

TADAM: Learning Timed Automata from Noisy Observations

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(equal contribution)



Introduction



system

communication protocol



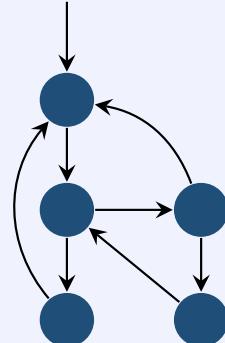
industrial process

Introduction



communication protocol

system → behavior model

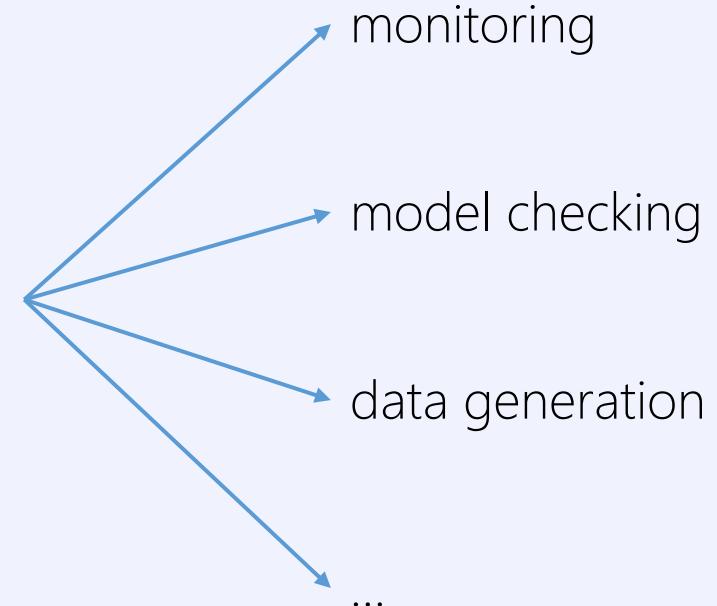
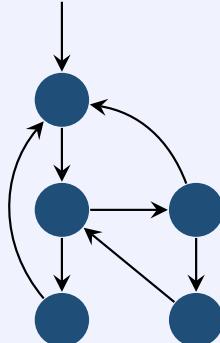


industrial process

Introduction

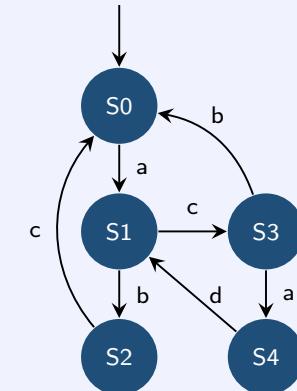


system → behavior model



Behavior model formalism

Automata formalism
Finite state automata (FSA)



Natural formalism for **discrete event system (DES)** modeling

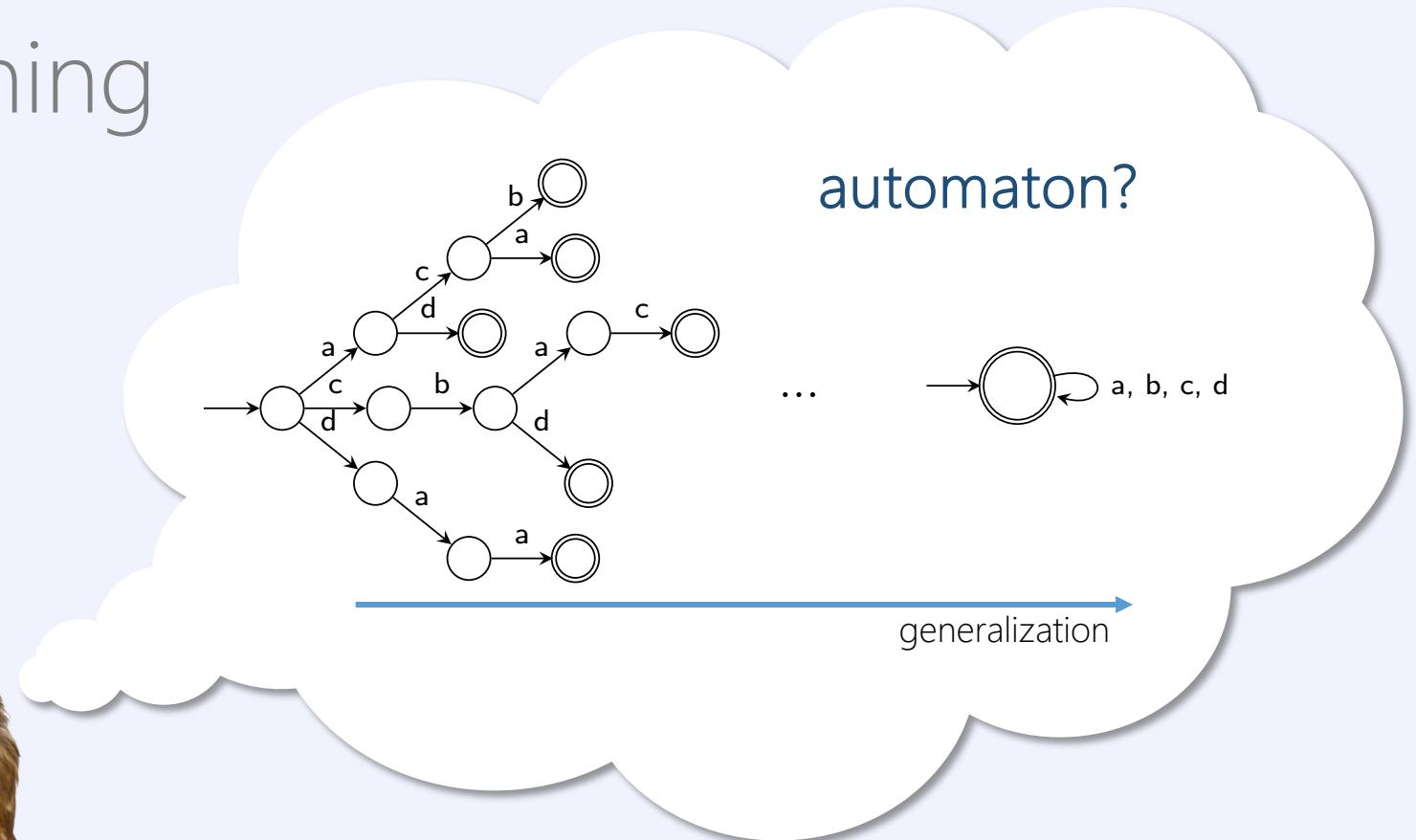
Human-understandable representation of the behavior of a system

Based on a **mathematical formalism** with **extensive literature** and with **software support**

Automata learning

data

```
a c b  
a d  
c b a c  
a c a  
d a a  
c b d
```



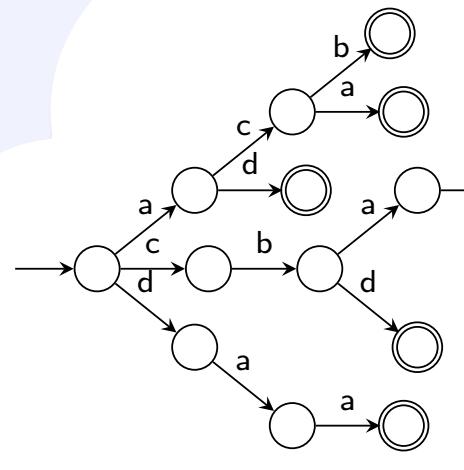
Automata learning



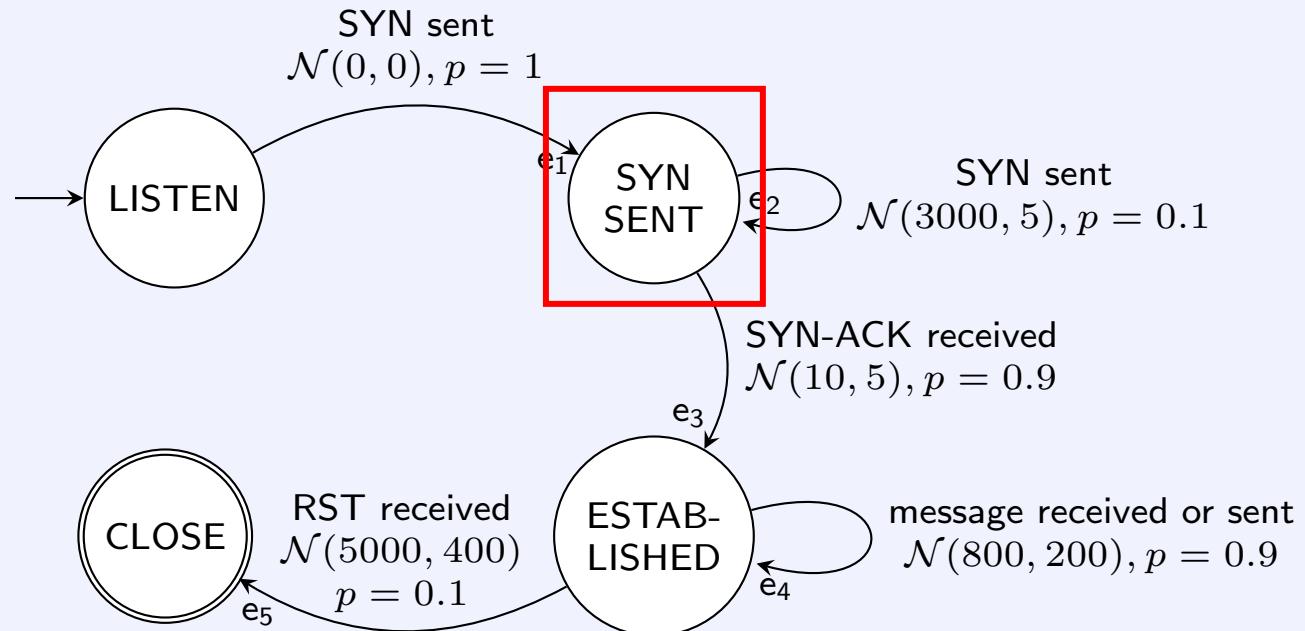
limited measurement accuracy,
probe configuration error
...

