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Nature of the Dipole Repeller

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Keywords : dipole repeller, primeval antimatter, dark matter, dark energy, runaway paradox, Sakharov model, bimetric model, negative mass, Janus cosmological model

Abstract : It's shown through the Janus cosmological model framework that the object responsible for the phenomenon called "dipole repeller" is a spheroidal cluster made exclusively with anti-hydrogen and anti-helium of dark matter of negative mass. Emitting photons of negative energy, this object is not observable with optical instruments, but it plays a crucial role in the structure of the world of negative dark matters which contains neither galaxies, nor stars, nor heavy elements, nor planets, nor biomolecules. This model thus proposes an important extension of the theory of dark matter by including the existence of negative masses in the universe, and thus opens up new perspectives for understanding the structure and evolution of the universe as a whole.

1 – Introduction

In 2017 Yehudi Hoffman, B. Tully, H.Courtois and D.Pomarède published the first very large scale map of the universe (Hoffman et al. [2017](#)), based not only on the position of galaxies, but also including the velocity field, subtracting the Hubble field from what emanated from the raw measurements of their redshift. The result is extremely impressive and will be, we believe, one of the major discoveries of observational cosmology today, comparable to that made by Edwin Hubble a century earlier. It was already known that the motions of a certain set of galaxies reflected a convergence toward a formation that had been given the name of Great Attractor. The analysis published in 2017 highlighted the action of another, larger formation, located beyond this one, to which the name Shapley Attractor was given. But the most singular result was to highlight a region, roughly diametrically opposed to these two formations, where no galaxies were found, only a large void around which the neighboring galaxies presented a flight motion, centered on this object. This formation was first given the name of dipole repeller. Then, deciding to couple it to the attractive formations, it was included in the name of Dipole Attractor. There is no doubt that the understanding of such a phenomenon, which cannot be qualified as an artifact, will require an important progress in our understanding of cosmic dynamics.

2 – Some attempts at interpretation.

Four years after this discovery, few attempts have been made to model this phenomenon. The article published by Neiser in [2020](#) does not focus on this question of the dipole repeller which follows from the hypotheses made. It is known that observations have highlighted a phenomenon of acceleration of the cosmic expansion (Perlmutter et al. [1999](#), Riess et al. [2004](#), Schmidt B.P. et al. [1998](#)). This implies the action of a component associated with a

negative pressure. One of the models considered is to suppose the existence, within the cosmos, of elements of negative mass, which would then exert a repulsive action on the components of positive mass. This hypothesis is at the center of the works corresponding to the references (Petit et al. [2014b](#), Petit et al. [2014a](#), Petit et al. [2019](#), Petit et al. [2018](#), Petit et al. [2021b](#), Petit et al. [2021a](#)). Neiser in [2020](#) makes numerous hypotheses in different fields, on the nature of the Big Bang, the quantum vacuum, the origin of the universe "which would have been created by the gravitational collapse of an antineutrino star". To the point that he recalls that the conclusions of his article remain speculative. Among these hypotheses is that antimatter would have a negative gravitational mass, which would allow, according to him, "the progressive formation of neutrino stars and stars made up of antineutrinos which would repel each other". This repulsive aspect of a primordial antimatter is also mentioned by Benoit-Lévy et al. in [2012](#), but without further justification. All these people are waiting for the result of the analysis of the behavior of the antimatter created in the LHC, slowed down and subjected to the earth gravitational field (Nieto et al. [1994](#)), aspects discussed by Nieto et al. in [1991](#). This is a question that we will address later. Heald evokes in [2020](#) the situation of Laniakea, pushed by the dipole repeller and pulled by the Shapley Attractor. Here again, a repulsion between matter and antimatter is evoked, which would have given rise to the present large-scale structure of the universe, organized around large voids. But we do not find any real model of the object that would be located at the center of this great void, nor why we would not receive any light signal from it. Vuyk appeals in [2018](#) to a hypothetical 5th force. Hoffman et al. in [2018](#) through numerical simulations reconstruct the ad hoc distribution of dark matter consistent with the observational data (Hoffman et al. [2017](#)). From this exploration we retain two explanatory schemes. That of the existence of hypothetical objects made of repulsive antimatter, unobservable and that of a gap in the dark matter distribution.

