Gaurav Chauhan
Roll No - 1
Section - D University RN - 2016748

Tutorial 2

①
```
void func (int n)
{
    int j=1, i=0;
    while (i<n)
    {
        i+=j;
        j++;
    }
}
```

Values after execution

1 time → i = 1
2 time → i = 1+2
3 time → i = 1+2+3
4 time → i = 1+2+3+4
for i^{th} time → $i = (1+2+3+4 \cdots i) < n$
$\Rightarrow i(i+1)/2 < n$
$i^2 < n \Rightarrow i < \sqrt{n}$

Time complexity $O(\sqrt{n})$

② Recurrence Relation

$F(n) = F(n-1) + F(n-2)$

Let $T(n)$ denote time complexity of $F(n)$. for $F(n-1)$ and $F(n-2)$ time will be $T(n-1)$ and $T(n-2)$. We've one more addition to sum as results. for $n \geq 1$

$T(n) = T(n-1) + T(n-2) + 1$ — (1)

for $n=0$ & $n=1$ no addition occurs

$T(0) = T(1) = 0$

Let $T(n-1) \approx T(n-2)$

Putting 2 in 1 we get

$T(n) = T(n-1) + T(n-1) + 1$

$= 2T(n-1) + 1$

Using Backward Substitution

$T(n-1) = 2T(n-2) + 1$

$T(n) = 2[2T(n-2)+1] + 1 = 4T(n-2) + 3$

We can substitute $T(n-2) = 2T(n-3) + 1$

$T(n) = 8T(n-3) + 7$

General equation

$T(n) = 2^k T(n-k) + (2^k - 1)$ — (3)

for $T(0)$, $n-k=0 \Rightarrow k=n$

Substituting values in (3)

$T(n) = 2^n \times T(0) + 2^n - 1$

$\Rightarrow 2^n + 2^n - 1$

$T(n) = O(2^n)$

Space Complexity ⇒ O(N)

Reason:
The function calls are executed sequentially Sequential execution guarantees that the stack size will exceed the depth of calls. for first F(n-1) it will make N stack frames, other F(n-2) will create N/2 so largest is N.

(3) O(n log n)

```
# include <iostream>
using namespace std;

int partition (int arr [], int start, int end)
{
    int pivot = arr [start];
    int count = 0;
    for (int i = start; i<= end; i++)
        if (arr [i] <= pivot)
            count++;
    int pivot_ind = start + count;
    swap (arr [pivot_ind], arr [start]);
    int i = start, j = end;
    while (i<pivot_ind && j>pivot_ind)
    {
        while (arr [i]<= pivot) i++;
```

```
    while (arr[j] > pivot) j--;
    if (i < pivot-ind && j > pivot-ind)
        swap (arr[i++], arr[j--]);
    }
}
return pivot-ind;
}
void quick (int arr[], int start, int end)
{
  if (start >= end) return;
  int p = partition (arr, start, end)
  quick (arr, start, p-1);
  quick (arr, p-1, end);
}

int main()
{
  int arr[] = {6, 8, 5, 2, 1}
  int n = 5;
  quick (arr, 0, n-1);
  return 0;
}
```

> O(N³)

```
int main ()
{
    int n=10
    for (int i=0; i<n; i++)
        for (int j=0; j<n; j++)
            for (int k=0; k<n; k++)
                printf (" . ");
    return 0;
}
```

> O(log(log(n)))

```
int countPrimes (int n)
{
  if (n<2) return 0;

bool [] nonprime = new bool [n];
nonprime [1] = true;
int numNonPrimes = 1;
for (int i=2; i<n; i++)
    {
      if (nonPrime [i]) continue;
    int j = i*2;
    while (j<n)
      if (! nonprime [j]) {
      nonprime [j] = true;
      numNonPrime ++;
}
```

jt = i; } }
return (n-1) - numNonPrime i
}

4. $T(n) = T(n/4) + T(n/2) + cn^2$

Using Master's theorem

We can assume $T(n/2) >= T(n/4)$
Equation can be rewritten as

$T(n) <= 2T(n/2) + cn^2$
$T(n) <= O(n^2)$
$T(n) = O(n^2)$
Also $T(n) >= cn^2 \Rightarrow T(n) >= O(n^2)$
$\Rightarrow T(n) = \Omega(n^2)$
$\because T(n) = O(n^2)$ & $T(n) = \Omega(n^2)$

$T(n) = O(n^2)$

5. for i = 1, inner loop is executed n times
for i = 2, inner loop is executed n/2 times
for i = 3, inner loop is executed n/3 times

It is forming a series :-
$n + n/2 + n/3 + \cdots n/n$

$\Rightarrow n(1 + 1/2 + 1/3 + \cdots 1/n)$
$\Rightarrow n \sum_{k=1}^{n} 1/k = n \log n$

$O(n \log n)$

6. for (int i=2 ; i<=n ; i = pow(i,k))
{
// same O(1) expression or statements
}

with iterations
i take values
for 1 iteration $\Rightarrow 2$
for 2 iteration $\Rightarrow 2^k$
for 3 iteration $\Rightarrow (2^k)^k$
for n iteration $\Rightarrow 2^{k(\log_k \log(n))}$

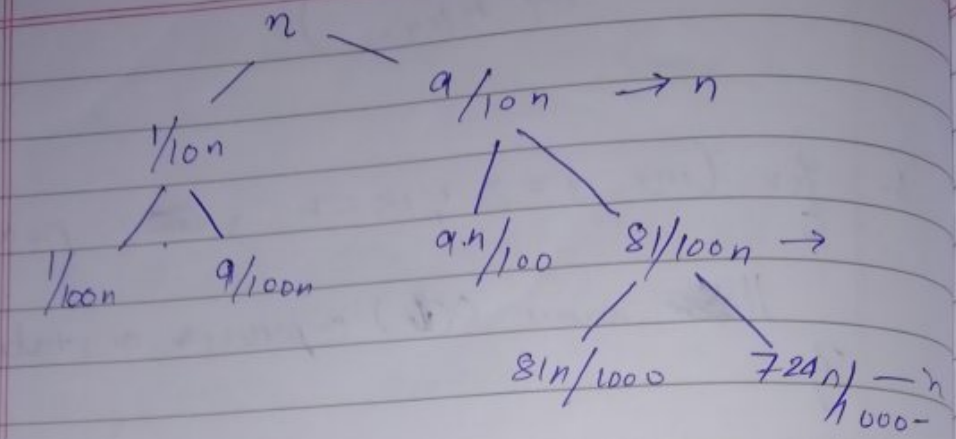
$\because$ last term must be less than equal to n

$2^{k \log_k (\log(n))} = 2^{\log n} = n$

Each iteration taken constant time
$\because$ Total iteration $= \log_k \log(n))$

Time complexity $= O(\log(\log(n)))$

7.



If we split in this manner
Recurrence relation = $T(n) = T(9n/10) + T(n/10) + O(n)$

first branch is of size $9n/10$ & second one is $n/10$.
Solving the above using recursion tree approach calculating values
At 1st level, value = n
At 2nd level, value = $9n/10 + n/10 = n$

value remain same at all levels i.e. n
Time Complexity = Summation of all values
= $O(n \log_{10/9} n)$ Upper bound
= $\Omega(n \log_{10} n)$ (lower bound)

$\Rightarrow O(n \log n)$