noisy
data

a c b
a d
c b a c
a c a
d a a
c b d

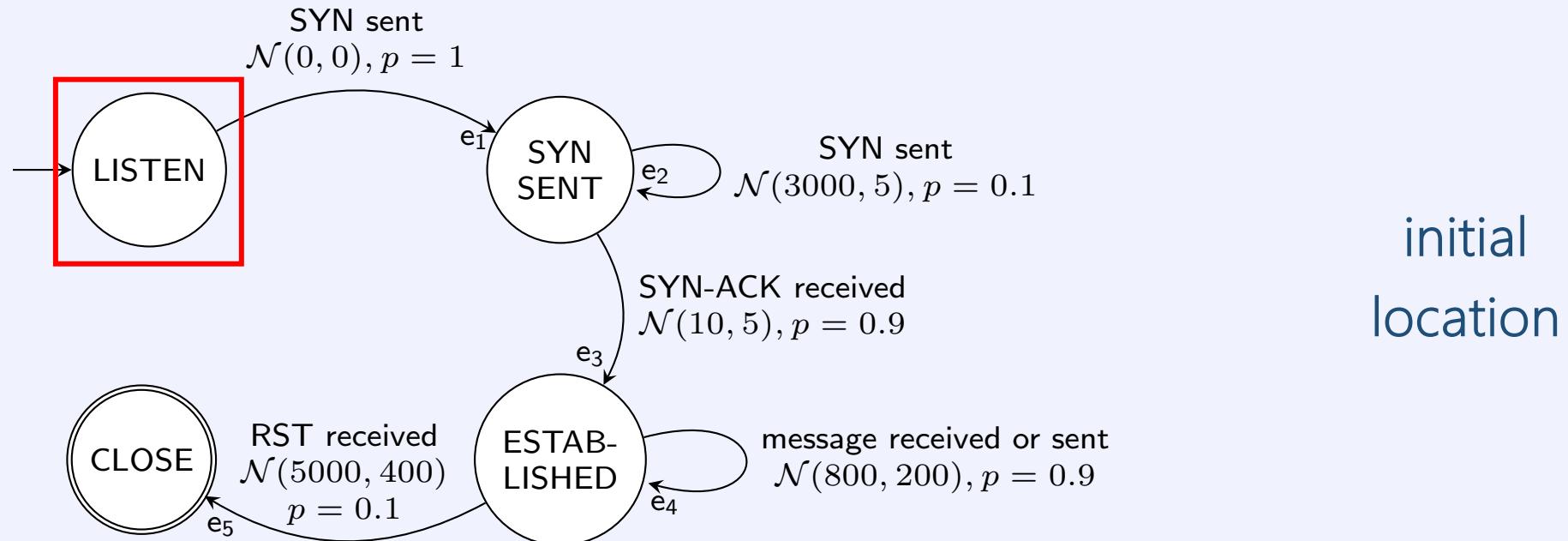


Probabilistic Real-Time Automaton



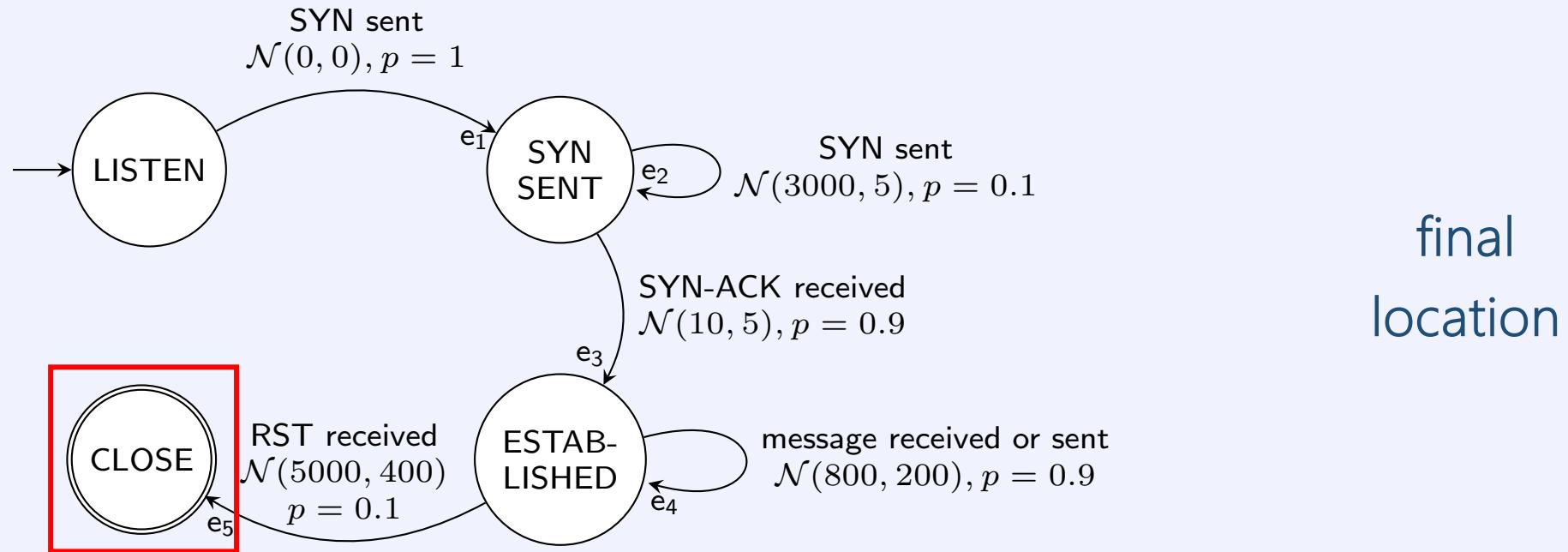
location
 $q \in \mathcal{Q}$

Probabilistic Real-Time Automaton

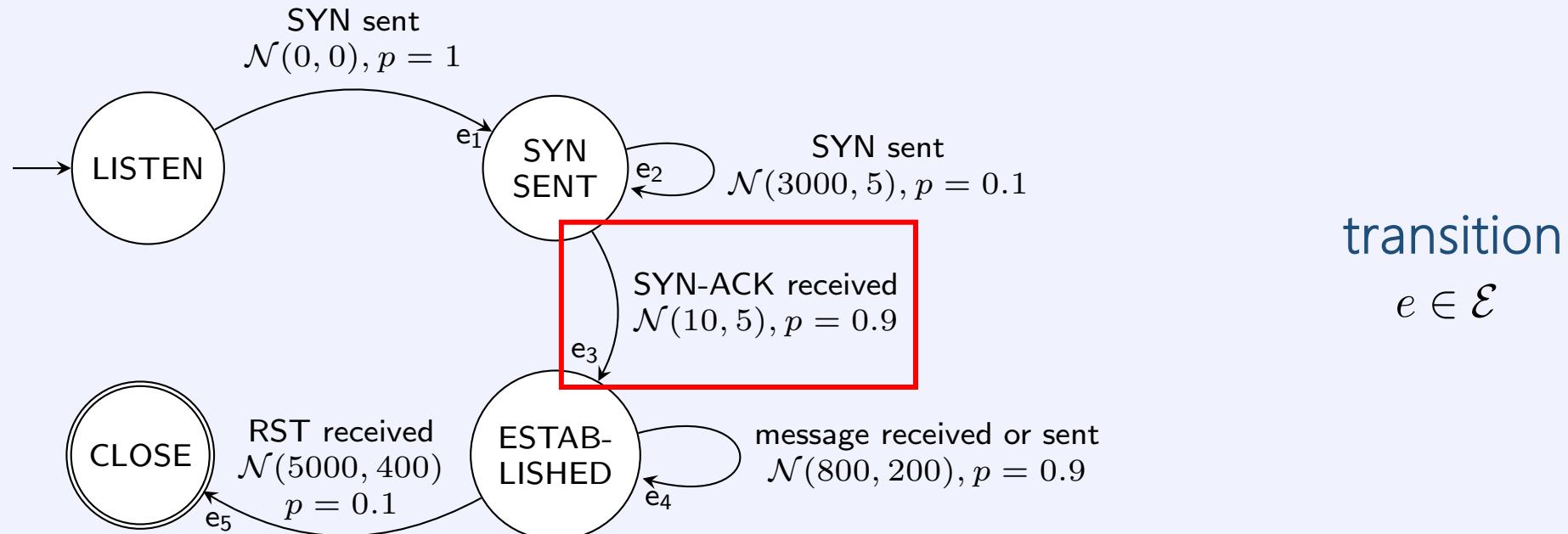


initial
location

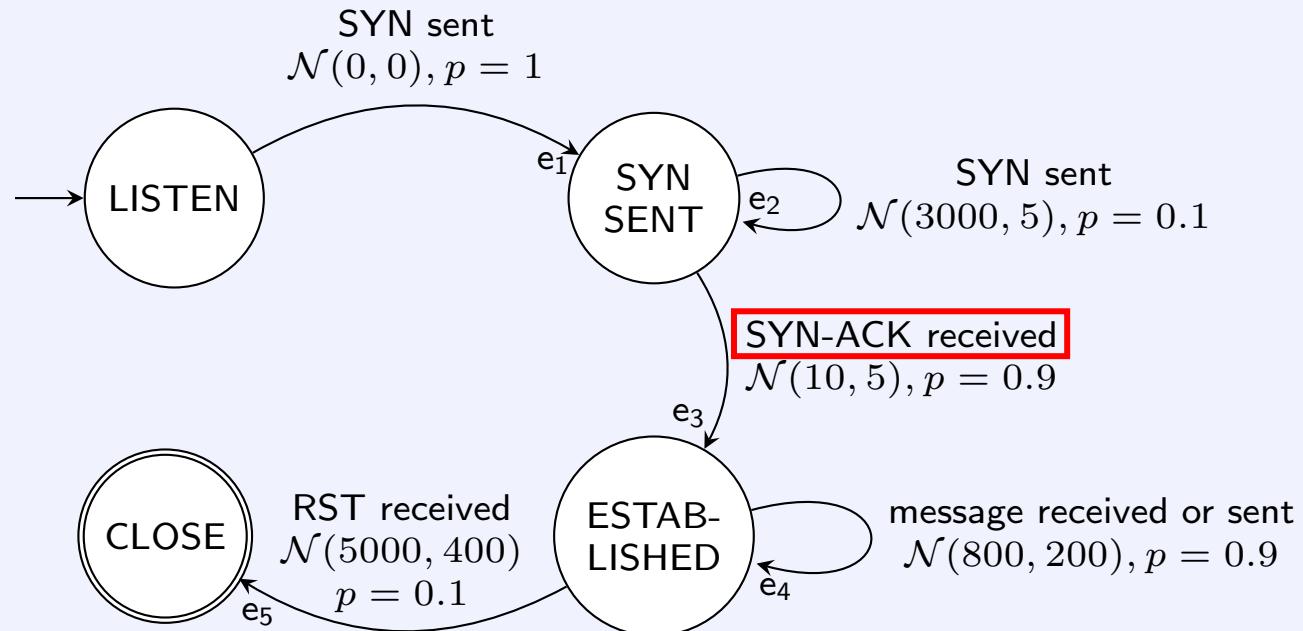
Probabilistic Real-Time Automaton



Probabilistic Real-Time Automaton



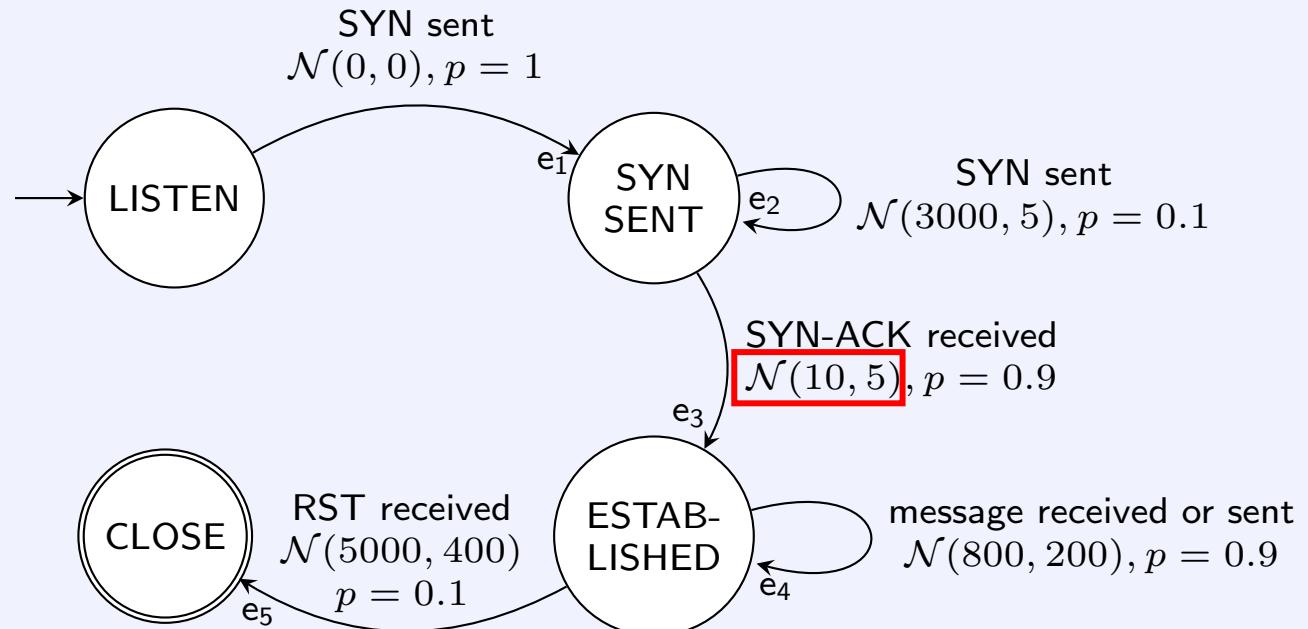
Probabilistic Real-Time Automaton



symbol

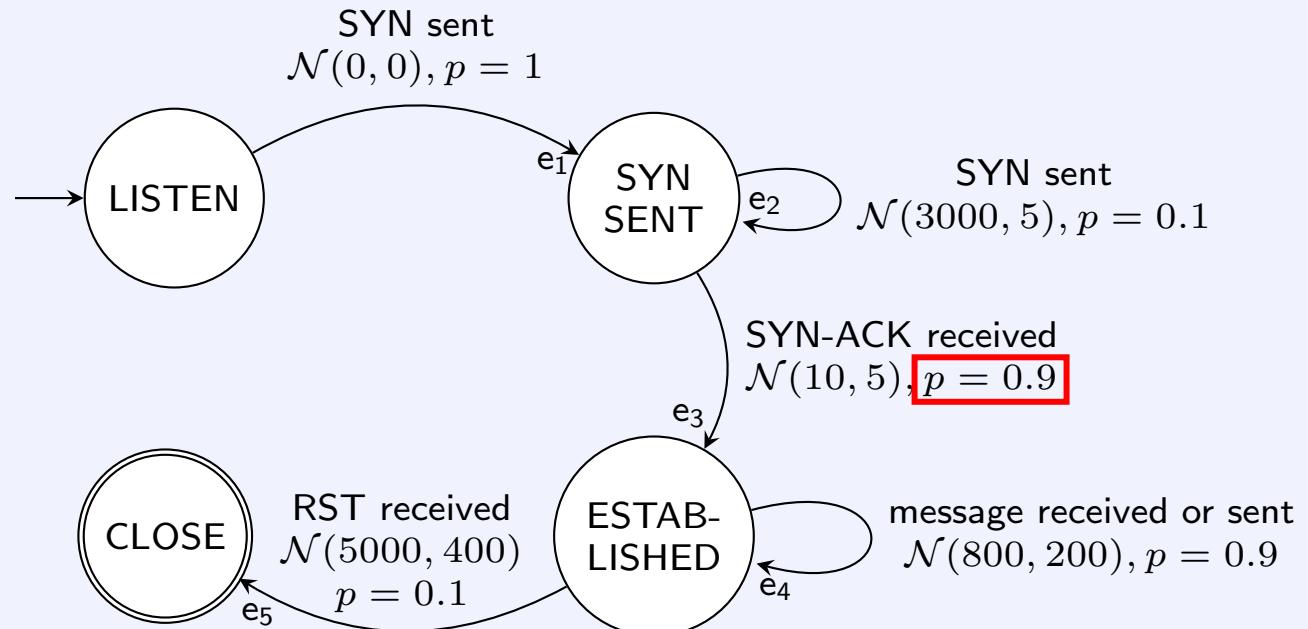
$$a \in \Sigma$$

Probabilistic Real-Time Automaton



delay
distribution
 $\mathcal{N}(\mu, \sigma)$

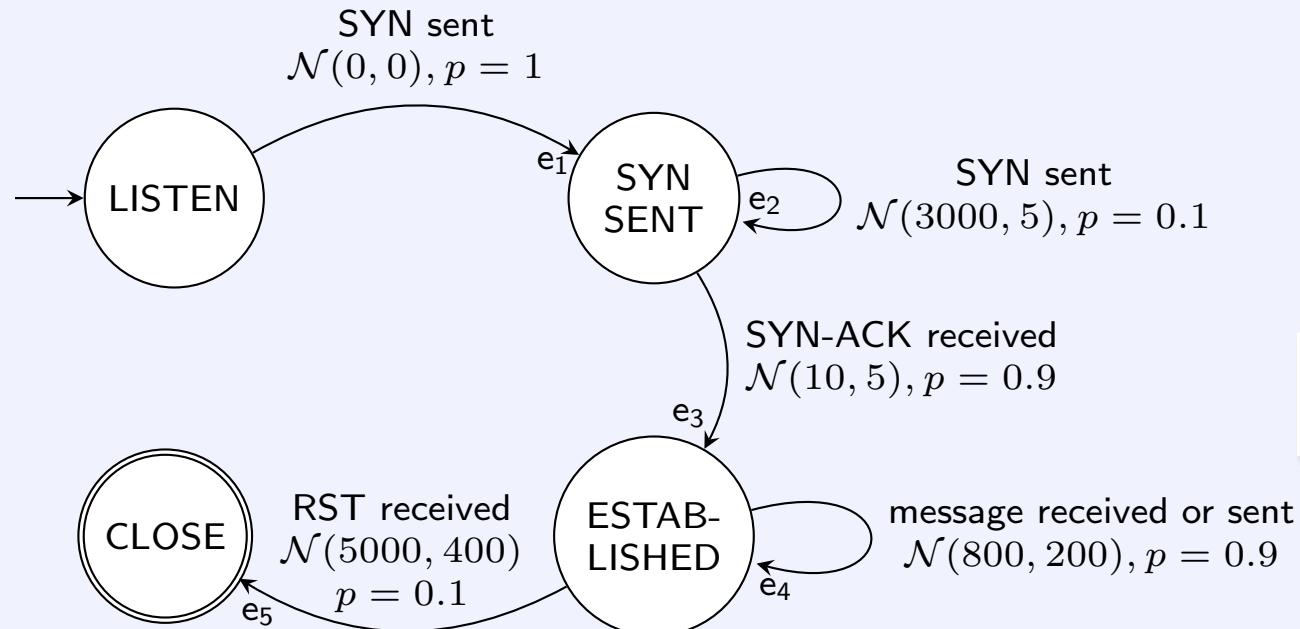
Probabilistic Real-Time Automaton



transition
probability

p

Probabilistic Real-Time Automaton



timed word

$(\text{SYN sent}, 0), (\text{SYN sent}, 2000), (\text{SYN-ACK received}, 10), (\text{RST received}, 6500)$

$w = \{(a, d), \dots\}$

Noise model

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

deletion



(SYN sent, 0),(SYN sent, 2000),(RST received, 6500)

Noise model

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

insertion
↓

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(SYN sent, 50),(RST received, 6500)

Noise model

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

transposition



(SYN sent, 0),(**SYN-ACK received, 10**),(SYN sent, 2000),(RST received, 6500)

Noise model

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

|
symbol repetition



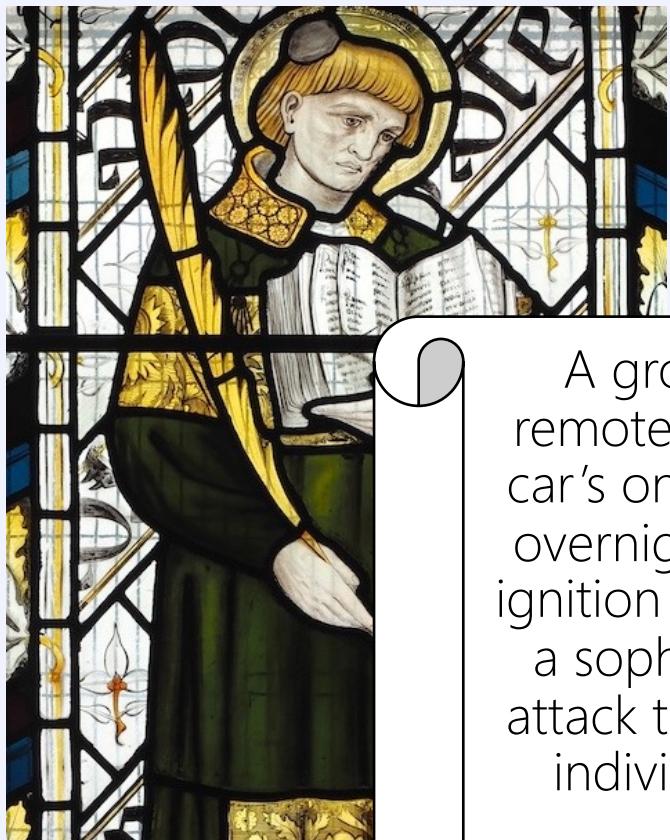
(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(SYN-ACK received, 7),(RST received, 6500)

Question



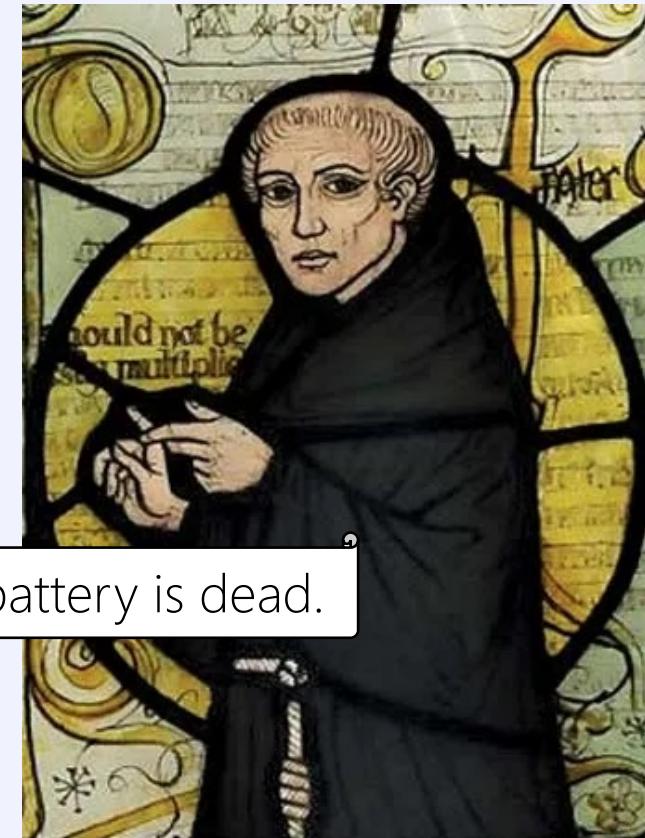
How to learn **timed automata** from **noisy** timed words?

Occam's razor



Why won't my car start?

A group of hackers remotely accessed your car's onboard computer overnight, disabling the ignition system as part of a sophisticated cyber-attack targeting random individuals to create chaos.



Your car battery is dead.

Occam's razor

The **simplest** model that fits the data is usually the correct one



MDL principle

Two-part description length

$$L(\mathcal{D}, \mathcal{A}) = L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$

MDL principle

Two-part description length

$$L(\mathcal{D}, \mathcal{A}) = L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$

global length
(# of bits needed)

MDL principle

Two-part description length

$$L(\mathcal{D}, \mathcal{A}) = L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$