3 – About the interpretation through a gap in the dark matter.

One is entitled to ask the question: can a gap in the dark matter produce the observed repulsion effect? Conceptually one is then tempted to start by considering a spherical gap in a uniform dark matter distribution. We then think that this system should obey the Poisson equation.

$$(1) \quad \frac{d^2\Psi}{dr^2} + \frac{2}{r} \frac{d\Psi}{dr} = 4\pi G \rho_{dm}$$

This equation is linear. We can therefore say that by superimposing two given density distributions ρ_1 and ρ_2 , the resulting gravitational potential is the sum of the potentials associated with these two distributions $\Psi = \Psi_1 + \Psi_2$.

Let us consider a uniform density distribution ρ_{dm}^{unif} , creating a potential Ψ_1 . We will create the gap by assuming that in a certain volume contained in a sphere we superimpose an equal and opposite density $-\rho_{dm}^{unif} < 0$ creating a potential Ψ_2 .

Let us start by calculating this one, solution of

$$(2) \quad \frac{d^2 \Psi_2}{dr^2} + \frac{2}{r} \frac{d\Psi_2}{dr} = -4\pi G \rho_{dm}^{unif}$$

This solution is :

$$(3) \quad \Psi_2 = -\frac{4\pi G \rho_{dm}^{unif}}{3} r^2 \quad \bar{g}_2 = \frac{8\pi G \rho_{dm}^{unif}}{3} \vec{r}$$

We thus obtain a repulsive gravitational field, proportional to the distance to the center of the sphere. It remains to calculate the gravitational field created by the uniform distribution, still considering the Poisson equation (1). Its solution is then :

$$(4) \quad \Psi_1 = \frac{4\pi G \rho_{dm}^{unif}}{3} r^2 \quad \bar{g}_1 = -\frac{8\pi G \rho_{dm}^{unif}}{3} \vec{r}$$

We obtain the same force field, of opposite sign. Consequently, by superimposing, inside the gap the gravitational field is null:

$$(5) \quad \bar{g} = \bar{g}_1 + \bar{g}_2 = 0$$

In any case the solution (4), of the Poisson equation, corresponding to a uniform distribution is a paradox. We have placed ourselves in a stationary or quasi-stationary situation, considering a uniform distribution of density ρ_{dm}^{unif} . Then, whatever the point chosen as the origin of the coordinates, we find a non-zero gravitational field \bar{g} , whose modulus increases proportionally to the distance to this point. We are faced with a paradox.

We are forced to go back to the construction of this Poisson equation. Unlike electromagnetism, this equation cannot be constructed by calculating the flow of the gravitational field through a closed surface, because of the non-nullity at infinity. In gravitation we will be forced to consider the Poisson equation as the linearized version of Einstein's equation in a very restrictive case: in a stationary (or quasi-stationary) situation and when we can describe the metric as a perturbation of a Lorentz metric :

$$(6) \quad g_{\mu\nu} = \eta_{\mu\nu} + \varepsilon \gamma_{\mu\nu}$$

The classical calculation gives :

$$(7) \quad \varepsilon \sum_{i=0}^3 \gamma_{00||i} = -\chi \varepsilon \rho_1$$

The gravitational potential is thus defined by:

$$(8) \quad \Psi = \frac{c^2}{2} \varepsilon \gamma_{00}$$

and equation (7) is then identified with the Poisson equation. But, in this approach, it should be noted that everything is based on the fact that we can consider a stationary metric solution, in the zero order, expressed in the form of a Lorentz metric, immediately associated to a portion of empty space. In the above, the perturbation of the metric is due to a density of finite extension. It is not possible to reconcile this approach on the basis of a non-empty, uniform and infinite density of order zero. The conclusion is that it is simply impossible to define a gravitational potential in a uniform matter distribution. One can consider that this remark is of little interest. It remains that this question had never been raised before. However, the problem of the existence of large voids in the large scale structure of the universe is not solved. Indeed, the gravitational instability produces clusters, not vacuums, so that we do not have a scheme for their formation.

