PGE 337 Data Analytics and Geostatistics

Lecture 12: Spatial Estimation

Lecture outline . . .

- Trend Modeling
- Kriging

Michael Pyrcz, The University of Texas at Austin

Introduction

General Concepts

Univariate

Bivariate

Spatial

- Calculation
- Variogram Modeling
- **Kriging**
- Simulation

Time Series

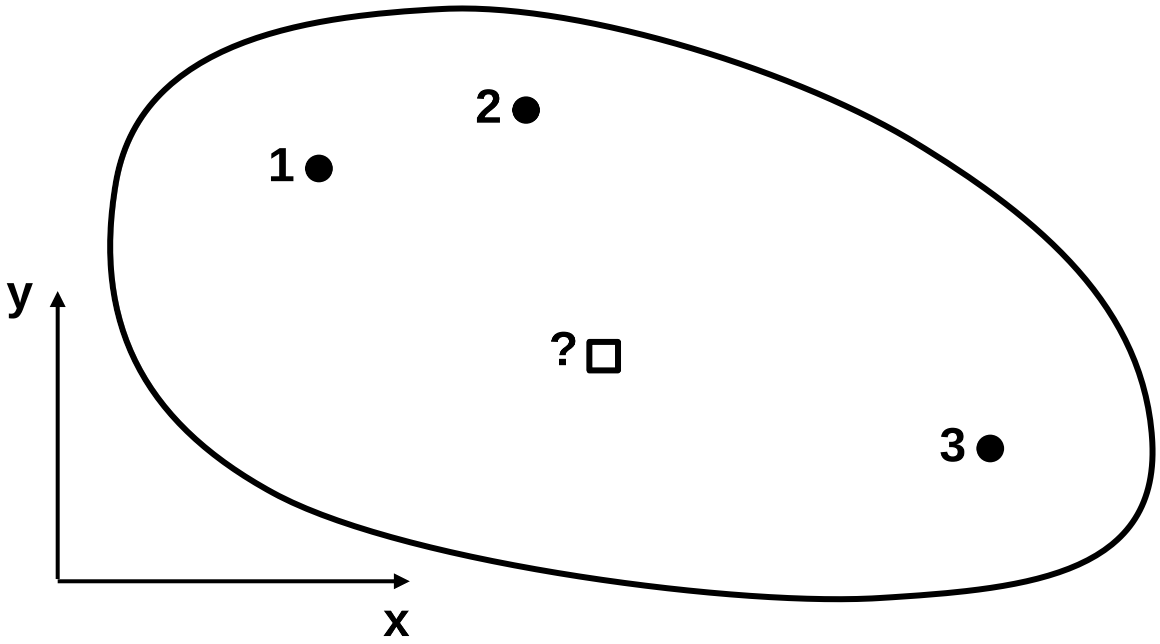
Machine Learning

Uncertainty Analysis

Motivation

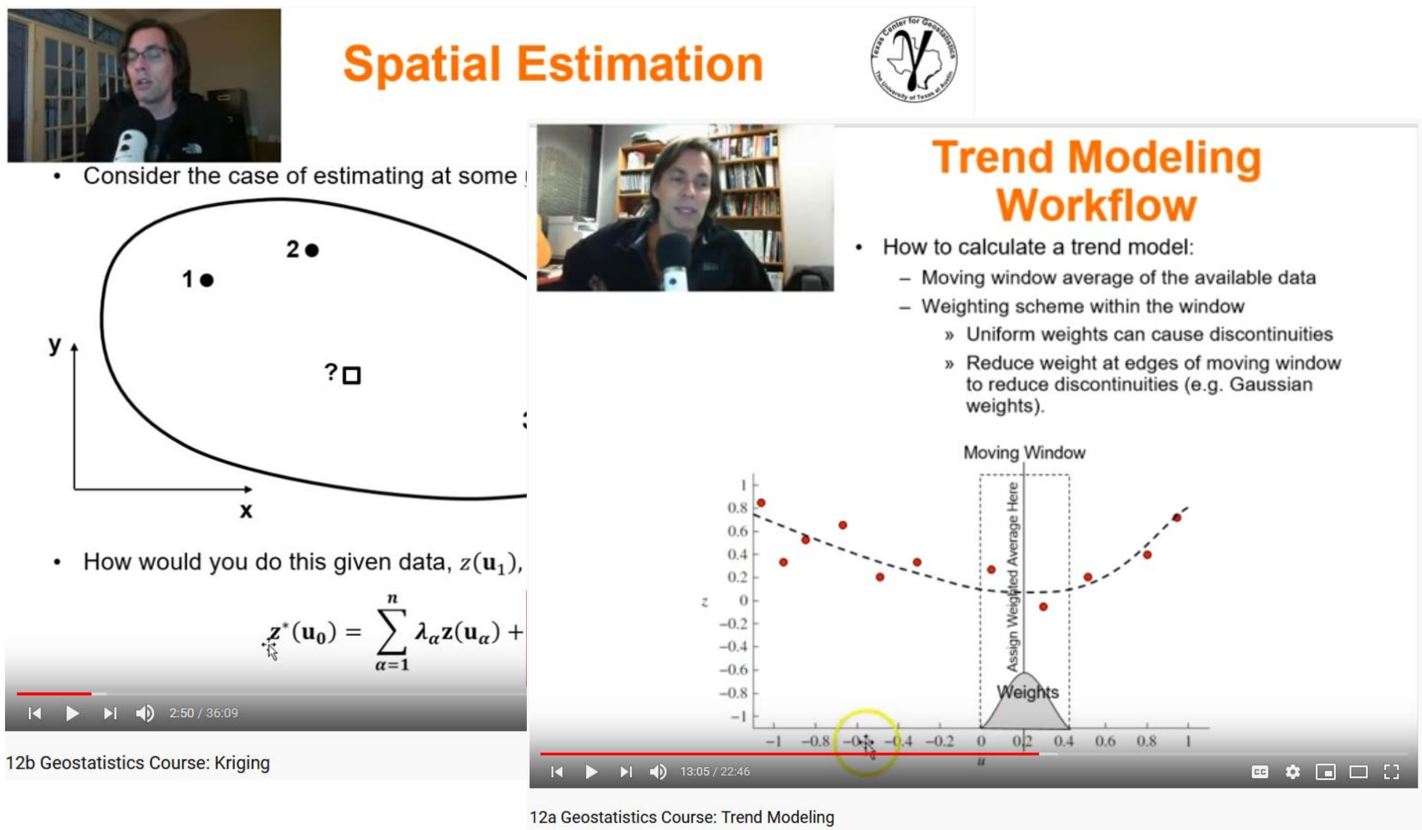
We need to make predictions away from sampled locations.

- To determine where to drill next, and to determine how to best develop a reservoir.



Resources

Reminder on recorded lectures.

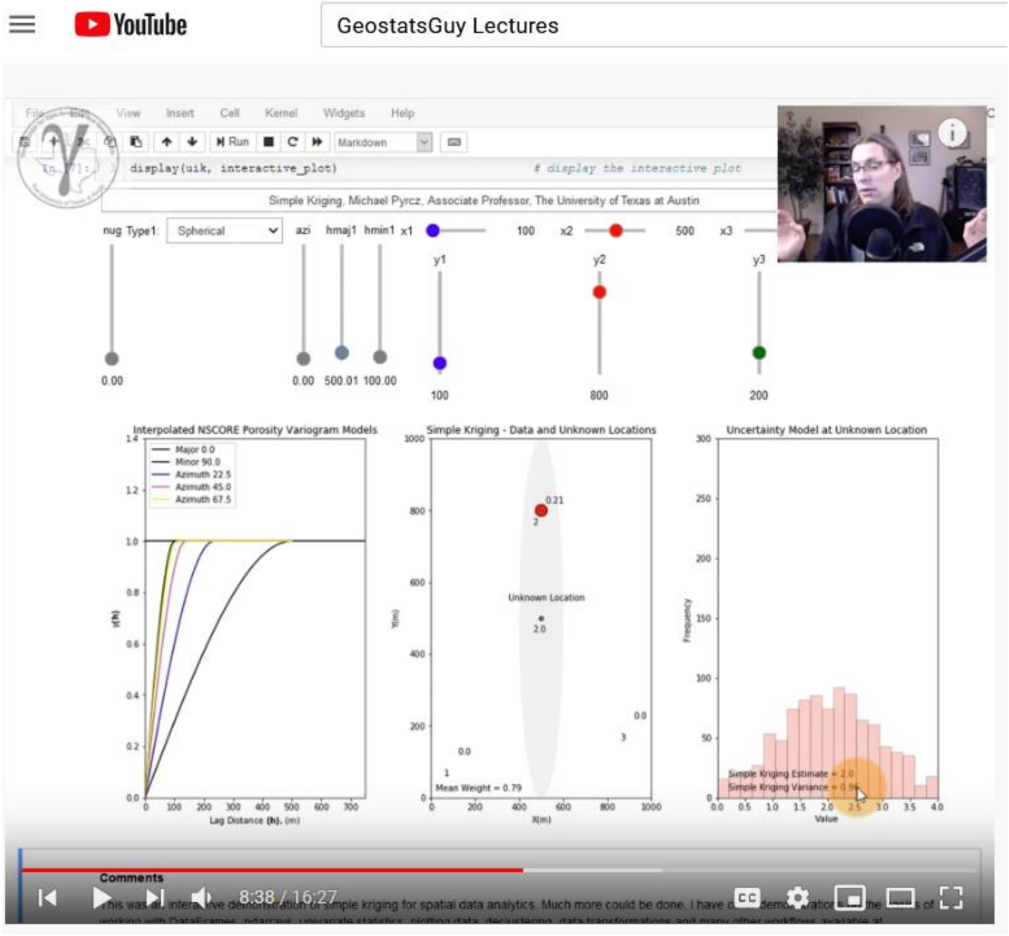


12b Geostatistics Course: Kriging

12a Geostatistics Course: Trend Modeling

Resources

- Added a recorded walk-though of the Interactive Kriging Demo in Python.



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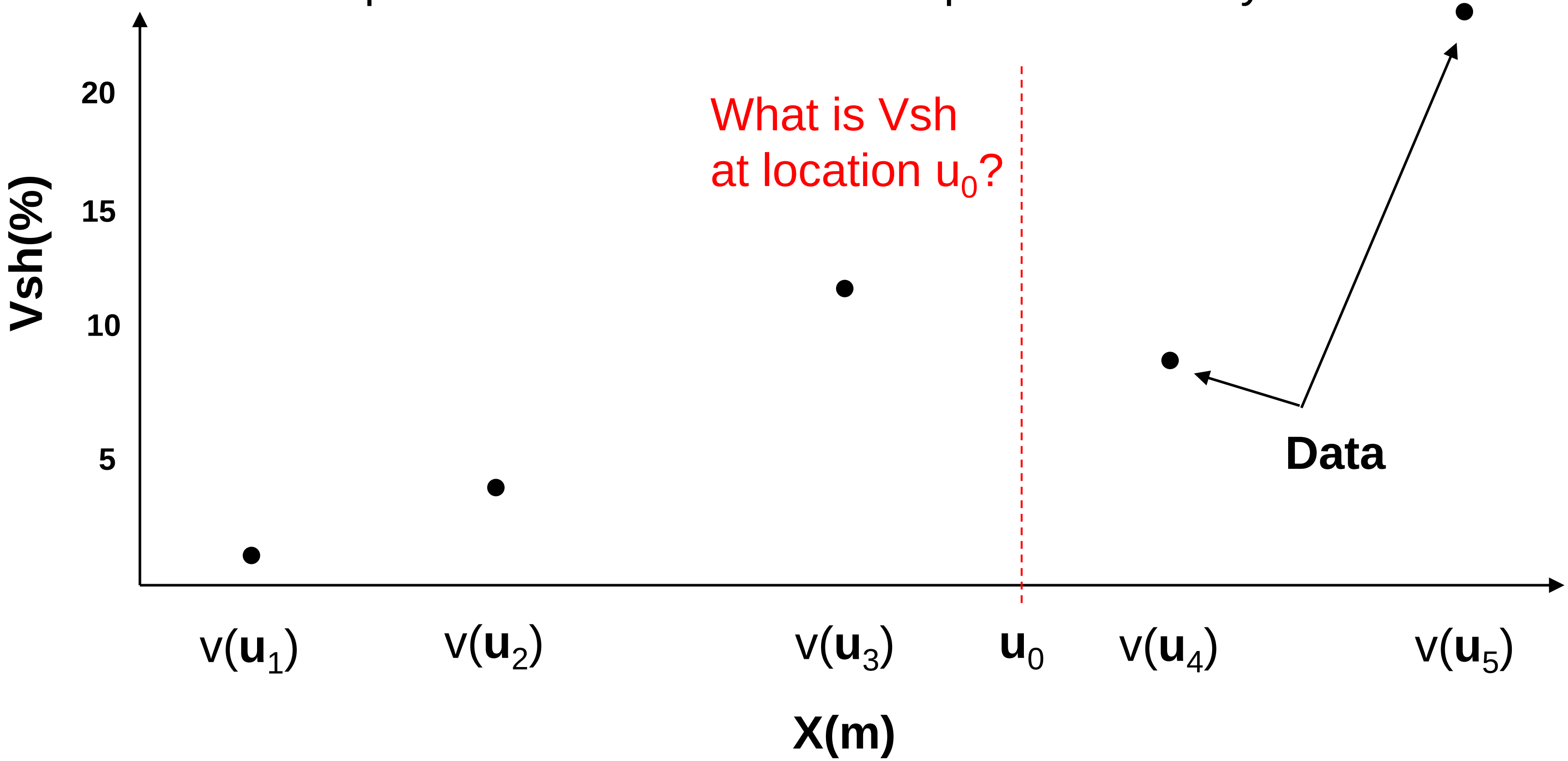
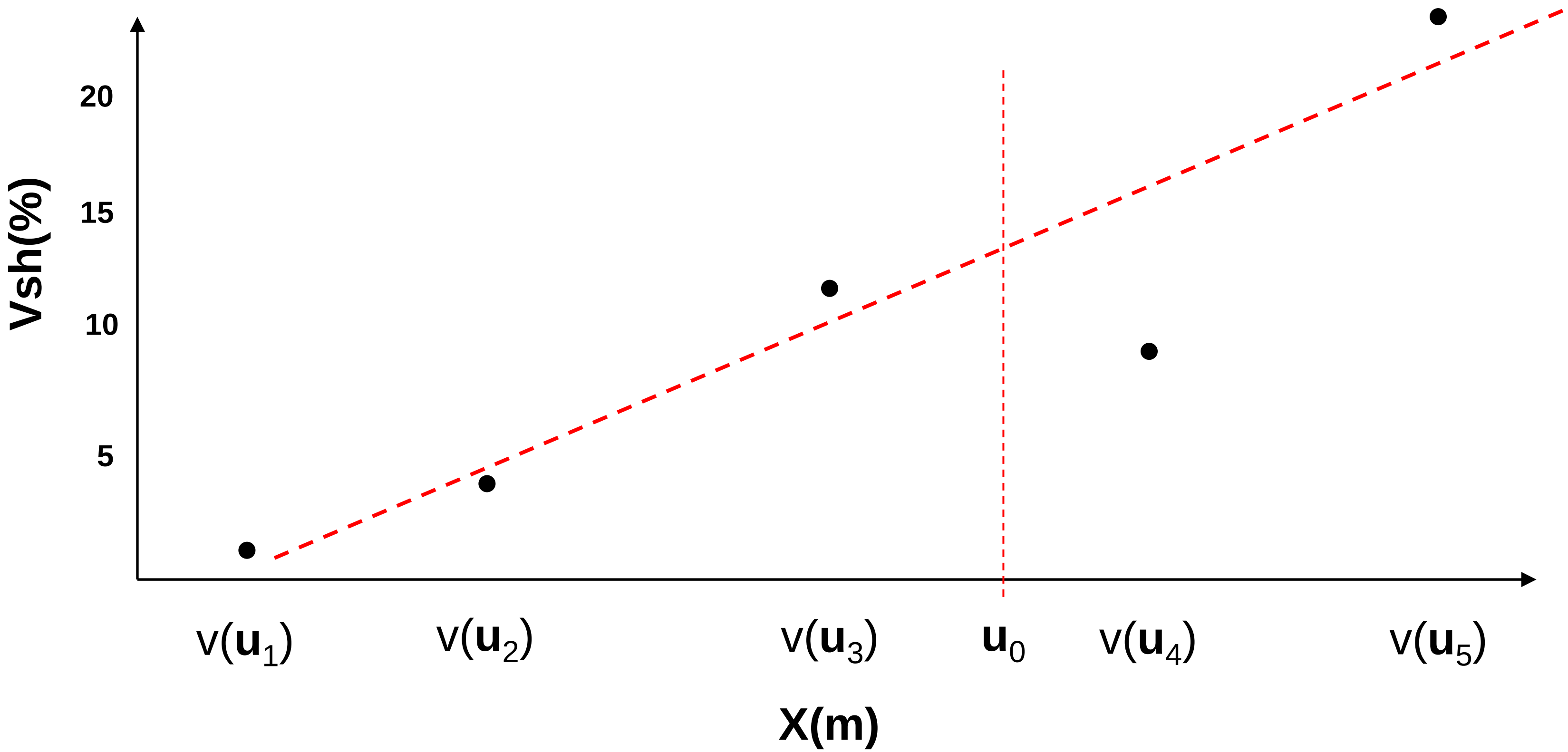
Trend and Residual Method