global length
(# of bits needed)

length of describing
the automaton

MDL principle

Two-part description length

$$L(\mathcal{D}, \mathcal{A}) = L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$

global length
(# of bits needed)

length of describing the
data encoded with
the automaton

length of describing
the automaton



MDL principle

Two-part description length

$$L(\mathcal{D}, \mathcal{A}) = L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$

global length
(# of bits needed)

length of describing the
data encoded with
the automaton

length of describing
the automaton



Minimum Description Length
principle

$$\mathcal{A}^* = \operatorname{argmin}_{\mathcal{A} \in \mathcal{A}} L(\mathcal{D}|\mathcal{A}) + L(\mathcal{A})$$

Automaton encoding

$$L(\mathcal{A}) =$$

Automaton encoding

- Locations

$$L(\mathcal{A}) = L_{\mathbb{N}}(|\mathcal{Q}|)$$

Automaton encoding

- Locations
- Alphabet

$$L(\mathcal{A}) = L_{\mathbb{N}}(|\mathcal{Q}|) + L_{\mathbb{N}}(|\Sigma|)$$

Automaton encoding

- Locations
- Alphabet
- Initial and accepting locations

$$L(\mathcal{A}) = L_{\mathbb{N}}(|Q|) + L_{\mathbb{N}}(|\Sigma|) + 2 \log_2(|Q|) +$$

Automaton encoding

- Locations
- Alphabet
- Initial and accepting locations
- For each transition:

$$L(\mathcal{A}) = L_{\mathbb{N}}(|Q|) + L_{\mathbb{N}}(|\Sigma|) + 2 \log_2(|Q|) +$$

$$\sum_{e \in \mathcal{E}} ($$

$$)$$

Automaton encoding

- Locations
- Alphabet
- Initial and accepting locations
- For each transition:
 - Source and destination locations

$$L(\mathcal{A}) = L_{\mathbb{N}}(|Q|) + L_{\mathbb{N}}(|\Sigma|) + 2 \log_2(|Q|) + \sum_{e \in \mathcal{E}} \left(2 \log_2(|Q|) \right)$$

Automaton encoding

- Locations
- Alphabet
- Initial and accepting locations
- For each transition:
 - Source and destination locations
 - Symbol

$$L(\mathcal{A}) = L_{\mathbb{N}}(|Q|) + L_{\mathbb{N}}(|\Sigma|) + 2 \log_2(|Q|) + \sum_{e \in \mathcal{E}} \left(2 \log_2(|Q|) + \log_2(|\Sigma|) \right)$$

Automaton encoding

- Locations
- Alphabet
- Initial and accepting locations
- For each transition:
 - Source and destination locations
 - Symbol
 - Guards' normal distributions parameters

$$\begin{aligned} L(\mathcal{A}) = & L_{\mathbb{N}}(|\mathcal{Q}|) + L_{\mathbb{N}}(|\Sigma|) + 2 \log_2(|\mathcal{Q}|) + \\ & \sum_{e \in \mathcal{E}} \left(2 \log_2(|\mathcal{Q}|) + \log_2(|\Sigma|) + \right. \\ & \quad \left. L_{\mathbb{N}}(\lfloor \mu_e \rfloor) + L_{\mathbb{N}}(\lfloor \sigma_e^2 \rfloor) \right) \end{aligned}$$