4 – The dipole repeller in the Janus cosmological model.

This translates into an introduction of negative masses in the cosmological model that satisfies the equivalence principle. This is impossible in the framework of classical general relativity, based on a single field equation, that of Einstein, with or without its cosmological constant:

$$(9) \quad R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} + \Lambda g_{\mu\nu} = \chi T_{\mu\nu}$$

Indeed the control particles, positive or negative, follow the same geodesics, whatever the source of the field. As a consequence, we get the following interaction scheme:

- The positive masses attract the positive and negative masses.
- The negative masses repel the positive and negative masses.

This leads to the runaway paradox (see Fig. 1), according to which a couple constituted by masses of opposite signs accelerates uniformly while keeping a constant kinetic energy.

RUNAWAY PHENOMENON :

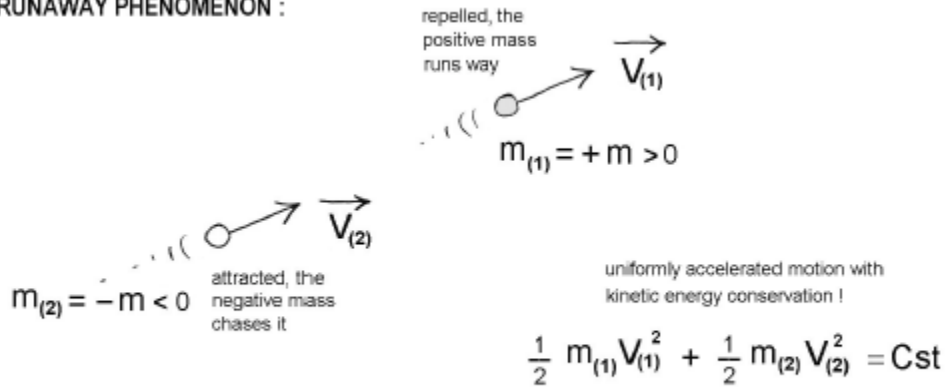


Fig.1 : The runaway paradox

When the authors Neiser in [2020](#), Benoit-Lévy et al. in [2012](#) and Nieto et al. in [1994](#) mention the existence of negative mass structures, these aspects are not considered, although they are fundamental. It is not possible to invoke the presence of negative masses without clarifying this point. In [2018](#), Farnes justifies what he calls himself a "toy model" by saying that this runaway effect could be the source of cosmic rays. To remain within the framework of a description using a single field equation, he is forced to invoke a hypothetical mechanism of continuous creation of negative mass so that this leads to the constancy of its value over time, which would then constitute an interpretation of the presence of the cosmological constant in the equation. But this scheme does not exclude the runaway phenomenon which he then considers as the source of the existence of particles of very high energy, so-called cosmic rays. Because of this undescribed mechanism of constant creation of negative mass the author only obscures the situation even more.

The Janus cosmological model (Petit et al. [2014b](#), Petit et al. [2014a](#), Petit et al. [2019](#), Petit et al. [2018](#), Petit et al. [2021b](#), Petit et al. [2021a](#)) takes up the idea of a bimetric description of the universe, initiated by Damour et al. in [2002](#), then taken up through articles published by Hossenfelder in [2006](#) and [2008](#). If the description done by Damour et al. in [2002](#) corresponds to branes whose points are connected by a hypothetical force field, the description done next by Hossenfelder in [2006](#) and [2008](#) is clearer and more constructed, geometrically. The system of the two field equations resulting from their construction, from a Lagrangian, satisfies the Bianchi conditions. The universe is then an M4 manifold equipped with two metrics, the first one producing the geodesics along which the positive masses and the positive energy photons move and the second one the geodesics along which the negative mass particles and the negative energy photons move. The disjoint character of these two systems of geodesics leads to the fact that each set of masses interacts with the other only through an antigravity force and that the negative masses are therefore invisible for an observer constituted of positive masses. The model was created by Hossenfelder in [2006](#) and [2008](#) at a time when the phenomenon of accelerating cosmic expansion was not recognized. In an effort to fit with what she considers as the standard model (of Friedmann) the author makes choices of signs which lead to a dissatisfaction of the principle of equivalence for the negative masses, which are then endowed with a negative gravitational mass, but a positive