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - In the presence of significant nonstationarity we would not rely 100% for spatial estimation on data + spatial continuity model



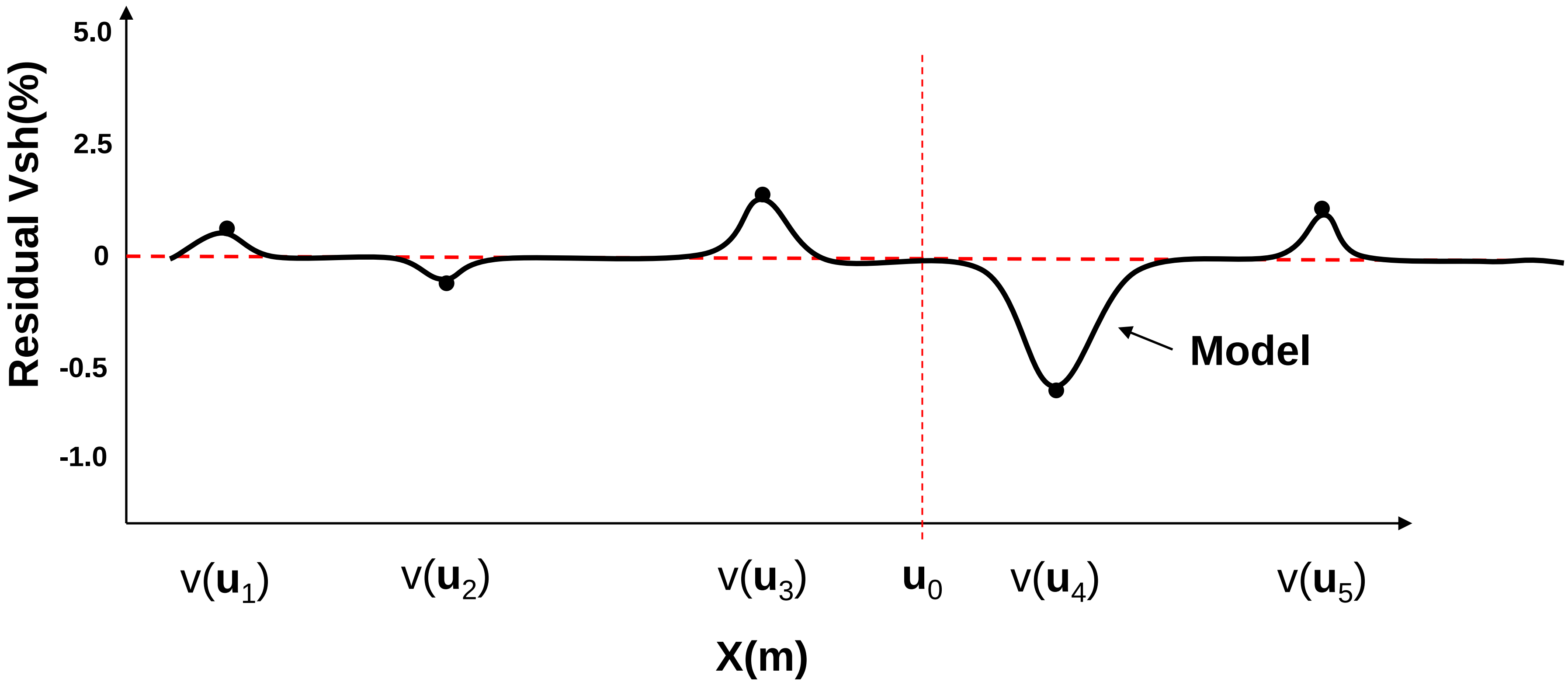
Trend and Residual Method

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - If we observe a trend, we should model the trend.



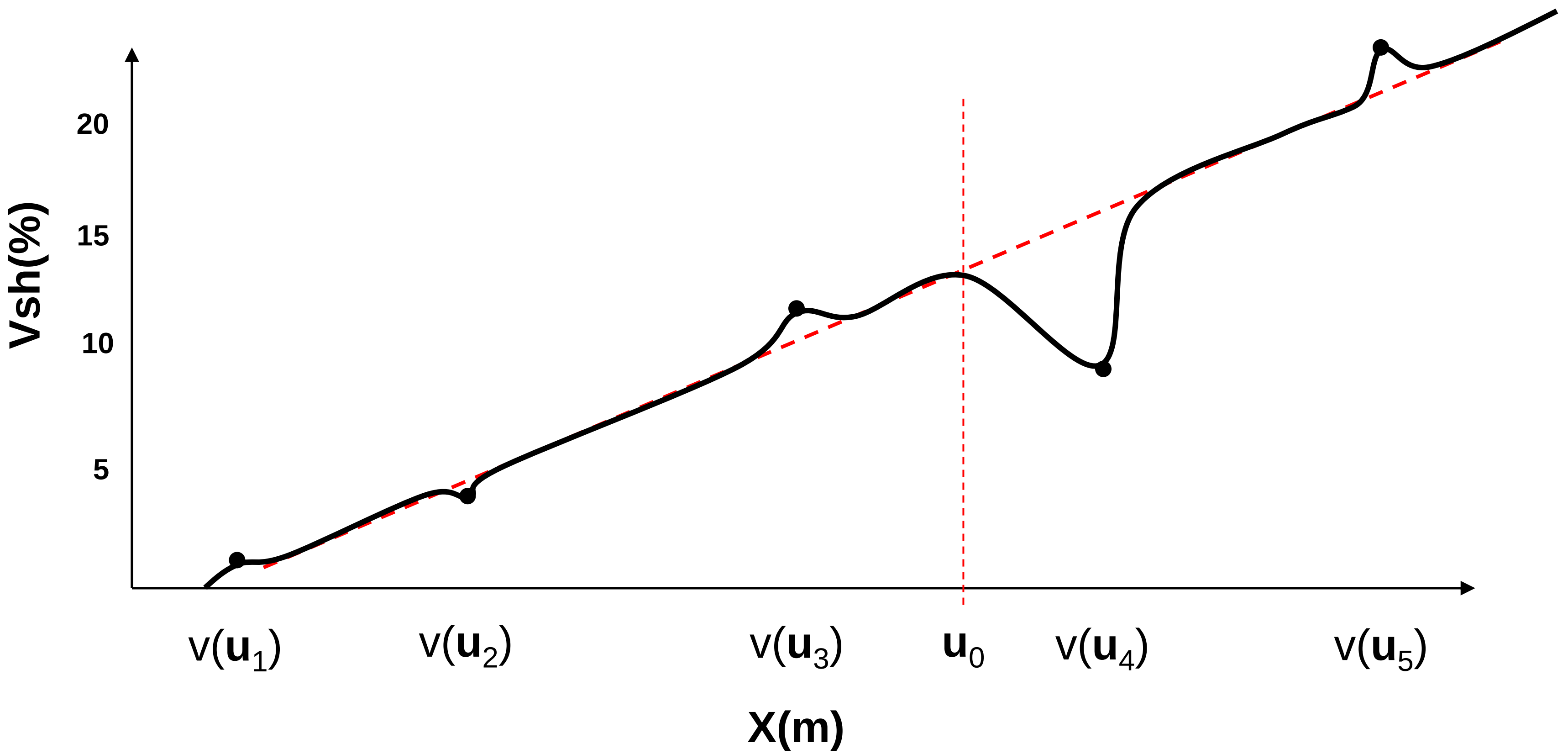
Trend and Residual Method

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - Then model the residuals stochastically.



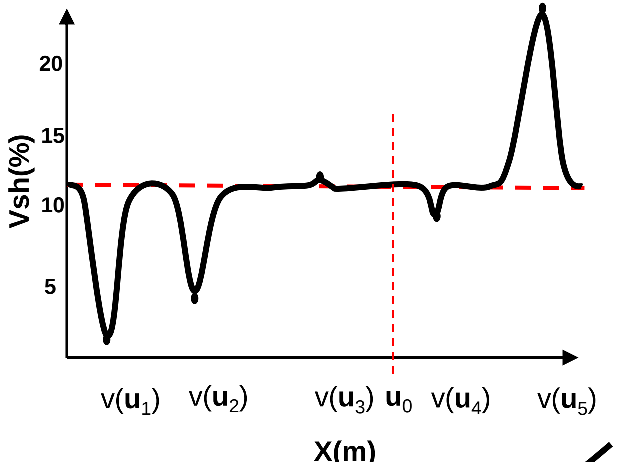
Trend and Residual Method

- Geostatistical spatial estimation methods will make an assumption concerning stationarity
  - Add the trend back to the modelled residuals

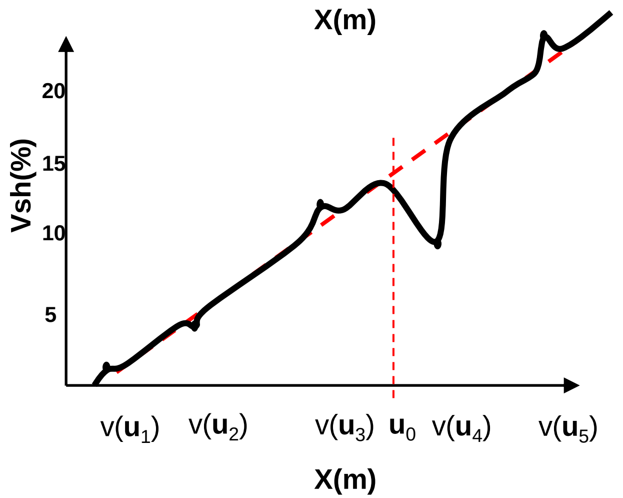


Trend and Residual Method

- How bad could it be if we did not model a trend?
- Geostatistical estimation would assume stationarity* and away from data we would estimate with the global mean (simple kriging)!



Model with stationary mean + data.

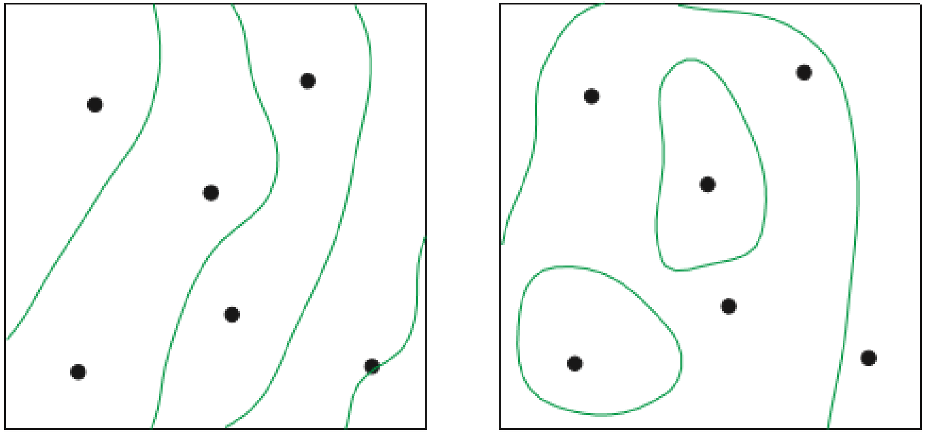


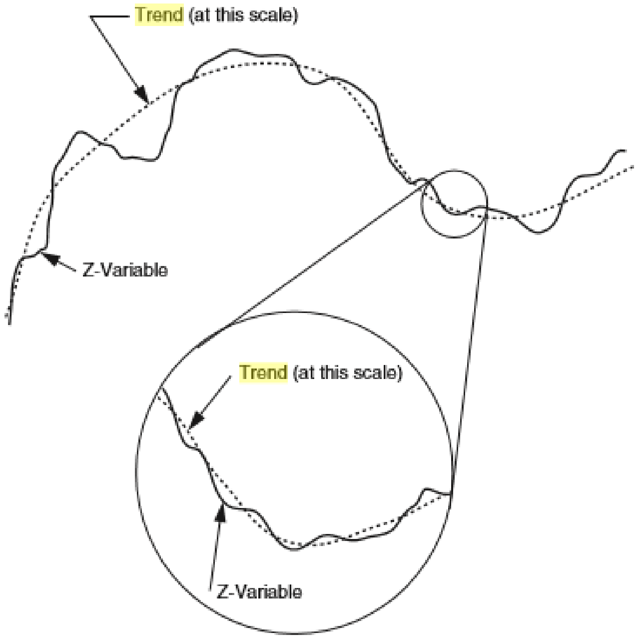
Model with mean trend model and residual + data.

***stationarity decision depends on type of method.**

Trend Modeling Method

- Trend Modeling
  – We must identify and model trends / nonstationarities





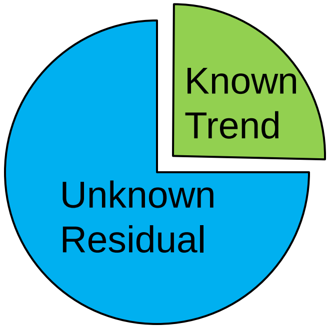
- While we discuss data-driven trend modeling here any **trend modeling should include data integration** over the entire asset team
  - Geology
  - Geophysics
  - Petrophysics
  - Reservoir Engineering

Images from Pyrcz and Deutsch (2014)

Trend Modeling Method

- Any variance in the trend is removed from the residual:

$$\boldsymbol{\sigma_X^2 = \sigma_{X_t}^2 + \sigma_{X_r}^2 + 2C_{X_t,X_r}}$$



  - if the $\boldsymbol{X_t \perp\!\!\!\perp X_r}$, $\boldsymbol{C_{X_t,X_r}} = 0$

$$\boldsymbol{\sigma_{X_r}^2 = \sigma_X^2 - \sigma_{X_t}^2}$$

– So if $\boldsymbol{\sigma_X^2}$ is the total variance (variability), and $\boldsymbol{\sigma_{X_t}^2}$ is the variability that is deterministically modelled, treated as known, and $\boldsymbol{\sigma_{X_r}^2}$ is the component of the variability that is treated as unknown.

