Probabilistic Programming with Deep Neural Networks

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https://andresmasegosa.github.io/
https://pgmlab.github.io/

This material jointly made with Thomas D. Nielsen (AAU).

Plan for this week

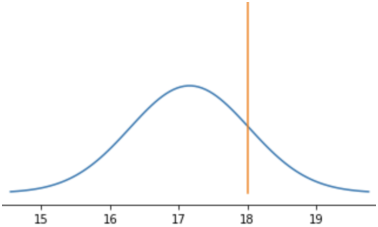
Day 1: Introduction to probabilistic programming languages

- Why do we need PPLs?
- Probabilistic programming in Pyro
- Hand-on exercises:
  - Probability Distributions in Pyro.
  - Probabilistic Models in Pyro.

Day 2: Probabilistic Models with Deep Neural Networks

- Uncertainty in Machine Learning
- Variational Inference
- Supervised/Unsupervised Learning
- Hand-on exercises
  - Bayesian Neural Networks
  - Variational Auto-encoders

What is a PPL?



Normal Distribution

What is a probabilistic model?

VQ-VAE-2

What is a PPL?

```python
import tensorflow as tf
import tensorflow_probability as tfp

# Pretend to load synthetic data set.
features = tfp.distributions.Normal(loc=0., scale=1.).sample(int(100e3))
labels = tfp.distributions.Bernoulli(logits=1.618 * features).sample()

# Specify model.
model = tfp.glm.Bernoulli()

# Fit model given data.
coeffs, linear_response, is_converged, num_iter = tfp.glm.fit(
    model_matrix=features[:, tf.newaxis],
    response=tf.cast(labels, dtype=tf.float32),
    model=model)
# ==> coeffs is approximately [1.618] (We're golden!)
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Probabilistic Programming Language (PPL)

- An attempt to **unify probabilistic modeling and general programming languages**.

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Probabilistic Programming Language (PPL)

- An attempt to **unify probabilistic modeling and general programming languages**.
- A programming paradigm to define **general probabilistic models (mixing deterministic + stochastic functions)**.
- Make probabilistic modeling more **applicable and powerful**.

Why PPLs?

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Why do we need PPLs?

