PGE 383 Lecture
Uncertainty Part II

Lecture outline . . .

- **What is Uncertainty?**
- **Types of Uncertainty**
- **Calculating Uncertainty**
- **Uncertainty Workflows**

Michael Pyrcz, Professor, University of Texas at Austin

What is Uncertainty?

What is Uncertainty?

Uncertainty is not an intrinsic property of the subsurface.

Uncertainty is a function of our ignorance.

Uncertainty is a model. We use the term “uncertainty model”.

Given the open earth systems we deal with, there is no objectively correct uncertainty model.

Uncertainty is represented by the dispersion / spread of a CDF representing a measure over a volume of the subsurface.

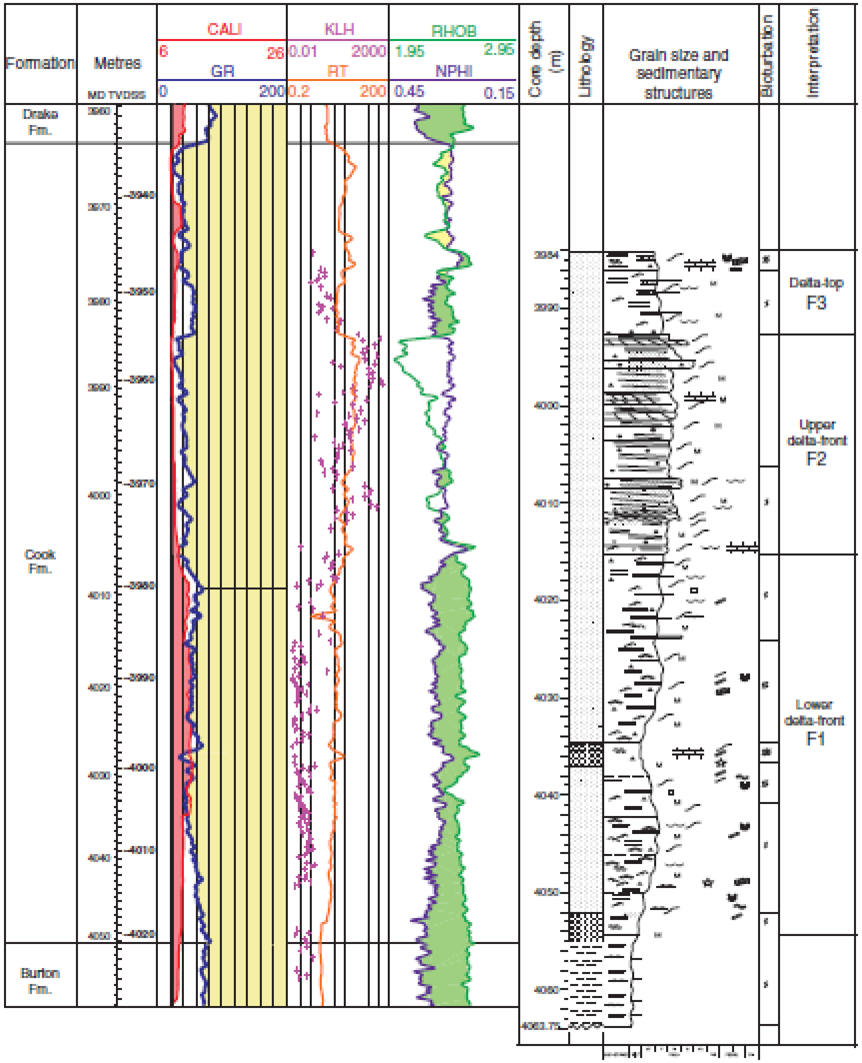
How Do We Represent Uncertainty?

Variance	$\text{Var}(Z) = \int_{-\infty}^{\infty} (z-m)^2 f(z)dz$	**Expected squared difference from mean.**
Dispersion Variance	$D^2(v,V) = \overline{\gamma}_{V,V} - \overline{\gamma}_{v,v}$	**Generalized variance accounting scale and heterogeneity.**
Entropy	$H(Z) = -\sum_{p}^{n} P(Z_i) \cdot ln\, P(Z_i)$	**Measure of uncertainty for categorical Variables.**
CDF	$F_Z(z)$	**The distribution, parametric or non-parametric, a list of samples from the distribution.**

Types of Uncertainty

Measurement / Interpretation Error.

