1. Proceso estacionario

Secuencia de variables aleatorias $\{y_t\}$ que cumple con las siguientes propiedades:

1. Media constante: $\forall t : \mathbb{E}[y_t] = \mu$.
2. Varianza constante: $\forall t : Var(y_t) = \sigma^2$
3. Función de autocovarianza: $\gamma_h = Cov(y_t, y_{t+h}) = f(|h|)$
   a) $Cov(y_1, y_4) = Cov(y_2, y_5)$

2. Esperanza condicional o expectativa en t

$$\mathbb{E}_t[y_t] = \mathbb{E}[y_s | s = 1, \ldots, t]$$

$$\mathbb{E}[XY | X = 1] = \mathbb{E}[Y | X = 1] = \mathbb{E}[Y] = \frac{1+10}{2}$$

$$\mathbb{E}[XY]$$

3. Modelo AR(1)

$$y_t = a_0 + a_1 y_{t-1} + \varepsilon_t$$

Vamos a utilizar una especificación de error de ruido blanco:

$$\varepsilon_t \sim N\left(0, \sigma_\varepsilon^2\right)$$

3.1. Media incondicional

$$\begin{aligned}
\mathbb{E}[y_t] &= \mathbb{E}[a_0 + a_1 y_{t-1} + \varepsilon_t] \\
&= \mathbb{E}[a_0] + \mathbb{E}[a_1 y_{t-1}] + \mathbb{E}[\varepsilon_t] \\
&= a_0 + a_1 \mathbb{E}[y_{t-1}] + \mathbb{E}[\varepsilon_t] \\
\mathbb{E}[y_t] &= a_0 + a_1 \mathbb{E}[y_t] \\
\mathbb{E}[y_t](1 - a_1) &= a_0 \\
\mathbb{E}[y_t] &= \frac{a_0}{1 - a_1}
\end{aligned}$$

3.2. Varianza incondicional

$$\begin{aligned}
Var(y_t) &= Var(a_0 + a_1 y_{t-1} + \varepsilon_t) \\
&= Var(a_1 y_{t-1} + \varepsilon_t) \\
&= a_1^2 Var(y_{t-1}) + Var(\varepsilon_t) \\
Var(y_t) - a_1^2 Var(y_t) &= \sigma_\varepsilon^2 \\
Var(y_t) &= \frac{\sigma_\varepsilon^2}{1 - a_1^2}
\end{aligned}$$

3.3. Media condicional

$$\begin{aligned}\mathbb{E}_t\left[y_{t+1}\right] &= \mathbb{E}_t\left[a_0 + a_1 y_t + \varepsilon_{t+1}\right] \\ &= a_0 + a_1 y_t + \mathbb{E}_t\left[\varepsilon_{t+1}\right] \\ &= a_0 + a_1 y_t\end{aligned}$$

3.4. Varianza condicional

$$\begin{aligned}Var_t\left(y_{t+1}\right) &= Var_t\left(a_0 + a_1 y_t + \varepsilon_{t+1}\right) \\ &= Var_t\left(a_1 y_t + \varepsilon_{t+1}\right) \\ &= Var_t\left(\varepsilon_{t+1}\right) \\ &= \sigma_\varepsilon^2\end{aligned}$$

3.5. Error de pronóstico

$$\begin{aligned}e_t\left(1\right) &= y_{t+1} - \mathbb{E}_t\left[y_{t+1}\right] \\ &= a_0 + a_1 y_t + \varepsilon_{t+1} - \left(a_0 + a_1 y_t\right) \\ &= \varepsilon_{t+1}\end{aligned}$$

$$\mathbb{E}\left[e_t\left(1\right)\right] = \mathbb{E}\left[\varepsilon_{t+1}\right] = 0$$

$$Var\left(e_t\left(1\right)\right) = Var\left(\varepsilon_{t+1}\right) = \sigma_\varepsilon^2$$