

Pág 35

1. Let $A = \{1, 3, 9\}$ and $B = \{1, 3, 6, 10\}$. Find each of the following ratios.

a) $A \cup B = \{1, 3, 4, 6, 9, 10\}$

b) $A \cap B = \{1, 3\}$

c) $A \setminus B = \{4, 9\}$

d) $B \setminus A = \{6, 10\}$

3. Find the following cardinalities

a) $|A|$ when $A = \{3, 5, 6, \dots, 37\}$.

$37 - 3 = 34$, then $|A| = 34$, since 1 is included

b) $|A|$ when $A = \{x \in \mathbb{Z} : -2 \leq x \leq 100\}$.

$100 + 2 = 102$, then $|A| = 103$, since 0 is included

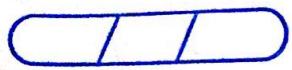
c) $|A \cap B|$ when $A = \{x \in \mathbb{N} : x \leq 20\}$ and $B = \{x \in \mathbb{N} : x \text{ is prime}\}$

$A \cap B = \{1, 3, 5, 7, 11, 13, 17, 19\}$

$|A \cap B| = 8$

4. Find a set of largest possible size that is a subset of both





$\{1, 2, 3, 4, 5\}$ and $\{2, 3, 6, 8, 10\}$

C = $\{2, 3\}$

7. Let $A = \{1, 2, 3, 4, 5\}$ and $B = \{2, 3, 4\}$. How many sets C have the property that $C \subseteq A$ and $B \subseteq C$.

$C = A$, $C = B$, $C = \{1, 2, 3, 4\}$ and $C = \{2, 3, 4, 5\}$, then 4 sets.

8. Let $A = \{1, 2, 3, 4, 5\}$, $B = \{3, 4, 5, 6, 7\}$, and $C = \{2, 3, 5\}$.

a) Find $A \cap B$

$$A \cap B = \{3, 4, 5\}$$

b) Find $A \cup B$

$$A \cup B = \{1, 2, 3, 4, 5, 6, 7\}$$

c) Find $A \setminus B$

$$A \setminus B = \{1, 2\}$$

d) Find $A \cap (B \cup C)$

$$B \cup C = \{2, 3, 4, 5, 6, 7\}$$

$$B \cup C = \{1, 8, 9, 10\}$$

$$A \cap (B \cup C) = \{1\}$$

10. Let $A = \{x \in \mathbb{N} : 3 \leq x \leq 13\}$, $B = \{x \in \mathbb{N} : x \text{ is even}\}$, and $C = \{x \in \mathbb{N} : x \text{ is odd}\}$.

a) Find $A \cap B$

$$A \cap B = \{4, 6, 8, 10, 12\}$$

b) Find $A \cup B$

$$A \cup B = \{2, 3, 4, 5, 6, 7, \dots\}$$

(Guia) A (a)

