FUZZY SETS AND FUZZY LOGIC

TOPIC 4

Fuzzy Sets

- Fuzzy concepts derive from fuzzy phenomena that commonly occur in the real world.

- For example, rain is a common natural phenomenon that is difficult to describe precisely since it can rain with varying intensity, anywhere from a light shower to a torrential downpour.

- Since the word rain does net adequately or precisely describe the wide variations in the amount and intensity of any rain event, "rain" is considered a fuzzy phenomenon.

Fuzzy Sets

- Often, the concepts formed in the human brain for perceiving, recognizing, and categorizing natural phenomena are also fuzzy.

- The boundaries of these concepts are vague. Therefore, the judging and reasoning that emerges from them are also fuzzy.

- For instance, "rain" might be classified as "light rain", "moderate rain", and "heavy rain" in order to describe the degree of raining. Unfortunately, it is difficult to say when the rain is light, moderate, or heavy, because the boundaries are undefined. **The concepts of "light", "moderate", and "heavy" are prime examples of fuzzy concepts themselves**.

Fuzzy Sets

- Sets are used often and almost unconsciously; we talk about a set of even numbers, positive temperatures, personal computers, fruits, and the like. For example, a classical set A of real numbers greater than 6 is a set with a crisp boundary, and it can be expressed as

$$A = \{x \mid x > 6\}$$

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  - where there is a clear, unambiguous boundary 6 such that if x is greater than this number, then x belongs to the set A; otherwise x does not belong to the set.
  - As an illustration, mathematically we can express a set of tall persons as a collection of persons whose height is more than 6 ft; this is the set denoted by previous equation, if we let A = "tall person" and x = "height". Yet, this is an unnatural and inadequate way of representing our usual concept of "tall person".

Fuzzy Sets

- Unlike the aforementioned conventional set, a fuzzy set expresses the degree to which an element belongs to a set. The characteristic function of a fuzzy set is allowed to have values between 0 and 1, which denotes the degree of membership of an element in a given set. If X is a collection of objects denoted generically by x, then a fuzzy set A in X is defined as a set of ordered pairs:

$$A = \{(x, \mu_A(x)) \mid x \in X\}$$

Fuzzy Sets

- Usually X is referred to as the universe of discourse, or, simply, the universe, and it may consist of discrete (ordered or nonordered) objects or continuous space. This can be clarified by the following examples. Let X = {San Francisco, Boston, Los Angeles} be the set of cities one may choose to live in. The fuzzy set C = "desirable city to live in" may be described as follows:

Let C = {(San Francisco, 0.9), (Boston, 0.8), (Los Angeles, 0.6)}.