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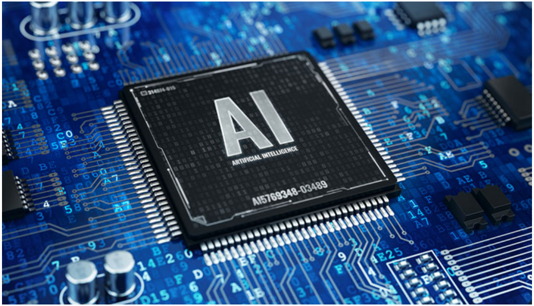
Why do we need PPLs?

- Reason 1: Try to democratize the development of AI systems.

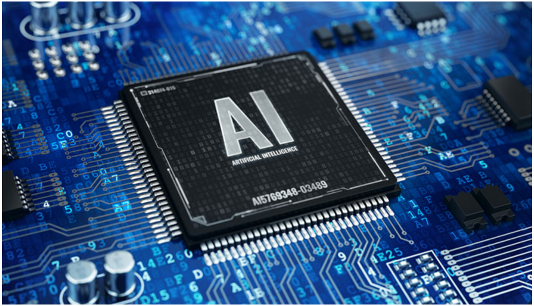
Why do we need PPLs?

- Reason 1: Try to democratize the development of AI systems.
- Reason 2: Try to make AI systems safer.

Reason 1: Try to democratize the development of AI systems.



DARPA's Fund Call for PPLs in Artificial Intelligence.



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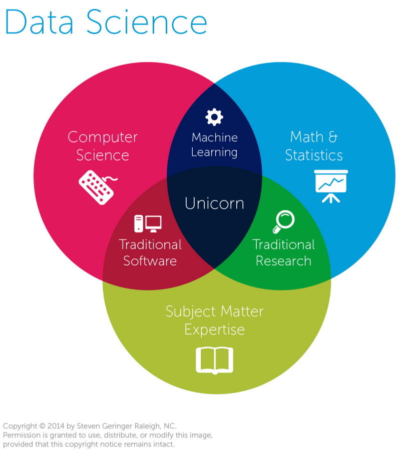
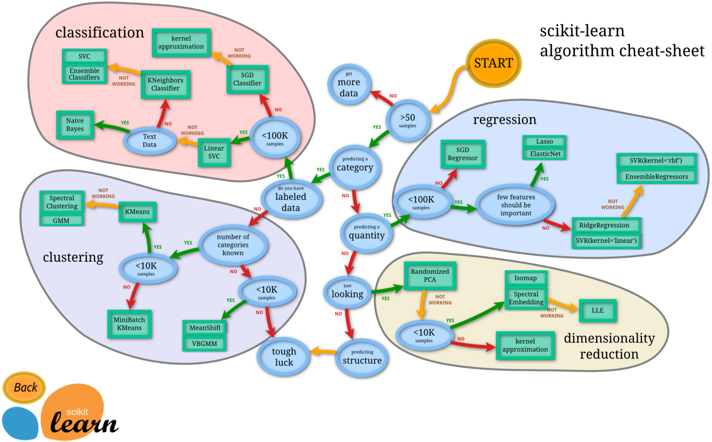


The development of **machine learning systems** requires enormous efforts.

- **It requires of highly qualified experts.**



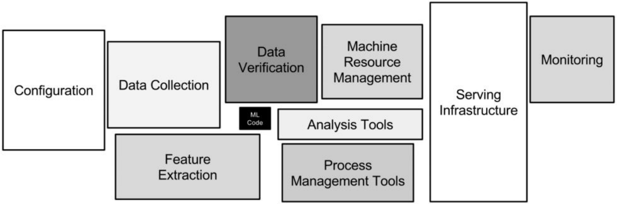
The development of machine learning applications requires enormous effort.

- It is necessary to have highly qualified experts.
- **It is difficult to find the ML model most suitable for an application.**

Why PPLs?

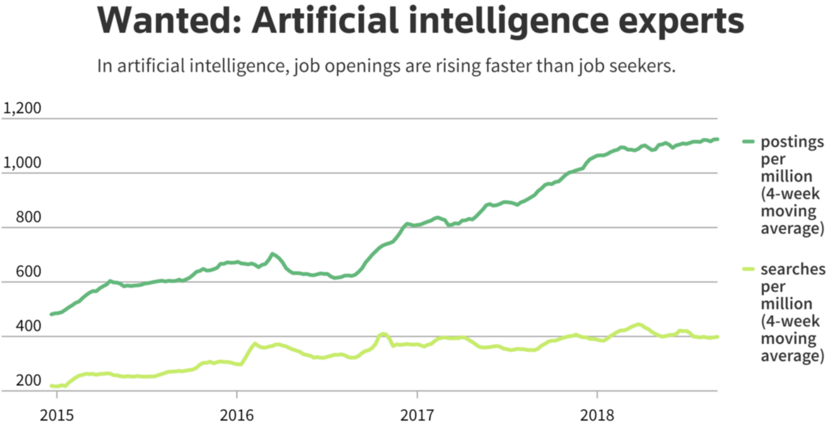
Hidden Technical Debt in Machine Learning Systems

D. Sculley, Gary Holt, Daniel Golovin, Eugene Davydov, Todd Phillips
{dsculley,gholt,dgg,edavydov,toddphillips}@google.com
Google, Inc.



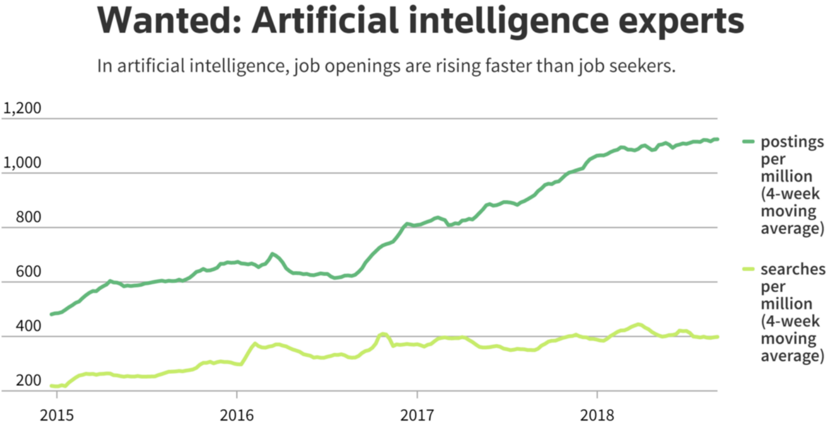
The development of machine learning applications requires enormous effort.

- It is necessary to have highly qualified experts.
- It is difficult to find the ML model most suitable for an application.
- **Programming a ML model is a complex task where many problems are intermingled**.



Consequences:

- Shortage of AI experts (and high salaries).



Consequences:

- Shortage of AI experts (and high salaries).
- Only big corporations have the resources for developing ML systems.

Why PPLs?

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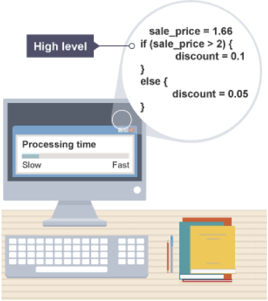
Similar situation than 50 years ago:

- People used to program in low-level programming languages.
- Programming was complex and demand high-expertise.
- Focus on application and low-level hardware details.

