Vibration Engineering Notes

Textbook: Engineering Vibration, 4th Edition, Daniel J. Inman

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Contents

Chapter 1 - Introduction to vibration and free response

Definition 1

Degree of freedom The degree of freedom of a system is the minimum number of displacement coordinates needed to represent the position of the systems mass at any instant of time.

Definition 2

Free response Free response refers to analysing the vibration of a system resulting from a non-zero inital displacement and/or velocity of the system with no external force or moment applied.

0.1 The spring-mass model

The fundamental kinematic quantities used to describe the motion of a particle are displacement, velocity, and acceleration vectors.

Definition 3

Kinematic Kinematic quantities are those that describe the motion of a particle without regard to the forces that cause the motion.

From physics, we know that: The motion of a mass with changing velocity is determined by the net force acting on the mass.

In fig. 1, the forces acting on the mass consist of the force of gravity pulling down (mg)

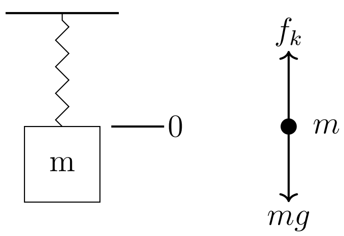

Figure 1. Single degree of freedom mass-spring system

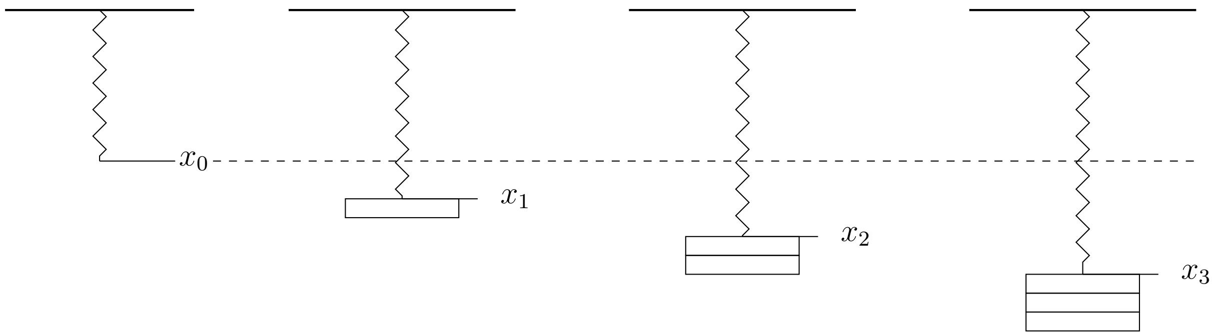

Figure 2. A schematic of a massless spring with no mass attached showing its static equilibrium position, followed by increments of increasing added mass illustrating the corresponding deflections.

and the *elastic-restoring* force of the spring pulling it back up (f_k)...

Definition 4

Constant of proportionality The slope of the straight line in the graph of force versus displacement of a spring.

The constant of proportionality can be easily determined for a spring using a simple experiment. Hanging a known mass on a spring and measuring the resulting displacement. This can be repeated for successivly hevier masses and a force displacement graph can be plotted.

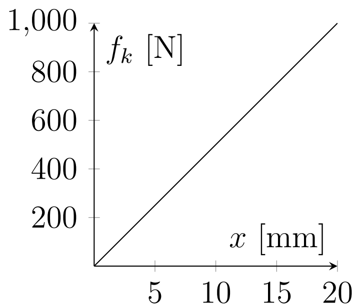

Figure 3. The static deflection curve for the spring in fig. 2

Chapter 2 - Response to harmonic excitation

Definition 5

Harmonic excitation Harmonic excitation refers to a sinusoidal external force of a single frequency applied to the system.

Definition 6

Resonance Resonance is the tendency of a system to absorb more energy when the driving frequency matches the systems natural frequency of vibration.

Harmonic excitations are a common source of external force applied to machines and structures. Rotating machines such as fans, electric motors, and reciprocating engines transmit a sinusoidally varying force to adjacent component's. In addition, the Fourier theorem indicates that many other forcing functions can be expressed as an infinite series of harmonic terms. Since the equations of motion considered here are linear, knowing the response to individual terms in the series allows the total repose to be represented as the sum of the response to the individual terms. This is the principle of superposition. In the way, knowing the response to a single harmonic input allows the calculation of the response to a variety of other input disturbances of periodic nature.

For now, lets consider the driving force, $F(t)$ to be of the form:

$$F(t) = F_0 \cos(\omega t). \tag{1}$$

Other forms could also have been chosen, but for now this will suffice.

Chapter 4 - Multiple-degree-of-freedom systems

More than one degree of freedom means more than one natural frequency. To keep record of each coordinate in the system, vectors are introduced and used along with matrices.