- Formation evaluation – tool tolerance, calibration error, approximations / assumptions
- Interpreter experience and prior model / assumptions
- How to integrate it?
  - Multiple data realizations in design of experiments

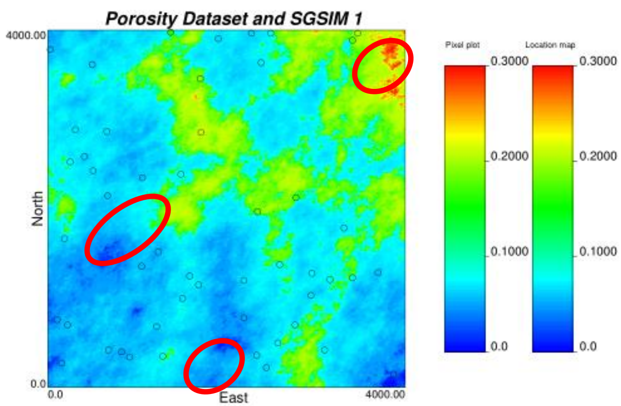


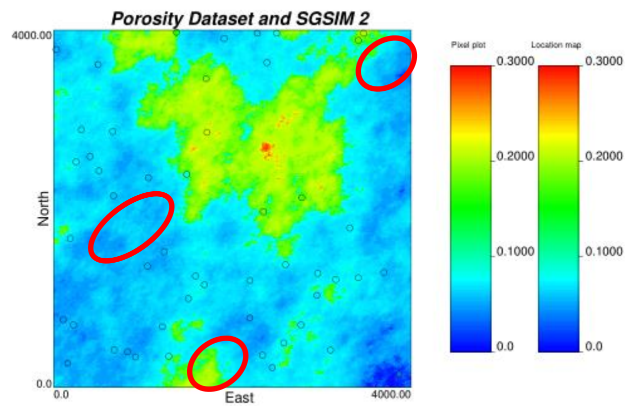
Cook Formation, Shallow Marine Sandstone form North Sea from Folkestad et al., (2012)

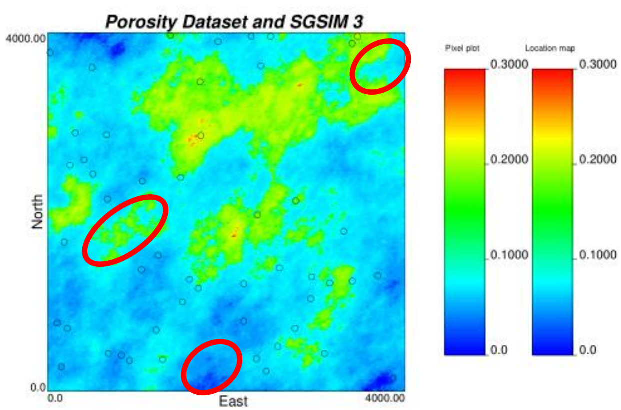
Types of Uncertainty

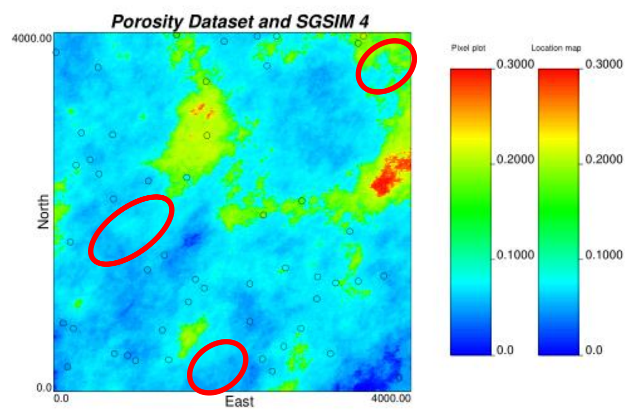
Interpolation Uncertainty

- Uncertainty due to spatial offset from sampled locations
- Integrate through multiple local realizations and scenarios