c) Find $B \cap C$

$$B \cap C = \{\}$$

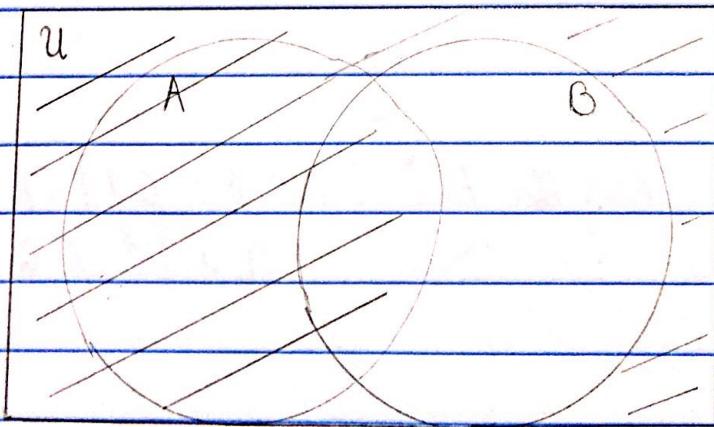
d) Find $B \cup C$

$$B \cup C = \mathbb{N}$$

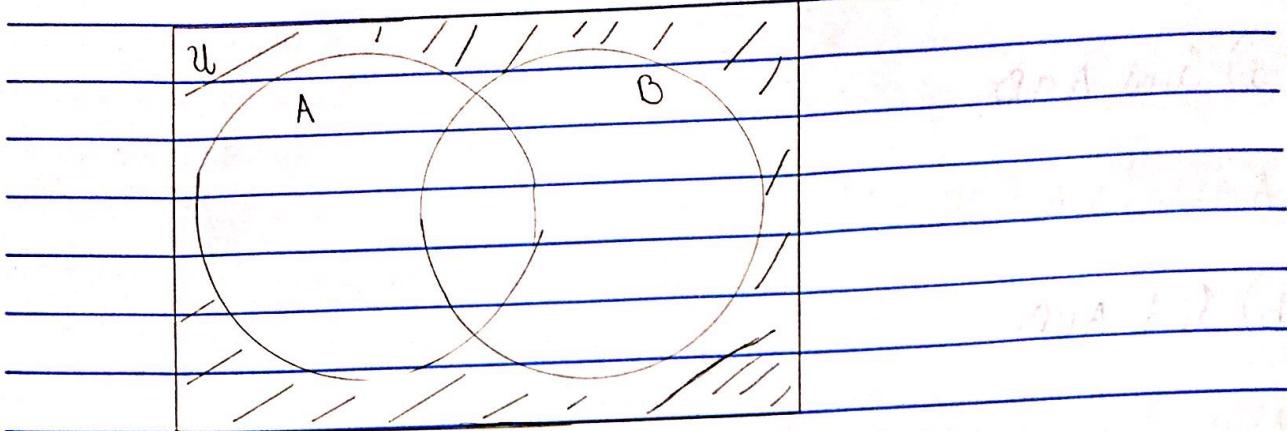
15. Draw a Venn diagram to represent each of the following.

a) $A \cup \bar{B}$

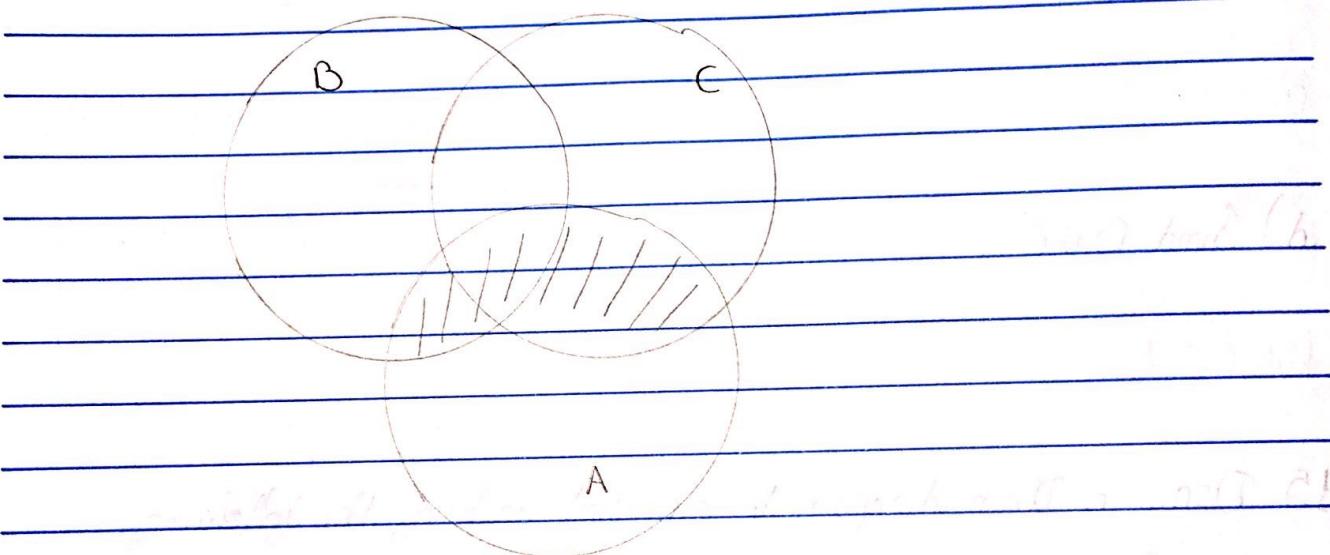
(Guia) A (b)