Why PPLs?



High-level programming languages brought many advantages:

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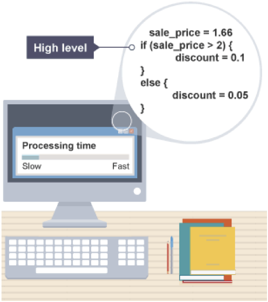

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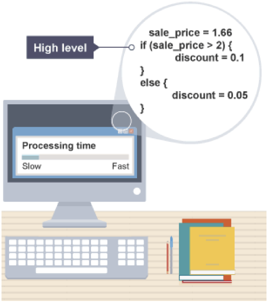

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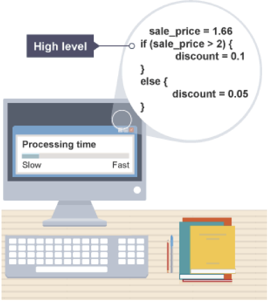

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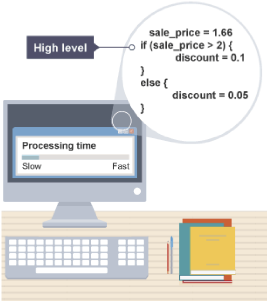

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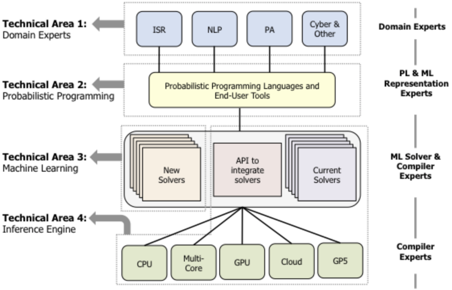
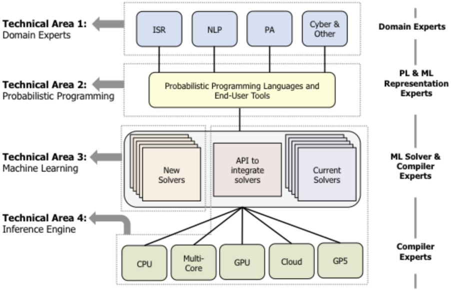
- "Democratization" of the software development.



PPLs as high-level programming languages for **machine learning systems**:

- Stacked architecture

Why PPLs?



PPLs as high-level programming languages for **machine learning systems**:

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- Different Domain Experts will code their models using the same language.

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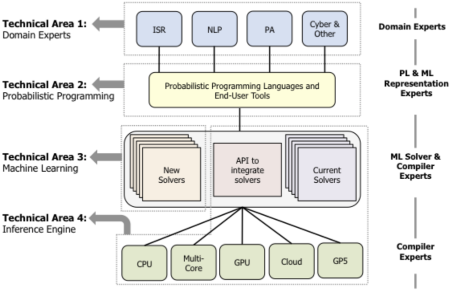

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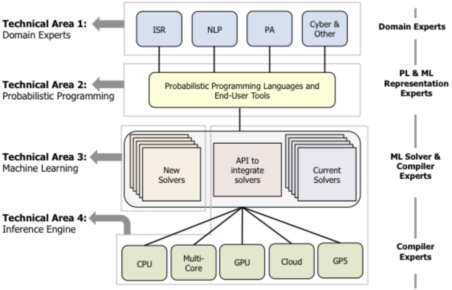

PPLs as high-level programming languages for **machine learning systems**:

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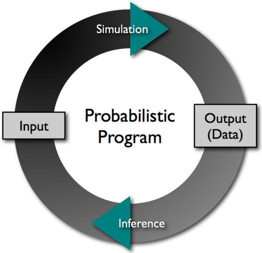
- Compile experts will focus on running these ML solvers on specialized hardware.

Why PPLs?



Benefits of PPLs:

- Simplify machine learning model code.

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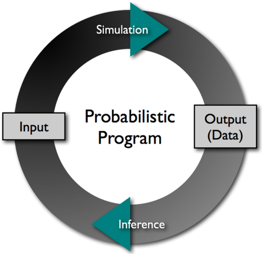

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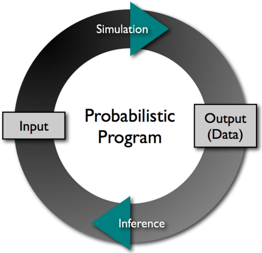

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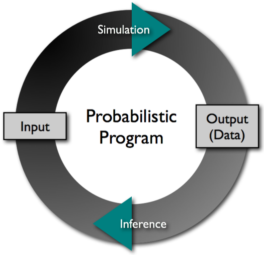

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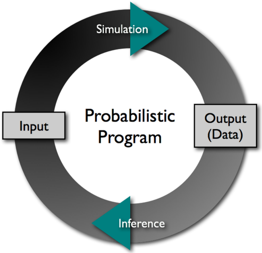

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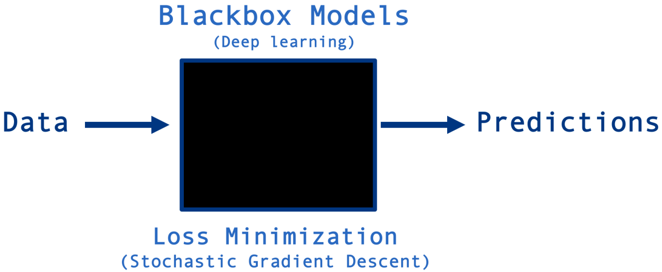
- "Democratization" of the development of ML systems.

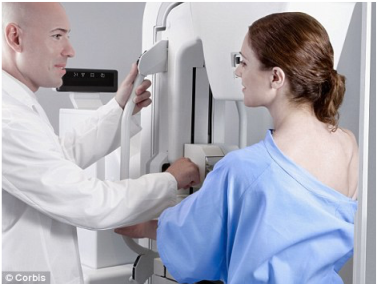