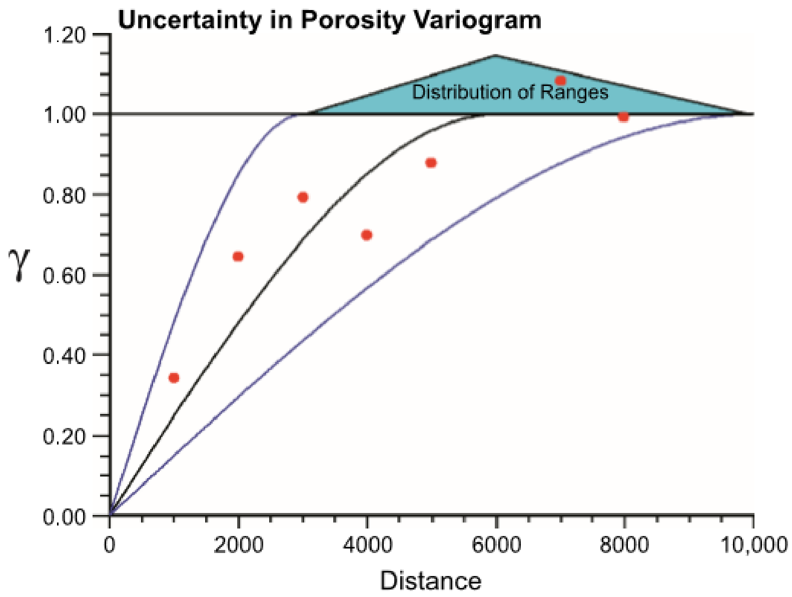




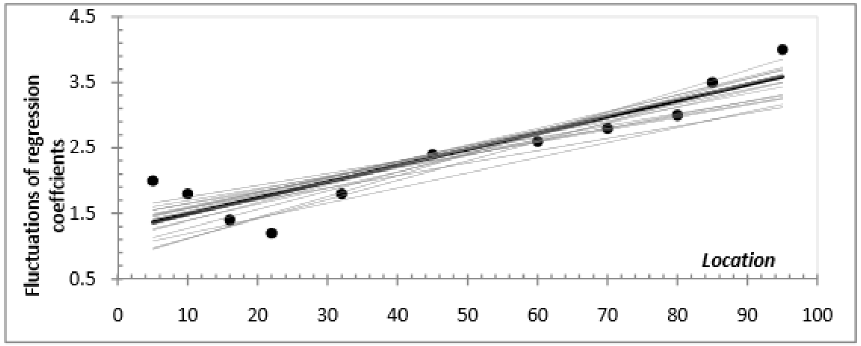
Types of Uncertainty

Parameter Uncertainty

- Uncertainty in the model choices
- Formulate distribution scenarios (could bootstrap for parameter realizations)



Distribution of Variogram Ranges (Pyrcz et al., 2006)

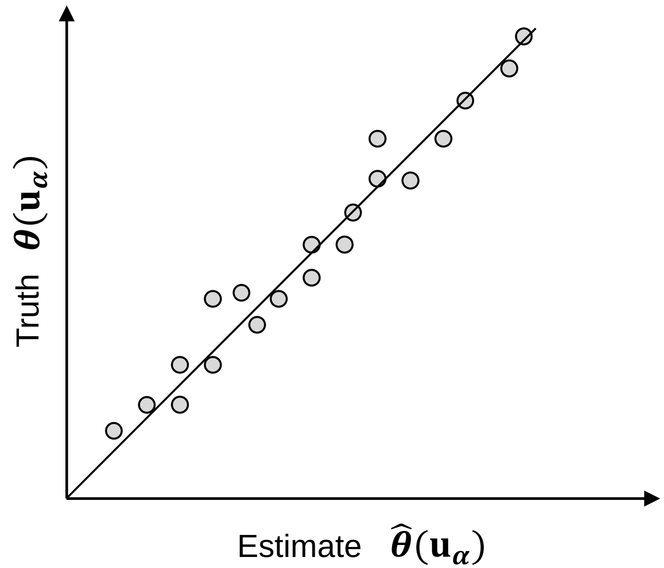


Trend Uncertainty (Villalba and Deutsch., 2010)