– Result: the more variability explained by the trend the less variability that remains as uncertain.

Additivity of Variance for Decomposing Trend and Residual

Can we partition variance of random variable Z between trend (X) and residual (Y)?

- Start with the variance of Z: $\sigma_Z^2 = E(Z^2) - [E(Z)]^2$

- Substitute: $Z = \mathrm{X} + \mathrm{Y}$

$$\sigma_{X+Y}^2 = E\left((X+Y)^2\right) - [E(X+Y)]^2$$

$$\sigma_{X+Y}^2 = E\left(X^2 + 2XY + Y^2\right) - [E(X) + E(Y)]^2$$

$$\sigma_{X+Y}^2 = E\left(X^2\right) + 2E(XY) + E\left(Y^2\right) - \left(E(X)^2 + 2E(X)E(Y) + E(Y)^2\right)$$

$$\sigma_{X+Y}^2 = \underbrace{\boxed{E\left(X^2\right) - E(X)^2}}_{\sigma_X^2} + \underbrace{\boxed{E\left(Y^2\right) - E(Y)^2}}_{\sigma_Y^2} + 2\underbrace{\boxed{\left(E(XY) - E(X)E(Y)\right)}}_{C_{XY}(0)}$$

- Note covariance: $C_{XY} = E(XY) - E(X)E(Y)$

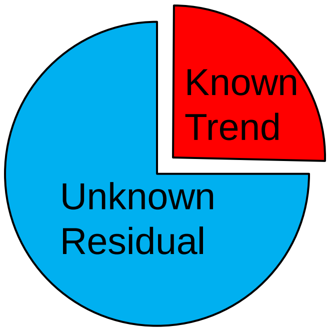
$$\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2 + 2C_{XY}(0) \quad \triangleleft$$ **Additivity of variance**

- If the $X \perp Y$, $C_{XY}(0) = 0$, then $\sigma_{X+Y}^2 = \sigma_X^2 + \sigma_Y^2$ $\triangleleft$ **In practice**

Definition
Deterministic Model

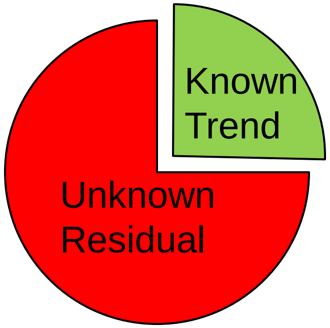
- Model that assumes perfect knowledge, without uncertainty

- Based on knowledge of the phenomenon or trend fitting to data
- Most subsurface models have a deterministic component (trend) to capture expert knowledge and to provide a stationary residual for geostatistical modeling.



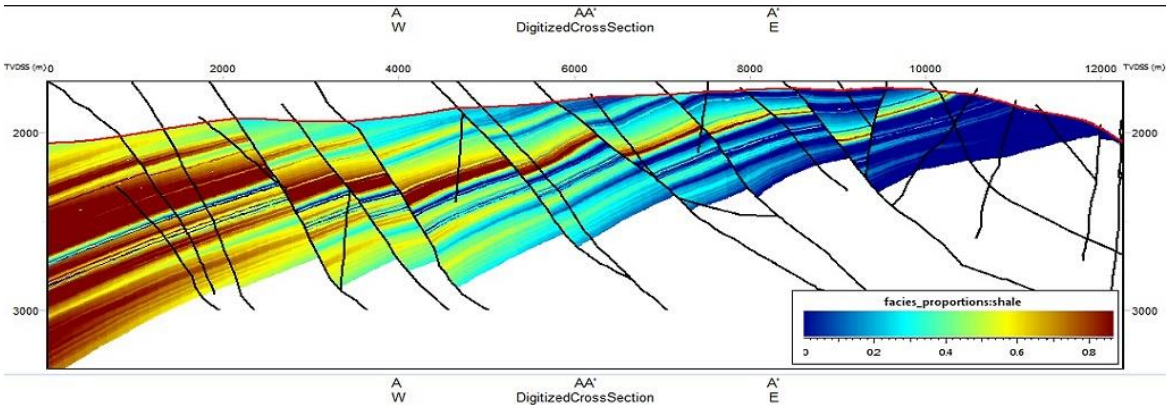
Definition Stochastic Model

- The unknown residual is modeled as a **stochastic model**
- Model that integrates uncertainty through the concept of random variables and functions
- Based primarily on data-driven statistics and various forms of integration of domain and local knowledge
- Most subsurface models have a stochastic component (residual) to quantify the uncertain component of the model (as opposed to the certain component from the trend model)



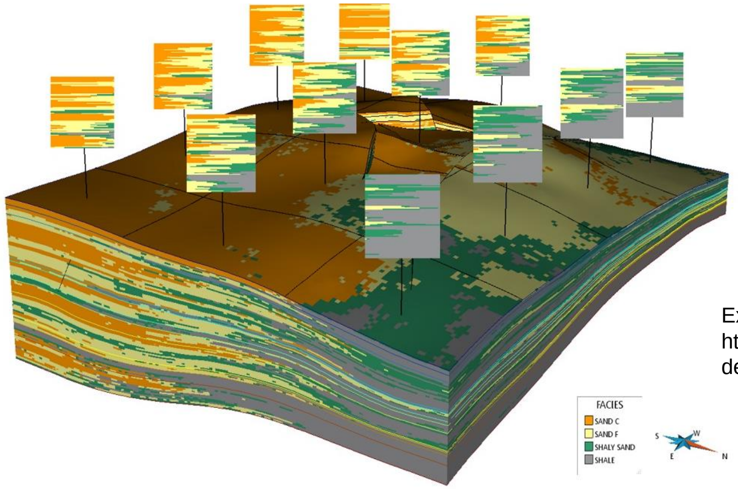
"Stochastic Residual"

Trend Modeling Methods



Example facies trend model Gocad SKUA. http://www.pdgm.com/getdoc/b24891f9-7470-4728-8cb7-0ddd7df196df/skua-facies-modeling/

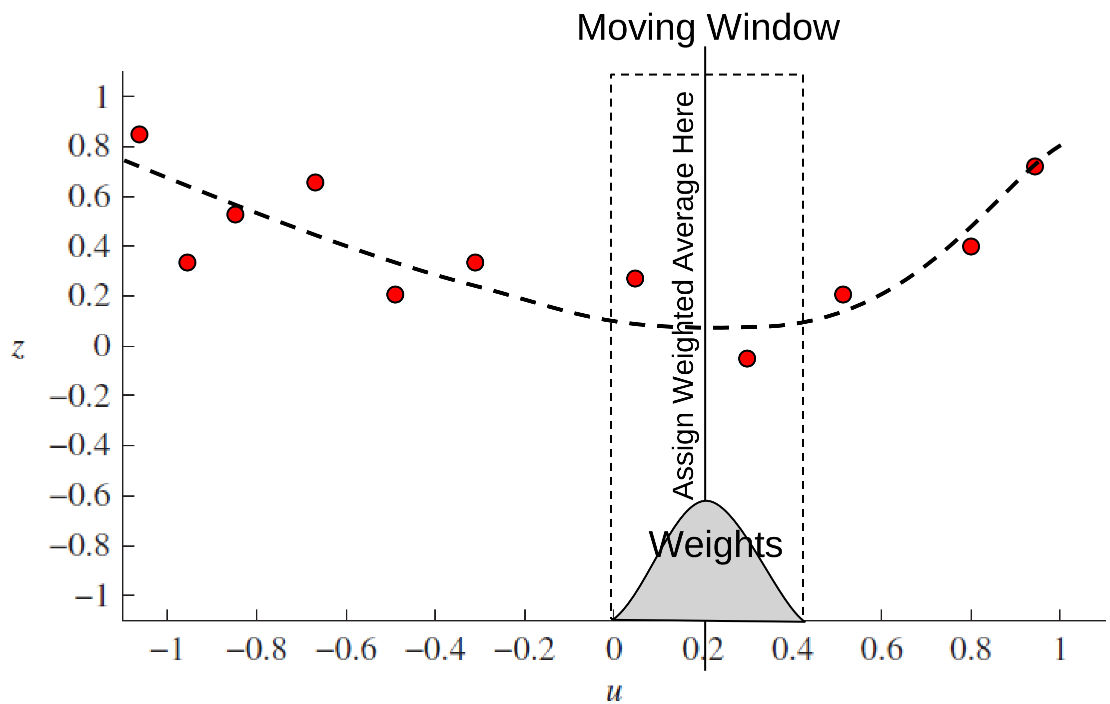
- Trend models:
  - Tend to be smooth, based on data and interpretation
  - May be complicated (see above)
  - Parameterized by vertical proportion curves (see below) and areal trend maps



Example facies trend model Gocad. http://www.pdgm.com/getdoc/bd9ab6b6-7dbb-4023-8f79-de34238d5136/skua-reservoir-data-analysis/

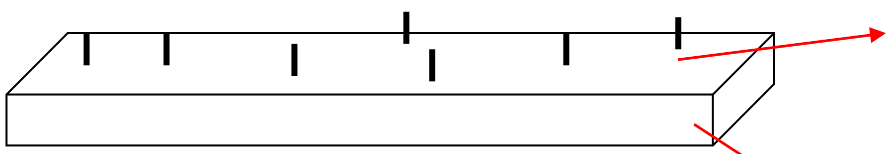
Trend Modeling Methods

- How to calculate a trend model:
  - Moving window average of the available data
  - Weighting scheme within the window
    - » Uniform weights can cause discontinuities
    - » Reduce weight at edges of moving window to reduce discontinuities (e.g. Gaussian weights).



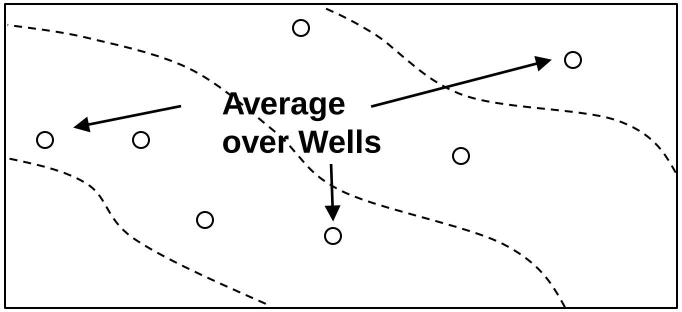
Trend 2D + 1D Workflow

- Calculate 2D Area and 1D Vertical trends:



Model and Well Data

2D Areal Trend From Well Averages

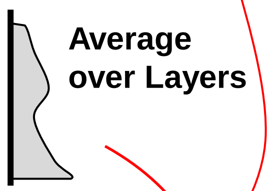


2D Trend Model

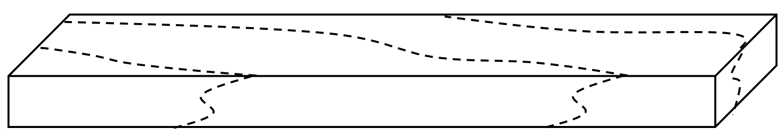
- Combine 1D and 2D ⇨ 3D.

$$\bar{X}(x,y,z) = \bar{X}(z) \cdot \frac{\bar{X}(x,y)}{\bar{X}}$$

1D Vertical Trend Layer Averages



1D Trend Model



3D Trend Model

Trend 2D + 1D Workflow

- How to calculate a trend model with sparse data
  - Break the problem up into a 1D and 2D trend inference problem and then combine to calculate a reasonable 3D trend
  - Calculate the 2D areal trend by interpolating over vertically averaged wells.
  - Calculate the 1D vertical trend by averaging layers
  - Combine the 1D vertical and the 2D areal trends:

3D Trend

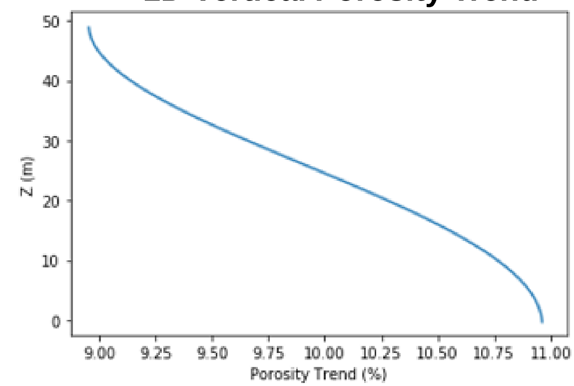
1D Vertical Trend

2D Areal Trend

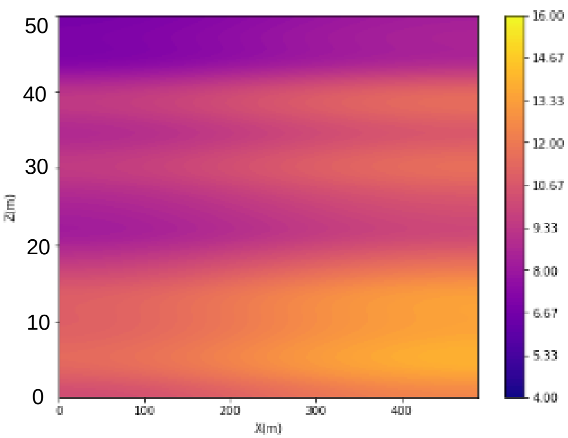
$$\overline{X}(x,y,z) = \overline{X}(z) \cdot \frac{\overline{X}(x,y)}{\overline{X}}$$