Reason 2: Try to make AI systems safer



Deep Learning Based AI Systems :

- **Issue 1**: Hard to **interpret**.
- **Issue 2**: No possible to know **how sure they are** in a particular prediction.
- Enormously **limit** the application of AI to many **real life problems**.

Today AI



AI in Health Care:

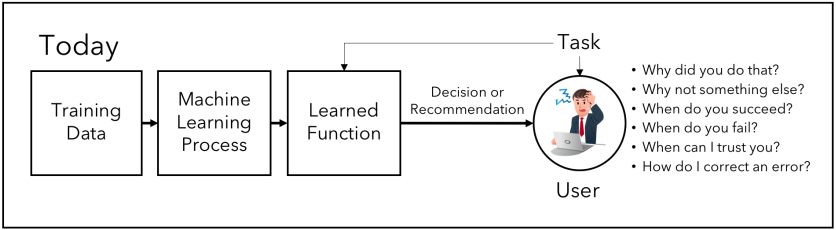
- Patients **need to know** why they are prescribed some treatment.
- An doctor can supervise the machine (**humans in the loop**).
- Extensible to any safe-critical system.

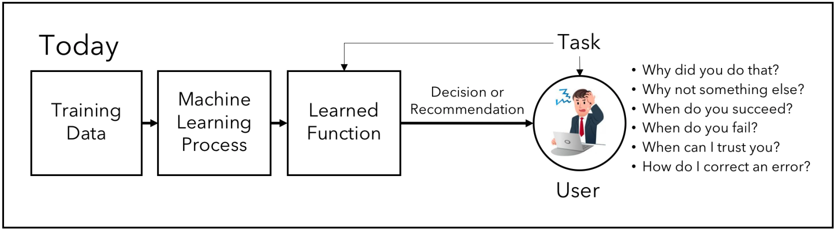
Today AI

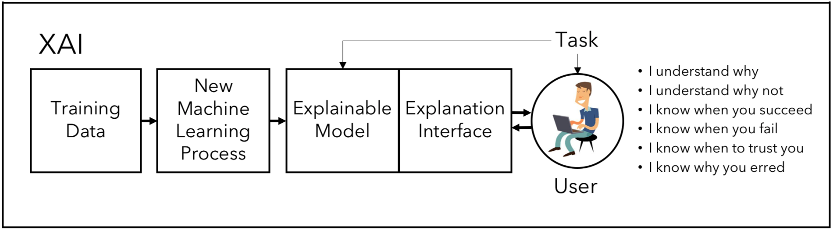
AI in Automated Systems:

- The system should detect when it is in a **completely new situation**.
- Let a **human take the control**.

XAI: Explainable AI




Today
Training Data
Machine Learning Process
Learned Function
Task
Decision or Recommendation
User
• Why did you do that?
• Why not something else?
• When do you succeed?
• When do you fail?
• When can I trust you?
• How do I correct an error?


XAI
Training Data
New Machine Learning Process
Explainable Model
Explanation Interface
Task
User
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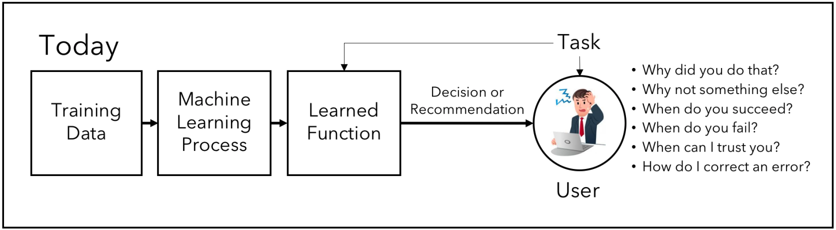
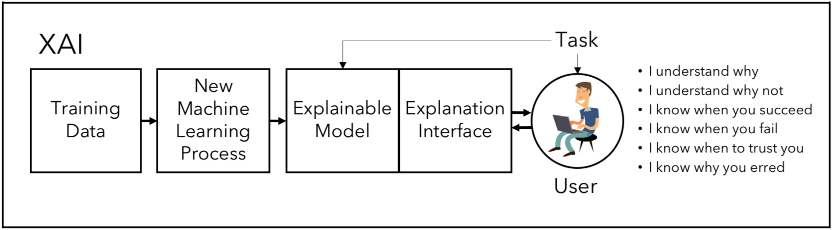
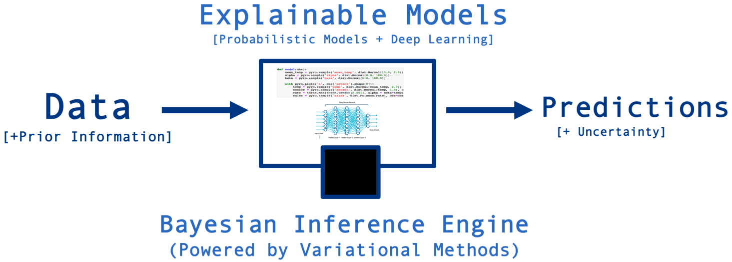




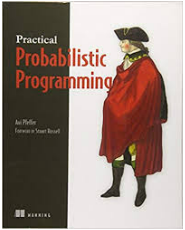
DARPA's Fund Call for XAI projects.



PPL based Systems :

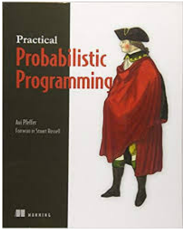
- Addressing Issue 1: PPLs provide **transparent** model description.
- Addressing Issue 2: PPLs provide **uncertainty estimation** both in models and predictions.

Brief Historical Review of PPLs



1st Generation of PPLs :

- Bugs, WinBugs, Jags, Figaro, etc.



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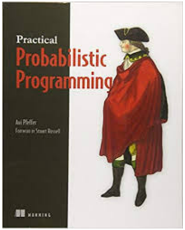

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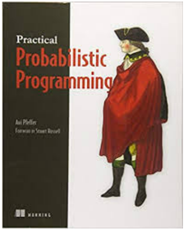

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PPLs

