Trabalho 8 (40 pontos)

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1. Calcule a integral definida

   (a) (4 pontos)

$$\int_0^{\frac{\pi}{6}} cosx\, sen^5 x\, dx$$

   (b) (4 pontos)

$$\int_0^{\frac{\pi}{3}} \frac{sen\, x}{cos^2 x}\, dx$$

   (c) (4 pontos)

$$\int_1^2 \frac{x^3+1}{x}\, dx$$

   (d) (4 pontos)

$$\int_0^1 xe^x\, dx$$

   (e) (4 pontos)

$$\int_0^1 \frac{x}{\sqrt{1+x^2}}\, dx$$

   (f) (4 pontos)

$$\int_1^e \frac{lnx}{x}\, dx$$

2. (a) (4 pontos) Desenhe a região plana limitada pela curva $y = senx$ e as retas $x = 0$, $x = \pi$ e $y = 0$.

   (b) (4 pontos) Calcule a área da região de item (a).

3. (a) (4 pontos) Desenhe a região plana limitada pela curva $y = \sqrt{x}$ e as retas $x = 3$ e $y = 0$.

   (b) (4 pontos) Calcule a área da região de item (a).

Aluna: Nurielly Oliveira Nascimento 11921BSI222

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1. Calcule a integral definida

a) $\int_0^{\pi/6} \cos x \,\operatorname{sen}^5 x \, dx$

$u = \operatorname{sen} x$
$du = (\operatorname{sen} x)' \cdot dx$
$du = \cos x \, dx$

$$\int u^5 \, du = \frac{u^6}{6} + K = \frac{\operatorname{sen}^6 x}{6} + K$$

$$\left[\frac{\operatorname{sen}^6 x}{6}\right]_0^{\pi/6} = \left[\frac{\operatorname{sen}^6 \frac{\pi}{6}}{6} - \frac{\operatorname{sen}^6 0}{6}\right] = \left(\frac{1}{2}\right)^6 \cdot \frac{1}{6} - 0$$

$$\left(\frac{1}{2^6}\right) \cdot \frac{1}{6} = \frac{1}{64} \cdot \frac{1}{6} = \frac{1}{384}$$

Resposta: $\frac{1}{384}$

b) $\int_0^{\pi/3} \frac{\operatorname{sen} x}{\cos^2 x} \, dx$

$$\int \frac{\operatorname{sen} x}{\cos^2 x} \, dx$$

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$u = \cos x$

$du = (\cos x)' \cdot dx$

$du = -\operatorname{sen} x \, dx$

$$\int \frac{1}{u^2} \cdot -du = -\int \frac{1}{u^2} du = -\left(-\frac{1}{u} + K\right)$$

$$-\left[\frac{1}{\cos x}\right]_0^{\frac{\pi}{3}} = \left[\frac{1}{\cos\frac{\pi}{3}} - \frac{1}{\cos 0}\right] = \frac{1}{\frac{1}{2}} - 1$$

$1 \cdot 2 + 1 = 1$

Resposta 1

c) $\displaystyle\int_1^2 \frac{x^3+1}{x} dx$

$$\int \frac{x^3+1}{x} dx = \int \frac{x^3}{x} dx + \int \frac{1}{x} dx$$

$$\int x^2 dx + \ln|x| + K = \frac{x^3}{3} + \ln|x|$$

$$\left[\frac{x^3}{3} + \ln x\right]_1^2 = \left[\frac{2^3}{3} + \ln 2\right] - \left[\frac{1^3}{3} + \ln 1\right]$$

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$$\left[\frac{8}{3}+\ln 2\right]-\left[\frac{1}{3}+0\right]=\frac{7}{3}+\ln 2$$

Resposta: $\frac{7}{3}+\ln 2$

d) $\int_0^1 xe^x\,dx$

$u = x$ $\qquad dv = e^x\,dx$

$du = (x)'\,dx$ $\qquad v = e^x + k$

$du = dx$

$$\int u\,dv = u.v - \int v.du$$
$$= x.e^x - \int e^x\,dx$$
$$= xe^x - e^x$$
$$= e^x(x-1)$$

$$\left[e^x(x-1)\right]_0^1 = \left[e^1(1-1)\right]-\left[e^0(0-1)\right]$$
$$= e.0 - [1.-1]$$
$$= 0 - [-1]$$
$$= +1$$
$$= 1$$

Resposta: 1

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e) $\int_0^1 \frac{x}{\sqrt{1+x^2}}\, dx$

$\int \frac{x}{\sqrt{1+x^2}}\, dx$

$u = (1+x^2)$

$du = (1+x^2)'\, dx$

$du = 2x\, dx$

$x\, dx = \frac{du}{2}$

$\int \frac{1}{\sqrt{u}} \cdot \frac{du}{2} = \frac{1}{2} \cdot \int u^{-\frac{1}{2}}\, du = \frac{1}{2} \cdot \frac{u^{\frac{1}{2}}}{\frac{1}{2}} + K$