inertial mass. In the Janus model, on the contrary, we try to satisfy the equivalence principle within the two populations, which leads to the system of equations:

$$(10a) \quad R_{\mu\nu}^{(+)} - \frac{1}{2}R^{(+)}g_{\mu\nu}^{(+)} = \chi \left[T_{\mu\nu}^{(+)} + \sqrt{\frac{g^{(-)}}{g^{(+)}}} \hat{T}_{\mu\nu}^{(-)} \right]$$

$$(10b) \quad R_{\mu\nu}^{(-)} - \frac{1}{2}R^{(-)}g_{\mu\nu}^{(-)} = -\chi \left[T_{\mu\nu}^{(-)} + \sqrt{\frac{g^{(+)}}{g^{(-)}}} \hat{T}_{\mu\nu}^{(+)} \right]$$

The tensors $\hat{T}_{\mu\nu}^{(+)}$ and $\hat{T}_{\mu\nu}^{(-)}$ correspond to “induced geometry”, i.e. the way each population contributes to the geometry of the other. Their form is determined by the Bianchi conditions. In their mixed form :

$$(11a) \quad \hat{T}_{\mu}^{(+)\nu} = \begin{pmatrix} \rho^{(+)}c^{(+)^2} & 0 & 0 & 0 \\ 0 & p^{(+)} & 0 & 0 \\ 0 & 0 & p^{(+)} & 0 \\ 0 & 0 & 0 & p^{(+)} \end{pmatrix}$$

$$(11b) \quad \hat{T}_{\mu}^{(-)\nu} = \begin{pmatrix} \rho^{(-)}c^{(-)^2} & 0 & 0 & 0 \\ 0 & p^{(-)} & 0 & 0 \\ 0 & 0 & p^{(-)} & 0 \\ 0 & 0 & 0 & p^{(-)} \end{pmatrix}$$

These geometrical conditions are only problematic in regions where the densities of matter, of positive or negative mass, are non-zero. By explaining these conditions, in the conditions of the Newtonian approximation they simply translate the fact that inside these masses, where the density is assumed to be constant, the pressure force balances the gravity force.

Outside these regions it is the vacuum. The second members are zero and the Bianchi conditions are satisfied identically because of the form of the first members,

themselves resulting from the presence of the terms $R^{(+)}\sqrt{-g^{(+)}}$ and $R^{(-)}\sqrt{-g^{(-)}}$ in action as first introduced by Damour et al. in [2002](#). The Janus model is based on a set of agreements with observation. What interests us here is a possible justification of the Great Repeller phenomenon. In its Newtonian approximation the system of the two coupled field equations leads to the following interaction laws:

- Masses of the same sign attract each other according to Newton's law
- Masses of opposite signs repel each other according to "anti-Newton"

The runaway phenomenon is thus eliminated and the principle of equivalence is satisfied in both populations. For negative masses, their gravitational masses and their inertial masses are negative and equal. It is then possible to do numerical simulations. If the absolute values of the two densities $\rho(+)>0$ and $\rho(-)<0$ the values of the thermal agitation velocities $\langle V(+)>=\langle V(-)>$ are equal we obtain a percolation, which does not fit with the observational data referring to the large scale structure of the universe (see Fig. 2).