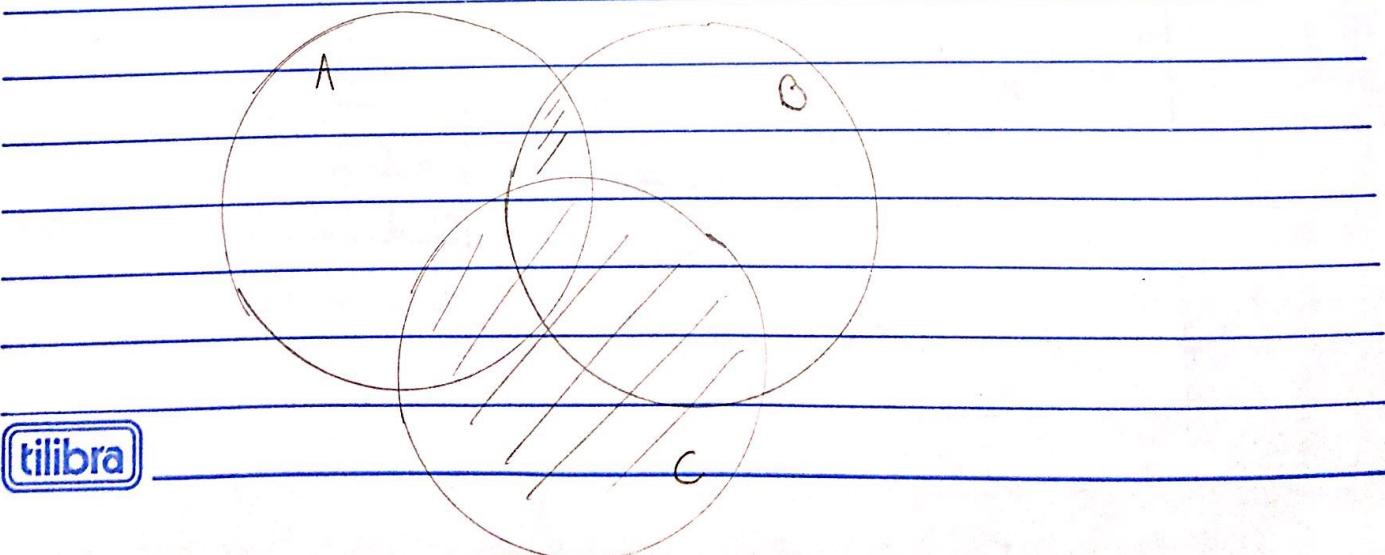
b) $(A \cup B)$



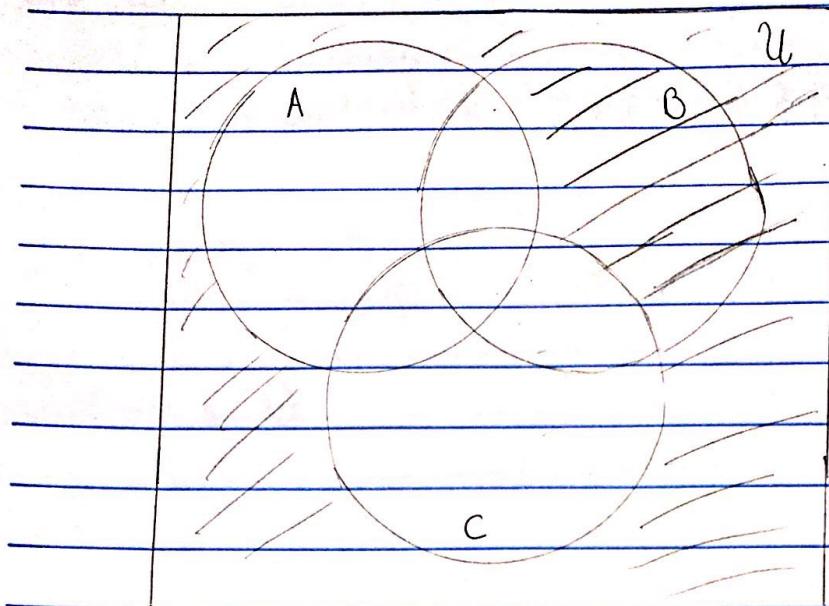
c) $A \cap (B \cup C)$



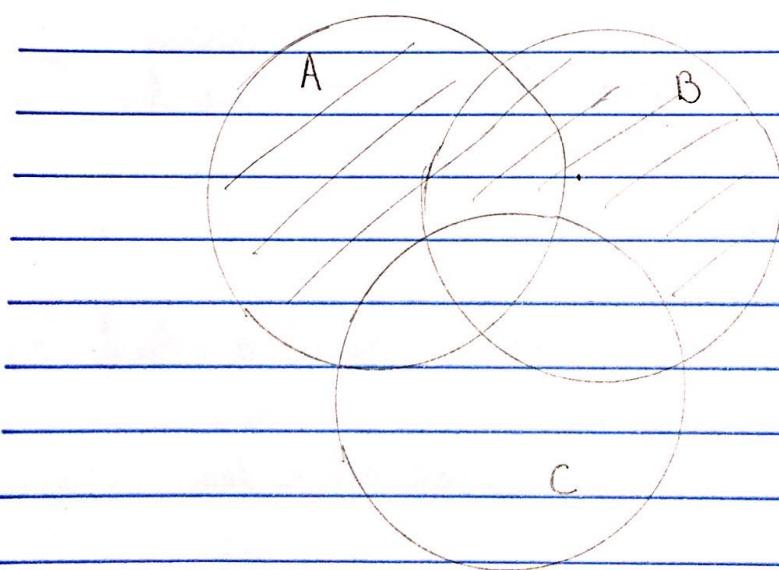
d) $(A \cap B) \cup C$



e) $\bar{A} \cap \bar{B} \cap \bar{C}$



f) $(A \cup B) \setminus C$



19 Let $A = \{1, 2, 3, 4, 5, 6\}$. Find all sets $B \in P(A)$ which have the property $\{2, 3, 5\} \subseteq B$.

A = $\{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{1,2\}, \{2,3\}, \{3,4\}, \{1,2,3\},$
 $\{3,4,5\}, \{4,5,6\}\}$

B = $\{\{1,2,3\}, \{2,3,5\}, \{3,4,5\}, \{4,5,6\}, \dots\}$

Page 55

1 Consider the function $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ given by

$$f(m) = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 4 & 1 & 3 & 2 \end{pmatrix}$$

a) Find $f(1)$

$$f(1) = 4$$

b) Find an element m in the domain such that $f(m) = 1$

$$f(m) = 1, m = 2$$

c) Find an element m of the domain such that $f(m) = m$

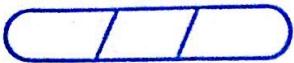
$$f(m) = m, m = 4$$

d) Find an element of the codomain that is not in the range

2 isn't part of the range

2 The following functions all have $\{1, 2, 3, 4, 5\}$ as both their domain and codomain. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

a) $f_1 = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 3 & 3 & 3 & 3 & 3 \end{pmatrix}$



This function is neither injective or surjective. Because 3 is the sum of more than one element and $\{1, 2, 4, 5\}$ aren't part of the range

b) $f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 3 & 1 & 5 & 4 \end{pmatrix}$

This function is bijective

c) $f(x) = 6 - x$