Global Mean

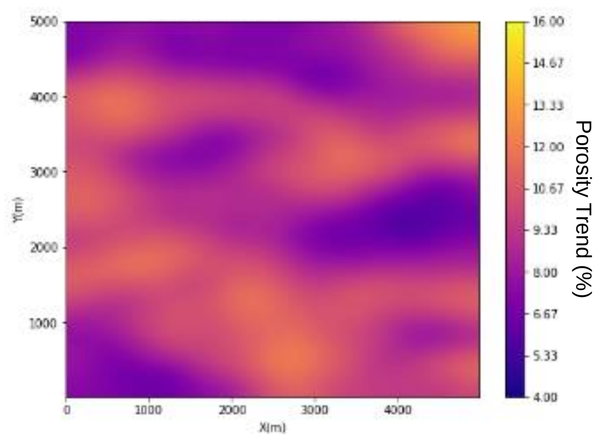
1D Vertical Porosity Trend



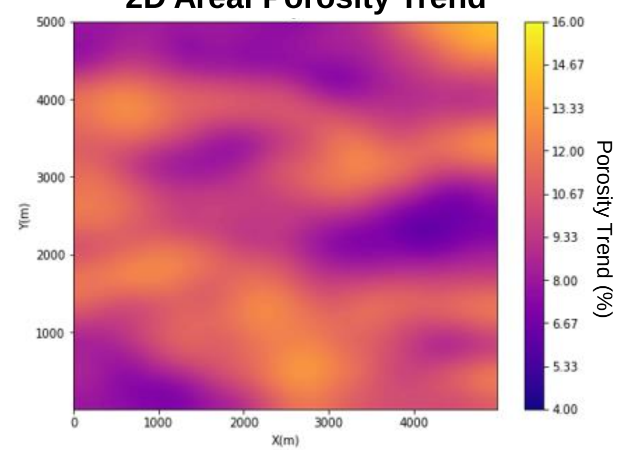
3D Porosity Trend, y = 2500m



3D Porosity Trend, z = 10m



2D Areal Porosity Trend

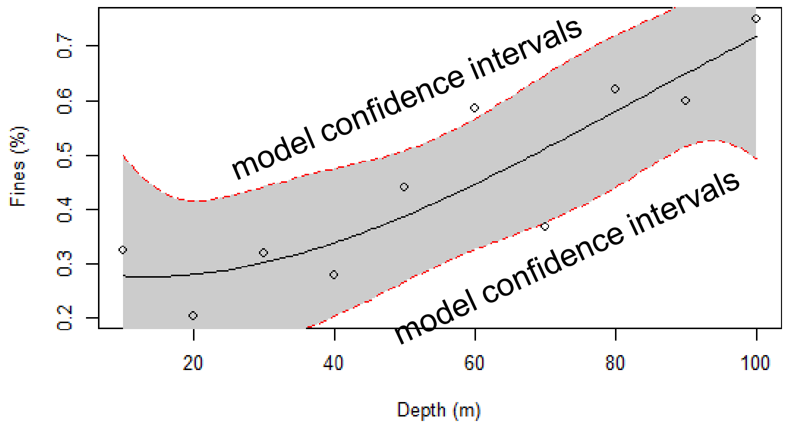


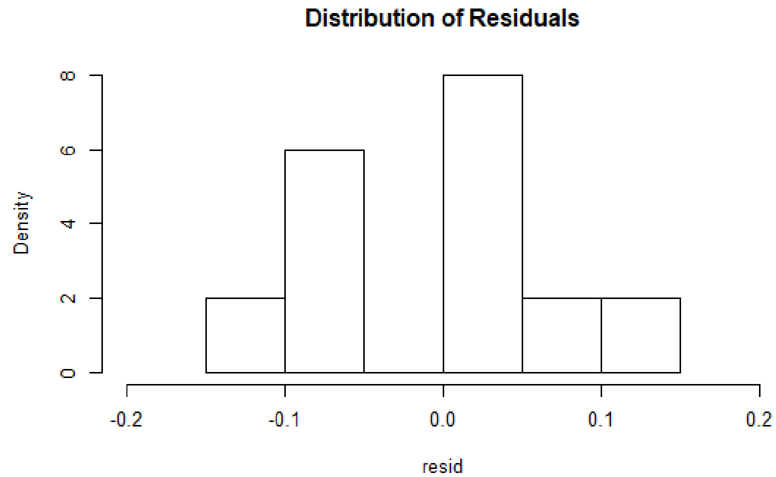
Trend Definition

- Observation of nonstationarity in any statistic, metric of interest
- A model of the nonstationarity in any statistic, metric of interest
- Typically modeled with support of data and expert knowledge in a deterministic manner (without uncertainty).

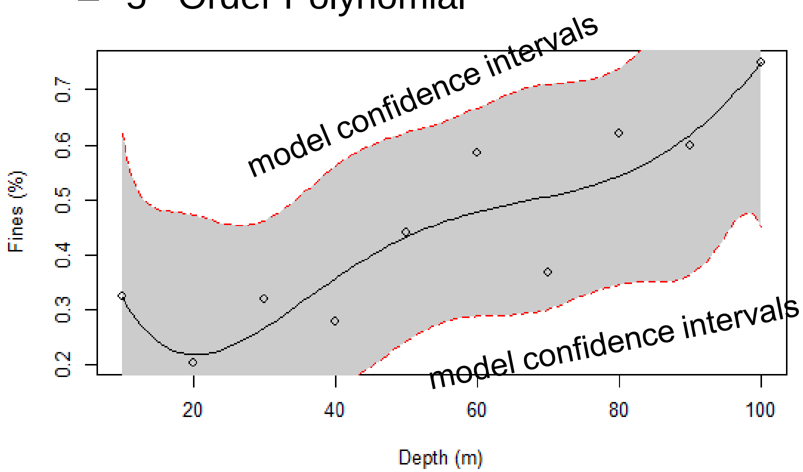
Overfitting

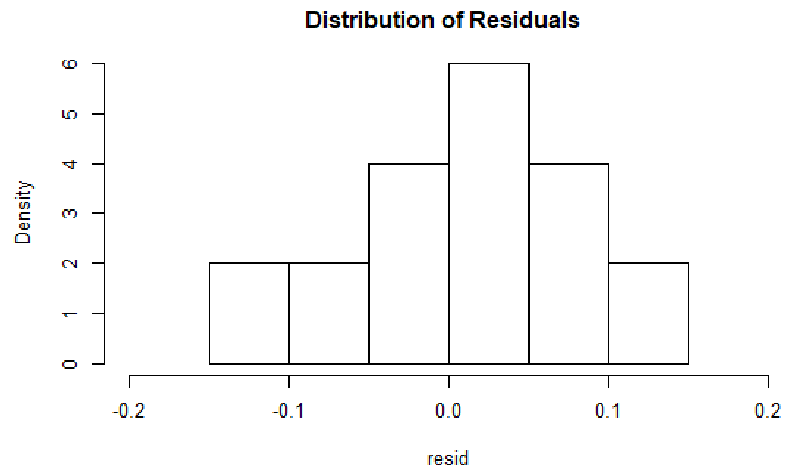
- Example of trend fits:
  - 3rd Ordered Polynomial





  - 5th Order Polynomial



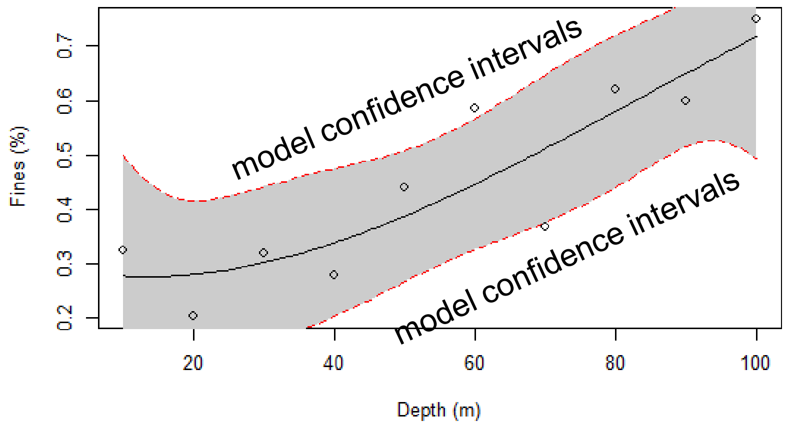


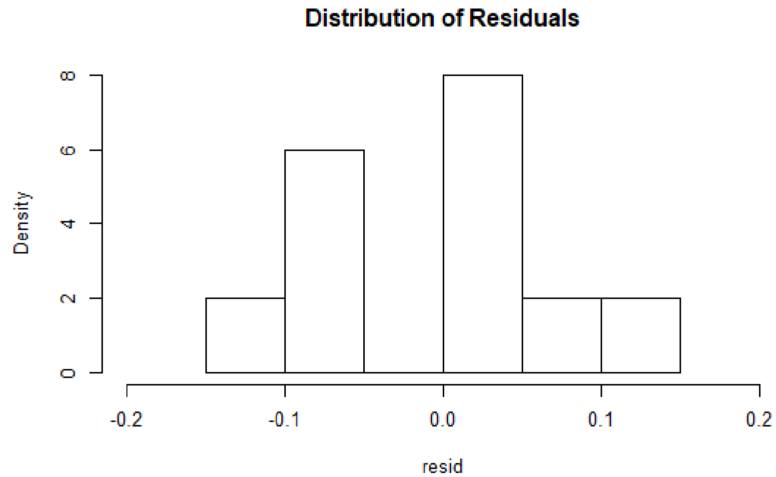
Overfit demonstration in R, code is here:
https://github.com/GeostatsGuy/geostatsr/blob/master/overfit.R

R code at Code/Overfit.R

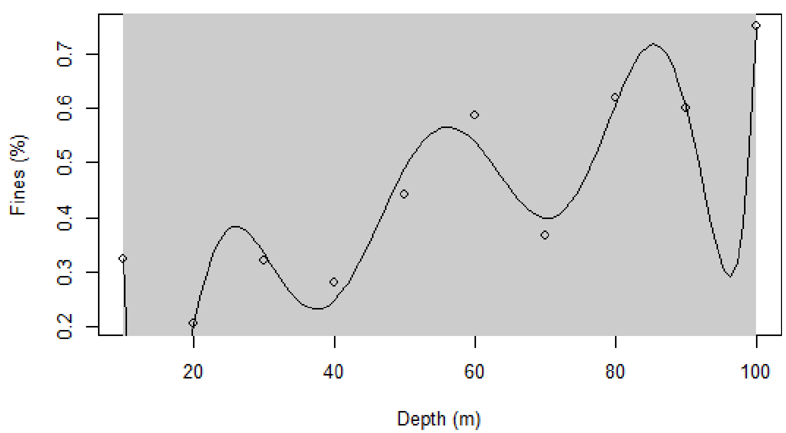
Overfitting

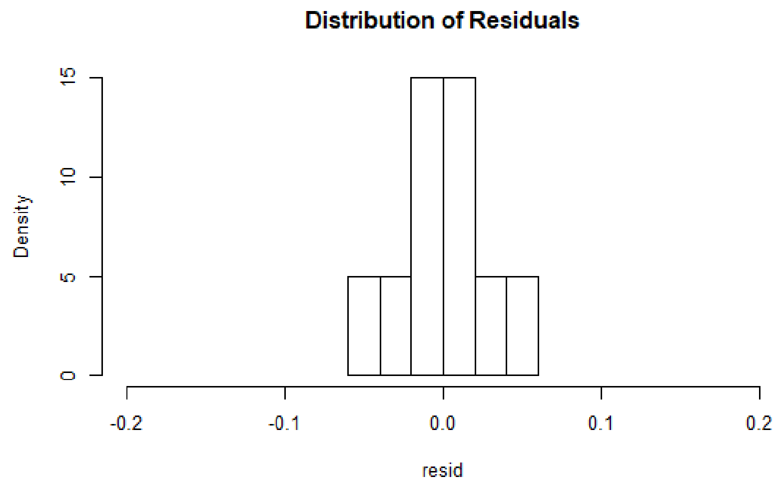
- Example of trend fits:
  - 3rd Ordered Polynomial





  - 8th Order Polynomial





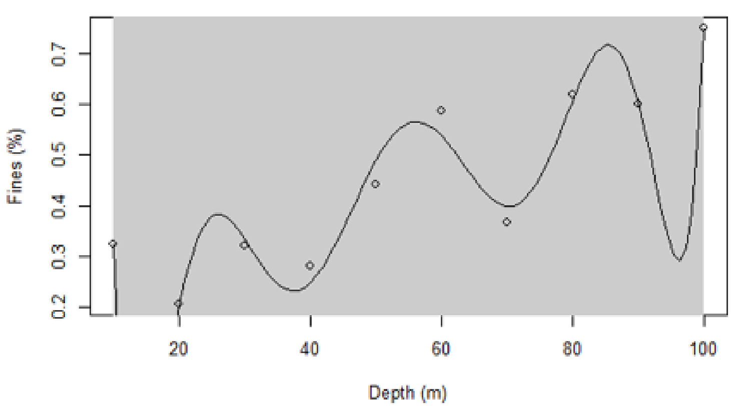
Overfit demonstration in R, code is here:
https://github.com/GeostatsGuy/geostatsr/blob/master/overfit.R

R code at Code/Overfit.R

Definition of Overfitting

- Overly complicated model to explain “idiosyncrasies” of the data, capturing data noise in the model
- More parameters than can be justified with the data

- Results in likely very high error away from the data
- But, results in low residual variance that cannot be defended with available data!
- High R^2 - proportion of variance explained
- Very accurate at the data! - Claim you know more than you do!



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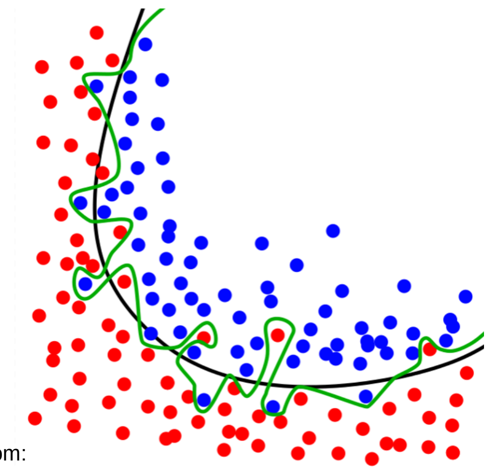

Overfit classification model example from:
https://en.wikipedia.org/wiki/Overfitting#/media/File:Overfitting.svg

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Lecture 12: Spatial Estimation

Lecture outline . . .

- **Kriging**

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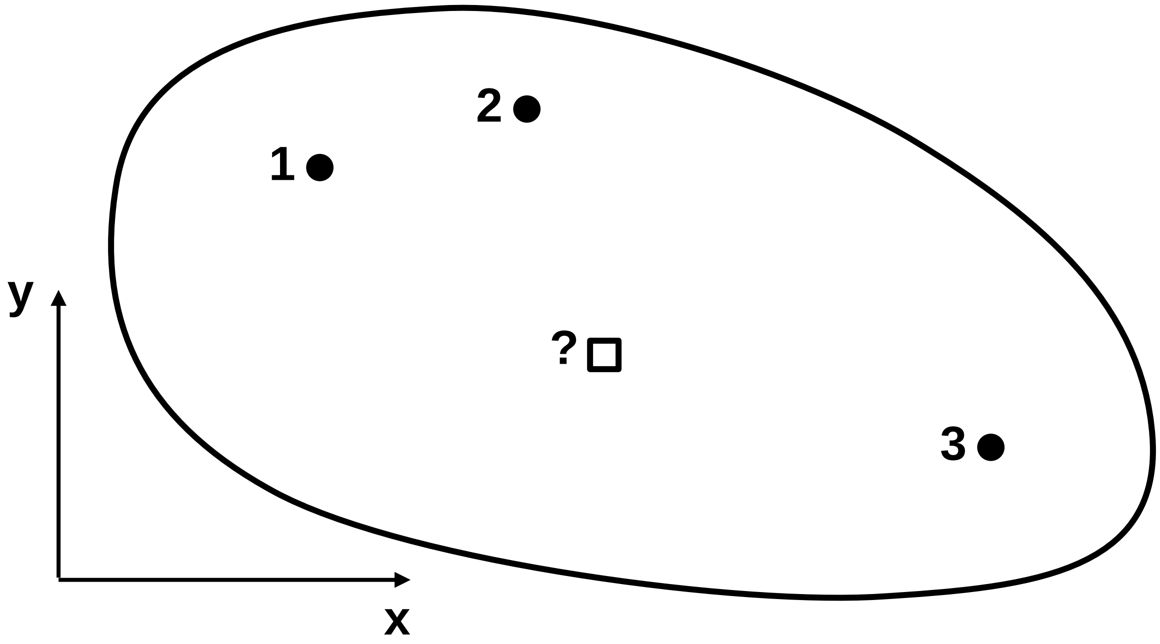
Spatial Estimation

- Consider the case of estimating at an unsampled location:

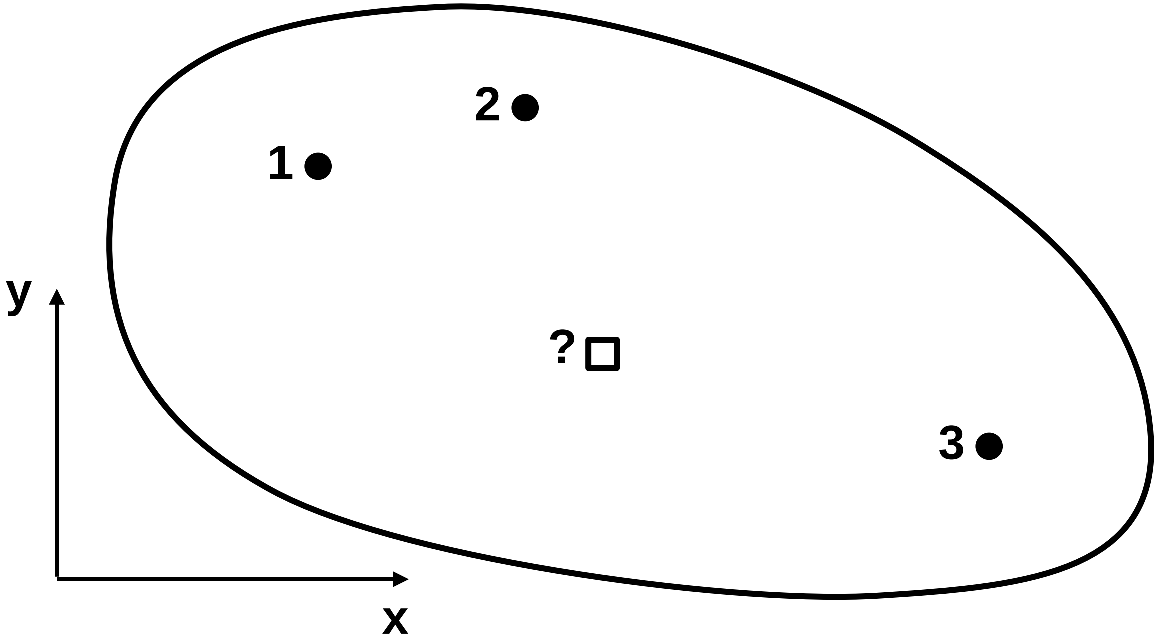


- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

- Note: z is the variable of interest (e.g. porosity etc.) and $\mathbf{u}_i$ is the data locations.

