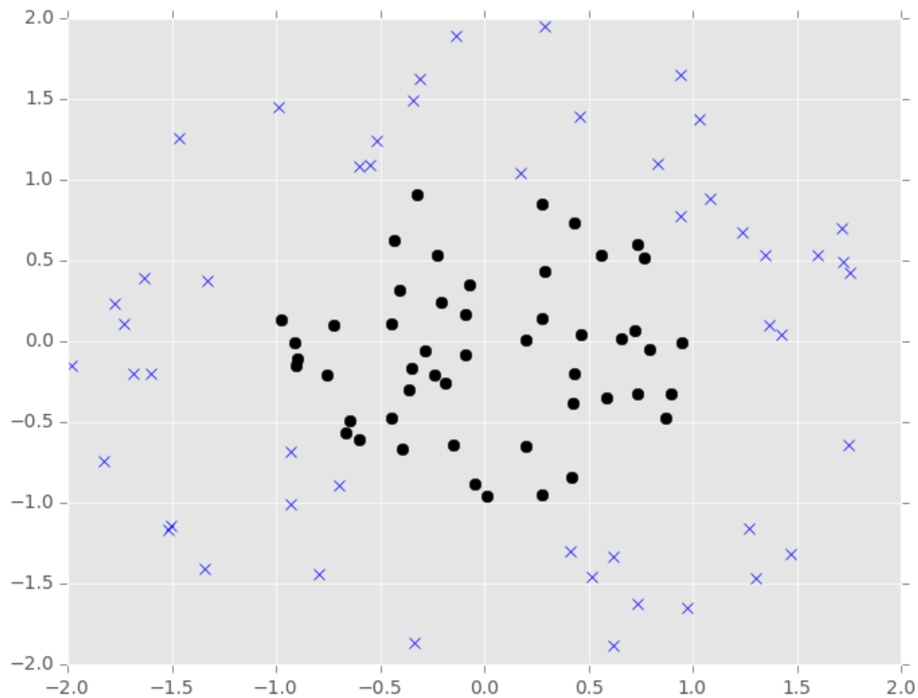
CS 189: Introduction to Machine Learning - Discussion 6

1. Circular Distributions

   Consider the following dataset where each point $x_n = (x_{1,n}, x_{2,n})$ is sampled *iid* and



   uniformly at random from two equiprobable (each equally likely) classes, a disk of radius 1 ($y_n = 1$) and a ring from 1 to 2 ($y_n = -1$).

   Qualitatively estimate and sketch the following quantities

   - The class conditional density $P(X \mid Y)$
   - The density of X
   - The conditional density of $P(Y \mid X)$
   - The Bayes Classifier
   - The Bayes Risk

   Now suppose you're given only one of the axes, namely $x_{1,n}$, re-estimate these quantities.

2. Multivariate Gaussian

   (a) **True/False** If X_1 and X_2 are both normally distributed and independent, then (X_1, X_2) must have multivariate normal distribution.

   (b) **True/False** If (X_1, X_2) has multivariate normal distribution, then X_1 and X_2 are independent.

   (c) **Affine Transformations** Suppose $\mathbf{X} = [X_1, X_2, \cdots, X_n]^T$ is a n-dimensional random vector which has multivariate Gaussian distribution. Given $\mathbf{X} \sim \mathcal{N}(\mu, \mathbf{\Sigma})$ and $\mathbf{y} = \mathbf{c} + \mathbf{BX}$ is an affine transformation of $\mathbf{X}$, where c is an $M \times 1$ vector of constants and $\mathbf{B}$ is a constant $M \times N$ matrix, what is the expectation and variance of $\mathbf{y}$?

3. Bayesian Decision Theory: Case Study - We're going fishing!

We want to design an automated fishing system that captures fish, classifies them, and sends them off to two different companies, Salmonites, Inc., and Seabass, Inc. For some reason we only ever catch salmon and seabass. Salmonites, Inc. wants salmon, and Seabass, Inc. wants seabass. Given only the weights of the fish we catch, we want to figure out what type of fish it is using machine learning!

Let us assume that the weight of both seabass and salmon are both normally distributed (univariate Gaussian), given by the p.d.f.:

$$P(x|\mu_i, \sigma_i) = \frac{1}{\sigma_i\sqrt{2\pi}} e^{-\frac{(x-\mu_i)^2}{2\sigma_i^2}}$$

We are given this data:

Data for salmon: $\{3, 4, 5, 6, 7\}$

Data for seabass: $\{5, 6, 7, 8, 9, 7+\sqrt{2}, 7-\sqrt{2}\}$

When we classify seabass incorrectly, it gets sent to Salmonites, Inc. who won't pay us for the wrong fish and sells it themselves. When we classify salmon incorrectly, it gets sent to SeaBass, Inc., who is nice and returns our fish. This situation gives rise to this loss matrix:

Predicted:

Truth:		**salmon**	**seabass**
	salmon	0	1
	seabass	2	0

a) First, compute the ML estimates for μ and σ for the univariate Gaussian in both the seabass and the salmon case. Also compute the empirical estimates of the priors. (Salmon = 1, Seabass = 2)

$\hat{\mu_1} =$ $\quad\quad\quad\quad$ $\hat{\mu_2} =$

$\hat{\sigma_1} =$ $\quad\quad\quad\quad$ $\hat{\sigma_2} =$

$\hat{\pi}_1 =$ $\quad\quad\quad\quad$ $\hat{\pi}_2 =$

What is significant about $\hat{\sigma_1}$ and $\hat{\sigma_2}$?

b) Next, find the decision rule when assuming a 0-1 loss function. Recall that a decision rule for the 0-1 loss function will minimize the probability of error.

c) Now, find the decision rule using the loss matrix above. Recall that a decision rule, in general, minimizes the risk, or expected loss.