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 3 & 2 & 1 \end{pmatrix}$$

f is bijective

d) $f(x) = \begin{cases} x/2 & \text{if } x \text{ is even} \\ (x+1)/2 & \text{if } x \text{ is odd} \end{cases}$

f isn't injective because $f(3) = 2 = f(1)$, and neither surjective since $\{3, 5\}$ aren't part of the range

4. The following functions all have domain $\{1, 2, 3, 4\}$ and codomain $\{1, 2, 3, 4, 5\}$. For each, determine whether it is (only) injective, (only) surjective, bijective, or neither injective nor surjective.

a) $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 5 & 4 \end{pmatrix}$

f is injective, but not surjective since 3 isn't part of the range.

b) $f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 1 & 2 & 3 & 2 \end{pmatrix}$

f isn't injective, because $f(2) = 2 = f(3)$, and neither surjective since $\{5, 4\}$ aren't part of the range

c) f(x) gives the number of letters in the English word for the number x. For example, $f(1) = 3$ since "one" contains three letters.

$$f(1) = 3$$

$$f(2) = 3$$

$$f(3) = 4$$

$$f(4) = 4$$

Since $f(1) = 3 = f(2)$, f is not injective, and because $\{1, 2, 5\}$ aren't part of the range, f is neither surjective.

5. Write out all functions $f: \{1, 2, 3\} \rightarrow \{a, b\}$ (using two-line notation)

$$\begin{array}{c|c} (1 \ 2 \ 3) & (1 \ 2 \ 3) & (1 \ 2 \ 3) & (1 \ 2 \ 3) \\ \hline (a \ a \ a) & (b \ b \ b) & (a \ b \ a) & (b \ a \ b) \end{array}$$

$$\begin{array}{c|c} (1 \ 2 \ 3) & (1 \ 2 \ 3) & (1 \ 2 \ 3) & (1 \ 2 \ 3) \\ \hline (b \ b \ a) & (a \ a \ b) & (a \ b \ b) & (b \ a \ a) \end{array}$$

a) How many functions are there?