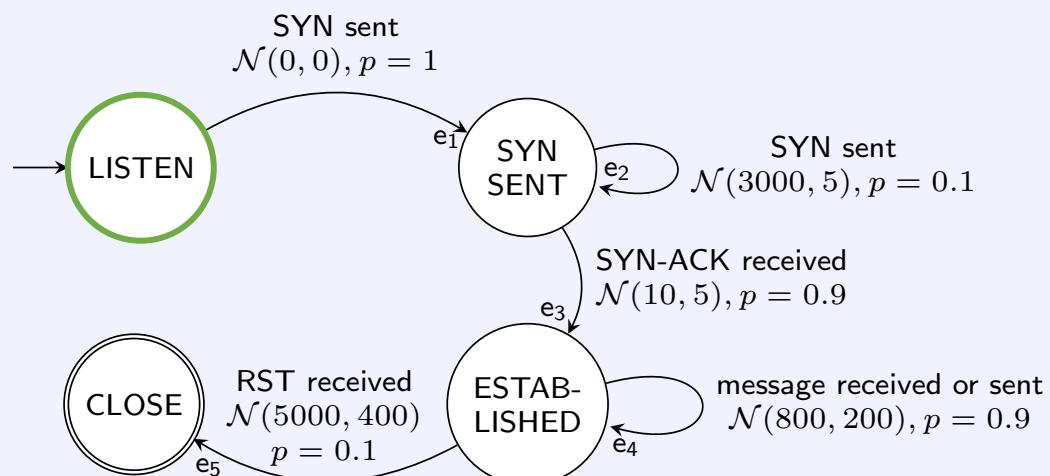
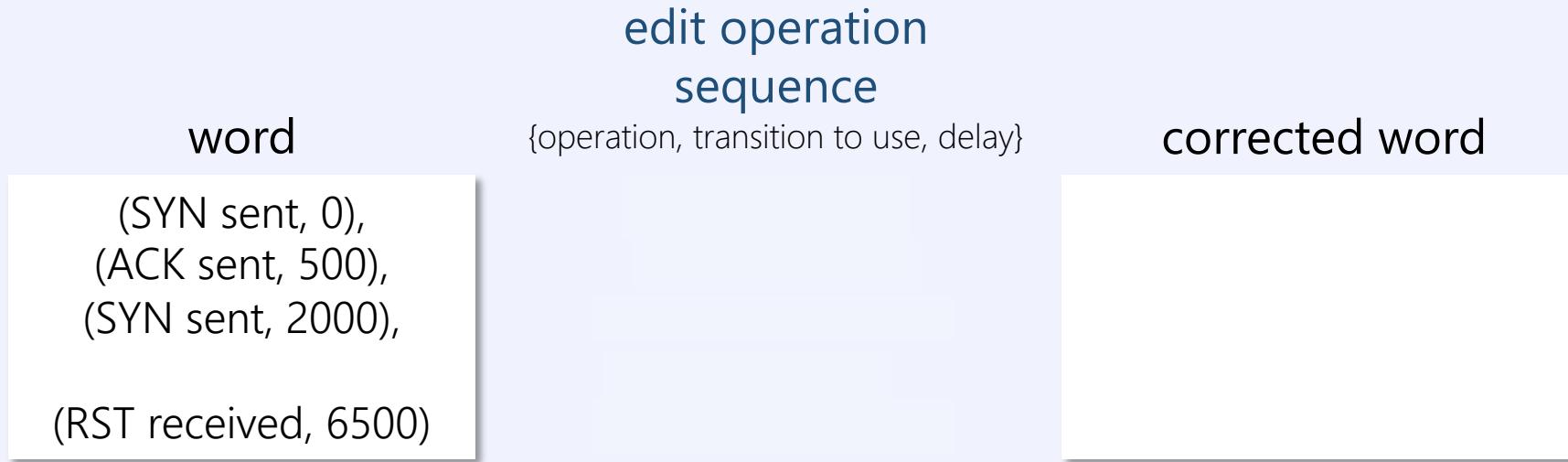
Data encoding

1. **Correct** the non-accepted words to **remove the noise**
2. Encode the corrected data

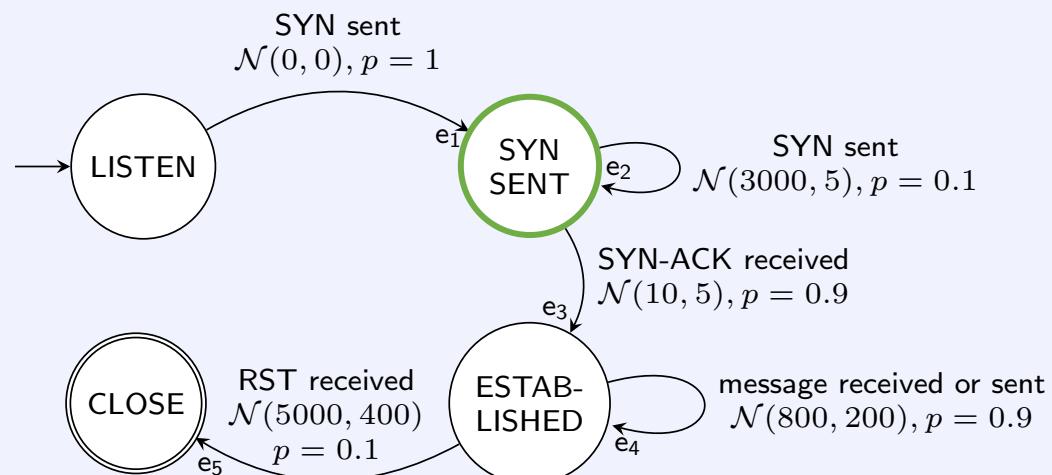
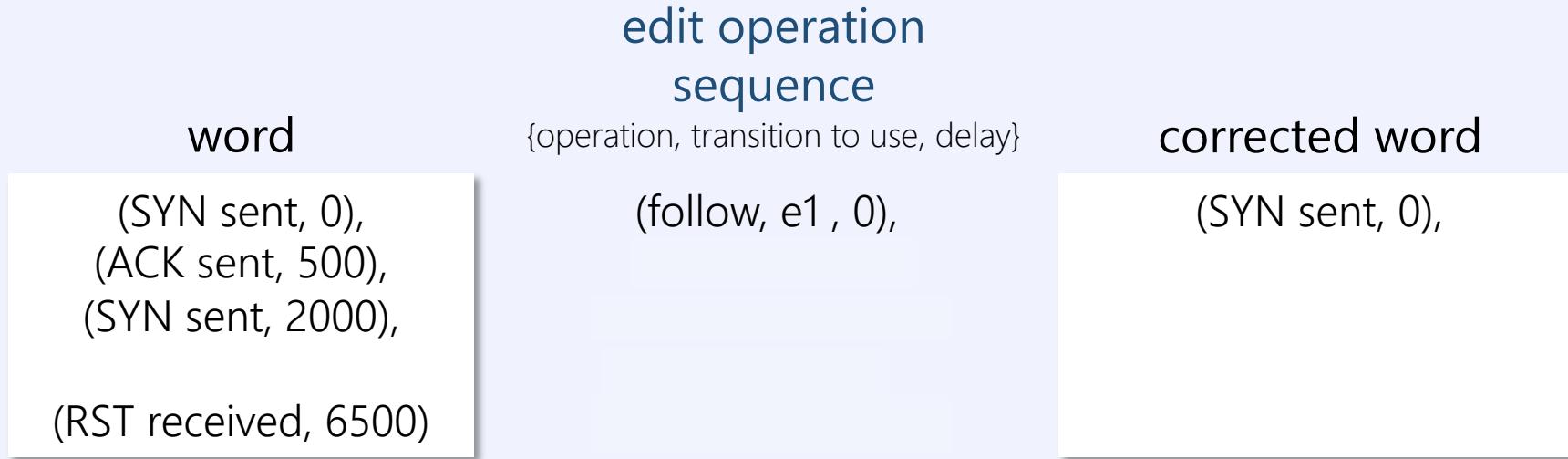
Data correction

Noise type	Correction operation
deletion $aabcad \rightarrow aacad$	add
insertion $aabcad \rightarrow aabcbad$	skip
transposition $aabcad \rightarrow aacbad$	transpose
symbol repetition $aabcad \rightarrow aabcaad$	deduplicate
-	follow

Data correction

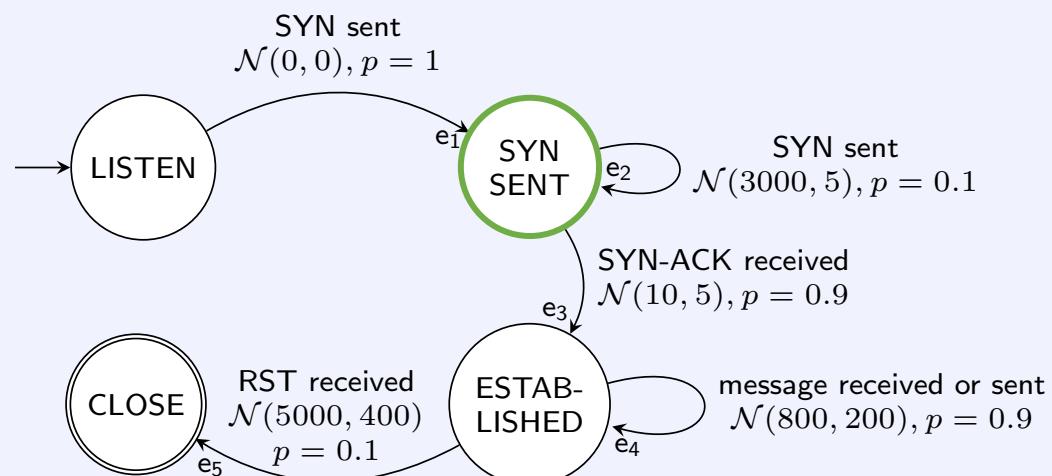


Data correction



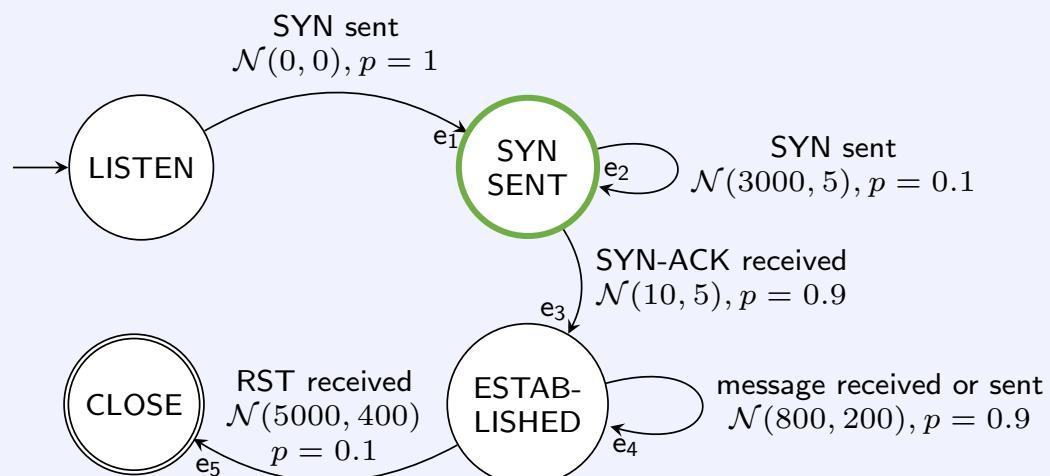
Data correction

word	edit operation sequence	corrected word
{ (SYN sent, 0), (ACK sent, 500), (SYN sent, 2000), (RST received, 6500)	{ (follow, e1 , 0), (skip, ε , 500), }	{ (SYN sent, 0), }



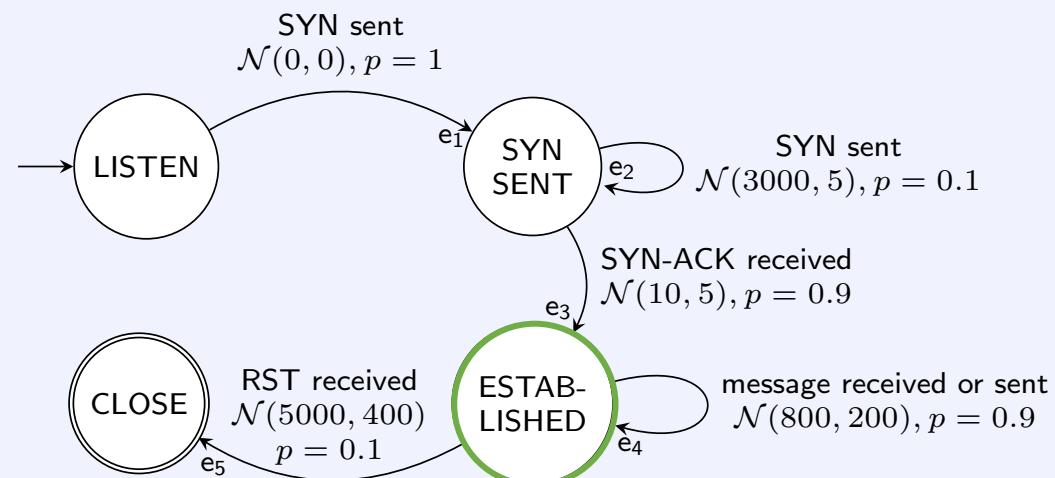
Data correction

word	edit operation sequence	corrected word
{ (SYN sent, 0), (ACK sent, 500), (SYN sent, 2000), (RST received, 6500)	{ (follow, e1 , 0), (skip, ε , 500), (follow, e2 , 2000),	{ (SYN sent, 0), (SYN sent, 2000),