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- **Restricted** probabilistic model family (i.e. factor graphs, conjuage exponential family, etc.)

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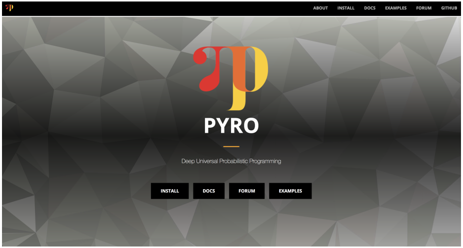
- Allow the inclusion of **deep neural networks**.
- Rely on **deep learning frameworks** (TensorFlow, Pytorch, Theano, etc).

Pyro

Pyro


ABOUT
INSTALL
DOCS
EXAMPLES
FORUM
GITHUB
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Deep Universal Probabilistic Programming
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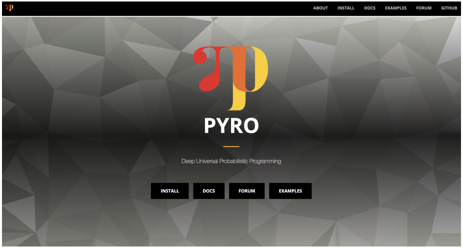

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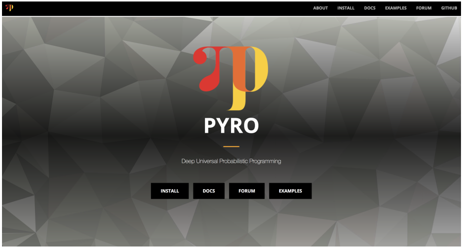
- Developed by **UBER** (the car riding company).



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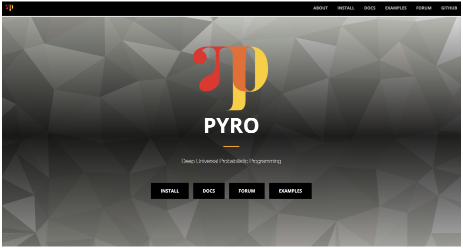

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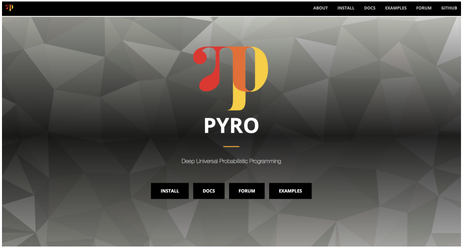

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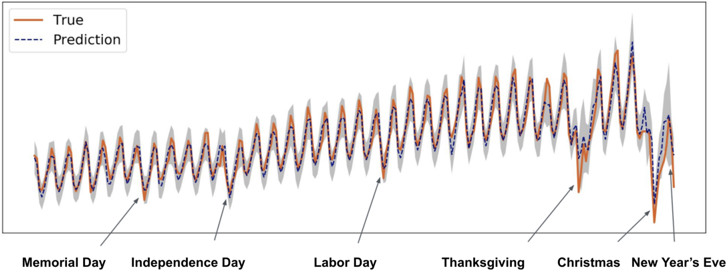
- Enable **GPU** accelaration and distributed learning.

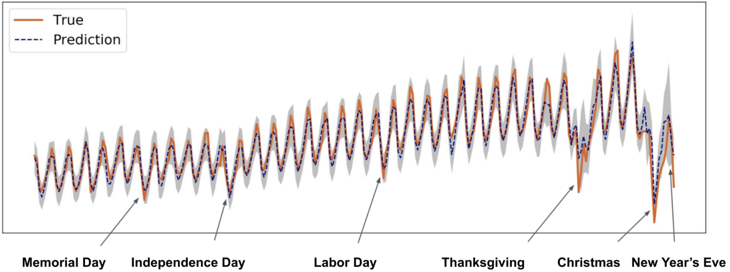
Pyro at UBER



Demand Prediction with Pyro:

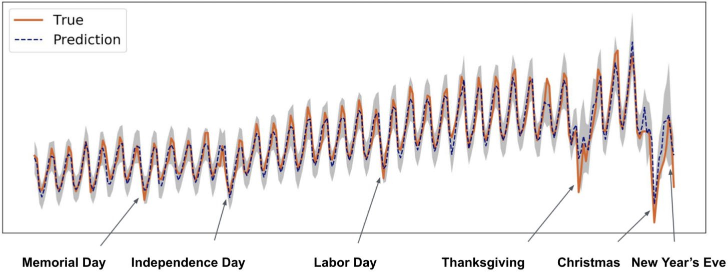
- Demand prediction is **critical** for user experience, resource allocation, etc.

Pyro at UBER



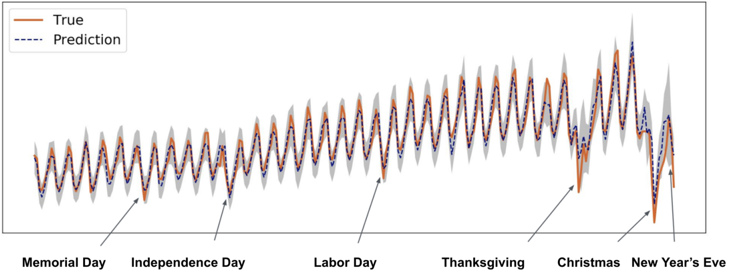
Demand Prediction with Pyro:

- Demand prediction is **critical** for user experience, resource allocation, etc.
- LSTM powerful for time series modelling.



Demand Prediction with Pyro:

- Demand prediction is **critical** for user experience, resource allocation, etc.
- LSTM powerful for time series modelling.
- Prediction at **special events** is challenging: weather, population growth, etc.



Demand Prediction with Pyro:

- Demand prediction is **critical** for user experience, resource allocation, etc.
- LSTM powerful for time series modelling.
- Prediction at **special events** is challenging: weather, population growth, etc.
- Bayesian LSTM provides **uncertainty estimation**.

Pyro's Distributions