The number of possible functions is $2^3 = 8$

b) How many are injective?

None

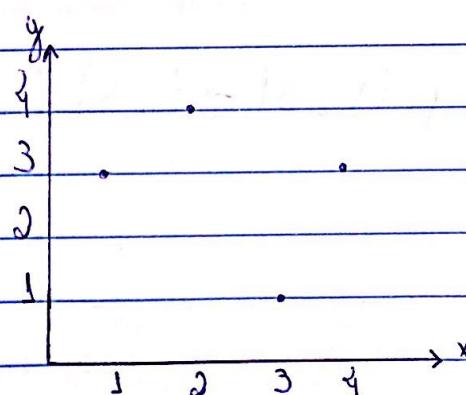
c) How many are surjective?

6, since 8 minus 2 functions that only have a or b on the codomain

d) How many are bijective?

None

8. Consider the function $f: \{1, 2, 3, 4\} \rightarrow \{1, 2, 3, 4\}$ given by the graph below.



a) Is f injective? Explain

Since $f(1) = 3 = f(3)$, f isn't injective

tilibra

b) Is f injective? Explain

Since 2 isn't part of the range, f isn't injective

c) Write the function using two-line notation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 3 \end{pmatrix}$$

12. For each function given below, determine whether or not the function is injective and whether or not the function is surjective.

a) $f: \mathbb{N} \rightarrow \mathbb{N}$ given by $f(n) = n + 3$

f is injective, but not surjective since $\{0, 1, 2, 3\}$ aren't part of the range

b) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(n) = n + 4$

f is surjective and injective

c) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(n) = 5n - 8$

f is injective, but not surjective since $\{3, 4, \dots\}$ aren't part of the range

d) $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given by $f(n) = \begin{cases} n/2 & \text{if } n \text{ is even} \\ (n+1)/2 & \text{if } n \text{ is odd} \end{cases}$

f is surjective, but not injective, since $f(2) = f(1)$

13 Let $A = \{1, 2, 3, \dots, 10\}$. Consider the function $f: P(A) \rightarrow \mathbb{N}$ given by $f(B) = |B|$. That is, f takes a subset of A as an input and outputs the cardinality of that set.

a) Is f injective? Prove your answer.

f is not injective since $f(\{1, 2, 3\}) = 3 = f(\{4, 5, 6\})$

b) Is f surjective? Prove your answer.

f isn't surjective since any subset of A is limited to 10 numbers

c) Find $f^{-1}(5)$

$f^{-1}(5) = \{\{1\}, \{2\}, \{3\}, \{4\}, \{5\}, \{6\}, \{7\}, \{8\}, \{9\}, \{10\}\}$

d) Find $f^{-1}(0)$

$f^{-1}(0) = \{\emptyset\}$

e) Find $f^{-1}(12)$.

$f^{-1}(12) = \emptyset$, because the set A has only 10 numbers

21 Consider the function $f: \mathbb{Z} \rightarrow \mathbb{Z}$ given $f(n) = \begin{cases} n+1 & \text{if } n \text{ is even} \\ n-3 & \text{if } n \text{ is odd} \end{cases}$



a) Is f injective? Prove your answer.

f is injective

$$\begin{aligned} f(a) &= f(b) \quad \text{or} \quad f(c) = f(d) \\ a+1 &= b+1 \quad c-3 = d-3 \\ a &= b \quad c = d \end{aligned}$$

b) Is f surjective? Prove your answer.

$$f(m) = m+1 \quad \text{if } m \in \mathbb{N}$$

$$y = m+1$$

$$m = y-1$$

$$m+1 = y$$

$$y = m$$

$$y = m-3$$

$$m = y+3$$

$$m-3 = y$$

$$y = m$$

f is surjective

30. Let $f: X \rightarrow Y$ be a function and $A, B \subseteq Y$ be subsets of the codomain.

a) Is $f^{-1}(A \cup B) = f^{-1}(A) \cup f^{-1}(B)$? Always, sometimes, or never?
Explain.

$$f: X \rightarrow Y \text{ and } A, B \subseteq Y$$

Always

$$f^{-1}: Y \rightarrow X \text{ and } A, B \subseteq Y$$

tilibra



b) Is $f^{-1}(A \cap B) = f^{-1}(A) \cap f^{-1}(B)$? Always, sometimes, or never?
Explain

$$f^{-1}: Y \rightarrow X$$

$$f^{-1}(A \cap B) = A - B$$

Always