II. TRANSFER OF RADIATION IN SPECTRAL LINES WIDENING OF LINES

CRITICAL REMARKS ON THE THEORY OF PRESSURE BROADENING OF SPECTRAL LINES

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There are two different approaches to the theory of pressure broadening of spectral lines, the statistical theory and the theory of impact broadening or collision damping. In the statistical theory one asks for the probability distribution of the microfield due to the perturbing ions. If the field of the perturbing ion is a Coulomb field, one gets — at least in a zero approximation — the well-known HOLTSMARK distribution, which varies as

$$W(F) \sim F^{-5/2}, \tag{1}$$

for large fields F. As HOLTSMARK pointed out himself, one should expect for the probability distribution of a vector in a homogeneous, isotropic medium, just as in the case of the velocity distribution, a Maxwell-GAUSS distribution instead. The reason one does not get a GAUSSIAN distribution lies in the fact that in the given problem the mean square of the field

$$\overline{F^2} = \int_0^\infty N . 4\pi r^2 dr . \frac{e^2}{r^4} \tag{2}$$

(N density of perturbing ions, r distance from the emitting atom) does not exist. And indeed, according to the HOLTSMARK distribution one gets an infinite mean square field :

$$\overline{F^2} = \int_0^\infty F^2\, W(F)\, dF \sim \int_0^\infty F^2 . F^{-5/2}\, dF \to \infty \tag{3}$$

This is a great difficulty in the HOLTSMARK theory as has been pointed out by CHANDRASEKHAR already 20 years ago. The mean square field is a measure of the energy density, and this density as a physical quantity cannot be infinite in any reasonable problem.

Now, how can this difficulty be overcome. The statistical theory as treated by HOLTSMARK is a static theory. The movements of the perturbing ions are neglected. If we take account of the always present movements of the ions, the mean square field $\overline{F^2}$ becomes finite. This may be performed for instance by introducing a sphere around the emitting atom. The fields of the particles outside the sphere change slowly so that they may be treated as being at rest. The fields of the particles inside the sphere change too rapidly as to give a contribution to the square field. In a very rough manner the radius of the sphere may be chosen as

$$r_0 = v\tau, \tag{4}$$

where v is the velocity and τ a characteristic time, say the duration of emission of the emitting atom. Then, the integration in (2) is only to be extended from $v\tau$ till ∞, and $\overline{F^2}$ becomes finite, even allowing for the velocity varying according to the MAXWELLIAN distribution.

It can be shown quite generally that if the mean square field is finite the probability function of the field must be GAUSSIAN. The problem is now shifted to the exact calculation of the mean square field which enters as the only free parameter in the GAUSSIAN distribution. If we assume the mean square field as being of the order of the square of the HOLTSMARK normal field F_0, we find the maximum of the distribution function in approximately the same position as before. But there is one striking difference : the wings of the spectral-lines vary, if we apply the HOLTSMARK theory to the linear STARK effect, as $\Delta\lambda^{-5/2}$ whereas according to the GAUSSIAN distribution as $\exp(-\alpha\Delta\lambda^2)$. Since the observed lines do not show this steep drop, we must conclude that statistical broadening in the wings is unimportant, and that the observed broadening must be due alone to the other, above mentioned factor, the impact broadening.

In the impact broadening picture an atom radiating a wave train suffers an encounter with another particle which quenches the wave or changes its phase appreciably. Whatever may be the statistics of these processes, the resulting absorption coefficient generally will be of the form of the socalled dispersion formula which varies as $\Delta\lambda^{-2}$ in the wings. The only parameter, as in the case of the statistical broadening, is the damping constant γ, and the task of any impact broadening theory is to compute γ.

If we turn again to the case of the linear STARK effect, there may be mentioned several theories. Among the most used theories is the theory of LINDHOLM. There are two major difficulties in the theory of LINDHOLM. First, there has to be introduced an arbitrary parameter in order to get a finite γ. Second, γ varies as $1/v$, which means that the lines would become wider if the temperature of the gas would be less, whereas one would expect the lines to become wider if the temperature would be higher, because the number of collisions increases with the velocity resp. the temperature. One of the latest impact broadening theories is the theory of

KOLB, GRIEM and SHEN, based on quantum mechanical calculations. Since in its simplest form the collision between a perturbing particle and an emitting atom is a 3 body-problem, the number of approximations inevitably introduced is not small, and there remains the difficult problem of estimating the possible errors.

In view of the differences between the results of the various theories, it seems to us that one should be happy if one had a γ being correct to a factor of 2. Furthermore, we believe that there are other possibilities of impact broadening theories not yet explored, one we are persuing presently.

Discussion

Eleonore TREFFTZ. — We (*) were quite happy with the KOLB-GRIEM considerations. It matches the contours of He-lines, split by STARK effect, over the whole wavelength region. Those were laboratory lines.

HUNGER. — The theory of GRIEM, KOLB and SHEN consists of 2 parts : the statistical part and the impact broadening part. If the statistical part is not right, because they used the corrected HOLTSMARK theory, it affects only the inner parts of the lines. In the wings, the impact broadening becomes dominant with increasing wavelength.

DE JAGER. — May I try to summarize this dispute ? You both agree that KOLB-GRIEM-SHEN theory is correct for $\Delta\lambda >$ say I Å, whereas Dr. HUNGER believes that in the line core the theory is not yet wholly certain.

Ten BRUGGENCATE. — We measured at Göttingen with great accuracy the wings of the Balmer lines Hα — Hδ. We found a very good agreement with the KOLB theory, especially that electron impact broadening is important.

FORBES. — Are line shifts to be expected either on the basis of KOLB's theory or from your own computations ? The LINDHOLM theory predicts a shift as well as a broadening.

HUNGER. — The measurement of the line-shift is so difficult, that one still cannot say with certainty if the line-shift predicted by LINDHOLM is real or not.

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