```
In [3]: normal = dist.Normal(0,1)
        normal

Out[3]: Normal(loc: 0.0, scale: 1.0)
```

Pyro's distributions (http://docs.pyro.ai/en/stable/distributions.html) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.

```
In [15]: sample = normal.sample()
         sample

Out[15]: tensor(0.4908)
```

Pyro's distributions (http://docs.pyro.ai/en/stable/distributions.html) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).

```
In [17]: sample = normal.sample(sample_shape=[3,4,5])
         sample.shape

Out[17]: torch.Size([3, 4, 5])
```

Pyro's distributions (`http://docs.pyro.ai/en/stable/distributions.html`) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).

```
In [19]: torch.sum(normal.log_prob(sample))

Out[19]: tensor(-85.1003)
```

Pyro's distributions (`http://docs.pyro.ai/en/stable/distributions.html`) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).
- Operations, like log-likelihood, are defined over tensors (with GPU acceleration powered by Pytorch).

```
In [9]: normal = dist.Normal(torch.tensor([1.,2.,3.]),1.)
        normal

Out[9]: Normal(loc: torch.Size([3]), scale: torch.Size([3]))
```

Pyro's distributions (http://docs.pyro.ai/en/stable/distributions.html) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).
- Operations, like log-likelihood, are defined over tensors (with GPU acceleration powered by Pytorch).
- Multiple distributions can be embedded in single object (to define efficient vectorized operations).

```
In [10]: normal.sample()

Out[10]: tensor([2.0592, 2.4035, 3.1918])
```

Pyro's distributions (`http://docs.pyro.ai/en/stable/distributions.html`) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).
- Operations, like log-likelihood, are defined over tensors (with GPU acceleration powered by Pytorch).
- Multiple distributions can be embedded in single object (to define efficient vectorized operations).

```
In [11]: normal.log_prob(normal.sample())
Out[11]: tensor([-0.9402, -1.2113, -2.3214])
```

Pyro's distributions (http://docs.pyro.ai/en/stable/distributions.html) :

- Wide range of distributions: Normal, Beta, Cauchy, Dirichlet, Gumbel, Poisson, Pareto, etc.
- Samples from the distributions are Pytorch's Tensor objects (i.e. multidimensional arrays).
- Operations, like log-likelihood, are defined over tensors (with GPU acceleration powered by Pytorch).
- Multiple distributions can be embedded in single object (to define efficient vectorized operations).

Now it is your turn!

Open the Notebook and Play around

- Test that you have installed the basic packages.
- Test that you can run the first lines of code.
- Play a bit with the code in Section 1 of the notebook.

`Day1/students_PPLs_Intro.ipynb`

`https://github.com/PGM-Lab/ASML-Tbilisi`

Pyro's Models

In [12]:
```python
def model():
    temp = pyro.sample('temp', dist.Normal(15.0, 2.0))
    return temp

print(model())
print(model())
```

```
tensor(12.3926)
tensor(22.5272)
```

Pyro's models (http://pyro.ai/examples/intro_part_i.html) :

- A probabilistic model is defined as a **stochastic function**.

```
In [12]: def model():
             temp = pyro.sample('temp', dist.Normal(15.0, 2.0))
             return temp

         print(model())
         print(model())

tensor(12.3926)
tensor(22.5272)
```

Pyro's models (http://pyro.ai/examples/intro_part_i.html) :

- A probabilistic model is defined as a **stochastic function**.
- Each random variable is associated to a **primitive stochastic function** using the construct **pyro.sample(...)**.

```
In [21]: def model():
             temp = pyro.sample('temp', dist.Normal(15.0, 2.0))
             sensor = pyro.sample('sensor', dist.Normal(temp, 1.0))
             return (temp, sensor)

         out1 = model()
         out1

Out[21]: (tensor(15.8576), tensor(16.9907))
```

Pyro's models (http://pyro.ai/examples/intro_part_i.html) :

- A **stochastic function** can be defined as a composition of **primitive stochastic functions**.

```
In [21]: def model():
             temp = pyro.sample('temp', dist.Normal(15.0, 2.0))
             sensor = pyro.sample('sensor', dist.Normal(temp, 1.0))
             return (temp, sensor)

         out1 = model()
         out1

Out[21]: (tensor(15.8576), tensor(16.9907))
```

Pyro's models (http://pyro.ai/examples/intro_part_i.html) :

- A **stochastic function** can be defined as a composition of **primitive stochastic functions**.
- We define the **joint probability distribution**:

$$p(\mathtt{sensor}, \mathtt{temp}) = p(\mathtt{sensor}|\mathtt{temp})p(\mathtt{temp})$$

Pyro's Inference

In [22]:
```python
#The observatons
obs = {'sensor': torch.tensor(18.0)}

def model(obs):
    temp = pyro.sample('temp', dist.Normal(15.0, 2.0))
    sensor = pyro.sample('sensor', dist.Normal(temp, 1.0),obs=obs['sensor'])
```

Pyro's inference (http://pyro.ai/examples/intro_part_ii.html) :

- We can introduce observations (e.g. sensor = 18.0).

```
In [27]: #Run inference
svi(model,guide,obs, plot=True)

#Print results
print("P(Temperature|Sensor=18.0) = ")
print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
print("")
```

Pyro's inference (http://pyro.ai/examples/intro_part_ii.html) :

- We can introduce observations (e.g. sensor = 18.0).
- We can query the **posterior probability distribution**:

$$p(\texttt{temp}|\texttt{sensor} = 18) = \frac{p(\texttt{sensor} = 18|\texttt{temp})p(\texttt{temp})}{\int p(\texttt{sensor} = 18|\texttt{temp})p(\texttt{temp})d\texttt{temp}}$$

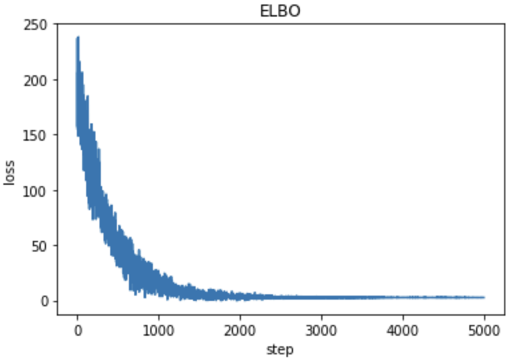
```
In [27]: #Run inference
svi(model,guide,obs, plot=True)

