CS 189: Introduction to Machine Learning - Discussion 3

1. Matrix calculus

   Let A be a $n \times n$ matrix, and let $x \in \mathbb{R}^n$. Find $\nabla_x(x^\top Ax)$.

2. Logistic Posterior with different variances

   We have seen in class that Guassian class conditionals with the same variance lead to a logistic posterior. Now we will consider the case when the class conditionals are Guassian, but have different variance, i.e.

$$\begin{aligned} X|Y = i &\sim \mathcal{N}(\mu_i, \sigma_i^2), \quad \text{where } i \in \{0, 1\} \\ Y &\sim \text{Bernoulli}(\pi) \end{aligned}$$

   Show that the posterior distribution of the class label given X is also a logistic function, however with a quadratic argument in X. Assuming 0-1 loss, what will the decision boundary look like (i.e., describe what the posterior probability plot looks like)?

3. MLE of the Laplace Distribution

Let X have a Laplace distribution with density

$$f(x|\mu, b) = \frac{1}{2b} \exp\left(-\frac{|x-\mu|}{b}\right)$$

Suppose that n samples $x_1, \ldots, x_n$ are drawn independently according to $f(x|\mu, b)$.

(a) Find the maximum likelihood estimate of μ.

(b) Find the maximum likelihood estimate of b.

(c) Assume that μ is given. Show that b_{MLE} is an unbiased estimator (to show that the estimator is unbiased, show that $\text{E}[b_{\text{MLE}} - b] = 0$).