Spatial Estimation

- Consider the case of estimating at an unsampled location:



$\mathbf{z}(\mathbf{u}_\alpha)$ **is the data values**

$z^*(\mathbf{u_0})$ **is an estimate**

λ_α **is the data weights**

m_z **is the global mean**

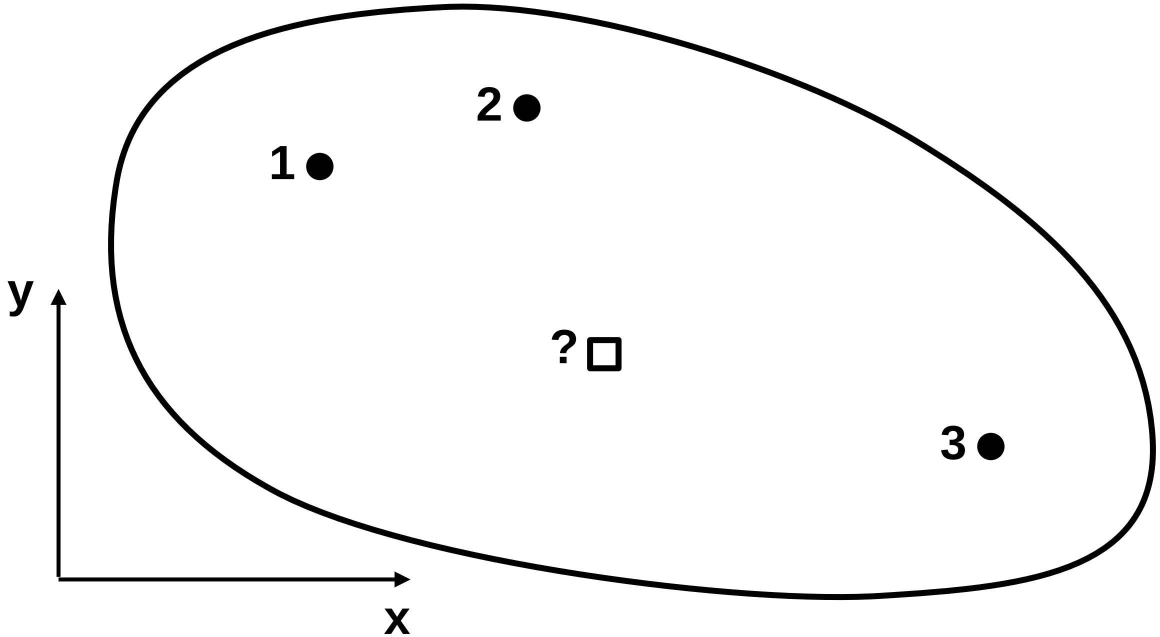
- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

$$z^*(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \mathbf{z}(\mathbf{u}_\alpha) + \boxed{\left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha\right) m_z}$$

Unbiasedness Constraint Weights sum to 1.0.

Spatial Estimation

- Consider the case of estimating at an unsampled location:



- How would you do this given data, $z(\mathbf{u}_1)$, $z(\mathbf{u}_2)$, and $z(\mathbf{u}_3)$?

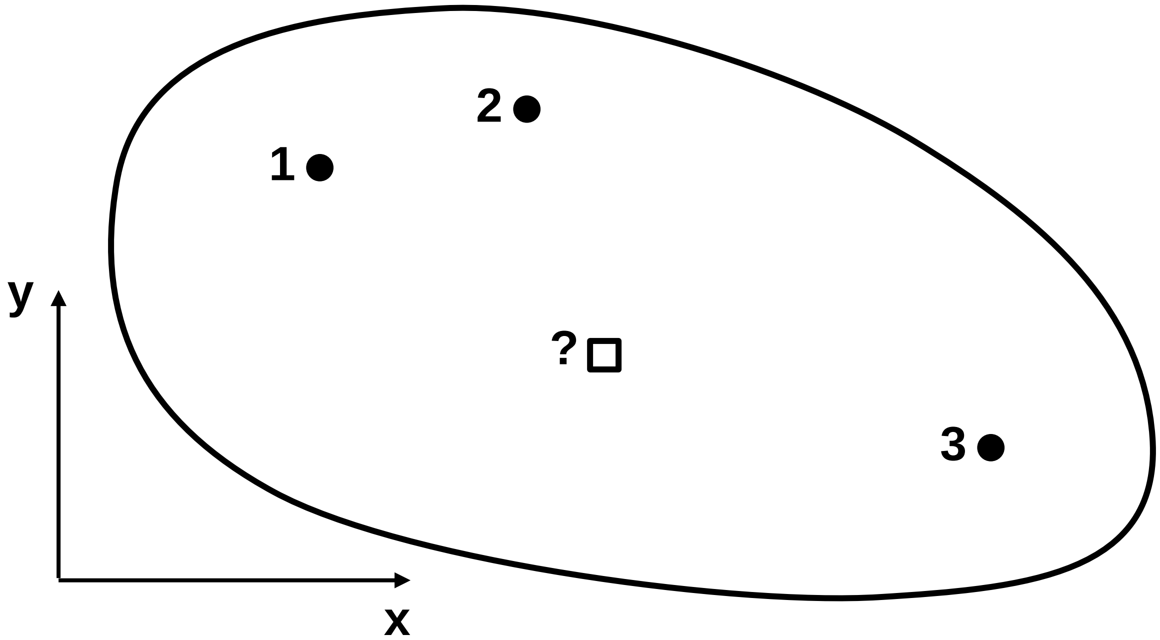
$$\boldsymbol{z}^*(\mathbf{u_0}) - \boldsymbol{m}_z(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \left(\mathbf{z}(\mathbf{u}_\alpha) - \boldsymbol{m}_z(\mathbf{u}_\alpha) \right)$$

In the case where the mean is non-stationary.

Given y$= \boldsymbol{z} - \boldsymbol{m}$, $\boldsymbol{y}^*(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \, \mathbf{y}(\mathbf{u}_\alpha)$ **Simplified with residual, y.**

Spatial Estimation

- Consider the case of estimating at an unsampled location:



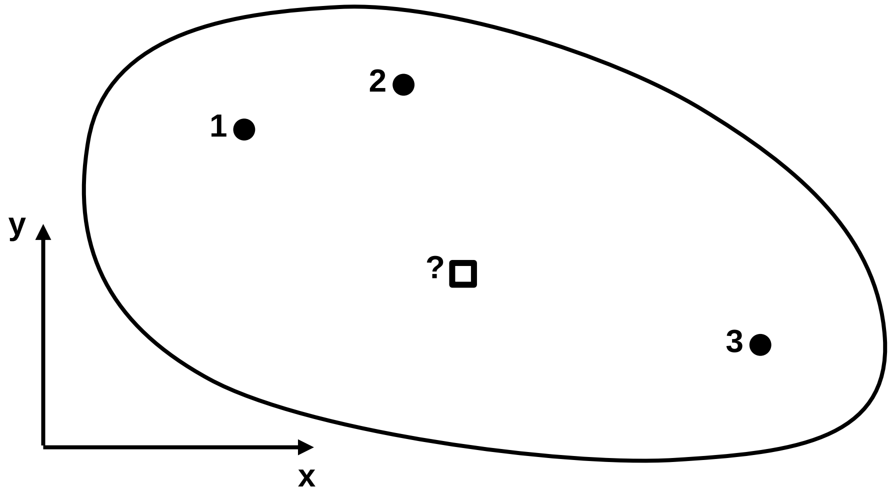
- Linear weighted, sound good. How do we get the weights? $\lambda_\alpha, \alpha = 1, \ldots, n$

$$y^*(\mathbf{u}_0) = \sum_{\alpha=1}^{n} \lambda_\alpha \, y(\mathbf{u}_\alpha)$$

Simplified with residual, y.

Spatial Estimation

- Consider the case of estimating at an unsampled location:



- Linear weighted, sound good. How do we get the weights? $\lambda_\alpha, \alpha = 1, \ldots, n$

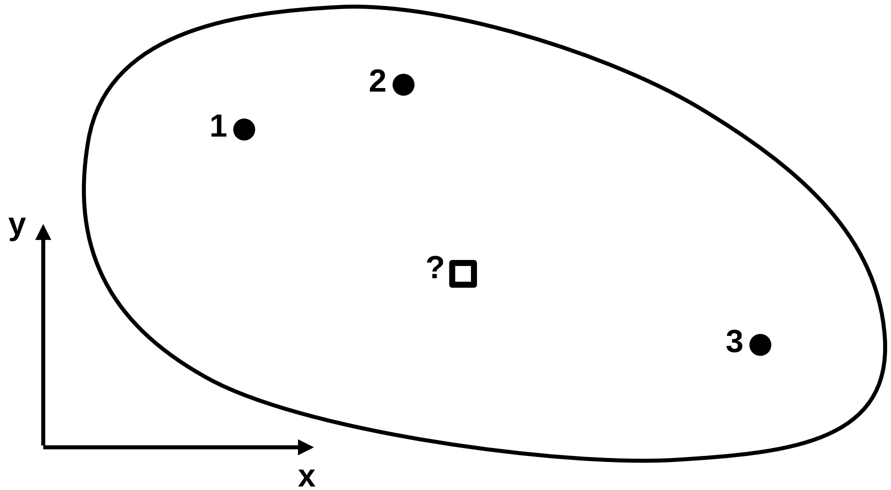
- Equal weighted / average? $\lambda_\alpha = {}^{1}/_{n}$

Equal weight average of data

- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at an unsampled location:



- How do we get the weights? $\lambda_\alpha, \alpha = 1, \dots, n$

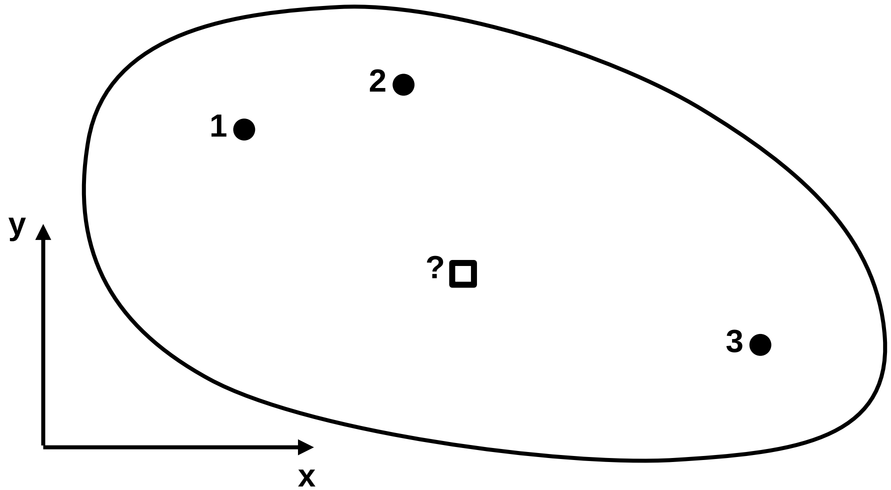
Inverse distance to power standardized so weights sum to 1.0.

- Inverse distance? $$\lambda_\alpha = \frac{1}{dist(\mathbf{u}_0, \mathbf{u}_\alpha)^p} \Big/ \sum_{\alpha=1}^{n} \lambda_\alpha$$

- What's wrong with that?

Spatial Estimation

- Consider the case of estimating at an unsampled location:



- How do we get the weights? $\lambda_\alpha, \alpha = 1, \ldots, n$

- It would be great to use weight that account for closeness (spatial correlation > distance alone), redundancy (once again with spatial correlation).

- How can we do that?

Derivation of Simple Kriging Equations

- Consider a linear estimator:

$$Y^*(\mathbf{u}) = \sum_{i=1}^{n} \lambda_i \cdot Y(\mathbf{u}_i)$$

where $Y(u_i)$ are the residual data (data values minus the mean) and $Y^*(u_i)$ is the estimate (add the mean back in when we are finished)

- The **estimation variance** is defined as:

Stationary Mean, Variogram

$$E\{Y\} = 0$$

$$2\gamma(\mathbf{h}) = E\left\{\left[Y(\mathbf{u}) - Y(\mathbf{u}+\mathbf{h})\right]^2\right\}$$

$$E\left\{\left[Y^*(u) - Y(u)\right]^2\right\} = \ldots$$

$$= E\left\{\left[Y^*(u)\right]^2\right\} - 2\, E\left\{Y^*(u)\, Y(u)\right\} + E\left\{\left[Y(u)\right]^2\right\}$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j E\left\{Y(u_i)\, Y(u_j)\right\} - 2 \sum_{i=1}^{n} \lambda_i E\left\{Y(u)\, Y(u_i)\right\} + C(0)$$

$$= \sum_{i=1}^{n} \sum_{j=1}^{n} \lambda_i \lambda_j C(u_i, u_j) - 2 \sum_{i=1}^{n} \lambda_i C(u, u_i) + C(0)$$

redundancy **closeness** **variance**

$C(\mathbf{u}_i, \mathbf{u}_j)$ – covariance between data i and j, $C(\mathbf{u}_i, \mathbf{u})$covariance between data and unknown location and $C(0)$ is the variance.

