作业 05 Mesh Parameterization

庄涛 PB15111679，ID：85
计算机科学与技术系，215 院 011 系 01 班
2018/04/01

一、要求

1) 实现 Floater 曲面参数化方法：实现网格参数化的方法（只要将边界固定在平面上的一个凸边界上，如正方形或圆周上），参考如下论文：
   M. Floater. Parameterization and smooth approximation of surface triangulations. CAGD, 1997.
   实现其中的三种参数化方法。

2) 实现曲面的纹理贴图，使用参数化结果，即可得到网格顶点的纹理坐标，比较不同的参数化对纹理映射结果的影响。可查阅 OpenGL 的 texture mapping 的相关资料即可完成。

3) 【Optional：可选，非强制要求】实现不固定边界的 ARAP 参数化方法：
   Ligang Liu, Lei Zhang, Yin Xu, Craig Gotsman, Steven J. Gortler. A Local/Global Approach to Mesh Parameterization.Computer Graphics Forum (Proc. Eurographics Symposium on Geometry Processing (SGP)), 27(5), 1495-1504, 2008.

二、环境

系统：windows10 x64
IDE：Vistual Studio 2010
外部依赖库：QT-5.5，eigen-3.3.2，glut

三、主要算法描述

3.1 Uniform Parameterization(Uniform)

对于均匀参数化而言，我们直接取

$$\lambda_{i_j} = \frac{1}{d_i}, j \in N(i)$$

即可，这里的 d_i 是顶点 v_i 的度。

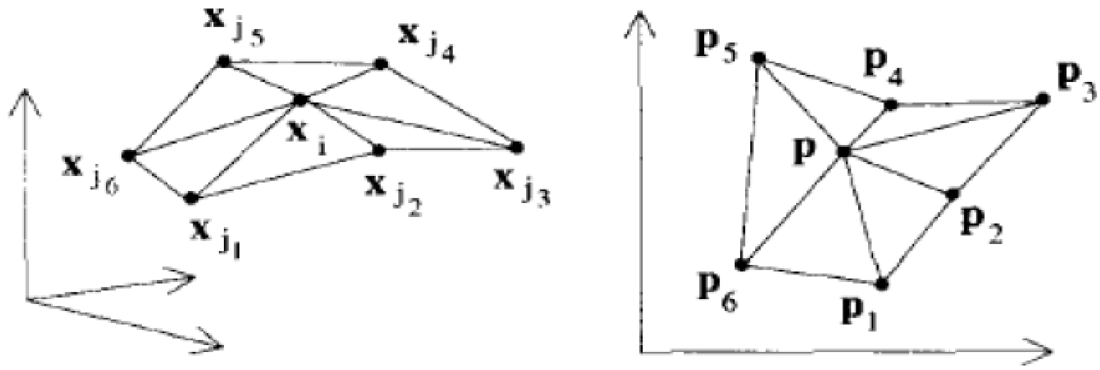
3.2 Weighted Least Squares Parameterization(WLS)

对于最小方差参数化，我们取

$$\lambda_{i_j} = \frac{1}{\|v_i - v_j\|} / \sum_{j \in N(i)} \frac{1}{\|v_i - v_j\|}$$

3.3 Shape-Preserving Parametrization(SP)

Step1 将顶点 i 的 1-邻域保角（保角度比例）保长度地映射到平面区域，见下图：



Step2 对每个 p_l 我们寻找 $\mathrm{r}(l)$ 使得，点 p 在 $\Delta p_l p_{r(l)} p_{r(l)+1}$ 的闭包中，并求出点 p 关于这个三角形的重心坐标（使用面积比即可）:$\delta_1, \delta_2, \delta_3$.