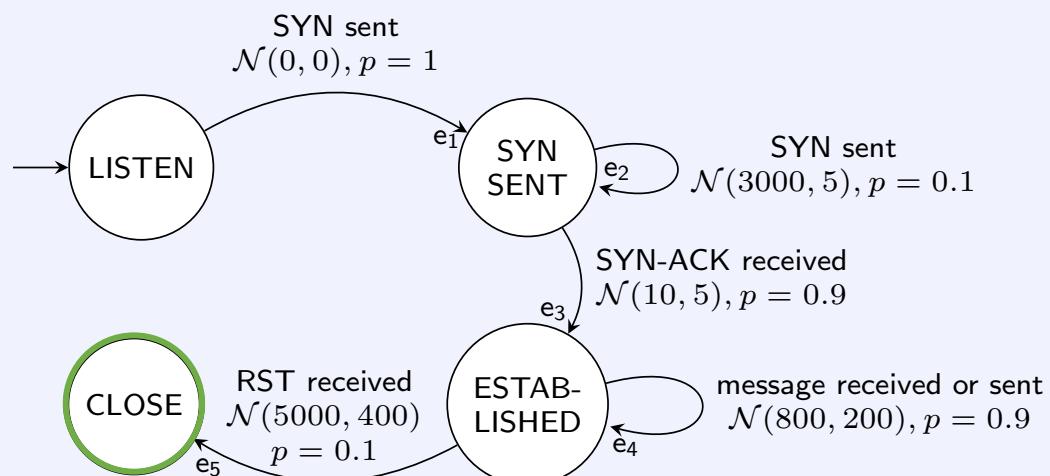
Data correction

word	edit operation sequence	corrected word
{operation, transition to use, delay}	(follow, e1 , 0), (skip, ε , 500), (follow, e2 , 2000), (add, e3 , 10),	(SYN sent, 0), (SYN sent, 2000), (SYN-ACK received, 10),



Data correction

word	edit operation sequence	corrected word
{ (SYN sent, 0), (ACK sent, 500), (SYN sent, 2000), (RST received, 6500)	{ (follow, e1 , 0), (skip, ε , 500), (follow, e2 , 2000), (add, e3 , 10), (follow, e5 , 6500)	{ (SYN sent, 0), (SYN sent, 2000), (SYN-ACK received, 10), (RST received, 6500)



Data correction

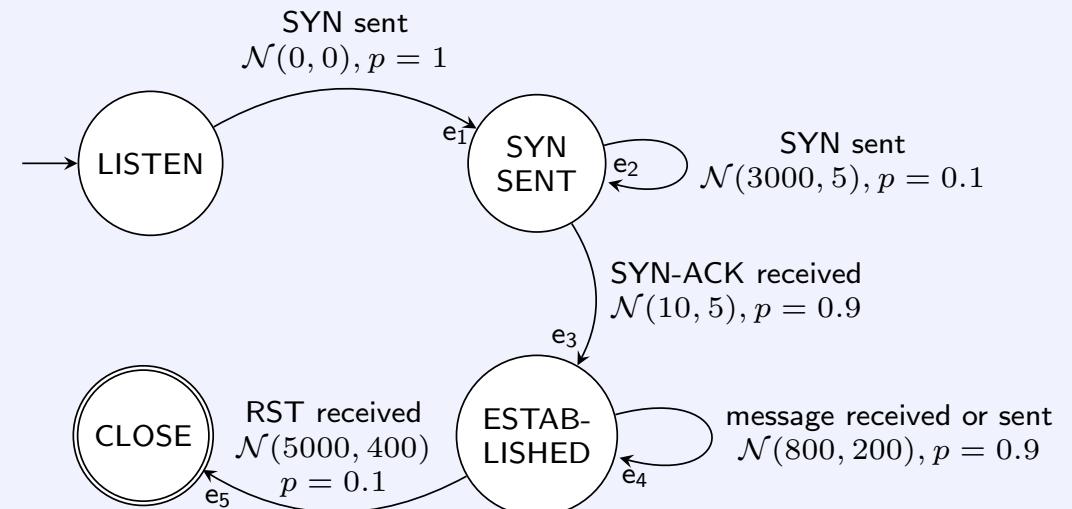
(SYN sent, 0),(ACK sent, 500),(SYN sent, 2000),(RST received, 6500)

↓
correction

(SYN sent, 0),(SYN-ACK received, 10),(RST received, 6500)

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

...



Many correction solutions possible...

Data correction

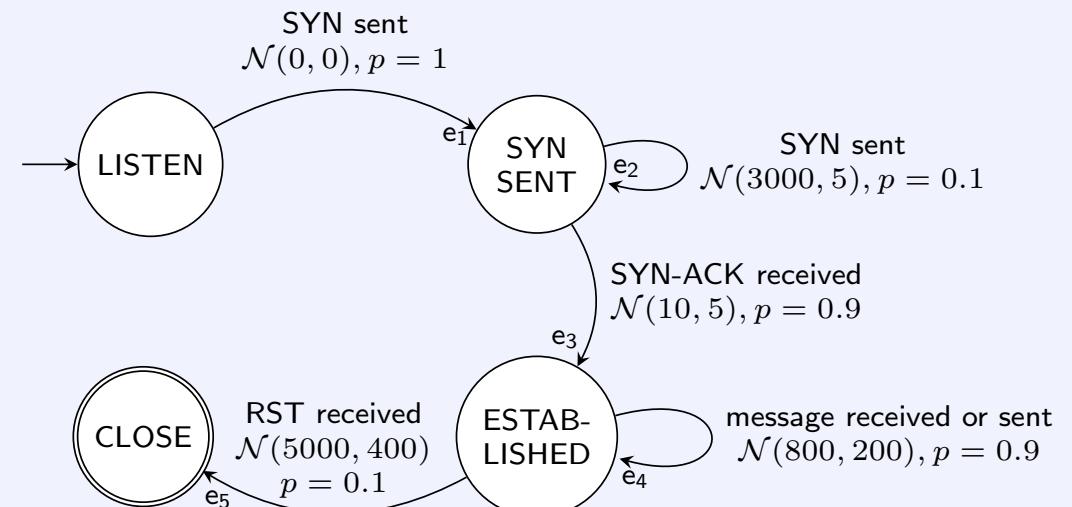
(SYN sent, 0),(ACK sent, 500),(SYN sent, 2000),(RST received, 6500)

↓
correction

(SYN sent, 0),(SYN-ACK received, 10),(RST received, 6500)

(SYN sent, 0),(SYN sent, 2000),(SYN-ACK received, 10),(RST received, 6500)

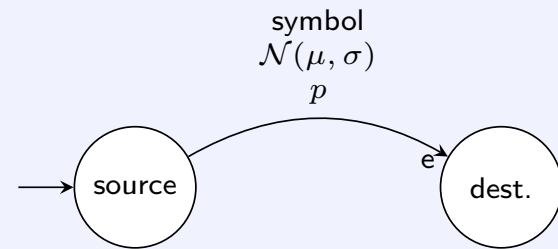
...



Many correction solutions possible...
We choose the one with the minimal description length

Data encoding

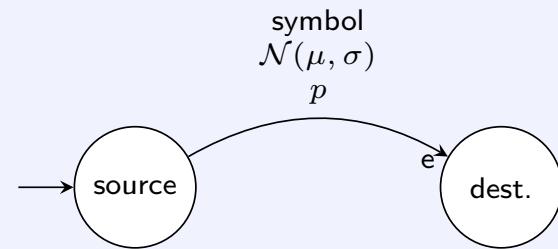
Cost of a **followed** pair (symbol, delay)
corrected with (operation, transition, delay) depends on



Data encoding

Cost of a **followed** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (follow),

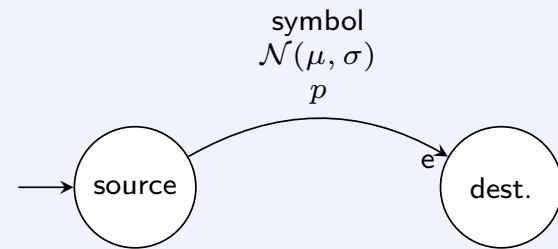


$$-\log_2 p(o)$$

Data encoding

Cost of a **followed** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (follow),
- The probability of the transition given the current state,

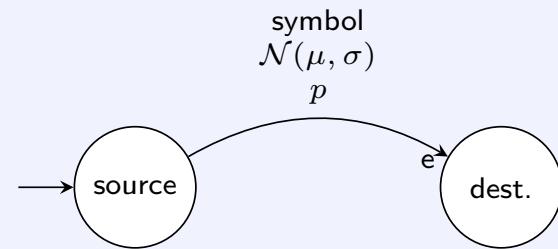


$$-\log_2 p(o) - \log_2 p(e|q_s(e))$$

Data encoding

Cost of a **followed** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (follow),
- The probability of the transition given the current state,
- The probability of the delay given the transition guard's parameters.

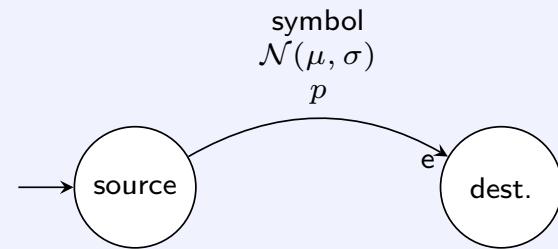


$$-\log_2 p(o) - \log_2 p(e|q_s(e)) - \log_2 p(d|e)$$

Data encoding

Cost of a **transposed** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (transpose),
- The probability of the transition given the current state,
- The probability of the delay given the transition guard's parameters.

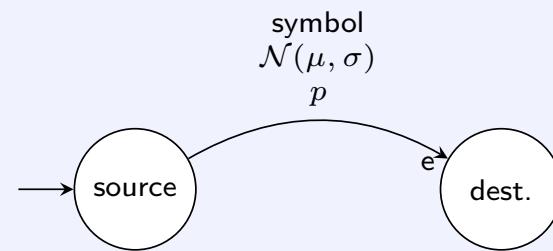


$$-\log_2 p(o) - \log_2 p(e|q_s(e)) - \log_2 p(d|e)$$

Data encoding

Cost of an **added** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (add),
- The probability of the transition given the current state,
- ~~The probability of the delay given the transition guard's parameters.~~

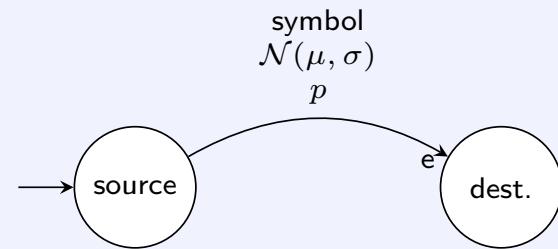


$$-\log_2 p(o) - \log_2 p(e|q_s(e))$$

Data encoding

Cost of a **deduplicated** pair (symbol, delay)
corrected with (operation, transition, delay) depends on

- The probability of the edit operation (deduplicate),
- ~~The probability of the transition given the current state,~~
- The probability of the delay given the transition guard's parameters.

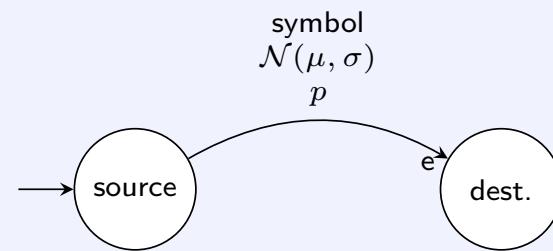


$$-\log_2 p(o) - \log_2 p(d|e)$$

Data encoding

Cost of a **skipped** pair (symbol, delay)
corrected with (operation, ε , delay) depends on

- The probability of the edit operation (skip),
- ~~The probability of the transition given the current state,~~
- ~~The probability of the delay given the transition guard's parameters.~~
- The cost of explicitly encoding the delay and the symbol.



$$-\log_2 p(o) + L_{\mathbb{N}}(d) + \log_2 |\Sigma|$$

Question



How to **find** the automaton with the **minimal
MDL cost**?