$\frac{1}{\cancel{2}} \cdot \sqrt{u}\, \cancel{2} + K = \sqrt{u} + K$

$\left[\sqrt{(1+x^2)}\right]_0^1 = \left[\sqrt{(1+1^2)} - \sqrt{(1+0^2)}\right]$

$\left[\sqrt{2} - 1\right] = \sqrt{2} - 1$

Resposta: $\sqrt{2} - 1$

f) $\int_1^e \frac{\ln x}{x}\, dx$

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$$\int \frac{\ln x}{x}\, dx$$

$u = \ln x$

$du = (\ln x)' \cdot dx$

$du = \frac{1}{x}\, dx$

$$\int u\, du = \frac{u^2}{2} + K = \frac{(\ln x)^2}{2} + K$$

$$\left[\frac{(\ln x)^2}{2}\right]_1^e = \left[\frac{(\ln e)^2}{2} - \frac{(\ln 1)^2}{2}\right]$$

$$\frac{(\ln e)^2}{2} - \frac{0^2}{2} = \frac{1}{2} - 0 = \frac{1}{2}$$

Resposta: $\frac{1}{2}$

2. a) Desenhe a região plana limitada pela curva $y = \operatorname{sen} x$ e as retas $x = 0$, $x = \pi$ e $y = 0$

$y = \operatorname{sen} x$

$\operatorname{sen} 0 = 0$

$\operatorname{sen} \pi = 0$

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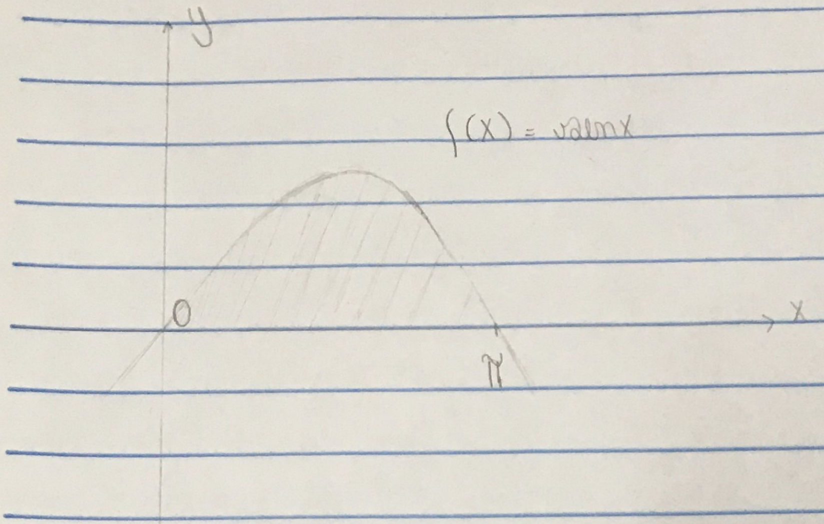

b) Calcule a área da região do item (a)

$A_R = \int_a^b f(x)\,dx \Rightarrow A_R = \int_0^\pi \operatorname{sen} x\,dx$

$\int \operatorname{sen} x\,dx = -\cos x + K$

$\Big[-\cos x\Big]_0^\pi = -\cos \pi - (-\cos 0)$

$= 1 - (-1) = 2$

Resposta: 2 u.a.

3. a) Desenhe a região plana limitada pela curva $y = \sqrt{x}$ e as retas $x = 3$ e $y = 0$

$y = \sqrt{x}$ $\qquad$ $0 = \sqrt{x}$

$y = \sqrt{3} \approx 1.73$ $\qquad$ $0 = x^{\frac{1}{2}}$

$x = 0$

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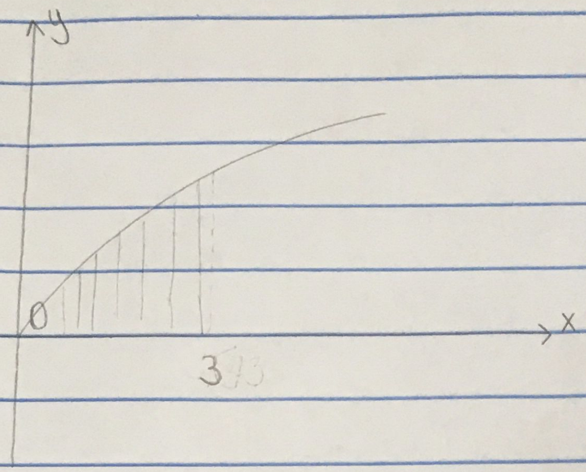

b) Calcule a área da região do item (a).

$$A_R = \int_a^b f(x)\,dx \Rightarrow A_R = \int_0^3 \sqrt{x}\,dx$$

$$\int \sqrt{x}\,dx = \int x^{\frac{1}{2}}\,dx = \frac{x^{\frac{1}{2}+1}}{\frac{1}{2}+1} = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2\sqrt{x^3}}{3}$$

$$\left[\frac{2\sqrt{(x^3)}}{3}\right]_0^3 = \left[\frac{2\sqrt{(3)^3}}{3} - \frac{2\sqrt{(0^3)}}{3}\right] = \frac{2\sqrt{27}-0}{3}$$

$$\frac{2\sqrt{3^2\cdot 3}}{3} = \frac{2\cdot\cancel{3}\sqrt{3}}{\cancel{3}} = 2\sqrt{3}$$

Resposta: $2\sqrt{3}$ u.a