Setp3 我们对每一个 l 定义

$$\mu_{k,l} = \begin{cases} \delta_1 & k = l \\ \delta_2 & k = r(l) \\ \delta_3 & k = r(l) + 1 \\ 0 & otherwise \end{cases}$$

则由 $\sum_{k=1}^{d_i} \mu_{k,l} = 1$，我们定义

$$\lambda_{i,j_k} = \frac{1}{d_i} \sum_{l=1}^{d_i} \mu_{k,l}, j_k \in N(i)$$

即得所求权值

3.4 As Rigid As Possible(ARAP)

In this part, we will consider another way to get the parametrization of the given mesh.Instead of map the mesh into an area with fixed boundary,we apply a local-global method named ARAP(as-rigid-as-possible),which will map the mesh into an area without a fixed boundary.

To begin with,we suppose the values of all the texture coordinates u have been known,then we compute the matrix S(u) for each face of the given mesh in the way

$$S_t(u) = \sum_{i=0}^{2} cot(\theta_t^i)(u_t^i - u_t^{i+1})(x_t^i - x_t^{i+1})^T$$

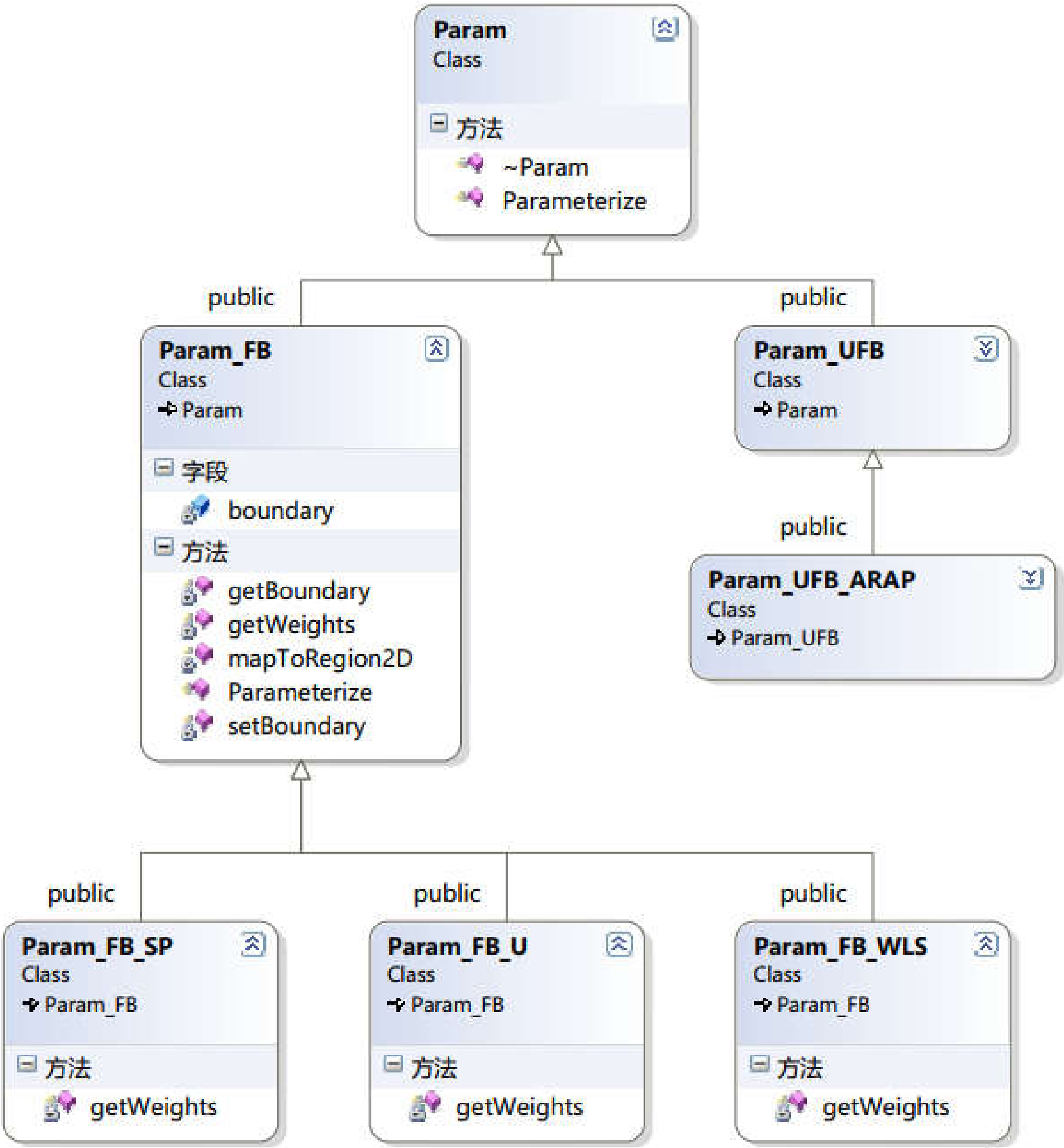
where t denotes the index of the face ,θ_t^i denotes the angle opposing the edge (i,i+1) in the face and x denotes the vertices of the face projected into $\mathbb{R}^2$ in the shape-preserving way.Then we can apply SVD to the matrix,and let matrix $L_t(u) = UV^T$.

