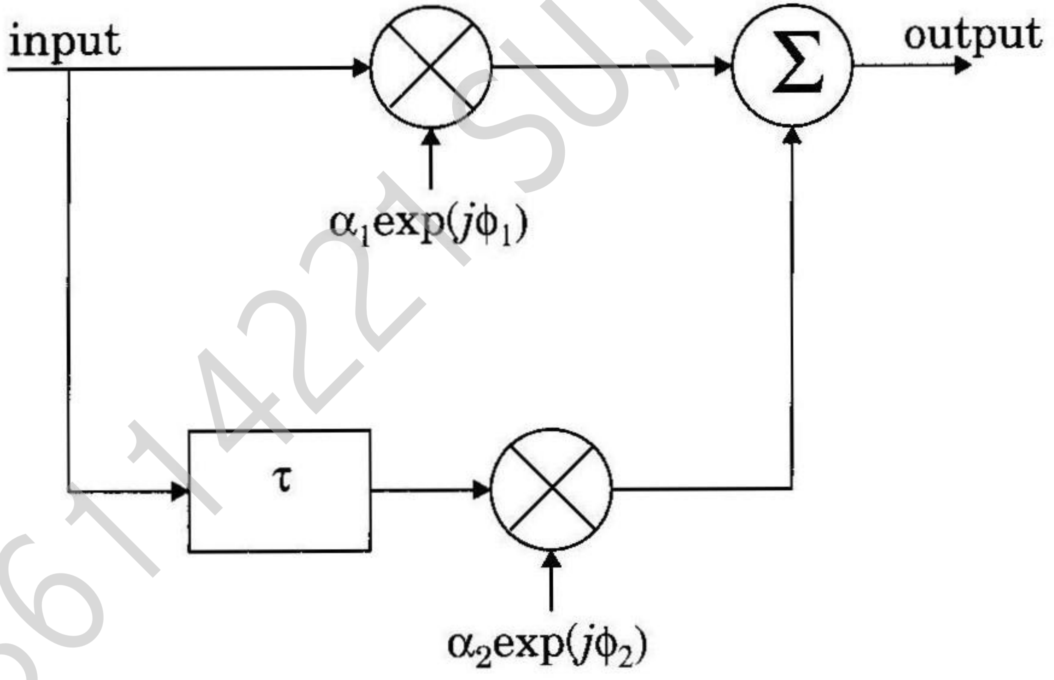

Figure 5.26 Two-ray Rayleigh fading model.

The simulation model for a time-varying frequency-selective channel model

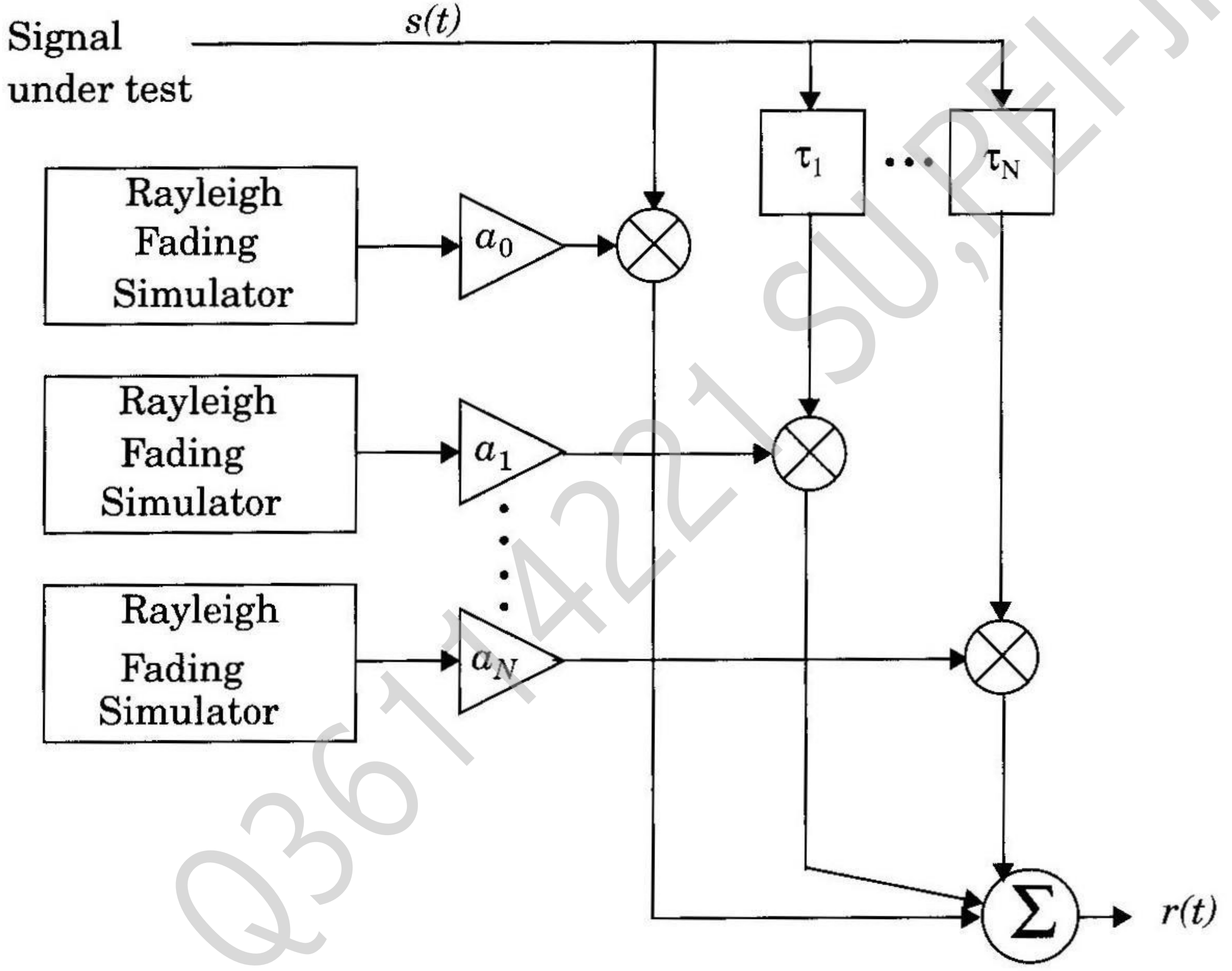

Figure 5.25 A signal may be applied to a Rayleigh fading simulator to determine performance in a wide range of channel conditions. Both flat and frequency selective fading conditions may be simulated, depending on gain and time delay settings.

Reference

- [1] J. G. Andrews, A. Ghosh, and R. Muhamed, *Fundamentals of WiMAX : Understanding Broadband Wireless Networking*. Westford, MA: Prentice Hall, 2007.
- [2] T. S. Rappaport, *Wireless Communications : Principles and Practice*, 2nd ed. Prentice Hall PTR, 2002.