#Print results
print("P(Temperature|Sensor=18.0) = ")
print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
print("")
```

Pyro's inference (http://pyro.ai/examples/intro_part_ii.html) :

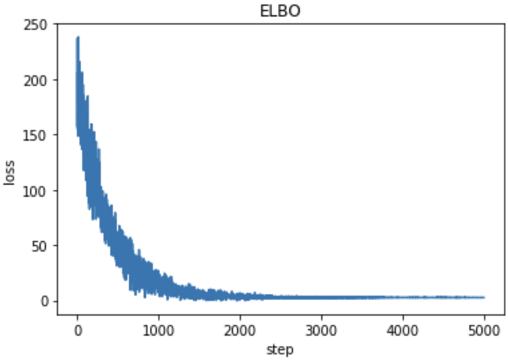
- We can introduce observations (e.g. sensor = 18.0).
- We can query the **posterior probability distribution**:

$$p(\texttt{temp}|\texttt{sensor} = 18) = \frac{p(\texttt{sensor} = 18|\texttt{temp})p(\texttt{temp})}{\int p(\texttt{sensor} = 18|\texttt{temp})p(\texttt{temp})d\texttt{temp}}$$

- Guide is an auxiliary method needed for inference (more details in the coming sessions).




```
P(Temperature|Sensor=18.0) =
Normal(loc: 17.39859390258789, scale: 0.9089401960372925)
```




```
P(Temperature|Sensor=18.0) =
Normal(loc: 17.39859390258789, scale: 0.9089401960372925)
```

Details on the inference method will be given on the following sessions.

Exercise 1: Change in the precision of the temperature sensor

- The precision of our temperature sensor is reflected in the **variance/scale** of the Normal distribution of the **sensor** variable.
- What happens if we get a **more precise temperature sensor**? Assume it has a variance/scale equal to 0.5.

`Day1/students_PPLs_Intro.ipynb`

`https://github.com/PGM-Lab/ASML-Tbilisi`

In [ ]:
```python
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    for i in range(0,obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(15.0, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (http://pyro.ai/examples/intro_part_ii.html) :

- What if we have a **bunch of observations**, $\mathrm{s} = \{s_1, \ldots, s_n\}$

```
In [ ]: #The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    for i in range(0,obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(15.0, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- What if we have a **bunch of observations**, $\mathrm{s} = \{s_1, \ldots, s_n\}$
- A random variable is created for each observation (using a **for-loop**).

```
In [ ]: #The observatons
        obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

        def model(obs):
            for i in range(0,obs['sensor'].shape[0]):
                temp = pyro.sample(f'temp_{i}', dist.Normal(15.0, 2.0))
                sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (http://pyro.ai/examples/intro_part_ii.html) :

- What if **we do not know the average temperature**?

```
In [28]: #The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    mean_temp = pyro.param('mean_temp', torch.tensor(15.0))
    for i in range(0,obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- What if **we do not know the average temperature**?
- We can introduce a **parameter** using **pyro.param** construct.

In [32]:
```
#Run inference
svi(model, guide, obs, num_steps=1000)

#Print results
print("Estimated Mean Temperature")
print(pyro.param("mean_temp").item())
```

```
Estimated Mean Temperature
19.129146575927734
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- And **learn the parameter** with the same general inference algorithm.

$$\mu_t = \arg\max_{\mu} \ln p(s_1, \ldots, s_n | \mu)$$

- Details about the inference algorithm will be given in the next sessions.

```
In [28]: #The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    mean_temp = pyro.param('mean_temp', torch.tensor(15.0))
    for i in range(0,obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- What if **we want to capture uncertainty about the estimation of the average temperature**?

In [176]:
```python
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))
    for i in range(obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- What if **we want to capture uncertainty about the estimation of the average temperature**?
- We can model this parameter with a **random variable**.

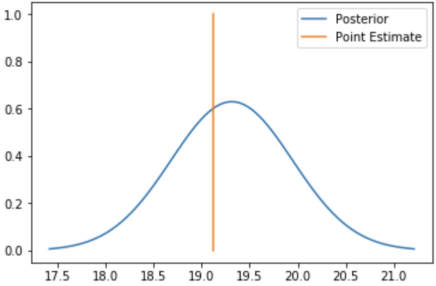
```
In [162]: import time

          #Run inference
          start = time.time()
          svi(model, guide, obs, num_steps=1000)

          #Print results
          print("P(mean_temp|Sensor=[18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]) =")
          print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
          print("")
          end = time.time()
          print(f"{(end - start)} seconds")

          P(mean_temp|Sensor=[18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]) =
          Normal(loc: 19.199871063232422, scale: 0.6046891212463379)

          10.298431873321533 seconds
```

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- And **learn the distribution** with the same general inference algorithm.

$$p(\mu_t | s_1, \ldots, s_{10})$$

- Details about the inference algorithm will be given in the next sessions.



f

Pyro's inference (`http://pyro.ai/examples/intro_part_ii.html`) :

- And **learn the distribution** with the same general inference algorithm.

$$p(\mu_t | s_1, \ldots, s_{10})$$

- Details about the inference algorithm will be given in the next sessions.