The next step is the global phase.In this step the values of u become unknown,waiting for being solved.After some mathematical deduction,we can get an equation with the similar form as we have discussed in the section of fixed boundary.The equation can be written as

$$\sum_{j\in N(i)} [cot(\theta_{ij})+cot(\theta_{ji})](u_i-u_j) = \sum_{j\in N(i)} [cot(\theta_{ij})L_{t(i,j)}+cot(\theta_{ji})L_{t(j,i)}](x_i-x_j)$$

for any i from 1 to n,where n denotes the amount of the vertices in the mesh including the boundary points and t(i,j) denotes the triangle where the edge (i,j) exists.

四、类图

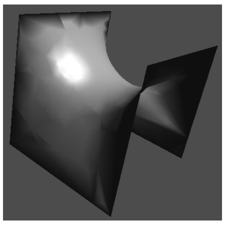
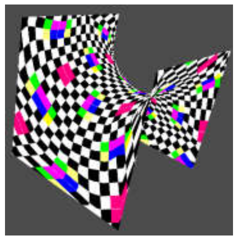
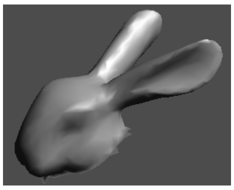
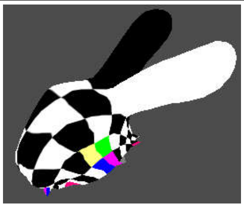
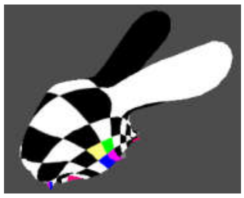
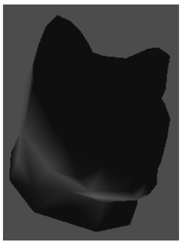
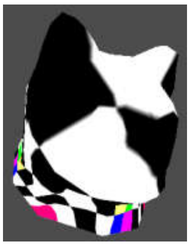
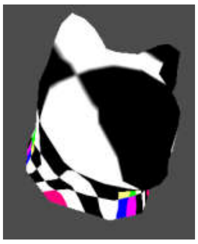
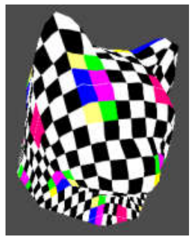
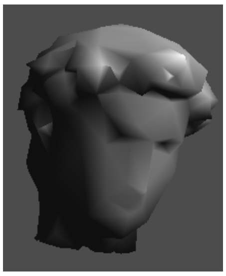
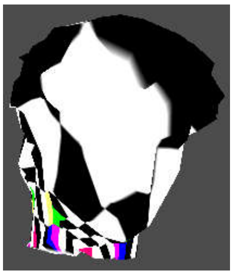
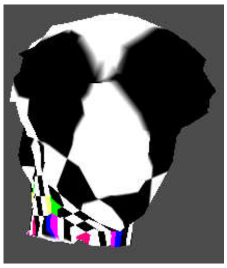
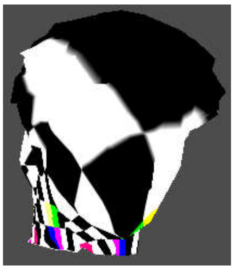
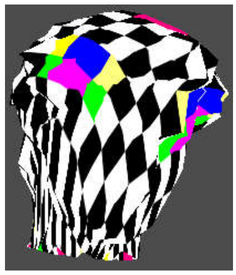