TADAM: MDL-based automata learning

Initialize an automaton $\hat{\mathcal{A}}$ with the data \mathcal{D}

→ Generate candidate automata by transforming $\hat{\mathcal{A}}$

For each candidate automaton \mathcal{A}

 Correct \mathcal{D} given \mathcal{A}

 Compute the cost $L(\mathcal{A}, \mathcal{D})$

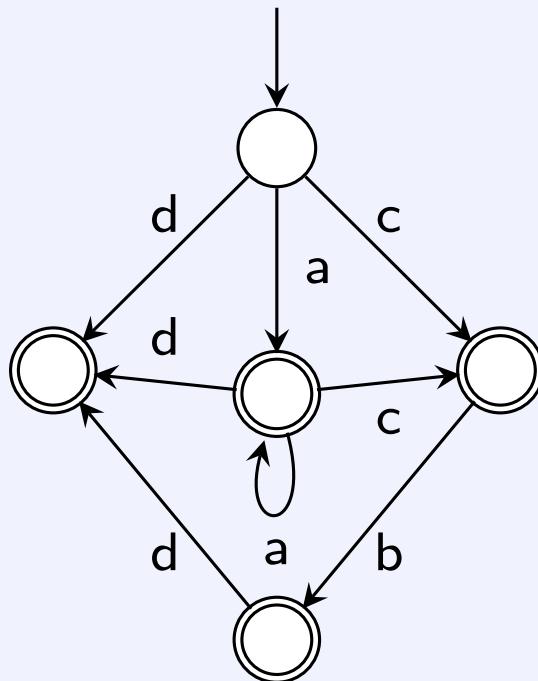
 Select the automaton with minimal cost as $\hat{\mathcal{A}}$

Return $\hat{\mathcal{A}}$ when the cost doesn't decrease anymore

Initialization

Markov initialization

a	c	b
a	d	
c	b	a
a	c	a
d	a	a
c	b	d



guards and probabilities omitted

Candidate automata generation

Automaton transformation operations:

- Location merge
- Location split
- Subpart deletion

One candidate per possible **transformation** and **position** in the automaton

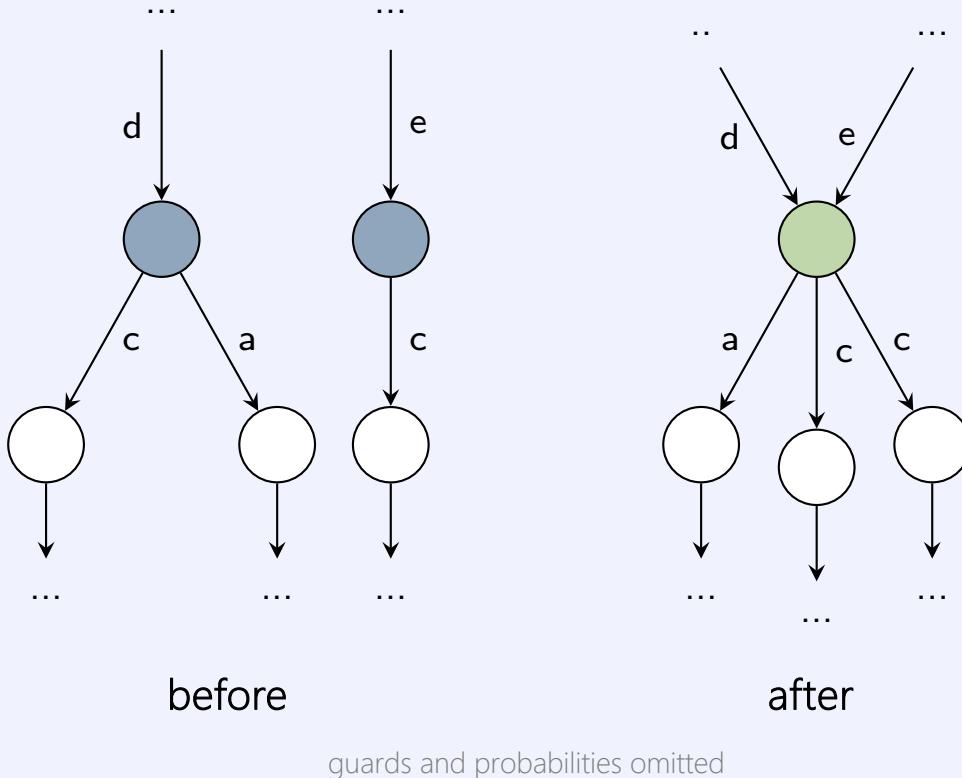
Location merge

Goal:

Reducing the size of the automaton and generalize the model
(reduces the model cost)

Side effect:

Increases the data cost



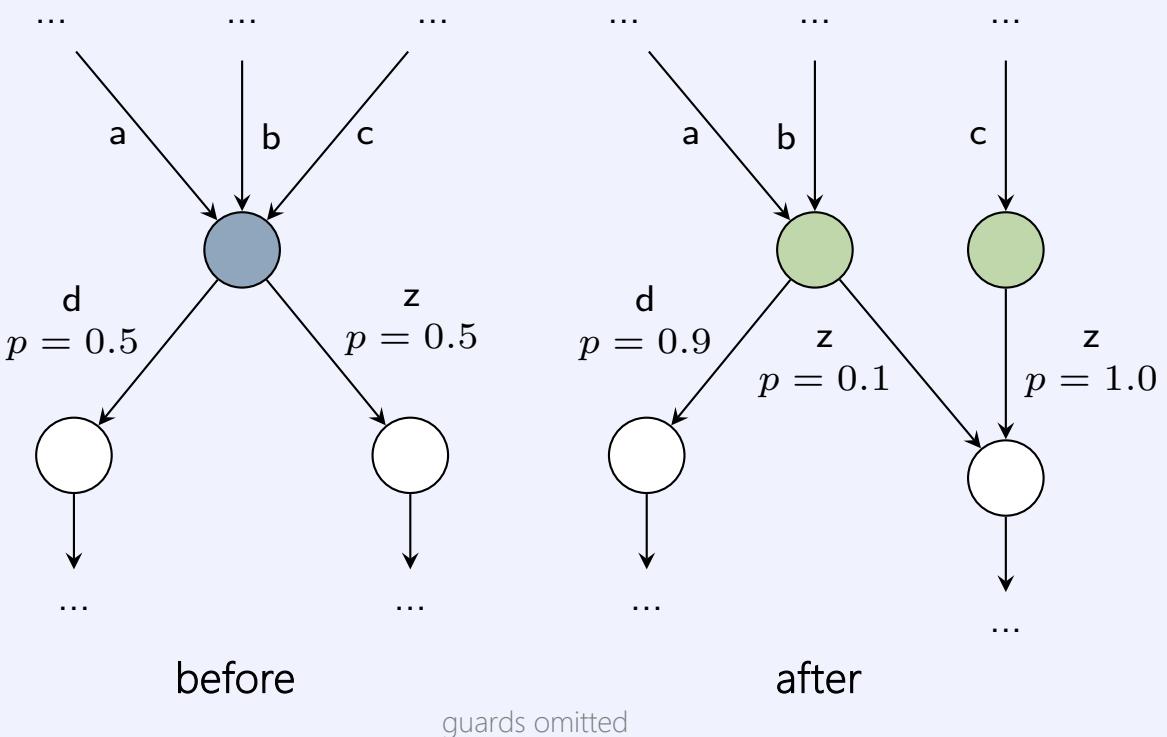
Location split

Goal:

Reducing the entropy of the
"next triggered transition" at
a given location
(reduces the data cost)

Side effect:

Increases the model cost



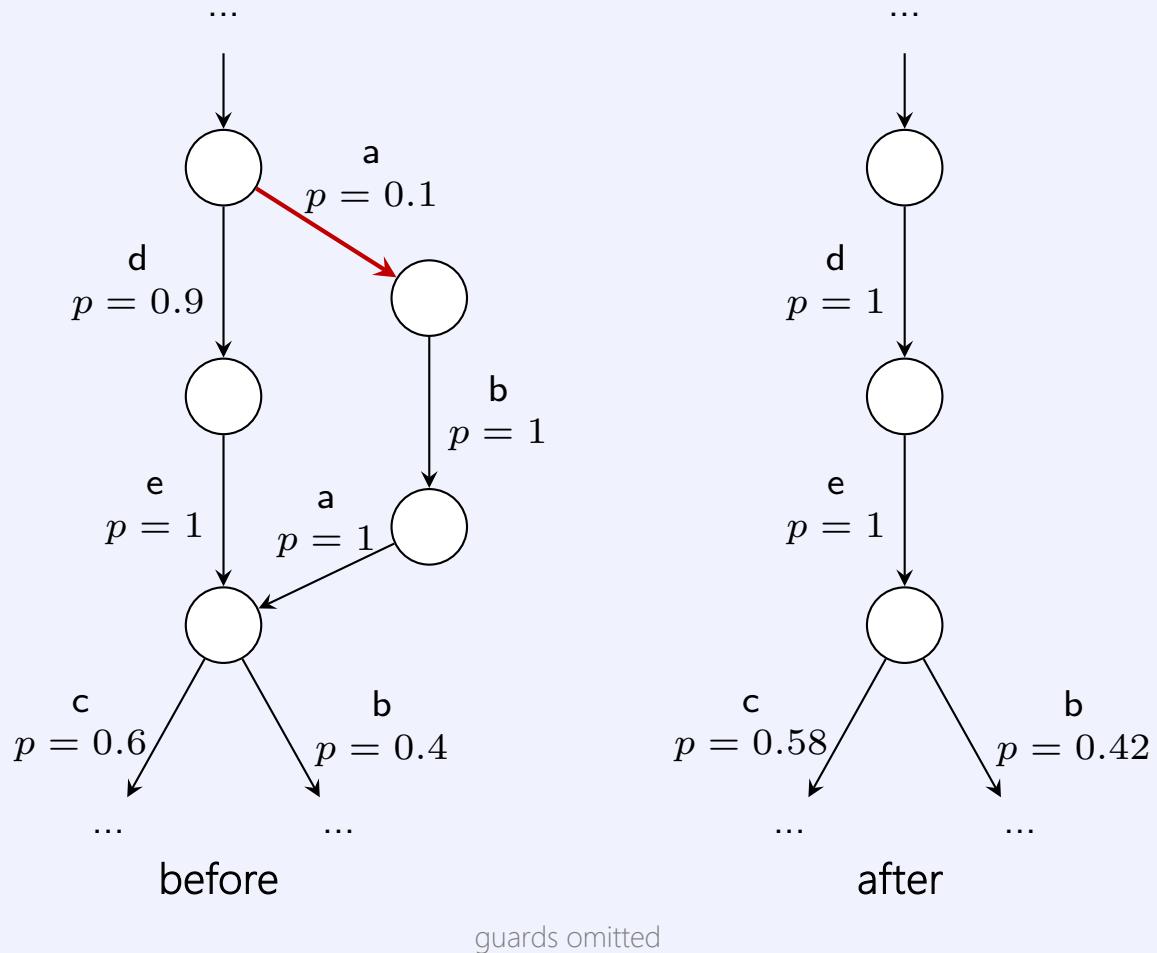
Subpart deletion

Goal:

Reducing the size of the automaton
(reduces the model cost)

Side effect:

Increases the data cost



Question



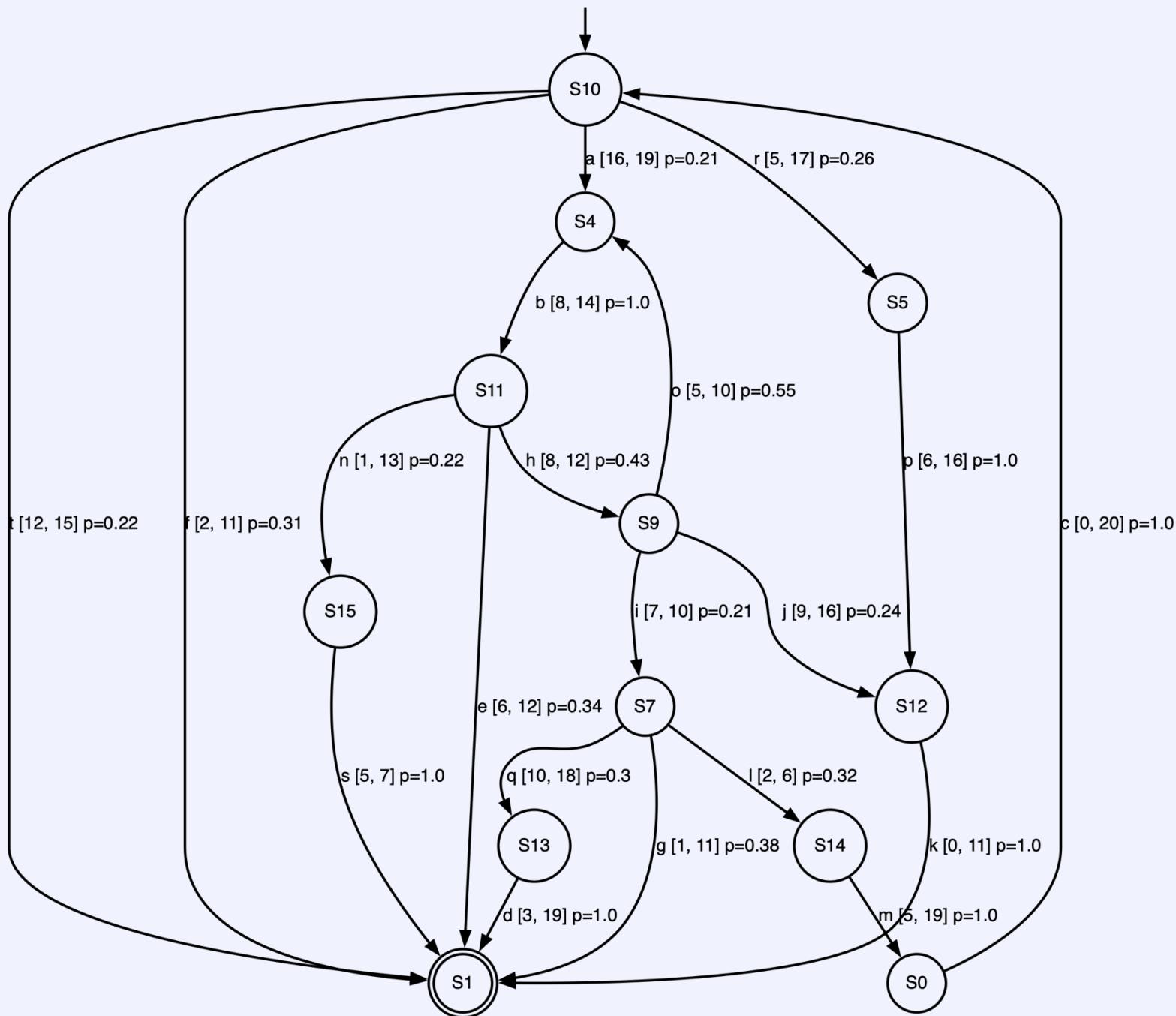
Does it work?