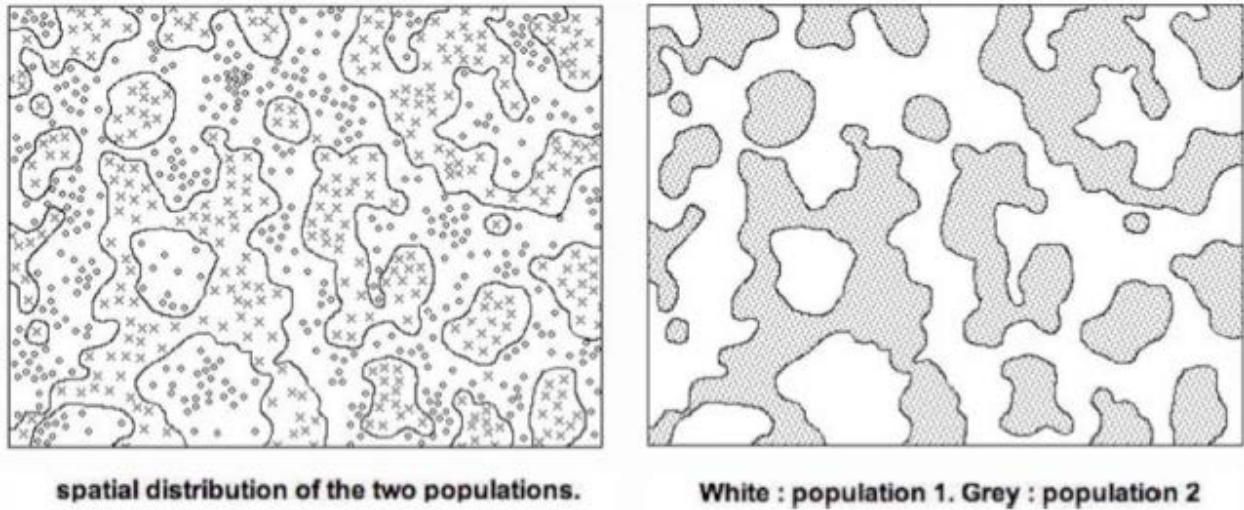


Fig.2 : Evolution of the system when $|\rho^{(-)}| = \rho^{(+)}$

On the other hand, if we assume that there is a strong dissymmetry between the two populations, so that $\rho(-) \gg \rho(+)$ then the pattern changes completely. As the Jeans times vary as the inverse of the square root of the density, the negative masses are shorter. These then form a regular network of spheroidal conglomerates. The positive mass is forced to occupy the remaining space, which gives it a lacunar structure (El-Ad et al. [1997](#)) comparable to a set of joined bubbles (see Fig. 3), a pattern repeated by Farnes in [2018](#).

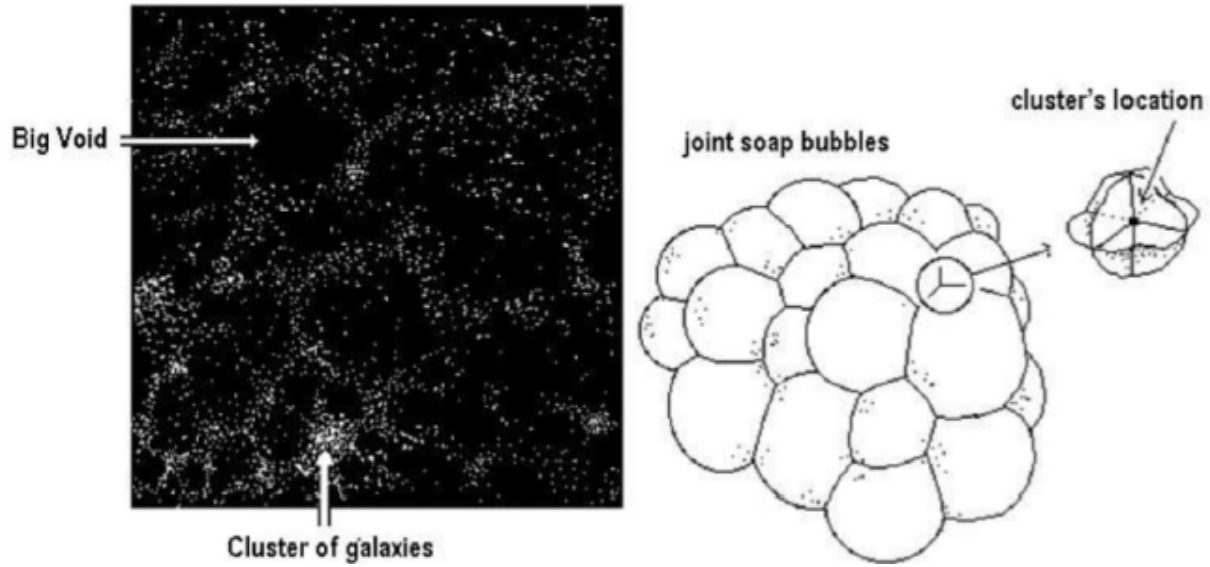


Fig.3 : Lacunar structure

On Fig. 4, there is the result of a 2D simulation (Petit [1995](#)) :