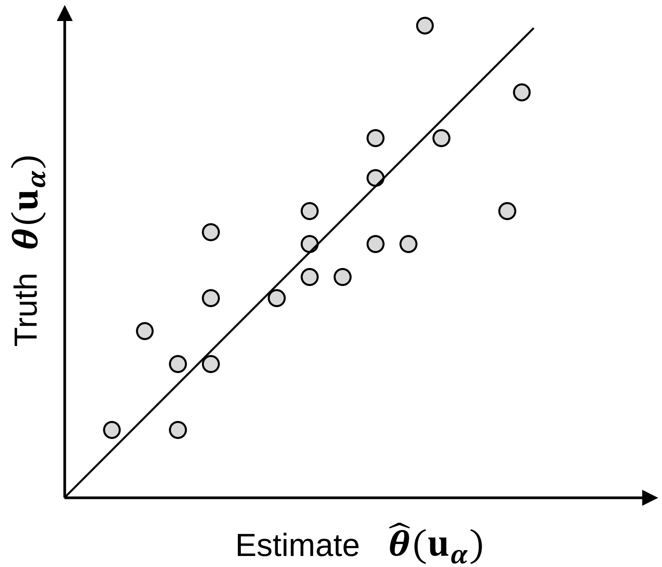
Types of Uncertainty

Model Error

- Errors in the prediction model are caused by data noise, missing information, poor model choice, etc.



Withheld testing data vs. estimates.

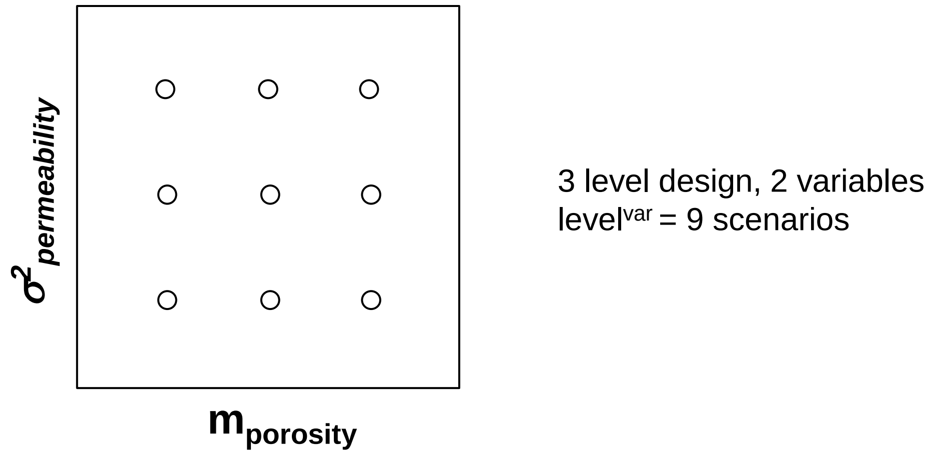


Withheld testing data vs. estimates.

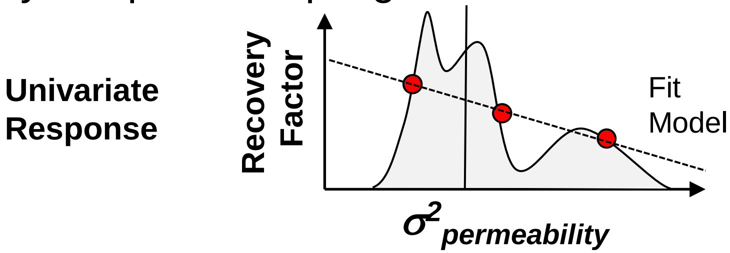
How is Uncertainty Calculated?

Uncertainty Space is Often Vast! We Do Out Best!

- Consider typical design of experiments over multiple features.