State of the Art

Algorithm	Strategy	Main limitation
TAG (Cornanguer et al., 2022)	Factorization on common sub-parts and location splits	
Timed k-Tail (Pastore et al., 2017)	Factorization on common sub-parts	No noise robustness strategy → requires clean data
RTI+ (Verwer et al., 2010)	Location merge based on likelihood test	

Experiments

Target automaton

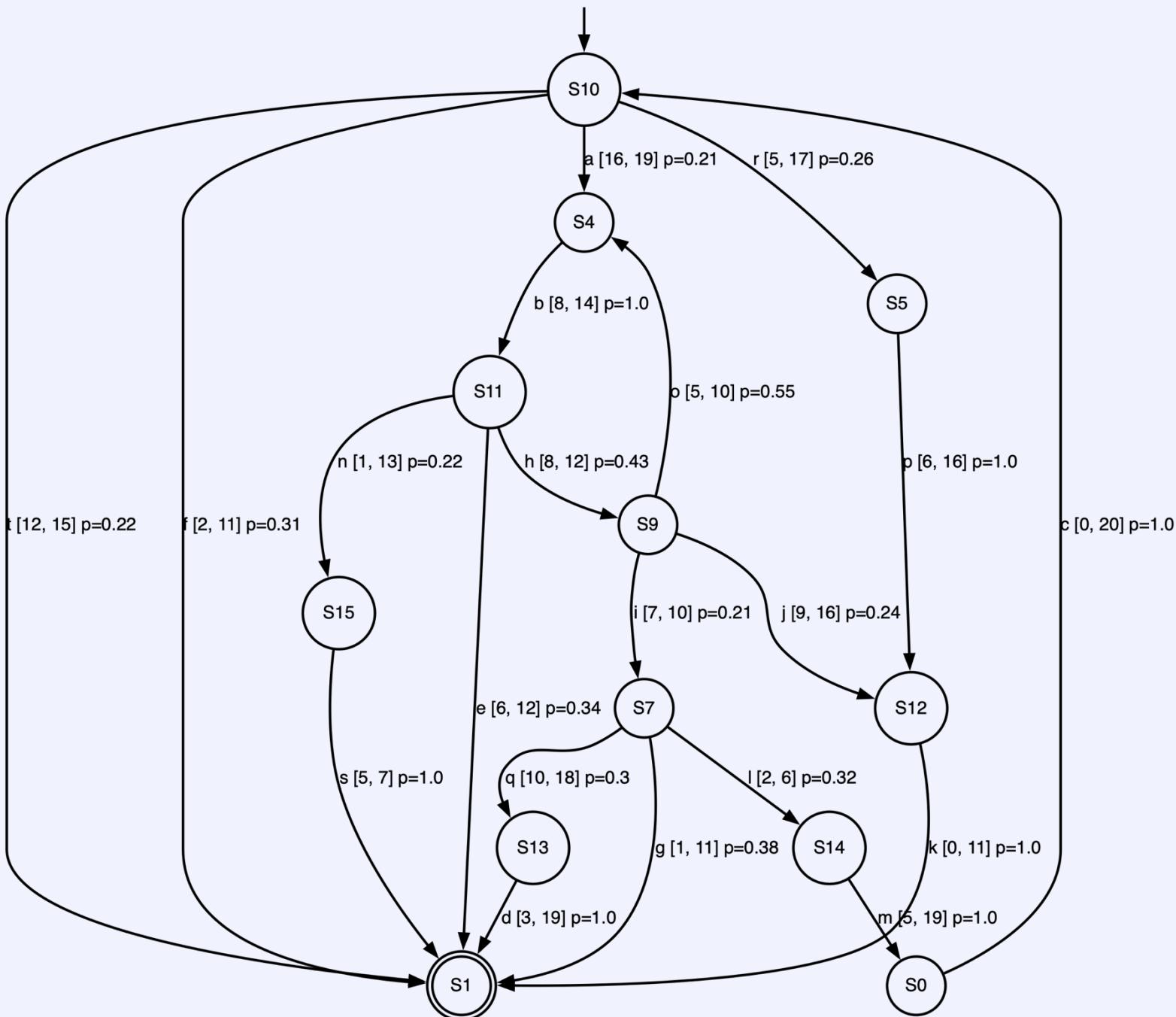


Experiments

Target automaton

Data:

- 500 timed words
- 2,5% of noisy events



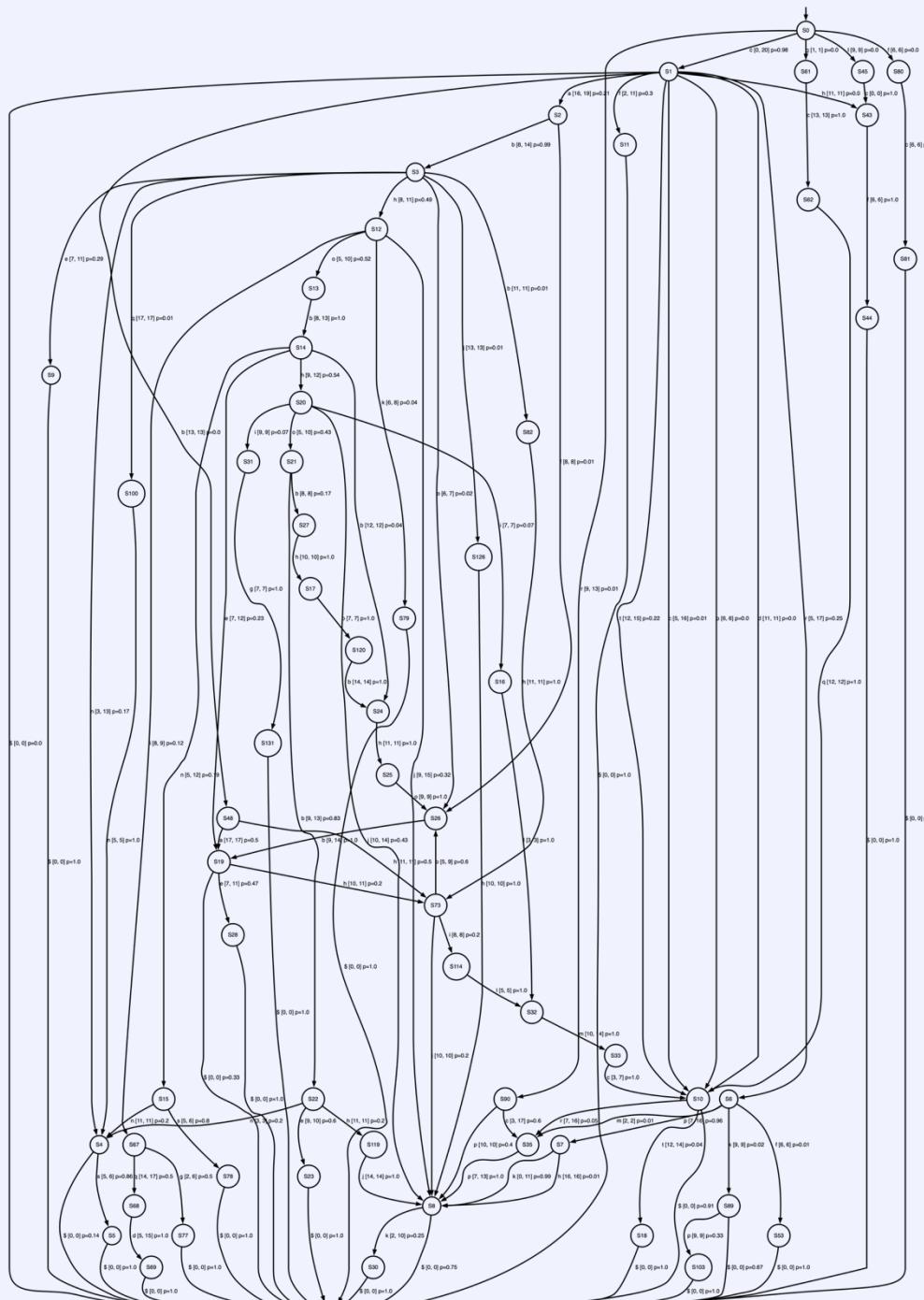
Experiments

TAG

learned automaton

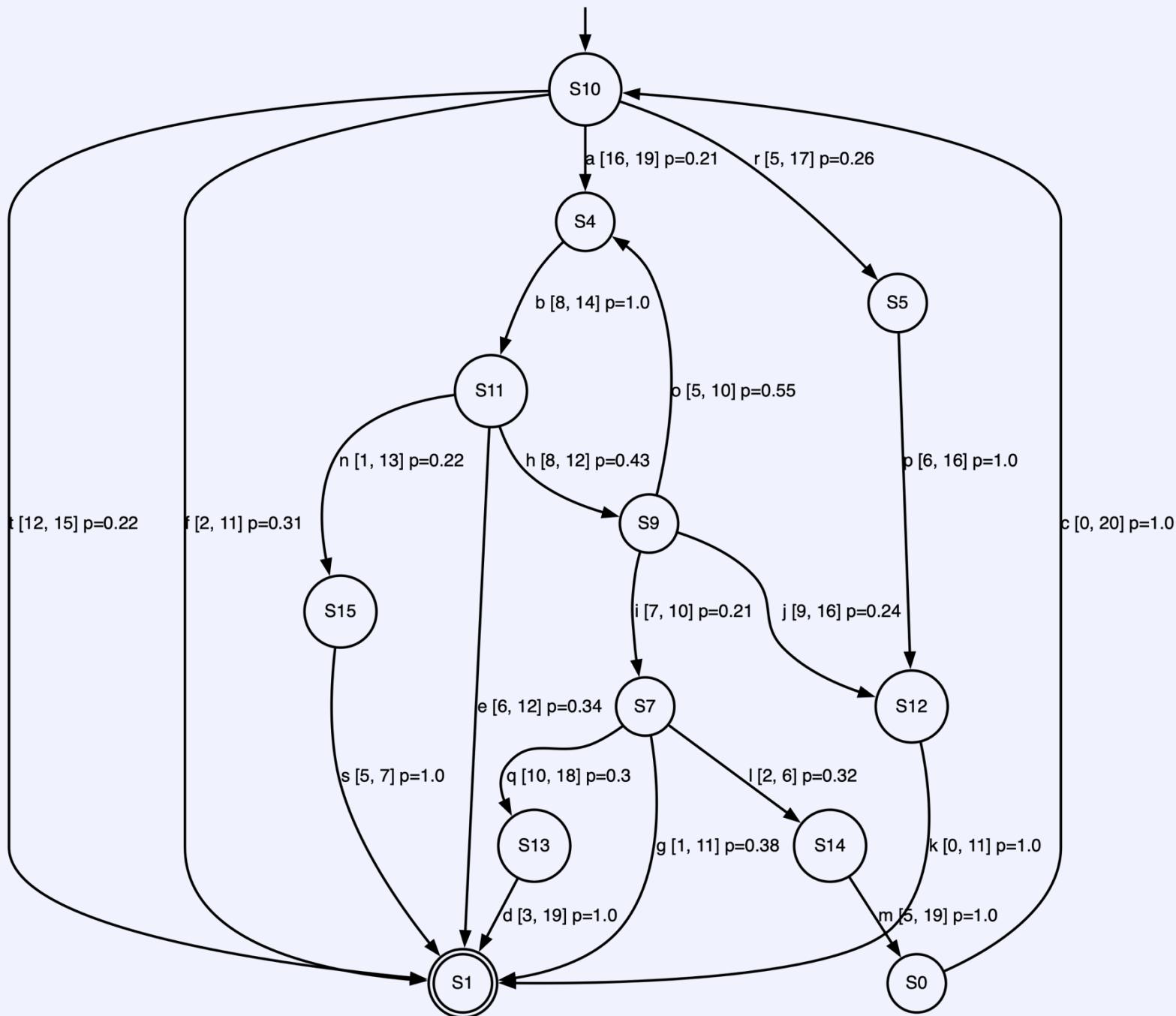
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Experiments