Derivation of Simple Kriging Equations

- Optimal weights $\lambda_i, i = 1, \ldots, n$ may be determined by taking partial derivatives of the error variance w.r.t. the weights

$$\frac{\partial[\quad\quad]}{\partial \lambda_i} = \sum_{j=1}^{n} \lambda_j C(\mathbf{u}_i, \mathbf{u}_i) - 2 \cdot C(\mathbf{u}, \mathbf{u}_i), i = 1, \ldots, n$$

and setting them to zero

$$\sum_{j=1}^{n} \lambda_j C(\mathbf{u}_i, \mathbf{u}_i) = \cdot\, C(\mathbf{u}, \mathbf{u}_i), i = 1, \ldots, n$$

- This system of *n* equations with *n* unknown weights is the simple kriging (SK) system

Kriging Definition

- Estimation approach that relies on linear weights that account for spatial continuity, data closeness and redundancy.
- Weights are unbiased and minimize the estimation variance.

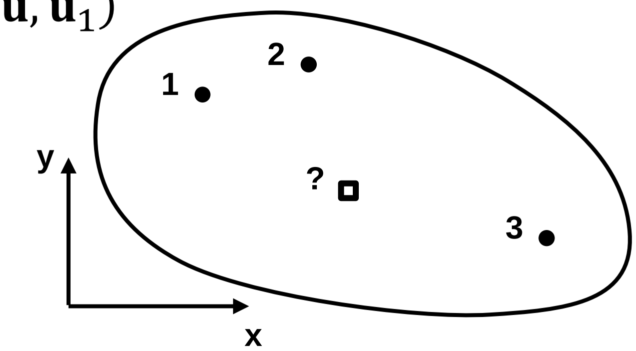
Simple Kriging System of Equations

There are three equations to determine the three weights:

$$\lambda_1 \cdot C(\mathbf{u}_1, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_1, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_1, \mathbf{u}_3) = C(\mathbf{u}, \mathbf{u}_1)$$

$$\lambda_1 \cdot C(\mathbf{u}_2, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_2, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_2, \mathbf{u}_3) = C(\mathbf{u}, \mathbf{u}_2)$$

$$\lambda_1 \cdot C(\mathbf{u}_3, \mathbf{u}_1) + \lambda_2 \cdot C(\mathbf{u}_3, \mathbf{u}_2) + \lambda_3 \cdot C(\mathbf{u}_3, \mathbf{u}_3) = C(\mathbf{u}, \mathbf{u}_1)$$



In matrix notation: Recall that $C(\mathbf{h}) = C(0) - \gamma(\mathbf{h})$

$$\underbrace{\begin{bmatrix} C(\mathbf{u}_1, \mathbf{u}_1) & C(\mathbf{u}_1, \mathbf{u}_2) & C(\mathbf{u}_1, \mathbf{u}_3) \\ C(\mathbf{u}_2, \mathbf{u}_1) & C(\mathbf{u}_2, \mathbf{u}_2) & C(\mathbf{u}_2, \mathbf{u}_3) \\ C(\mathbf{u}_3, \mathbf{u}_1) & C(\mathbf{u}_3, \mathbf{u}_2) & C(\mathbf{u}_3, \mathbf{u}_3) \end{bmatrix}}_{\text{redundancy}} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \underbrace{\begin{bmatrix} C(\mathbf{u}, \mathbf{u}_1) \\ C(\mathbf{u}, \mathbf{u}_2) \\ C(\mathbf{u}, \mathbf{u}_3) \end{bmatrix}}_{\text{closeness}}$$

redundancy

Covariance between all combinations of data locations, $\mathbf{u}_\alpha$.

closeness

Covariance between all data locations, $\mathbf{u}_\alpha$, and the unknown location, $\mathbf{u}$, combinations of data.

Notation Reminder

Locations of the data:

$$\mathbf{u}_\alpha, \alpha = 1, \dots, n$$

Data values at those locations:

$$y(\mathbf{u}_\alpha), \alpha = 1, \dots, n$$

Properties of Simple Kriging

- Solution exists and is unique of matrix $\left[C(v_i, v_j) \right]$ is positive definite

- Kriging estimator is unbiased: $E\left\{ \left[Z - Z^* \right] \right\} = 0$

- Minimum error variance estimator (just try to pick weights, you won't bet it)

- Best Linear Unbiased Estimator

- Provides a measure of the estimation (or kriging) variance (uncertainty in the estimate):

$$\sigma_E^2(\mathbf{u}) = C(0) - \sum_{\alpha=1}^{n} \lambda_\alpha C(\mathbf{u} - \mathbf{u}_\alpha) \qquad \boldsymbol{\sigma_E^2} \rightarrow [\mathbf{0}, \boldsymbol{\sigma_x^2}]$$

Properties of Simple Kriging

- Exact interpolator: at data location

- Kriging variance can be calculated before getting the sample information, homoscedastic!

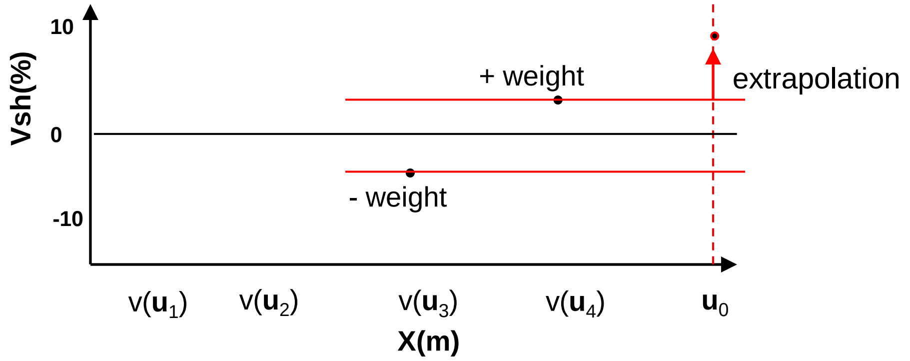
- Kriging takes into account:
  - distance of the information: $C(\mathbf{u},\mathbf{u}_i)$
  - configuration of the data: $C(\mathbf{u}_i,\mathbf{u}_j)$
  - structural continuity of the variable being considered: $C(\mathbf{h})$

- The smoothing effect of kriging can be forecast – we will return to this with simulation.

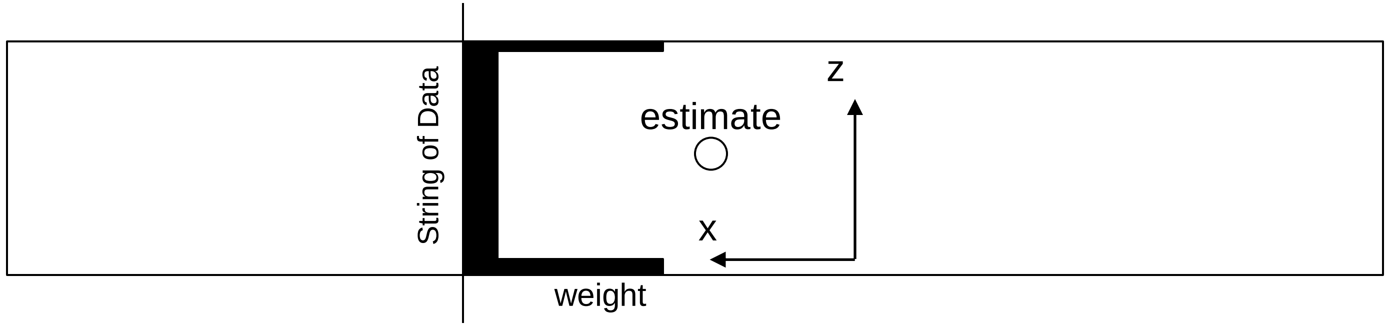
Properties of Simple Kriging

- Outside range of the data, simple kriging weights all equal 0.0. The best estimate is the provided mean!

- Screened data will sometimes have negative weights! This allows kriging to extrapolate.

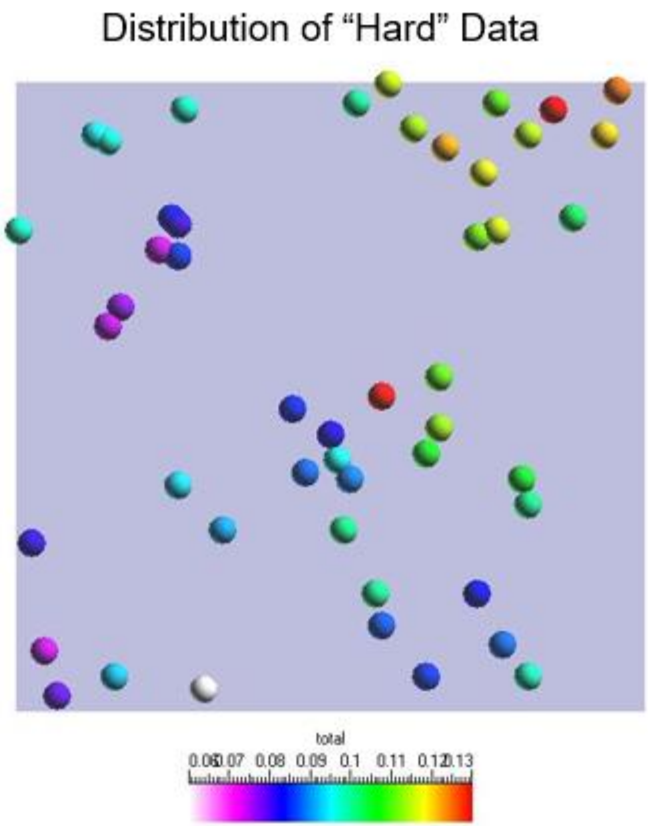


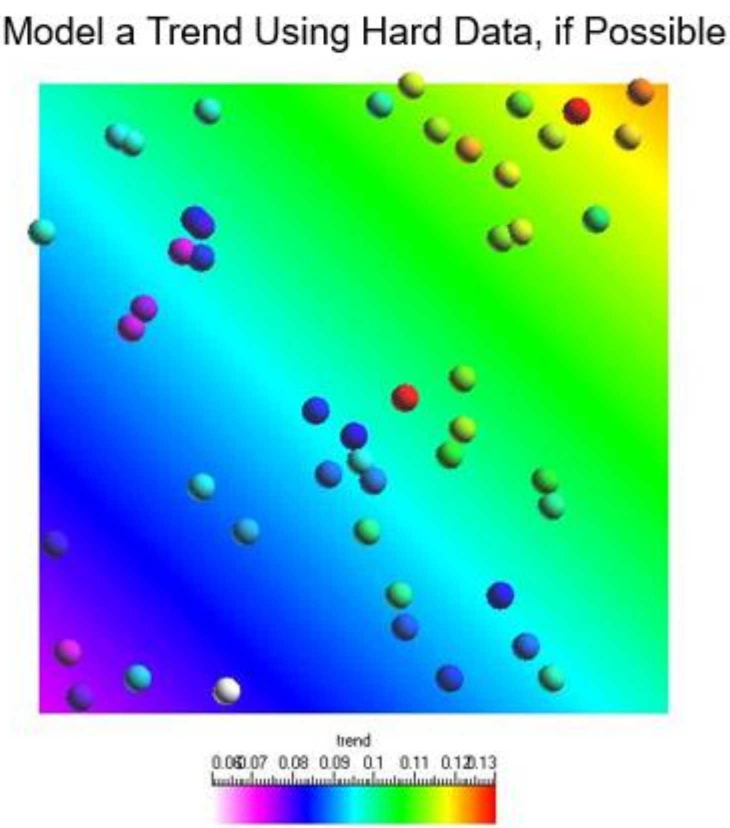
- Strings of data will have an artifact known as the string effect.



Kriging Estimation Example

- Kriging residual with trend modeling workflow



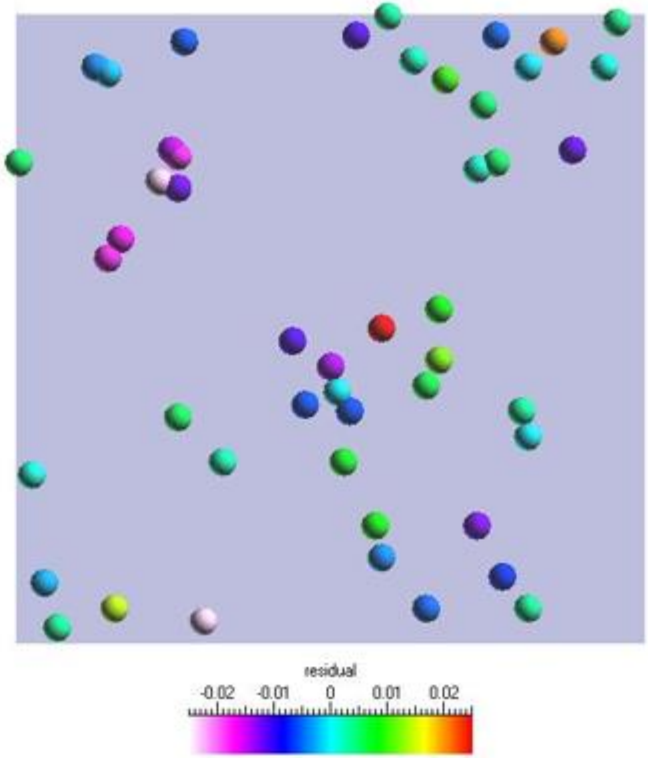


Example by William Meddaugh, Midwestern State University

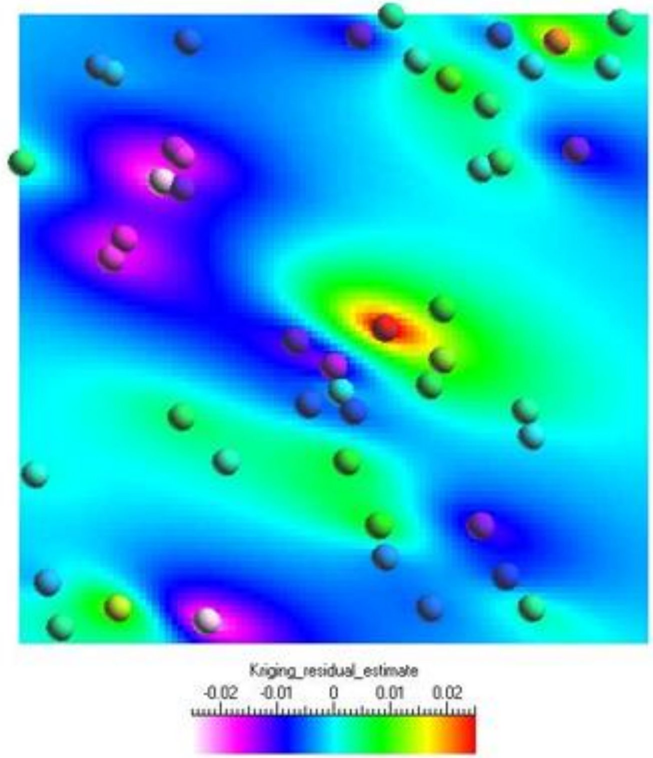
Kriging Estimation Example

- Kriging residual with trend modeling workflow

Calculate Residual at Hard Data Locations:
Residual=Actual Value – Trend Model Value