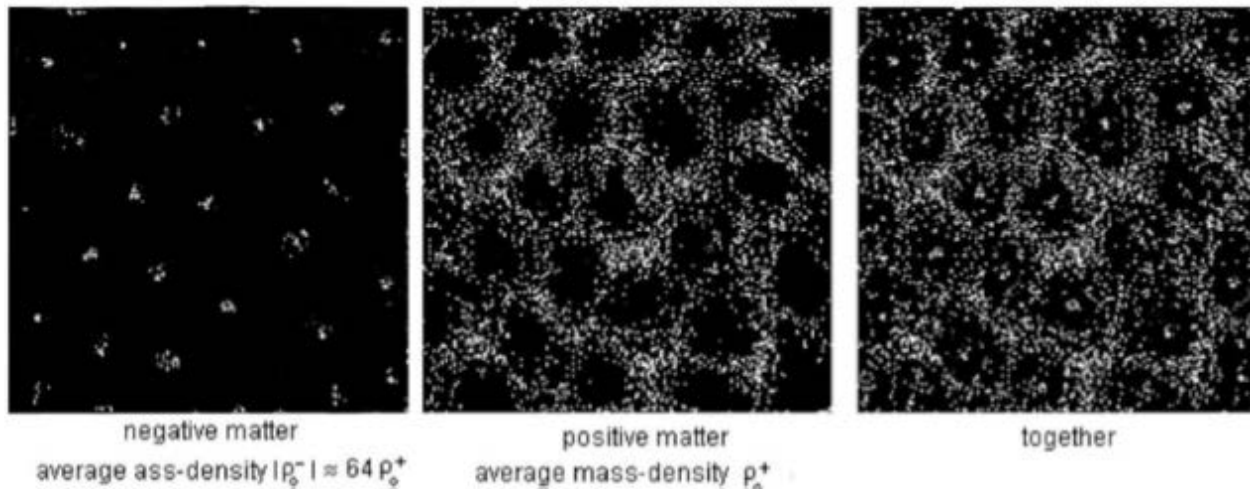


Fig.4 : Evolution when $|\rho^{(-)}| \gg \rho^{(+)}$

Teams with adequate means will easily extend this to 3D. As developed by Petit et al. through articles referenced in [2014b](#), [2021b](#), [2021a](#), the negative mass content accounts for both the dark matter and the dark energy, which is identified with the contribution $\rho^{(-)} c^{(-)2} < 0$ and is therefore responsible for the acceleration of the expansion of positive masses. According to this model, the phenomenon of the dipole repeller translates the presence, within this great void, of a spheroidal conglomerate of negative mass, invisible, because it emits photons of negative energy that our eyes and our measuring instruments cannot capture.

5 – Nature of the objects of negative masses

This question has been examined by Petit et al. in [2014b](#) and the detailed calculations can be found in the article published by Petit et al. in [2021a](#). In the theory of dynamical groups (Souriau [1970](#)) the link between geometric structure and content is translated by the nature of the associated dynamical group. The General Relativity goes with the restricted Poincaré group, where we keep only the set of terms which do not reverse time. It is thus the orthochron subgroup of the complete Poincaré group, built from the orthochron Lorentz subgroup. Here after its matrix representation:

$$(12) \quad \begin{pmatrix} L_o & C \\ 0 & 1 \end{pmatrix}$$

C being the space-time translation vector. It is a group of dimension 10. By making this group act on the dual of its Lie algebra, or momentum space, one makes appear the energy E, the momentum p and the spin s, as attributes of the particle, of purely geometric essence. In order to endow the particles with n quantum charges, it is necessary to endow the space with n additional dimensions $\{\zeta_0, \zeta_1, \dots, \zeta_n\}$ and move on to the group below, where we have included its action.