0.2 Two-degree-of-freedom model (undamped)

In moving from single-degree-of-freedom systems to two or more degrees of freedom, two important physical phenomena result.

1. The first important difference is that ta two-degree-of-freedom system will have two natural frequencies
2. The second important phenomenon is that of a mode shape, which is not present in single-degree-of-freedom systems. A mode shape is a vector that described the relative motion between the two masses or between tow degrees of freedom.

These important concepts of multiple natural frequencies and mode shapes are intimately tied to the mathematical concepts of eigenvalues and eigenvectors of computational matrix theory.

Figure 4 shows a simple two-degree-of-freedom system. The two masses are connected in series by two springs. Each mass only has a single degree of freedom because it can only move in a single direction. However, when considering the effect of the masses on one another, to completely described the system, two coordinates, namely x_1 and x_2 are needed. That is to say, the two coordinates depend on each other. The position of m_2 cannot be fully described without the position of m_1.

Figure 5 shows a single mass with two degrees of freedom. The mass can move in both the x_1 and x_2 directions. The mass is connected to a fixed wall by two springs.

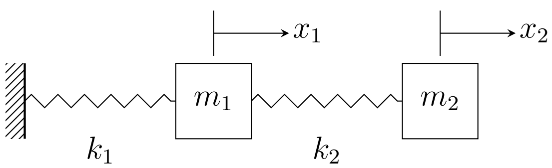

Figure 4. Single degree of freedom mass-spring system

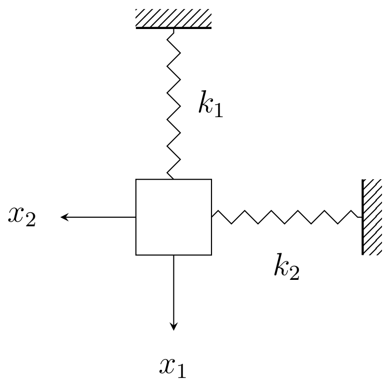

Figure 5. A single mass with two degrees of freedom, i.e. the mass moves along both the x_1 and x_2 directions

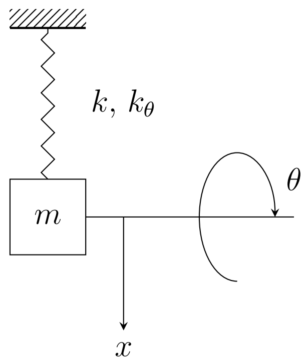

Figure 6. A single mass with one translation and one rotational degree of freedom.

Each of the figures above, namely figs. 4 to 6 shows a two-degree-of-freedom system. Each of these systems requires more than one coordinate to describe the vibration.

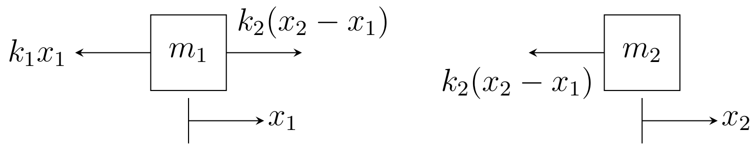

Figure 7. Free-body diagrams of each mass in the system of fig. 4 indicating the restoring force provided by the springs.

Summing forces on each mass in the horisontal direction yields:

$$\begin{aligned} m_1\ddot{x}_1 &= -k_1x_1 + k_2(x_2 - x_1) \\ m_2\ddot{x}_2 &= -k_2(x_2 - x_1), \end{aligned} \tag{2}$$

Rearranging the equations above yields:

$$\begin{aligned} m_1\ddot{x}_1 + (k_1 + k_2)x_1 - k_2x_2 &= 0 \\ m_2\ddot{x}_2 - k_2x_1 + k_2x_2 &= 0. \end{aligned} \tag{3}$$

Equation (3) consist of two coupled second-order ordinary differential equations with constant coefficients, each of which requires two initial conditions to solve. Hence these two coupled equations are subject to the four initial conditions:

$$\begin{aligned} x_1(0) &= x_{10} \\ \dot{x}_1(0) &= \dot{x}_{10} \\ x_2(0) &= x_{20} \\ \dot{x}_2(0) &= \dot{x}_{20}, \end{aligned} \tag{4}$$

where the constants x_{10}, $\dot{x}_{10}$, x_{20}, and $\dot{x}_{20}$ are the initial displacements and velocities of the two masses. These initial conditions are assumed to be known or given and provide the four constants of integration needed to solve the two second-order-differential equations for the free response of each mass. There are multiple different ways to solve the above equations, however, a convenient method of solving this system is to use vectors and matrices. The vector approach t solving this simple two-DOF problem is also readily extendable to systems with an arbitrary finite number of DOFs and is compatible with computer programming.

We define the vector $\mathbf{x}(t)$ to be the column vector of consisting of the two responses of interest:

$$\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}. \tag{5}$$