Perform Variography and Krig the Residuals

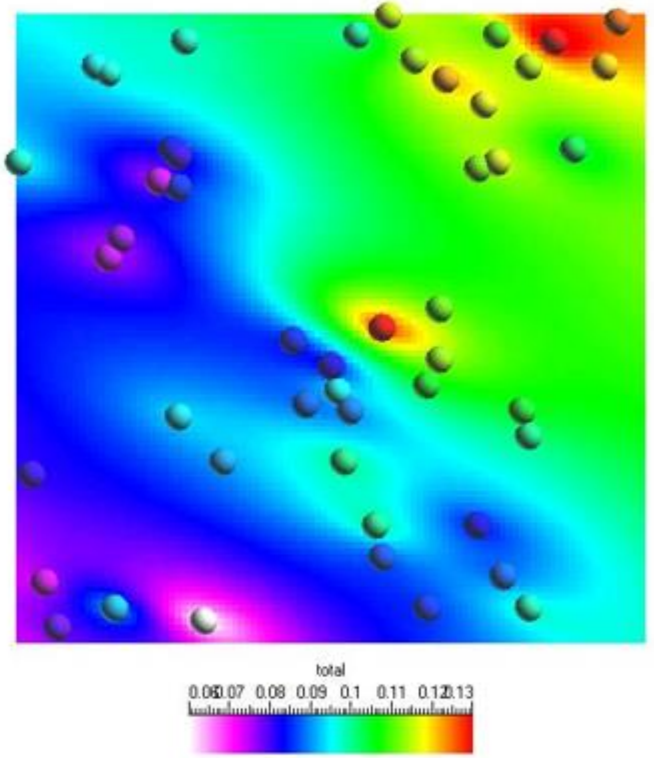


Example by William Meddaugh, Midwestern State University

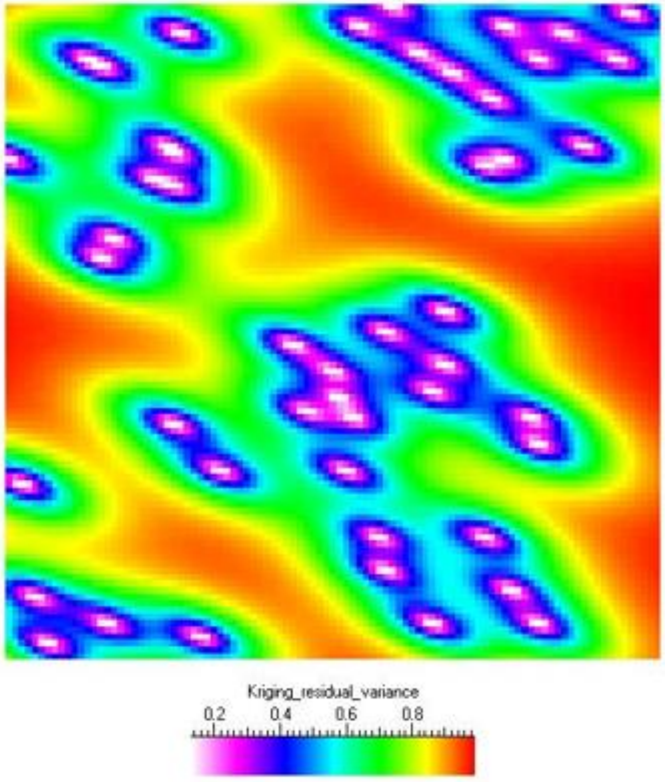
Kriging Estimation Example

- Kriging residual with trend modeling workflow

Add Back the Trend:
Estimate=Kriged Residuals + Trend



Kriging Variance



Example by William Meddaugh, Midwestern State University

Simple Kriging Exercise in Excel

File at Examples/Simple_Kriging_Demo.xls

Simple Kriging Demonstration

Michael Pyrcz, Geostatistics at Petroleum and Geosystems Engineering, University of Texas at Austin (mpyrcz@austin.utexas.edu)

1. Data and Estimate Locations and Values

Point	x	y	value	residual
1	60	80	0.1	-0.040
2	25	50	0.12	-0.020
3	80	10	0.2	0.060
unknown	50	50		
mean			0.140	

2. Distance Matrix

0.00	46.10	72.80	31.62
46.10	0.00	68.01	25.00
72.80	68.01	0.00	50.00

3. Variogram Model

Nugget	0
Spherica	1
Range	300

4. Variogram Matrix

0.000	0.229	0.357	0.158
0.229	0.000	0.334	0.125
0.357	0.334	0.000	0.248

5. Covariance Matrix

1.000	0.771	0.643	0.842
0.771	1.000	0.666	0.875
0.643	0.666	1.000	0.752

6. Inverse Left Side

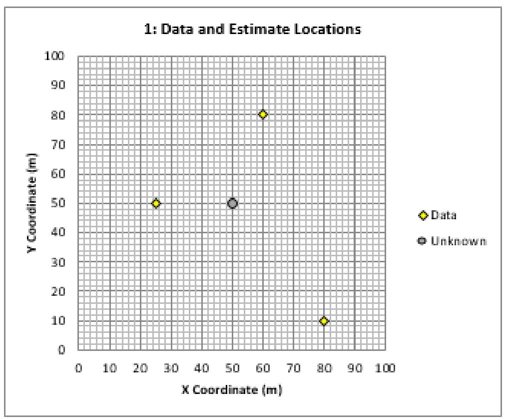
2.667	-1.644	-0.621
-1.644	2.810	-0.813
-0.621	-0.813	1.941

7. Weights

	0.341
	0.462
	0.225
Sum Weights	1.029
Mean Weight	-0.029

8. Kriging Results

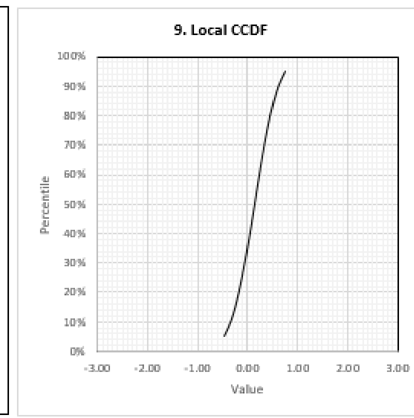
Kriging Estimate	0.131
Kriging Variance	0.139



Legend

	Information
	User Input
	Calculation
	Redundancy Measure
	Closeness Measures

p-value	$F^{-1}(p)$
5%	-0.48
10%	-0.35
15%	-0.26
20%	-0.18
25%	-0.12
30%	-0.06
35%	-0.01
40%	0.04
45%	0.08
50%	0.13
55%	0.18
60%	0.22
65%	0.27
70%	0.33
75%	0.38
80%	0.44
85%	0.52
90%	0.61
95%	0.74



Description

This sheet provides an illustration of Simple Kriging at a single estimated location.

Step 1: Input the data locations and values, the unknown simulated location. At any point these locations and values may be changed to observed their influence on the simulation.
Step 2: The distance matrix is automatically calculated, that is the distance between the data and the unknown locations.
Step 3: Enter the model of spatial continuity in the form of an isotropic spherical variogram and nugget effect (contributions should sum to one). This model may be changed at any time to observed sensitivities to spatial continuity.
Step 4: Variogram matrix is calculated by applying the distance matrix to the isotropic variogram model.
Step 5: Covaraince matrix is calculated by subtracing the variogram from the variance (1 for standard normal distribution). This is applied to improve numerical stability as a diagonally dominant matrix is more readily invertable.
Step 6: The left hand side of the covariance matrix is inverted.
Step 7: The inverted left handside matrix is multiplied by the right hand side matrix to calculate the simple kriging weights.
Step 8: The kriging estimate and kriging variance are calculated with the weights and covariances.
Step 9: With the Gaussian assumption the complete local conditional cumulative distribution function is available.

Excel file available at: https://github.com/GeostatsGuy/ExcelNumericalDemos/blob/master/Simple_Kriging_Demo.xlsx

Simple Kriging Exercise in Excel

- Some ideas for experimenting with simple kriging. Do the following and pay attention to the weights, the estimate and the estimation variance.

1. Set points 1 and 2 closer together.

2. Put point 1 behind point 2 to create screening.

3. Put all points outside the range.

4. See the range very large.

Ordinary Kriging

Add the constraint of : $\sum_{\alpha=1}^{n} \lambda_\alpha = 1.0$

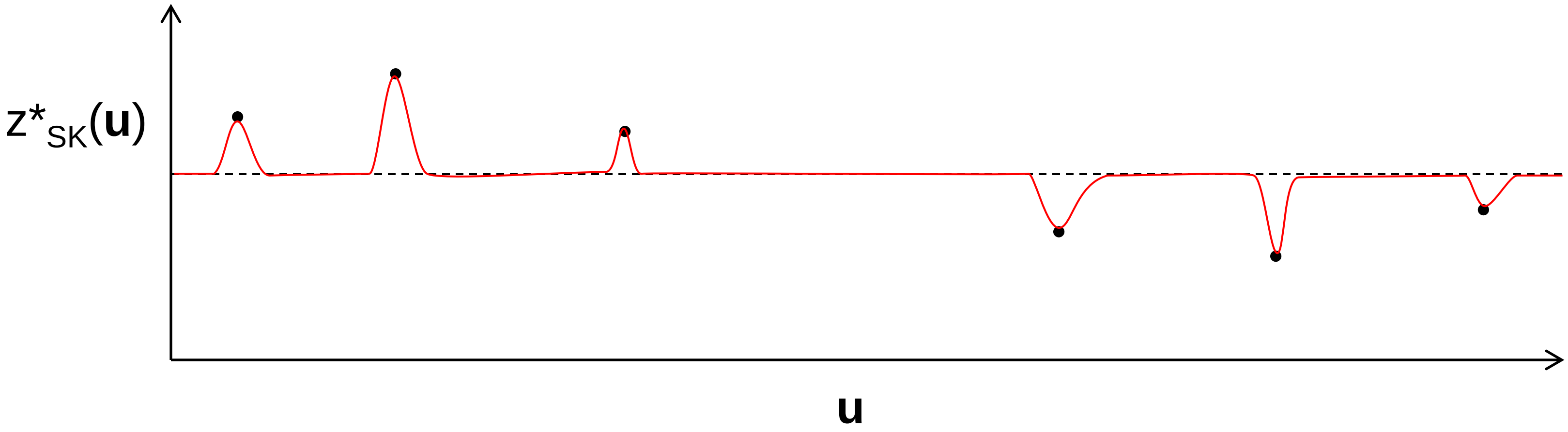
$$\begin{bmatrix} C(\mathbf{u}_1,\mathbf{u}_1) & C(\mathbf{u}_1,\mathbf{u}_2) & C(\mathbf{u}_1,\mathbf{u}_3) & 1 \\ C(\mathbf{u}_2,\mathbf{u}_1) & C(\mathbf{u}_2,\mathbf{u}_2) & C(\mathbf{u}_2,\mathbf{u}_3) & 1 \\ C(\mathbf{u}_3,\mathbf{u}_1) & C(\mathbf{u}_3,\mathbf{u}_1) & C(\mathbf{u}_3,\mathbf{u}_3) & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} \lambda_1 \\ \lambda_2 \\ \lambda_3 \\ \mu \end{bmatrix} = \begin{bmatrix} C(\mathbf{u}_0,\mathbf{u}_1) \\ C(\mathbf{u}_0,\mathbf{u}_2) \\ C(\mathbf{u}_0,\mathbf{u}_3) \\ 1 \end{bmatrix}$$

Recall that $C(\mathbf{h}) = C(0) - \gamma(\mathbf{h})$

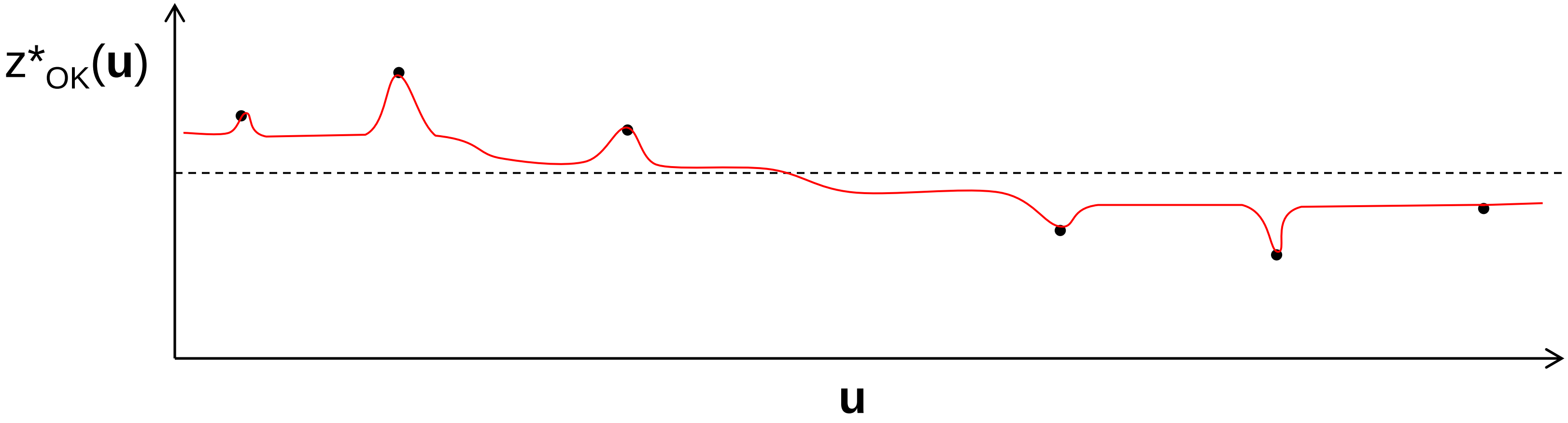
$$z^*(\mathbf{u_0}) = \sum_{\alpha=1}^{n} \lambda_\alpha \mathbf{z}(\mathbf{u}_\alpha) + \cancelto{0}{\left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha\right)} m_z$$

With ordinary kriging the mean does not need to be known. Ordinary kriging estimates the mean locally!

Simple Kriging vs. Ordinary Kriging



Beyond the range of correlation, Simple Kriging estimates the global mean.



Beyond the range of correlation, Ordinary Kriging estimates with an estimated local mean. Relaxes the stationary mean assumption.

Kriging Summary

- Kriging is a procedure for constructing a minimum error variance linear estimate at a location where the true value is unknown
- The main controls on the kriging weights are:
  – closeness of the data to the location being estimated
  – redundancy between the data
  – the variogram

- Simple Kriging (SK) does not constrain the weights and works with the residual from the mean
- Ordinary Kriging (OK) constrains the sum of the weights to be 1.0, therefore, the mean does not need to be known
- There are many different types of kriging:
  – e.g. universal kriging fits a parametric trend model over location while calculating the optimum weights.