Defining Conditional Independences in Pyro

In [176]:
```
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))
    for i in range(obs['sensor'].shape[0]):
        temp = pyro.sample(f'temp_{i}', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample(f'sensor_{i}', dist.Normal(temp, 1.0), obs=obs['sensor'][i])
```

Pyro's cond. independeces (http://pyro.ai/examples/svi_part_ii.html) :

- Sensor variables are **independent** given temperature mean, μ_t.

$$p(s_1, t_1, ..., s_{10}, t_{10} | \mu_t) = \prod_{i=1}^{10} p(s_i, t_i | \mu_t)$$

In [37]:
```
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1])}

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))
    with pyro.plate('a', obs['sensor'].shape[0]):
        temp = pyro.sample('temp', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample('sensor', dist.Normal(temp, 1.0), obs=obs['sensor'])
```

Pyro's cond. independeces (http://pyro.ai/examples/svi_part_ii.html) :

- Sensor variables are **independent** given temperature mean, μ_t.

$$p(s_1, t_1, ..., s_{10}, t_{10}|\mu_t) = \prod_{i=1}^{10} p(s_i, t_i|\mu_t)$$

- We can use **Pyro's plate construct** to introduce this independence.

```
In [165]: #Run inference
start = time.time()
svi(model, guide, obs, num_steps=1000)

#Print results
print("P(mean_temp|Sensor=[18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]) =")
print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
print("")
end = time.time()
print(f"{(end - start)} seconds")

P(mean_temp|Sensor=[18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]) =
Normal(loc: 19.300748825073242, scale: 0.6379732489585876)

2.81210994720459 seconds
```

Pyro's cond. independeces (`http://pyro.ai/examples/svi_part_ii.html`) :

- We get large gains in efficiency due to **vectorized operations**.
- Execution time without *plate* is over 10s.

Now it is your turn!

- Exercise 2: The role of the **number of observations** in learning.
- Exercise 3: The role of the **prior distribution** in learning.

`Day1/students_PPLs_Intro.ipynb`

Toy Example: Ice-cream shop

Defining Machine Learning models with PPLs:

- We have an ice-cream shop and we record the ice-cream sales and the average temperature of the day.
- We know temperature affects the sales of ice-creams.
- We want to precisely find out how temperature affects ice-cream sales.

In [49]:
```
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]),
       'sales': torch.tensor([46., 47., 49., 44., 50., 54., 51., 52., 49., 53.])}

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))

    with pyro.plate('a', obs['sensor'].shape[0]):
        temp = pyro.sample('temp', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample('sensor', dist.Normal(temp, 1.0), obs=obs['sensor'])
```

Ice-cream Shop Model:

- We have observations from temperature and sales.

Pyro

In [43]:
```
#The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19.5, 20.1]),
       'sales': torch.tensor([46., 47., 49., 44., 50., 54., 51., 52., 49., 53.])}

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))

    with pyro.plate('a', obs['sensor'].shape[0]):
        temp = pyro.sample('temp', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample('sensor', dist.Normal(temp, 1.0), obs=obs['sensor'])
        sales = pyro.sample('sales', dist.Poisson(??????), obs=obs['sales'])
```

Ice-cream Shop Model:

- We have observations from temperature and sales.
- Sales are modeled with a **Poisson distribution**.

```
In [201]: #The observatons
obs = {'sensor': torch.tensor([18., 18.7, 19.2, 17.8, 20.3, 22.4, 20.3, 21.2, 19
       'sales': torch.tensor([46., 47., 49., 44., 50., 54., 51., 52., 49., 53.])

def model(obs):
    mean_temp = pyro.sample('mean_temp', dist.Normal(15.0, 2.0))
    alpha = pyro.sample('alpha', dist.Normal(0.0, 100.0))
    beta = pyro.sample('beta', dist.Normal(0.0, 100.0))

    with pyro.plate('a', obs['sensor'].shape[0]):
        temp = pyro.sample('temp', dist.Normal(mean_temp, 2.0))
        sensor = pyro.sample('sensor', dist.Normal(temp, 1.0), obs=obs['sensor']
        rate = torch.max(torch.tensor(0.001), alpha + beta*temp)
        sales = pyro.sample('sales', dist.Poisson(rate), obs=obs['sales'])
```

Ice-cream Shop Model:

- We have observations from temperature and sales.
- Sales are modeled with a **Poisson distribution**.
- The rate of the Poisson **linearly depends of the real temperature**.

In [48]:
```
#Run inference
svi(model, guide, obs, num_steps=1000)

#Print results
print("Posterior Temperature Mean")
print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
print("")
print("Posterior Alpha")
print(dist.Normal(pyro.param("alpha_mean").item(), pyro.param("alpha_scale").item()))
print("")
print("Posterior Beta")
print(dist.Normal(pyro.param("beta_mean").item(), pyro.param("beta_scale").item()))
```

```
Posterior Temperature Mean
Normal(loc: 19.311052322387695, scale: 0.6258021593093872)

Posterior Alpha
Normal(loc: 19.773971557617188, scale: 1.8541947603225708)

Posterior Beta
Normal(loc: 1.5178951025009155, scale: 0.1155082955956459)
```

Ice-cream Shop Model:

- We run the **(variational) inference engine** and get the results.

In [48]:
```
#Run inference
svi(model, guide, obs, num_steps=1000)

#Print results
print("Posterior Temperature Mean")
print(dist.Normal(pyro.param("mean").item(), pyro.param("scale").item()))
print("")
print("Posterior Alpha")
print(dist.Normal(pyro.param("alpha_mean").item(), pyro.param("alpha_scale").item()))
print("")
print("Posterior Beta")
print(dist.Normal(pyro.param("beta_mean").item(), pyro.param("beta_scale").item()))
```

```
Posterior Temperature Mean
Normal(loc: 19.311052322387695, scale: 0.6258021593093872)

Posterior Alpha
Normal(loc: 19.773971557617188, scale: 1.8541947603225708)

Posterior Beta
Normal(loc: 1.5178951025009155, scale: 0.1155082955956459)
```

Ice-cream Shop Model:

- We run the **(variational) inference engine** and get the results.
- With PPLs, we only care about modeling, **not about the low-level details** of the machine-learning solver.

Exercise 4: Introduce Humidity in the Icecream shop model

- Assume we also have a bunch of humidity senor measurements.
- Assume the sales are also linearly influenced by the humidity.
- **Extend the above model in order to integrate all of that**.

`Day1/students_PPLs_Intro.ipynb`