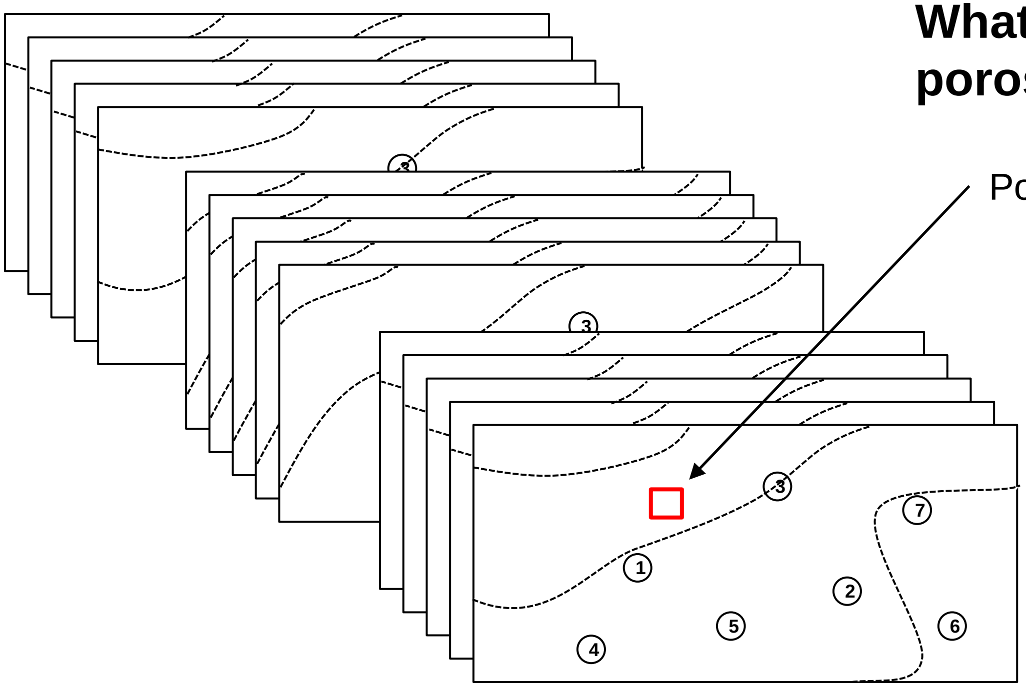
- Consider there are typically are around 10 or more uncertain variables.
  - 3^{10} = 59,049 scenarios x 10 realizations of each scenario = 590,490 models
  - Variable screening is important!
  - 3 level may still poor sampling



How is Uncertainty Calculated?

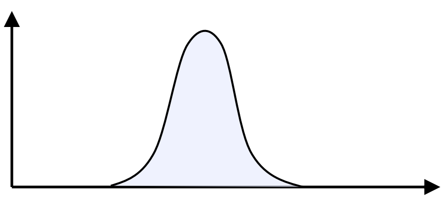
We have represented the “uncertainty model” through scenarios and realizations:

- We can ask any question of the model by considering all scenarios and realizations jointly.



What is the uncertainty in porosity at location u_1?

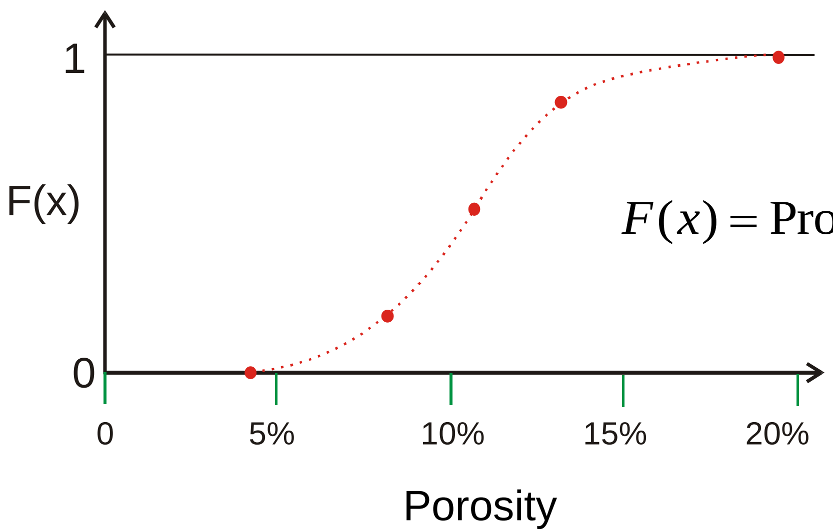
Porosity($\mathbf{u}_1$) = F(Φ; $\mathbf{u}_1$)



How is Uncertainty Calculated?