$$(13) \quad \begin{pmatrix} 1 & 0 & \phi \\ 0 & L_o & C \\ 0 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} \zeta \\ \xi \\ 1 \end{pmatrix} = \begin{pmatrix} \zeta + \phi \\ L_o \xi + C \\ 1 \end{pmatrix} \quad \text{with } \xi = \begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} \quad \zeta = \begin{pmatrix} \zeta^0 \\ \zeta^1 \\ \dots \\ \zeta^n \end{pmatrix} \quad \phi = \begin{pmatrix} \phi^0 \\ \phi^1 \\ \dots \\ \phi^n \end{pmatrix}$$

The action on the momentum space adds to the particle n quantum numbers q_i , including the electric charge, which are all constants. By introducing the group:

$$(14) \quad \begin{pmatrix} \mu & 0 & \phi \\ 0 & L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with } \mu = \pm 1$$

We translate geometrically the concept of antimatter, the inversion of quantum charges (C-symmetry) going hand in hand with the inversion of movements in additional dimensions.

If we want to introduce negative energies, it is enough to pass from the restricted Poincaré group L_o , orthochronic, to the full group λL_o with $\lambda = \pm 1$. Immediately we find that the time-inversion ($\lambda = -1$) goes with energy and mass-inversion. The corresponding dynamic group is according to the article published by Petit et al. in [2014b](#) :

$$(15) \quad \begin{pmatrix} \lambda\mu & 0 & \phi \\ 0 & \lambda L_o & C \\ 0 & 0 & 1 \end{pmatrix} \quad \text{with} \quad \begin{array}{l} \lambda = \pm 1 \\ \mu = \pm 1 \end{array}$$

In this model the C-symmetry exists both in the world of positive and negative masses. There are thus two antimatter.

- The first one, C-symmetric ($\lambda = 1; \mu = -1$) of the ordinary matter, of positive mass
- The second, PT-symmetric ($\lambda = -1; \mu = 1$) of ordinary matter, of negative mass.

It is the first one that we make appear in laboratory. The model therefore predicts that this one will behave like ordinary matter in the earth's gravity field.

6 – About the lack of observation of primordial antimatter.

If we suppose, as A. Sakharov ([1967](#), [1979](#), [1980](#)), that the couple :

- Matter of positive mass
- Antimatter of positive mass

is formed from quarks and antiquarks of positive energy, and that a couple :

- Matter of negative mass
- Antimatter of negative mass

was formed from quarks and antiquarks of negative energy, and that in addition the synthesis of the matter of positive mass, in the first couple, was faster, whereas it is that of the antimatter of negative mass, in the second couple which was it one leads on the idea that the objects, in the center of the great voids of the large-scale structure of the universe, whose presence is betrayed by the phenomenon of the dipole repeller, would be constituted of antimatter, that is to say of antiprotons, anti-neutrons and antielectrons of negative mass. These would then form spheroidal objects made of antihydrogen (light elements) of negative mass comparable to immense protostars with a cooling time superior to the age of the universe, which could not be the seat of fusion reaction, thus generating neither stars, nor galaxies, nor planets.

7 – Observations specifically related to these objects.

As presented by Petit in [1995](#), these negative mass objects decrease the luminosity of the background sources by negative lensing. But the luminosities of galaxies at $z > 7$ are indeed weak. The classical approach is to consider them as dwarfs. But these could be galaxies of normal size and mass, so the luminosity would be affected by this phenomenon. Finer measurements, in large voids, could reveal a threshold that would allow access to the diameter of these formations.

8 – Conclusion.

We have examined the very few attempts to model the dipole repeller phenomenon. Then we have moved on to the interpretation of the Janus cosmological model. We recall the origin and the essential aspects of this model, based on the hypothesis that the universe has a content of negative masses, such that the principle of equivalence is satisfied in both populations and that eliminates the runaway phenomenon and produces the same effects attributed so far to dark matter and dark energy. In this model the dominant negative mass forms a regular set of spheroidal clusters, which repel the positive mass by confining it in the residual space, giving it a lacunar structure. One of these can then produce the phenomenon of dipole repeller.

Data availability

The data underlying this article are included within the article and in its online supplementary material.

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