Target automaton

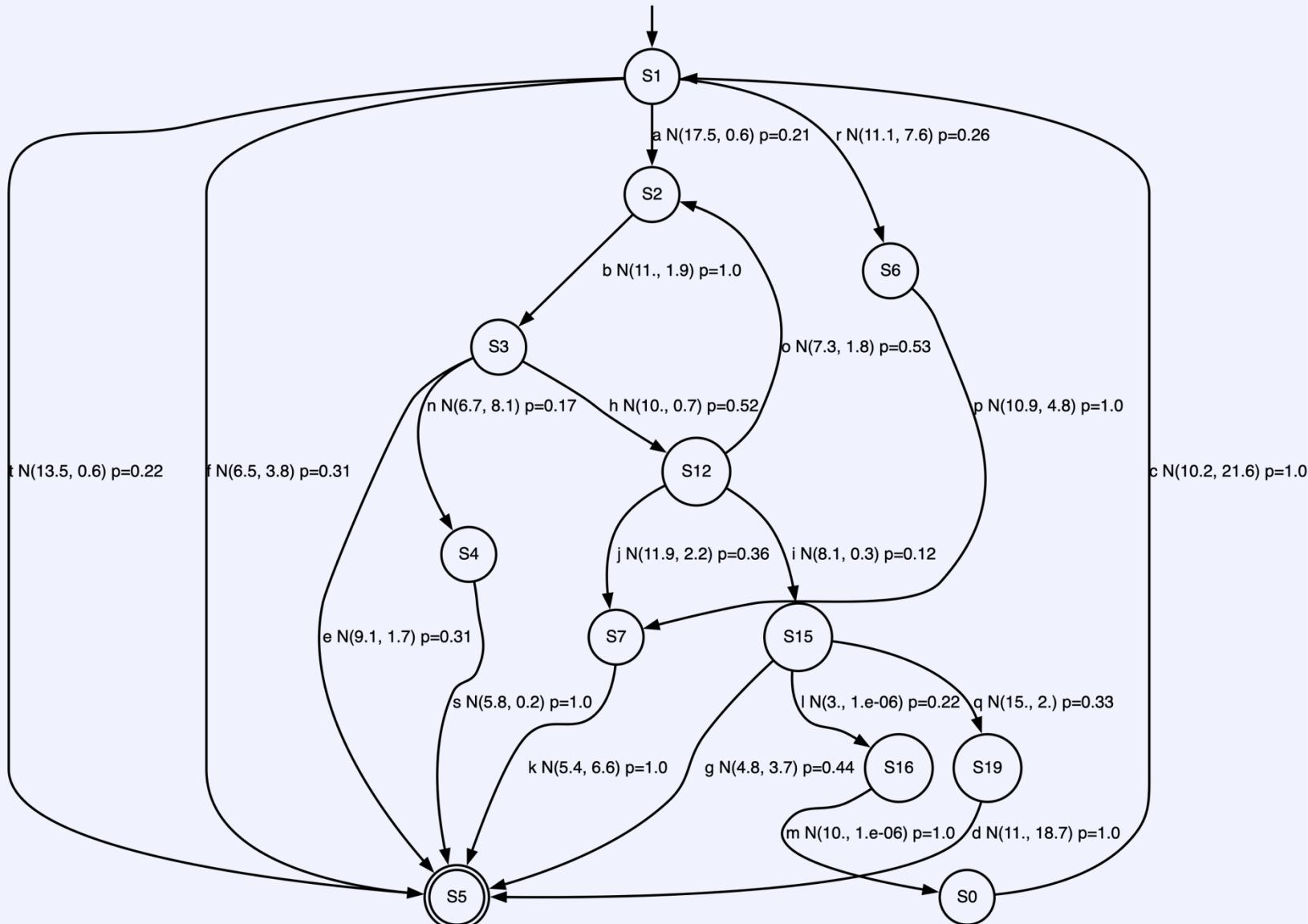


Experiments

TADAM learned automaton

Data:

- 500 timed words
- 2,5% of noisy events



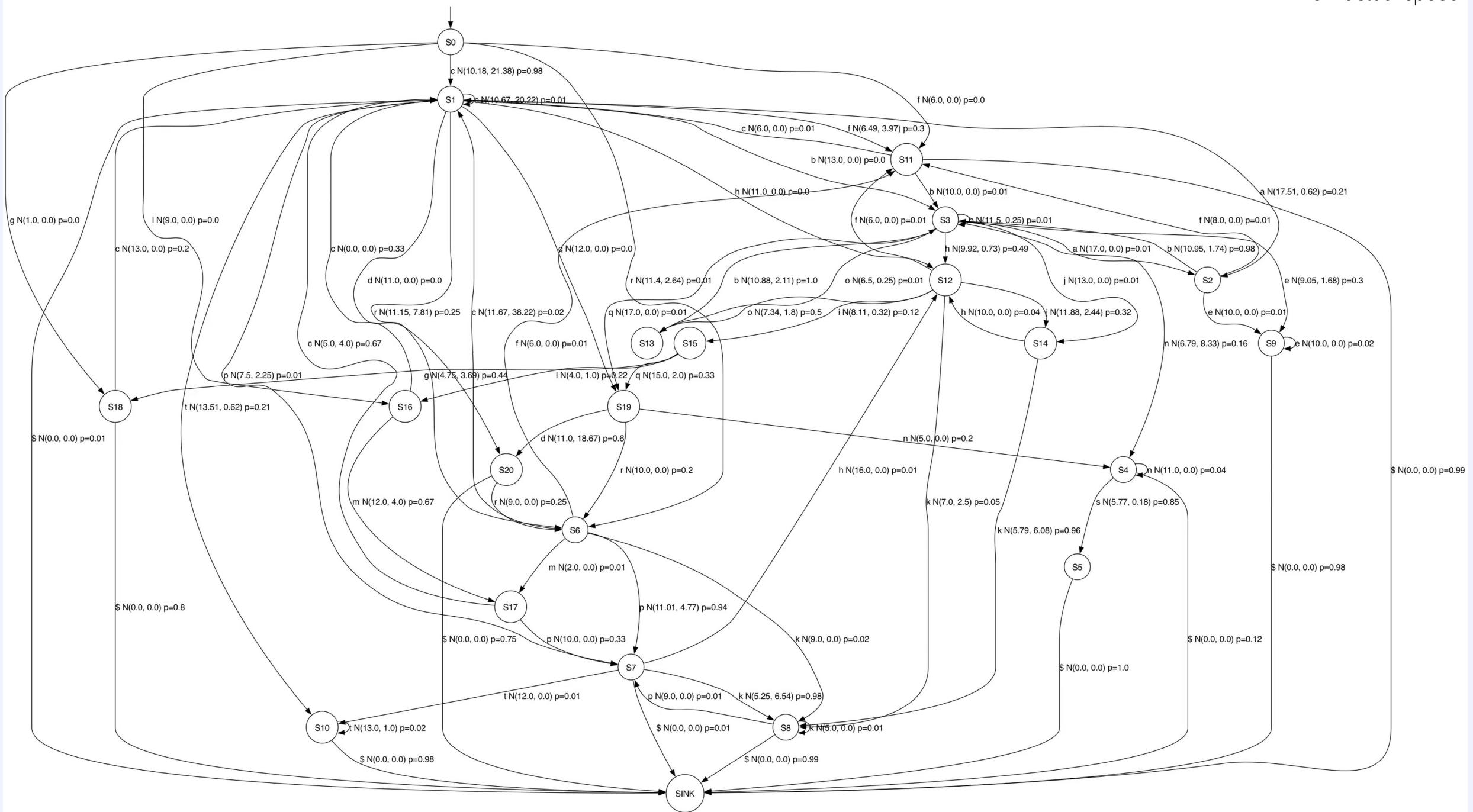
Experiments

TADAM

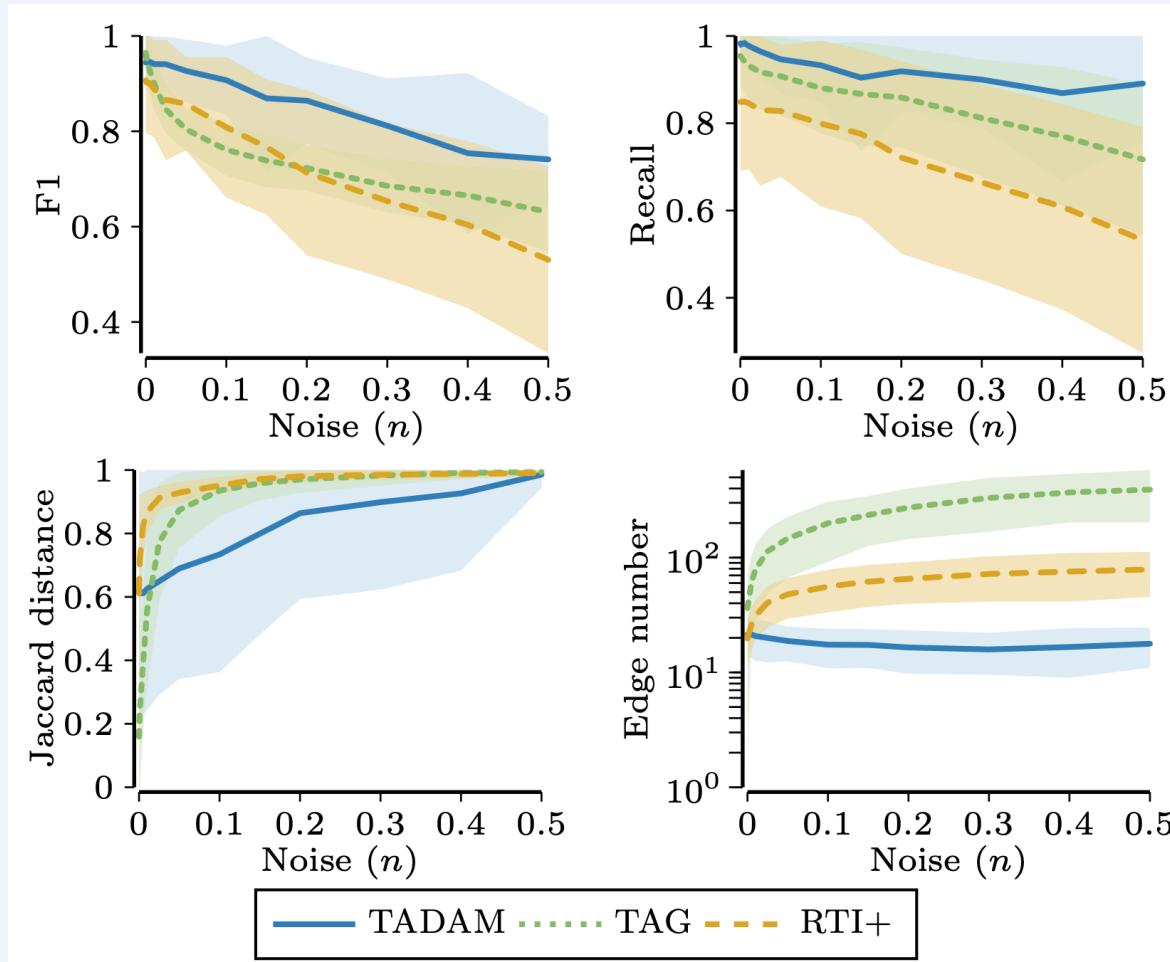
learned automaton

Data:

- 500 timed words
- 2,5% of noisy events



Experiments



Noise robustness
on synthetic data

Experiments

	Learner	AU-ROC	TPR	FPR	F1
TA learners	TADAM	0.982	0.998	0.025	0.705
	TAG	0.891	1	0.142	0.298
	RTI+	0.790	1	0.292	0.171
	Hidden Markov Model	0.608	0.640	0.085	0.288

Anomaly detection performances
on HDFS dataset¹

Experiments

	Learner	AU-ROC	TPR	FPR	F1	
TA learners	TADAM	0.982	0.998	0.025	0.705	very high detection rate and less false alarms
	TAG	0.891	1	0.142	0.298	
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Anomaly detection performances
on HDFS dataset¹

Experiments

	Learner	AU-ROC	TPR	FPR	F1	
TA learners	TADAM	0.982	0.998	0.025	0.705	 very high detection rate and less false alarms
	TAG	0.891	1	0.142	0.298	 overfit on the training data and don't generalize well
	RTI+	0.790	1	0.292	0.171	
	Hidden Markov Model	0.608	0.640	0.085	0.288	

Anomaly detection performances
on HDFS dataset¹

Experiments

	Learner	AU-ROC	TPR	FPR	F1	
TA learners	TADAM	0.982	0.998	0.025	0.705	very high detection rate and less false alarms
	TAG	0.891	1	0.142	0.298	overfit on the training data and don't generalize well
	RTI+	0.790	1	0.292	0.171	not expressive enough
	Hidden Markov Model	0.608	0.640	0.085	0.288	

Anomaly detection performances
on HDFS dataset¹

Conclusions

Contributions:

- A compression-based (MDL) score to avoid overfitting
- An explicit modelization of the noise

Experiments show that TADAM

- is far more robust to noise
- learns smaller models
- has better performances on real-world classification and anomaly detection tasks

Thank you!

Contributions:

- A compression-based (MDL) score to avoid overfitting
- An explicit modelization of the noise



 Fos-R/TADAM

`pip install tadam-learner`

Experiments show that TADAM

- is far more robust to noise
- learns smaller models
- has better performances on real-world classification and anomaly detection tasks

See you at the poster session!