Frequentist Approach

- Sample and pool data samples and formulate a CDF (assumption of stationarity)
- Local data, analog data, calibrated data etc.



$$F(x) = \mathrm{Prob}\{X \leq x\}$$

How is Uncertainty Calculated?

Bayesian Approach

- Formulate prior belief
- Calibrate new data, information into a likelihood
- Update to calculate a posterior

$$\text{Prob}\,\{\,A \mid B\,\} = \frac{\text{Prob}\,\{\,B \mid A\,\} \times \text{Prob}\,\{A\}}{\text{Prob}\,\{\,B\,\}}$$

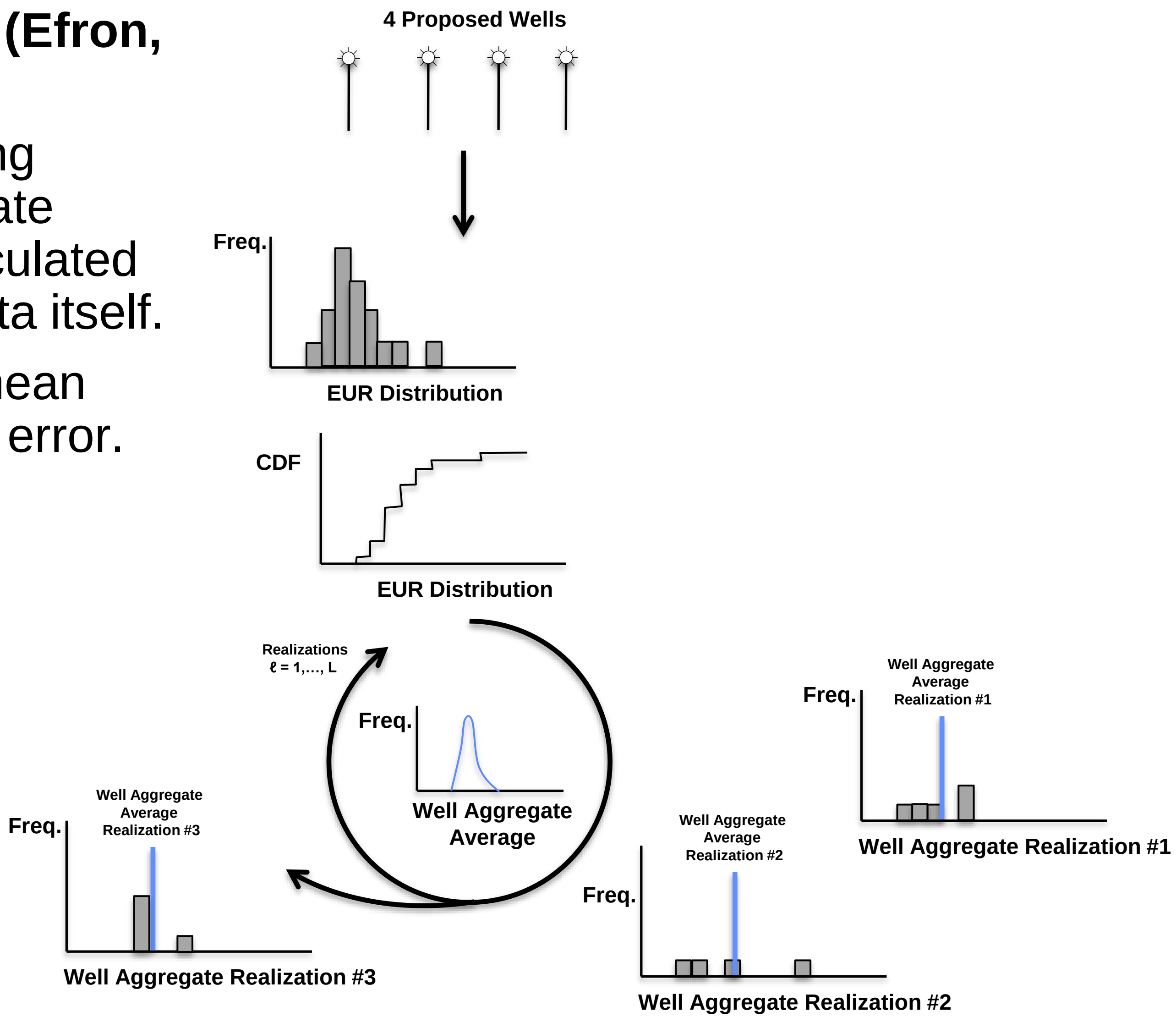
$$\text{Posterior} = \frac{\text{Likelihood} \times \text{Prior}}{\text{Evidence}}$$