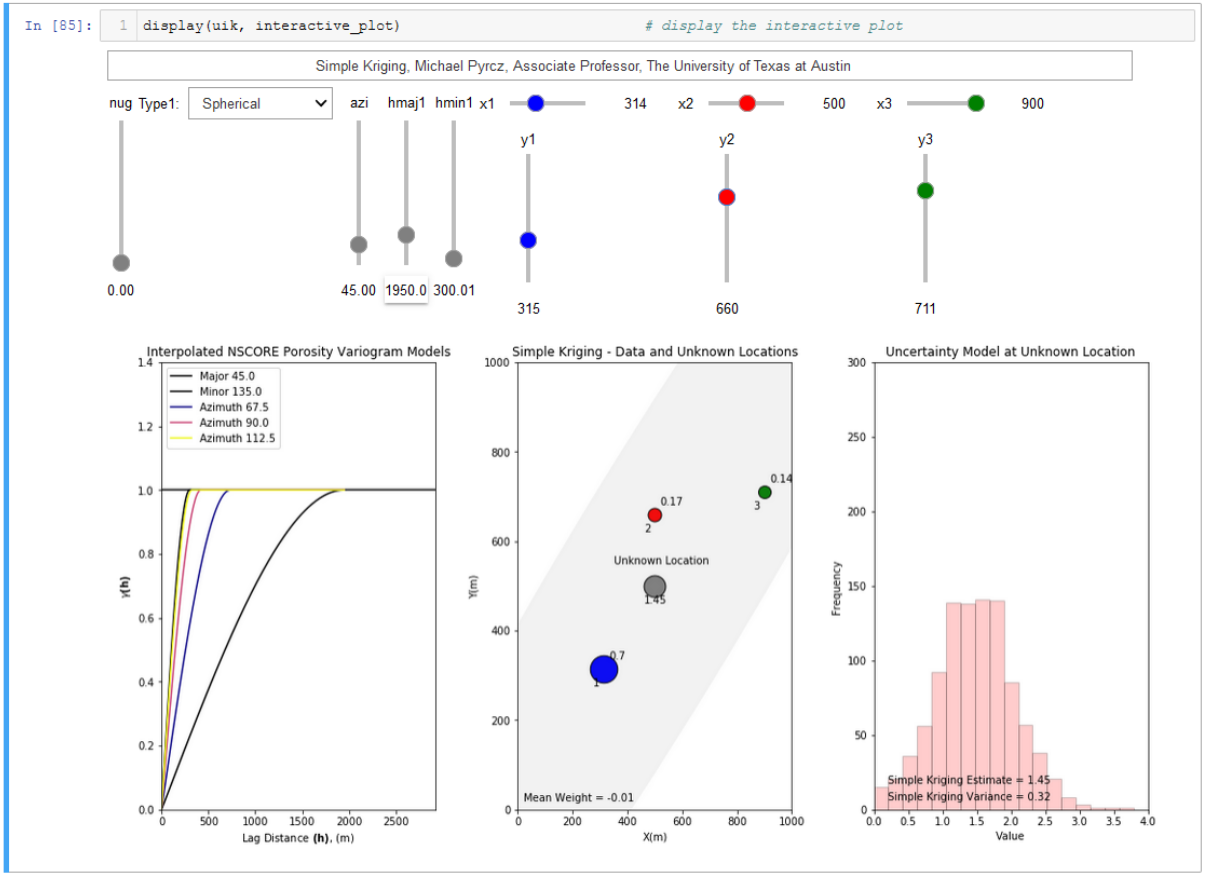
Slide modified from Clayton Deutsch’s MinE 612 Course

Kriging in Python

Interactive Kriging Demonstration in Python

Walkthrough:

- Change the variogram parameters.
- Change the data locations.
- Investigate:
    - closeness
    - redundancy
    - mean weight
    - screening effect



- File: Interactive_Simple_Kriging.ipynb

Spatial Uncertainty Hands-on

Here's an opportunity for experiential learning with Simple Kriging for spatial uncertainty. The kriging estimation variance is very useful.

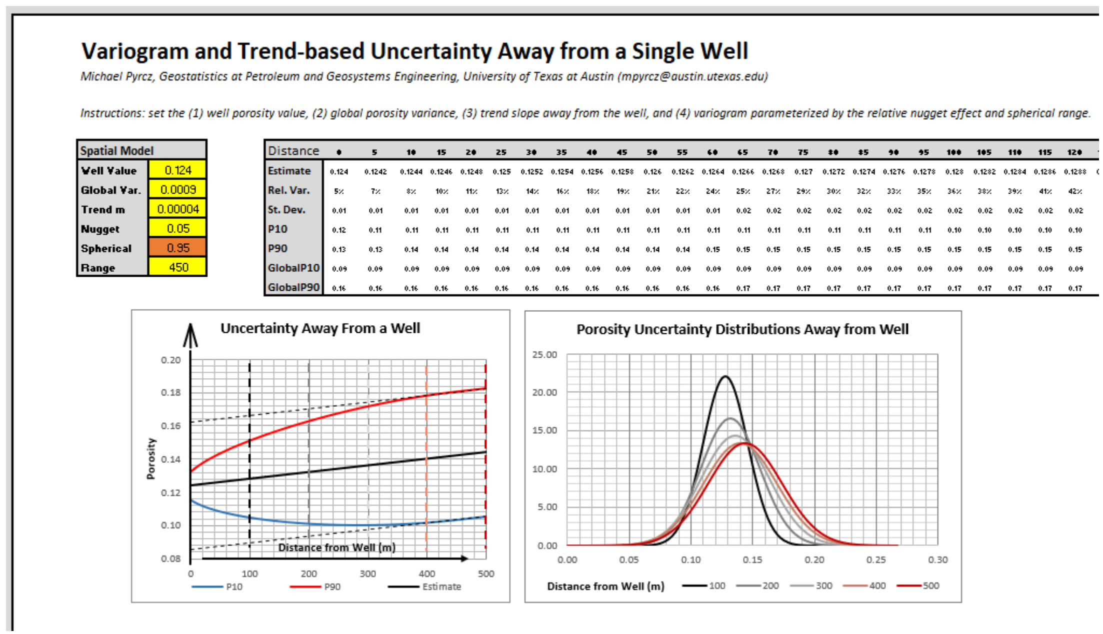
- **Things to try:**

Pay attention to the kriging uncertainty P10, mean and P90 away from the well as you:

1. Change the spatial continuity range.

2. Add and adjust the nugget effect.

3. Modify the trend slope.



Spatial Model	
Well Value	0.124
Global Var.	0.0009
Trend m	0.00004
Nugget	0.05
Spherical	0.95
Range	450

Distance	0	5	10	15	20	25	30	35	40	45	50	55	60
Estimate	0.124	0.1242	0.1244	0.1246	0.1248	0.125	0.1252	0.1254	0.1256	0.1258	0.126	0.1262	0.1264
Rel. Var.	5%	7%	8%	10%	11%	13%	14%	16%	18%	19%	21%	22%	24%
St. Dev.	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01	0.01
P10	0.12	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.11
P90	0.13	0.13	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.14	0.15
GlobalP10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
GlobalP90	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16	0.16

Distance	65	70	75	80	85	90	95	100	105	110	115	120
Estimate	0.1266	0.1268	0.127	0.1272	0.1274	0.1276	0.1278	0.128	0.1282	0.1284	0.1286	0.1288
Rel. Var.	25%	27%	29%	30%	32%	33%	35%	36%	38%	39%	41%	42%
St. Dev.	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02
P10	0.11	0.11	0.11	0.11	0.11	0.11	0.11	0.10	0.10	0.10	0.10	0.10
P90	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15	0.15
GlobalP10	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09	0.09
GlobalP90	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17	0.17

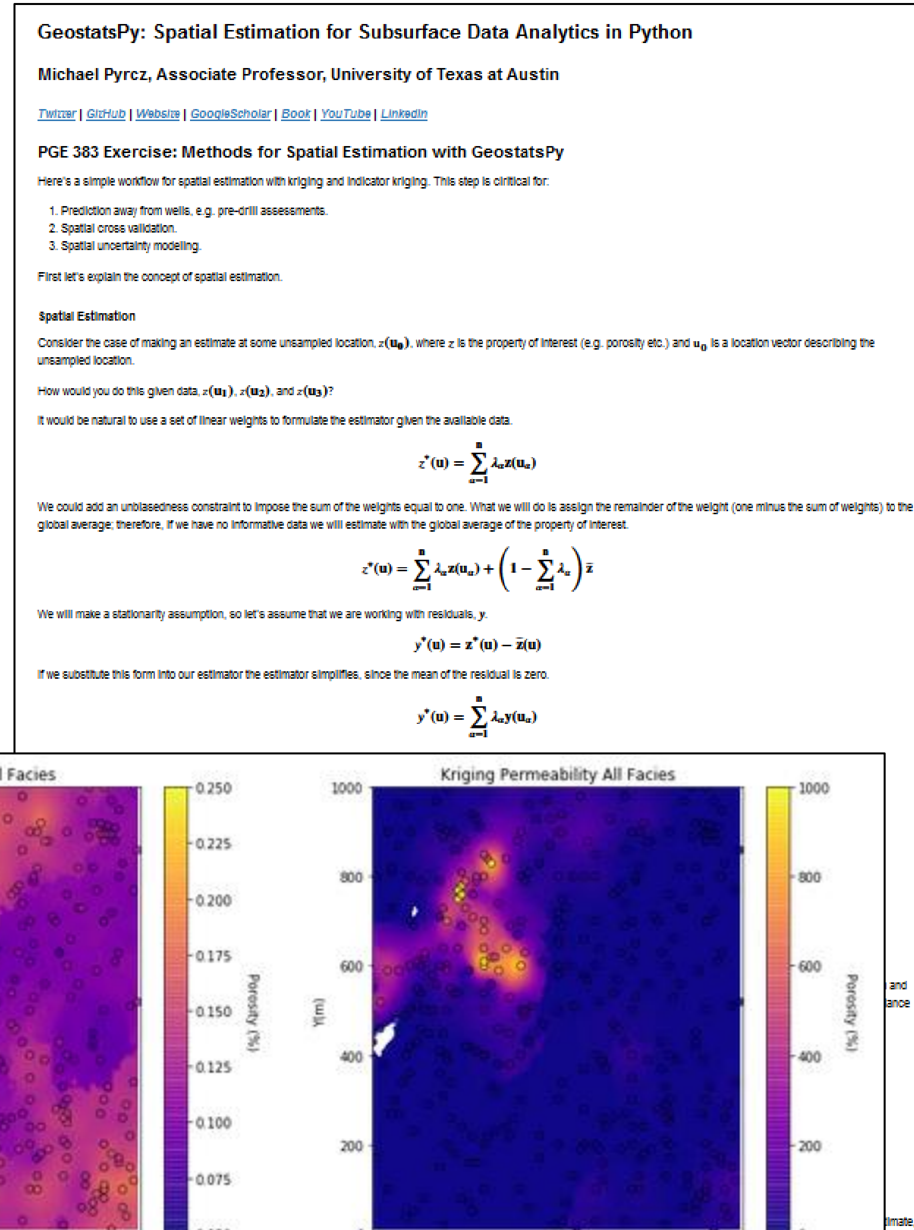
Kriging in Python

Kriging Workflow in Python

Walkthrough and try to:

- Change the variogram and search parameters.

- File is: GeostatsPy_kriging.ipynb



GeostatsPy: Spatial Estimation for Subsurface Data Analytics in Python

Michael Pyrcz, Associate Professor, University of Texas at Austin

Twitter | GitHub | Website | GoogleScholar | Book | YouTube | LinkedIn

PGE 383 Exercise: Methods for Spatial Estimation with GeostatsPy

Here's a simple workflow for spatial estimation with kriging and indicator kriging. This step is critical for:

1. Prediction away from wells, e.g. pre-drill assessments.
2. Spatial cross validation.
3. Spatial uncertainty modeling.

First let's explain the concept of spatial estimation.

Spatial Estimation

Consider the case of making an estimate at some unsampled location, $z(\mathbf{u_0})$, where z is the property of interest (e.g. porosity etc.) and $\mathbf{u_0}$ is a location vector describing the unsampled location.

How would you do this given data, $z(\mathbf{u_1})$, $z(\mathbf{u_2})$, and $z(\mathbf{u_3})$?

It would be natural to use a set of linear weights to formulate the estimator given the available data.

$$z^*(\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha z(\mathbf{u}_\alpha)$$

We could add an unbiasedness constraint to impose the sum of the weights equal to one. What we will do is assign the remainder of the weight (one minus the sum of weights) to the global average; therefore, if we have no informative data we will estimate with the global average of the property of interest.

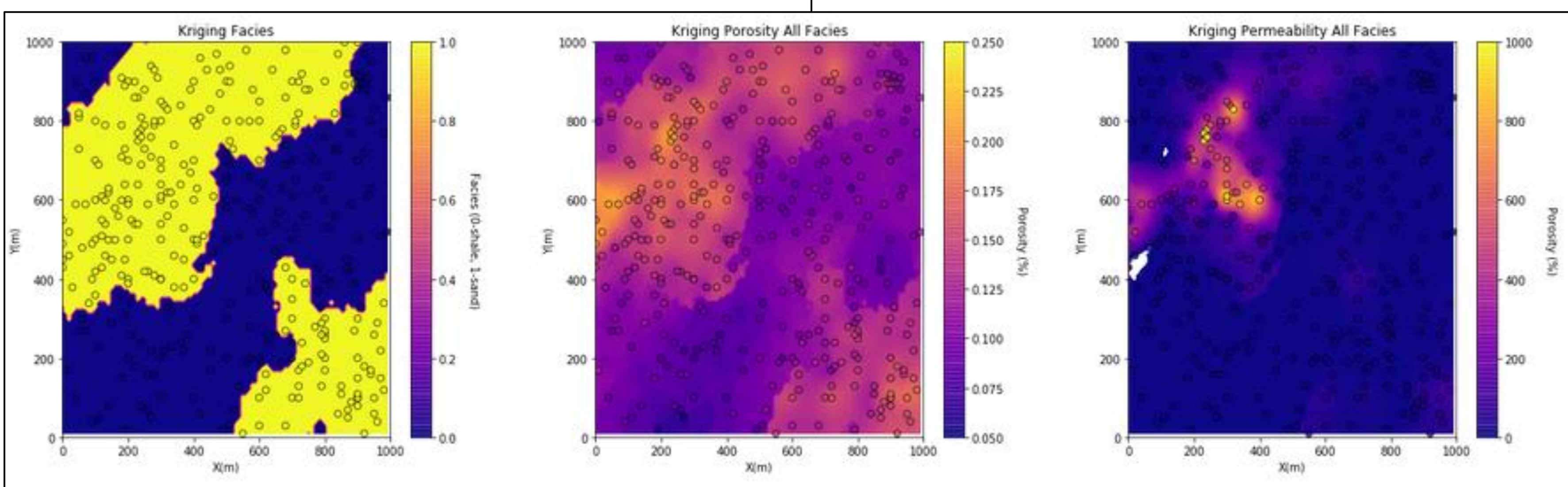
$$z^*(\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha z(\mathbf{u}_\alpha) + \left(1 - \sum_{\alpha=1}^{n} \lambda_\alpha\right) \bar{z}$$

We will make a stationarity assumption, so let's assume that we are working with residuals, y.

$$y^*(\mathbf{u}) = z^*(\mathbf{u}) - \bar{z}(\mathbf{u})$$

If we substitute this form into our estimator the estimator simplifies, since the mean of the residual is zero.

$$y^*(\mathbf{u}) = \sum_{\alpha=1}^{n} \lambda_\alpha y(\mathbf{u}_\alpha)$$



Review of Main Points

- Simple kriging (SK) is linear regression with some special properties:
  - Gives the mean and variance of conditional normal distribution
  - Best linear estimate for mean squared error criterion and variogram model
- Estimation variance is expected squared difference between estimate and truth that accounts for:
  - Initial variance if no data are available, the stationary variance of the property
  - The redundancy between the data
  - The closeness of the data to what is being estimated
- We derive simple kriging to minimize the error variance in expected value
- The use of SK estimates directly is somewhat limited, but it is used extensively under a multivariate Gaussian model for inference of conditional means and variances
  - We will discuss more next about simulation.

PGE 337 Data Analytics and Geostatistics

Lecture 12: Spatial Estimation

Lecture outline . . .

- Trend Modeling
- Kriging

Introduction

General Concepts

Univariate

Bivariate

Spatial

Calculation

Variogram Modeling

Kriging

Simulation

Time Series

Machine Learning

Uncertainty Analysis

Michael Pyrcz, The University of Texas at Austin