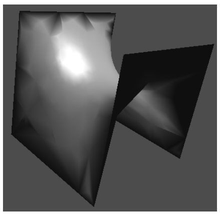
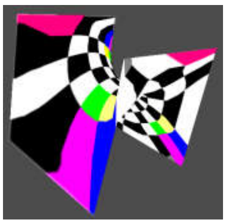
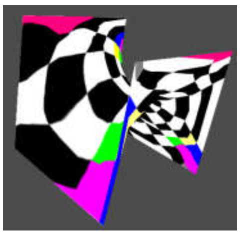
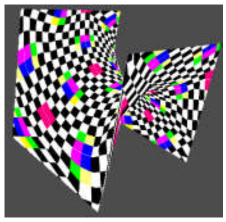
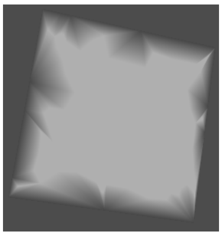
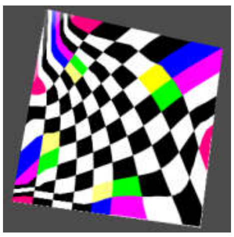
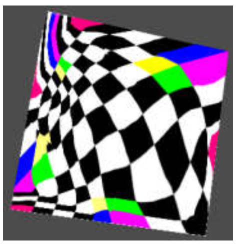
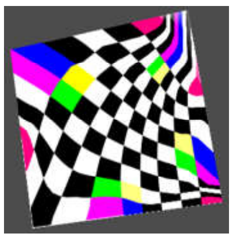
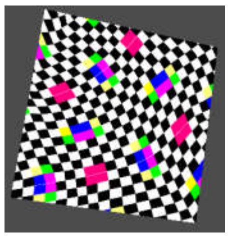


Param 只有一个接口 Parameterize，用于对 mesh 参数化，派生了 Param_FB 固定边界（fixed boundary）的参数化方法类和 Param_UFB 非固定边界的参数化方法类。

Param_FB 派生了 Param_FB_SP 保形、Param_FB_U 一致、Param_FB_WLS 加权最小方差。三者只定义了 getWeights 方法，主要流程都定义在了 Param_FB 中。

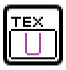
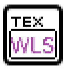
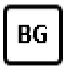
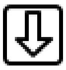
Param_UFB 派生了 Param_UFB_ARAP。

五、结果演示

样例	U	WLS	SP	ARAP
				
				
				
				
				
				

附录

1. 按键

	打开 obj 文件，可以是网格，也可以是封闭曲线
	打开纹理文件
	Uniform Parameterization
	Weighted Least Squares Parameterization
	Shape-Preserving Parameterization
	As Rigid As Possible Parameterization
	设置背景颜色
	另存为
	局部法（迭代）
	全局法
	求封闭曲线的极小曲面
	撤销

2. 封闭曲线文件格式

如

```
// xxxx.obj
v 0 0 0
v 0 0 1
v 1 0 1
v 1 0 0
v 1 1 0
v 1 1 1
v 0 1 1
v 0 1 0

f 1 2 3 4 5 6 7 8
```