How is Uncertainty Calculated?

Bootstrap Approach (Efron, 1982)

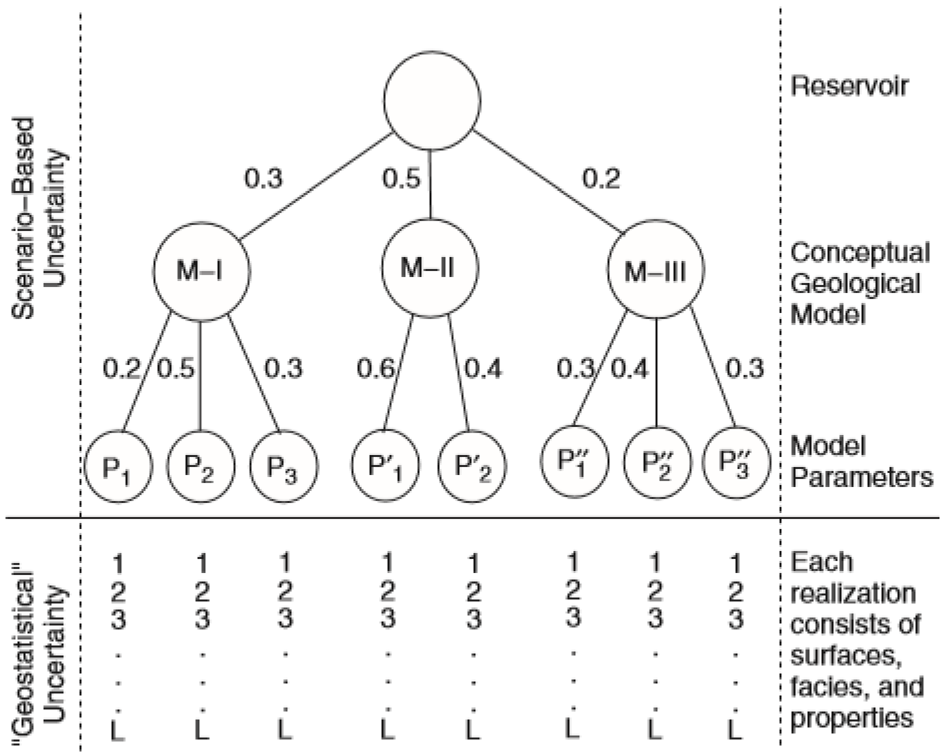
- Statistical resampling procedure to calculate uncertainty in a calculated statistic from the data itself.
- For uncertainty in mean solution is standard error.

$$\sigma_x^2 = \frac{\sigma_s^2}{n}$$



Uncertainty Best Practice

1. Seek out all significant uncertainty sources
2. Assign scenarios when needed with associated de-biased probabilities
3. Include data realizations if needed.
4. Also include stochastic realizations to account for spatial uncertainty.
5. Need enough models the uncertainty space is vast.
6. Document / defend choices.



Uncertainty exploration scheme (Pyrcz and Deutsch, 2014)